Nuclear Physics Summer School
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Some Results-1

A new exact symmetry for baryons as $N_c \to \infty$

- A $SU(6)_c$ spin-flavor symmetry that connects the six states $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$, $s\uparrow$, $s\downarrow$.
- Baryons form an irrreducible representation of the spin-flavor algebra.
- Relates octet N, Λ, Σ, Ξ and decuplet $\Delta, \Sigma^*, \Xi^*, \Omega$.

Some Results-2

Can compute $1/N_c$ corrections in a systematic expansion, and the expansion is useful for $N_c = 3$

- $1/N_c$ corrections can be classified by their spin-flavor transformation properties
- Relations obtained to various orders in $1/N_c$. $1/N_c = 1/3$ factors evident in the experimental data.
- $1/N_c$ corrections comparable in size to $SU(3)$ breaking corrections due to m_s

Some Results-3

- Provides a deeper understanding of the success of quark models.
- Many results obtained in the nonrelativistic quark model, bag model, or Skyrme model, can be proven in QCD to order $1/N_c$ or $1/N_c^2$.
- $SU(6)_c$ is the underlying symmetry that relates quark models to each other and to QCD.

Some Results–4

Tells you how to consistently apply chiral perturbation theory to baryons

- N_c and Δ states have to be treated together
- Cancellations in chiral loops
- Form of $SU(3)$ symmetry breaking due to m_s is constrained by the $1/N_c$ expansion.
- Provides new insights into $SU(3)$ breaking.

Some Results-5

New predictions for heavy baryon properties

- Relates heavy quark baryons to the nucleon
- Compute masses and pion couplings of the Λ_c, Λ_b, etc Results are in good agreement with experiment.
- Can combine $1/N_c$ and $1/m_Q$ expansions

Some Results-6

New predictions for excited baryons
(Carlson, Carone, Goity, Schat, Lebed, Pirjol, Yan)

Nucleon Potential

- Explains the size of terms in the nucleon potential
- Gives Wigner supermultiplet symmetry in light nuclei

Large-N_c Baryons

$$\epsilon_{i_1 i_2 i_3 \cdots i_{N_c}} \; q^{i_1} q^{i_2} q^{i_3} \cdots q^{i_{N_c}}$$

bound state of N_c quarks
completely antisymmetric in color

completely symmetric in the quarks, since Fermi-statistics compensated by color antisymmetry.

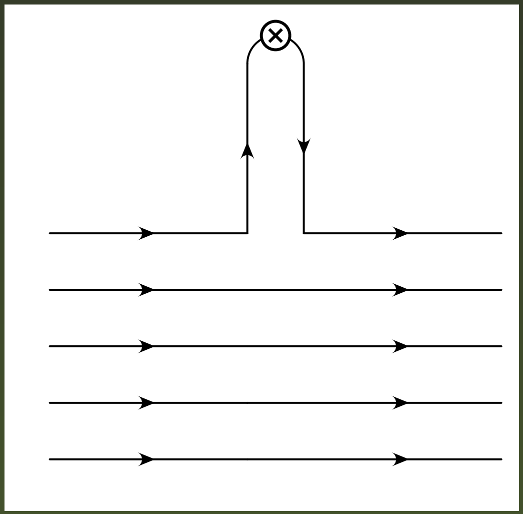
N_c Counting Rules for Baryons

Baryon is made of N_c quarks.

- Baryon mass is order N_c
- Baryon size is order $\Lambda_{\mathrm{QCD}}^{-1}$ (order one)
- Baryon-meson coupling is $\leq \sqrt{N_c}$
- Each extra meson costs $1/\sqrt{N_c}$
- One-body matrix element $\langle B|\,\bar{q}\,\Gamma\,q\,|B\rangle \leq N_c$
- Two-body matrix element $(\bar{q}\,\Gamma\,q\,\bar{q}\,\Gamma\,q) \leq N_c^2$

Baryon-Meson Couplings

baryon-meson vertex $\sim O\left(\sqrt{N_c}\right)$



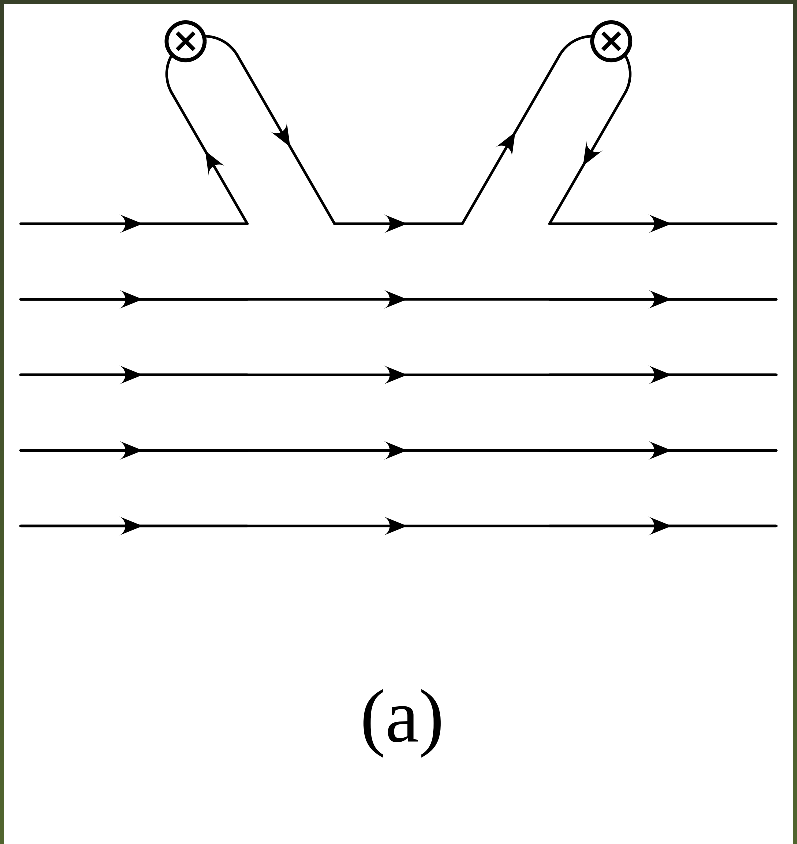
$$N_c\left(\frac{1}{\sqrt{N_c}}\right)$$

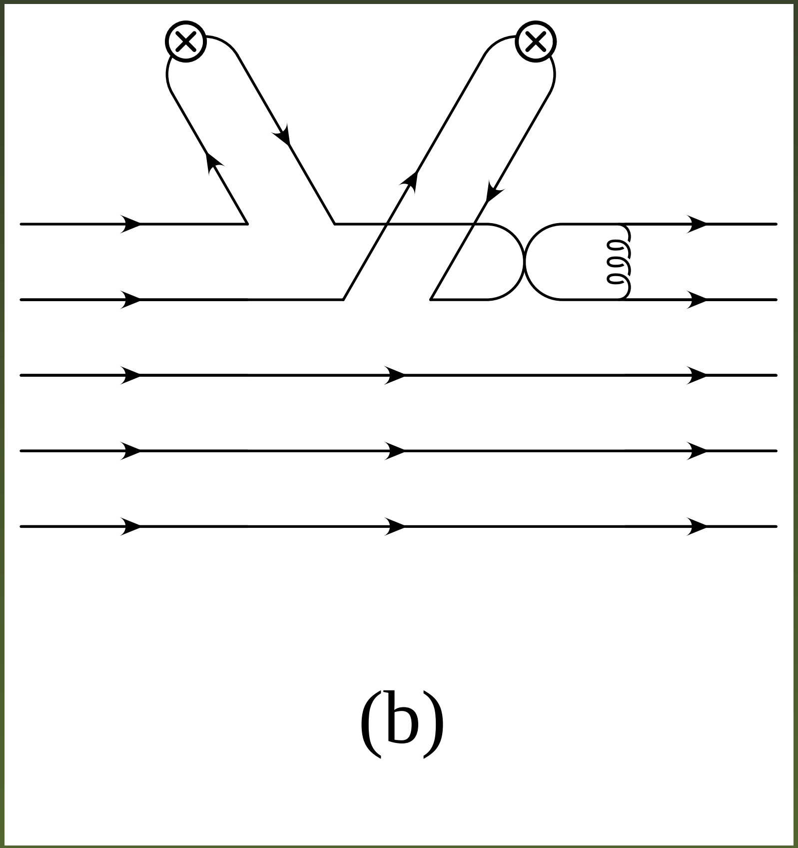
$\otimes = \bar{q}q/\sqrt{N_c}$ creates a meson with unit amplitude

Baryon-Meson Scattering

baryon $+$ meson $\rightarrow$ baryon $+$ meson $\sim O(1)$

$$N_c \left(\frac{1}{\sqrt{N_c}}\right)^2 \qquad\qquad N_c^2 \left(\frac{1}{\sqrt{N_c}}\right)^2 \left(\frac{1}{N_c}\right)$$





Baryon-Pion Scattering

- $M_{\text{baryon}} \sim O(N_c)$, so baryon acts as heavy static source
- Baryon propagator

$$\frac{i\,(\not{P} + M)}{P^2 - M^2} \rightarrow \frac{i}{k \cdot v}\left(\frac{1 + \not{v}}{2}\right) \rightarrow \frac{i}{E}$$

[Not only for pions. Argument is cleanest in this case.]

- $BB'\pi$ vertex $\sim O(\sqrt{N_c})$

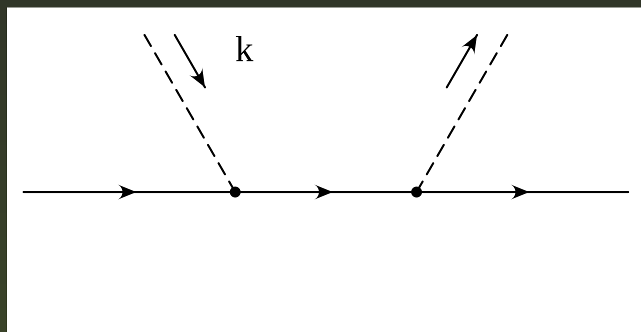
$$\frac{\partial_\mu \pi^a}{f_\pi} \left(A^{\mu a}\right)_{B'B}$$

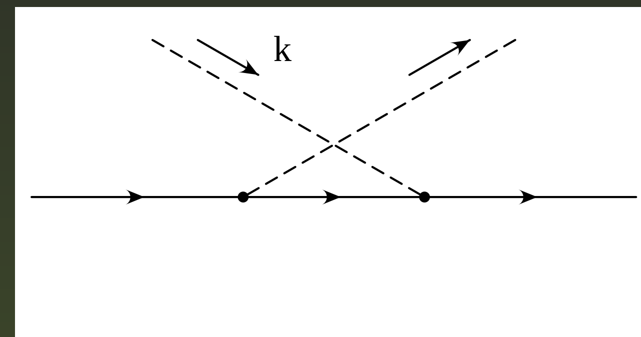
$$\left(A^{\mu a}\right)_{B'B} = \langle B' | \bar{q} \gamma^\mu \gamma_5 \tau^a q | B \rangle \sim O(N_c)$$

- $N_c \to \infty$ limit

$$\frac{\partial^i \pi^a}{f_\pi} \left(A^{ia}\right)_{B'B}$$

$$A^{ia} \equiv g N_c X^{ia}$$





$$N_c \left[X^{ia}, X^{jb}\right] \leq O(1)$$

$$X^{ia} = X_0^{ia} + \frac{1}{N_c} X_1^{ia} + \frac{1}{N_c^2} X_2^{ia} + \dots$$

$$\left[X_0^{ia}, X_0^{jb}\right] = 0$$

Spin-Flavor Symmetry

- Consistency conditions for scattering of low-energy pions with baryons at large-N_c leads to derivation of contracted spin-flavor symmetry for baryons
- Consistency of large-N_c power counting rules for baryon-meson scattering amplitudes and vertices leads to non-trivial constraints on $1/N_c$ corrections to large-N_c baryon matrix elements

Contracted Spin-Flavor Algebra

$$\left[J^i, I^a\right] = 0,$$

$$\left[J^i, J^j\right] = i\epsilon^{ijk} J^k, \qquad \left[I^a, I^b\right] = i\epsilon^{abc} I^c,$$

$$\left[J^i, X_0^{ja}\right] = i\epsilon^{ijk} X_0^{ka}, \quad \left[I^a, X_0^{ib}\right] = i\epsilon^{abc} X_0^{ic},$$

$$\boxed{\left[X_0^{ia}, X_0^{jb}\right] = 0}$$

Induced Representations

So the starting point is to work out the irreducible representations of the contracted symmetry, and then classify the $1/N_c$ corrections.

Standard theory of induced representations (e.g. Mackey) gives the Skyrme model.

Infinite dimensional unitary representations

Large-N_c Skyrme Model

- Simple understanding of spin-flavor generator X_0^{ia} as collective coordinate

$$X_0^{ia} = \mathrm{tr}\, A\tau^i A^{-1}\tau^a$$

- Contracted spin-flavor symmetry for baryons in $N_c \to \infty$ limit realized exactly since

$$\left[X_0^{ia}, X_0^{bj}\right] = 0.$$

Quark Model

The SU(6) generators are

$$\begin{aligned} J^i &= q^\dagger \, \frac{\sigma^i}{2} \, q \\ T^a &= q^\dagger \, \frac{\tau^a}{2} \, q \\ G^{ia} &= q^\dagger \, \frac{\sigma^i}{2} \frac{\tau^a}{2} \, q \end{aligned}$$

with $\left[q, q^\dagger\right] = 1$.

This gives

$$\begin{aligned}
[J^i, G^{jb}] &= i\epsilon^{ijk}\ G^{kb} \\
[I^a, G^{jb}] &= i\epsilon^{abc}\ G^{jb} \\
[G^{ia}, G^{jb}] &= \frac{i}{4}\epsilon^{ijk}\delta^{ab}\ J^k + \frac{i}{4}\delta^{ij}\epsilon^{abc}\ T^c
\end{aligned}$$

Let

$$G^{ia} = N_c X^{ia},$$

and take the limit $N_c \to \infty$. This reduces to the QCD symmetry derived earlier.

$N_F = 2$

$$J = I = \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \cdots,\ \frac{N_c}{2}$$

$$N, \Delta, \ldots$$

An infinite tower of degenerate states as $N_c \to \infty$.

Known that the Skyrme and Quark models were equivalent as $N_c \to \infty$. By explicit calculation, and by a trick.

$J = \frac{1}{2}$ $SU(3)_F$ rep

$$
\begin{array}{ccccccccccccccccccc}
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 & & & & & 1 & & 2 & & 2 & & 2 & & 1 & & & & & \\
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 & 1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 & \\
1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 \\
 & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 &
\end{array}
$$

$\longleftarrow \frac{1}{2}\left(N_c + 1\right)$ weights $\longrightarrow$

$J = \frac{3}{2}$ $SU(3)_F$ **rep**

$$
\begin{array}{ccccccccccccccccccc}
 & & & & & & 1 & & 1 & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 2 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 3 & & 3 & & 2 & & 1 & & & & \\
 & & & 1 & & 2 & & 3 & & 4 & & 3 & & 2 & & 1 & & & \\
 & & 1 & & 2 & & 3 & & 4 & & 4 & & 3 & & 2 & & 1 & & \\
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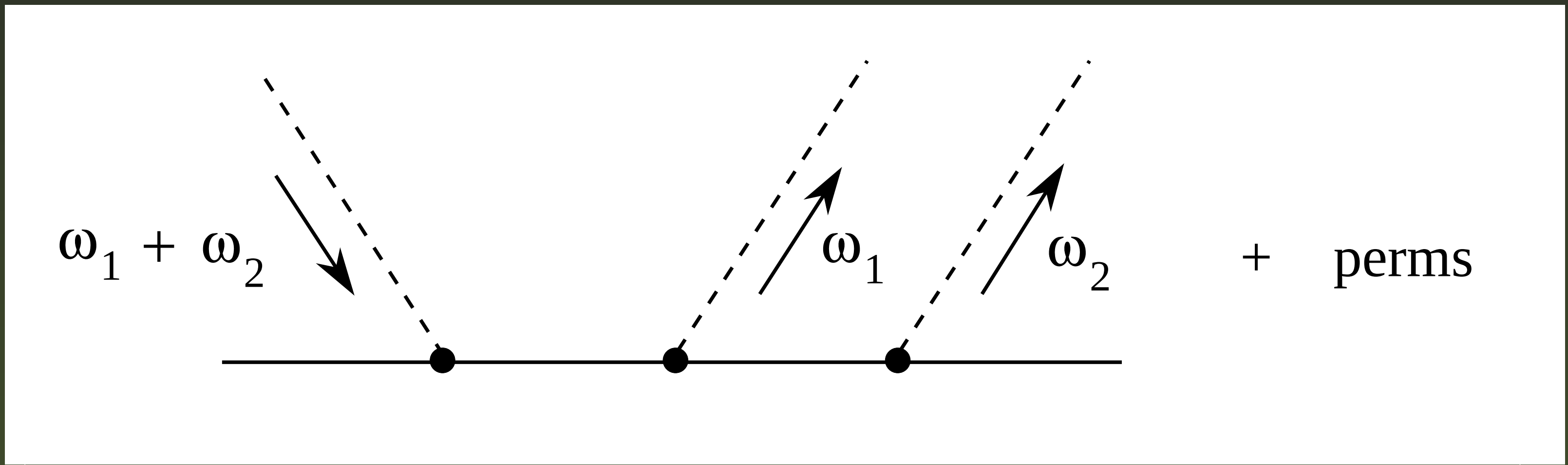
1 & & 2 & & 3 & & 4 & & 4 & & 4 & & 4 & & 3 & & 2 & & 1 \\
 & 1 & & 2 & & 3 & & 3 & & 3 & & 3 & & 3 & & 2 & & 1 & \\
 & & 1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 & & \\
 & & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & &
\end{array}
$$

$$\longleftarrow \tfrac{1}{2}\left(N_c - 1\right) \longrightarrow$$

$1/N_c$ CORRECTIONS



$$\text{Graph} \quad \propto \quad N_c^{3/2}\left[X^{ia},\left[X^{jb},X^{kc}\right]\right]$$
$$+\ N_c^{3/2}\left[X^{ia},\left[X^{jb},\left[X^{kc},\Delta M\right]\right]\right]$$

Feynman diagrams give multiple commutators

Baryon propagator

$$\frac{i}{E - \Delta M} \to \frac{i}{E}\left(1 + \frac{\Delta M}{E} + \cdots\right)$$

Expand the vertex in $1/N_c$

$$X = X_0 + \frac{1}{N_c} X_1 + \frac{1}{N_c^2} X_2 + \ldots$$

Find

$$\left[X_0^{ia}, \left[X_0^{jb}, X_1^{kc}\right]\right] + \left[X_0^{ia}, \left[X_1^{jb}, X_0^{kc}\right]\right] \;=\; 0$$

$$\left[X_0^{ia}, \left[X_0^{jb}, \left[X_0^{kc}, \Delta M\right]\right]\right] \;=\; 0$$

Results (two flavors)

$$\begin{aligned} X_1 &\propto X_0 \\ \Delta M &\propto \frac{J^2}{N_c} \end{aligned}$$

$$X = X_0 + \frac{1}{N_c} X_1 + \frac{1}{N_c^2} X_2 + \dots$$

No $1/N_c$ corrections to ratio of pion couplings such as $g_{\pi NN}/g_{\pi N\Delta}$.

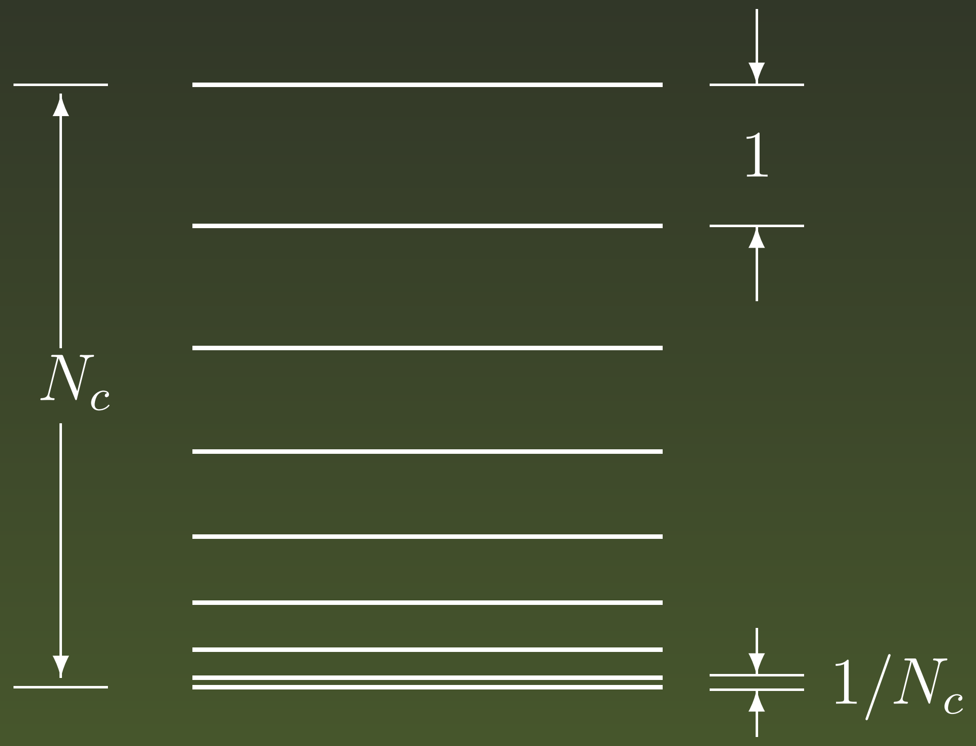
Pion Couplings

Explains why the Skyrme/Quark model predictions work well for the ratios, but not for the absolute values. The ratios are the same as in QCD to order $1/N_c^2$.

	Theory	Experiment
$g_{\pi N\Delta}$	13.2	20.3
$g_{\pi NN}$	8.9	13.5
$g_{\pi N\Delta}/g_{\pi NN}$	1.48	1.5

(From Adkins, Nappi, Witten)

Hyperfine Mass Splittings



The $1/N_c$ corrections are small only in a part of the irreducible representation.

Form of $1/N_c$ Expansion ($N_F = 2$)

$$N_c^{?}\ \mathcal{P}\left(X_0^{ia}, \frac{J^i}{N_c}, \frac{I^a}{N_c}\right)$$

Or one can use

$$N_c^{?}\ \mathcal{P}\left(\frac{G^{ia}}{N_c}, \frac{J^i}{N_c}, \frac{I^a}{N_c}\right)$$

Two equivalent representations.

Form of $1/N_c$ Expansion ($N_F = 3$)

$N_F = 3$ using isospin flavor symmetry only

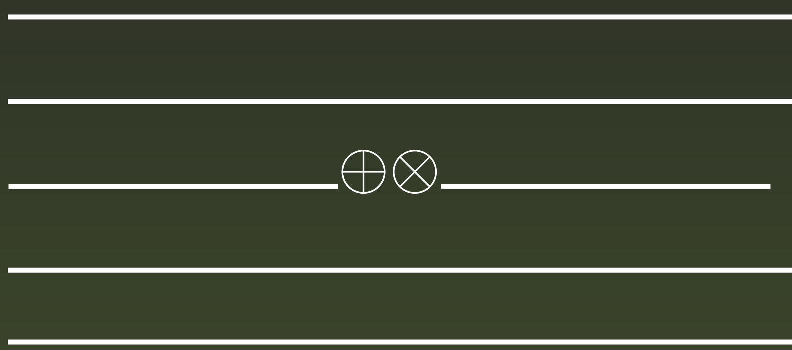
$$N_c \, \mathcal{P}\left(X_0^{ia}, \frac{J^i}{N_c}, \frac{I^a}{N_c}, \frac{S}{N_c} \right)$$

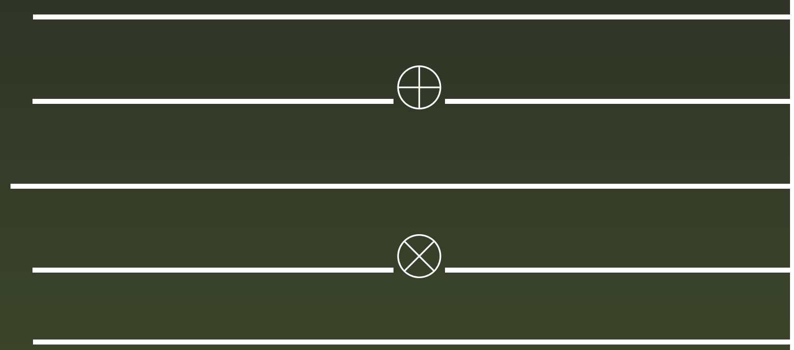
where S is the strangeness. Thus $1/N_c$ constrains the form of $SU(3)$ breaking.

n-body Quark Operators

- 0-body: $\mathbb{1}$
- 1-body:
$$J^i = q^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1} \right) q$$
$$I^a = q^\dagger \left(\mathbb{1} \otimes \frac{\tau^a}{2} \right) q$$
$$G^{ia} = q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\tau^a}{2} \right) q$$
$$N_c = q^\dagger q$$
- 2-body: $\{J^i, G^{ja}\}$

$$\vdots$$





$$G^{ia} = \sum_{\ell=1}^{N_c} q_\ell^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\tau^a}{2} \right) q_\ell, \qquad J^i I^a = \sum_{\ell,\ell'} \left(q_\ell^\dagger \frac{\sigma^i}{2} q_\ell \right) \left(q_{\ell'}^\dagger \frac{\tau^a}{2} q_{\ell'} \right)$$

Note that commutators reduce n-body $\longrightarrow$ $(n-1)$-body.

Operator Analysis

The general solution of the consistency conditions is to expand a given QCD quantity Q as

$$Q = N_c^? \mathcal{P}\left(\frac{G^{ia}}{N_c}, \frac{J^i}{N_c}, \frac{T^a}{N_c},\right)$$

where $\mathcal{P}$ is a polynomial.
Agrees with the digrammatic analysis: each extra quark needs a gluon-exchange $\Rightarrow 1/N_c$

$SU(3)_F$

$$
\begin{array}{c}
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \\
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1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1
\end{array}
$$

$$
\langle T^a \rangle \sim \begin{cases} O(1) & \text{a=1,2,3} \\ O(\sqrt{N_c}) & \text{a=4,5,6,7} \\ O(N_c) & \text{a=8} \end{cases}
\qquad
\langle G^{ia} \rangle \sim \begin{cases} O(N_c) & \text{a=1,2,3} \\ O(\sqrt{N_c}) & \text{a=4,5,6,7} \\ O(1) & \text{a=8} \end{cases}
$$

$SU(6)$ Operator Identities

$2\ \{J^i, J^i\} + 3\ \{T^a, T^a\} + 12\ \{G^{ia}, G^{ia}\} = 5N\,(N+6)$	$(0,0)$
$d^{abc}\ \{G^{ia}, G^{ib}\} + \frac{2}{3}\ \{J^i, G^{ic}\} + \frac{1}{4}\ d^{abc}\ \{T^a, T^b\} = \frac{2}{3}\,(N+3)\ T^c$	$(0,8)$
$\{T^a, G^{ia}\} = \frac{2}{3}\,(N+3)\ J^i$	$(1,0)$
$\frac{1}{3}\ \{J^k, T^c\} + d^{abc}\ \{T^a, G^{kb}\} - \epsilon^{ijk} f^{abc} \{G^{ia}, G^{jb}\} = \frac{4}{3}\,(N+3)\ G^{kc}$	$(1,8)$
$-12\ \{G^{ia}, G^{ia}\} + 27\ \{T^a, T^a\} - 32\ \{J^i, J^i\} = 0$	$(0,0)$
$d^{abc}\ \{G^{ia}, G^{ib}\} + \frac{9}{4}\ d^{abc}\ \{T^a, T^b\} - \frac{10}{3}\ \{J^i, G^{ic}\} = 0$	$(0,8)$
$4\ \{G^{ia}, G^{ib}\} = \{T^a, T^b\} \qquad (27)$	$(0,27)$
$\epsilon^{ijk}\ \{J^i, G^{jc}\} = f^{abc}\ \{T^a, G^{kb}\}$	$(1,8)$
$3\ d^{abc}\ \{T^a, G^{kb}\} = \{J^k, T^c\} - \epsilon^{ijk} f^{abc}\ \{G^{ia}, G^{jb}\}$	$(1,8)$
$\epsilon^{ijk}\ \{G^{ia}, G^{jb}\} = f^{acg} d^{bch}\ \{T^g, G^{kh}\} \qquad (10+\overline{10})$	$(1, 10+\overline{10})$
$3\ \{G^{ia}, G^{ja}\} = \{J^i, J^j\} \qquad (J=2)$	$(2,0)$
$3\ d^{abc}\ \{G^{ia}, G^{jb}\} = \{J^i, G^{jc}\} \qquad (J=2)$	$(2,8)$

Operator Reduction Rule $N_F = 3$

- All operators in which two flavor indices are contracted using δ^{ab}, d^{abc}, or f^{abc} or two spin indices on G's are contracted using δ^{ij} or ϵ^{ijk} can be eliminated.

Baryon Masses

- Combined expansion in $1/N_c$ and $SU(3)$ flavor symmetry breaking
- Flavor symmetry breaking expansion extends to 3^{rd} order in $SU(3)$ breaking
- For $N_c = 3$, only need to keep the expansion till 3-body operators

$$M = M^1 + M^8 + M^{27} + M^{64}$$

Jenkins & Lebed

Baryon Masses $N_F = 3$

$$M^1 = N_c \mathbb{1} + \frac{1}{N_c} J^2$$

$$M^8 = T^8 + \frac{1}{N_c}\left\{J^i, G^{i8}\right\} + \frac{1}{N_c^2}\left\{J^2, T^8\right\}$$

$$M^{27} = \frac{1}{N_c}\left\{T^8, T^8\right\} + \frac{1}{N_c^2}\left\{T^8, \left\{J^i, G^{i8}\right\}\right\}$$

$$M^{64} = \frac{1}{N_c^2}\left\{T^8, \left\{T^8, T^8\right\}\right\}$$

- 8 independent operators $\leftrightarrow$ 8 masses:
  N, Λ, Σ, Ξ, Δ, Σ^*, Ξ^*, Ω

Baryon Mass Hierarchy $N_F = 3$

- Unknown coefficient multiplies each operator: order in $1/N_c$ and in $SU(3)$ flavor breaking predicted
- $\mathbf{8}$, $\mathbf{27}$, $\mathbf{64}$ operators are first, second, third order in $SU(3)$ flavor breaking parameter $\epsilon \sim m_s/\Lambda_{\rm QCD} \sim 30\%$
- Order in $1/N_c$ given by explicit factor of $1/N_c$ times leading N_c-dependence of operator matrix element $\langle \mathcal{O} \rangle$

Baryon Mass Hierarchy $N_F = 3$

- Each operator contributes to unique linear combination of masses

$$\frac{J^2}{N_c}: \qquad \frac{1}{8}\left(2N + \Lambda + 3\Sigma + 2\Xi\right) - \frac{1}{10}\left(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega\right)$$

- Define dimensionless quantity

$$\frac{\sum B_i}{\sum |B_i|/2}$$

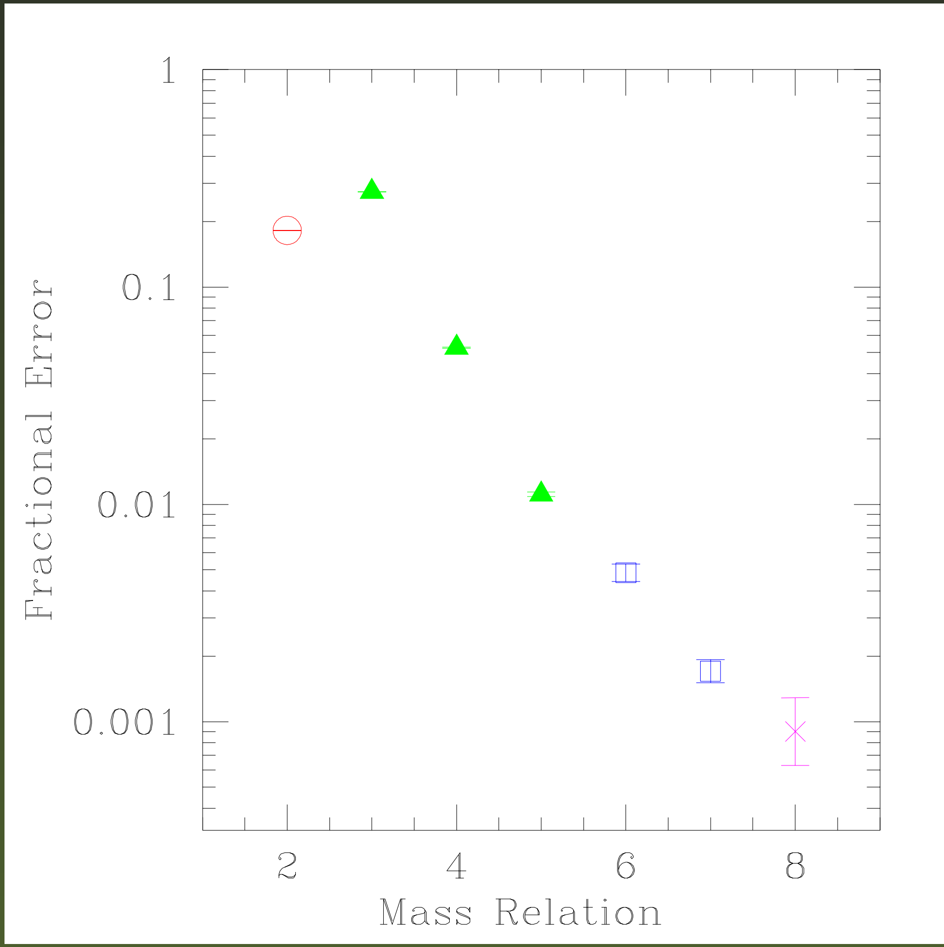
Theory: $1/N_c^2$ Expt: $(18.21 \pm 0.03)\%$

Baryon Mass Hierarchy

Mass Splitting	$1/N_c$	Flavor	Expt.
$\frac{5}{8}(2N+3\Sigma+\Lambda+2\Xi)-\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	N_c	1	*
$\frac{1}{8}(2N+3\Sigma+\Lambda+2\Xi)-\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	$1/N_c$	1	$18.21\pm0.03\%$
$\frac{5}{2}(6N-3\Sigma+\Lambda-4\Xi)-(2\Delta-\Xi^*-\Omega)$	1	ϵ	$20.21\pm0.02\%$
$\frac{1}{4}(N-3\Sigma+\Lambda+\Xi)$	$1/N_c$	ϵ	$5.94\pm0.01\%$
$\frac{1}{2}(-2N-9\Sigma+3\Lambda+8\Xi)+(2\Delta-\Xi^*-\Omega)$	$1/N_c^2$	ϵ	$1.11\pm0.02\%$
$\frac{5}{4}(2N-\Sigma-3\Lambda+2\Xi)-\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c$	ϵ^2	$0.37\pm0.01\%$
$\frac{1}{2}(2N-\Sigma-3\Lambda+2\Xi)-\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c^2$	ϵ^2	$0.17\pm0.02\%$
$\frac{1}{4}(\Delta-3\Sigma^*+3\Xi^*-\Omega)$	$1/N_c^2$	ϵ^3	$0.09\pm0.03\%$

(From Jenkins & Lebed)

Baryon Mass Hierarchy



$$\frac{1}{N_c^2} : \frac{\epsilon}{N_c} : \frac{\epsilon}{N_c^2} : \frac{\epsilon}{N_c^3} : \frac{\epsilon^2}{N_c^2} : \frac{\epsilon^2}{N_c^3} : \frac{\epsilon^3}{N_c^3}$$

Isospin Splittings

Jenkins and Lebed

Get relations that work to 0.1 MeV accuracy. Clear evidence for the $1/N$ hierarchy. Many relations cannot be tested because the baryon masses are not well-measured.

One prediction, that the Coleman-Glashow relation

$$(p-n)-\left(\Sigma^+-\Sigma^-\right)+\left(\Xi^0-\Xi^-\right)$$

should work more accurately, because it is of order $\epsilon\epsilon'/N_c$ has been confirmed recently due to a more accurate measurement of the Ξ^0 mass.

Baryon Axial Vector Couplings

Octet (B) and Decuplet (T) baryons have an interaction

$$
\begin{aligned}
&2D\ \mathrm{Tr}\ \bar{B}S^\mu \left\{\mathcal{A}_\mu, B\right\} + 2F\ \mathrm{Tr}\ \bar{B}S^\mu \left[\mathcal{A}_\mu, B\right] \\
&+C\ \left(\bar{T}^\mu \mathcal{A}_\mu B + \bar{B}\mathcal{A}_\mu T^\mu\right) + 2H\ \bar{T}^\mu S^\nu \mathcal{A}_\nu T_\mu
\end{aligned}
$$

Large N_c predicts (to an accuracy $1/N_c^2$)

$$
F/D = 2/3, \qquad C = -2D, \qquad H = -3F\ .
$$

which agrees with experimental fit using chiral perturbation theory.

A more detailed analysis including $SU(3)$ breaking gives a good description of the data.
One result (to all orders in $SU(3)$ breaking) is that the pion coupling has the form

$$g = N_c \left(A + B \frac{S}{N_c} + \ldots \right)$$

$$\begin{aligned} g(\Sigma^* \to \Sigma\pi) - g(\Delta \to N\pi) &= g(\Xi^* \to \Xi\pi) - g(\Sigma^* \to \Sigma\pi) \\ g(\Sigma^* \to \Sigma\pi) &= g(\Sigma^* \to \Lambda\pi). \end{aligned}$$

Isovector Magnetic Moments

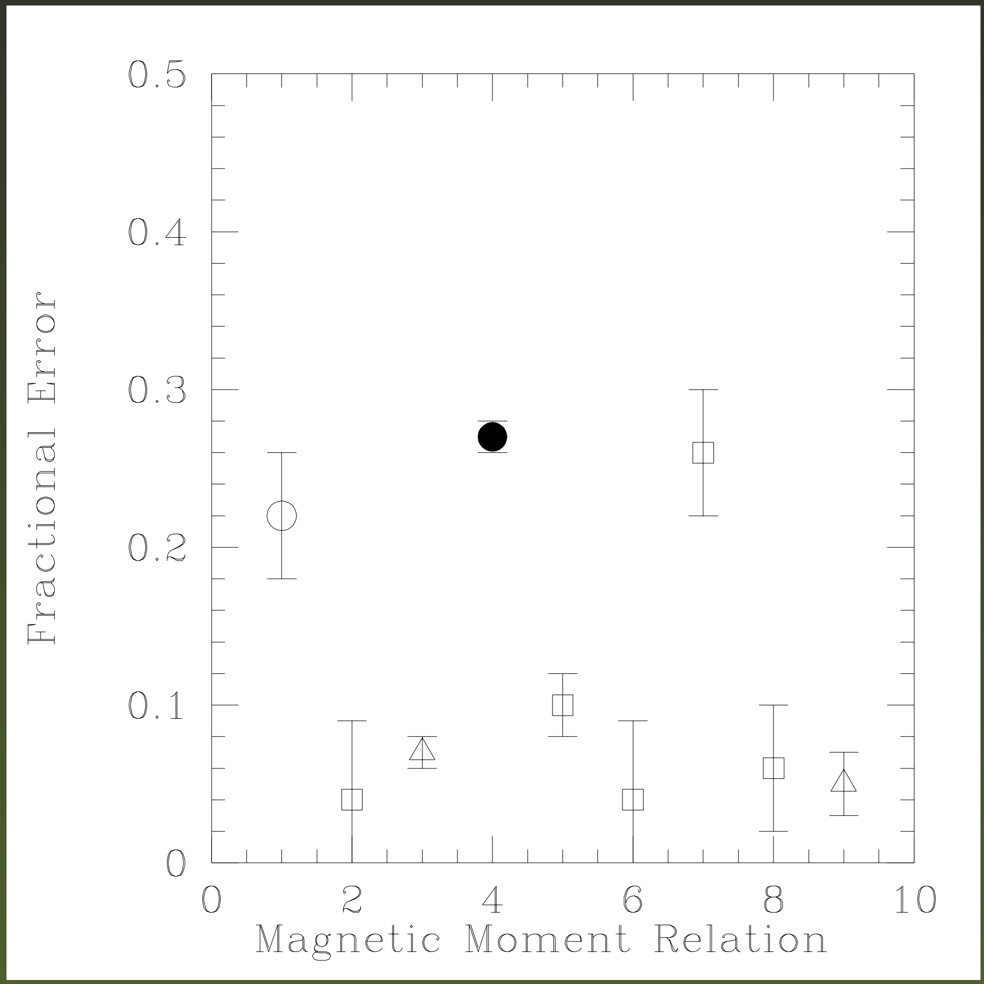
	Isovector	$1/N_c$	Flavor	Expt.
V1	$(p-n)-3(\Xi^0-\Xi^-)=2(\Sigma^+-\Sigma^-)$	$1/N_c$	–	$10\pm 2\%$
V2	$\Delta^{++}-\Delta^- = \frac{9}{5}(p-n)$	$1/N_c$	–	
V3	$\Lambda\Sigma^{*0} = -\sqrt{2}\Lambda\Sigma^0$	$1/N_c$	–	
V4	$\Sigma^{*+}-\Sigma^{*-} = \frac{3}{2}(\Sigma^+-\Sigma^-)$	$1/N_c$	–	
V5	$\Xi^{*0}-\Xi^{*-} = -3(\Xi^0-\Xi^-)$	$1/N_c$	–	
V6	$\sqrt{2}(\Sigma\Sigma^{*+}-\Sigma\Sigma^{*-}) = (\Sigma^+-\Sigma^-)$	$1/N_c$	–	
V7	$\Xi\Xi^{*0}-\Xi\Xi^{*-} = -2\sqrt{2}(\Xi^0-\Xi^-)$	$1/N_c$	–	
V8	$-2\Lambda\Sigma^0 = (\Sigma^+-\Sigma^-)$	$1/N_c$	–	$11\pm 5\%$
V9	$p\Delta^+ + n\Delta^0 = \sqrt{2}(p-n)$	$1/N_c$	–	$3\pm 3\%$
V10$_1$	$(\Sigma^+-\Sigma^-) = (p-n)$	1	–	$27\pm 1\%$
V10$_2$	$(\Sigma^+-\Sigma^-) = \left(1-\frac{1}{N_c}\right)(p-n)$	1	ϵ	$13\pm 2\%$

Isoscalar Magnetic Moments

Isoscalar	$1/N_c$	Flavor	Expt.
$(p+n)-3(\Xi^0+\Xi^-)=-3\Lambda+\frac{3}{2}(\Sigma^++\Sigma^-)-\frac{4}{3}\Omega^-$	$1/N_c^2$	–	$4\pm5\%$
$\Delta^{++}+\Delta^-=3(p+n)$	$1/N_c^2$	–	
$\frac{2}{3}(\Xi^{*0}+\Xi^{*-})=\Lambda+\frac{3}{2}(\Sigma^++\Sigma^-)-(p+n)+(\Xi^0+\Xi^-)$	$1/N_c^2$	–	
$\Sigma^{*+}+\Sigma^{*-}=\frac{3}{2}(\Sigma^++\Sigma^-)+3\Lambda$	$1/N_c^2$	–	
$\frac{3}{\sqrt{2}}(\Sigma\Sigma^{*+}+\Sigma\Sigma^{*-})=3(\Sigma^++\Sigma^-)-(\Sigma^{*+}+\Sigma^{*-})$	$1/N_c^2$	–	
$\frac{3}{\sqrt{2}}(\Xi\Xi^{*0}+\Xi\Xi^{*-})=-3(\Xi^0+\Xi^-)+(\Xi^{*0}+\Xi^{*-})$	$1/N_c^2$	–	
$5(p+n)-(\Xi^0+\Xi^-)=4(\Sigma^++\Sigma^-)$	$1/N_c$	–	$22\pm4\%$
$(p+n)-3\Lambda=\frac{1}{2}(\Sigma^++\Sigma^-)-(\Xi^0+\Xi^-)$	$1/N_c$	ϵ	$7\pm1\%$

Additional Relations

Isoscalar/Isovector Relations	$1/N_c$	Flavor	Expt.
$(\Sigma^+ + \Sigma^-) - \frac{1}{2}(\Xi^0 + \Xi^-) = \frac{1}{2}(p+n) + 3\left(\frac{1}{N_c} - \frac{2}{N_c^2}\right)(p-n)$	1	ϵ	$10 \pm 3\%$
$\Delta^{++} = \frac{3}{2}(p+n) + \frac{9}{10}(p-n)$	$1/N_c^2$	$-$	$21 \pm 10\%$



$\bigcirc = 1/N_c,\ \square = 1/N_c^2,\ \triangle = \epsilon/N_c$

$\Delta \to N\gamma$

(Jenkins, Ji, AM)

Two helicity amplitudes, and one finds

$$\begin{aligned} \frac{A_{3/2}}{A_{1/2}} &= \sqrt{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right) \\ &= 1.73(1.89 \pm 0.10) \end{aligned}$$

Equivalently,

$$\begin{aligned} \frac{E2}{M1} &= \mathcal{O}\left(\frac{1}{N_c^2}\right) \\ &= -0.025 \pm 0.005 \end{aligned}$$

Nucleon-Nucleon Potential

D.B. Kaplan & AM

$$V_{NN} =$$
$$V_0^0 + V_\sigma^0 \sigma_1 \cdot \sigma_2 + V_{LS}^0 \mathbf{L} \cdot \mathbf{S} + V_T^0 S_{12} + V_Q^0 Q_{12}$$
$$+(V_0^1 + V_\sigma^1 \sigma_1 \cdot \sigma_2 + V_{LS}^1 \mathbf{L} \cdot \mathbf{S} + V_T^1 S_{12} + V_Q^1 Q_{12}) \tau_1 \cdot \tau_2$$

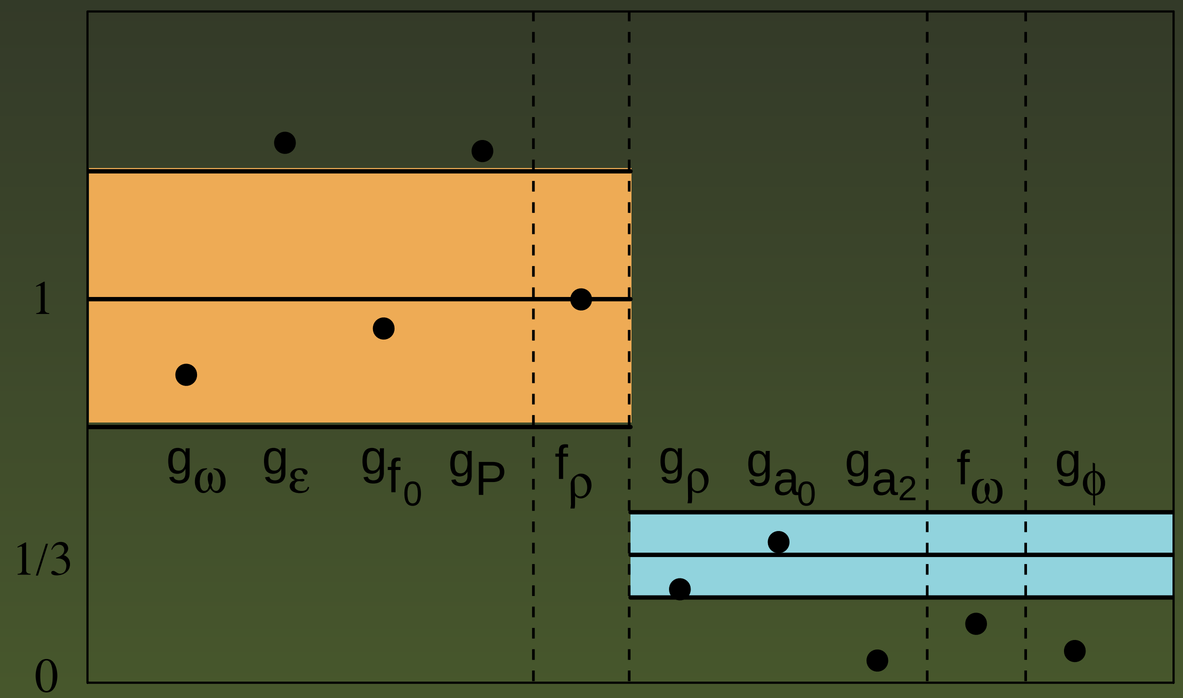
where

$$\begin{aligned} S_{12} &\equiv 3\sigma_1 \cdot \hat{\mathbf{r}}\ \sigma_2 \cdot \hat{\mathbf{r}} - \sigma_1 \cdot \sigma_2 \\ Q_{12} &= \frac{1}{2}\{(\sigma_1 \cdot \mathbf{L}), (\sigma_2 \cdot \mathbf{L})\} \end{aligned}$$

Isospin	V_0	V_σ	V_{LS}	V_T	V_Q
$\mathbf{1}\cdot\mathbf{1}$	N_c	$1/N_c$	$1/N_c$	$1/N_c$	$1/N_c^3$
$\tau_\mathbf{1}\cdot\tau_\mathbf{2}$	$1/N_c$	N_c	$1/N_c$	N_c	$1/N_c$

The potential in the large N_c limit has Wigner supermultiplet symmetry under which the $p\uparrow$ $p\downarrow$, $n\uparrow$, $n\downarrow$ transform as a $\mathbf{4}$ of $SU(4)$.

Fit to parameters in the Nijmegen potential



$I = J$ Rule

Mattis, Braaten

Mattis' $I = J$ Rule and its generalization:
Couplings are of order $N^{1-|I-J|/2}$.

e.g the ρ is $I = 1$, so the dominant ρ coupling is $J = 1$, i.e. magnetic moment-like (F_2 form factor). The ω has $I = 0$, and its dominant coupling is $J = 0$, i.e. charge-like (F_1).

For the ρ, $F_2/F_1 \sim 3$
For the ω, $F_1/F_2 \sim 3$.

Heavy Baryons

Jenkins

Form a $\overline{\mathbf{3}}$ Λ_Q and Ξ_Q and a $\mathbf{6}$, Σ_Q, Ξ'_Q, Ω_Q (and their spin-3/2 partners).

Heavy-quark hyperfine splittings $(\Sigma^*_Q - \Sigma_Q)$ are 150 MeV for the c, and 60 MeV for the b

Light-quark hyperfine splittings $(\Xi'_Q - \Xi_Q)$ are 150 MeV.

In this case, results before the measurements.

- Can relate the pion couplings of heavy baryons to those of the p up to corrections of order $1/N_c$ (rather than $1/N_c^2$).

- Obtain mass relations for heavy baryons, e.g.

$$\frac{1}{3}\left(\Sigma_Q + \Sigma_Q^*\right) = \frac{2}{3}\left(\Delta - N\right) + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

- The Ξ_c' mass was predicted to be 2580 ± 2.1 MeV before the measurement of 2576.5 ± 2.3 MeV.

- Predictions for other c baryon masses, and all the b baryon masses in terms of the Λ_b mass.

Conclusions

- $1/N_c$ expansion useful and predictive for QCD baryons, and most of the spin-flavor structure of baryons can be understood using the $1/N_c$ expansion.
- $1/N_c$ hierarchy evident in baryon masses, axial couplings and magnetic moments
- Intricate pattern of spin-flavor breaking since $1/N_c$ and $SU(3)$ breaking comparable. Restricts the form of $SU(3)$ breaking, and so is important in understanding baryon chiral perturbation theory
- Provides a unifying symmetry that connects QCD with various models such as the quark and Skyrme model.