

Matching the Equation of State of Dense Neutron-Rich Matter Constrained by Terrestrial Experiments and Astrophysical Observations

Bao-An Li



Collaborators:

Farrooh J. Fattoyev, Indiana University

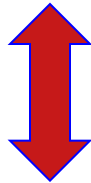
Plamen G. Krastev, Harvard University

William G. Newton, Texas A&M University-Commerce

Jun Xu, Chinese Academy of Sciences

Naibo Zhang, TAMU-Commerce & Shandong University

EOS parameter space restricted by terrestrial experiments & theories



EOS constraints from M_{\max} , R and tidal polarizability of neutron stars

Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

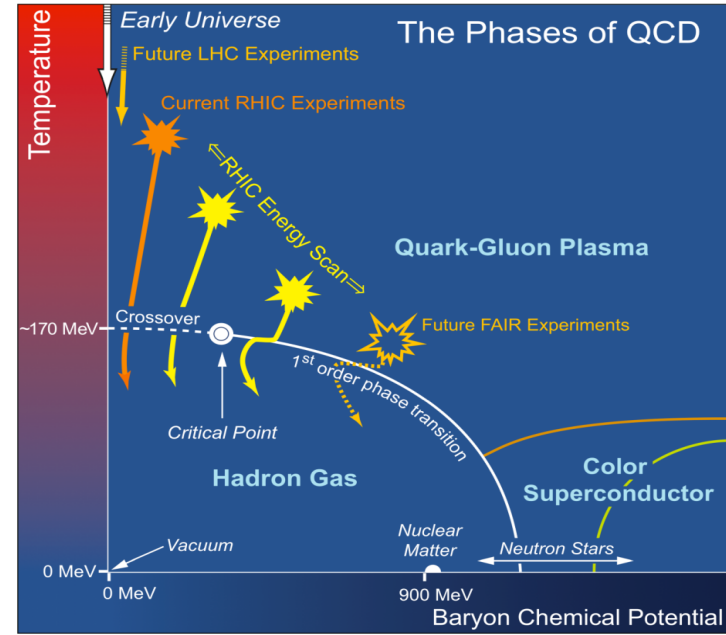
symmetry energy

Isospin asymmetry δ

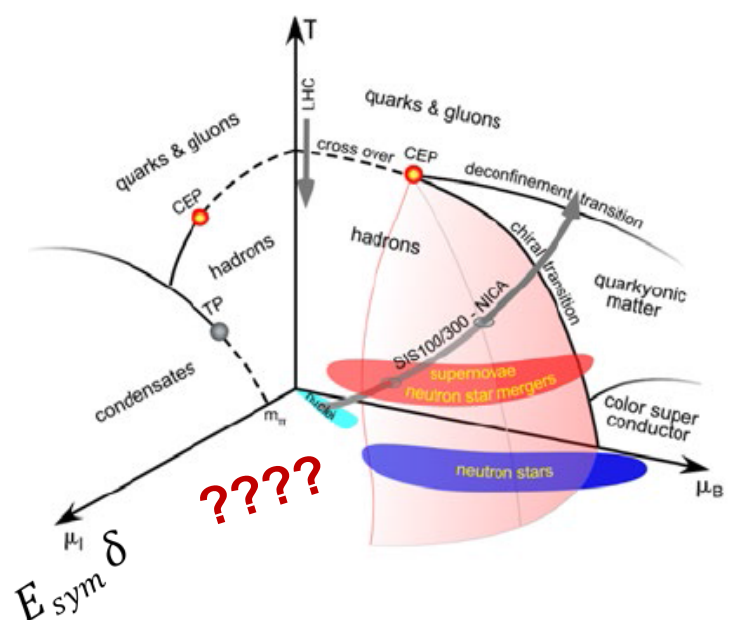
$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{sym}(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter



density



????

???

New opportunities
Isospin asymmetry
 $\delta = (\rho_n - \rho_p) / \rho$

Parameterizing the EOS of symmetric matter and symmetry energy for the NS core

$$E_0(\rho) \approx E_0(\rho_0) + \frac{K_0}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{J_0}{6} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3,$$

$$E_{\text{sym}}(\rho) \approx E_{\text{sym}}(\rho_0) + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{\text{sym}}}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{J_{\text{sym}}}{6} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3$$

Naturally approach asymptotically their Taylor expansions near the saturation density

Current status of the restricted EOS parameter space:

Low density: $K_0 = 240 \pm 20$, $E_{\text{sym}}(\rho_0) = 31.7 \pm 3.2$ and $L = 58.7 \pm 28.1$ MeV

High density: $-400 \leq K_{\text{sym}} \leq 100$, $-200 \leq J_{\text{sym}} \leq 800$, and $-800 \leq J_0 \leq 400$ MeV

Why do not you use piecewise polytropes at high densities (HD)?

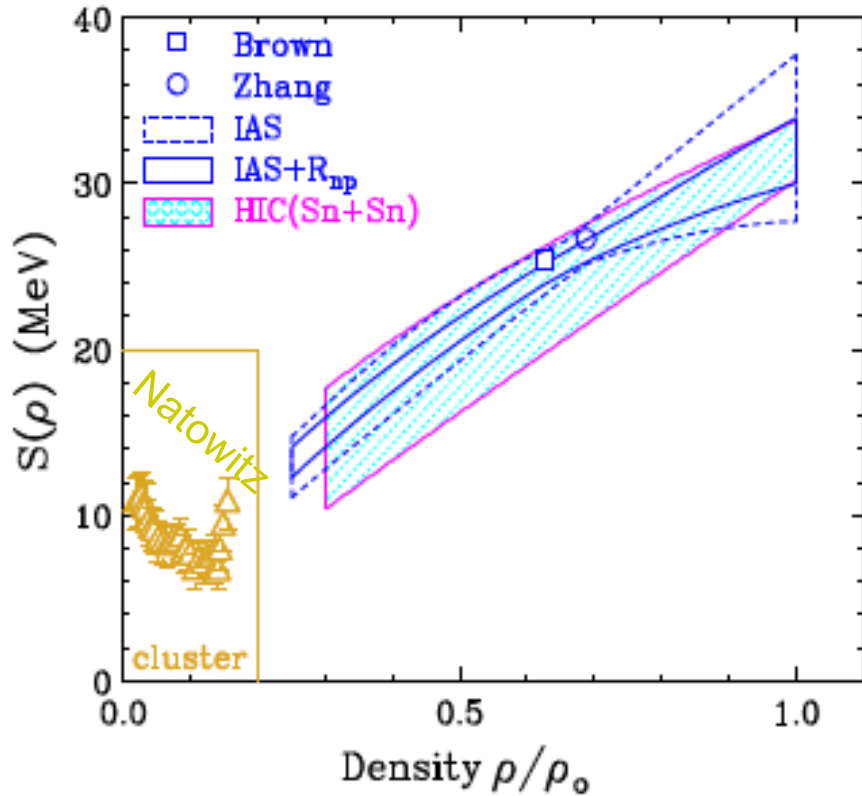
- (1) The ranges of HD parameters K_{sym} , J_{sym} and J_0 are so large that they cover
- (2) The polytropes: pressures at several fiducial densities have NO isospin dependence
- (3) Need a parameterization facilitating the extraction of high density symmetry energy

The minimum model of neutron stars: The pressure in the npe μ matter:

$$\begin{aligned} P(\rho, \delta) &= P_0(\rho) + P_{\text{asy}}(\rho, \delta) = \rho^2 \left(\frac{\partial E}{\partial \rho} \right)_{\delta} + \frac{1}{4} \rho_e \mu_e \\ &= \rho^2 \left[E'(\rho, \delta = 0) + E'_{\text{sym}}(\rho) \delta^2 \right] + \frac{1}{2} \delta (1 - \delta) \rho E_{\text{sym}}(\rho), \end{aligned}$$

What is the density dependence of symmetry energy?

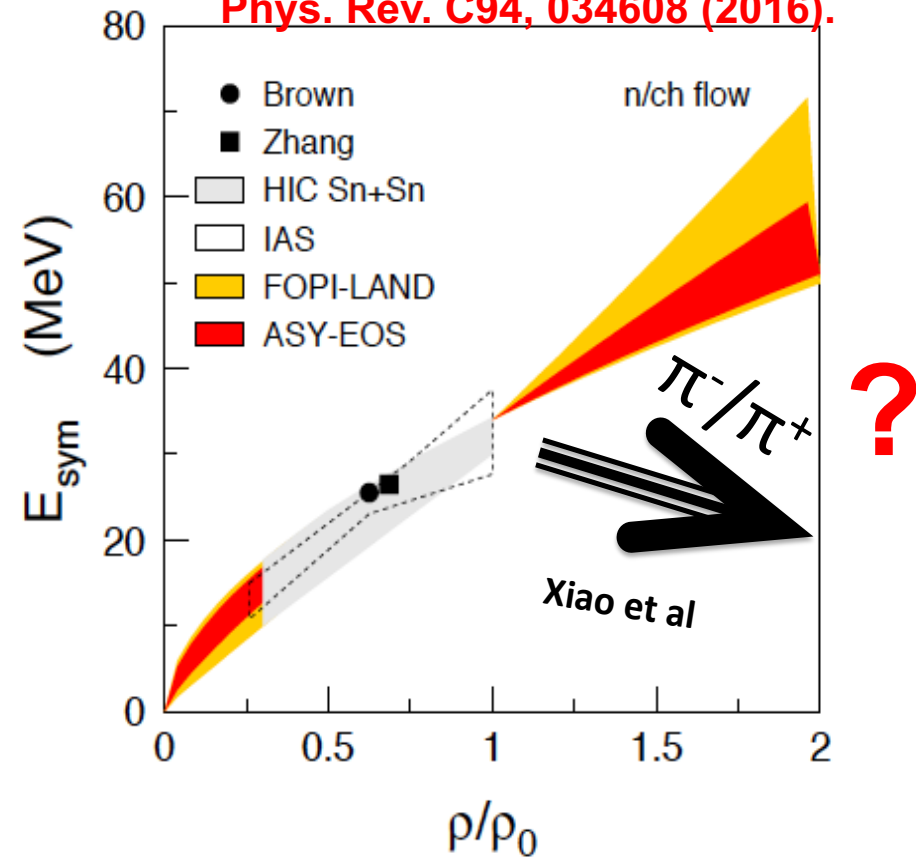
Indications of experiments (model dependent)



Betty Tsang et al., PRC 86, 105803 (2012).

Chuck Horowitz et al., JPG: 41 (2014) 093001

P. Russotto et al. (ASY-EOS Coll),
Phys. Rev. C94, 034608 (2016).



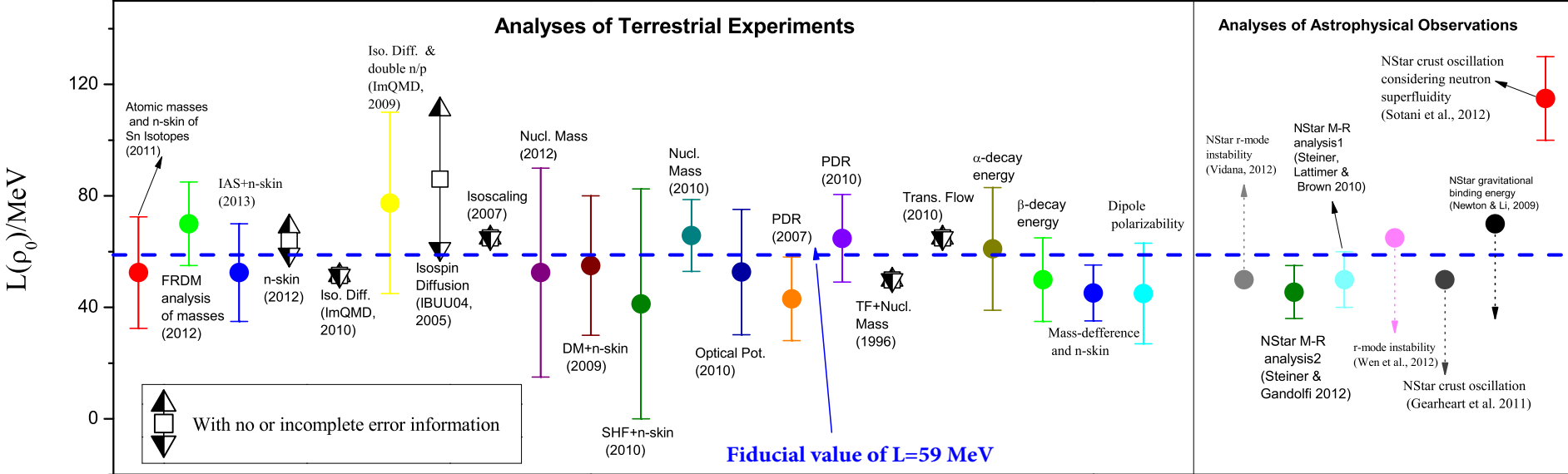
Z.G. Xiao et al, based on GSI/FOPI data
Phys. Rev. Lett. 102, 062502 (2009).

Constraints on $E_{\text{sym}}(\rho_0)$ and L based on 29 analyses of data

Fiducial values
as of Aug. 2013

$$E_{\text{sym}}(\rho_0) \approx 31.6 \pm 2.66 \text{ MeV}$$

$$L \approx 2 E_{\text{sym}}(\rho_0) = 59 \pm 16 \text{ MeV}$$



[Bao-An Li](#) and [Xiao Han](#), Phys. Lett. B727 (2013) 276-281

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left(\frac{\rho}{\rho_0} - 1 \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho}{\rho_0} - 1 \right)^2 + \frac{J_{\text{sym}}}{162} \left(\frac{\rho}{\rho_0} - 1 \right)^3$$

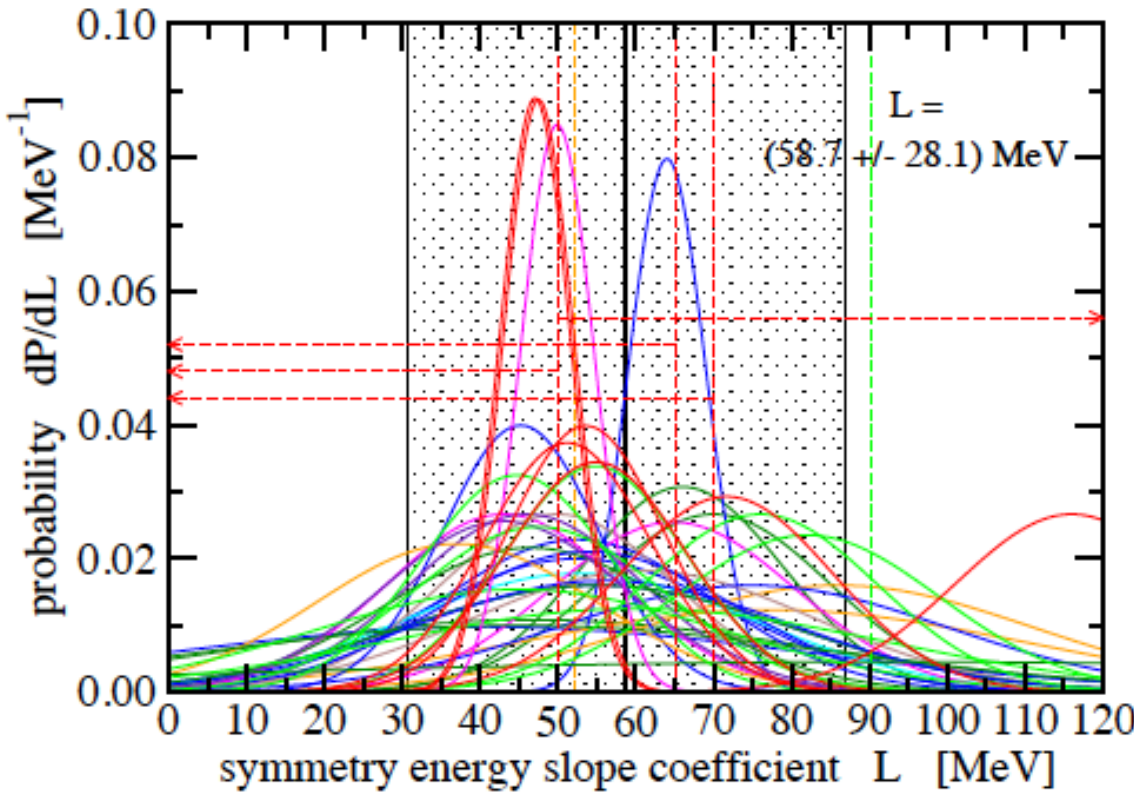
Constraints on $E_{\text{sym}}(\rho_0)$ and L based on 53 analyses of data

**Fiducial values
as of Oct. 12, 2016**

$$E_{\text{sym}}(\rho_0) \approx 31.7 \pm 3.2 \text{ MeV}$$

$$L = 58.7 \pm 28.1 \text{ MeV}$$

- Assuming:
- (1) Gaussian distribution of L
 - (2) Democratic principle
(treat & trust everyone/publication equally)



M. Oertel, M. Hempel, T. Klähn, S. Typel

Review of Modern Physics 89 (2017) 015007

Isoscalar Excitation Modes of Nuclear Resonance

$$K_0 = 240 \pm 20 \text{ MeV}$$

$$E_{ISGMR} \approx \sqrt{\frac{K_A}{m \langle r^2 \rangle}}$$

$$E_{ISGDR} \approx \sqrt{\frac{3}{7} \frac{K_A + (27/25)\epsilon_F}{m \langle r^2 \rangle}}$$



$$K_A = K_{vol} + K_{surf}A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}}$$



Isospin dependence of incompressibility

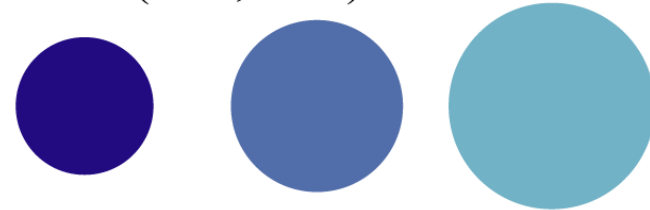
$$K_\tau = K_{sym} - 6L(\rho_0) - \frac{J_0 L(\rho_0)}{K_0}$$

$$K_\tau = -550 \pm 100 \text{ MeV}$$

Nothing conclusive about K_{sym}

Isoscalar Giant Resonances:

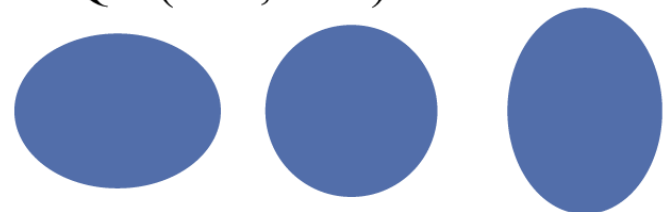
ISGMR (T=0, L=0)



ISGDR (T=0, L=1)



ISGQR (T=0, L=2)

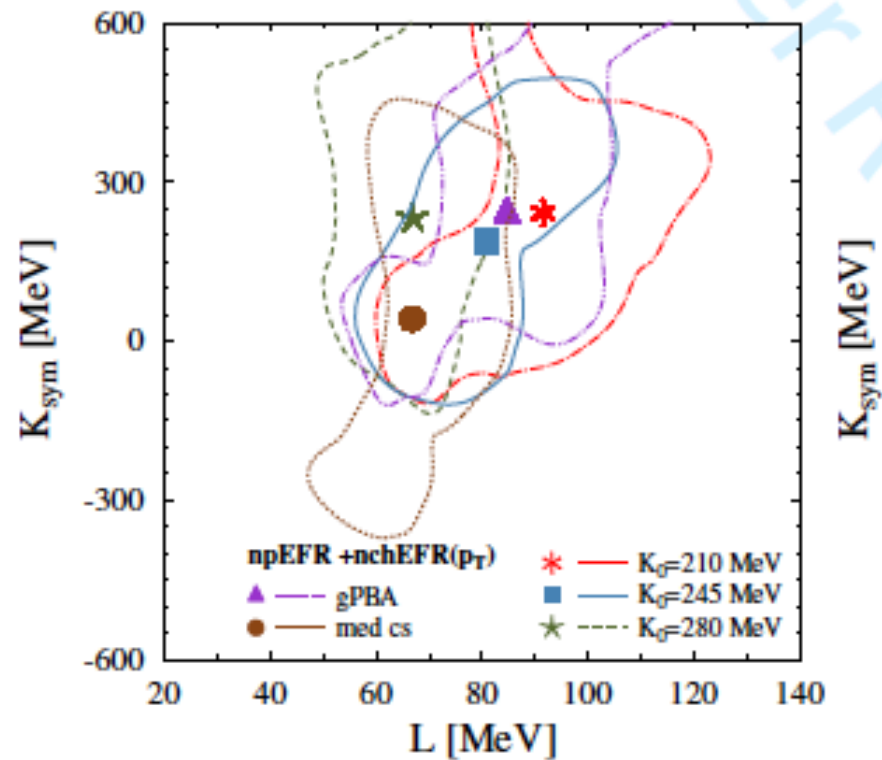


QMD analysis of GSI data on neutron-proton relative elliptical flow

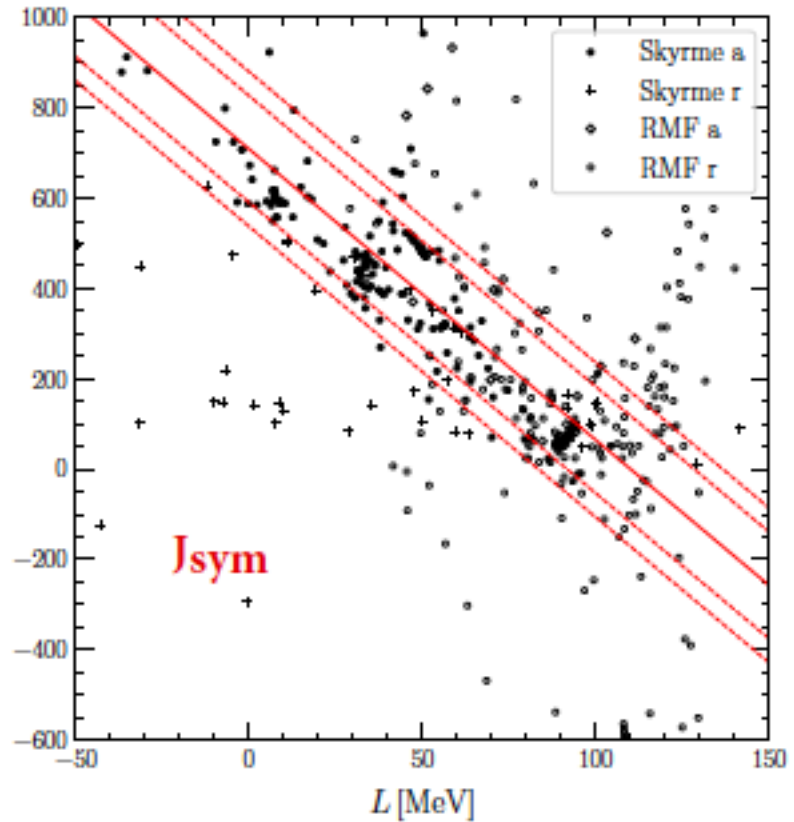
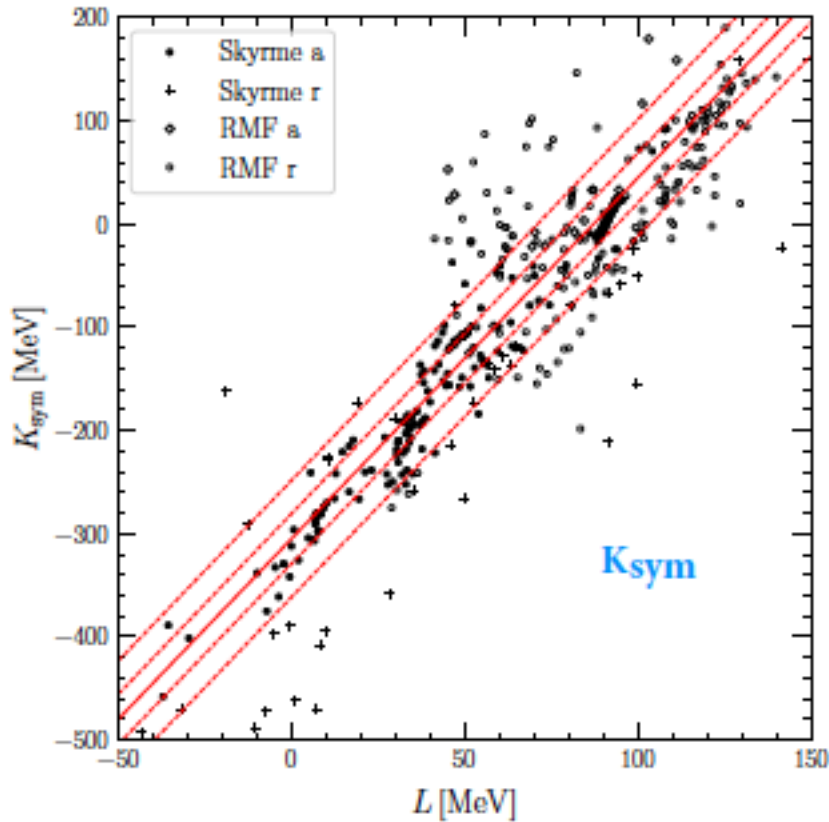
Dan Cozma, Euro Phys. J. A 54: 40 (2018).

$$L = 85 \pm 22(\text{exp}) \pm 20(\text{th}) \pm 12(\text{sys}) \text{ MeV}$$

$$K_{\text{sym}} = 96 \pm 315(\text{exp}) \pm 170(\text{th}) \pm 166(\text{sys}) \text{ MeV}.$$



Systematics from over 520 Skyrme+RMF energy density functionals

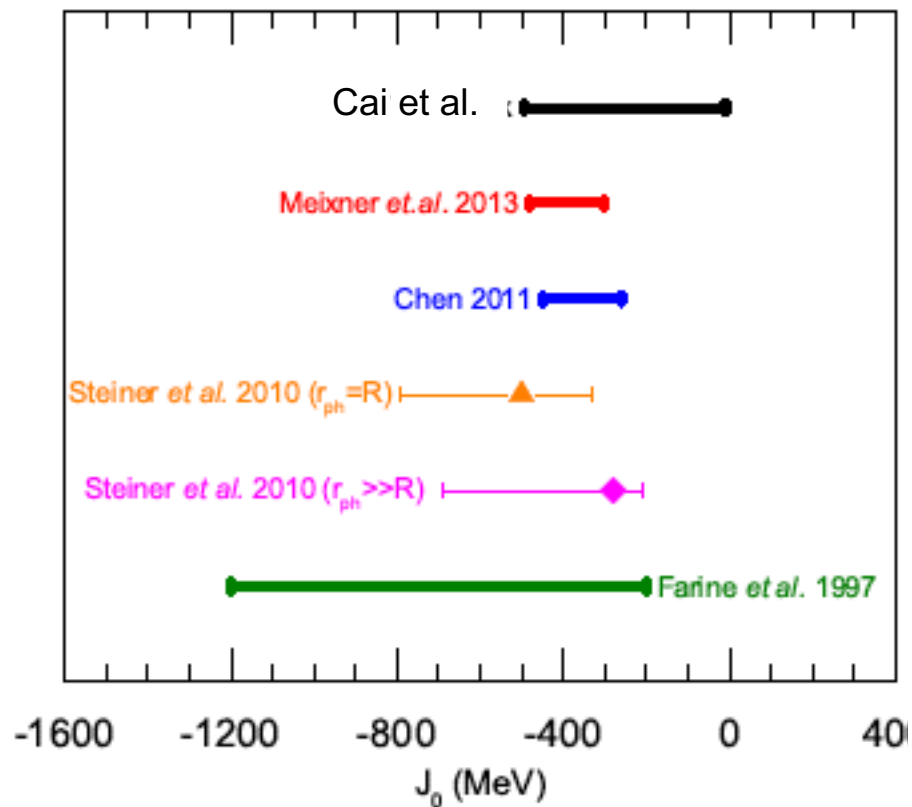


[Ingo Tews](#), [James M. Lattimer](#), [Akira Ohnishi](#), [Evgeni E. Kolomeitsev](#)
Astrophysics J. 848, 105 (2017)

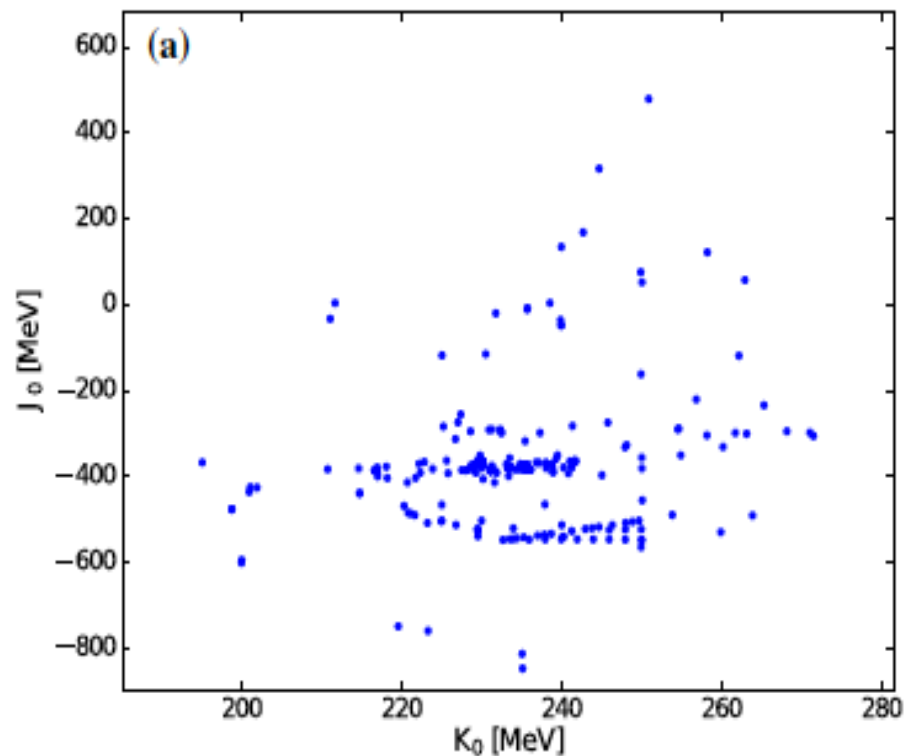
$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left(\frac{\rho}{\rho_0} - 1 \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho}{\rho_0} - 1 \right)^2 + \frac{J_{\text{sym}}}{162} \left(\frac{\rho}{\rho_0} - 1 \right)^3$$

Skewness J_0 of symmetric matter

Indications from model analyses of data



173 Skyrme+101 RMF predictions



N.B. Zhang et al., Nucl. Sci. Tech., 28, 181 (2017)

P. Danielewicz, R. Lacey, & W.G. Lynch, Science, 298, 1592 (2002)

B.J. Cai & L.W. Chen, Nucl. Sci. Tech., 28, 185 (2017)

A. W. Steiner, J.M. Lattimer & E. F. Brown, APJ, 722, 33 (2010).

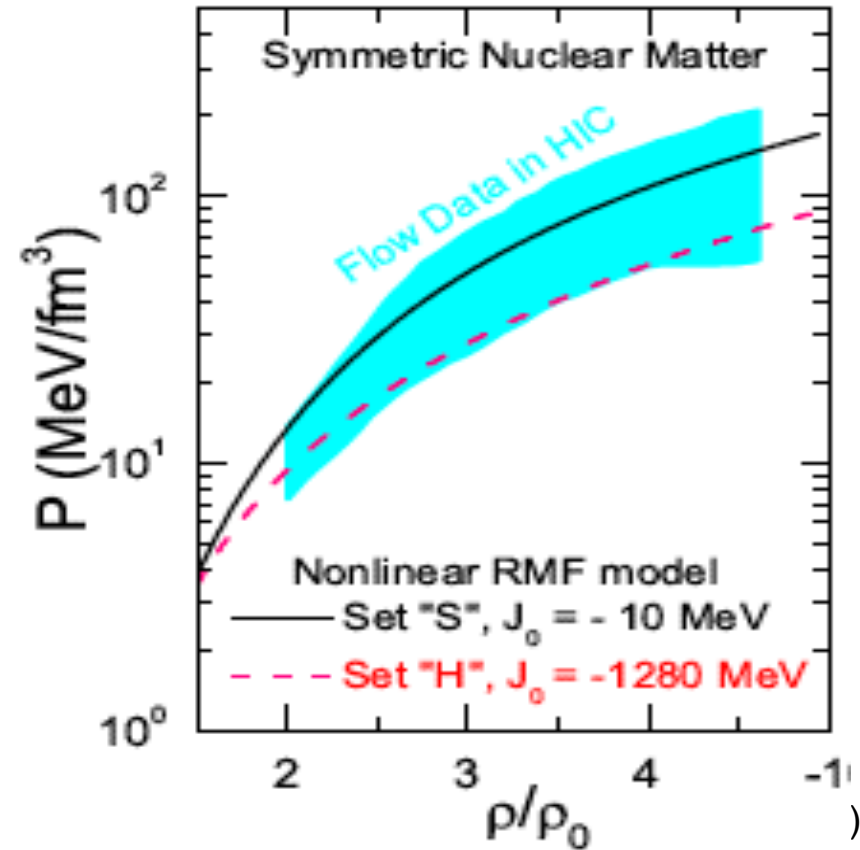
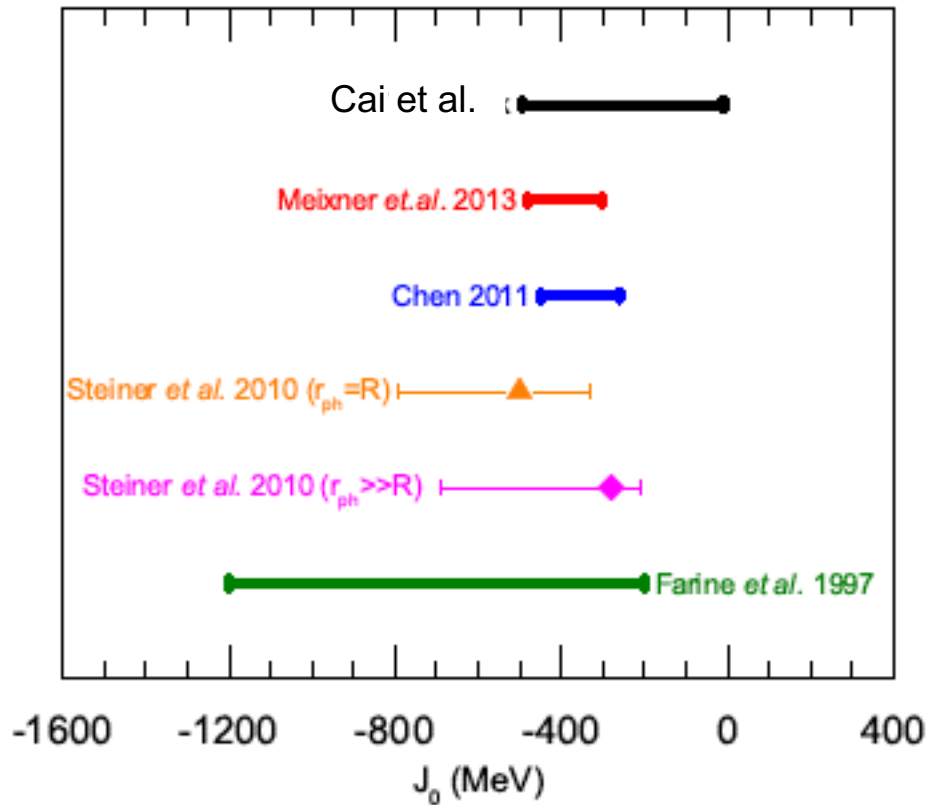
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Skewness J_0 of symmetric matter

Indications of model analyses of data



B.J. Cai & L.W. Chen, Nucl. Sci. Tech., 28, 185 (2017)

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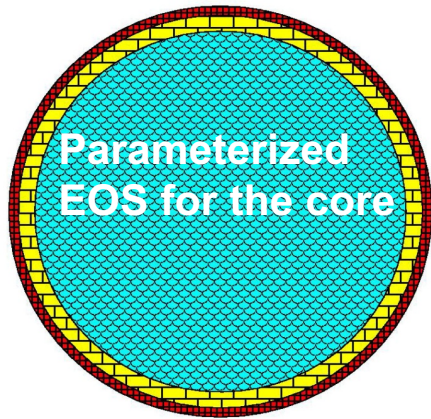
P. Danielewicz, R. Lacey, & W.G. Lynch, Science, 298, 1592 (2002)

Current status of the restricted EOS parameter space:

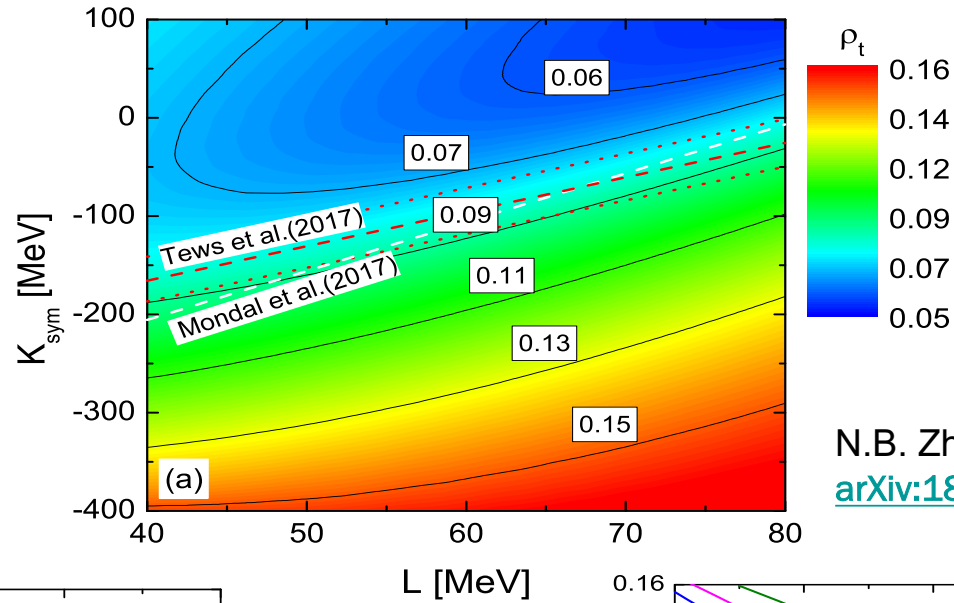
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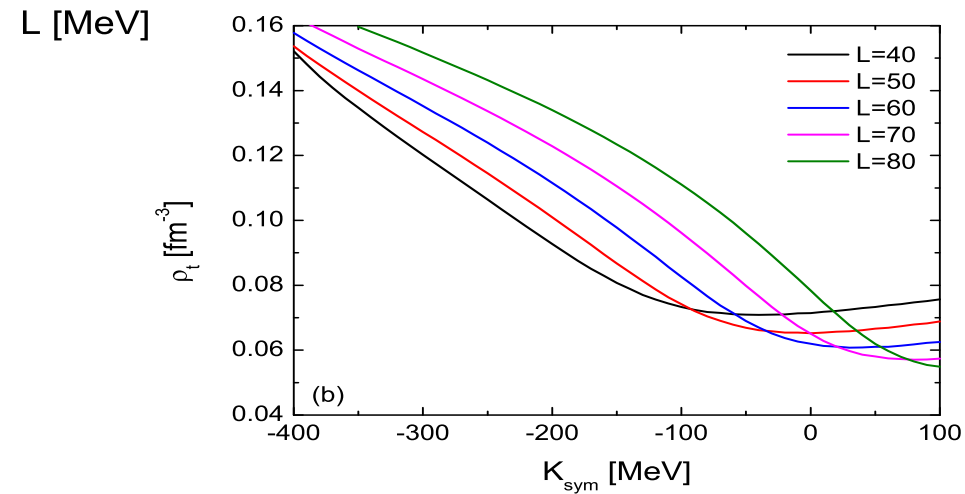
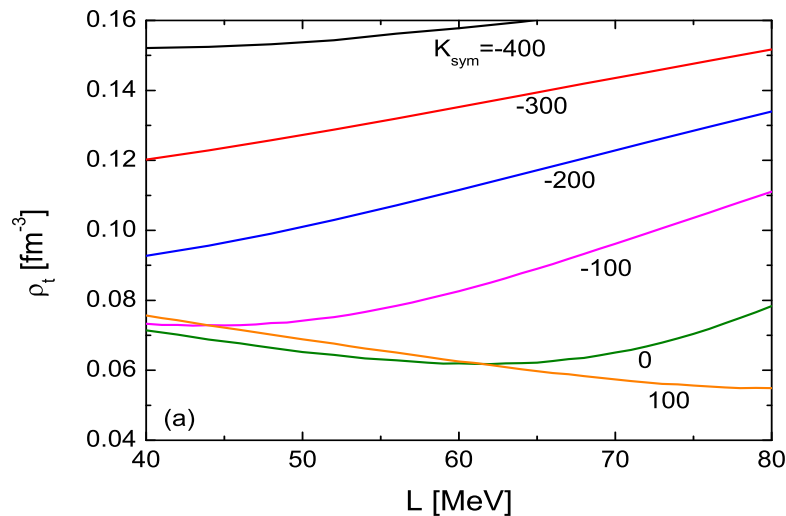
Effects of symmetry energy on the crust-core transition density



NV+BPS EOS for the crust



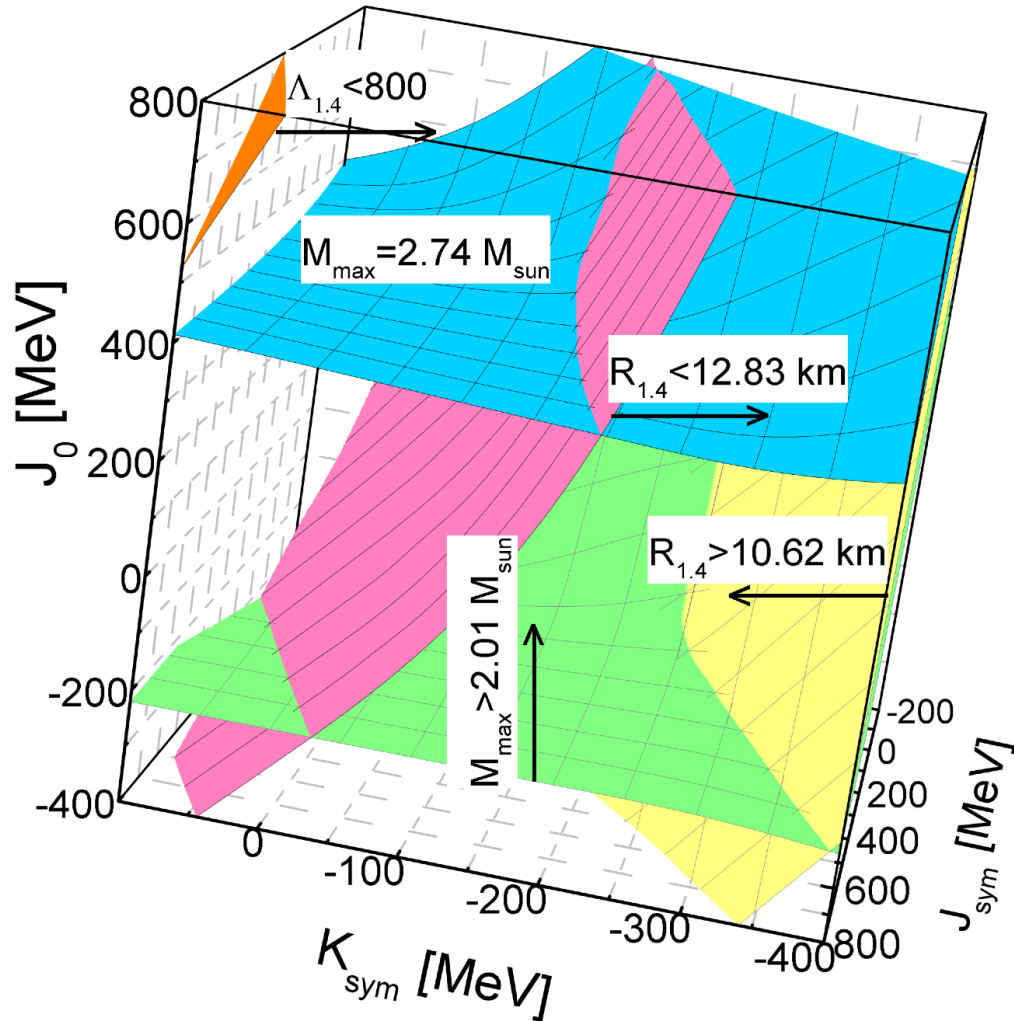
N.B. Zhang, B.A. Li and J. Xu,
[arXiv:1801.06855](https://arxiv.org/abs/1801.06855)



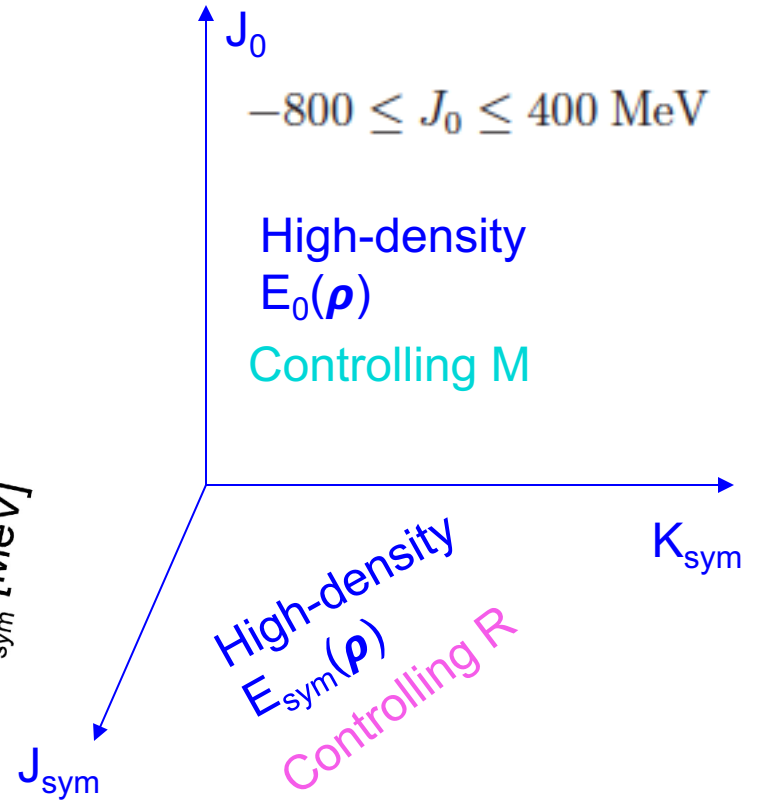
At the crust-core transition: Incompressibility in neutron stars at β equilibrium = 0

$$K_\mu = \rho^2 \frac{d^2 E_0}{d\rho^2} + 2\rho \frac{dE_0}{d\rho} + \delta^2 \left[\rho^2 \frac{d^2 E_{\text{sym}}}{d\rho^2} + 2\rho \frac{dE_{\text{sym}}}{d\rho} - 2E_{\text{sym}}^{-1} \left(\rho \frac{dE_{\text{sym}}}{d\rho} \right)^2 \right]$$

Astrophysical constraints on the high-density EOS parameters in 3D



Low-density parameters are fixed at their most probable values

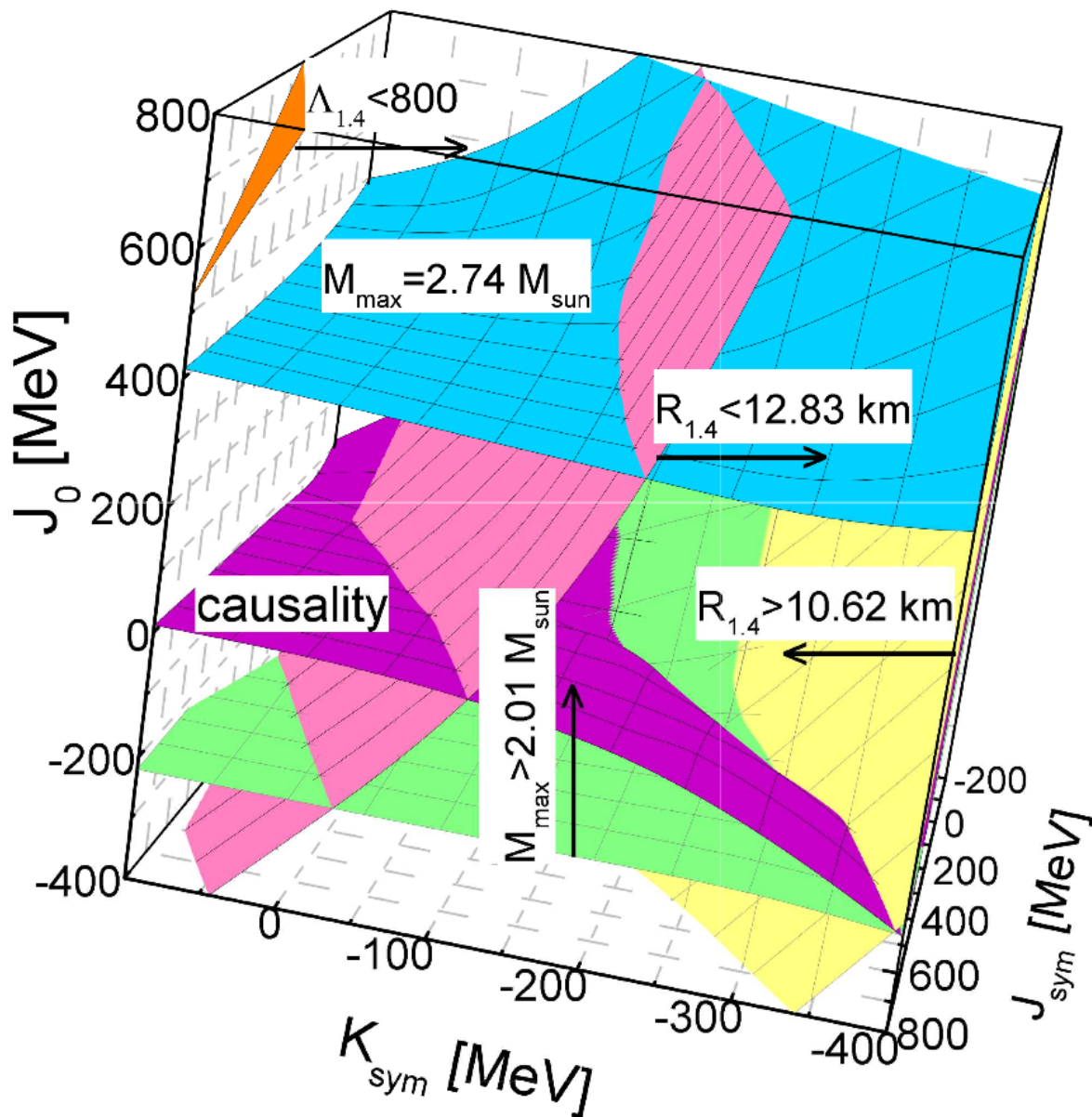


Degeneracy:

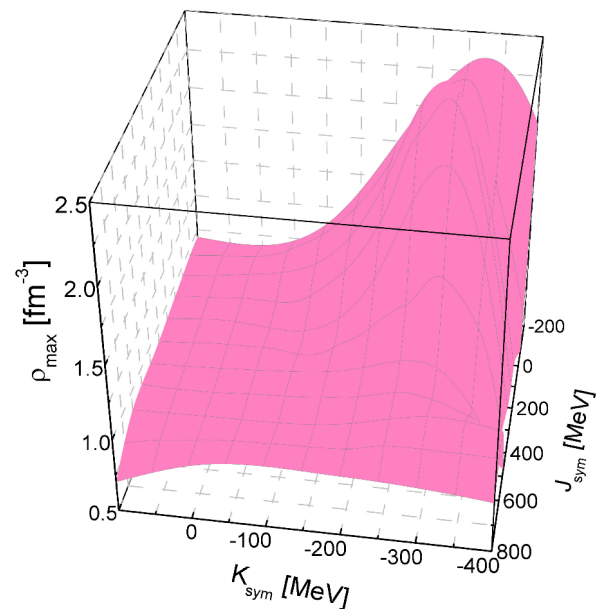
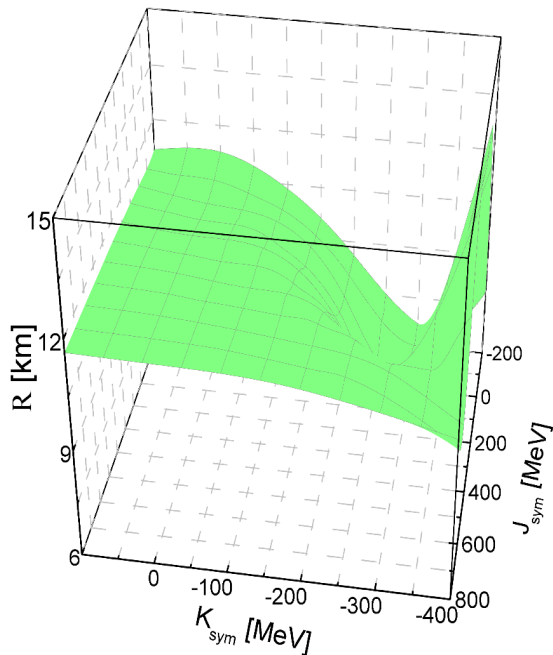
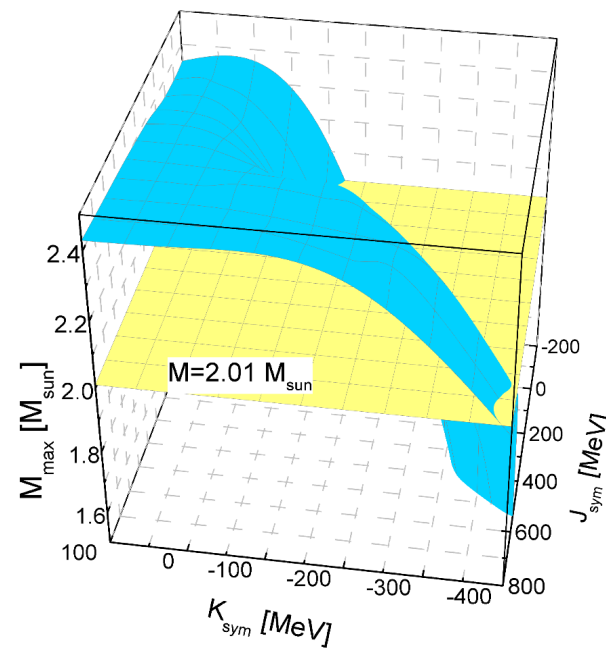
Intersections \rightarrow different EOS parameters giving identical M&R, same R different M or same M different R, **have to fix E_{sym} at HD to break it**

Causality, not the blue sky, is the upper limit!

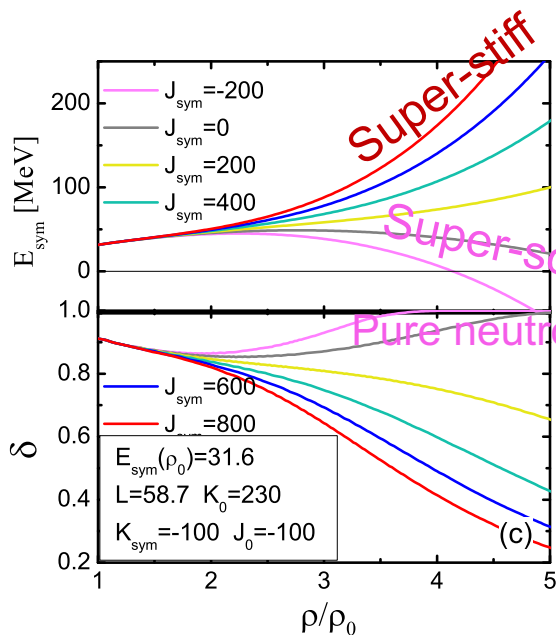
It is impossible to form a stable hyper-massive NS with $M=2.74M_{\odot}$.



The maximum mass, radius and density on the causality surface



symmetry energy



Isospin asymmetry profile at beta equilibrium

Isospin separation instability:

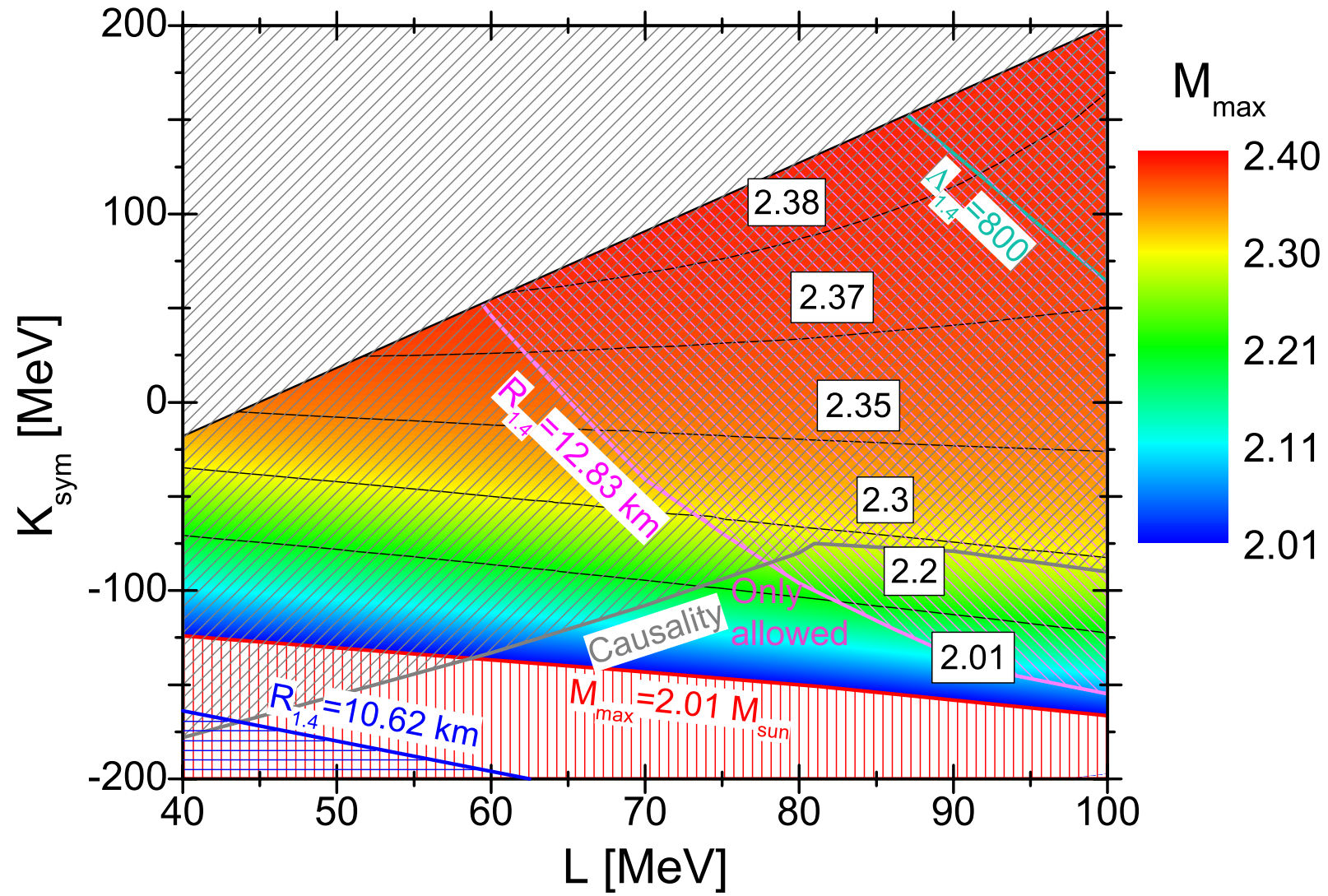
If $E_{\text{sym}} < 0$, replace $\delta=0$ with $1+$ (-1)
 SNM=PNM+PPM to lower energy
 Tensor force in n-p due to ρ meson exchange can make this happen

$$E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}(\rho)\delta^2$$

Direct URCA limit

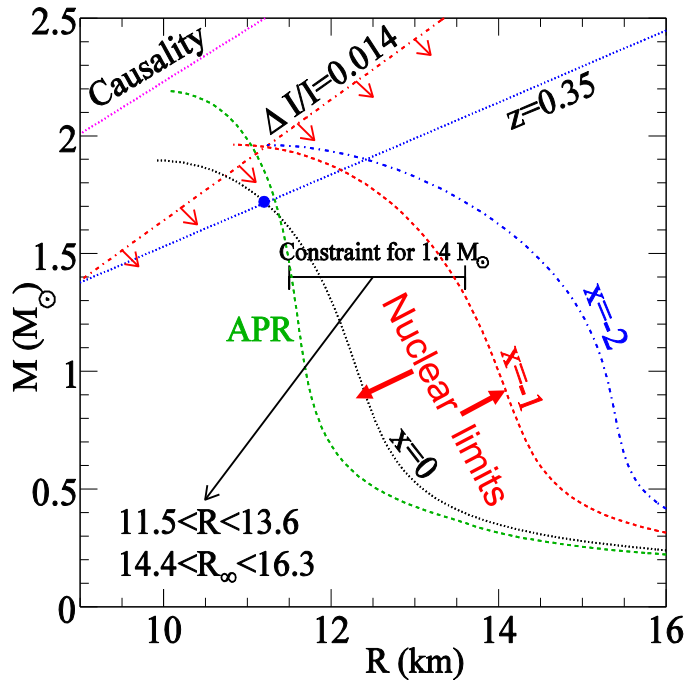
Canonical NSs have high core density, but almost pure neutrons \rightarrow slow cooling

Setting $J_0=0$, $J_{\text{sym}}=0$, parameterizing the EOS quadratically in ρ and δ in 2D



Constraining the radii of neutron stars with terrestrial experiments

Bao-An Li and Andrew W. Steiner, Phys. Lett. B642, 436 (2006)



APR: $K_0=269$ MeV.

Radii of neutron stars inferred from observations

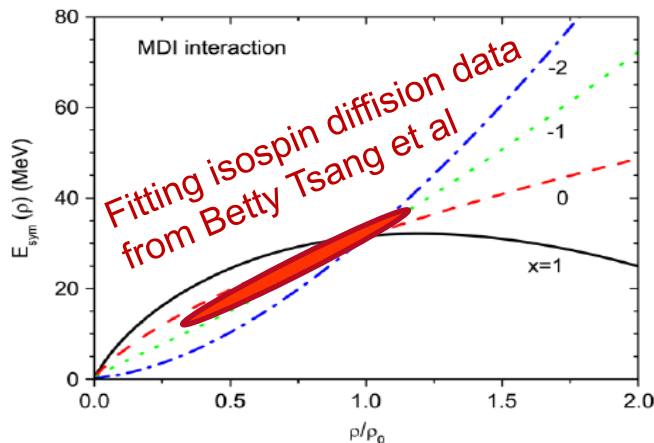
- (a) thermal emissions from quiescent neutron star low-mass X-ray binaries (qLMXBs)
- (b) photospheric radius expansion (PRE) bursts with H and/or He atmosphere models

| Model | $R_{1.4}$ 90% confidence range |
|-----------------------------|--------------------------------|
| Alt/H+He QLMXB; $z = 0$ PRE | 11.13 – 12.33 |
| $z = 0$ PRE only | 11.56 – 12.64 |
| Base, QLMXB only | 11.01 – 11.94 |
| Alt, QLMXB only | 10.62 – 11.50 |
| H+He, QLMXB only | 11.29 – 12.83 |
| Alt/H+He, QLMXB only | 11.24 – 12.59 |

J.M. Lattimer and A.W. Steiner,
European Physics Journal A50, 40 (2014)

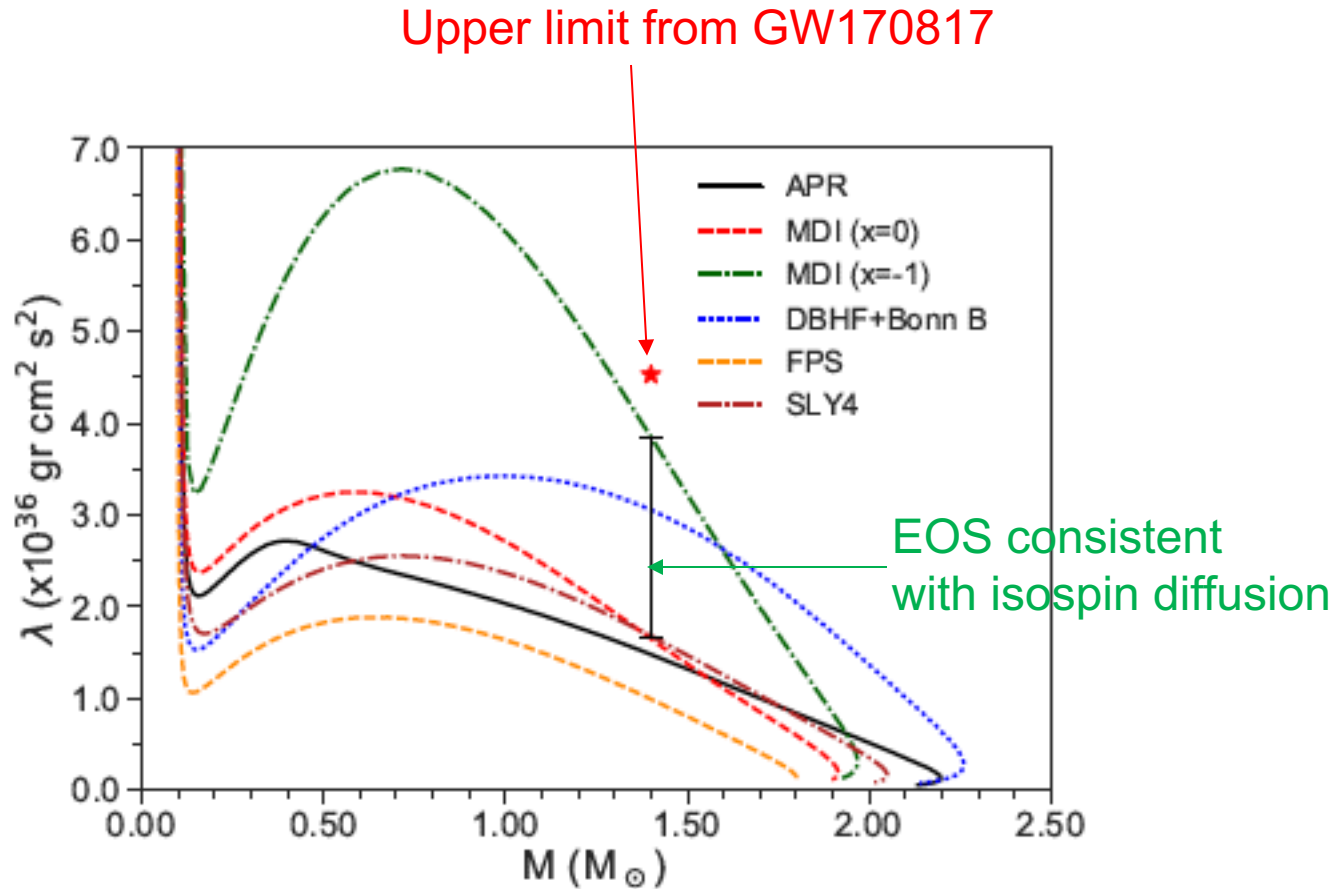
Review of techniques & controversies:

M.C. Miller & F.K. Lamb,
European Physical Journal A 52, 63 (2016).



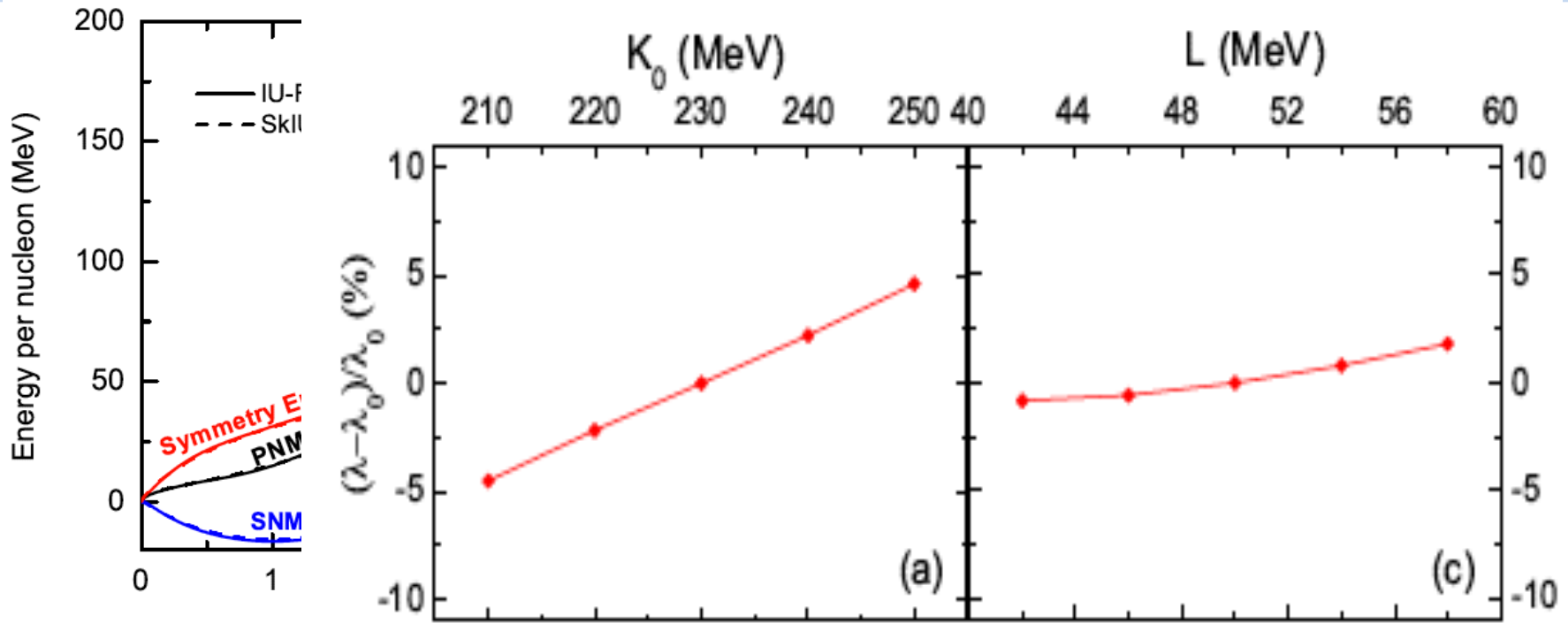
L.W. Chen, C.M. Ko and B.A. Li,
Phys. Rev. Lett 94, 32701 (2005)

Imprints of nuclear E_{sym} on the tidal deformability of neutron stars



[Plamen G. Krastev](#), [Bao-An Li](#) [arXiv:1801.04620](#)

Signatures of S,L($n>n_0$) in GW signals: Tidal deformability and mergers



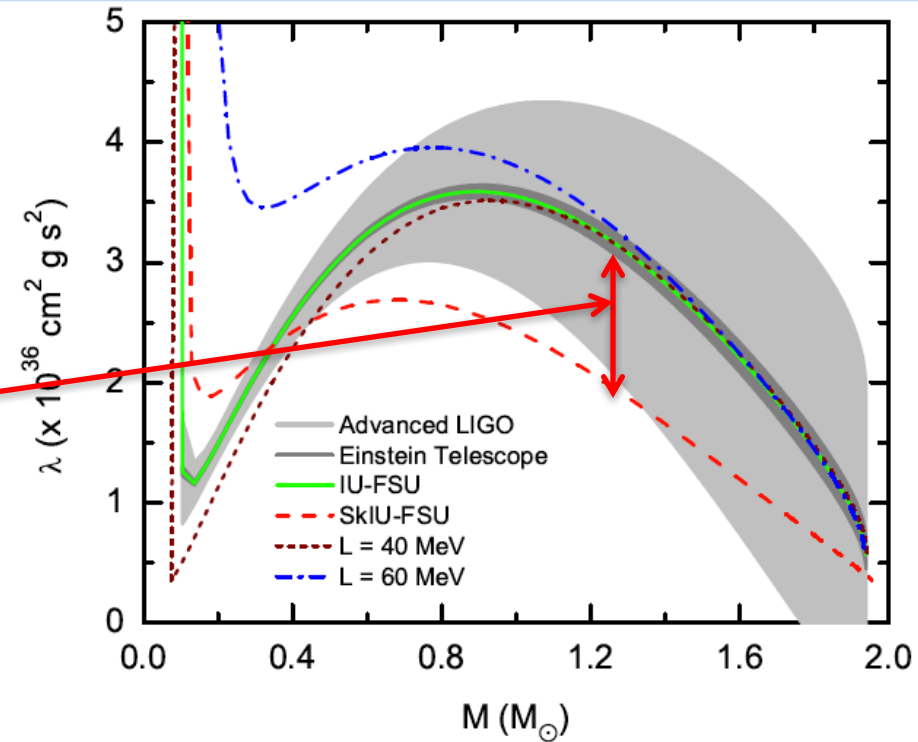
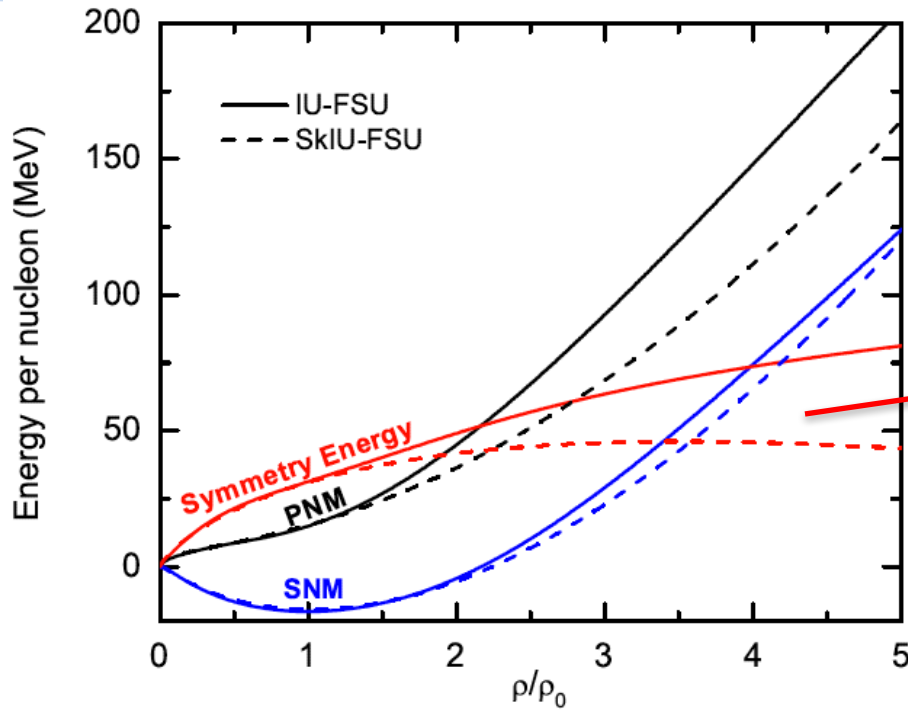
F. Fattoyev, J. Carvajal, W.G. Newton and B.A. Li, PRC87, 15806 (2013)

Λ of light NS is sensitive to L

Λ of canonical and more massive NS is sensitive to high-density E_{sym} but not L

- Detector sensitivities assuming optimally oriented, equal mass binary at $D=100$ Mpc
- Damour, Nagar, PRD81, 084016 (2010)
- Damour, Nagar, Villain, PRD85, 123007 (2012)
- Hinderer et al, PRD 81, 123016 (2010)

Signatures of $S, L (n > n_0)$ in GW signals: Tidal deformability and mergers



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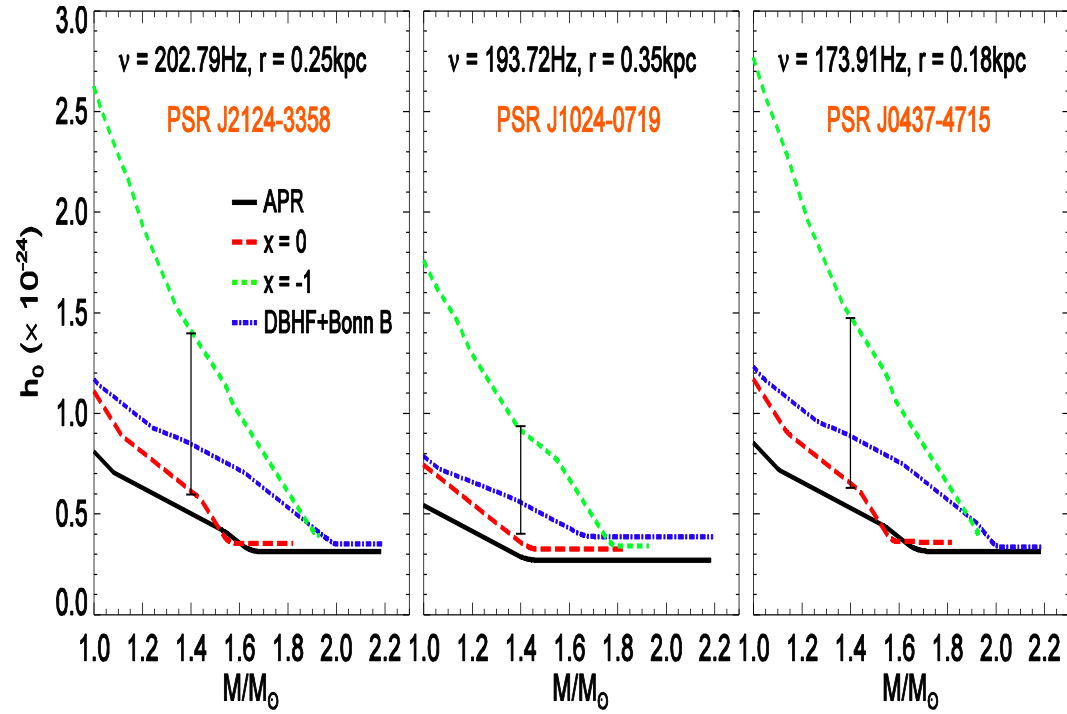
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- Damour, Nagar, Villain, PRD85, 123007 (2012)
- Hinderer et al, PRD 81, 123016 (2010)

Nuclear constraints on the strength of gravitational waves

Plamen Krastev, Bao-An Li and Aaron Worley, Phys. Lett. B668, 1 (2008).



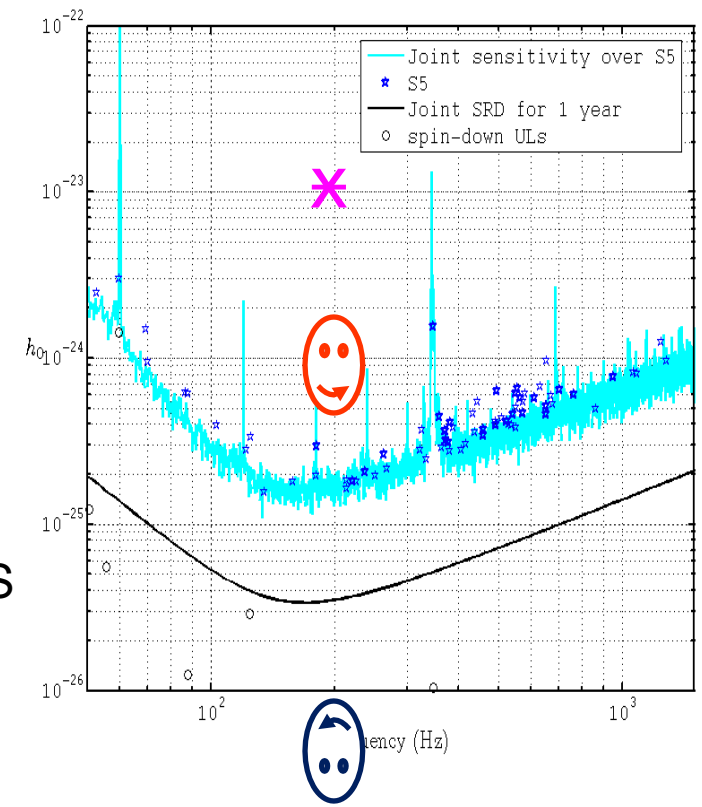
σ is the breaking strain of the neutron star crust

$$\sigma = [10^{-5} - 10^{-2}] \quad \text{or } 0.1$$

Depends on the symmetry energy and structure of NS

$$\Phi_{22,max} = 2.4 \times 10^{38} \text{g cm}^2 \left(\frac{\sigma}{10^{-2}}\right) \left(\frac{R}{10\text{km}}\right)^{6.26} \left(\frac{1.4M_\odot}{M}\right)^{1.2}$$

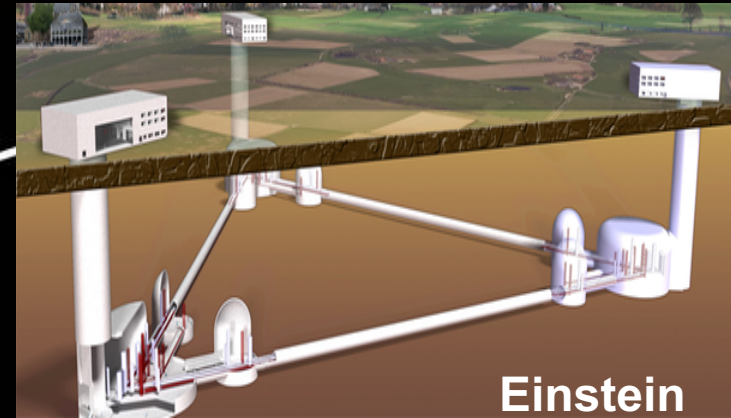
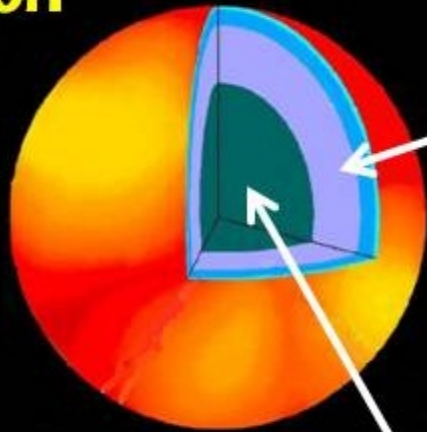
Compare with the latest upper limits from LIGO+GEO observations



Summary

- (1) Truly **multi-messenger approach** to probe the EOS of dense neutron-rich matter
= astrophysical observations + terrestrial experiments + theories + ...
- (2) The upper limit $\Lambda(1.4M_{\odot}) < 800$ from GW170817 is consistent but less restrictive than the existing constraints on the EOS from the M_{\max} and $R_{1.4}$

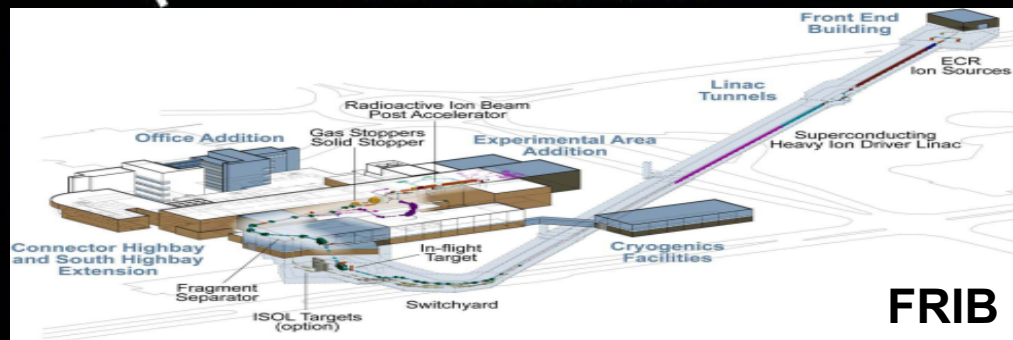
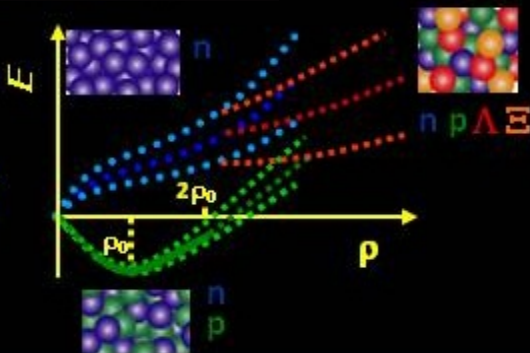
ASTRO-X Observation



Einstein

Experiments

EOS Theory



FRIB

EOS of dense neutron-rich matter: a major scientific thrust of

(0) 2015 US and 2017 Europe Long Range Plan for Nuclear Physics

(1) High-energy rare isotope beam facilities around the world

(2) Neutron Star Interior Composition Explorer (NICER of NASA, launched on June 3rd, to take science data on July 13th, 2017) and various x-ray satellite (Chandra, LOFT, XMM-Newton, etc)

(3) Various gravitational wave detectors

Among the promising observables of high-density symmetry energy:

- π^-/π^+ , neutron-proton differential flow in heavy-ion collisions
- Radii of neutron stars
- Neutrino flux of supernova explosions
- Strain amplitude and frequency of gravitational waves from spiraling neutron star binaries and/or oscillations/rotations of deformed pulsars

Topical Issue on Nuclear Symmetry Energy
edited by Bao-An Li, Àngels Ramos,
Giuseppe Verde and Isaac Vidaña

Euro Phys. Jour. A, Vol. 50, No. 2 (2014)

Can the symmetry energy become negative at high densities?

Yes, it happens when the tensor force due to ρ exchange in the T=0 channel dominates

At high densities, the energy of pure neutron matter can be lower than symmetric matter leading to negative symmetry energy

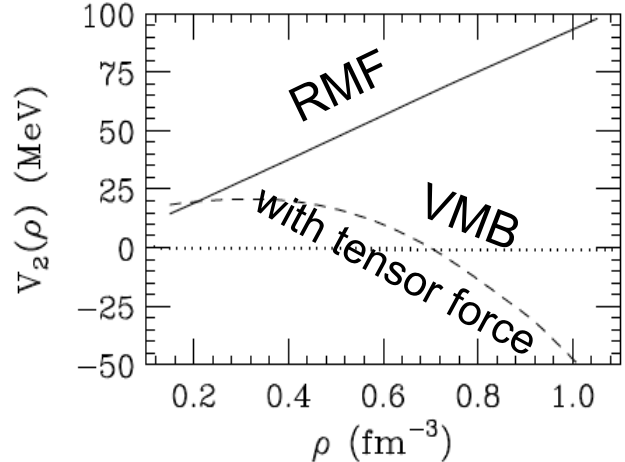
Pandharipande V R and Garde V K 1972 *Phys. Lett. B* 39 608
 Wiringa R B, Fiks V and Fabrocini A 1988 *Phys. Rev. C* 38 1010
 Kutschera M 1994 *Phys. Lett. B* 340 1

Example: proton fractions with interactions/models leading to negative symmetry energy

M. Kutschera et al., *Acta Physica Polonica B*37 (2006)

$$x = 0.048 [E_{sym}(\rho) / E_{sym}(\rho_0)]^3 (\rho / \rho_0) (1 - 2x)^3$$

Potential part of the symmetry energy

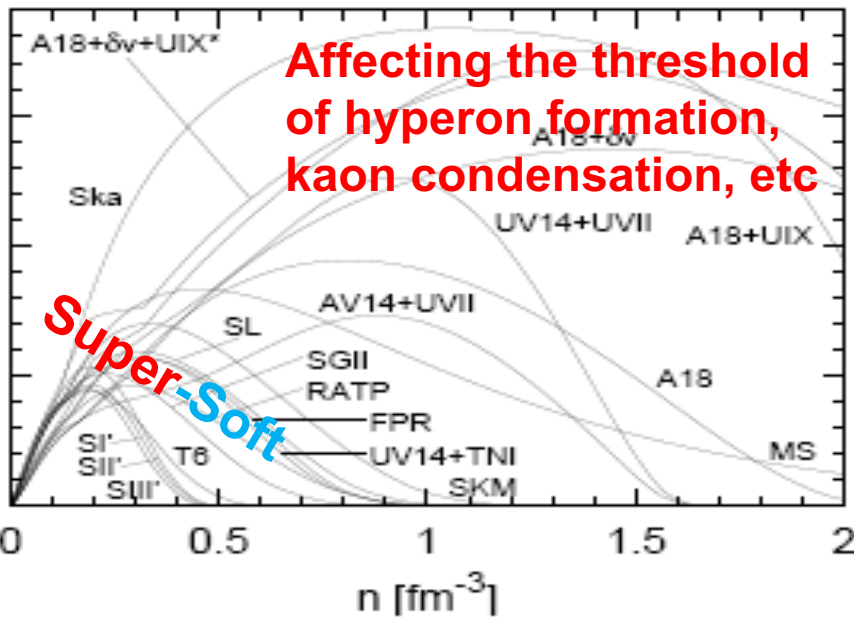


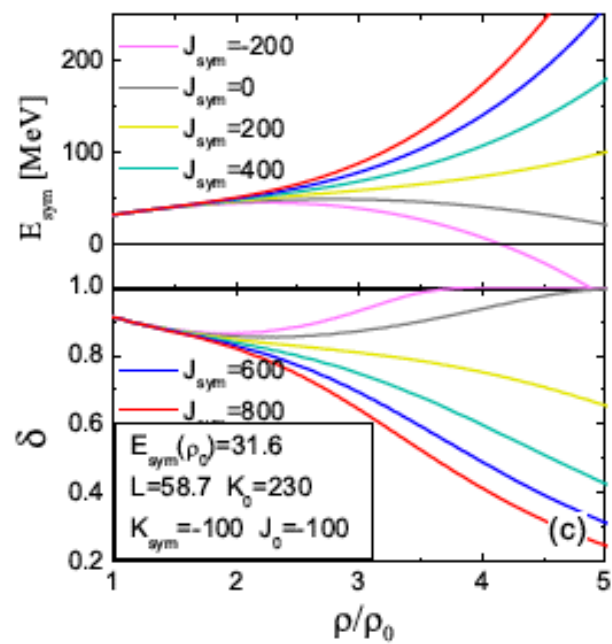
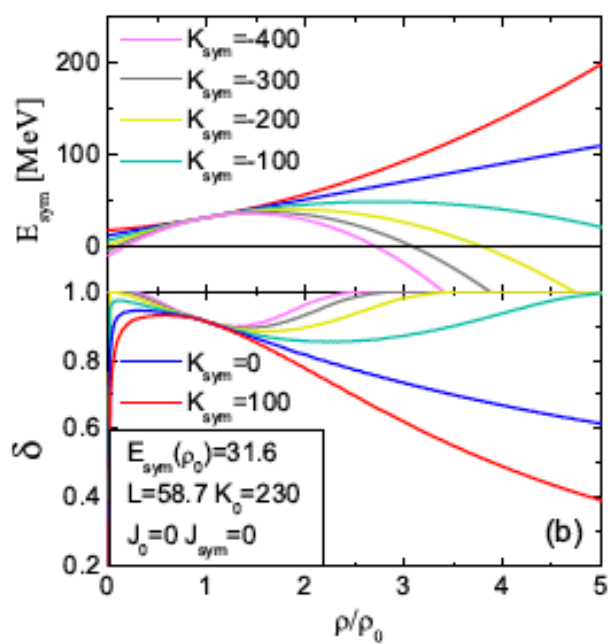
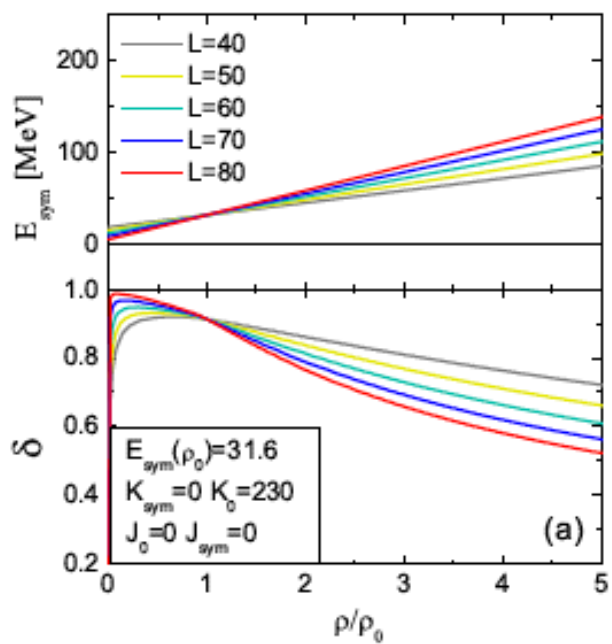
Tensor force and/or 3-body force can make E_{sym} negative at high densities

3-body force effects in Gogny or Skyrme HF

$$V_d = t_0(1 + x_0 P_\sigma) \rho^\alpha \delta(r)$$

$$E_{sym}^{TBF} = -(1 + 2x_0) \frac{t_0}{8} \rho^{\alpha+1}$$



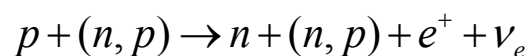
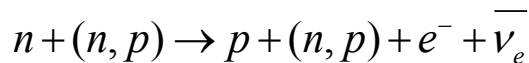


The proton fraction x at β -equilibrium in proto-neutron stars is determined by

$$x = 0.048 [E_{sym}(\rho) / E_{sym}(\rho_0)]^3 (\rho / \rho_0) (1 - 2x)^3$$

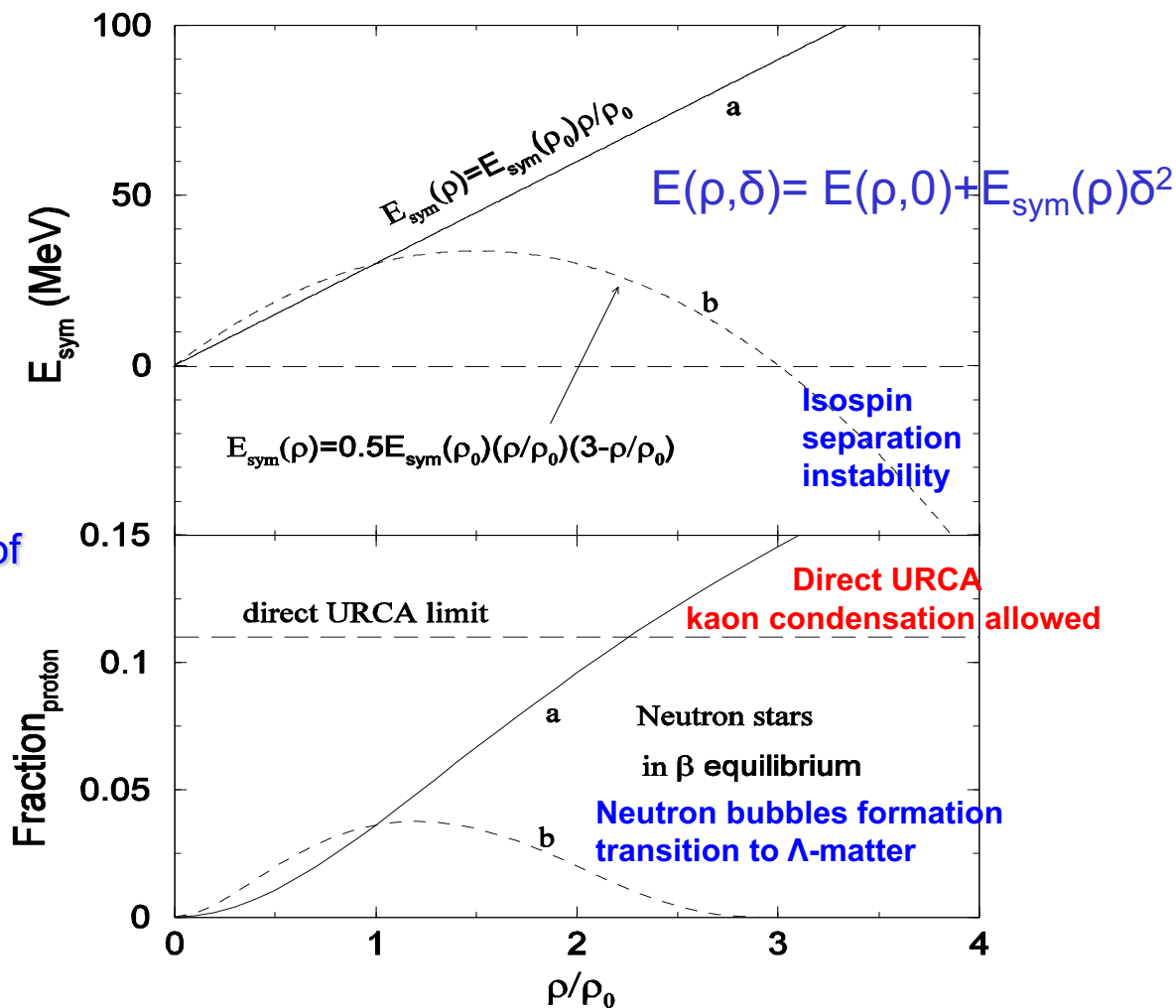
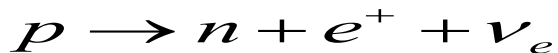
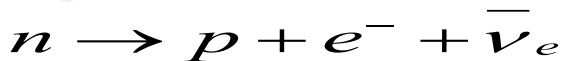
The critical proton fraction for direct URCA process to happen is $X_p = 0.14$ for npe μ matter obtained from energy-momentum conservation on the proton Fermi surface

Slow cooling: modified URCA:



Consequence: long surface thermal emission up to a few million years

Faster cooling by 4 to 5 orders of magnitude: direct URCA



News Release on Jan. 12, 2006:

Astronomers discover the fastest-spinning neutron-star

Scienceexpress

Report

A Radio Pulsar Spinning at 716 Hz

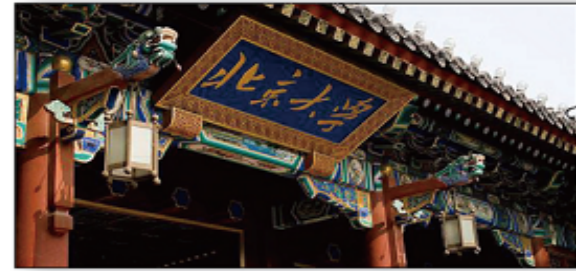
Science 311, 1901 (2006).

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We have discovered a 716-Hz eclipsing binary radio pulsar in the globular cluster Terzan 5 using the Green Bank Telescope. It is the fastest-spinning neutron star ever found, breaking the 23-year-old record held by the 642-Hz pulsar B1937+21. The difficulty in detecting this pulsar, due to its very low flux density and high eclipse fraction (~40% of the orbit), suggests that even faster-spinning neutron stars exist. If the pulsar has a mass less than 2 M_⊙, then its radius is constrained by the spin rate to be <16 km. The short period of this pulsar also constrains models that suggest gravitational radiation, through an r-mode instability, limits the maximum spin frequency of neutron stars.

spinning pulsars could have larger radii. Recently, Li and Steiner (18) have derived a radius range of 11.5–13.6 km for a 1.4 M_⊙ neutron star, based on terrestrial laboratory measurements of nuclear matter. For a 1.4 M_⊙ neutron star, we find an upper limit of 14.4 km, which is in agreement with their result. These radius constraints are more robust than those obtained through observations of neutron star thermal emission, which is faint, difficult to measure, and whose characterization depends on uncertain atmosphere models (19). Although in principle a radius measurement could constrain the unknown equation of state of dense matter, PSR J1748–2446ad does not rule out any particular existing models, since the pulsar mass is unknown. It is unlikely that a



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