Weak Interactions in the Nucleus I

Summer School, Tennessee

June 2003

Using the nucleus to search for new physics

Summary

Historical Introduction

Dirac Equation. E&M and Weak Int.

Non-VA forces in weak decays
→ Measure e-ν correlation

Non-Unitarty of the CKM matrix
→Isospin Breaking
→Measure β asymmetry with UCN

Time-Reversal Invariance Violation
→Measure TRIV correlation

The Weak Interaction: A Drama in Many Acts

- 1890's: Roentgen discovers β rays

Thought Uranium salts were affected by the sun but rainy Paris soon helped showing otherwise.

- 1920's: Pauli proposes ν

To explain continuous β spectrum: only way to save conservation of energy.

- 1950's Parity Violation

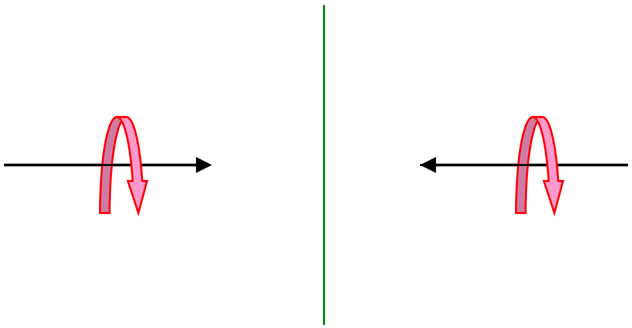
To explain identical properties of θ and τ particles. Then clearly proven in Madame Wu's experiment.

- 1960's CP-Violation

Parity Violation

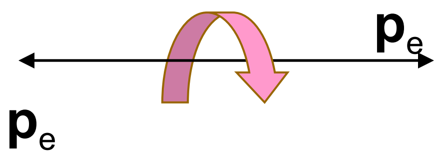
- $x \rightarrow -x$
- $p \rightarrow -p$
- $r \times p \rightarrow r \times p$
- $J \rightarrow J$

Parity Mirror



Madame Wu's experiment:
Polarize ^{60}Co and look at the direction of the emitted β's.

In a Parity-symmetric world we would see as many electrons emitted in the direction of **J** as opposite **J**.



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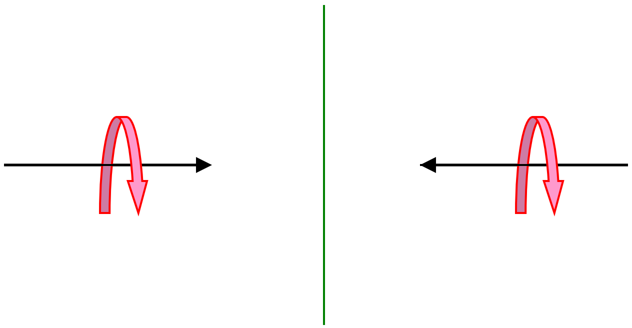

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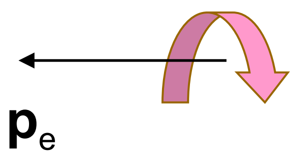
But in the real world we see only electrons emitted in the direction opposite **J**.



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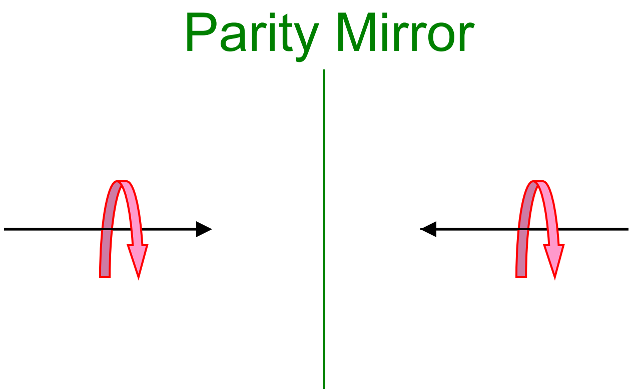

Other experiments:
Look at the helicity of neutrinos.

In a Parity-symmetric world we would see as many ν's with left-handed helicity (**p** opposite **S**) as right-handed helicity (**p** parallel to **S**).

Parity Violation

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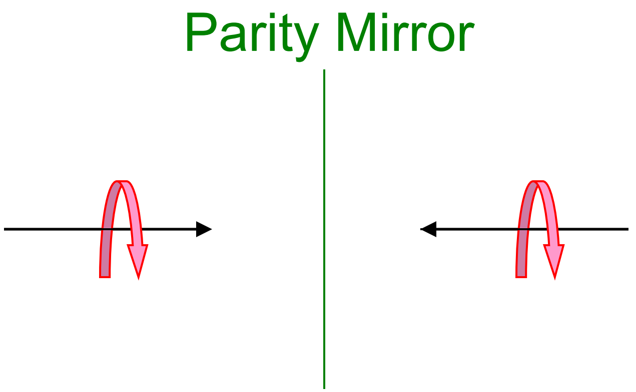

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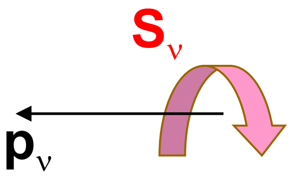

Schroedinger Equation

How do we get a wave equation that yields conservation of energy and the correct deBroglie relations between particles and associated waves?

$$\frac{p^2}{2m} + V = E \qquad\qquad p = \hbar k$$

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$$\frac{p^2}{2m} + V = E \qquad\qquad p = \hbar k$$

$$\left.\begin{array}{l} H \to i\hbar \dfrac{\partial}{\partial t} \\[2ex] p \to \dfrac{\hbar}{i} \nabla \end{array}\right\rangle \qquad \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \Psi = i\hbar \frac{\partial}{\partial t} \Psi$$

Schroedinger Equation: perturbation theory

$$H_0 \varphi_n = E_n \varphi_n \qquad (H_0 + V)\psi = i\hbar \frac{\partial}{\partial t}\psi$$

How do we get ψ?

$$\psi(x,t) = \sum_n a_n(t)\varphi_n(x)\, e^{-iE_n t/\hbar}$$

Replacing in Schrodinger's Equation and integrating:

$$a_n(t) = \frac{1}{i\hbar}\int_{-T}^{T} dt \int d^3x\, \underbrace{\varphi^*{}_n(x)\, e^{iE_n t/\hbar}}\; V\, \underbrace{\varphi_i(x)\, e^{-iE_i t/\hbar}}$$

Lorentz-invariant form:

$$T_{fi} = \frac{1}{i\hbar}\int d^4x \;\; \varphi^*{}_n(x)\, V\, \varphi_i(x)$$

Schroedinger Equation: decaying rate

If V(x) is time-independent: $$T_{fi} = V_{fi}\, 2\pi\delta(E_f - E_i)$$

The transition probability per unit time:

$$W_{fi} = \frac{|T_{fi}|^2}{T} = \frac{2\pi}{\hbar}|V_{fi}|^2\, \delta(E_f - E_i)$$

In a decay, like *n→p e ν* we have to sum over final states

FERMI's golden rule:

$$\boxed{W_{fi} = \frac{2\pi}{\hbar}|V_{fi}|^2 \frac{dN}{dE}}$$

Examples of phase-space calculations

Using the neutron mean-life ($\tau \approx 900$ s) estimate the anti-neutrino absorption cross section on protons $\underline{\nu}$+p → n+e^+:

$$W_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \frac{dN}{dE}$$

For neutron decay:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} |V_{fi}|^2 \int \frac{d^3 p_\nu}{(2\pi\hbar)^3} \frac{d^3 p_e}{(2\pi\hbar)^3} \delta(E_{\max} - E_e - E_\nu)$$

For anti-neutrino absorption (Kamland):

$$\sigma \propto \frac{2\pi}{\hbar} |V_{fi}|^2 \int \frac{d^3 p_e}{(2\pi\hbar)^3} \delta(E_e - E_\nu - M_p - M_n)$$

Dirac Equation

How do we get a wave equation that is relativistically correct?

$$p^2 + m^2 = E^2$$

Dirac showed that one can start with a linear equation

$$\frac{\hbar}{i}\left(\alpha_1 \frac{\partial}{\partial x_1} + \alpha_2 \frac{\partial}{\partial x_2} + \alpha_3 \frac{\partial}{\partial x_3}\right)\Psi + \beta m \Psi = i\hbar \frac{\partial}{\partial t}\Psi$$

from where one regains $\left(-\hbar^2 \nabla^2 + m^2\right)\Psi = -\hbar^2 \frac{\partial^2}{\partial t^2}\Psi$

for which the coefficients and the wave function can not be simply scalars

$$\boxed{\begin{aligned} \alpha_i \alpha_k + \alpha_k \alpha_i &= 2\delta_{ik} \\ \alpha_i \beta + \beta \alpha_i &= 0 \\ \beta^2 &= 1 \end{aligned}}$$

Dirac Equation

The matrices alphas have to be at minimum of dimension 4:

$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}$$

where σ_i are the Pauli matrices

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The wave function now has 4 components. For a free particle with ***p=0***:

$$\Psi_1 = e^{-imt/\hbar}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Psi_2 = e^{-imt/\hbar}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \Psi_3 = e^{+imt/\hbar}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \Psi_4 = e^{+imt/\hbar}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

We write it in terms of 2-comp spinors. For a free particle with ***p=0***:

$$\chi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \chi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$E > 0 \qquad u = \begin{pmatrix} \chi^s \\ 0 \end{pmatrix}$$

$$E < 0 \qquad \mathrm{v} = \begin{pmatrix} 0 \\ \chi^s \end{pmatrix}$$

For ***p ≠ 0***:

$$Hu = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & -m \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = E \begin{pmatrix} u_A \\ u_B \end{pmatrix} \qquad \Rightarrow \begin{cases} \sigma . p\, u_B = (E - m) u_A \\ \sigma . p\, u_A = (E + m) u_B \end{cases}$$

$$E > 0 \qquad u = \begin{pmatrix} \chi^s \\ \left(\dfrac{\sigma \cdot p}{E + m} \right) \chi^s \end{pmatrix} \qquad E < 0 \qquad u = \begin{pmatrix} \left(\dfrac{-\sigma \cdot p}{|E| + m} \right) \chi^s \\ \chi^s \end{pmatrix}$$

Dirac Eq. and E&M

Dirac Equation without E&M

$$\gamma^{\mu} p_{\mu} \Psi + \beta m \Psi = 0$$

where

$$\boxed{\begin{aligned} \gamma^{i} &= \beta \alpha_{i} \\ \gamma^{0} &= \beta \\ \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} &= 2g^{\mu\nu} \end{aligned}}$$

with E&M (for electron)

$$\gamma^{\mu}(p_{\mu} + \frac{e}{c} A_{\mu})\Psi + \beta m \Psi = 0$$

This is equivalent to the previous plus an interaction:

$$\gamma^{\mu} \frac{e}{c} A_{\mu} \Psi$$

Quantizing the fields

Schroedinger Equation
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

Take: $\Psi(x,t) = \sum_n b_n(t)\psi_n(x); \qquad H\psi_n(x) = E_n\psi_n(x)$

Then: $\dfrac{db_n}{dt} = \dfrac{-i}{\hbar} b_n E_n$

The Hamiltonian that yields the previous is:

$$H = \int d^3x\, \Psi^*(x,t)\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi(x,t)$$

Interpreting b_n as an operator:

$$\left[b_n, b_{n'}^{+}\right] = \delta_{nn'} \qquad \hat{H} = \sum E_n b_n^{+} b_n$$

Quantizing the fields

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Quantizing the fields

For bosons: $\left[b_n, b_{n'}^{+}\right] = \delta_{nn'}$

For fermions: $\left\{b_n, b_{n'}^{+}\right\} = \delta_{nn'}$

The various wave functions are generated by applying the creator operator on the vacuum state:

$$|C_1, t> = \int d^3x \, C_1(x) \Psi^{+}(x,t) |0>$$

Current-current interaction

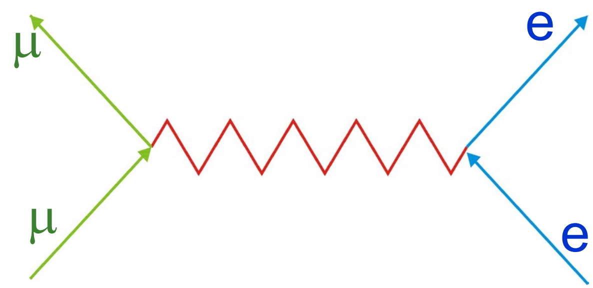
E&M interaction in Dirac's Equation

$$\hat{H}_{\text{int}} = -\int d^3x \, \boxed{\overline{\Psi}(x,t)} \left(\frac{e}{c} \gamma^\mu A_\mu \right) \boxed{\Psi(x,t)}$$

The vector potential should satisfy Maxwell's equations:

$$\square^2 A_\mu = j_\mu \rightarrow A_\mu(q_\mu) = -\frac{j_\mu}{q^2} \qquad j_\mu \Leftrightarrow \overline{\Psi}(x,t) \frac{e}{c} \gamma^\mu \Psi(x,t)$$

Example of Feynman diagram: e-μ scattering.



$$\mathrm{M} = -\overline{\Psi}_\mu \gamma_\mu \Psi_\mu(p) \frac{e^2}{q^2} \overline{\Psi}_e \gamma^\mu \Psi_e$$

E&M vs. WEAK

Same order of magnitude:
g 0.22 ≈ e

E&M $$-\overline{\Psi}_\mu \gamma_\mu \Psi_\mu(p) \frac{e^2}{q^2} \overline{\Psi}_e \gamma^\mu \Psi_e$$

Weak $$-\overline{\Psi}_\mu \gamma_\mu \boxed{(1-\gamma_5)} \Psi_\mu(p) \frac{g^2}{q^2 - \boxed{M_W^2}} \overline{\Psi}_e \gamma^\mu \boxed{(1-\gamma_5)} \Psi_e$$

W's are left handed

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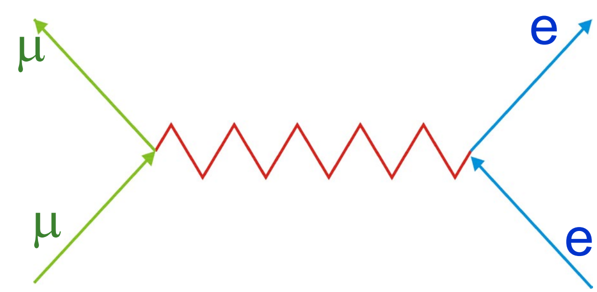


E&M vs. Weak

Order of magnitude of the Weak coupling at very low energies:

$$g^2 \frac{q^2}{M_W{}^2} \approx g^2 \left(\frac{5}{80{,}000} \right)^2$$

Order of magnitude of the E&M coupling :

$$e^2$$

Ratio Weak/E&M:

$$\approx 10^{-7}$$

E&M vs. Weak: helicity

Helicity is defined as: $$H = \frac{\vec{\sigma}.\vec{p}}{|p|}$$

The helicity of leptons produced in Weak decays: $$H\,(1\text{-}\gamma_5)\psi \approx -\frac{p}{E}(1\text{-}\gamma_5)\psi$$

Konopinski's argument:

instant velocity against momentum can only be $\pm c$ $$(\Delta t \to 0 \quad \Rightarrow \Delta E \to \infty)$$

$$|a|^2(+c) + (1-|a|^2)(-c) = \mathrm{v}$$

Then helicity: $$|a|^2(-1) + (1-|a|^2)(+1) = -\,\mathrm{v}/c$$

Allowed approximation

$$\gamma_\mu \psi \approx (1, \frac{\vec{p}}{m})\psi \Rightarrow H \approx \sum_i \tau_i = T$$

(0⁺,1⁻)

$$\gamma_\mu \gamma_5 \psi \approx (\frac{\vec{p}\cdot\vec{\sigma}}{m}, \vec{\sigma})\psi \Rightarrow H \approx \sum_i \tau_i \vec{\sigma}_i$$

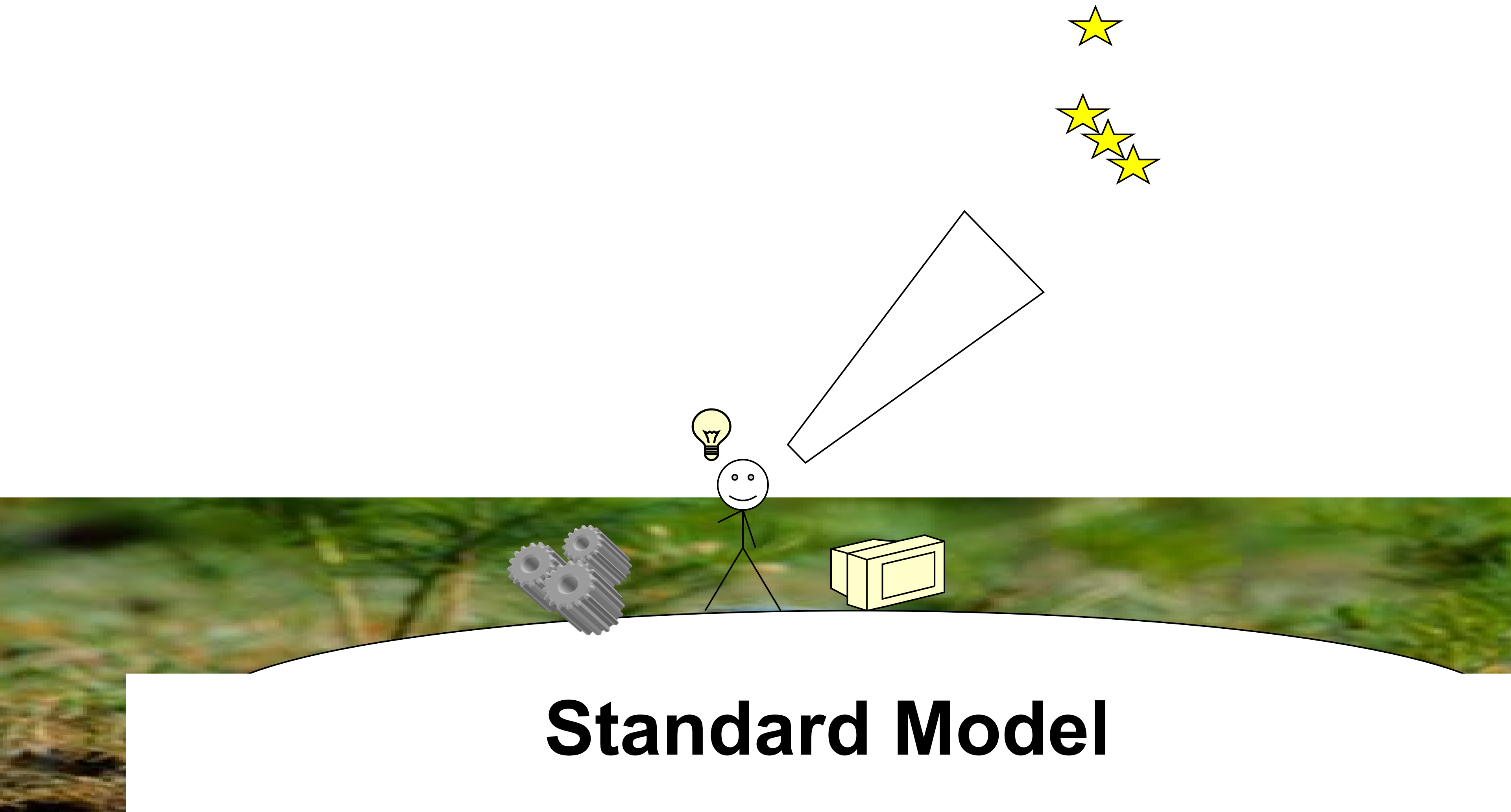
(0⁻,1⁺)

As a result, two types of allowed transitions:

Fermi: $J_f = J_i; \quad I_f = I_i; \quad \pi_f = \pi_i;$

Gamow-Teller: $J_f = J_i \otimes 1; \quad I_f = I_i \otimes 1; \quad \pi_f = \pi_i;$

Looking for Physics Beyond the Standard Model



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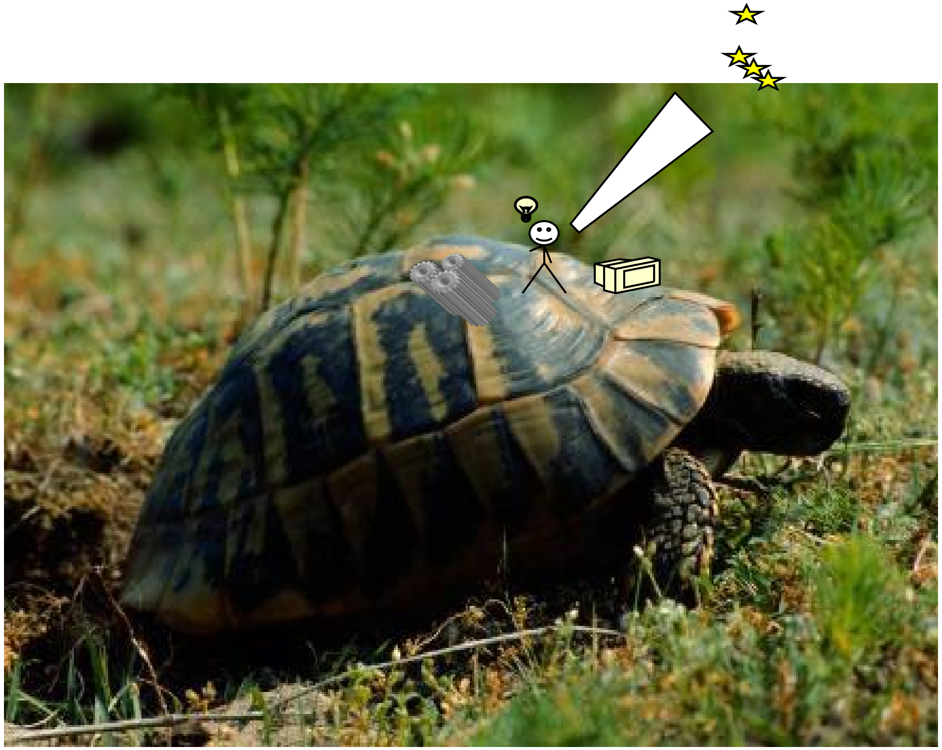
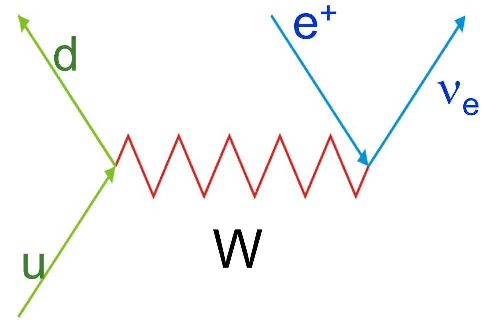
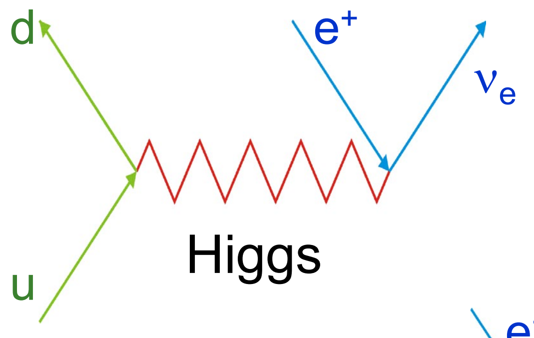


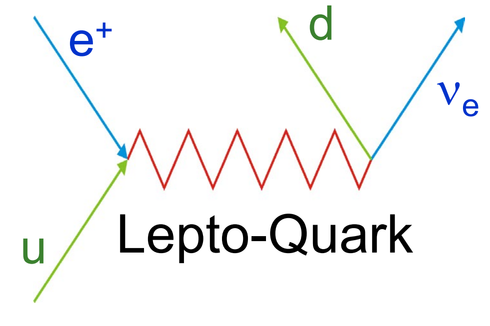
Non-VA currents in Weak decays

Are weak decays carried only by W's?





Or is there something new?



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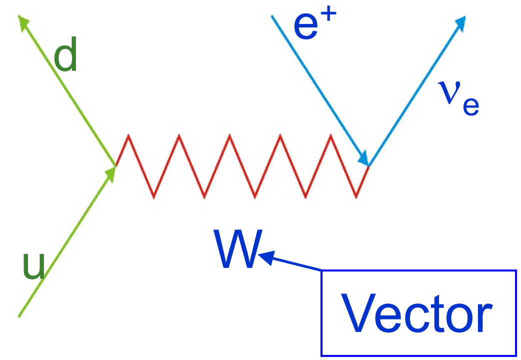

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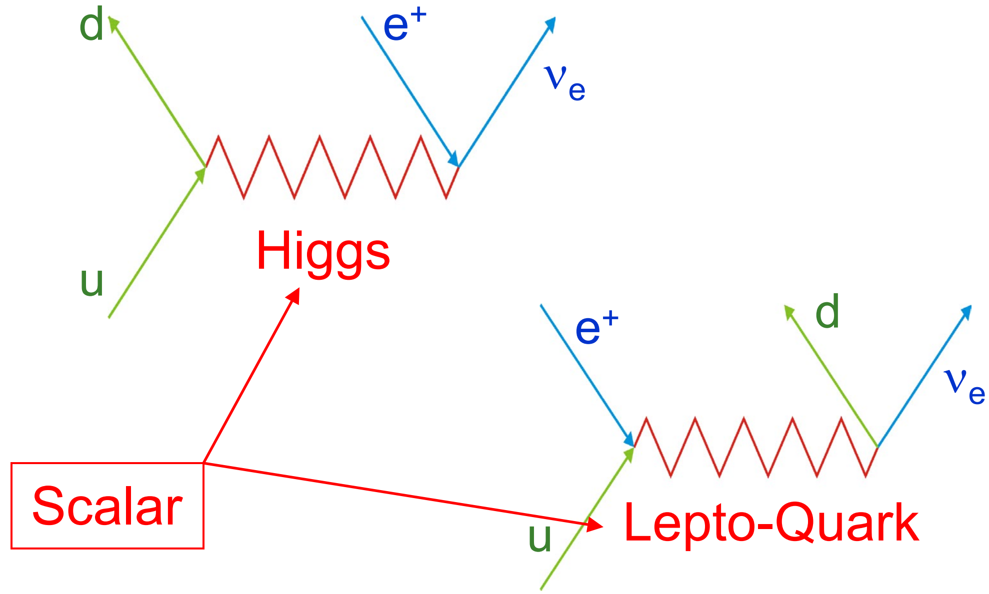
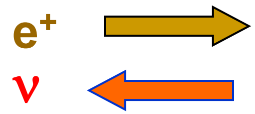


Detecting Scalar currents in weak decays

The e-ν correlation depends strongly on the nature of the carrier (we take a $0^+ \rightarrow 0^+$ transition).

spins have to couple to zero

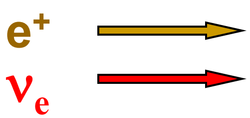


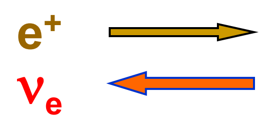
spins

Standard Model
Vector Currents

New Physics?
Scalar Currents

momenta

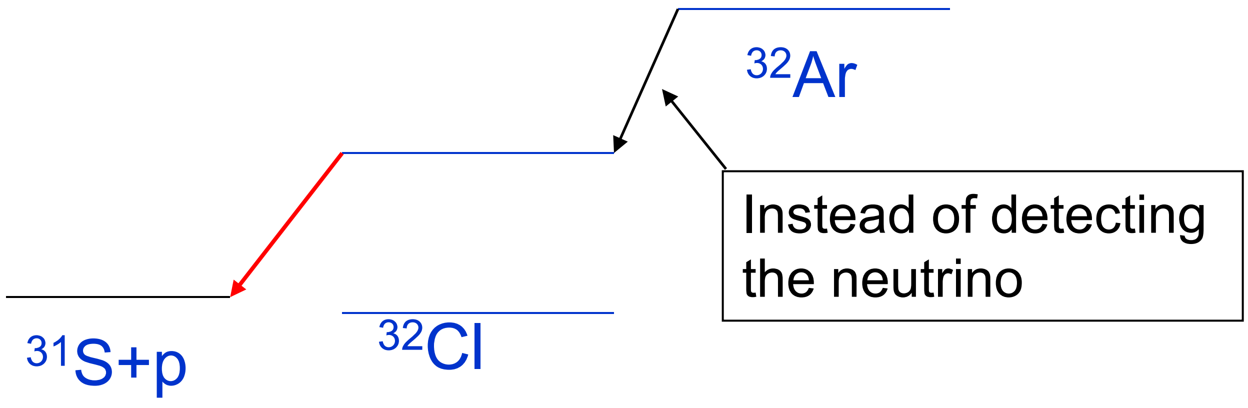




$dW/d\Omega = 1 + \mathbf{p}_e . \mathbf{p}_\nu / E_e E_\nu$

$dW/d\Omega = 1 - \mathbf{p}_e . \mathbf{p}_\nu / E_e E_\nu$

A trick to avoid detecting the neutrino



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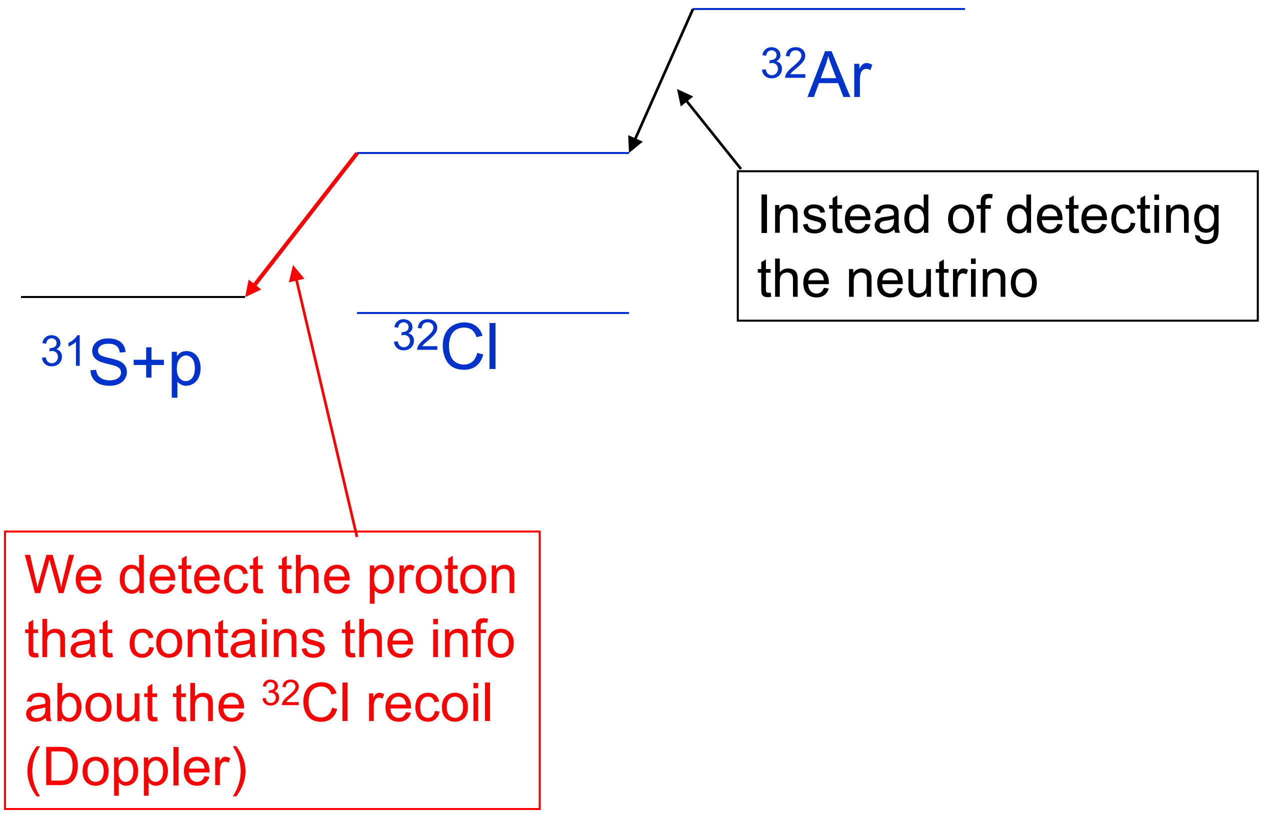

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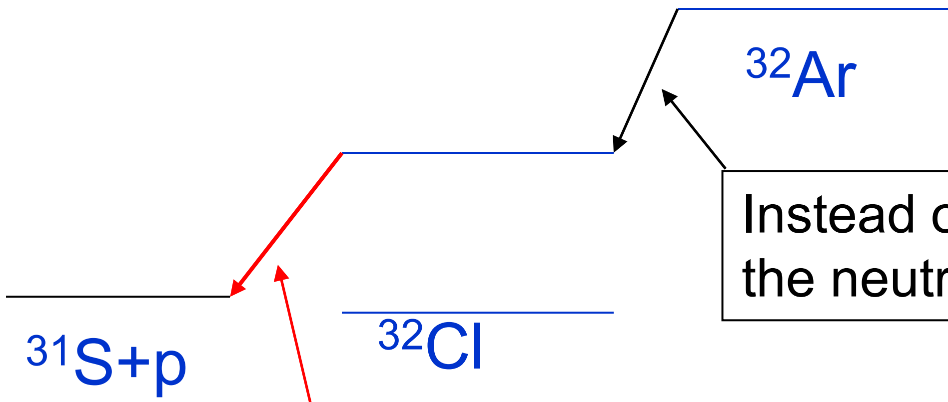
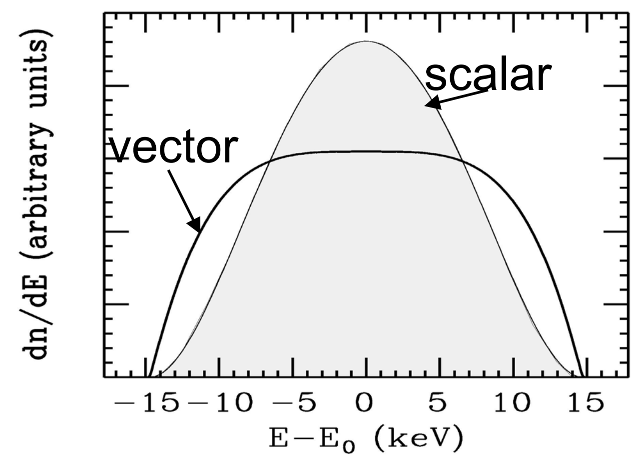


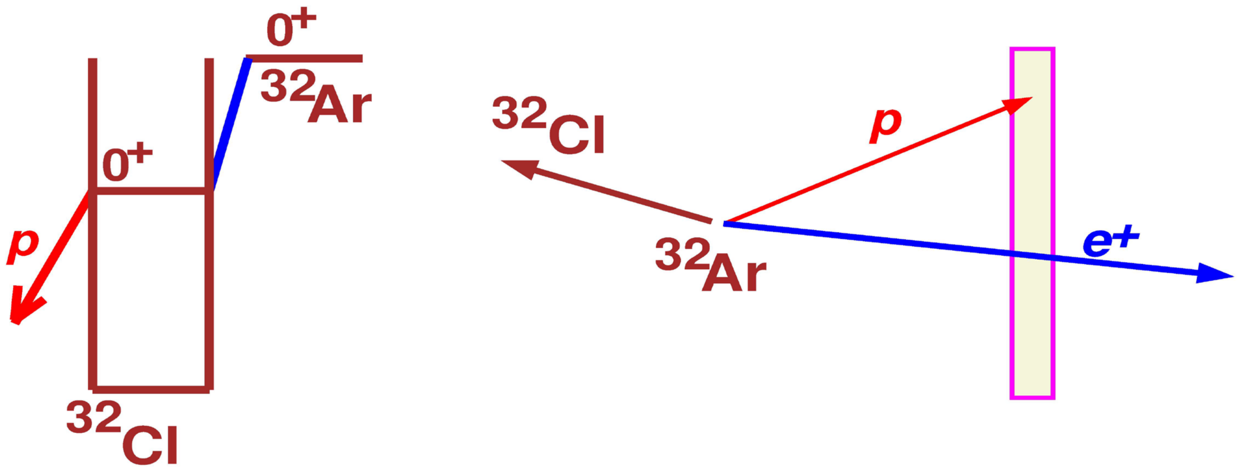
Instead of detecting the neutrino

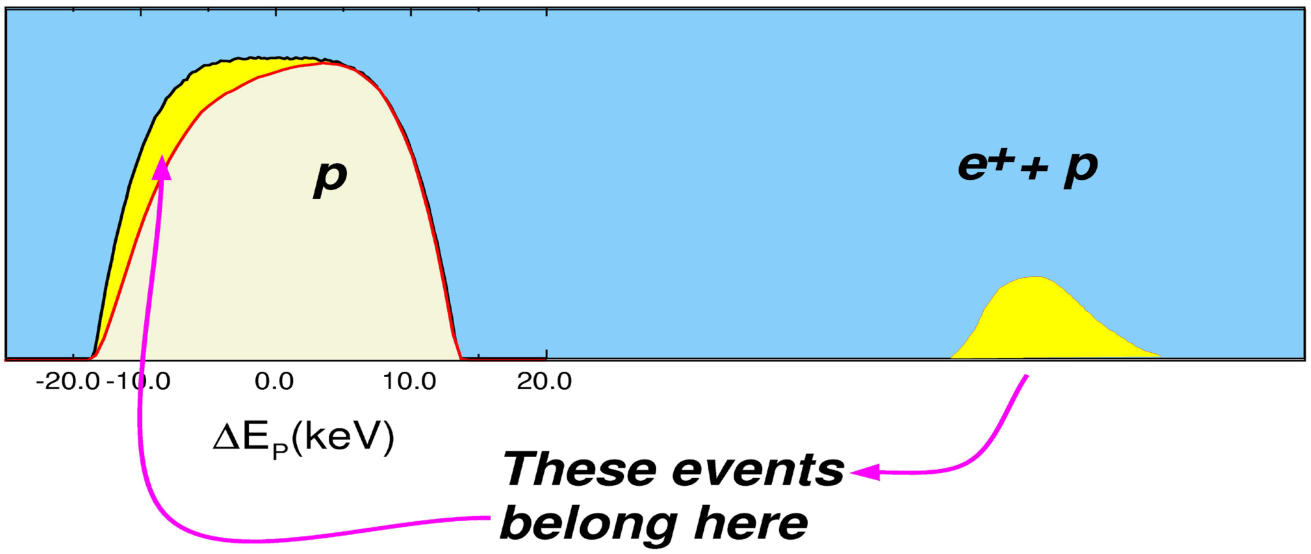
We detect the proton that contains the info about the ^{32}Cl recoil (Doppler)

Monte-Carlo calculation of proton energy



Problem: Summing with positrons distorts the shape of the proton peak





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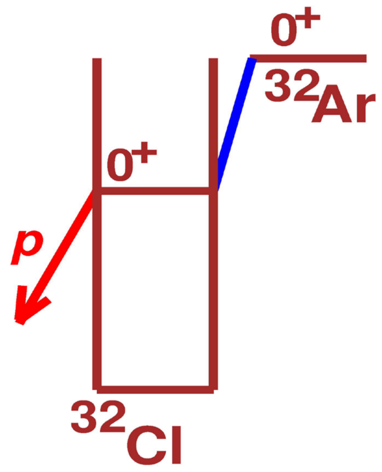
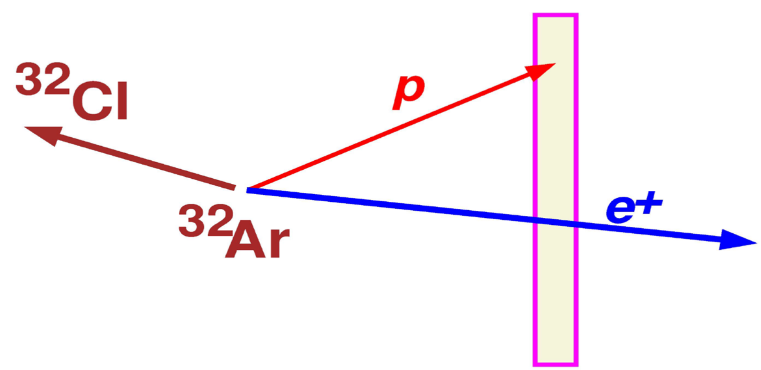
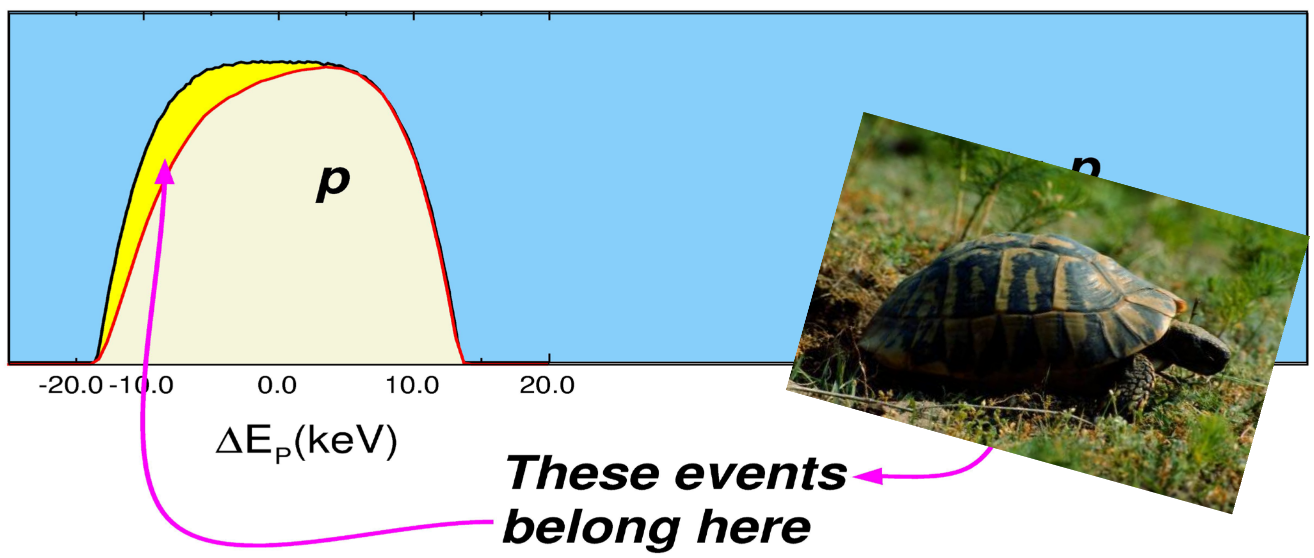
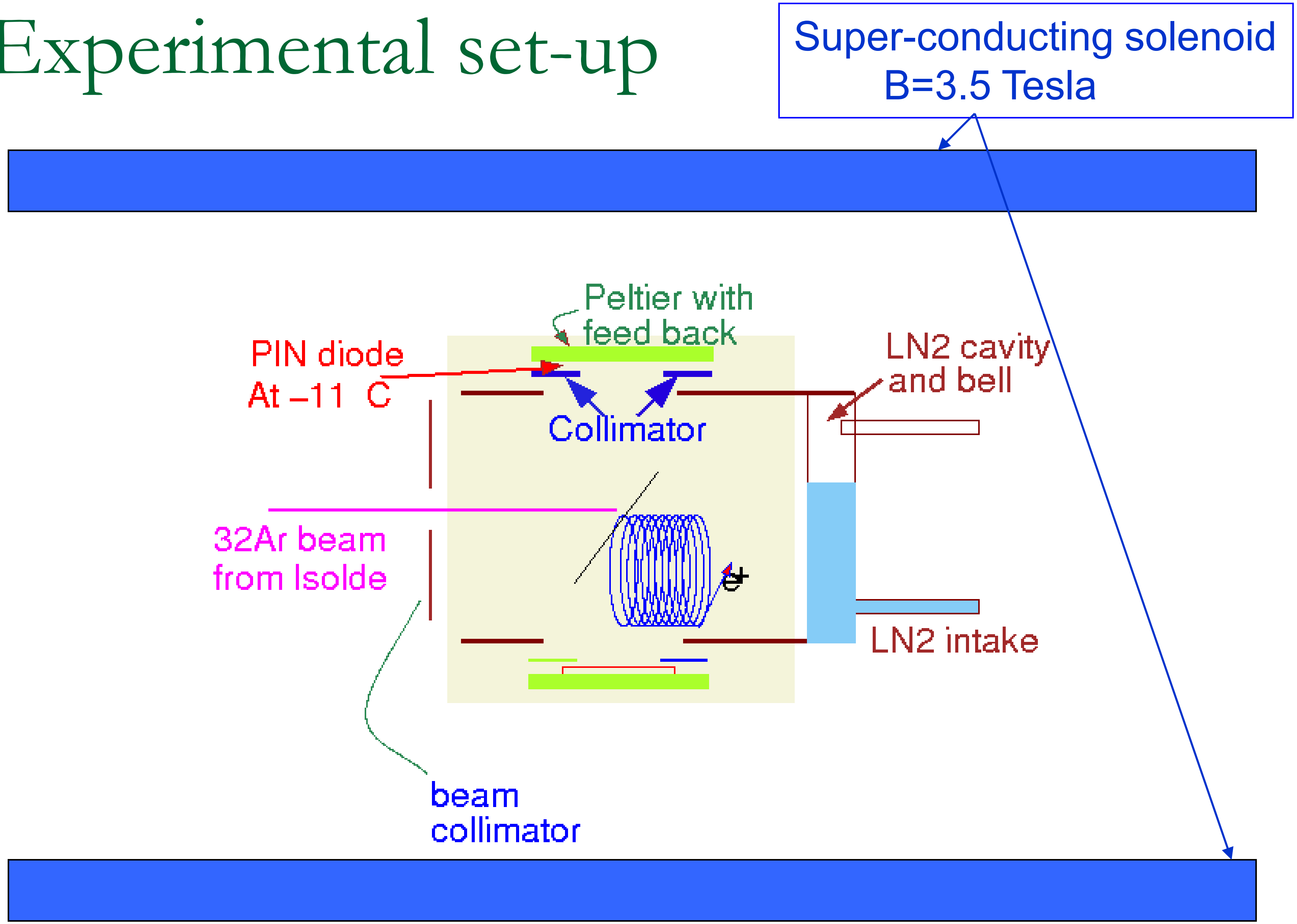


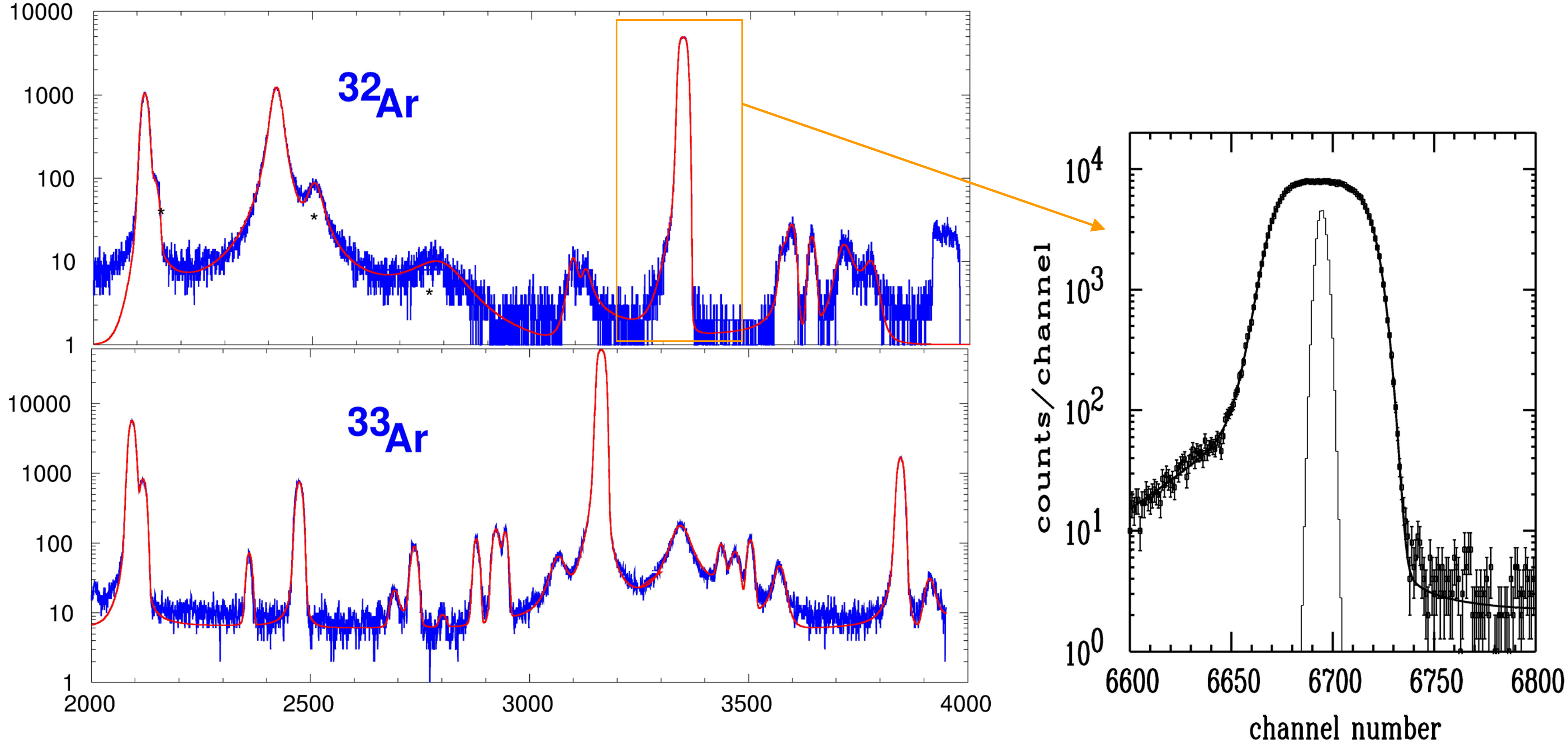




Experimental set-up

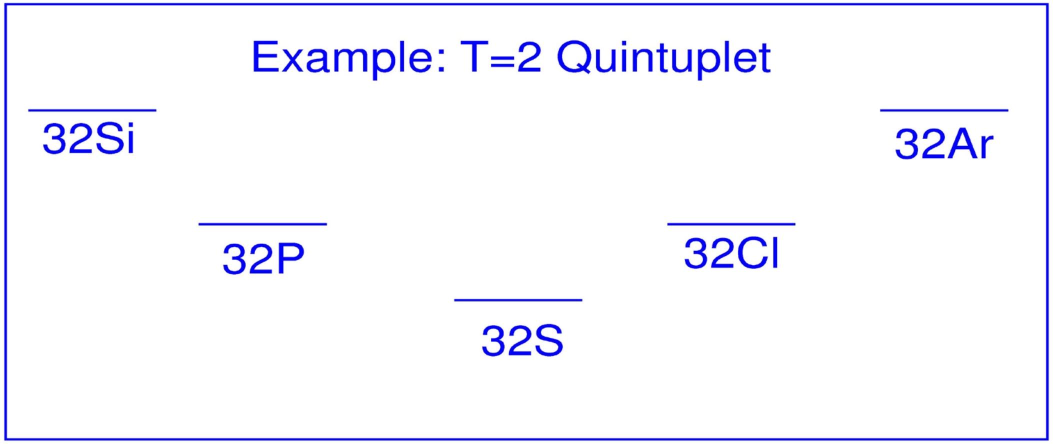


Full *R-matrix* fit to beta-delayed proton spectra from ^{32}Ar and ^{33}Ar



Isobaric Multiplet Mass Equation

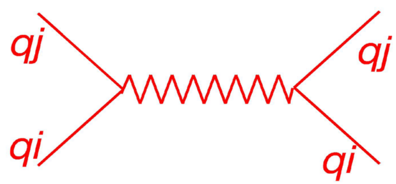
Masses of an Isospin mulitplet should follow a parabola



Proof (Wigner, Weinberg-Treiman):

$M = \langle \Psi | \, H \, | \Psi \rangle = \langle T, T_z \, | \, H_o + H_c \, | \, T, T_z \rangle$

$H_c = \Sigma \, q_i \, q_j \, f(r_{ij})$

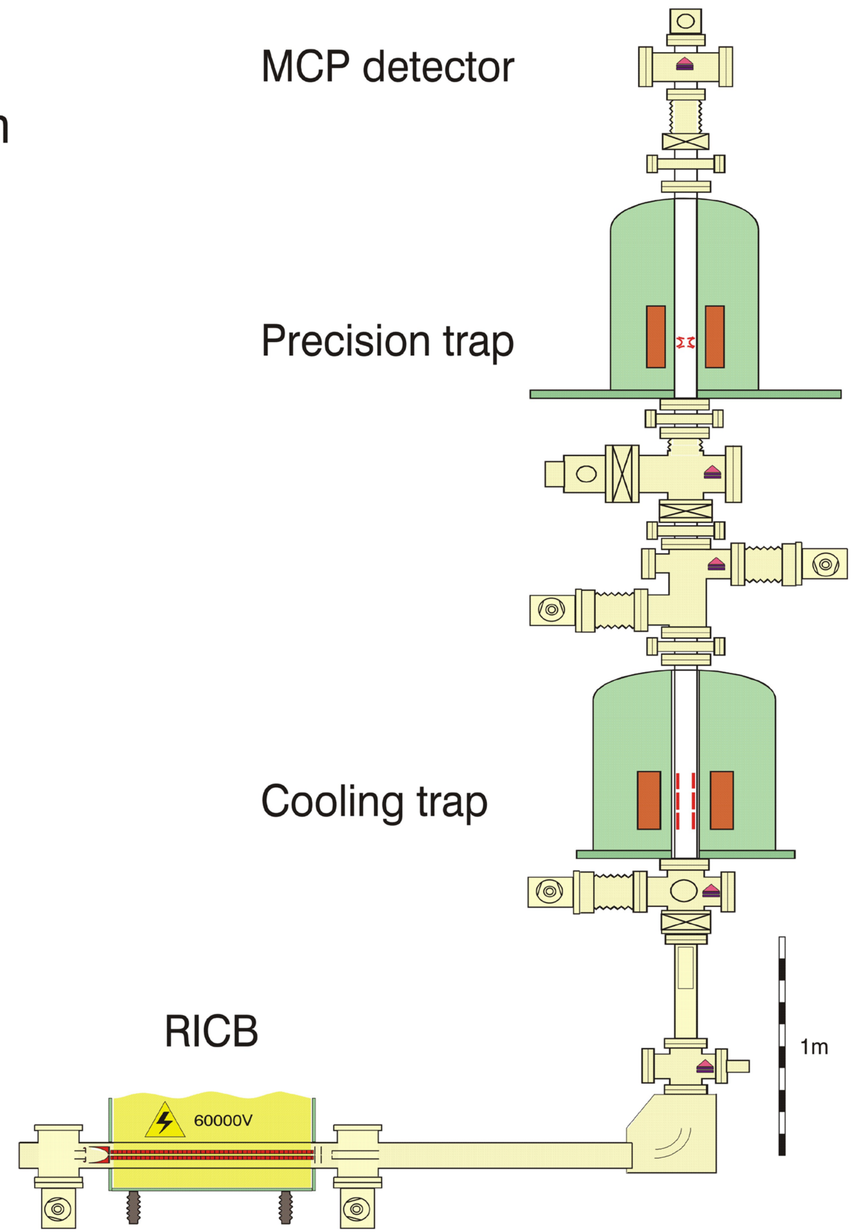


Then: $M = a + b\,T_z + c\,T_z^2$

Problem: Isol-trap fellows measured a mass of ^{33}Ar and found in disagreement with parabola for A=33 system.

Experimental Setup

- Penning trap mass spectrometer
- Principle based on measurement of $\omega_c = qB/m$
- Installed at the ISOLDE facility at CERN
- Typical relative mass uncertainty: $u(m)/m = 10^{-7}$
- Relative mass accuracy: $u_{syst}(m)/m < 10^{-8}$
- Applicable to all elements and to short-lived radioactive nuclides



RFQ ion beam cooler and buncher (RICB)

Function: Deceleration, cooling, bunching

Cooling Penning trap

- Function: Mass-selective cooling
- Resolving power typically $R = 10^4$

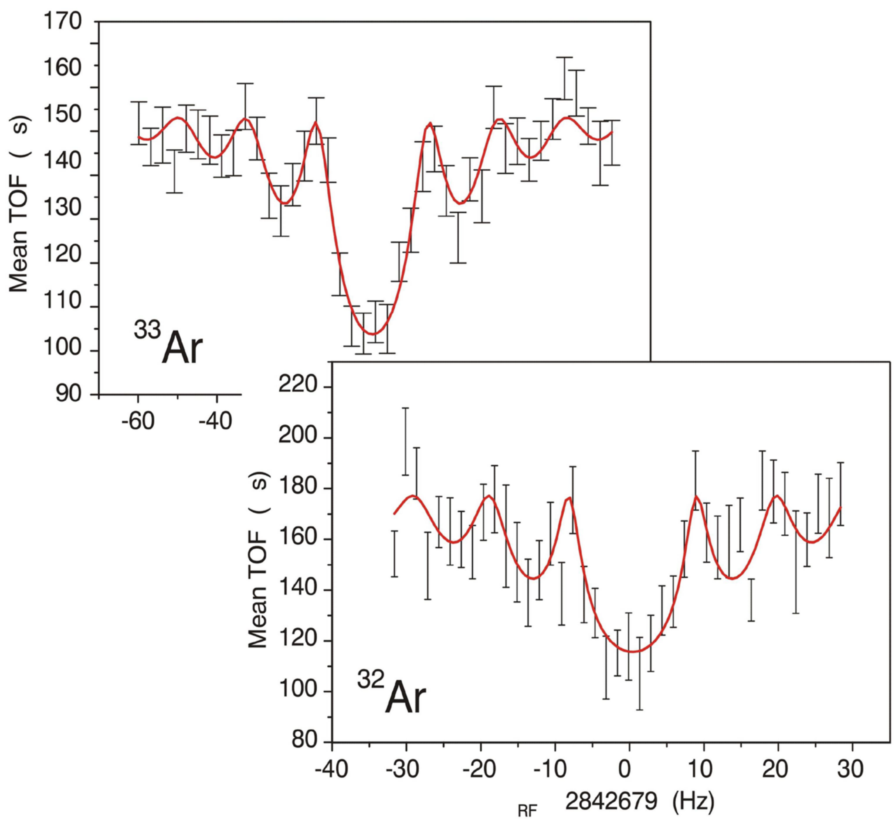
Precision Penning trap

- Function: Precision mass measurement
- Resolving power $R = 10^6$ at $t_{exc} = 1$ s

Results: Argon measurements 2000/2001

Isotopes measured: ^{34}Ar, ^{33}Ar, ^{32}Ar (n-deficient)

^{42}Ar, ^{43}Ar, ^{44}Ar, ^{45}Ar (n-rich)



Test of the IMME for the A =33, T =3/2 quartet (^{33}P, ^{33}S, ^{33}Cl, ^{33}Ar):

Quadratic fit yields ^{2} = 10

Cubic term required d = 2.75(88) keV

Highlight: ^{32}Ar

Half-life $T_{1/2}$ = 98 ms

Mass uncertainty u(m) 3 keV

(to be evaluated)

Problem: Isol-trap fellows measured a mass of ^{33}Ar and found in disagreement with parabola for A=33 system.

Solution: we found out the mass of $^{33}Cl(T=3/2)$ they were using was incorrect (Pyle et al. PRL **88**, 122501 (2002).)

Using the correct mass for $^{33}Cl(T=3/2)$ one obtains an excellent agreement with the Isospin parabola.

Assuming the parabola works for A=32 one obtains M(^{32}Ar)= -2197.0+-4.2 keV

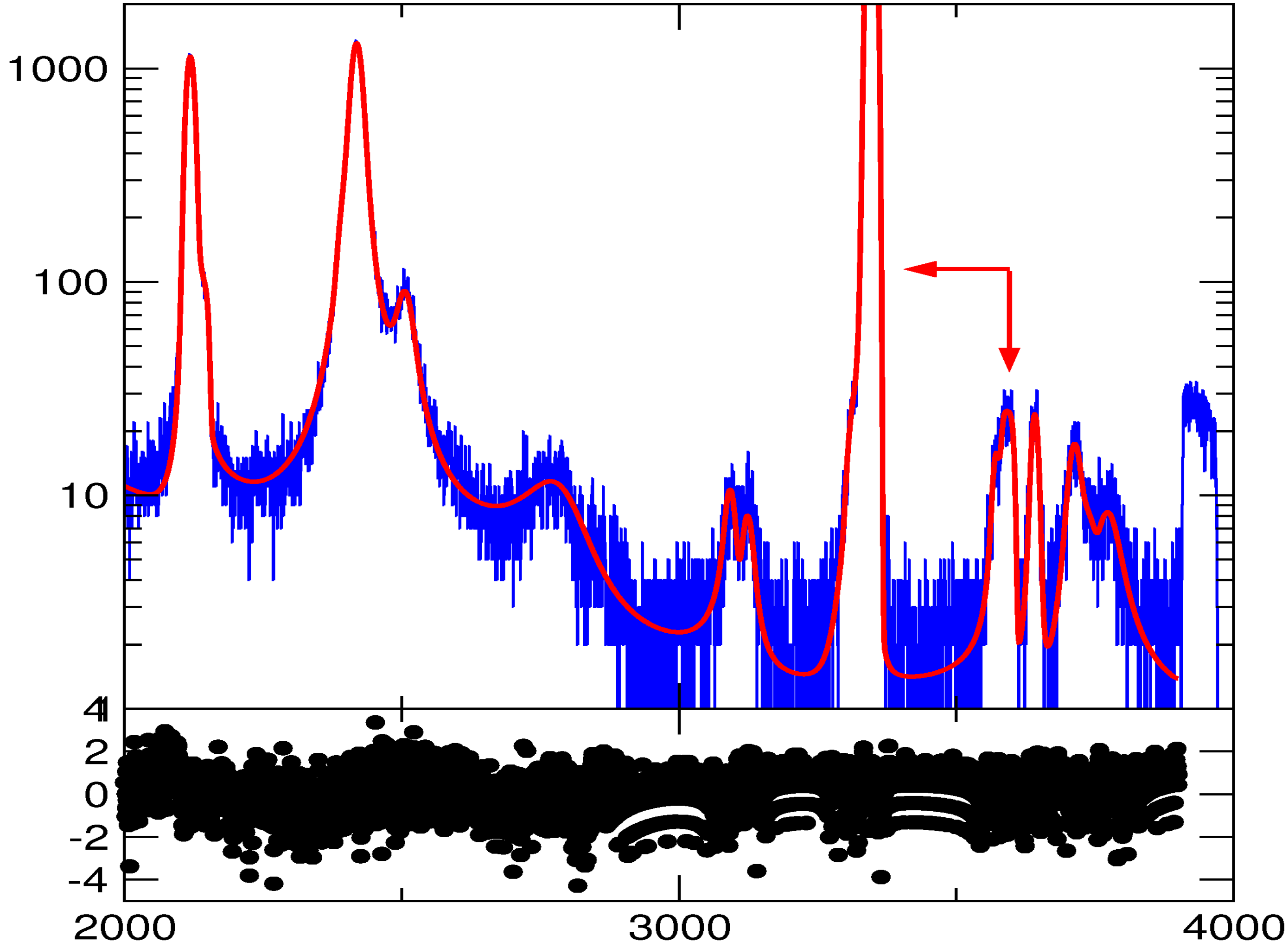
The Isol-trap new determination of the mass of ^{32}Ar is: -2200.1 +- 1.8 keV.

New Isol-trap data shows excellent agreement with the Isospin parabola but several quantities that affect our determination of the (e,ν) correlation have changed.

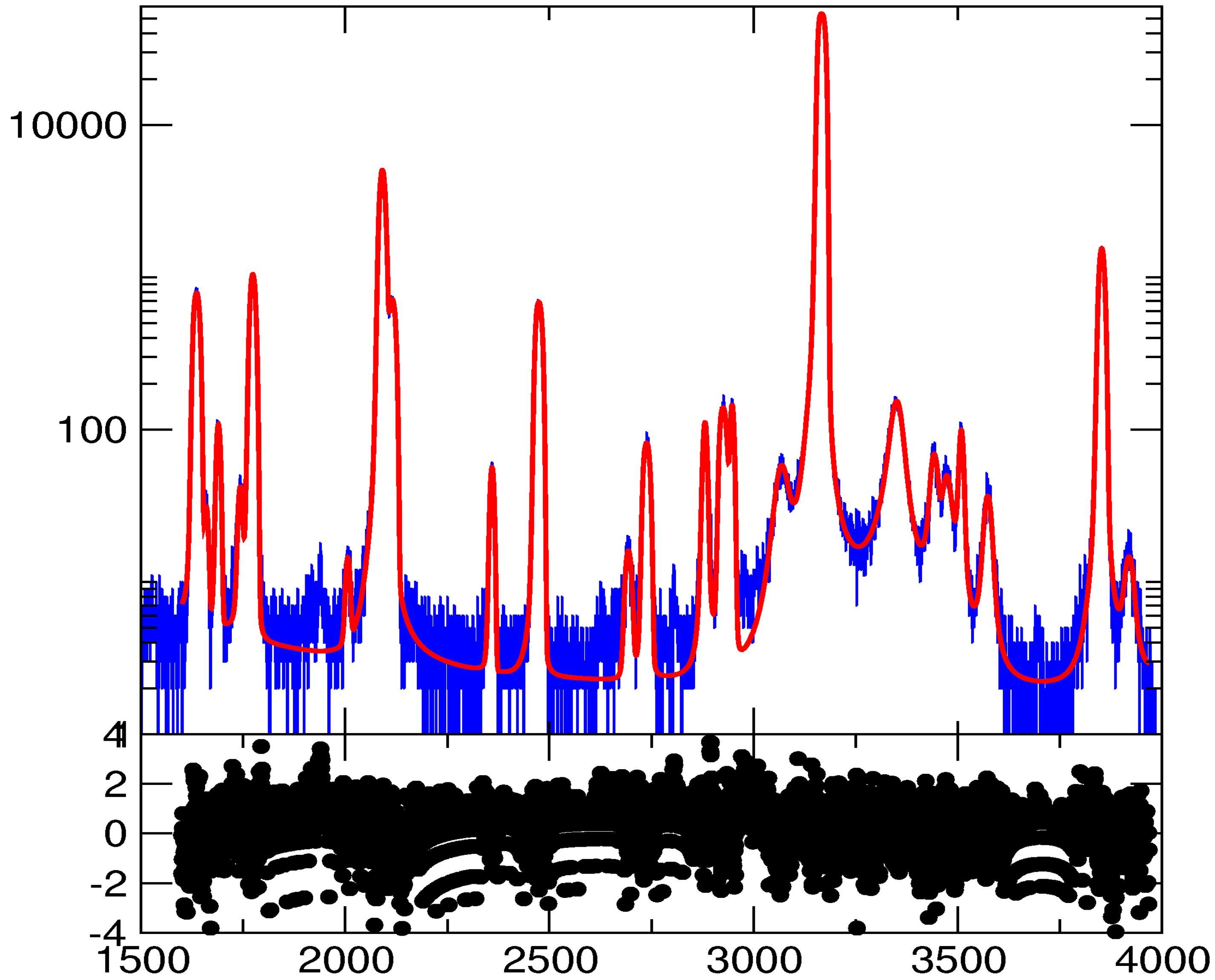
Q_{EC}, Energy calibration

We are presently re-doing all the data analysis to extract the correlation coefficient and systematic uncertainties.

Isospin Configuration Mixing



Widths and spins of ^{33}Cl from decay of ^{33}Ar



In $0^+ \rightarrow 0^+$ nuclear β decay:

$$H = G_W/\sqrt{2}\,(H_S + H_V)$$

$$H_S = (\bar{\Psi}_p \Psi_n)\,(\bar{\Psi}_e (C_S + \gamma_5 C'_S)\,\Psi_\nu)$$

$$H_V = (\bar{\Psi}_p \gamma^\mu \Psi_n)\,(\bar{\Psi}_e (C_V \gamma_\mu + C'_V \gamma_\mu \gamma_5)\,\Psi_\nu)$$

Consequence: decay rate for $0^+ \rightarrow 0^+$ decays

$$dW = dW_0\left(1 + a\,\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b\,\frac{\Gamma m_e}{E_e} + \langle \vec{J} \rangle . \ldots\right)$$

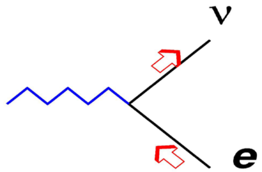
$$a = \frac{C_V^2 + C_V'^2 - C_S^2 - C_S'^2}{C_V^2 + C_V'^2 + C_S^2 + C_S'^2}$$

$$b = \frac{-2Re(C_S C_V^* + C'_S C_V'^*)}{C_V^2 + C_V'^2 + C_S^2 + C_S'^2}$$

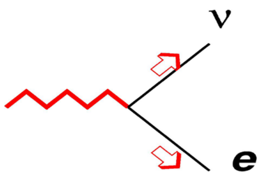
A pedestrian approach: Why are *a* and *b* sensitive to *Tensor* and *Scalar* contributions to Nuclear Beta Decay?

Example: $0^+ \rightarrow 0^+$ nuclear β decay

Vector:

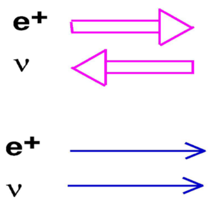


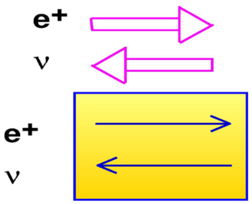
Scalar:



spins

momenta





$$dW/d\Omega(\text{Vector}) = 1 + \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e \, E_\nu}$$

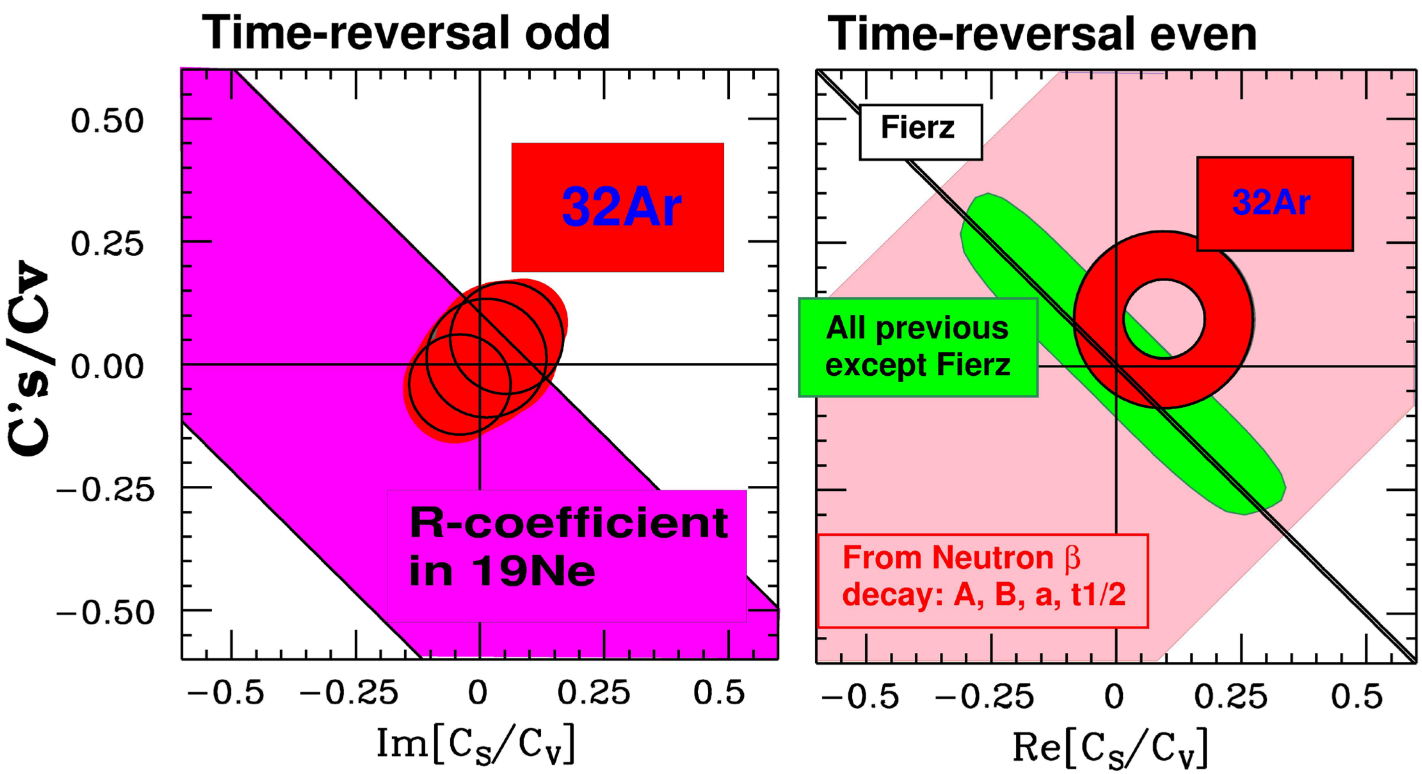
$$dW/d\Omega(\text{Scalar}) = 1 - \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e \, E_\nu}$$

b **comes as in interference between V and S**

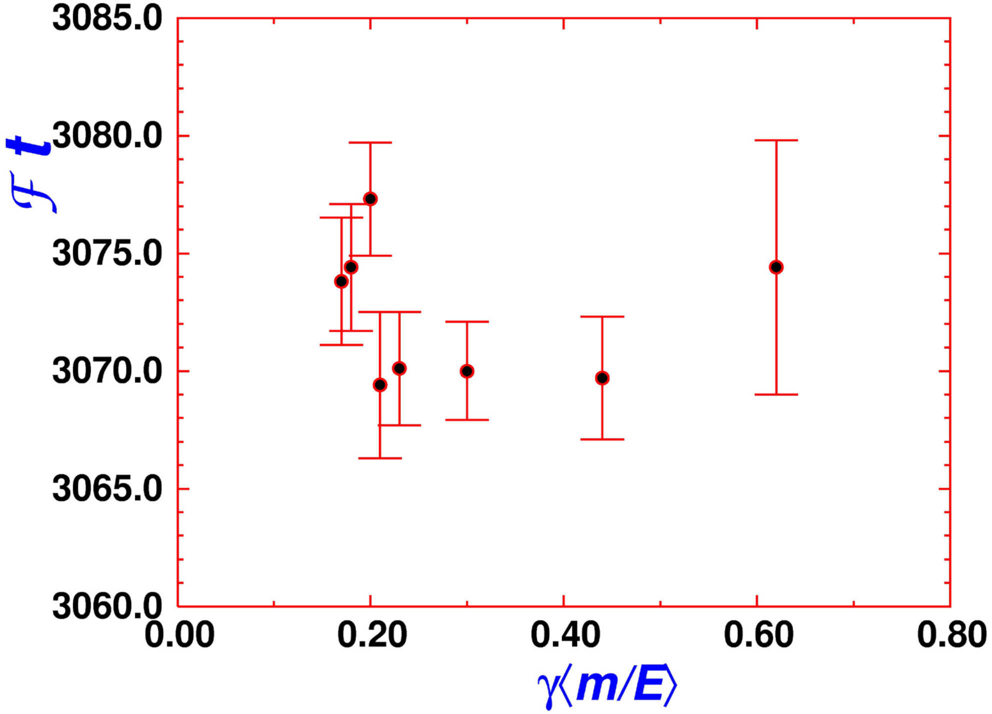
$$H_V = (\overline{\Psi}_p \gamma^\mu \Psi_n)\ (\overline{\Psi}_e^L (C_V \gamma_\mu) \Psi_\nu^L) + \ldots$$

$$H_S = (\overline{\Psi}_p \Psi_n)\ (\overline{\Psi}_e^R (C_S + C_S') \Psi_\nu^L) + \ldots$$

Limits for scalar couplings



$$\mathcal{F}t = \mathcal{F}t_0\,(1 + b\,\gamma\,\langle m/E\rangle)$$



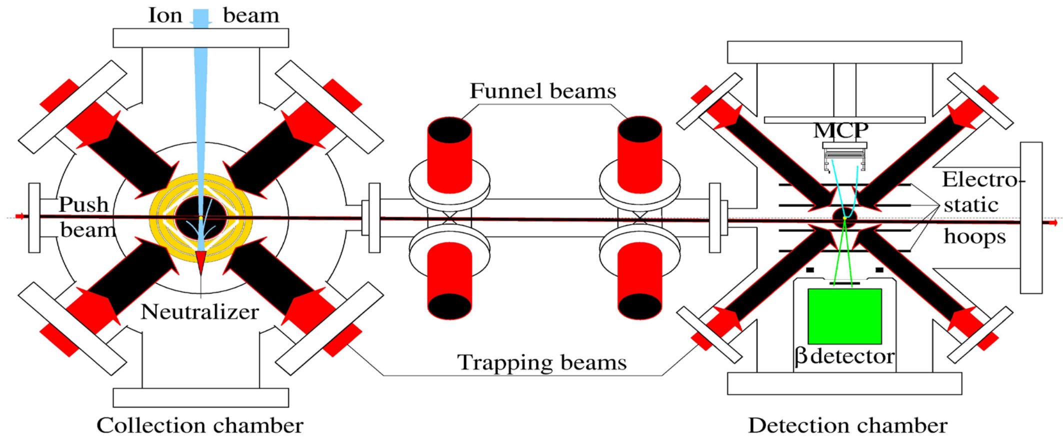
From the slope in this plot one extracts:

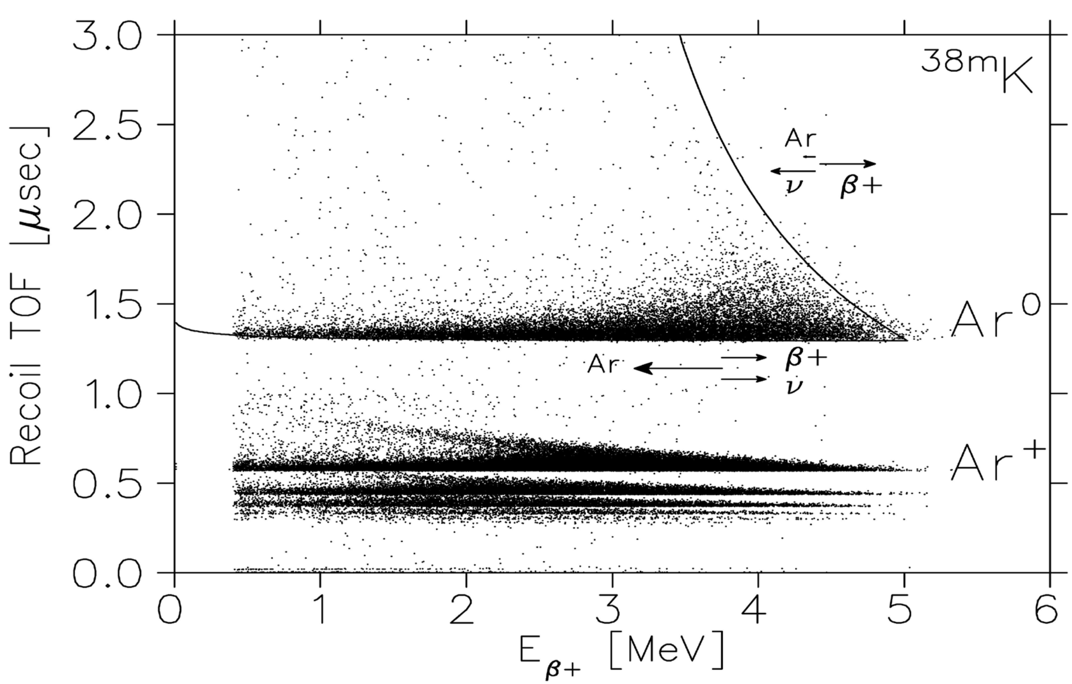
$$b = (3.4 \pm 2.9)\text{x } 10^{-3}$$

or,

$$C_S + C_S' \approx (1.7 \pm 1.5)\text{x } 10^{-3}$$

TRIUMF Neutral Atom Trap
Search for Scalars

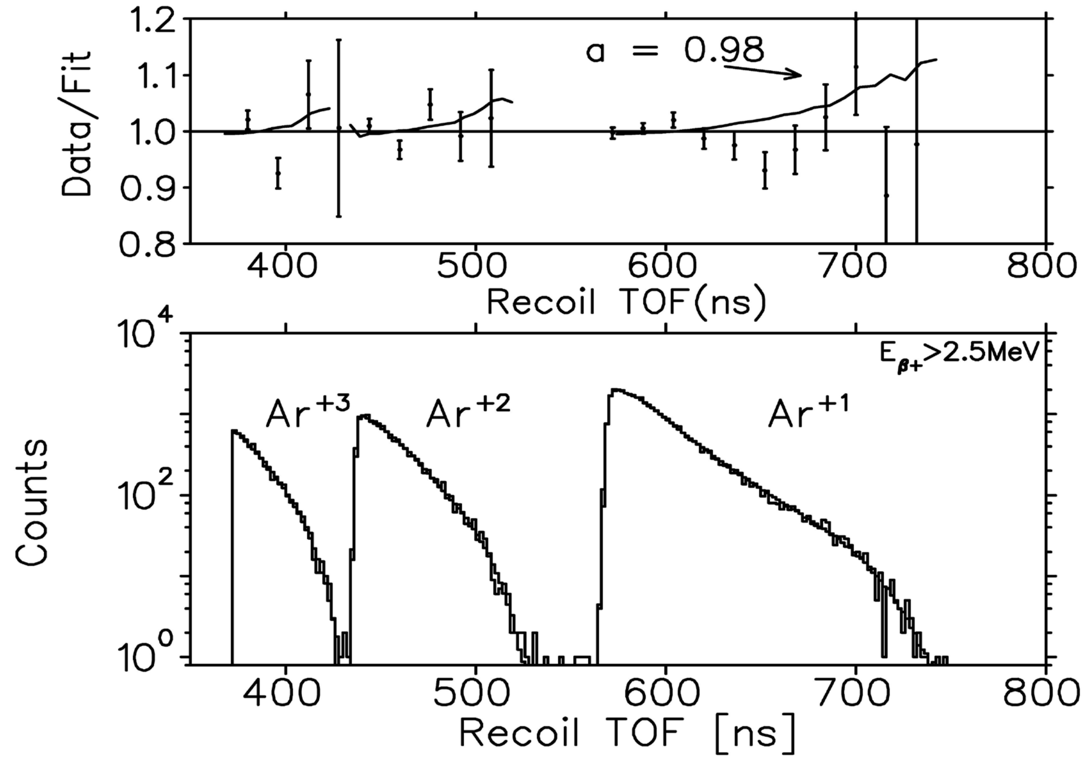




TRIUMF Neutral Atom Trap

^{38m}K $0^+ \to 0^+$ $\beta^+ - \nu$ correlation

$$W[\theta] = 1 + bm/E + a\, v/c \cos\theta$$



PRELIMINARY:

$a = 0.992 \pm 0.008$ (stat) ± 0.005 (syst)

$\rightarrow M_S/(G_S/G_W) > 200$ GeV/c^2

[Oct 00 a= ? $\pm$ 0.003 (stat) $\pm$ 0.003? (syst)]

Adelberger et al. ^{32}Ar [P.R.L. 83 (1999) 1299]: