Dense Matter EOS for Mergers

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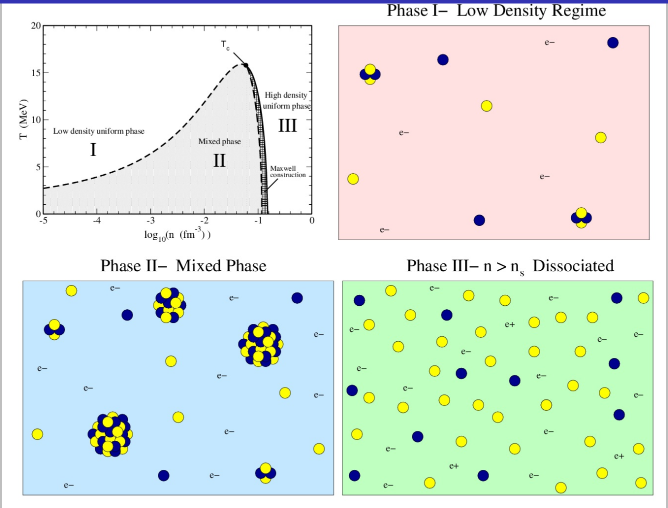
What astrophysical phenomena contend with

	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
n/n_s	10^{-8} - 10	10^{-8} - 10	10^{-8} - 10
$T(\mathrm{MeV})$	0 - 30	0 - 50	0 - 100
Y_e	0.35 - 0.45	0.01 - 0.3	0.01 - 0.6
$S(k_B)$	0.5 - 10	0 - 10	0 - 100

Table: Ranges of baryon number density n, temperature T, net electron fraction $Y_e = n_e/n$, and entropy per baryon S encountered in the indicated astrophysical phenomena. The nuclear equilibrium density $n_s \simeq 0.16\ \mathrm{fm}^{-3}$.

Phases of dense matter



The upper left figure shows the boundary separating the three phases illustrated in the other pictures. Figure courtesy Matthew Carmell.

Sweet and sour spots of the EOS approaches

Sub-nuclear density inhomogeneous phase ($n_b < 0.1\ \mathrm{fm}^{-3}$)

- Nuclear statistical equilibrium (NSE), single-nucleus approximation, full ensemble, virial expansion, and molecular dynamics, ...
- Matching of NSE results to others, Excluded volumes lack attractive interactions, fugacities exceed unity in the virial method, ...

Near-nuclear density homogeneous phase ($0.1 < n_b < 0.3\ \mathrm{fm}^{-3}$)

- Microscopic: (R)Brueckner-Hartree Fock, variational, Greens function Monte Carlo, chiral effective theory, ...
- Phenomenological: Nonrelativistic zero- or finite range potential models, relativistic mean field theory and extensions, ...
- Convergence of methods, extensions to the high density region problematic in some cases, ...

Supra-nuclear phase with or without phase transitions ($n_b > 0.3\ \mathrm{fm}^{-3}$)

- Non-nucleonic degrees of freedom, hybrid approaches for inclusion of quarks, acausality in NR approaches, ...

EOS for the post-merger remnant

Some questions:

- How are the mass and radii affected due to finite entropy, composition, trapped neutrinos, magnetic field, and rotation (rigid or differential)?
- What are the relevant relaxation times for deleptonization, cooling, rigidization of rotation, subsidence to a black hole, etc.?
- How would emission of gravity waves be influenced by the aforementioned effects?
- Can upgraded LIGO's detect gravity waves from post-merger remnants such as hypermassive neutron stars?

Clusters in the inhomogeneous phase

- For densities $n \leq 10^{-2}\ \mathrm{fm}^{-3}$, and temperatures T not exceeding their $B.E/A$, clusters of light nuclei, such as $\alpha, \mathrm{d}, \mathrm{t},$ *etc.*, are permitted in matter.
- The treatment of clusters is afforded by the viral expansion approach that includes bound and continuum states, and provides corrections to the ideal gas result. When applicable, this approach is model independent as experimental data where available is input to theory.

In terms of the partition function $\mathcal{Q}$, the pressure $P = \frac{T}{V} \log \mathcal{Q}$, and is expressed in terms of the fugacities $z_i = \exp(\mu_i/T)$ (i=N, d, α, *etc.*,) and the 2nd virial coefficients b_2 which are simple integrals involving thermal weights and elastic scattering phase shifts.

Sample references:

E. Beth and G. E. Uhlenbeck, Physica **4** (1937) 915

R. Venugopalan and M. Prakash, Nucl. Phys. A **546** (1992) 718

C. J. Horowitz and A. Schwenk, Nucl. Phys. A **776** (2006) 55

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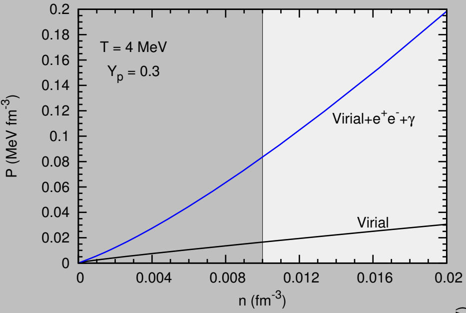
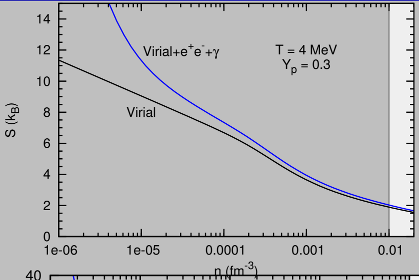
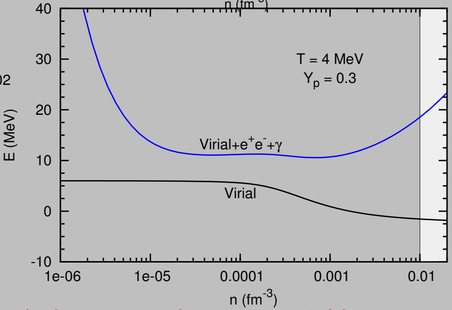
Clusters of light nuclei & their thermal properties



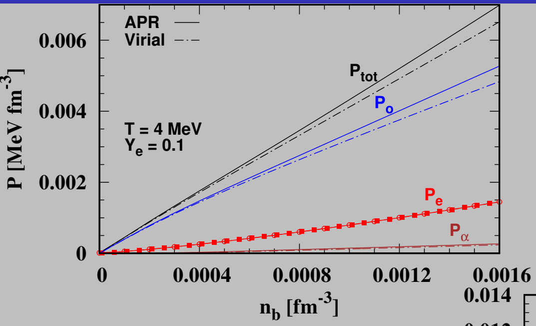




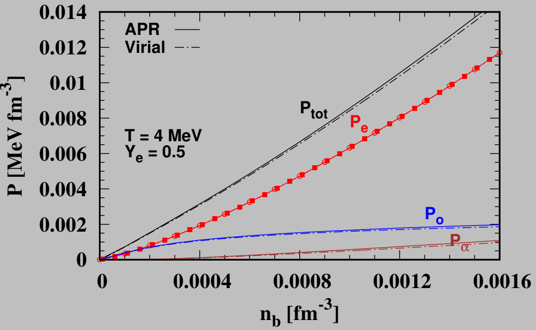
Leptons treated as ideal gases as contributions from interactions are smaller by $\mathcal{O}(\alpha \simeq 1/137)$

Note the contributions of leptons and photons to the state variables.

Virial vs APR

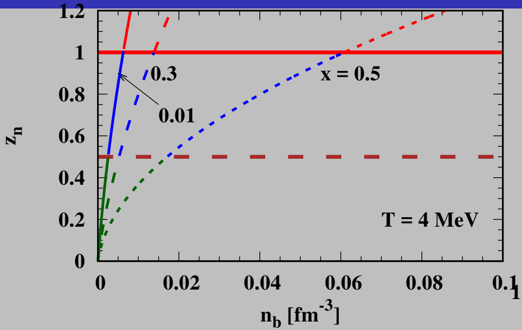


Results of APR are from the excluded volume approach as in Lattimer & Swesty.

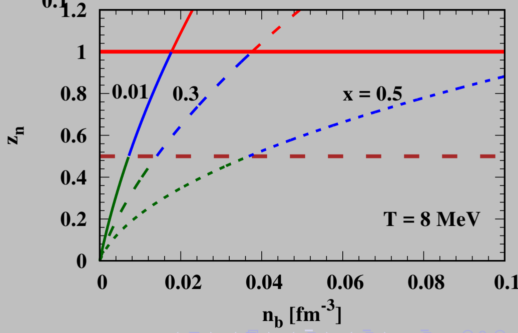


Discrepancy decreases with increasing T.

Limitations of the virial approach



Fugacities $z_i = \exp(\mu_i/T) > 1$ for many values of T, $x = n_p/n_b$ and n_b.



Development of alternative approaches (mean field, EFT, etc.) indicated and is in progress.

Effects of finite entropy on the structure of neutron stars

Neutrino-free beta-equilibrated nucleonic matter

Model	S	$\frac{M_{\rm max}}{M_\odot}$	R (km)	$\frac{n_c}{n_0}$	P_c $\left(\frac{\rm MeV}{\rm fm^{-3}}\right)$	T_c (MeV)	λ ($x10^2$)	I ($M_\odot$ km^2)
BPAL32	0	1.93	10.1	7.7	590.2	0		90.1
	2	1.97	10.9	6.9	482.8	71.5	0.53	100.2
SL32	0	2.1	10.6	6.8	689.9	0		107.1
	2	2.2	11.6	5.8	532.2	103.2	1.11	127.1
MRHA	0	1.86	10.6	7.3	484.9	0		85.6
	2	1.9	11.2	6.6	419.6	58.8	0.56	94.22
GM	0	2.0	10.9	7.1	545.8	0		100.6
	2	2.04	11.6	6.4	458.2	62.6	0.47	110.6

$M_{max}(S) = M_{max}(0)\ [1 + \lambda\ S^2 + \cdots]$; R's will be larger than quoted.

M. Prakash et al., Phys. Rep. 280, 1, (1997).

Effects of trapped ν's on the structure of neutron stars

Beta-equilibrated nucleonic matter with $Y_{\rm Le} = 0.4$

Model	S	$\frac{M_{\rm max}}{M_\odot}$	R (km)	$\frac{n_c}{n_0}$	P_c $\left(\frac{\rm MeV}{\rm fm^{-3}}\right)$	T_c (MeV)	λ ($x10^2$)	I ($M_\odot$ ${\rm km}^2$)
BPAL32	0	1.86	10.1	7.6	609.6	0		82.4
	2	1.91	10.8	6.7	503.7	63.7	0.63	92.4
MRHA	0	1.78	10.3	7.5	514.1	0		76.3
	2	1.84	10.9	6.8	448.3	54.6	0.75	85.3
GM	0	1.94	10.5	7.4	595.8	0		90.13
	2	1.98	11.2	6.7	496.6	59.0	0.58	100.1

For each S, $M_{max}(Y_{Le} = 0.4) < M_{max}(Y_\nu = 0)$;
R's will be larger than quoted.

M. Prakash et al., Phys. Rep. 280, 1, (1997).

Effects of trapped ν's on the structure of neutron stars

Beta-equilibrated hyperonic matter with $Y_\nu = 0$ & $Y_{\rm Le} = 0.4$

Model	S	$\frac{M_{\rm max}}{M_\odot}$	R (km)	$\frac{n_c}{n_0}$	P_c $\left(\frac{\rm MeV}{\rm fm^{-3}}\right)$	T_c (MeV)	λ ($x10^2$)	I ($M_\odot$ km^2)
MRHA	0	1.41	10.4	8.4	310.3	0		51.0
	2	1.43	10.9	7.8	283.9	39.2	0.36	54.5
GM	0	1.54	10.8	7.7	311.3	0		63.2
	2	1.57	11.3	6.9	269.5	41.4	0.47	69.2
MRHA	0	1.58	10.5	7.7	355.9	0		63.1
	2	1.60	11.1	6.9	299.7	36.9	0.3	64.9
GM	0	1.77	11.1	6.6	334.8	0		83.8
	2	1.78	11.7	6.2	296.7	37.0	0.1	88.5

For each S, $M_{max}(Y_{Le} = 0.4)$ about $0.2 M_\odot$ larger than $M_{max}(Y_\nu = 0)$;
R's will be larger than quoted.

M. Prakash et al., Phys. Rep. 280, 1, (1997).

Effects of rotation on the structure of neutron stars

Bozzola, Sterigioulas & Bauswein, arXiv: 1709.02787

From a study of uniformly and differentially rotating stars, BSB report the following "universal" relations:

$$\frac{M_B}{M_B^*} = 1 + 0.51 \left(\frac{cJ}{GM_B^{*2}}\right)^2 - 0.28 \left(\frac{cJ}{GM_B^{*2}}\right)^4$$

$$\frac{M_B}{M_B^*} = 0.93 \frac{M_G}{M_G^*} + 0.07$$

$$\frac{M_G}{M_G^*} = 1 + 0.29 \left(\frac{cJ}{GM_B^{*2}}\right)^2 - 0.10 \left(\frac{cJ}{GM_B^{*2}}\right)^4$$

Notation:

$M_G^* := M_{G,max}^{TOV}$ & $M_B^* := M_{B,max}^{TOV}$: Maximum gravitational and baryon masses for non-rotating models with $J = 0$.

M_G & M_B are for rotating models with $J \neq 0$.

Kerr parameter $a = \frac{cJ}{GM^2}$

How much additional mass can rotation support?

Bozzola, Sterigioulas & Bauswein, arXiv: 1709.02787

Study includes several EOS's (for cold and neutrino-free cases) including "soft" and "stiff" varieties.

Self-bound strange quark stars buck the trend yielding larger $\frac{M_B}{M_B^*}$ vs $\frac{cJ}{GM_B^{*2}}$:

$$\frac{M_B}{M_B^*} = 1 + 0.87\left(\frac{cJ}{GM_B^{*2}}\right)^2 - 0.60\left(\frac{cJ}{GM_B^{*2}}\right)^4$$

Uniform (differential) rotation can increase the maximum allowed mass (before mass shedding) by up to $\sim 25\%$ (50%).

Caveats:

Study restricted to stationary and axi-symmetric space-time; differential rotation studied for a 1-parameter and 3-parameter rotation laws (for details, see BSB). Analysis for realistic rotation laws extracted from dynamical simulations promised in future work.

Effects of magnetic fields on the structure of neutron stars

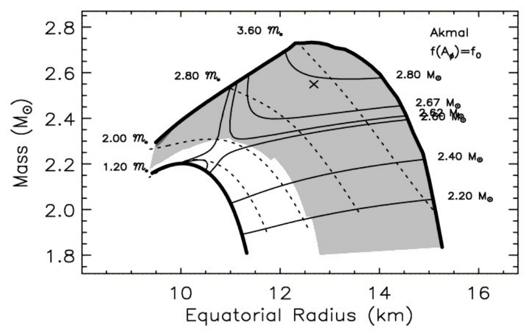
Poloidal fields; M_{max} larger than that for uniform rotation (see "x" in figure).

Increase over $M_{max}(B=0) \sim 24\%$.

Light solid curves: Sequences of constant M_B.

Light dotted curves: Sequences of constant $\mathcal{M}$, dipole magnetic moment.

$B_c = 2.7 \times 10^{18}$ G & $B_{pole} = 2.1 \times 10^{18}$ G.



EOS significantly affected by Landau quantization and magnetic moment interactions only for $B > 5 \times 10^{18}$ G.

Stability analysis (as for rotation) still lacking.

Cardall, Prakash & Lattimer, ApJ, 554, 322, (2001)

Effects of magnetic fields on stars with hyperons

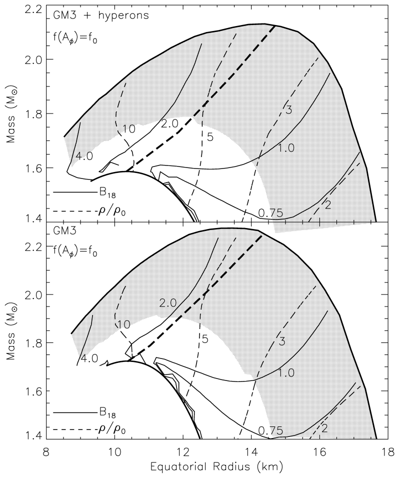
Poloidal fields; M_{max} larger than that for uniform rotation (see "x" in figure).

Increases over $M_{max}(B = 0) \sim 20\%$ are similar.

Heavy dashed curves: Sequences of constant M_B.

Thin solid curves: Sequences of constant B_{max}, in 10^{18} G.

Thin dashed lines: Contours of maximum energy density.



Broderick, Prakash & Lattmer, Phys. Lett. B 531 (2002) 167.

Outlook

- An appealing and defensible treatment of the EOS above about twice nuclear density including possible phase transitions.
- Beyond the virial treatment of low-density inhomogeneous phase.
- More extensive studies of differential rotational laws.
- Study of magnetic field generation in conjunction with differential rotation and convection.