Constraints on Neutron Star Sttructure and Equation of State from GW170817

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INT-JINA GW170817
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GW170817 Source Properties

90% confidence intervals

$$D = 40^{+8}_{-14}\ \mathrm{Mpc}$$

Chirp mass

$$\mathcal{M} = 1.188^{+0.004}_{-0.002} M_\odot$$

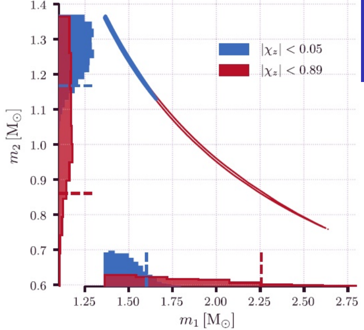
$$m_1 = 1.42^{+0.18}_{-0.06} M_\odot$$

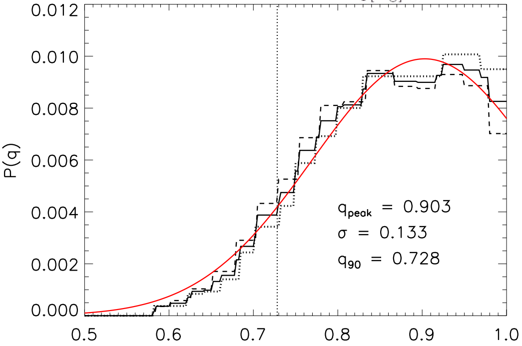
$$m_2 = 1.29^{+0.07}_{-0.13} M_\odot$$

$$q = \frac{m_2}{m_1} = 0.90^{+0.10}_{-0.17}$$

The binary tidal deformability

$$\bar{\Lambda} < 800$$



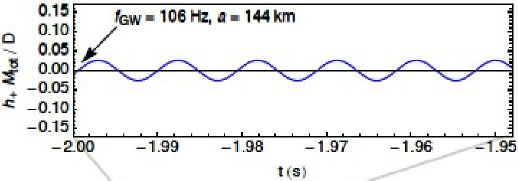


Probable Black Hole Formation in GW170817

- The GRB suggests a black hole formed within 1.75 s.
- Large ejected mass estimates imply any black hole formation was not prompt, but delayed by tenths of a second because a substantial disc wind was necessary.
- Most of the ejecta is inferred to have very high opacity, suggesting synthesis of nuclides between the 2nd and 3rd r-process peak. This implies low electron fractions in most of the ejecta, incompatible with long-term ($\gtrsim 0.3$ s) neutrino absorption and a long-lived neutron star.
- A long-lived but metastable neutron star supported by high rotation would pump large amounts of spin-down energy into the remnant, incompatible with the weak GRB and inferred moderate remant kinetic energy.
- Simulations show that there was too much angular momentum initially in the remnant for a uniformly-rotating star; it was differentially rotating.

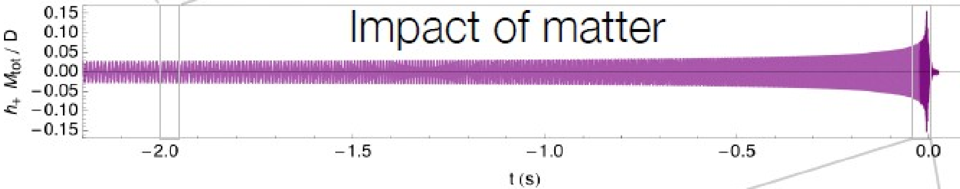
Maximum Mass Constraint

- Pulsar observations imply that slowly rotating neutron stars have a maximum mass $M_{max} \gtrsim 2M_\odot$.
- A uniformly rotating star has $M_{max,u} \simeq 1.17 - 1.20 M_{max}$. *Supramassive* stars, with $M_{max} \leq M \leq M_{max,u}$, are metastable but have long $t >> 0.1$ s lifetimes.
- A differentially-rotating star likely has $M_{max,d} \sim 1.5 M_{max}$. *Hypermassive* stars, with $M_{max,u} < M < M_{max,d}$, are metastable with short $t \sim 0.1$ s lifetimes.
- The chirp mass of GW170817, $\mathcal{M} = 1.18^{+0.04}_{-0.02} M_\odot$, means the total inspiralling mass $M_{tot} = m_1 + m_2$ is between $2.72 M_\odot$ $(q = m_2/m_1 = 1)$ and $2.78 M_\odot$ $(q = 0.7)$.
- Corrections for gravitational binding energy and mass loss suggest that $2.28 M_\odot \lesssim M_{rem} \lesssim 2.53 M_\odot$.
- To not initially be stabilized by uniform rotation implies $M_{max} \lesssim M_{rem}/1.17 \lesssim 2.16 M_\odot$.



Hard to modify inspiral: transfer of ~10^{46} erg at ~100 Hz modifies phase by 10^{-3} radians (Crust shattering, Tsang et al 1110.0467)

Impact of matter

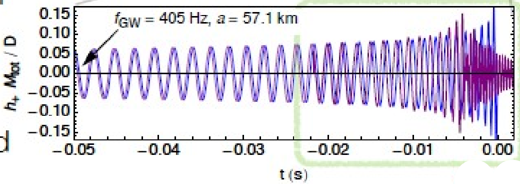


Tidal interactions lead to accumulated phase shift at higher frequencies.

$$\delta\Phi_t = -\frac{117}{256}\frac{(1+q)^4}{q^2}\left(\frac{\pi f_{GW} G\mathcal{M}}{c^3}\right)^{5/3}\bar{\Lambda}$$

For the final coalescence, numerical simulations are required

credit: Jocelyn Read

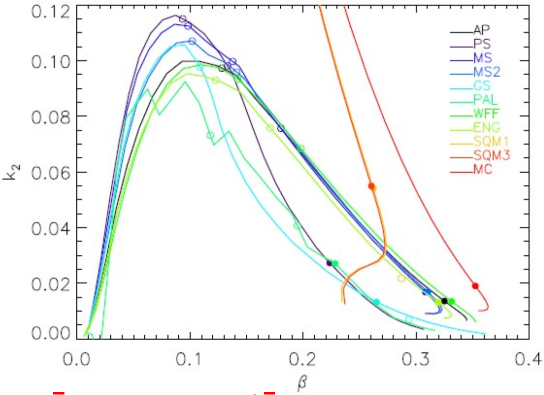


Tidal Deformability

Tidal deformability λ is the ratio between the induced dipole moment Q_{ij} and the external tidal field E_{ij}, $Q_{ij} \equiv -\lambda E_{ij}$,.

k_2 is the dimensionless Love number. It is convenient to work with the dimensionless $\bar{\lambda}$

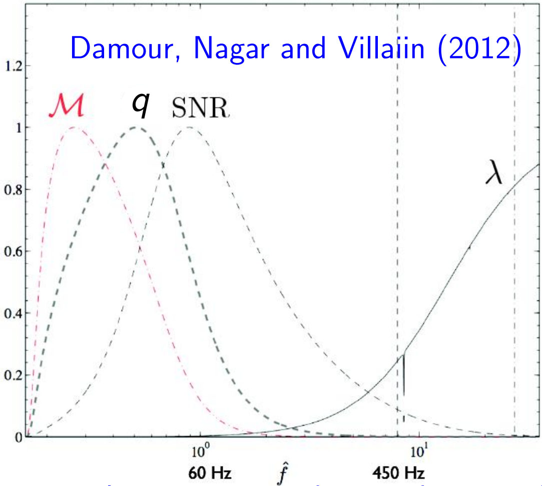
$$\bar{\lambda} = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \left(\frac{Rc^2}{GM} \right)^5$$



For a binary neutron star, the relevant quantity is $(q = m_2/m_1)$

$$\bar{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\bar{\lambda}_1 + (12 + q)q^4 \bar{\lambda}_2}{(1 + q)^5}.$$

When We Know What



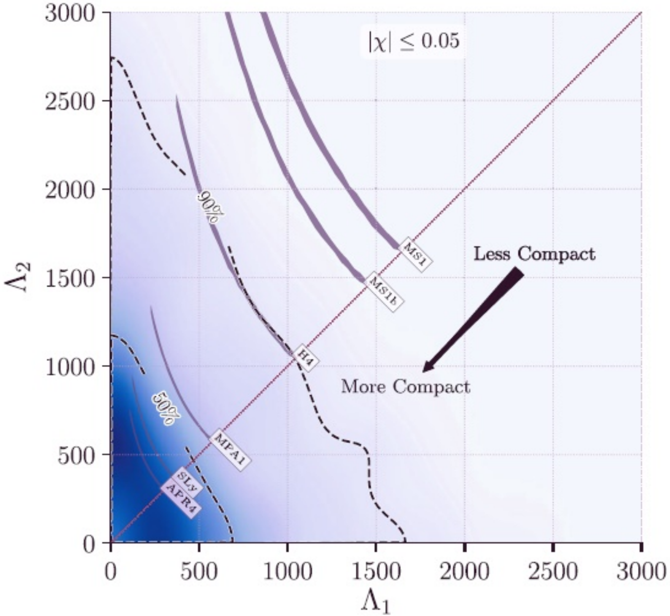
There are also spin-spin and spin-orbit contributions to $\delta\Phi$.
For spins aligned with $\vec{L}$, they act oppositely to $\delta\Phi_T$.
In a post-Newtonian expansion, $\delta\Phi_S$ is characterized by a single spin parameter β, primarily determined around 50 Hz.

LIGO/VIRGO Parameter Determination

Although there are 11 free wave-form parameters to post-Newtonian order, LIGO/VIRGO used 13 to fit their data:

- Sky location (2)
- Distance (1)
- Inclination (1)
- Coalescence time (1)
- Coalescence phase (1)
- Polarization (1)
- Component masses (2)
- Spin parameters (2)
- Tidal parameters (2)

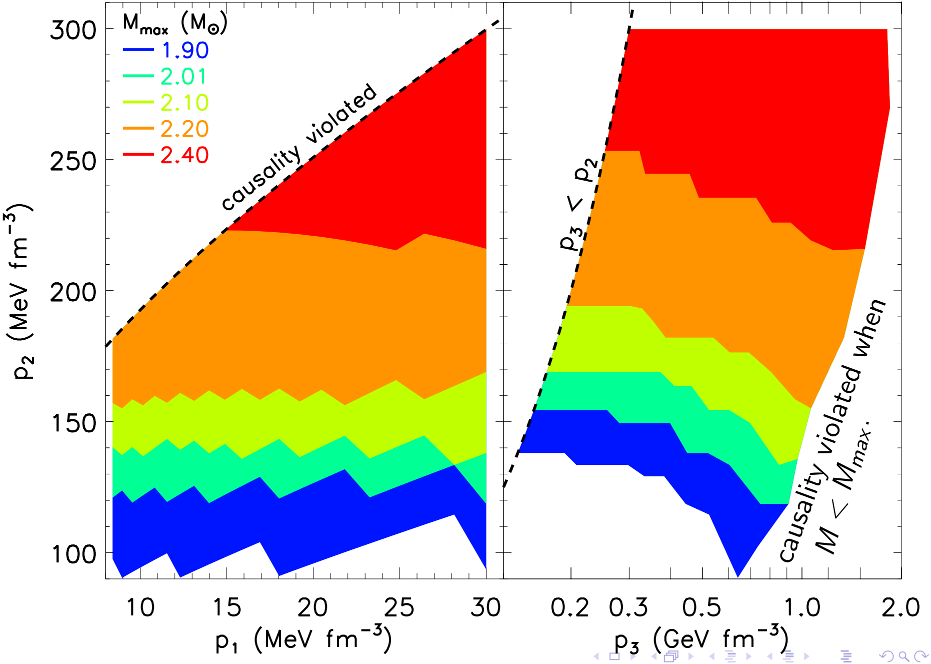
GW170817 Tidal Deformability Constraints



LIGO/VIRGO (2017)

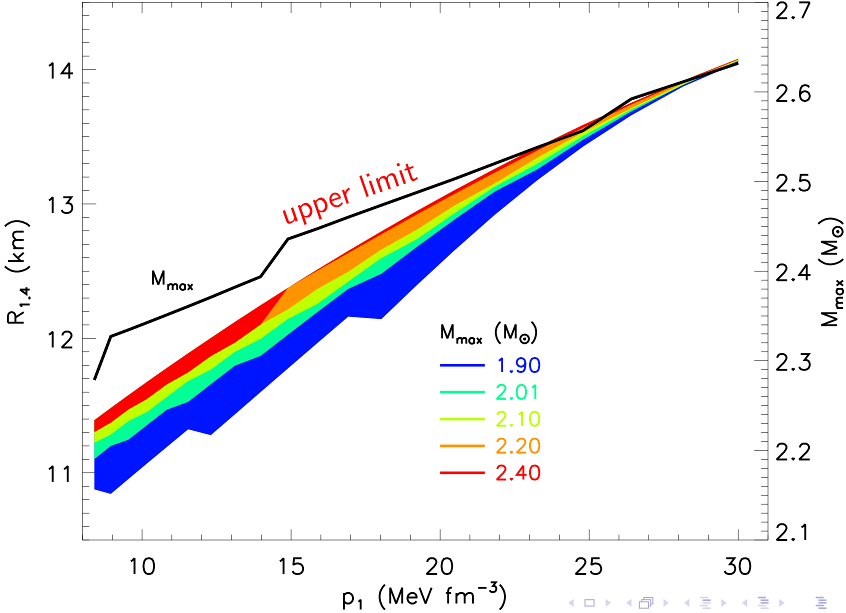
Piecewise Polytropic Equations of State

- For many reasons, it's believed neutron stars have hadronic crusts; the EOS is well-determined below $n_0 \sim 0.5 n_s$.
- $n_0 = n_s/2.7$, $p_0 = 0.2177$ MeV fm^{-3}, $\varepsilon_0 = 56.24$ MeV fm^{-3}.
- Read et al. found that $M - R$ is well-approximated with an EOS above n_0 containing as few as 3 polytropic segments.
- Read et al. found optimal upper boundaries (n_1, n_2, and $n_3 = 1.85 n_s$, $3.7 n_s$, and $7.4 n_s$) globally fit wide varieties of hadronic EOSs, leaving just 3 EOS parameters: p_1, p_2, and p_3.
- Neutron matter theory, nuclear experiment, and the unitary gas suggest that 8.4 MeV fm$^{-3} < p_1 <$ 20 MeV fm^{-3}, but we extend the upper limit to 30 MeV fm^{-3}. These limits imply $32 < S_v/\mathrm{MeV} < 38$ and $39 < L/\mathrm{MeV} < 85$.
- The parameters p_2 and p_3 are limited from above by causality and below by a maximum mass $1.9 M_\odot < M_{max} < 2.4 M_\odot$.
- The parameters p_1, p_2 and p_3 are uniformly sampled.

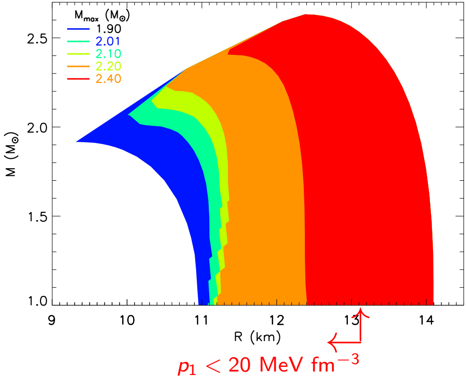


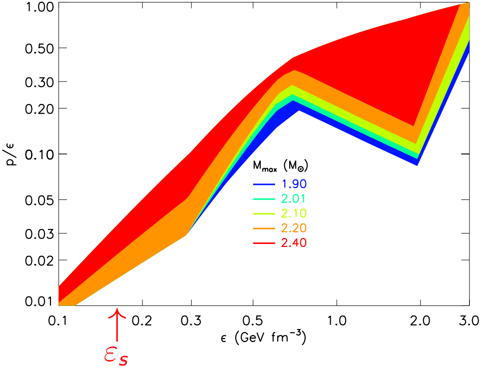
M_max (M_⊙)
1.90
2.01
2.10
2.20
2.40
causality violated
$p_3 < p_2$
causality violated when $M < M_{max}$.
p_2 (MeV fm^{-3})
p_1 (MeV fm^{-3})
p_3 (GeV fm^{-3})

The Radius-Pressure-M_{max} Correlations

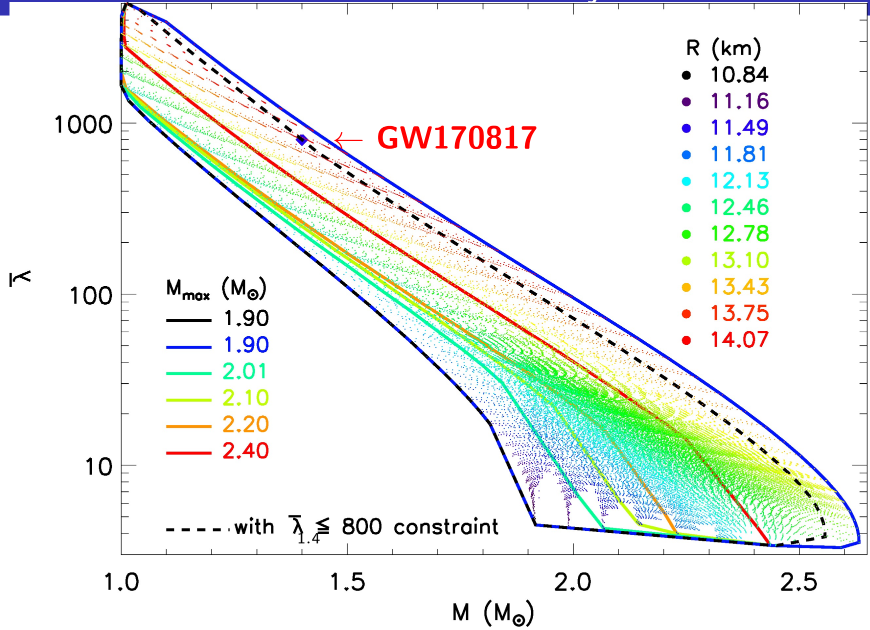


$M - R$ and EOS Constraints





Dimensionless Tidal Deformability



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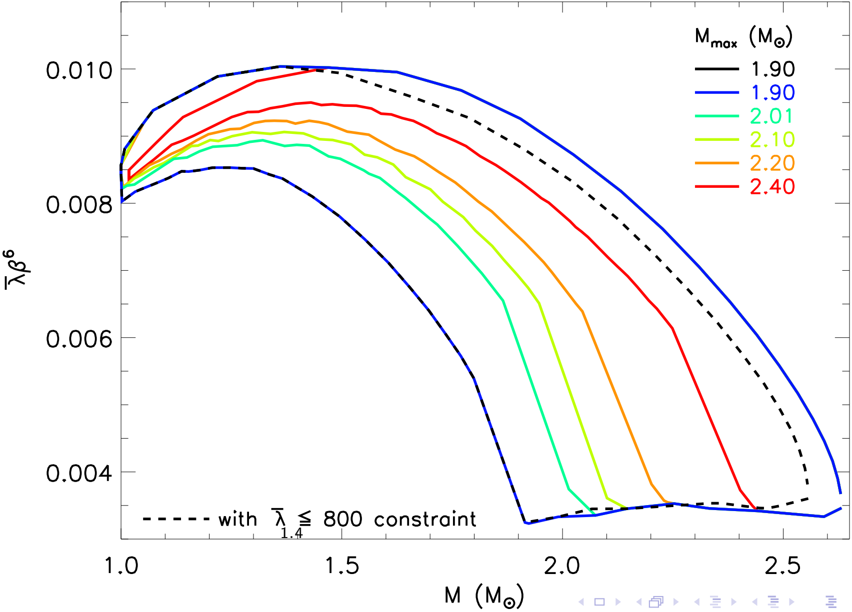


Using the $\bar{\lambda} \propto \beta^{-6}$ Correlation

Given that $k_2 \propto \beta^{-1}$ it is inevitable that $\bar{\lambda} \simeq a\beta^{-6}$. In the GW170817 mass range, $1.1 < M/M_\odot < 1.6$, piecewise polytropes give $a = 0.0093 \pm 0.0007$.

Furthermore, in this mass range, R is insensitive to M. As long as $M_{max} \gtrsim 2M_\odot$, $\Delta R = R_{1.6} - R_{1.1} < 0.46$ km, $< \Delta R >= -0.07$ km and $\sqrt{< \Delta R^2 >} = 0.11$ km. $-1.117 \leq (c^2/G)dR/dM \leq 0.261$, and $< dR/dM >= -0.134G/c^2$.

With the assumptions $\bar{\lambda} = a\beta^{-6}$ and $R_M = R_{1.4}$, one finds

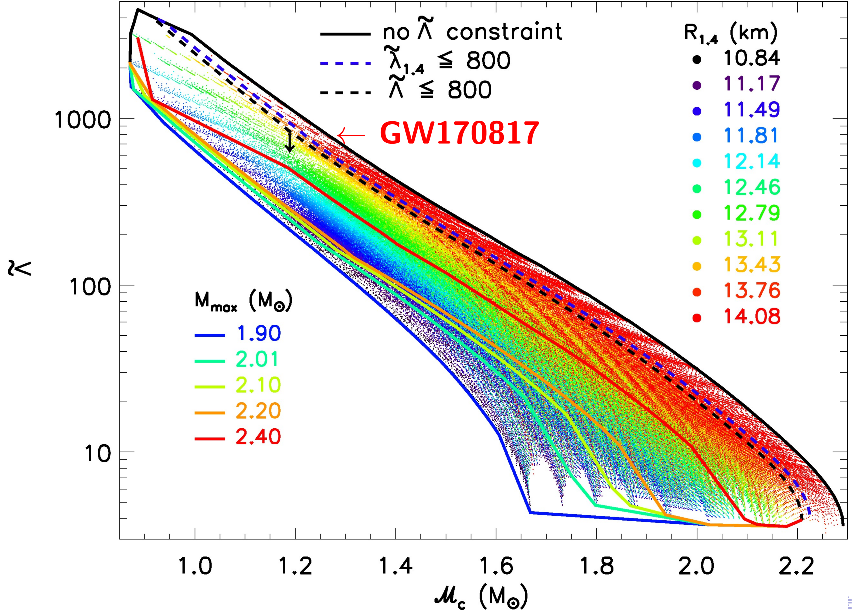
$$\bar{\Lambda} = \frac{16a}{13}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \frac{q^{8/5}}{(1+q)^{26/5}}(12 - 11q + 12q^2).$$

This is remarkably insensitive to q:

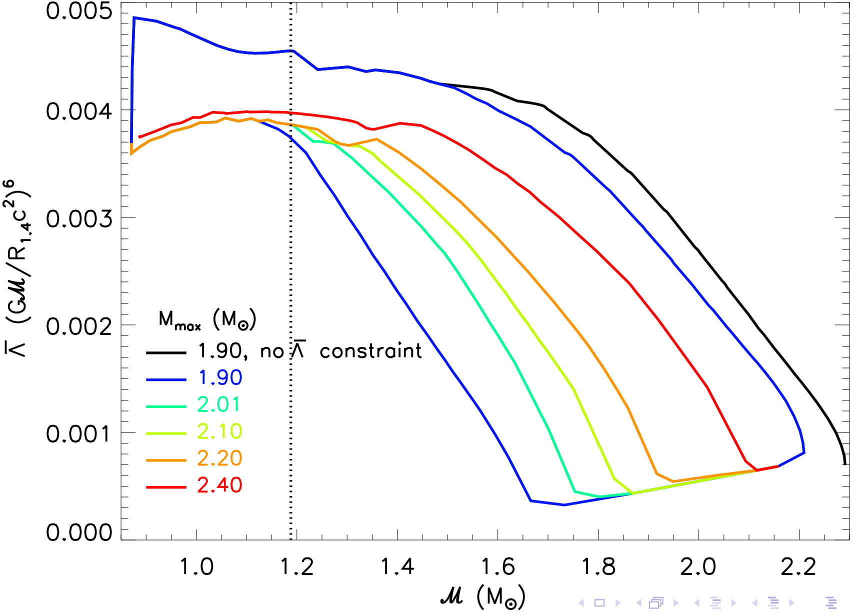
$$\frac{\partial\bar{\Lambda}}{\partial q} = \frac{16a}{65}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \frac{(1-q)q^{3/5}}{(1+q)^{31/5}}(96 - 263q + 96q^2),$$

which vanishes when $q = 1$. $\bar{\Lambda}(q = 0.7)/\bar{\Lambda}(q = 1) = 1.02$.

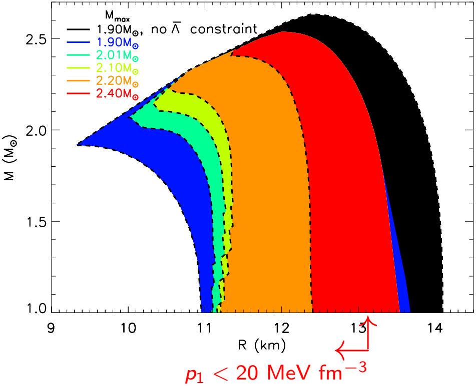
Dimensionless Binary Tidal Deformability

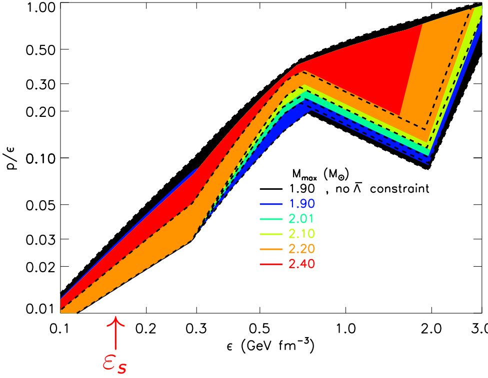


Dimensionless Binary Tidal Deformability

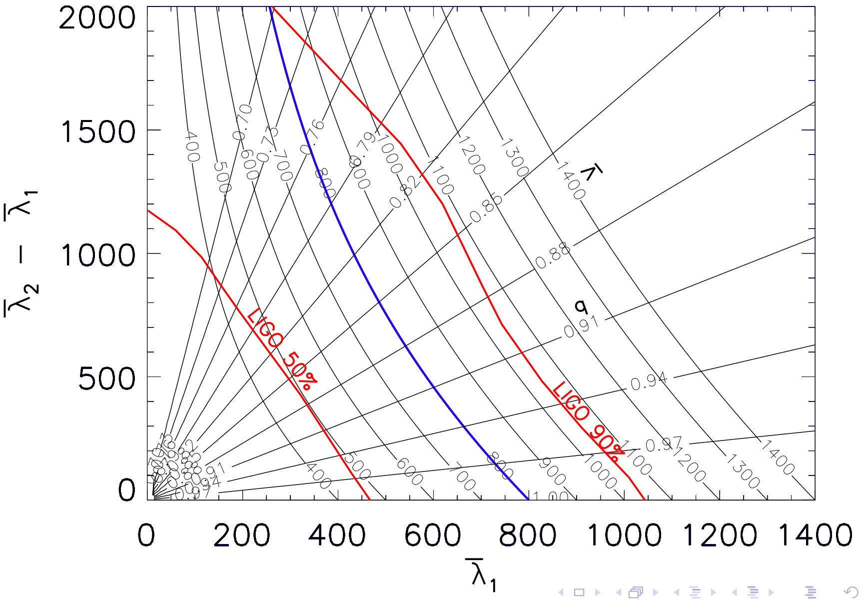


Modified $M-R$ and EOS Constraints





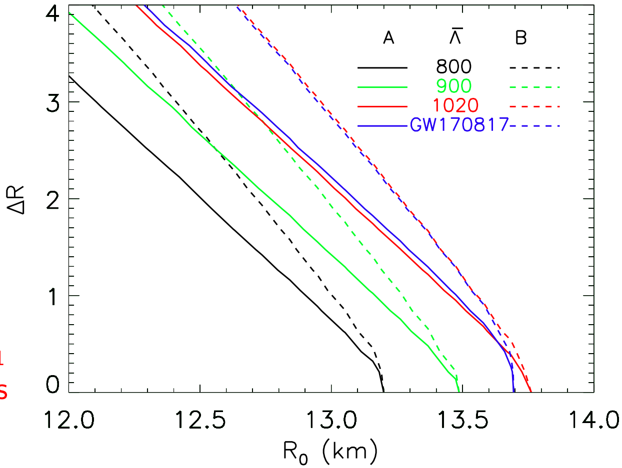
Tidal Deformabilities



The Bias of Uncorrelated Deformabilities

Randomly selecting R_1 and R_2 over a range of 3-4 km is similar to randomly selecting $\bar{\lambda}_1$ and $\bar{\lambda}_2$ within their natural ranges of 1000 or 2000 (model B).

Instead, randomly selecting $\bar{\lambda}_1$ and utilizing $\bar{\lambda}_2 = q^{-6}\bar{\lambda}_1$ (model A) decreases the 90% confidence contour of $\bar{\Lambda}$ by 100-150.



Conclusions from GW170817

- A constraint on $\bar{\Lambda}$ corresponds to a constraint on the neutron star radius in the GW170817 mass range:
$$R \simeq (3.69 \pm 0.04)\bar{\Lambda}^{1/6}(\mathcal{M}/M_\odot)\ \mathrm{km}.$$
$$dR \simeq 0.22(d\bar{\Lambda}/100)\ \mathrm{km}$$
- This correlation between $\bar{\Lambda}$ and R is tight because $\bar{\Lambda}$ is insensitive to q, a poorly-determined quantity.
- The quoted constraint $\bar{\Lambda} < 700 - 800$ is not justified by the $\bar{\lambda}_1 - \bar{\lambda}_2$ constraints; its too small by 250-350 due to $\bar{\lambda}_1 - \bar{\lambda}_2$ correlations, even considering hybrid (twin) stars..
- Spin priors with negative values correspond to spins anti-aligned with $\vec{L}$, which is physically improbable except for systems formed by capture. Such priors overestimate $\bar{\Lambda}$.
- Failure to include the natural correlation between $\bar{\lambda}_1$ and $\bar{\lambda}_2$, and that $\bar{\lambda}_2 \geq \bar{\lambda}_1$, overestimates $\bar{\Lambda}$ by 100-150
- An upper limit to M_{max} does not constrain neutron star radii.