Heavy Quarks

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Reference

- A.V. Manohar and M.B. Wise,
  Heavy Quark Physics,
  Cambridge University Press (2000)

Introduction

QCD describes the dynamics of quarks, and has a non-perturbative scale $\Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$.

Simplifications when $m_Q \gg \Lambda_{\mathrm{QCD}}$

A single heavy quark interacting with light particles can be described by an effective field theory known as HQET.

Applied to c and b quarks. The t-quark decays via $t \to bW$ before it forms hadrons. The width in the standard model is $\Gamma_t \approx 1.5\,\mathrm{GeV}$

NRQCD

Systems with two heavy quarks (such as J/ψ, Υ or $\bar{t}t$ near threshold) are described by a completely different effective theory, NRQCD [non-relativistic QCD].

Velocity Superselection Rule

UCSD

Consider a heavy quark Q interacting with light degrees of freedom, such as light quarks and gluons.

$$v^\mu = \frac{p^\mu}{m_Q}, \qquad \delta v^\mu = \frac{\delta p^\mu}{m_Q} \sim \frac{\Lambda_{\rm QCD}}{m_Q} \longrightarrow 0$$

Quark has a constant velocity.

$$[x, v] = \frac{1}{m_Q}\,[x, p] = \frac{i\hbar}{m_Q} \longrightarrow 0.$$

Cannot simultaneously have a well-defined position and momentum, but can have a well-defined position and velocity.

Spin-Flavor Symmetry

In the $m_Q \to \infty$ limit: static color $\mathbf{3}$ source in the rest frame $v^\mu = (1, 0, 0, 0)$.

Strong interactions flavor blind
$\Rightarrow$ HQ flavor symmetry
Symmetry breaking $\propto 1/m_b - 1/m_c$

Color coupling is color electric charge. The magnetic interaction is $\propto 1/m_Q$ for a pointlike spin-1/2 fermion (not true for the proton). $\Rightarrow$ HQ spin symmetry
Symmetry breaking $\propto 1/m_Q$.

Combining gives HQ $SU(4)$ spin-flavor symmetry:
$b\uparrow$, $b\downarrow$, $c\uparrow$ and $c\downarrow$ transform as a $\mathbf{4}$

QCD Lagrangian

$$\mathcal{L} = \sum_{i=c,b,t} \bar{Q}_i \left(i \not{D} - m_{Q_i} \right) Q_i + \mathcal{L}_{\text{light}}$$

The Lagrangian has m_Q term, and no well-defined $m_Q \to \infty$ limit. It describes the interactions of Q at all energies, including those greater than m_Q.

Want to consider an effective theory valid for momenta smaller than m_Q, which makes the simplications of low momentum manifest.

Should have an expansion in $1/m_Q$

HQET Lagrangian

HQET Lagrangian:

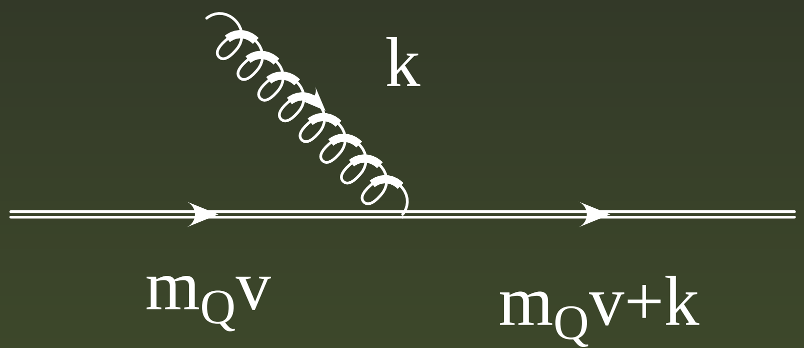
$$\mathcal{L}_{\rm HQET} = \mathcal{L}_0 + \frac{1}{m_Q}\mathcal{L}_1 + \frac{1}{m_Q^2}\mathcal{L}_2 + \ldots$$

$\mathcal{L}_0$ has spin-flavor symmetry,

$1/m_Q$ terms are symmetry breaking corrections.

Quark Propagator

Look at the quark propagator:



$$i\frac{\not{p}+m_Q}{p^2-m_Q^2+i\epsilon}$$

$p = mv + k$, where k is called the residual momentum, and is of order $\Lambda_{\rm QCD}$.

HQET Propagator

$$i\frac{m_Q \not{v} + \not{k} + m_Q}{(m_Q v + k)^2 - m_Q^2 + i\epsilon}$$

Expanding this in the limit $k \ll m_Q$ gives

$$i\frac{1+\not{v}}{2k\cdot v + i\epsilon} + \mathcal{O}\left(\frac{k}{m_Q}\right) = i\frac{P_+}{k\cdot v + i\epsilon} + \mathcal{O}\left(\frac{k}{m_Q}\right),$$

with a well defined limit.

$$P_+ \equiv \frac{1+\not{v}}{2}$$

Projectors

$$P_+ = \frac{1 + \not{v}}{2}, \qquad P_- = \frac{1 - \not{v}}{2},$$

In the rest frame,

$$P_+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_+^2 = P_+, \; P_-^2 = P_-, \; P_+ + P_- = 1, \; P_+ P_- = 0 \; P_- P_+ = 0.$$

Gluon Vertex

UCSD

The quark-gluon vertex

$$-igT^a\gamma^\mu \to -igT^a P_+\gamma^\mu P_+ = -igT^a v^\mu,$$

using the identity

$$P_+\gamma^\mu = v^\mu + \gamma^\mu P_-$$

In the rest frame: the coupling is purely that of an electric charge.

HQET L_0

HQET Lagrangian:

$$\mathcal{L} = \bar{h}_v(x)\,(iD\cdot v)\,h_v(x)\,,$$

$h_v(x)$ is the quark field in the effective theory and satisfies

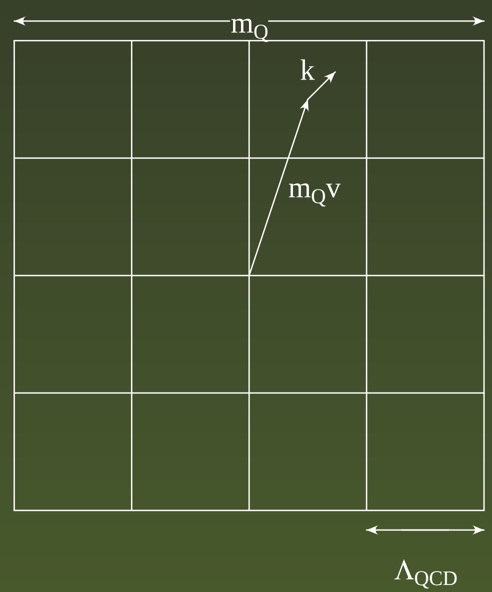
$$P_+ h_v(x) = h_v(x)\,.$$

h_v annihilates quarks with velocity v, but does not create antiquarks

Dividing up momentum space

v appears explictly in the HQET Lagrangian.

h_v describes quarks with velocity v, and momenta within $\Lambda_{\rm QCD}$ of $m_Q v$.



quarks with velocity $v' \neq v$ are far away in the EFT

Feynman rules

$$\mathcal{L} = \bar{h}_v \left(iD \cdot v \right) h_v$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu$$

The $\partial \cdot v$ term gives a propagator

$$\frac{iP_+}{k \cdot v}$$

The $A \cdot v$ term gives a vertex

$$-igT^a v^\mu$$

Manifest Spin-Flavor Symmetry

$$\mathcal{L}_0 = \sum_{f=c,b,t} \bar{h}_{fv}\,(iD\cdot v)\,h_{fv},$$

has manifest spin-flavor symmetry, since $D\cdot v$ does not depend on the spin or the flavor of the heavy quark.

Light degrees of freedom

UCSD

Hadrons containing a single heavy quark contain Q, and light quarks and gluons [light degrees of freedom ℓ].

D^+ meson has a c quark, $\bar{d}$ quark, plus $\bar{q}q$ pairs and gluons. Quantum numbers of ℓ are the same as the $\bar{d}$.

Total angular momentum $\mathbf{J}$ is conserved

$\mathbf{S}_Q$ is conserved as $m_Q \to \infty$

Define $\quad \mathbf{S}_\ell \equiv \mathbf{J} - \mathbf{S}_Q$

Spin of the light degrees of freedom

Multiplet Structure

$\mathbf{J}^2 = j(j+1)$, $\mathbf{S}_Q^2 = s_Q(s_Q+1)$, $\mathbf{S}_\ell^2 = s_\ell(s_\ell+1)$

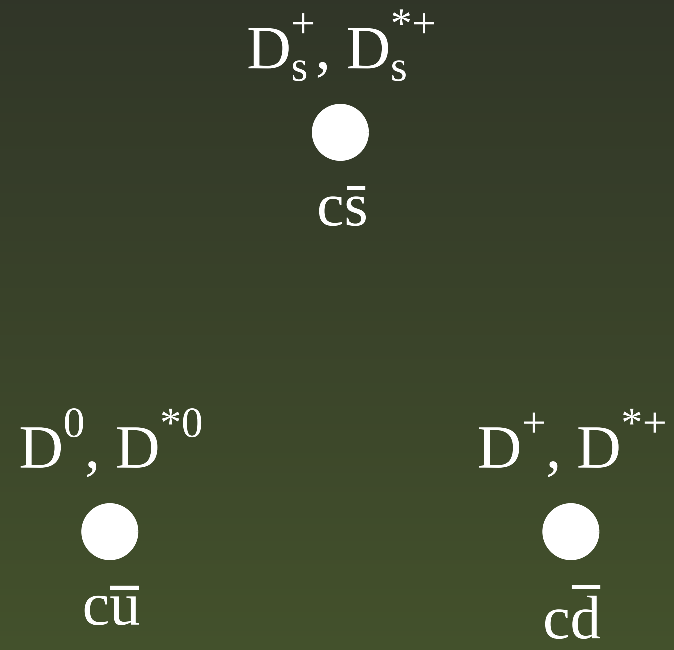
$s_Q = 1/2$, so heavy hadrons are in degenerate multiplets with $j = s_\ell \pm 1/2$, unless $s_\ell = 0$, in which case there is a single $j = 1/2$ multiplet.

Ground state mesons: Q and a light antiquark $\bar{q}$, so $s_\ell = 1/2$.
$j = 0 \oplus 1$ and negative parity, since quarks and antiquarks have opposite parity

Degenerate 0^- and 1^- mesons which form a flavor $\bar{\mathbf{3}}$

Called $H^{(Q)}$

Ground State Mesons



$\bar{B}_s^0$, $\bar{B}^-$, $\bar{B}^0$ (spin-0) $\bar{B}_s^{*0}$, $\bar{B}^{*-}$, $\bar{B}^{*0}$ (spin-1)

NOTE: $c \in D$, $b \in \bar{B}$

Ground State Baryons

One Q and qq. Either (anti)symmetric in both flavor and spin.

Spin $\left(\frac{1}{2}\times\frac{1}{2}\right)_S = 1$ Flavor $(\mathbf{3}\times\mathbf{3})_S = \mathbf{6}$

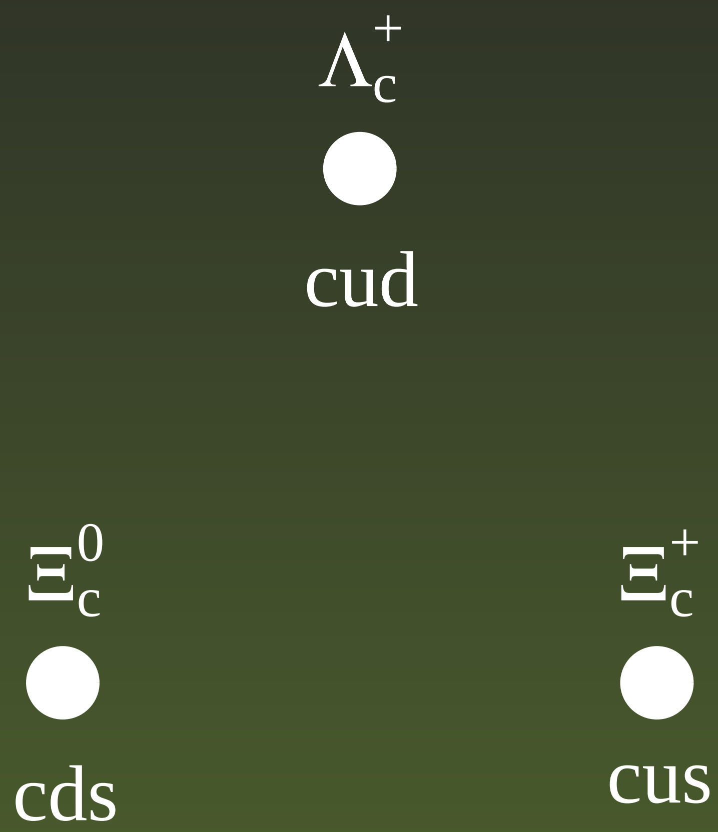
Spin $\left(\frac{1}{2}\times\frac{1}{2}\right)_A = 0$ Flavor $(\mathbf{3}\times\mathbf{3})_A = \bar{\mathbf{3}}$

$s_\ell = 0 \Rightarrow j = 1/2$ baryons which are flavor $\bar{\mathbf{3}}$

$s_\ell = 1 \Rightarrow j = 1/2, 3/2$ baryons which are flavor $\mathbf{6}$

$\Lambda^{(Q)}$ $\Sigma^{(Q)}, \Sigma^{*(Q)}$

$\bar{3}$ Baryons



$\Lambda_b^0, \ \Xi_b^-, \ \Xi_b^0$

6 Baryons

$\Sigma_c^0, \Sigma_c^{*0}$ — cdd

$\Sigma_c^+, \Sigma_c^{*+}$ — cud

$\Sigma_c^{++}, \Sigma_c^{*++}$ — cuu

$\Xi_c'^{\,0}, \Xi_c^{*0}$ — cds

$\Xi_c'^{\,+}, \Xi_c^{*+}$ — cus

$\Omega_c^0, \Omega_c^{*0}$ — css

$$\Sigma_b^{-,0,+}, \ \Xi_b^{-,0}, \ \Omega_b^-$$

Quark Field

Write the quark field as

$$\begin{aligned} Q(x) &= e^{-im_Q v\cdot x}\left[h_v(x) + \mathcal{Q}_v(x)\right] \\ &= e^{-im_Q t}\begin{pmatrix} h_v(x) \\ \mathcal{Q}_v(x) \end{pmatrix} \end{aligned}$$

$p = m_Q v + k$, so the x dependence of $h_v(x)$ is k.

$$\begin{aligned} \not{v} h_v(x) &= h_v(x), \\ \not{v} \mathcal{Q}_v(x) &= -\mathcal{Q}_v(x) \end{aligned}$$

Tree-level Lagrangian

At tree-level, one does not have to worry about renormalization effects:

$$\begin{aligned}\mathcal{L} &= \bar{Q}\left(i\not{D} - m_Q\right)Q \\ &= \left(\bar{h}_v + \bar{\mathcal{Q}}_v\right)e^{im_Q v\cdot x}\left(i\not{D} - m_Q\right)e^{-im_Q v\cdot x}\left(h_v + \mathcal{Q}_v\right) \\ &= \left(\bar{h}_v + \bar{\mathcal{Q}}_v\right)\left(i\not{D} - m_Q + m_Q\not{v}\right)\left(h_v + \mathcal{Q}_v\right) \\ &= \left(\bar{h}_v + \bar{\mathcal{Q}}_v\right)\left(i\not{D}h_v + \left[i\not{D} - 2m_Q\right]\mathcal{Q}_v\right)\end{aligned}$$

The result can be simplified using

$$P_+\gamma^\mu P_+ = v^\mu,\;\; P_-\gamma^\mu P_- = -v^\mu,\;\; P_+\gamma^\mu P_- = \gamma_\perp^\mu,\;\; P_-\gamma^\mu P_+ = \gamma_\perp^\mu,$$

where the $\perp$ projector is defined for any vector A by

$$A_\perp^\mu \equiv A^\mu - v^\mu \;\, v\cdot A.$$

$$\mathcal{L} = \bar{h}_v \left(iv \cdot D\right) h_v - \mathcal{Q}_v \left(iv \cdot D + 2m_Q\right) \mathcal{Q}_v + \bar{h}_v i \not{D}_\perp \mathcal{Q}_v + \bar{\mathcal{Q}}_v i \not{D}_\perp h_v$$

Quadratic in $\mathcal{Q}_v$:

$$\mathcal{L} = \bar{h}_v \left(iv \cdot D\right) h_v + \bar{h}_v i \not{D}_\perp \frac{1}{2m_Q + iv \cdot D} i \not{D}_\perp h_v .$$

The last term can be expanded in a power series in $1/m_Q$,

$$\frac{1}{2m_Q + iv \cdot D} = \frac{1}{2m_Q} - \frac{1}{4m_Q^2} iv \cdot D + \ldots \; .$$

$1/m_Q$ Lagrangian

The effective Lagrangian to order $1/m_Q$ is thus

$$\mathcal{L} = \bar{h}_v \left(iv \cdot D \right) h_v + \frac{1}{2m_Q} \bar{h}_v i \not{D}_\perp \, i \not{D}_\perp h_v .$$

This can be rewritten using the identities

$$\gamma^\alpha \gamma^\beta = g^{\alpha\beta} - i\sigma^{\alpha\beta}, \;\; \left[D^\alpha, D^\beta \right] = igG^{\alpha\beta}$$

$$\begin{aligned} \mathcal{L} \;\; &= \;\; \bar{h}_v \left(iv \cdot D \right) h_v + \frac{1}{2m_Q} \bar{h}_v \left(iD_\perp \right)^2 h_v - \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v \\ & \quad + \mathcal{O}\left(\frac{1}{m_Q^2} \right) \end{aligned}$$

$$\mathcal{L} = \bar{h}_v \left(iv \cdot D \right) h_v + \frac{1}{2m_Q} \bar{h}_v \left(iD_\perp \right)^2 h_v - c_F \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v$$

The $(iD_\perp)^2$ term violates flavor symmetry at order $1/m_Q$

$g\sigma_{\alpha\beta}G^{\alpha\beta}$ term violates spin and flavor symmetry at order $1/m_Q$

One can carry out the expansion to higher order in $1/m_Q$ to obtain the tree level HQET Lagrangian.

Masses

Hadron mass mass in effective theory is $M_H - m_Q$.

- Lowest order: all hadrons degenerate, mass m_Q
- Order one: Hadron mass

$$\langle H_Q | \mathcal{H}_0 | H_Q \rangle \equiv \bar{\Lambda}$$

where H_0 = Hamiltonian from lowest order Lagrangian (including the light degrees of freedom).
$\bar{\Lambda}$ has different values for each multiplet:
$\bar{\Lambda}$ for mesons, $\bar{\Lambda}_\Lambda$, $\bar{\Lambda}_\Sigma$

$\lambda_1, \ \lambda_2$

- Order $1/m_Q$:

$$\frac{\mathcal{H}_1}{m_Q} = -\frac{\mathcal{L}_1}{m_Q} = -\frac{1}{2m_Q}\bar{h}_v \left(iD_\perp\right)^2 h_v + c_F \frac{g}{4m_Q}\bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v .$$

Define two non-perturbative parameters

$$\begin{aligned} \lambda_1 &= \left\langle H_Q \right| \bar{h}_v \left(iD_\perp\right)^2 h_v \left| H_Q \right\rangle, \\ 8\left(\mathbf{S}_Q \cdot \mathbf{S}_\ell\right) \lambda_2 &= \left\langle H_Q \right| \bar{h}_v g \sigma_{\alpha\beta} G^{\alpha\beta} h_v \left| H_Q \right\rangle, \end{aligned}$$

In the rest frame:

$$\begin{aligned} \left(iD_\perp\right)^2 &= -\mathbf{p}^2 \\ \sigma_{\alpha\beta} G^{\alpha\beta} &= -2\boldsymbol{\sigma} \cdot \mathbf{B} \end{aligned}$$

Meson Masses

UCSD

$$m_H = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} + c_F \frac{2\lambda_2 \mathbf{S}_Q \cdot \mathbf{S}_\ell}{m_Q}$$

$\mathbf{S}_Q \cdot \mathbf{S}_\ell = (J^2 - S_Q^2 - S_\ell^2)/2$, so

$$\begin{aligned} m_B &= m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2}{2m_b} \\ m_{B^*} &= m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2}{2m_b} \\ m_D &= m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2}{2m_c} \\ m_{D^*} &= m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} + \frac{\lambda_2}{2m_c} \end{aligned}$$

Note that heavy quark symmetry implies that $\bar{\Lambda}$, λ_1 and λ_2 have the same value in the b and c systems (upto renormalization)

$$0.49\ \mathrm{GeV}^2 = m_{B*}^2 - m_B^2 = 4\lambda_2 = m_{D*}^2 - m_D^2 = 0.55\ \mathrm{GeV}^2,$$

up to corrections of order $1/m_{b,c}$.

$$90 \pm 3\mathrm{MeV} = m_{B_s} - m_{B_d} = \bar{\Lambda}_s - \bar{\Lambda}_d = m_{D_s} - m_{D_d} = 99 \pm 1\mathrm{MeV}$$
$$345 \pm 9\mathrm{MeV} = m_{\Lambda_b} - m_B = \bar{\Lambda}_\Lambda - \bar{\Lambda}_d = m_{\Lambda_c} - m_D = 416 \pm 1\mathrm{MeV}$$

$\bar{\Lambda}$, λ_1 and λ_2 will occur elsewhere

c-Meson Spectrum

	(MeV)		J^P	s_ℓ
D^+	1869.3 ± 0.5	$c\bar{d}$	0^-	$1/2$
D^{*+}	2010.0 ± 0.5		1^-	
D^0	1864.6 ± 0.5	$c\bar{u}$	0^-	$1/2$
D^{*0}	2006.7 ± 0.5		1^-	
D_s^+	1968.5 ± 0.6	$c\bar{s}$	0^-	$1/2$
D_s^{*+}	2112.4 ± 0.7		1^-	
D_0^*		$c\bar{q}$	0^+	$1/2$
D_1^*	2461 ± 50		1^+	
D_1	2422.2 ± 1.8	$c\bar{q}$	1^+	$3/2$
D_2^*	2458.9 ± 2.0		2^+	

c-Baryon Spectrum

Λ_c^+	2284.9 ± 0.6	$c[ud]$	$1/2^+$	0
Ξ_c^+	2465.6 ± 1.4	$c[us]$	$1/2^+$	0
Ξ_c^0	2470.3 ± 1.8	$c[ds]$	$1/2^+$	0
Σ_c^{++}	2452.8 ± 0.6	$c(uu)$	$1/2^+$	1
Σ_c^{*++}	2519.4 ± 1.5		$3/2^+$	
Σ_c^+	2453.6 ± 0.9	$c(ud)$	$1/2^+$	1
Σ_c^{*+}			$3/2^+$	
Σ_c^0	2452.2 ± 0.6	$c(dd)$	$1/2^+$	1
Σ_c^{*0}	2517.5 ± 1.4		$3/2^+$	
$\Xi_c^{\prime +}$	2573.4 ± 3.3	$c(us)$	$1/2^+$	1
Ξ_c^{*+}	2644.6 ± 2.1		$3/2^+$	
$\Xi_c^{\prime 0}$	2577.3 ± 3.4	$c(ds)$	$1/2^+$	1
Ξ_c^{*0}	2643.8 ± 1.8		$3/2^+$	
Ω_c^0	2704 ± 4	$c(ss)$	$1/2^+$	1
Ω_c^{*0}			$3/2^+$	

b-Meson Spectrum

	(MeV)		J^P	s_ℓ
$\bar{B}^0$	5279.2 ± 1.8	$b\bar{d}$	0^-	$1/2$
$\bar{B}^{*0}$	5324.9 ± 1.8		1^-	
$\bar{B}^-$	5278.9 ± 1.8	$b\bar{u}$	0^-	$1/2$
$\bar{B}^{*-}$	5324.9 ± 1.8		1^-	
$\bar{B}_s^0$	5369.3 ± 2.0	$b\bar{s}$	0^-	$1/2$
$\bar{B}_s^{*0}$			1^-	
$\bar{B}_0^*$		$b\bar{q}$	0^+	$1/2$
$\bar{B}_1^*$			1^+	
$\bar{B}_1$		$b\bar{q}$	1^+	$3/2$
$\bar{B}_2^*$			2^+	

b-Baryon Spectrum

UCSD

Λ_b^0	5624 ± 9	$b[ud]$	$1/2^+$	0
Ξ_b^0		$b[us]$	$1/2^+$	0
Ξ_b^-		$b[ds]$	$1/2^+$	0
Σ_b^+		$b(uu)$	$1/2^+$	1
Σ_b^{*+}			$3/2^+$	
Σ_b^0		$b(ud)$	$1/2^+$	1
Σ_b^{*0}			$3/2^+$	
Σ_b^-		$b(dd)$	$1/2^+$	1
Σ_b^{*-}			$3/2^+$	
$\Xi_b^{\prime 0}$		$b(us)$	$1/2^+$	1
Ξ_b^{*0}			$3/2^+$	
$\Xi_b^{\prime -}$		$b(ds)$	$1/2^+$	1
Ξ_b^{*-}			$3/2^+$	
Ω_b^-		$b(ss)$	$1/2^+$	1
Ω_b^{*-}			$3/2^+$	

Meson Field

$Q\bar{q}$ mesons $\to$ field $H_v^{(Q)}$ (4×4 matrix, bispinor)

Pseudoscalar $P_v^{(Q)}(x)$ and vector $P_{v\mu}^{*(Q)}(x)$

Vector particles have a polarization vector ϵ_μ, with $\epsilon \cdot \epsilon = -1$, and $v \cdot \epsilon = 0$.

$$H_v^{(Q)} = \frac{1 + \not{v}}{2} \left[\not{P}_v^{*(Q)} + i P_v^{(Q)} \gamma_5 \right].$$

$$H_v^{(Q)} = \frac{1+\not{v}}{2}\left[\not{P}_v^{*(Q)} + iP_v^{(Q)}\gamma_5\right].$$

$$\not{v}H_v^{(Q)} = H_v^{(Q)}, \qquad H_v^{(Q)}\not{v} = -H_v^{(Q)}.$$

using $v \cdot P_v^{*(Q)} = 0$

Conjugate field:

$$\bar{H}_v^{(Q)} = \gamma^0 H_v^{(Q)\dagger} \gamma^0 = \left[\not{P}_v^{*(Q)\dagger} + i P_v^{(Q)\dagger} \gamma_5 \right] \frac{1 + \not{v}}{2},$$

which also transforms as a bispinor,

$$H_{v_r}^{(Q)} = \begin{pmatrix} 0 & iP_{v_r}^{(Q)} - \boldsymbol{\sigma} \cdot \mathbf{P}_{v_r}^{*(Q)} \\ 0 & 0 \end{pmatrix},$$

$$\begin{aligned} \mathbf{S}_Q H_{v_r}^{(Q)} &= \frac{1}{2}\boldsymbol{\sigma}_{\mathbf{4\times4}} H_{v_r}^{(Q)} \\ \mathbf{S}_\ell H_{v_r}^{(Q)} &= -\frac{1}{2} H_{v_r}^{(Q)} \boldsymbol{\sigma}_{\mathbf{4\times4}} \end{aligned}$$

Rotations:

$$\delta P_{v_r}^{(Q)} = 0, \qquad \delta \mathbf{P}_{v_r}^{(Q)} = \delta\boldsymbol{\theta} \times \mathbf{P}_{v_r}^{*(Q)}$$

HQ Spin:

$$\delta P_{v_r}^{(Q)} = -\frac{1}{2}\delta\boldsymbol{\theta}\cdot\mathbf{P}_{v_r}^{(Q)}, \qquad \delta\mathbf{P}_{v_r}^{*(Q)} = \frac{1}{2}\delta\boldsymbol{\theta}\times\mathbf{P}_{v_r}^{*(Q)} - \frac{1}{2}\delta\boldsymbol{\theta} P_{v_r(Q)}.$$

Under finite heavy quark spin transformations,

$$H_v^{(Q)} \to D(R)_Q H_v^{(Q)},$$

where $D(R)$ is the rotation matrix in the spinor representation for the rotation R.

Normalization of States

$$\langle H(p', \varepsilon') | H(p, \varepsilon) \rangle = 2E_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{\varepsilon\varepsilon'},$$

Mass dimension -1

HQET states eigenstates of the $m_Q \to \infty$ theory and labelled by v and k, with $v \cdot k = 0$. They differ from full QCD states.

$$\langle H(v', k', \varepsilon') | H(v, k, \varepsilon) \rangle = 2v^0 (2\pi)^3 \delta_{vv'} \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\varepsilon\varepsilon'}.$$

Usually take $k = 0$. States have mass dimension $-3/2$

$$|H(p)\rangle \;=\; \sqrt{m_H}\left[|H(v)\rangle + \mathcal{O}\left(\frac{1}{m_Q}\right)\right]$$

Similarly

$$\begin{aligned}
\overline{u}(p,s)\gamma^\mu u(p,s) &= 2p^\mu \\
\overline{u}(v,s)\gamma^\mu u(v,s) &= 2v^\mu \\
u(p,s) &= \sqrt{m_H}\,u(v,s)
\end{aligned}$$

$\bar{B} \to D^{(*)}$ Form Factors

Semileptonic $b \to c$ decays via the weak current $\bar{c}\,\gamma_\mu P_L\, b$

Decay form-factors are defined by:

$$
\begin{aligned}
\left\langle D(p')\right| V^\mu \left|\bar{B}(p)\right\rangle &= f_+(q^2)\,(p+p')^\mu + f_-(q^2)\,(p-p')^\mu\,, \\
\left\langle D^*(p',\epsilon)\right| V^\mu \left|\bar{B}(p)\right\rangle &= g(q^2)\epsilon^{\mu\nu\alpha\tau}\epsilon^*_\nu\,(p+p')_\alpha\,(p-p')_\tau\,, \\
\left\langle D^*(p',\epsilon)\right| A^\mu \left|\bar{B}(p)\right\rangle &= -if(q^2)\epsilon^{*\mu} \\
&\quad -i\epsilon^*\cdot p\left[a_+(q^2)\,(p+p')^\mu + a_-(q^2)\,(p-p')^\mu\right],
\end{aligned}
$$

where $q = p - p'$

Six form-factors

w

Label states by v and v', and use

$$w = v \cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

The allowed kinematic range for w is

$$0 \leq w - 1 \leq \frac{\left(m_B - m_{D^{(*)}}\right)^2}{2 m_B m_{D^{(*)}}}$$

The zero-recoil point, at which $D^{(*)}$ is at rest in the $\bar{B}$ rest frame, is $w = 1$ (maximum q^2)

Better to use:

$$\frac{\langle D(p')|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_D}} = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu,$$

$$\frac{\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta,$$

$$\frac{\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} = -ih_{A_1}(w)(w+1)\epsilon^{*\mu} + ih_{A_2}(w)(\epsilon^*\cdot v)v^\mu$$

$$+ih_{A_3}(w)(\epsilon^*\cdot v)v'^\mu.$$

$$q_{\rm light}^2 \sim (\Lambda_{\rm QCD} v - \Lambda_{\rm QCD} v')^2 = 2\Lambda_{\rm QCD}^2 (1-w)$$

HQ symmetry should hold if:

$$2\Lambda_{\rm QCD}^2 \, (w-1) \ll m_{b,c}^2.$$

The heavy meson form factors are expected to vary on the scale $q_{\rm light}^2 \sim \Lambda_{\rm QCD}^2$, i.e. on the scale $w \sim 1$.

QCD matrix elements are of the form:

$$\left\langle H^{(c)}(p')\right| \bar{c}\,\Gamma\, b \left| H^{(b)}(p)\right\rangle$$

At leading order in $1/m_{c,b}$ and $\alpha_s(m_{c,b})$:

$$\left\langle H^{(c)}(v')\right| \bar{c}_{v'}\,\Gamma\, b_v \left| H^{(b)}(v)\right\rangle$$

use spurion trick $\Gamma \to D(R)_c \Gamma D(R)_b^{-1}$

$$\bar{c}_{v'}\,\Gamma\, b_v = \mathrm{Tr}\; X \bar{H}_{v'}^{(c)}\,\Gamma\, H_v^{(b)},$$

$$X = X_0 + X_1 \not{v} + X_2 \not{v}' + X_3 \not{v} \not{v}',$$

where the coefficients are functions of $w = v \cdot v'$.

Isgur-Wise Function

Use $X = -\xi(w)$:

$$\begin{aligned}
\left\langle D(v')\right| \bar{c}_{v'}\,\gamma_\mu\, b_v \left|\bar{B}(v)\right\rangle &= \xi(w)\,[v_\mu + v'_\mu], \\
\left\langle D^*(v',\epsilon)\right| \bar{c}_{v'}\,\gamma_\mu\gamma_5\, b_v \left|\bar{B}(v)\right\rangle &= -i\xi(w)\,[(1+w)\epsilon^*_\mu - (\epsilon^*\cdot v)v'_\mu], \\
\left\langle D^*(v',\epsilon)\right| \bar{c}_{v'}\,\gamma_\mu\, b_v \left|\bar{B}(v)\right\rangle &= \xi(w)\,\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^{\alpha}v^{\beta}.
\end{aligned}$$

Six form-factors in terms of one Isgur-Wise function

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w),$$

$$h_-(w) = h_{A_2}(w) = 0$$

Normalization of $\xi(1)$

Consider the forward matrix element of the vector current $\bar{b}\gamma^\mu b$ between $\bar{B}$ meson states. Setting $v' = v$, and letting $c \to b$, $D \to \bar{B}$,

$$\frac{\langle \bar{B}(p)|\bar{b}\gamma_\mu b|\bar{B}(p)\rangle}{m_B} = \langle \bar{B}(v)|\bar{b}_v\gamma_\mu b_v|\bar{B}(v)\rangle = 2\ \xi(w=1)\, v_\mu .$$

where ξ for $b \to b$ is the same as for $b \to c$.

So $\xi(1) = 1$.
This fixes the absolute normalization and allows one to determine V_{cb}.

Radiative Corrections

Can be computed in a systematic way.

$\bar{B} \to D^{(*)}$ at zero-recoil

$$\bar{c}\gamma_\mu b \;=\; C_1^{(V)}\bar{c}_{v'}\gamma_\mu b_v + C_2^{(V)}\bar{c}_{v'}v_\mu b_v + C_3^{(V)}\bar{c}_{v'}v'_\mu b_v$$

and

$$\bar{c}\gamma_\mu\gamma_5 b \;=\; C_1^{(A)}\bar{c}_{v'}\gamma_\mu\gamma_5 b_v + C_2^{(A)}\bar{c}_{v'}v_\mu\gamma_5 b_v + C_3^{(A)}\bar{c}_{v'}v'_\mu\gamma_5 b_v.$$

At the zero-recoil point:

$$\begin{aligned}\bar{c}\gamma_\mu b &= \eta_V\bar{c}_{v'}\gamma_\mu b_v\\ \bar{c}\gamma_\mu\gamma_5 b &= \eta_A\bar{c}_{v'}\gamma_\mu\gamma_5 b_v\end{aligned}$$

$$\eta_V = 1 + \frac{\alpha_s(\mu)}{\pi}\left[-2 + \left(\frac{m_b + m_c}{m_b - m_c}\right)\ln\left(\frac{m_b}{m_c}\right)\right].$$

$$\eta_A = 1 + \frac{\alpha_s(\mu)}{\pi}\left[-\frac{8}{3} + \frac{(m_b + m_c)}{(m_b - m_c)}\ln\left(\frac{m_b}{m_c}\right)\right].$$

(Known to two-loops)

$1/m$ Corrections

$$\mathcal{L} = \bar{h}_v \left(iv \cdot D\right) h_v + \frac{1}{2m_Q} \bar{h}_v \left(iD_\perp\right)^2 h_v - c_F \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v$$

$$c_F(\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{9/(33-2n_f)}$$

Reparameterization Invariance

UCSD

$$\begin{aligned} p_Q &= m_Q v + k, \\ v &\to v + \varepsilon/m_Q, \\ k &\to k - \varepsilon. \end{aligned}$$

Since $v^2 = 1$, $v \cdot \varepsilon = 0$.
Also $\not{v} Q_v = Q_v$ so the change in the field:

$$Q_v \to Q_v + \delta Q_v,$$

δQ_v satisfies

$$\left(\not{v} + \frac{\not{\varepsilon}}{m_Q} \right) (Q_v + \delta Q_v) = Q_v + \delta Q_v.$$

so that

$$(1 - \not{v})\delta Q_v = \frac{\not{\varepsilon}}{m_Q} Q_v.$$

One can choose:

$$\delta Q_v = \frac{\not{\varepsilon}}{2m_Q} Q_v.$$

[Not unique, one can always make field redefinitions]
L invariant under

$$\begin{aligned} v &\rightarrow v + \varepsilon/m_Q, \\ Q_v &\rightarrow e^{i\varepsilon\cdot x}\left(1 + \frac{\not{\varepsilon}}{2m_Q}\right) Q_v, \end{aligned}$$

$$\mathcal{L}_0 \rightarrow \mathcal{L}_0 + \frac{1}{m_Q}\bar{Q}_v \left(i\varepsilon \cdot D\right) Q_v,$$
$$\mathcal{L}_1 \rightarrow \mathcal{L}_1 - \frac{1}{m_Q}\bar{Q}_v \left(i\varepsilon \cdot D\right) Q_v.$$

so that the kinetic energy is not renormalized.

Other connections that follow form reparameterization invariance:

$$c_S = 2c_F - 1, \qquad \boldsymbol{\sigma} \cdot \nabla \times E$$

E.g. relates matching coefficients of leading order and $1/m$ operators, and their anomalous dimensions.

Luke's Theorem

Can compute $1/m$ corrections to meson form-factors. Two sources of $1/m$ corrections, those from the Lagrangian, and from the current. So one has

$$T\left(\mathcal{L}_1, J_0\right), \qquad J_1$$

where

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{m}\mathcal{L}_1 + \ldots, \qquad J = J_0 + \frac{1}{m}J_1 + \ldots$$

Luke's theorem: no $1/m$ corrections to the form-factor at zero recoil.

V_{cb}

Experimentally, measure $\bar{B} \to D^*$ which determines $|V_{cb}\mathcal{F}(1)|$.

$$\mathcal{F}(1) = \eta_A + 0 + \mathcal{O}\left(\frac{1}{m^2}\right)$$

$\eta_A = 0.96$, and $1/m^2 \approx -0.05$, so

$$\mathcal{F}(1) = 0.91 \pm 0.05$$

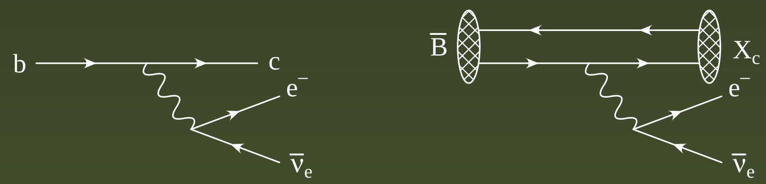
and from this one finds

$$|V_{cb}| = [\,38.6 \pm 1.5(\text{exp}) \pm 2.0(\text{th})\,] \times 10^{-3},$$

Inclusive Semileptonic Decay

Inclusive decay due to

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma^\mu P_L b\;\; \bar{e}\gamma_\mu P_L \nu_e.$$



The inclusive decay rate is

$$\frac{d\Gamma}{dq^2\, dE_e\, dE_{\nu_e}} = \int \frac{d^4 p_e}{(2\pi)^4} \int \frac{d^4 p_{\nu_e}}{(2\pi)^4}\; 2\pi\delta(p_e^2) 2\pi\delta(p_{\nu_e}^2)\theta(p_e^0)\theta(p_{\nu_e}^0)$$

$$\times \delta(E_e - p_e^0)\delta(E_{\nu_e} - p_{\nu_e}^0)\delta(q^2 - (p_e + p_{\nu_e})^2)$$

$$\times \sum_{X_c} \sum_{\substack{\text{lepton}\\ \text{spins}}} \frac{\left|\left\langle X_c e \bar{\nu}_e \right| H_W \left| \bar{B} \right\rangle\right|^2}{2m_B} (2\pi)^4 \delta^4(p_B - (p_e + p_{\nu_e}) - p_{X_c}),$$

Calculation similar to deep inelastic scattering. Do an operator product expansion in powers of $1/m_Q$ rather in powers of $1/Q$. [No twist here].

Leading term like free-quark decay

No $1/m_Q$ correction

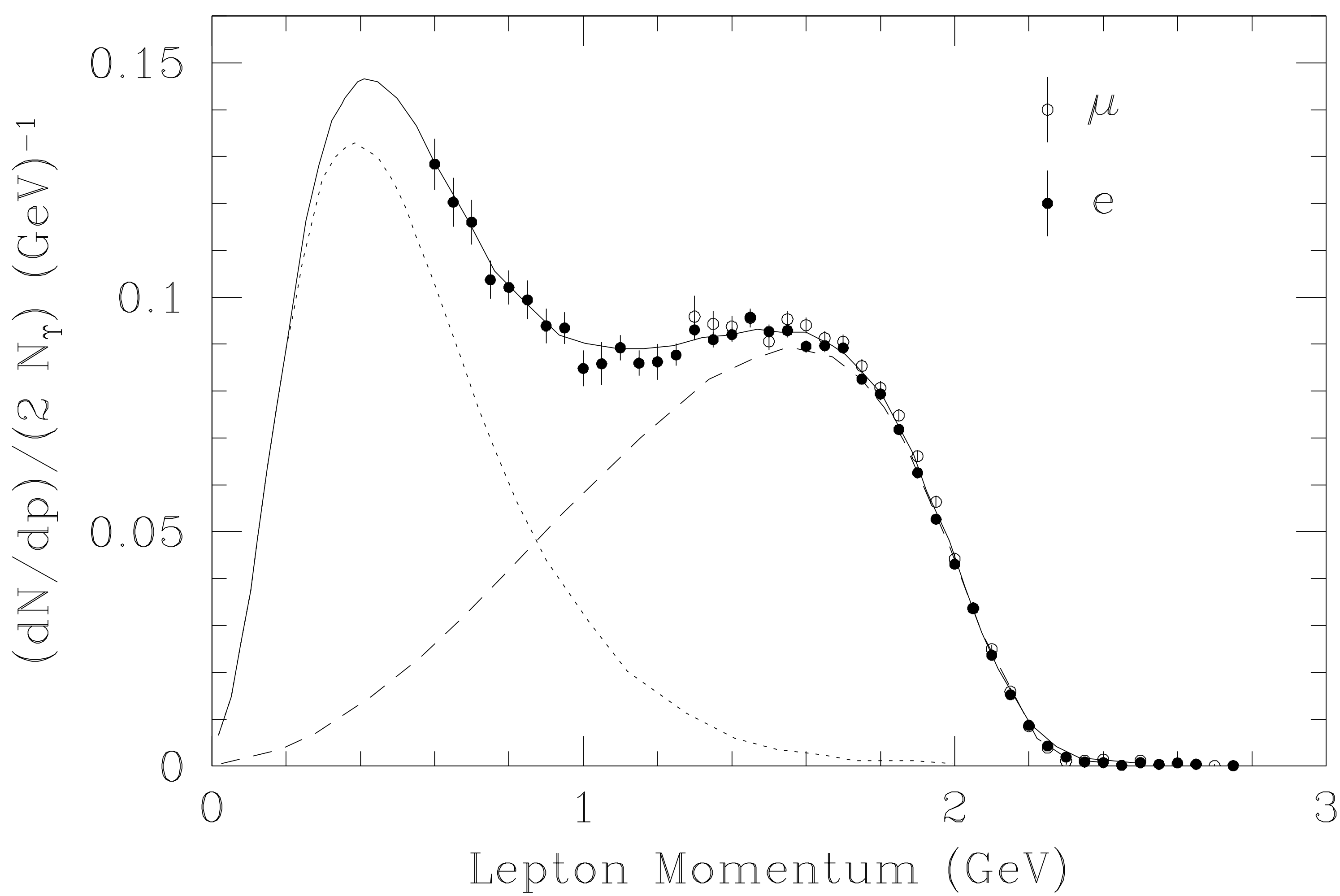
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[1 + \frac{\lambda_1}{2m_b^2} + \frac{3\lambda_2}{2m_b^2}\left(2\rho\frac{d}{d\rho} - 3\right)\right] f(\rho),$$

where

$$f(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho.$$

For $b \to u$:

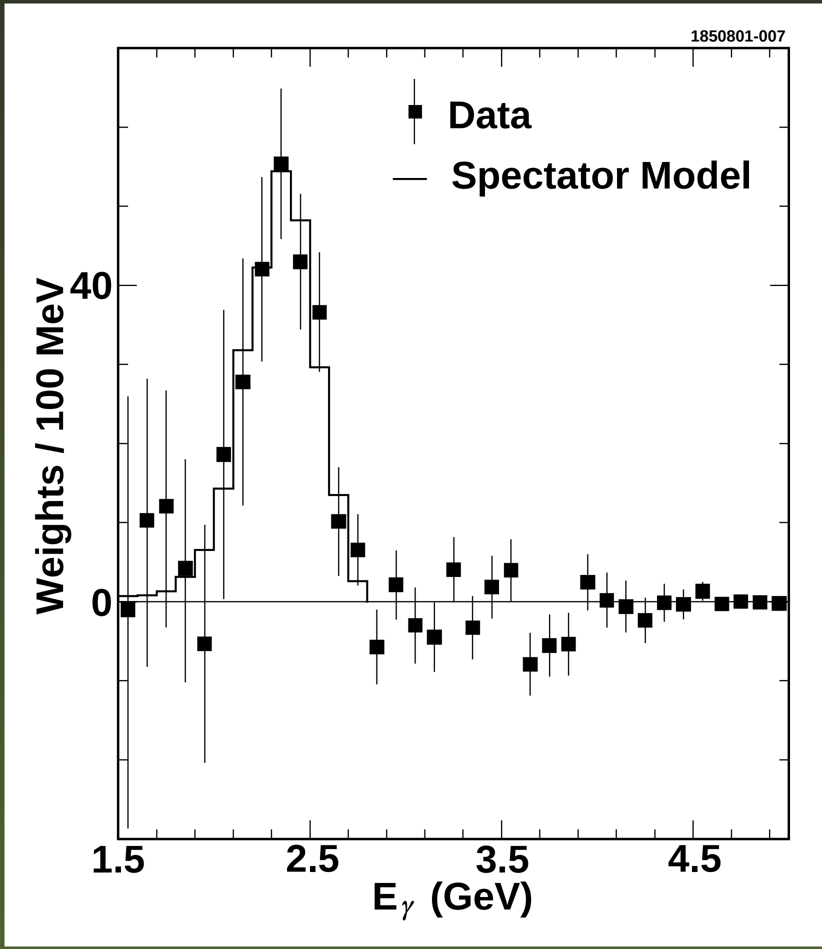
$$\Gamma = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3}\left[1 + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2}\right].$$



0.15
0.1
0.05
0
(dN/dp)/(2 N$_\Upsilon$) (GeV)$^{-1}$
0
1
2
3
Lepton Momentum (GeV)
μ
e

$b \to s\gamma$

CLEO $b \to s\gamma$ photon spectrum



- $\mathcal{B}(b \to s\gamma) = (3.21 \pm 0.43 \pm 0.27 \pm 0.15) \times 10^{-4}$
- $\langle E_\gamma \rangle = 2.346 \pm 0.04$ GeV
- $\left\langle E_\gamma^2 \right\rangle - \langle E_\gamma \rangle^2 = 0.0226 \pm 0.008$ GeV

Standard model BR is $(3.28 \pm 0.33) \times 10^{-4}$

Summary

A lot of high precision data to come.