Extracting Neutron Star Radii from the Tidal Deformability of GW170817

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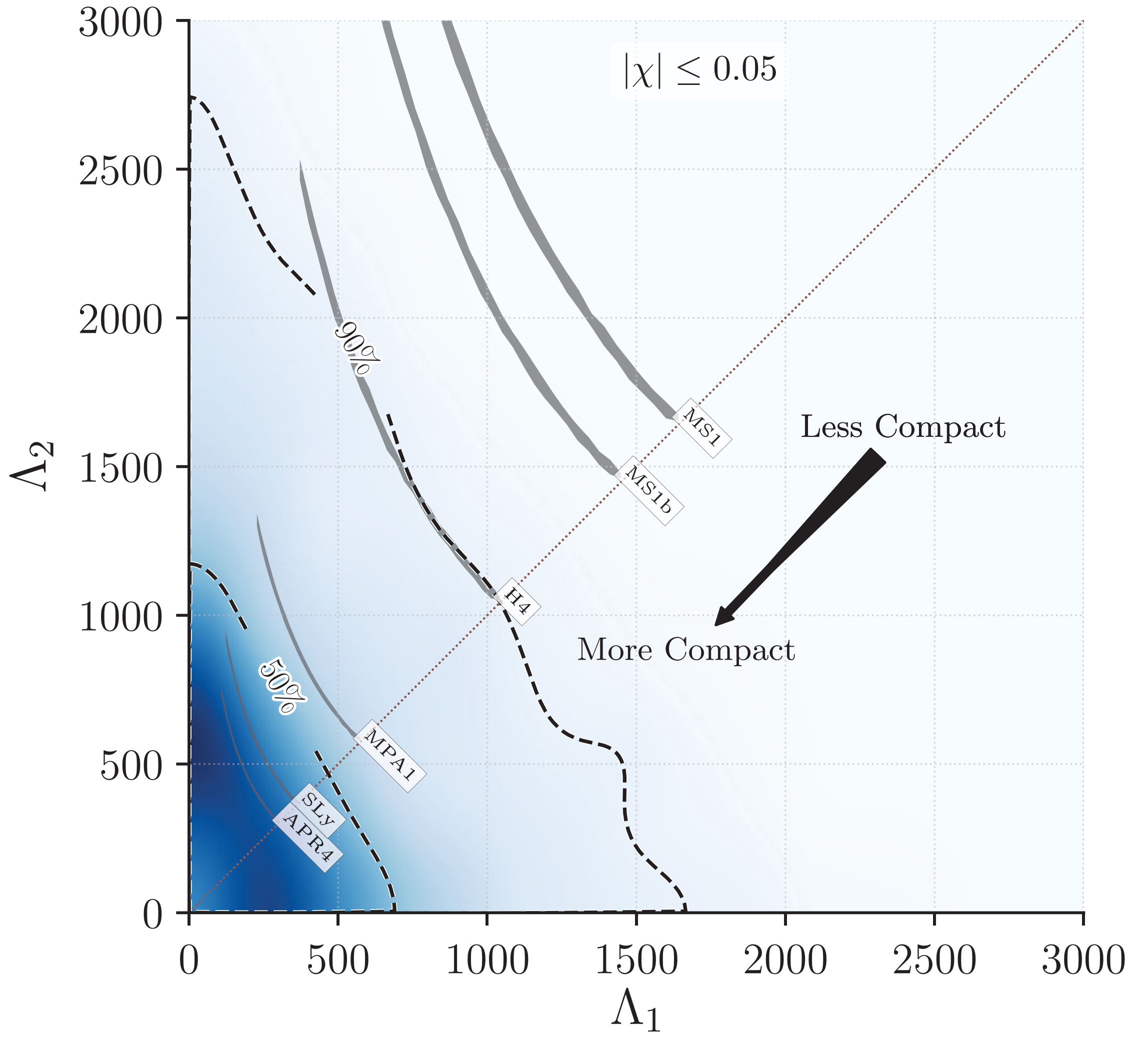
First measurement of $\tilde{\Lambda}$ from GW170817

- Constraint is on the combined, effective tidal deformability:

$$\tilde{\Lambda} < 800$$

- Measurement "disfavors EOS that predict less compact stars"



Abbott et al. 2017 (PRL)

What does $\tilde{\Lambda}$ measure?

$$\widetilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\,\Lambda_1 + (m_2 + 12m_1)m_2^4\,\Lambda_2}{(m_1 + m_2)^5}$$

- $\Lambda = \frac{2}{3}k_2\left(\frac{R}{m}\right)^5$
- k_2 depends on the EOS and compactness
- k_2 ~0.05-0.15 (Hinderer 2008; Hinderer et al. 2010; Postnikov et al. 2010)

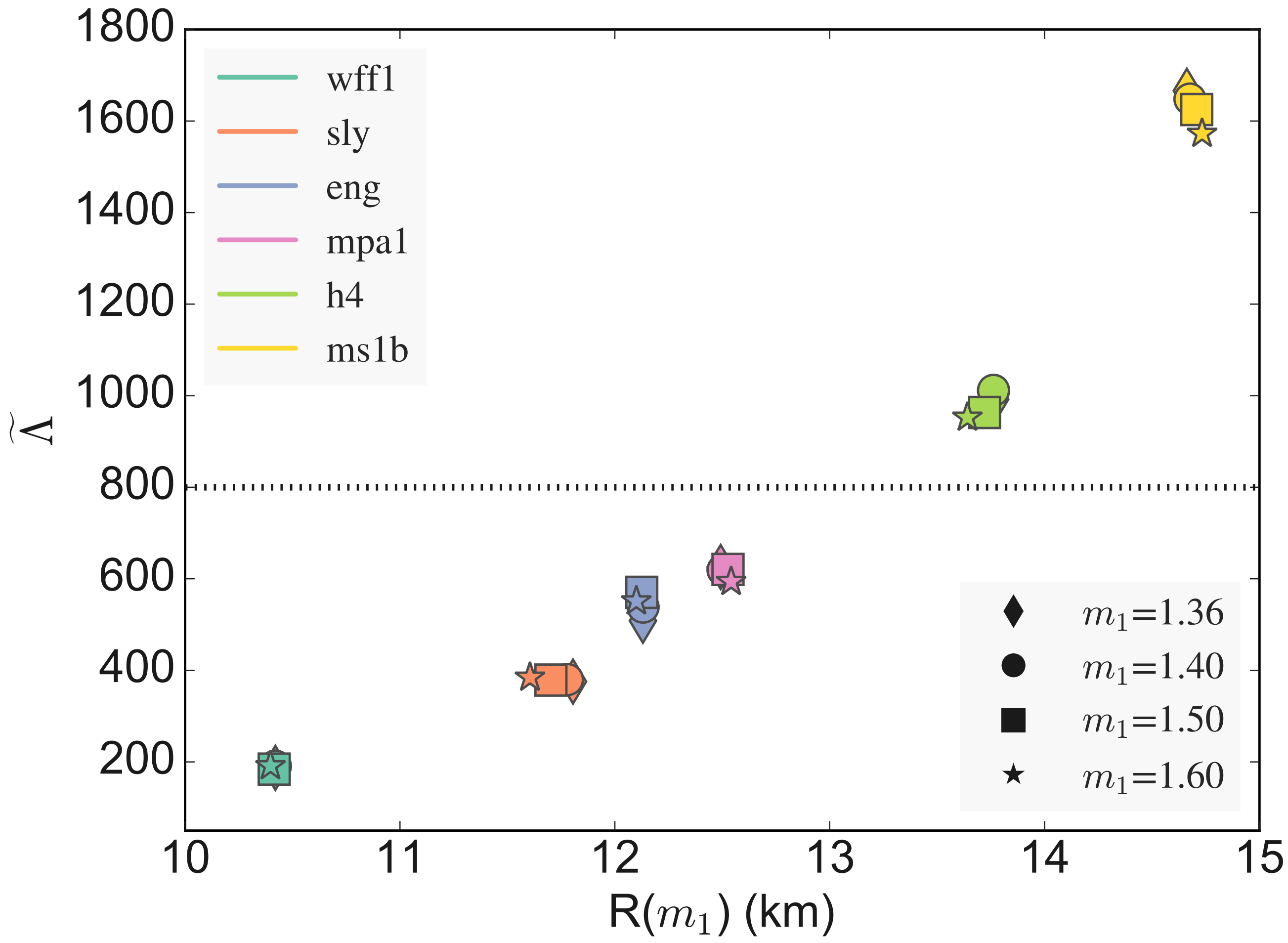
What does $\tilde{\Lambda}$ measure?

$$\widetilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\,(C_1)^5 + (m_2 + 12m_1)m_2^4\,(C_2)^5}{(m_1 + m_2)^5}\left(\frac{2}{3}k_2\right)$$

- Expectation: $\tilde{\Lambda}$ measures a mass-weighted compactness

Evidence of a universal relation between $\tilde{\Lambda}$ and R

- Λ_1, Λ_2 are calculated for a range of m_1 and m_2
- All combinations obey observed chirp mass, M_c=1.188 $M_\odot$
- GW170817 probes radius directly, not compactness!



Raithel, Özel, and Psaltis (in prep).

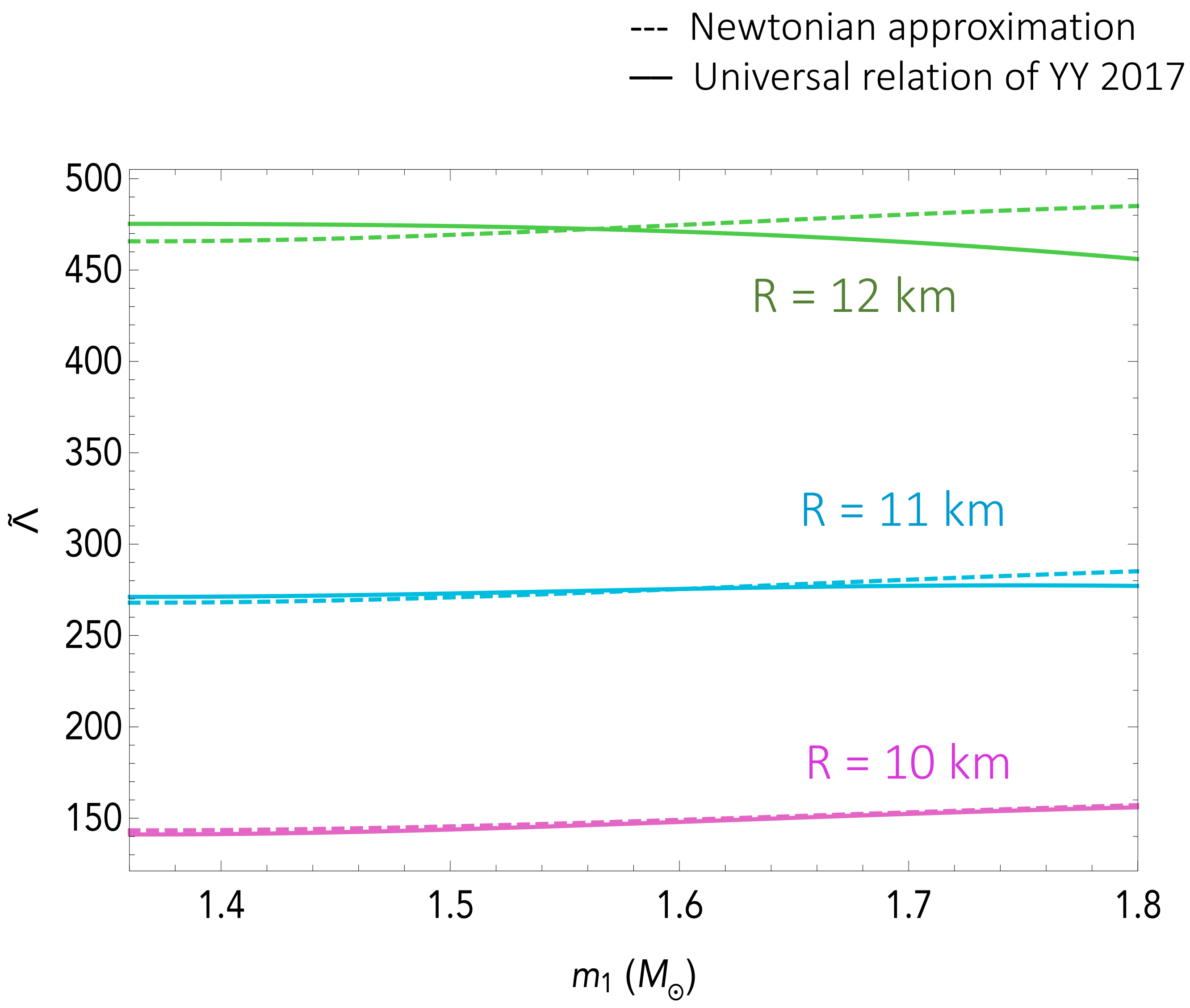
Analytic origin

- Use Newtonian expression for each star:

$$\Lambda_N = \frac{15-\pi^2}{3\pi^2}\left(\frac{Rc^2}{Gm}\sqrt{1-\frac{2Gm}{Rc^2}}\right)^5$$

- Or in terms of universal relations of YY 2017:

$$C = a_0 + a_1 \ln\Lambda + a_2(\ln\Lambda)^2$$



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Analytic origin

- Expand combined effective deformability, assuming q = (1-ε)

$$\tilde{\Lambda}_N = \frac{15-\pi^2}{3\pi^2}\xi^{-5}(1-2\xi)^{5/2} \times \left[1 - \frac{3}{108}(1-2\xi)^{-2}\left(10-94\xi+83\xi^2\right)\epsilon^2\right] + \mathcal{O}(\epsilon^3)$$

Coefficient

Deviation away from equal mass ratio

$$\xi = \frac{2^{1/5}GM_c}{Rc^2}$$

"Effective" compactness: Depends only on R

Expansion results

- Component mass dependence enters at $O(\varepsilon^2)$, and only as deviation away from equal mass ratio
- Dependence on mass even weaker for larger radii

TABLE 1

$\widetilde{\Lambda}_N$ EXPANSION TERMS FOR THE CHIRP MASS MEASURED FROM GW170817.

Radius	Coefficient	Expansion
$R = 10$ km	143.4	$1 + 0.045\left(\frac{1-q}{1-0.7}\right)^2 + \mathcal{O}\left(\frac{1-q}{1-0.7}\right)^3$
$R = 11$ km	268.0	$1 + 0.029\left(\frac{1-q}{1-0.7}\right)^2 + \mathcal{O}\left(\frac{1-q}{1-0.7}\right)^3$
$R = 12$ km	465.8	$1 + 0.020\left(\frac{1-q}{1-0.7}\right)^2 + \mathcal{O}\left(\frac{1-q}{1-0.7}\right)^3$
$R = 13$ km	764.6	$1 + 0.014\left(\frac{1-q}{1-0.7}\right)^2 + \mathcal{O}\left(\frac{1-q}{1-0.7}\right)^3$

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$\tilde{\Lambda}$ is a direct probe of NS radius!

How do we make further use of deformability measurements?

- To infer the underlying equation of state: need Bayesian inference
- We use parametric EOS, with 5 piecewise polytropes

$$P(\,EOS \mid \{M_c, \tilde{\Lambda}\}\,) \;=\; P_{pr}(\,P_1, \ldots, P_5\,)\; P(\,\{M_c, \tilde{\Lambda}\} \mid EOS)$$

5 piecewise polytropes

Priors

- Nuclear physics information
- Hydrostatic stability
- Causality
- $1.97\,M_\odot < M_{max} < 2.33\,M_\odot$
- $0.7 < q < 1.0$

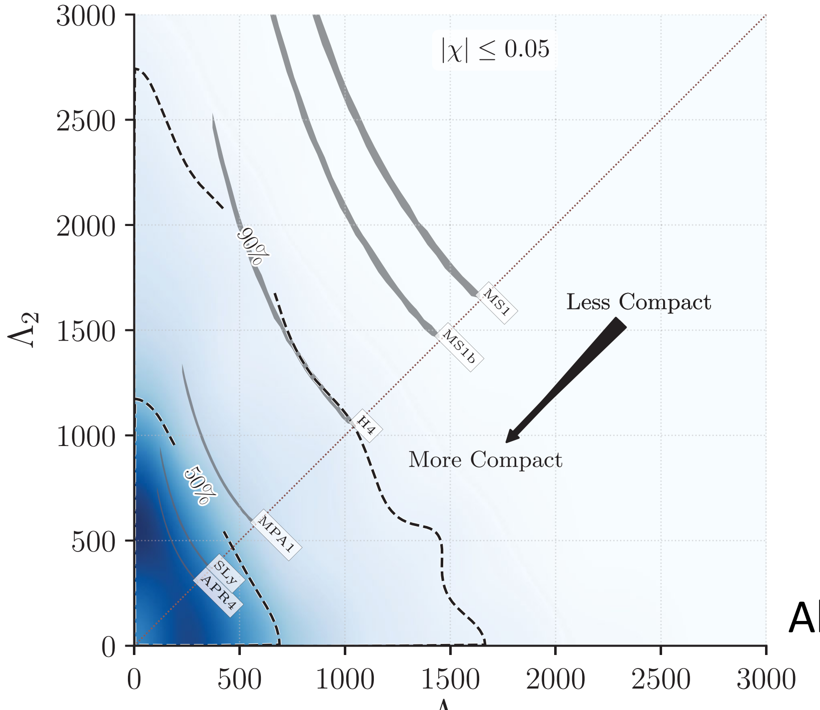
Rezzolla et al. 2018
Margalit & Metzger 2017

Example inferences

1. $\tilde{\Lambda}$ is a zero centered Gaussian, with σ = 490
2. $\tilde{\Lambda}$ is a Gaussian centered at 400 with σ = 240
3. No data at all (just priors)

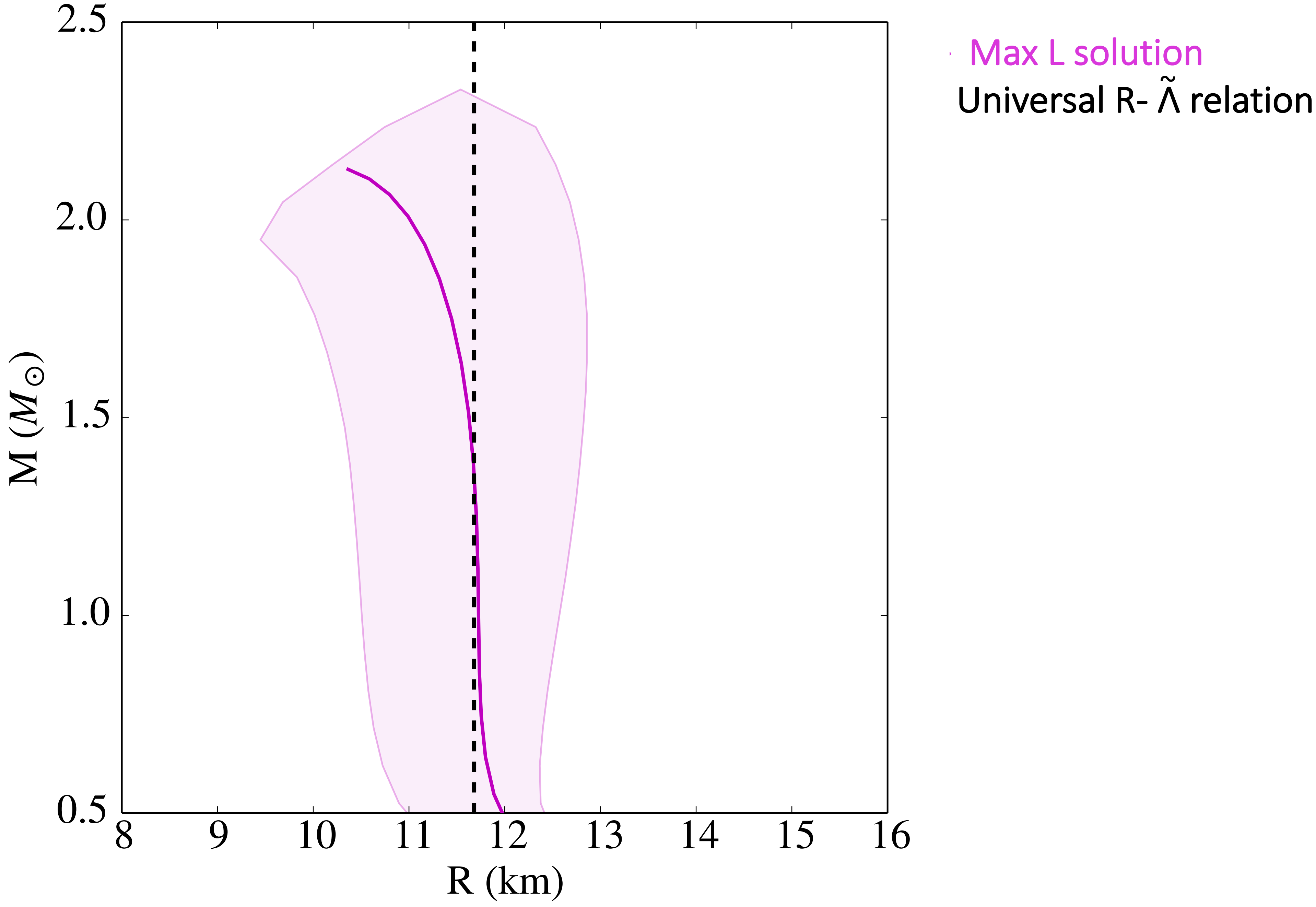
Both give 90% interval at $\tilde{\Lambda}$ = 800

Also use an asymmetric Gaussian for the chirp mass: M_c = $1.188^{+0.004}_{-0.002}$ $M_\odot$

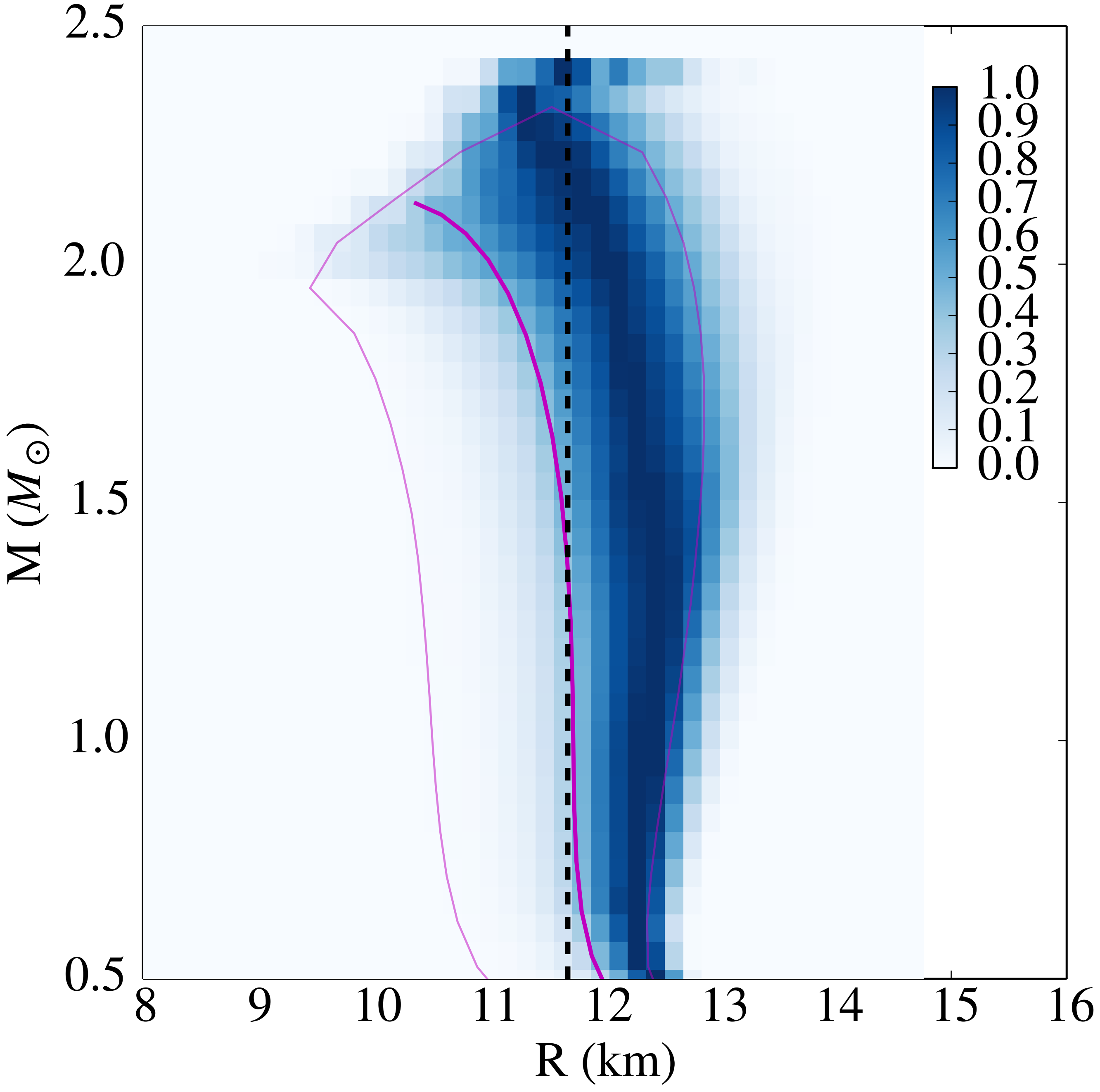


Abbott et al. 2017

$\tilde{\Lambda}$ = 400



$\tilde{\Lambda} = 400$



Max L solution

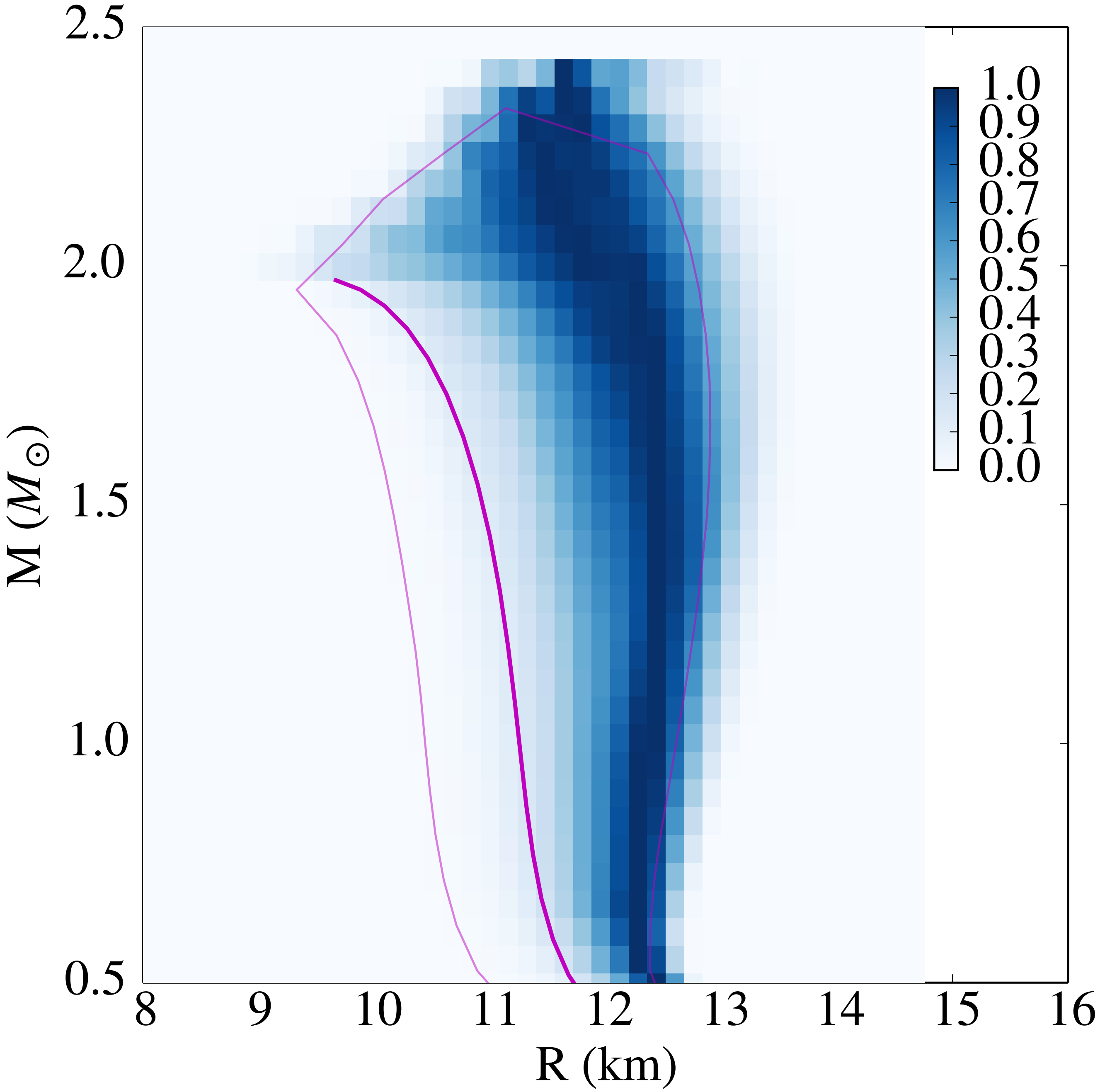
Universal R- $\tilde{\Lambda}$ relation

Marginalized results

Marginalized to see "density" of results

i.e., integrated the priors over mass-radius volume

$\tilde{\Lambda} = 0$



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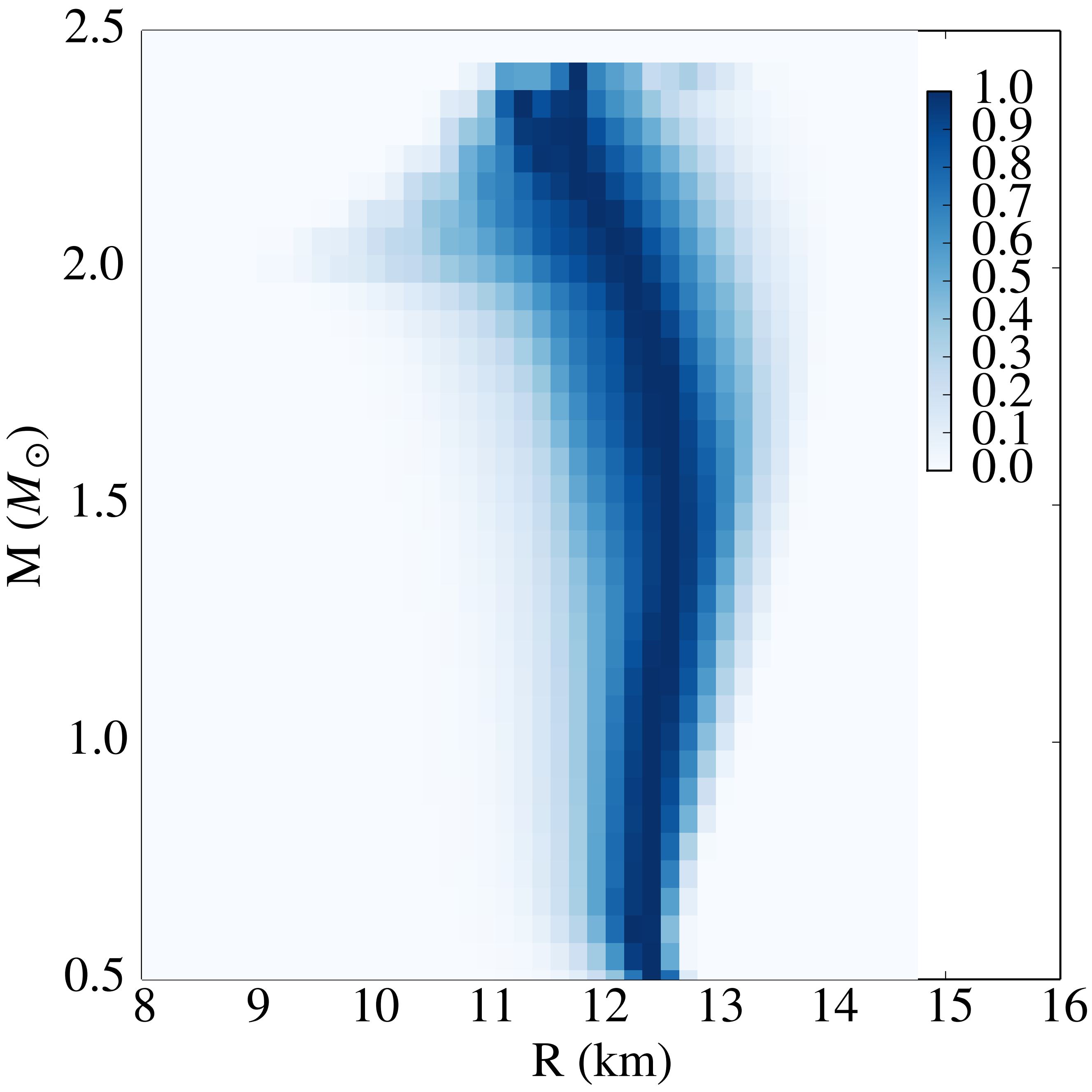

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Conclusions

- $\tilde{\Lambda}$ probes the radii of the neutron stars
- Full Bayesian inference is required to get the underlying EOS
- Additional measurements of $\tilde{\Lambda}$ will provide new constraints on the EOS