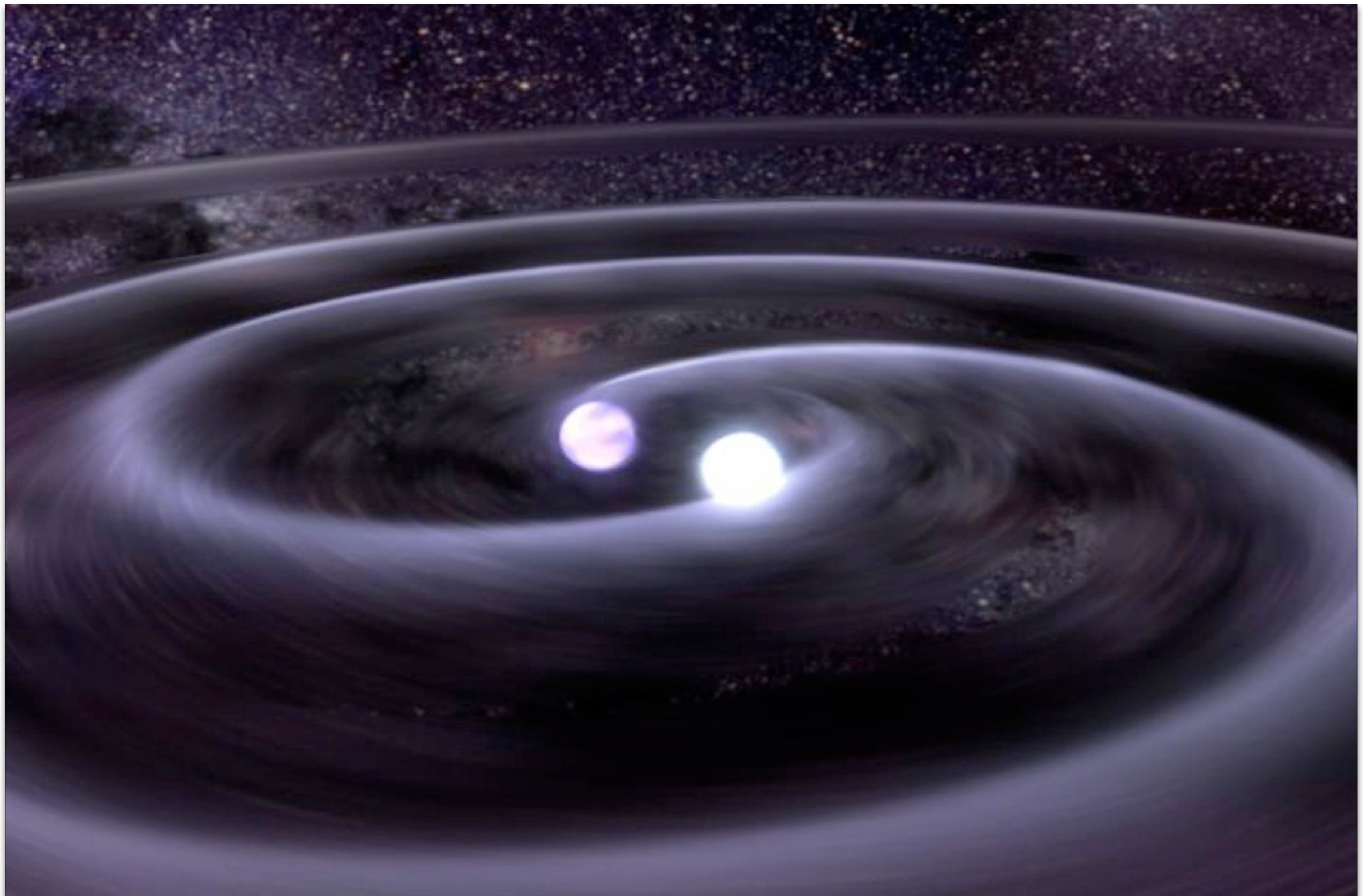
Improving waveform models for measuring the parameters of GW170817



Ben Lackey, Michael Pürrer, Andrea Taracchini, Sylvain Marsat

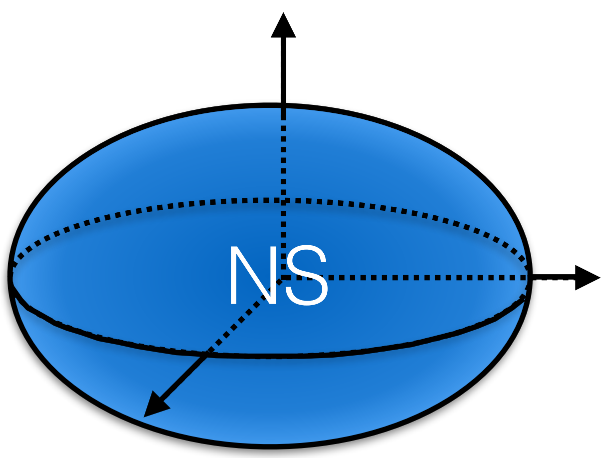
Albert Einstein Institute-Potsdam, Germany

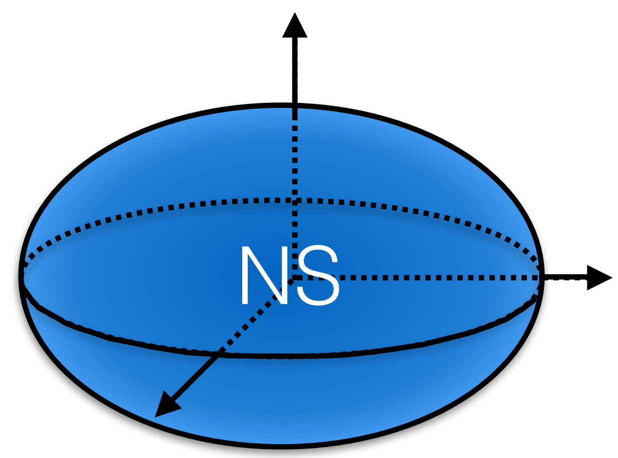
University of Washington, 12 March 2018

Measuring matter effects with BNS inspiral

- Tidal field $\mathcal{E}_{ij}$ from companion star induces a quadrupole moment Q_{ij} in the NS
- Amount of deformation depends on stiffness of EOS via the tidal deformability Λ:

$$Q_{ij} = -\Lambda(\mathrm{EOS}, m) m^5 \mathcal{E}_{ij}$$

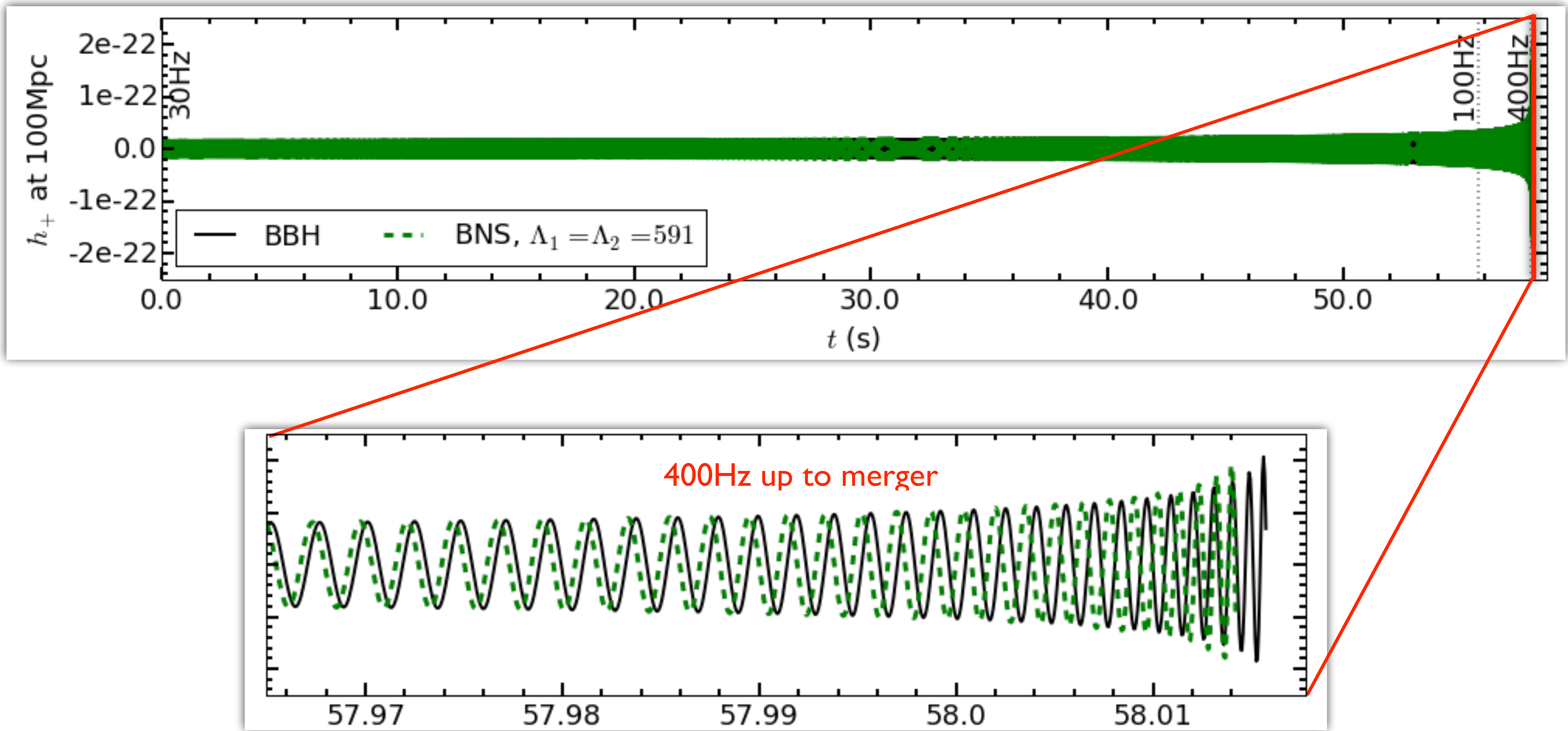




Measuring matter effects with BNS inspiral

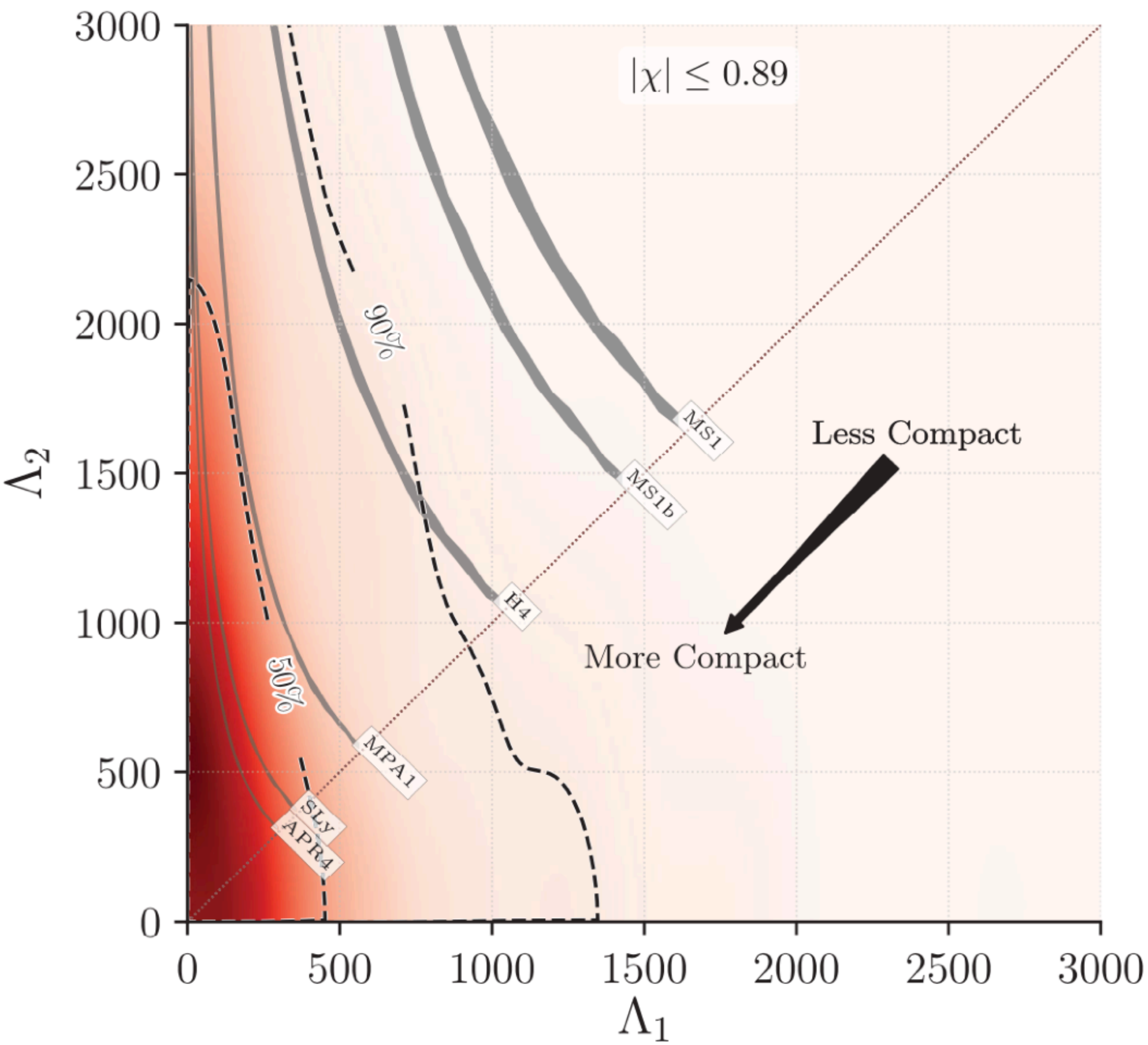
- Tidal effect enters at 5th post-Newtonian order
- Results in phase shift of ~10 radians

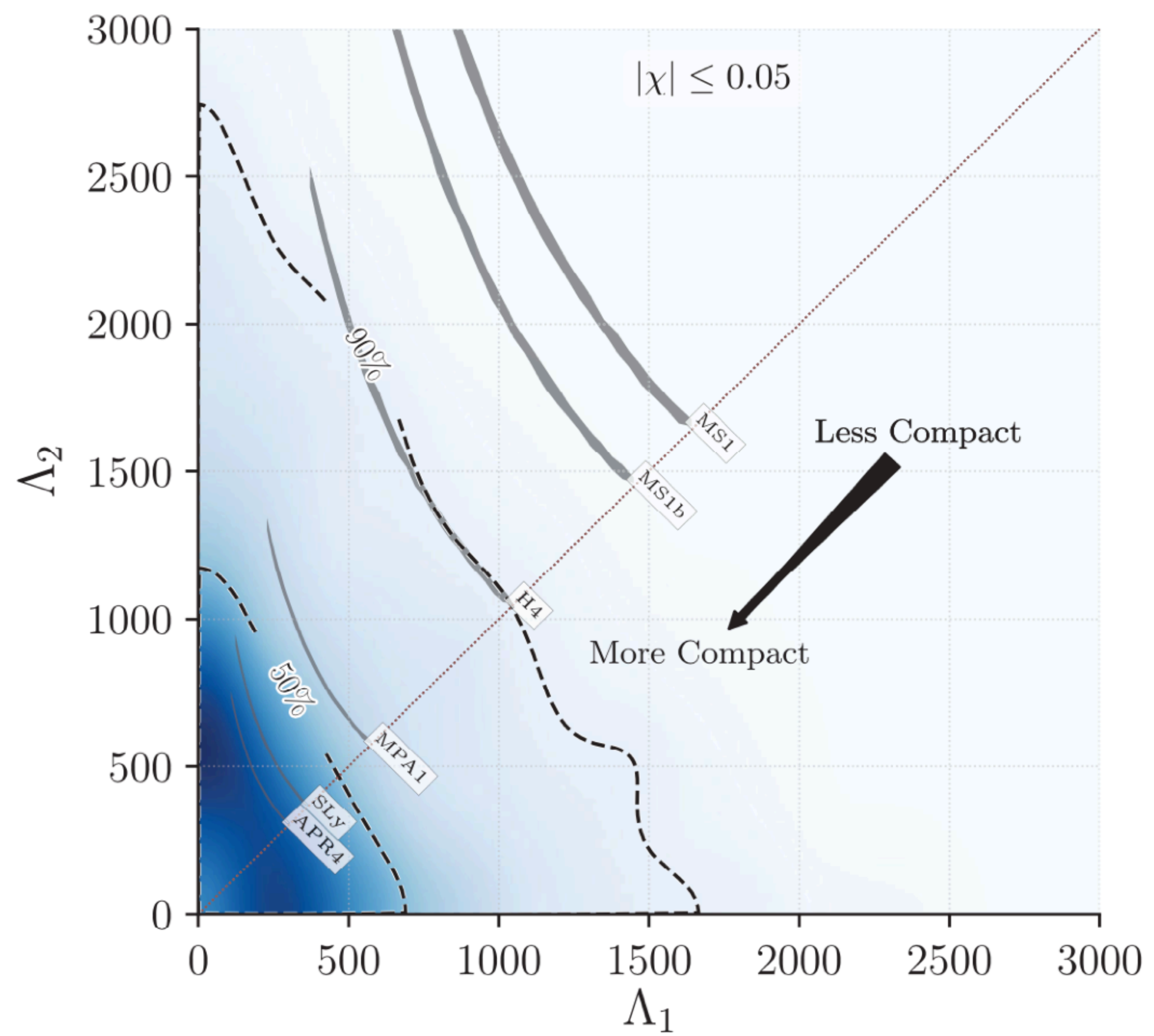
$$\Phi(f) = 0\mathrm{PN}(f;\mathcal{M})\,[1 + \underbrace{1\mathrm{PN}(f;\eta)}_{(v/c)^2} + 1.5\mathrm{PN}(f;\eta,S_1,S_2) + \cdots + \underbrace{3.5\mathrm{PN}(f;\eta,S_1,S_2)}_{(v/c)^7} + \underbrace{5\mathrm{PN}(f;\eta,\Lambda_1,\Lambda_2)}_{(v/c)^{10}}]$$



Initial results for GW170817

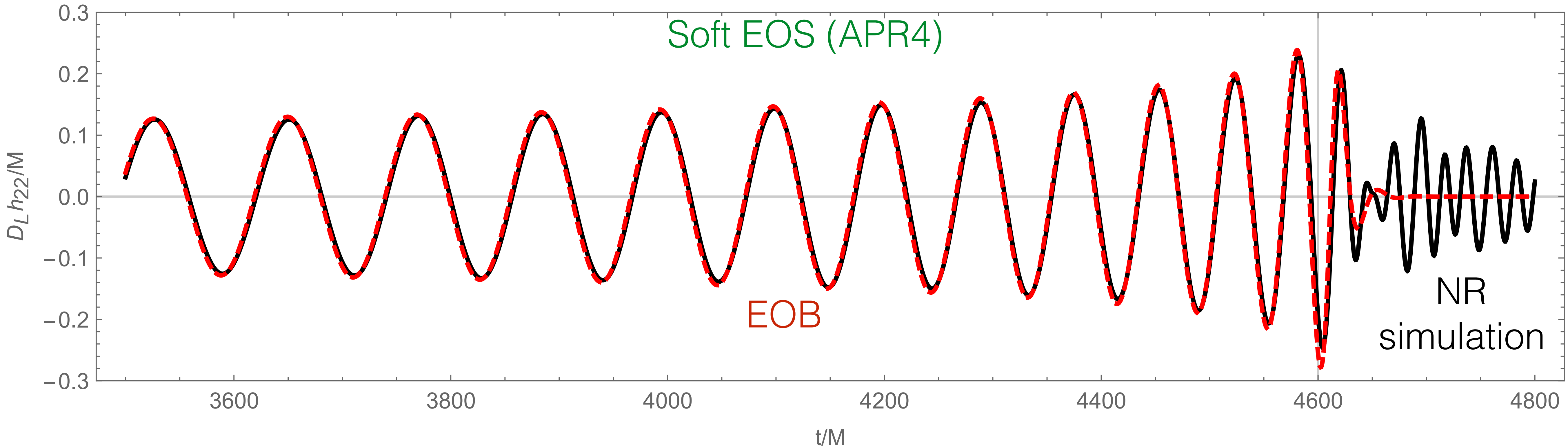
- Initial results used fast, frequency-domain PN waveform (TaylorF2)
- But, waveform errors and unmodeled effects can affect results
- Results should be updated in <1 month





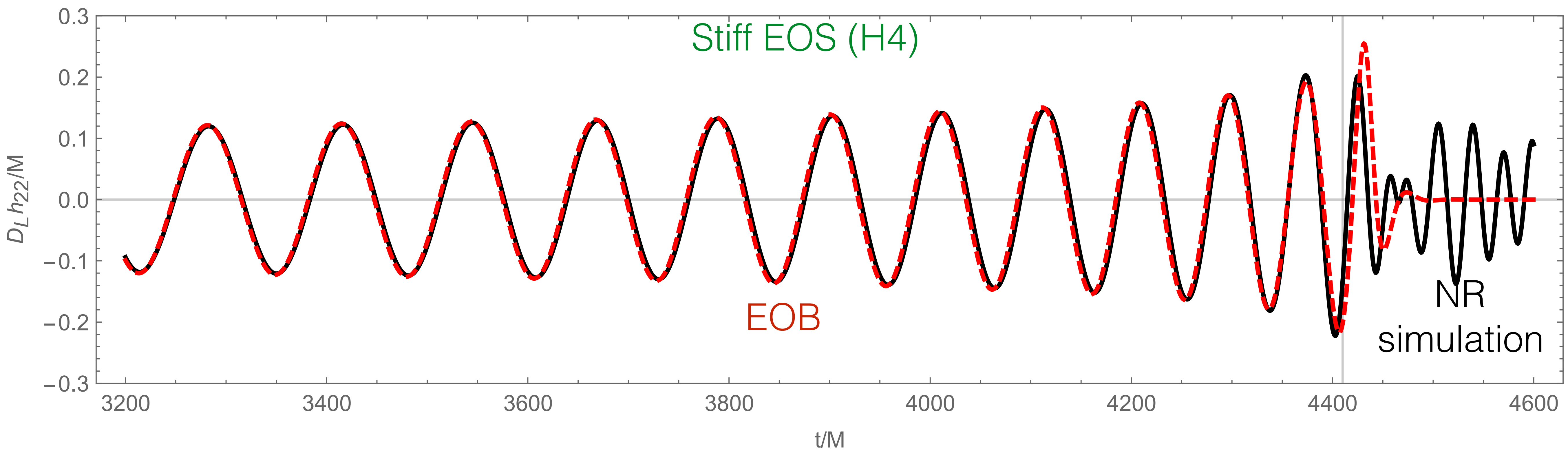
Spin-tidal-EOB waveform model

- Effective-one-body model with aligned spin and dynamic tides
  T. Hinderer et al. PRL 116, 181101 (2016)
  - $\ell = 2, 3$ tidal parameters
  - $\ell = 2, 3$ f-mode frequencies
  - Spin-induced quadrupole moment (2PN effect)
  - End is tapered to agree with numerical BNS simulations
  - 14-dimensional parameter space (1 mass, 1 spin, 5 matter parameters per NS)



Spin-tidal-EOB waveform model

- Effective-one-body model with aligned spin and dynamic tides
  T. Hinderer et al. PRL 116, 181101 (2016)
  - $\ell = 2, 3$ tidal parameters
  - $\ell = 2, 3$ f-mode frequencies
  - Spin-induced quadrupole moment (2PN effect)
  - End is tapered to agree with numerical BNS simulations
  - 14-dimensional parameter space (1 mass, 1 spin, 5 matter parameters per NS)



Spin-tidal-EOB waveform model

- Effective-one-body model with aligned spin and dynamic tides
  T. Hinderer et al. PRL 116, 181101 (2016)
  - $\ell = 2, 3$ tidal parameters
  - $\ell = 2, 3$ f-mode frequencies
  - Spin-induced quadrupole moment (2PN effect)
  - End is tapered to agree with numerical BNS simulations
  - 14-dimensional parameter space (1 mass, 1 spin, 5 matter parameters per NS)
- Can reduce number of parameters to 5: $\mathbf{x} = \{q, S_1, S_2, \Lambda_1, \Lambda_2\}$
  - Rescale waveform with total mass ***M***, and just use mass ratio ***q***
  - Universal relations relate $\ell = 3$ tidal parameter, $\ell = 2, 3$ f-mode frequencies, spin-induced quadrupole to $\ell = 2$ tidal parameter (Kent Yagi. PRD 89, 043011 (2014))

Surrogate model overview

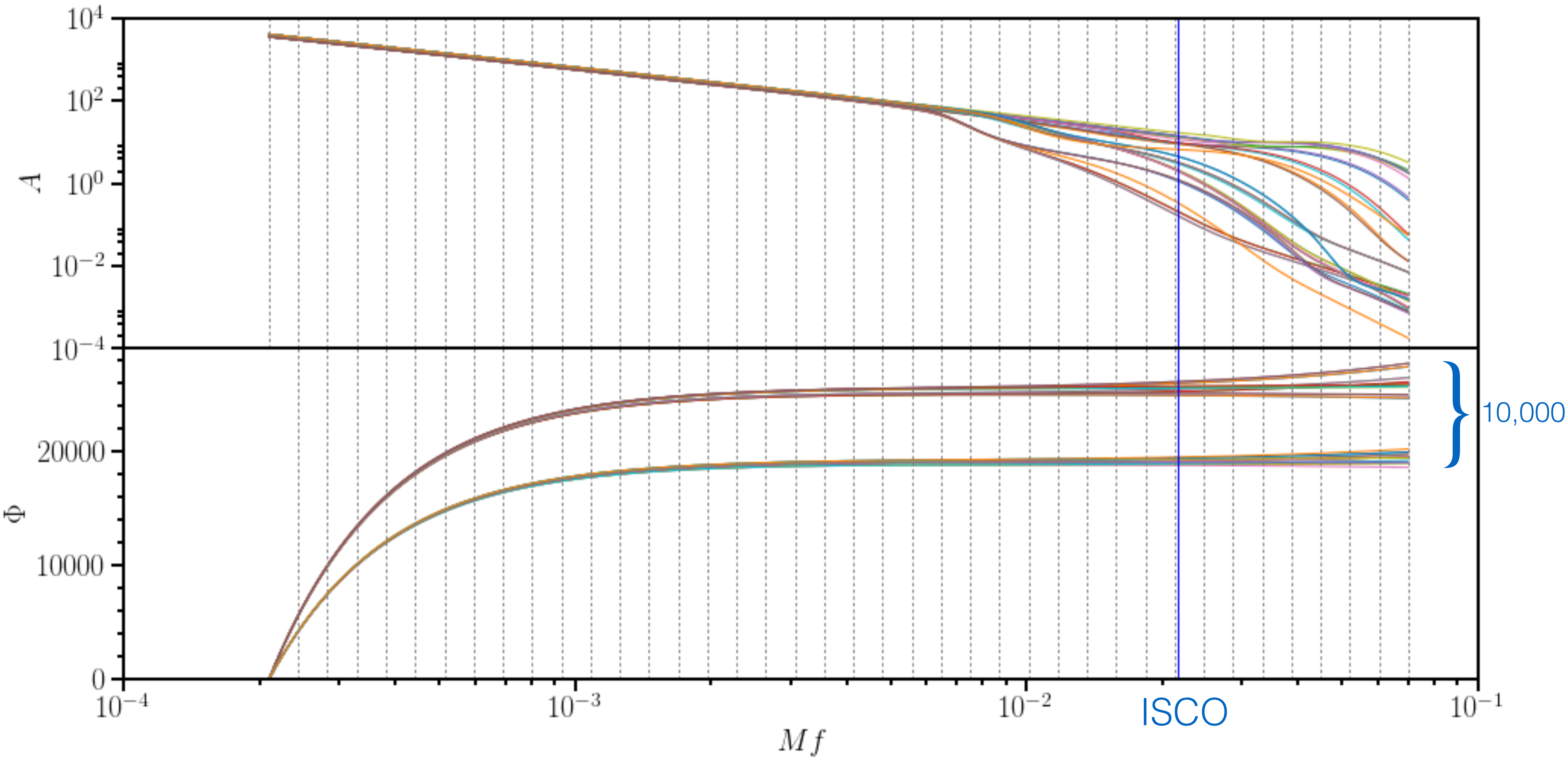
- Interpolate amplitude and phase of the accurate waveform as a function of frequency ***f*** and waveform parameters $\mathbf{x}$:

$$\tilde{h}(f;\mathbf{x}) = A(f;\mathbf{x})e^{i\Phi(f;\mathbf{x})}$$

  - Interpolate between frequency nodes F_j with cubic splines
  - Interpolate amplitude and phase at nodes F_j as functions of $\mathbf{x}$
    - Rectangular grid is prohibitive in 5-dimensions ($\sim 10^5$ points)
    - Gaussian Process Regression (GPR) does not require rectangular grid

Frequency-domain waveforms

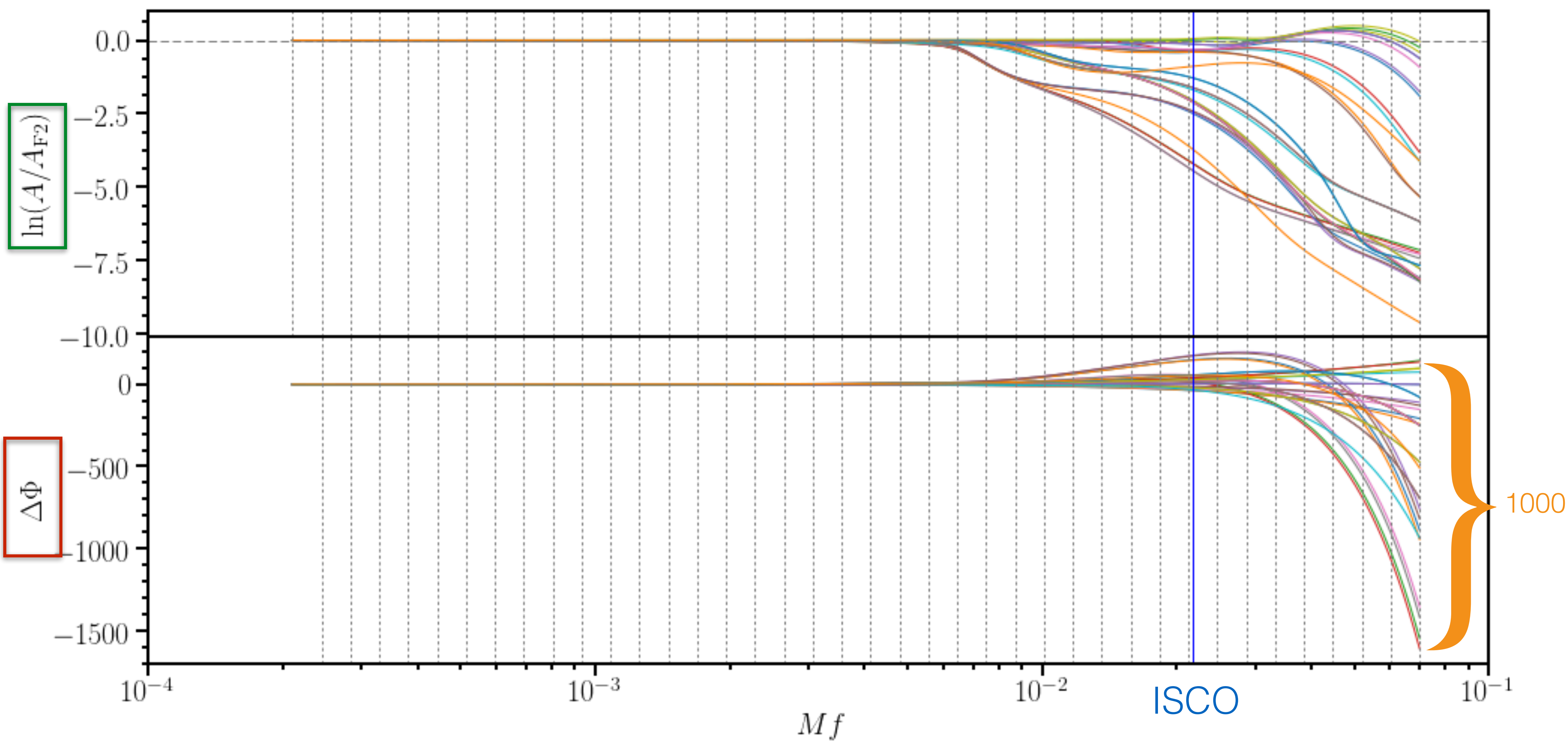
- Phase of waveform spans ~10,000 radians over parameter space
- Requires interpolation accuracy of 10^{-5} for 0.1rad phase error



Frequency-domain residual

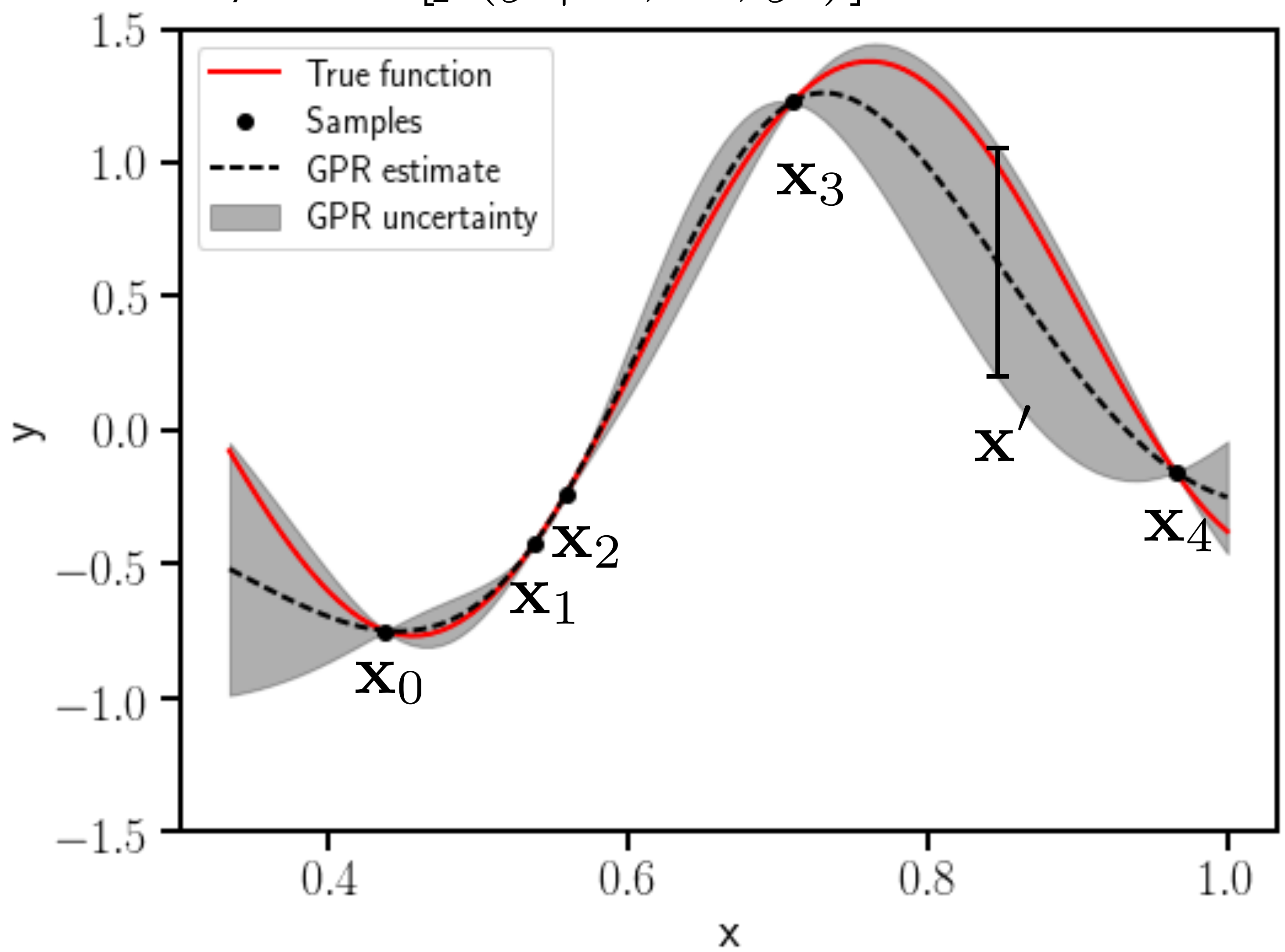
- Phase of residual relative to TaylorF2 spans ~100 radians
- Requires interpolation accuracy of 10^{-3}–10^{-4} for 0.1rad phase error

$$\tilde{h} = \tilde{h}_{\mathrm{F2}} e^{\Delta \ln A + i \Delta\Phi}$$



Gaussian Process Regression (GPR)

- GPR assumes the values of a function y_i at the points $\mathbf{x}_i$ are random variables that follow a multivariate Gaussian: $p(y_i|\mathbf{x}_i) = \mathcal{N}(0, k(\mathbf{x}_i, \mathbf{x}_j))$
  - Covariance between points $k(\mathbf{x}_i, \mathbf{x}_j)$
  - Can estimate the functional value y' at $\mathbf{x}'$ given sampled data $(\mathbf{x}_i, y_i)$
    - Mean: $E[p(y'|\mathbf{x}_i, \mathbf{x}', y_i)]$
    - Uncertainty: $\mathrm{Var}[p(y'|\mathbf{x}_i, \mathbf{x}', y_i)]$



Gaussian Process Regression (GPR)

- GPR assumes the values of a function y_i at the points $\mathbf{x}_i$ are random variables that follow a multivariate Gaussian: $p(y_i|\mathbf{x}_i) = \mathcal{N}(0, k(\mathbf{x}_i, \mathbf{x}_j))$
  - Covariance between points $k(\mathbf{x}_i, \mathbf{x}_j)$
  - Can estimate the functional value y' at $\mathbf{x}'$ given sampled data $(\mathbf{x}_i, y_i)$
    - Mean: $E[p(y'|\mathbf{x}_i, \mathbf{x}', y_i)]$
    - Uncertainty: $\mathrm{Var}[p(y'|\mathbf{x}_i, \mathbf{x}', y_i)]$
- Covariance (kernel) here assumes function is twice differentiable:

$$k(x_i, x_j) = \boxed{\sigma_f^2} k_{\mathrm{Matern}}^{\nu=5/2}(r) + \boxed{\sigma_n^2} \delta_{ij}$$

$$k_{\mathrm{Matern}}^{\nu=5/2}(r) = \left(1 + \sqrt{5}r + \frac{5r^2}{3}\right) \boxed{\exp\left(-\sqrt{5}r\right)}$$

Correlation decreases with distance

$$r^2 = (x - x')^T M (x - x')$$

$$M = \mathrm{diag}(\boxed{\ell_q^{-2}, \ell_{S_1}^{-2}, \ell_{S_2}^{-2}, \ell_{\Lambda_1}^{-2}, \ell_{\Lambda_2}^{-2}})$$

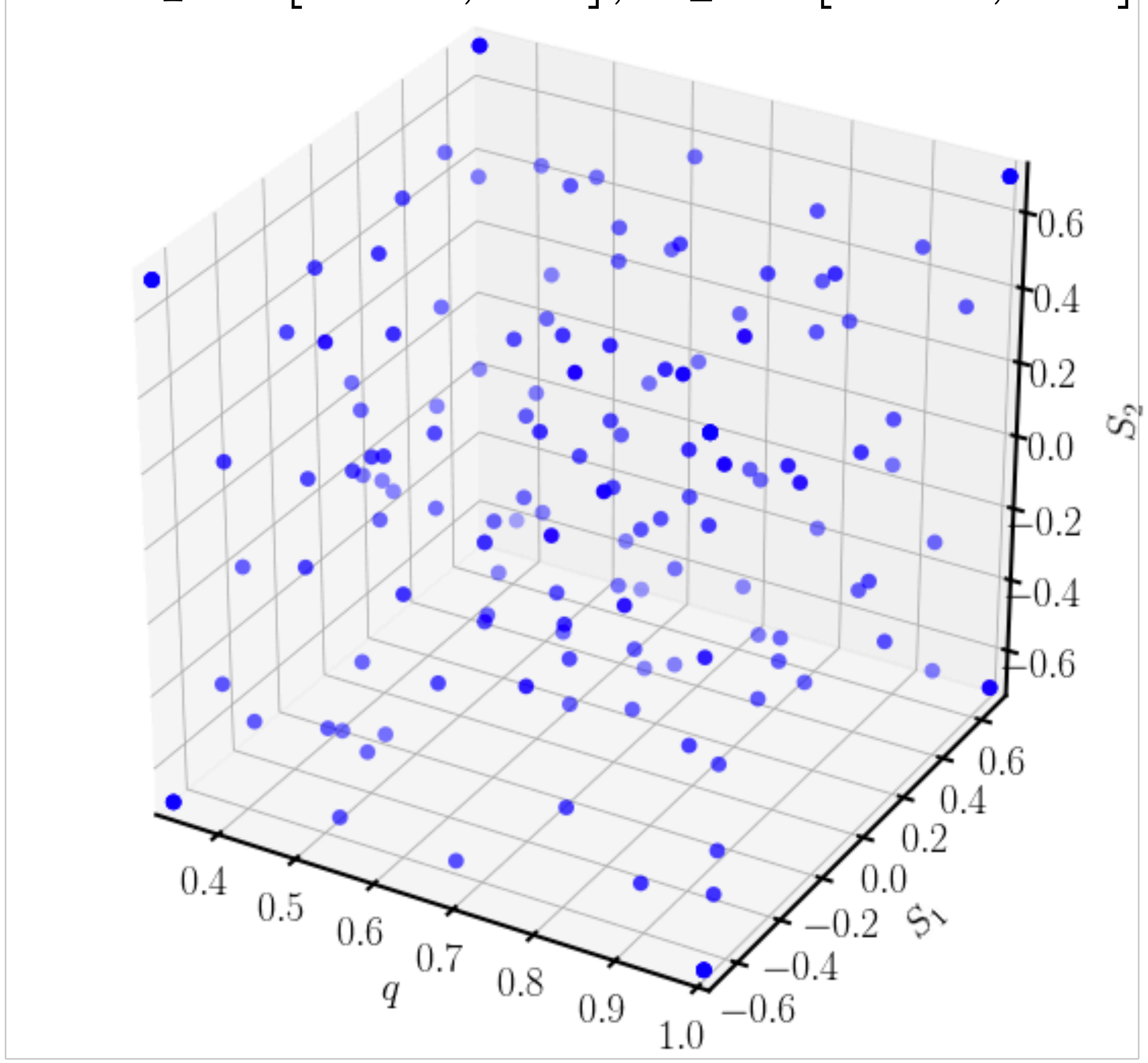
Tunable hyperparameters

Initial training set

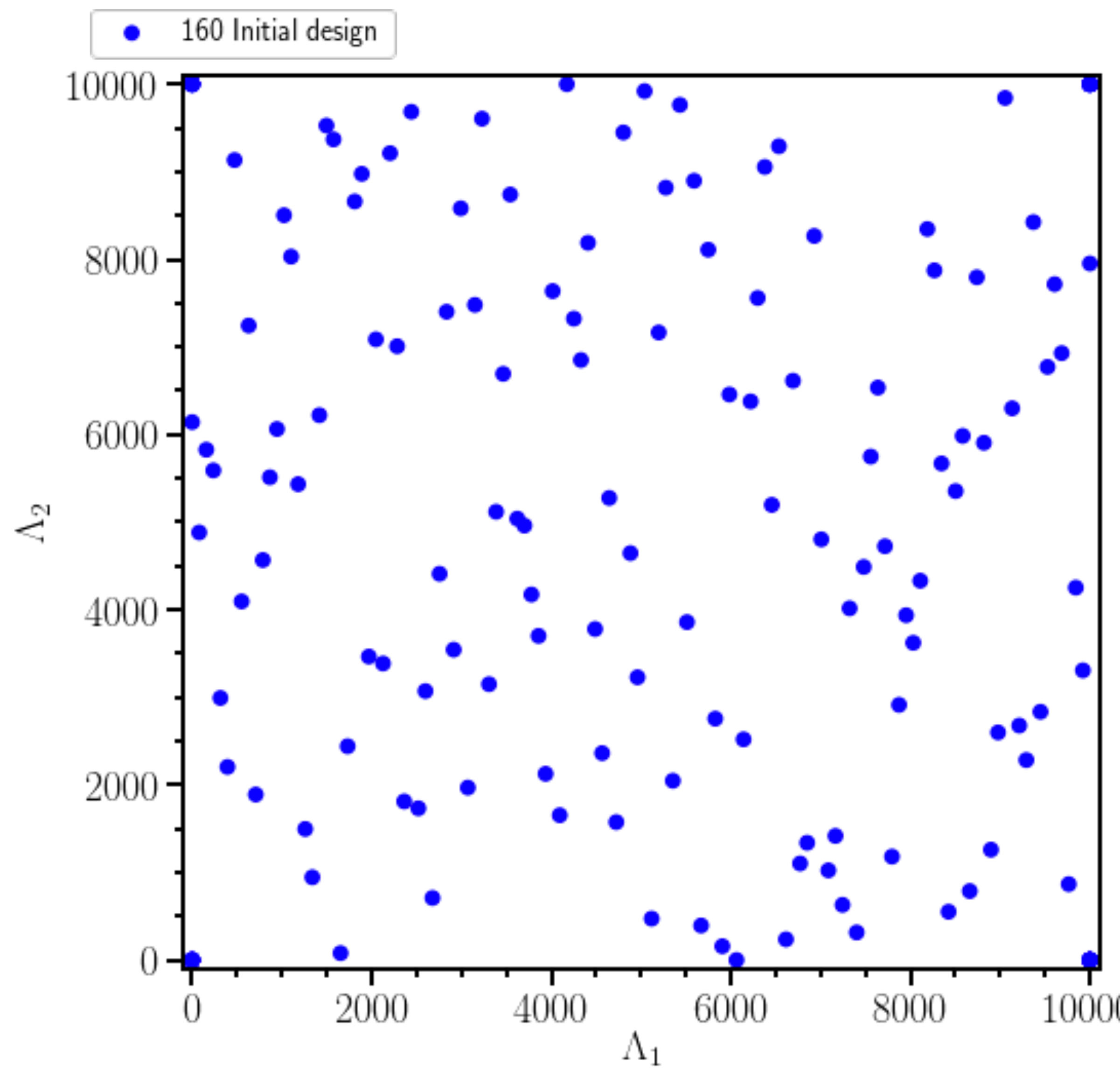
- Draw 128 points from space-filling latin hypercube design (LHD)
- Add the 32 corners of parameter space

$$q \in [1/3, 1]$$

$$S_1 \in [-0.5, 0.5], S_2 \in [-0.5, 0.5]$$

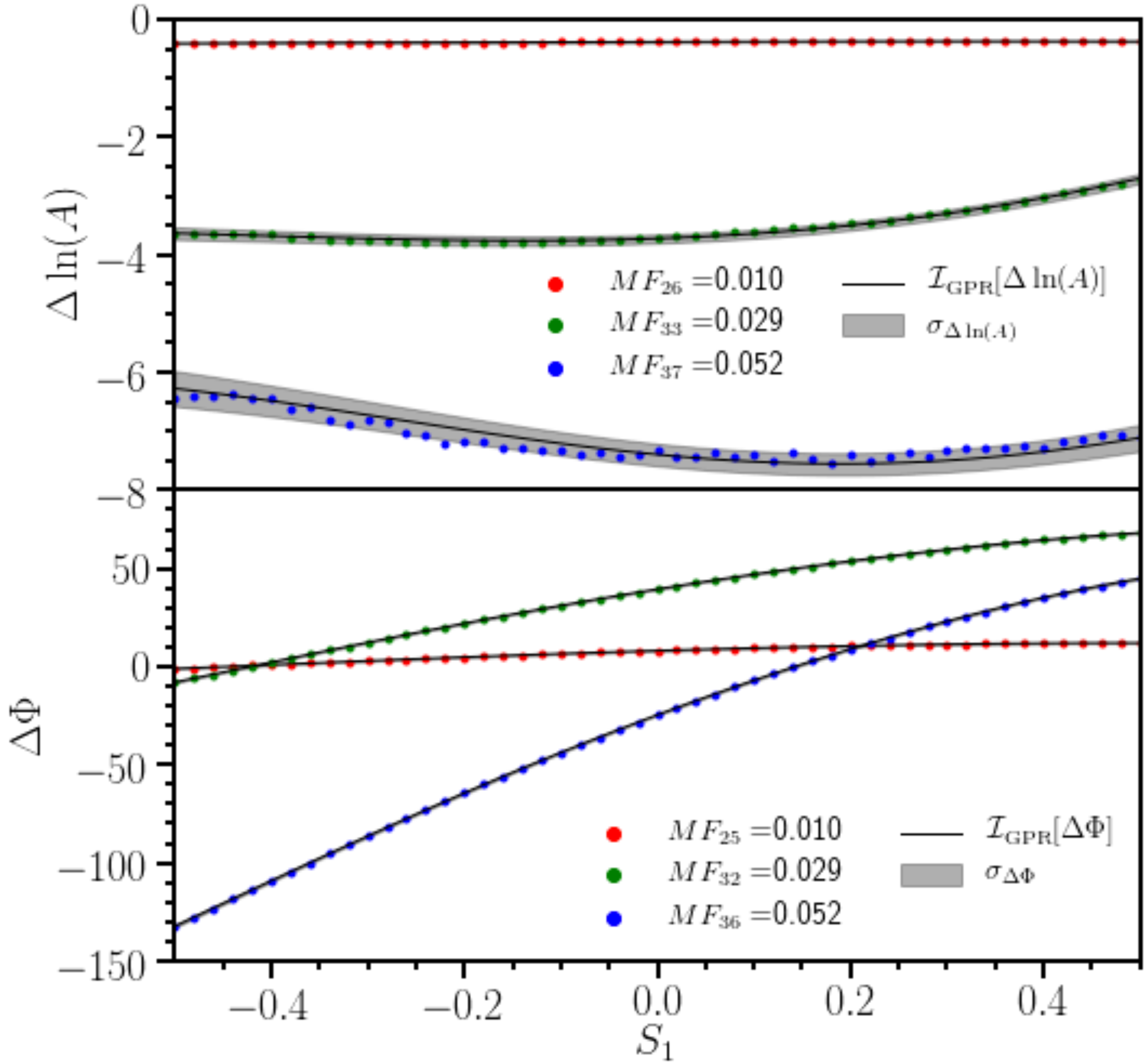


$$\Lambda_1 \in [0, 5000], \Lambda_2 \in [0, 5000]$$



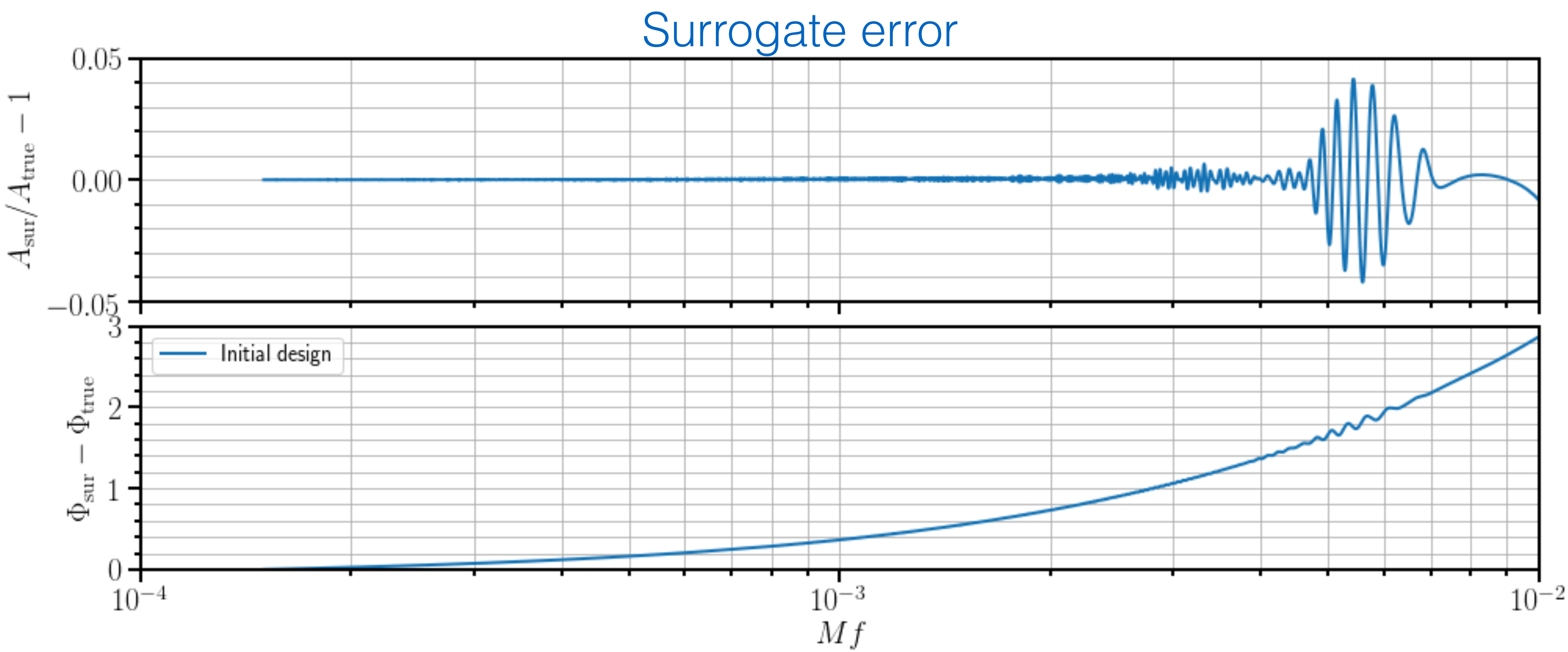
Gaussian Process Regression (GPR)

- Taking 1d slice through parameter space



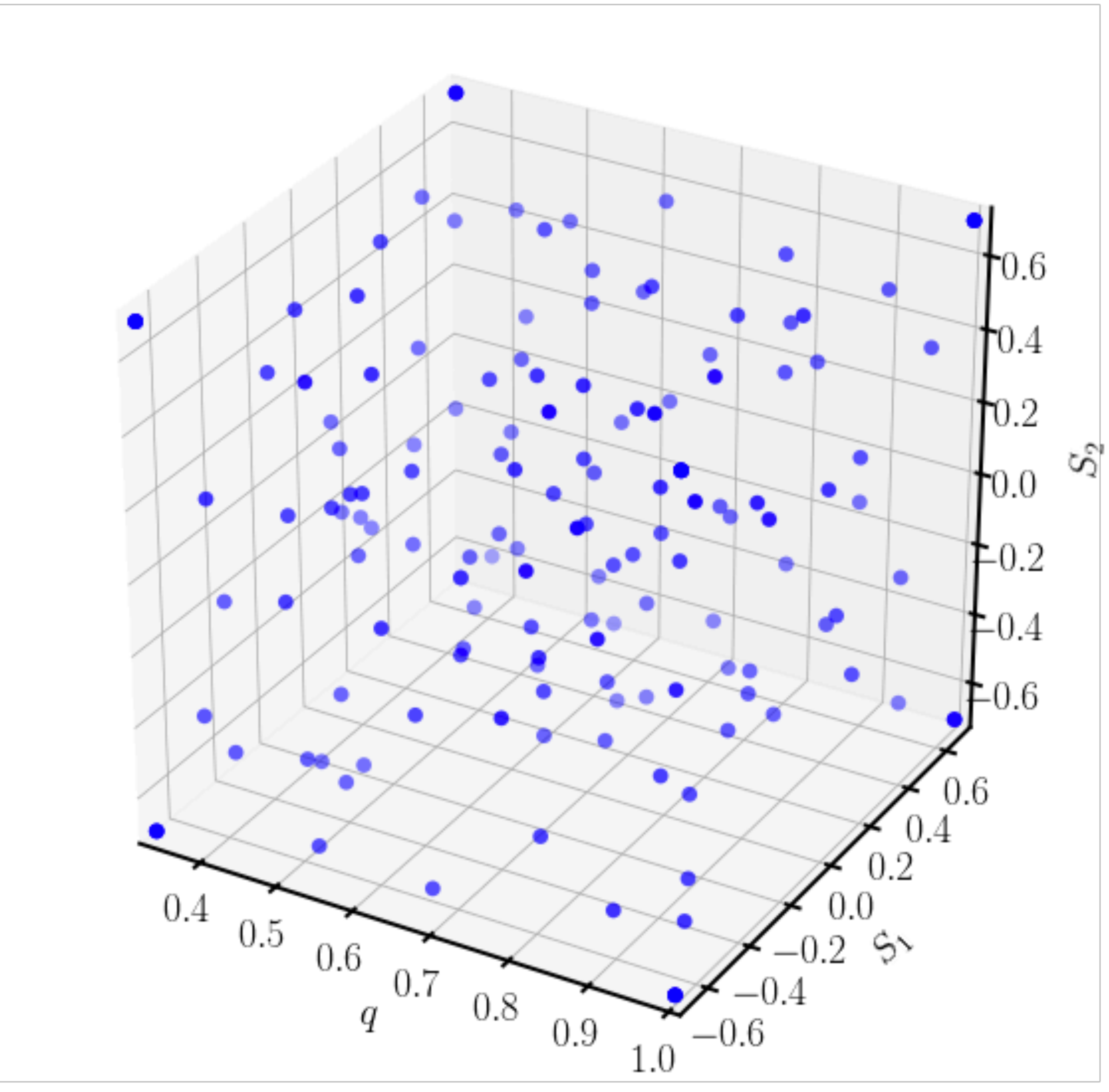
Surrogate of initial training set

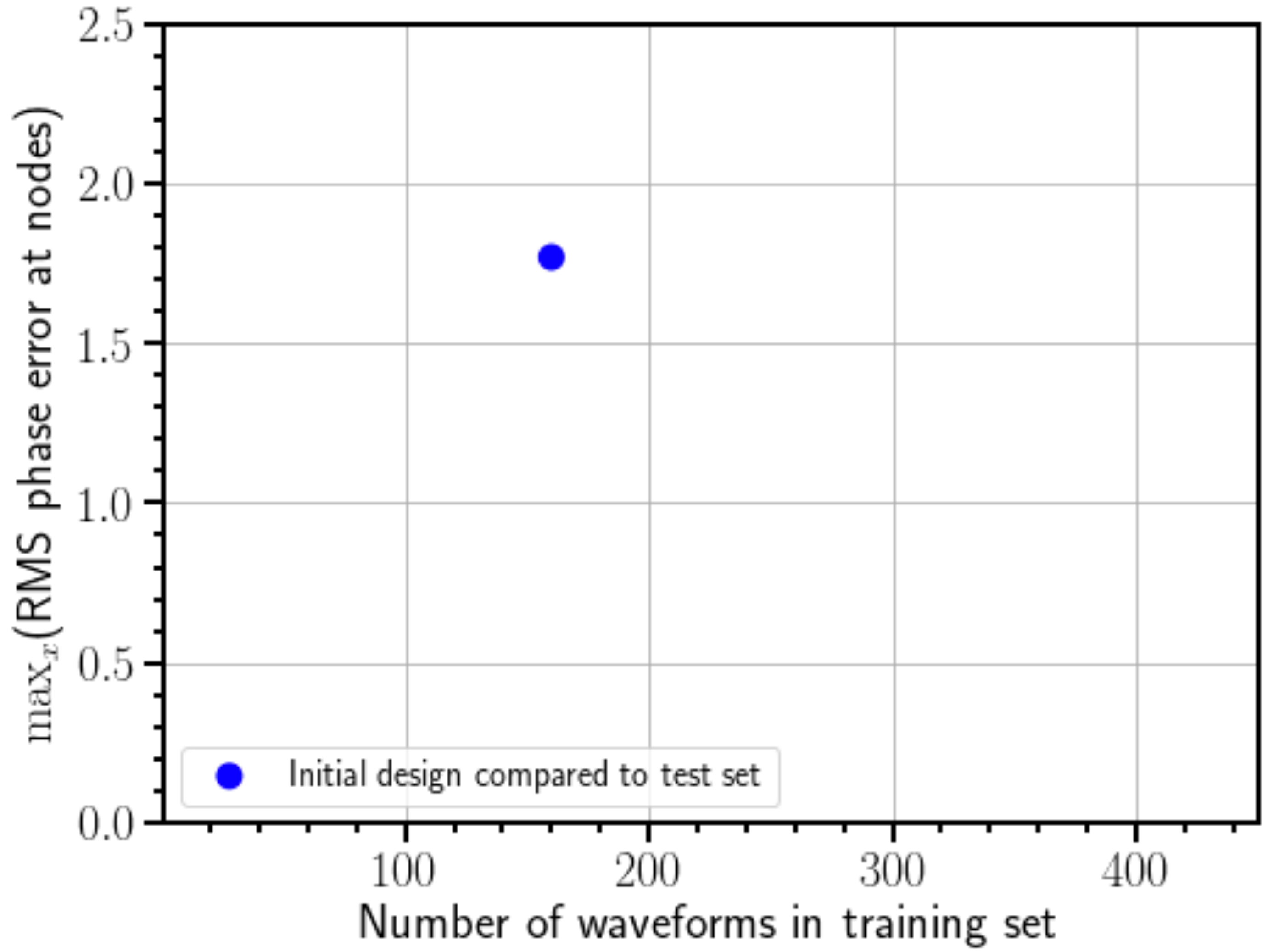
- Compared surrogate to 1000 test-set waveforms
- Maximum phase error of 2.9 rad



Updating surrogate with active learning

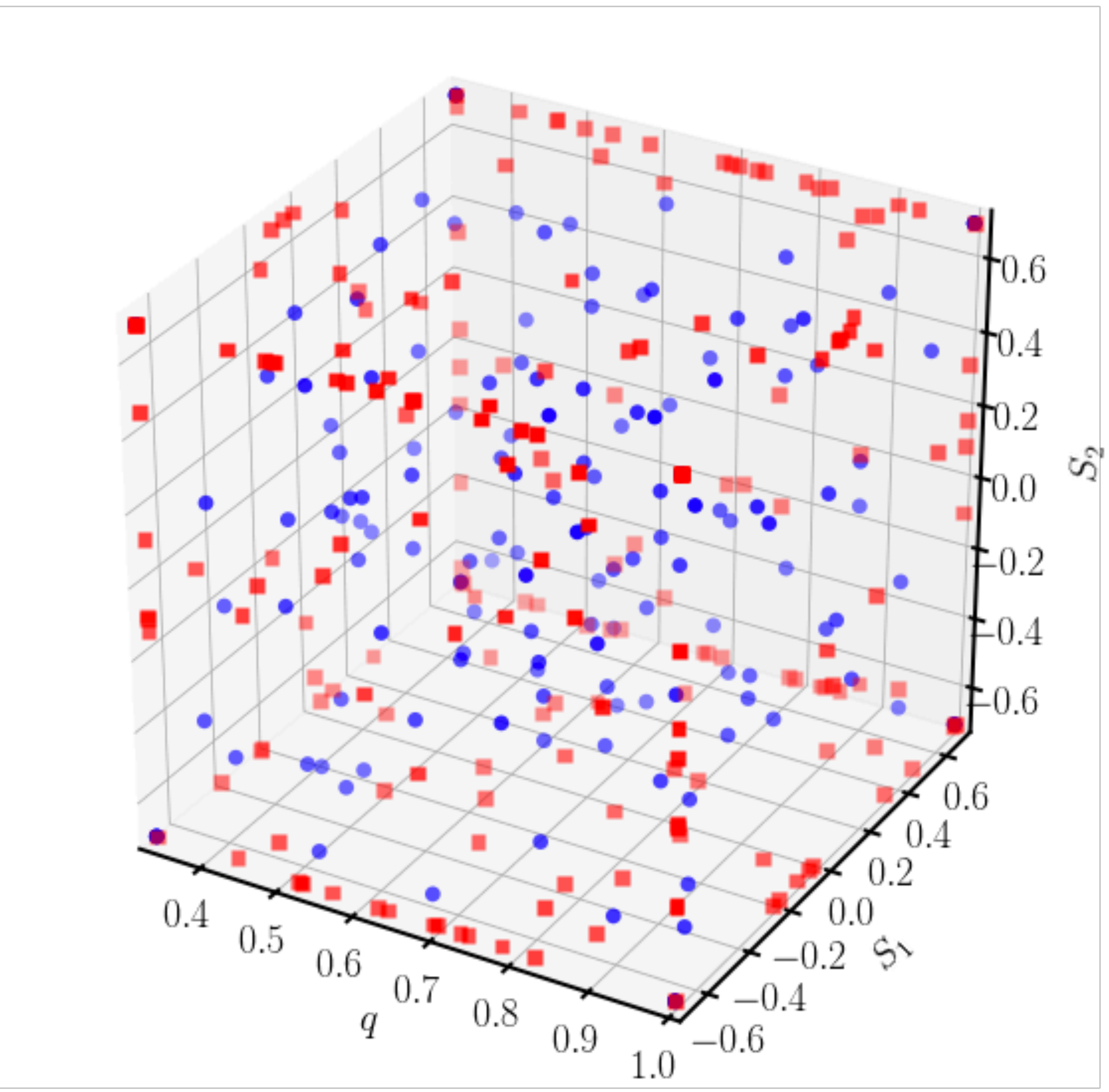
- GPR provides inexpensive estimate of surrogate model error

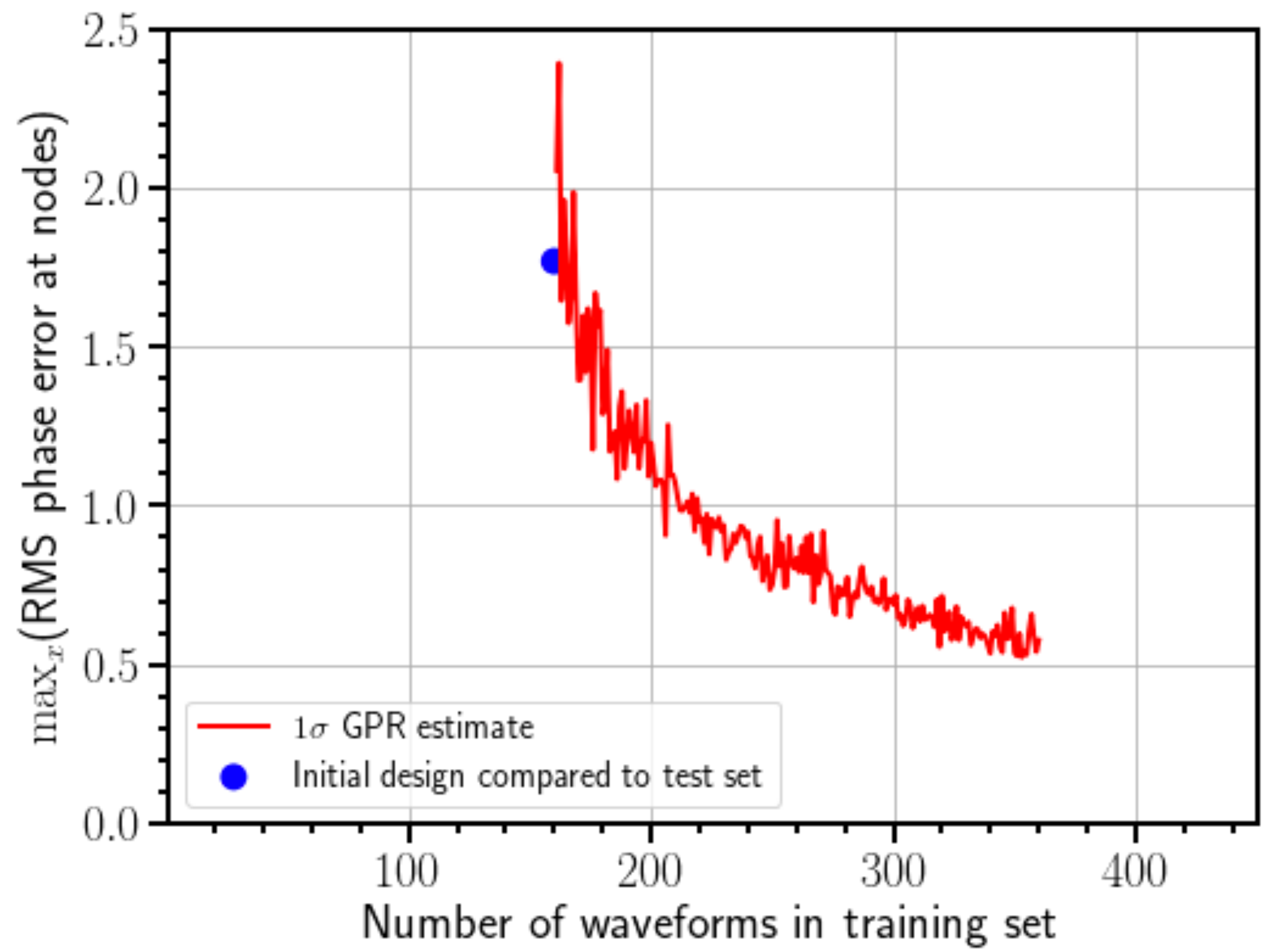




Updating surrogate with active learning

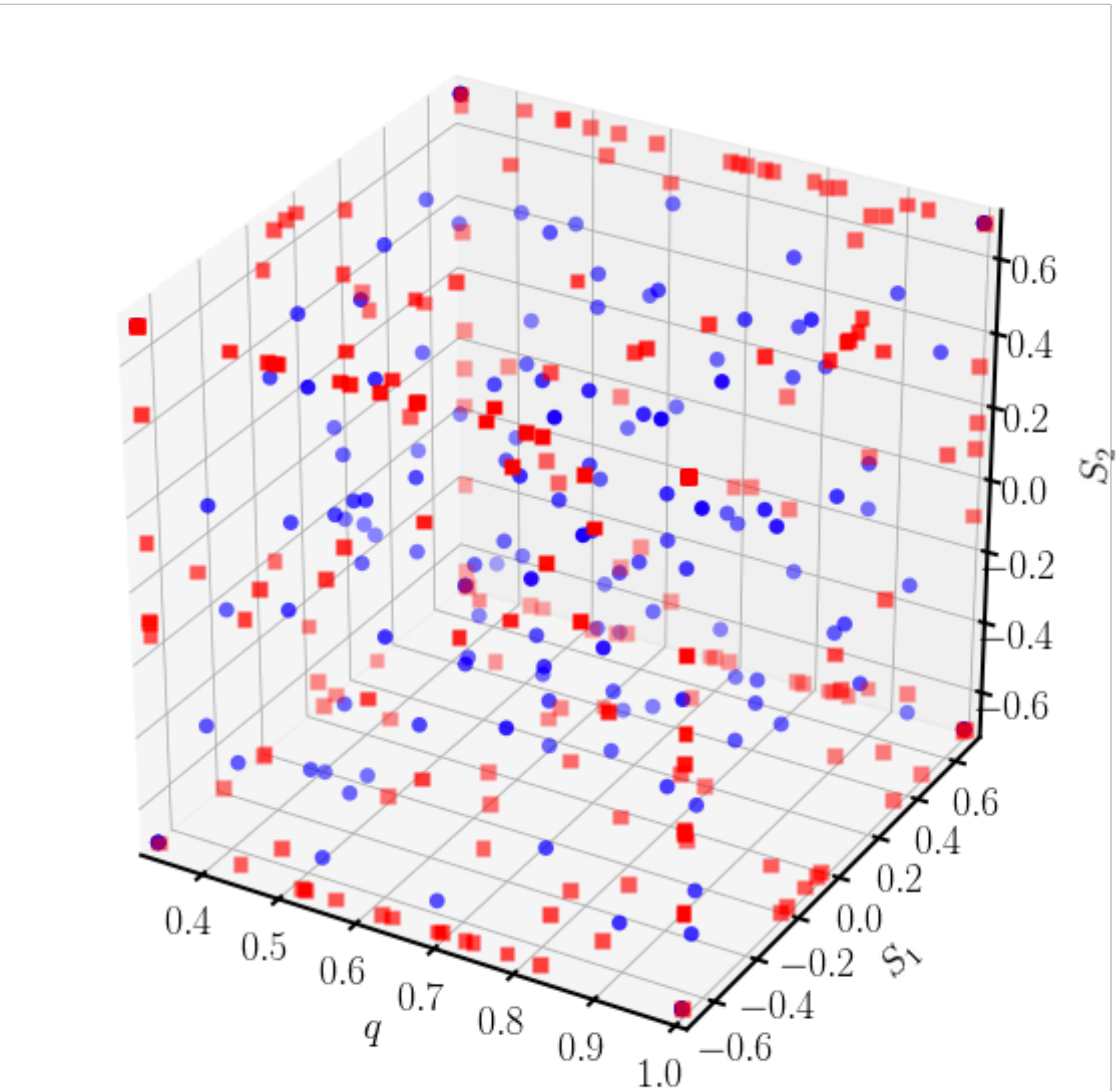
- GPR provides inexpensive estimate of surrogate model error
    1. Iteratively choose 200 new points that maximize the GPR error

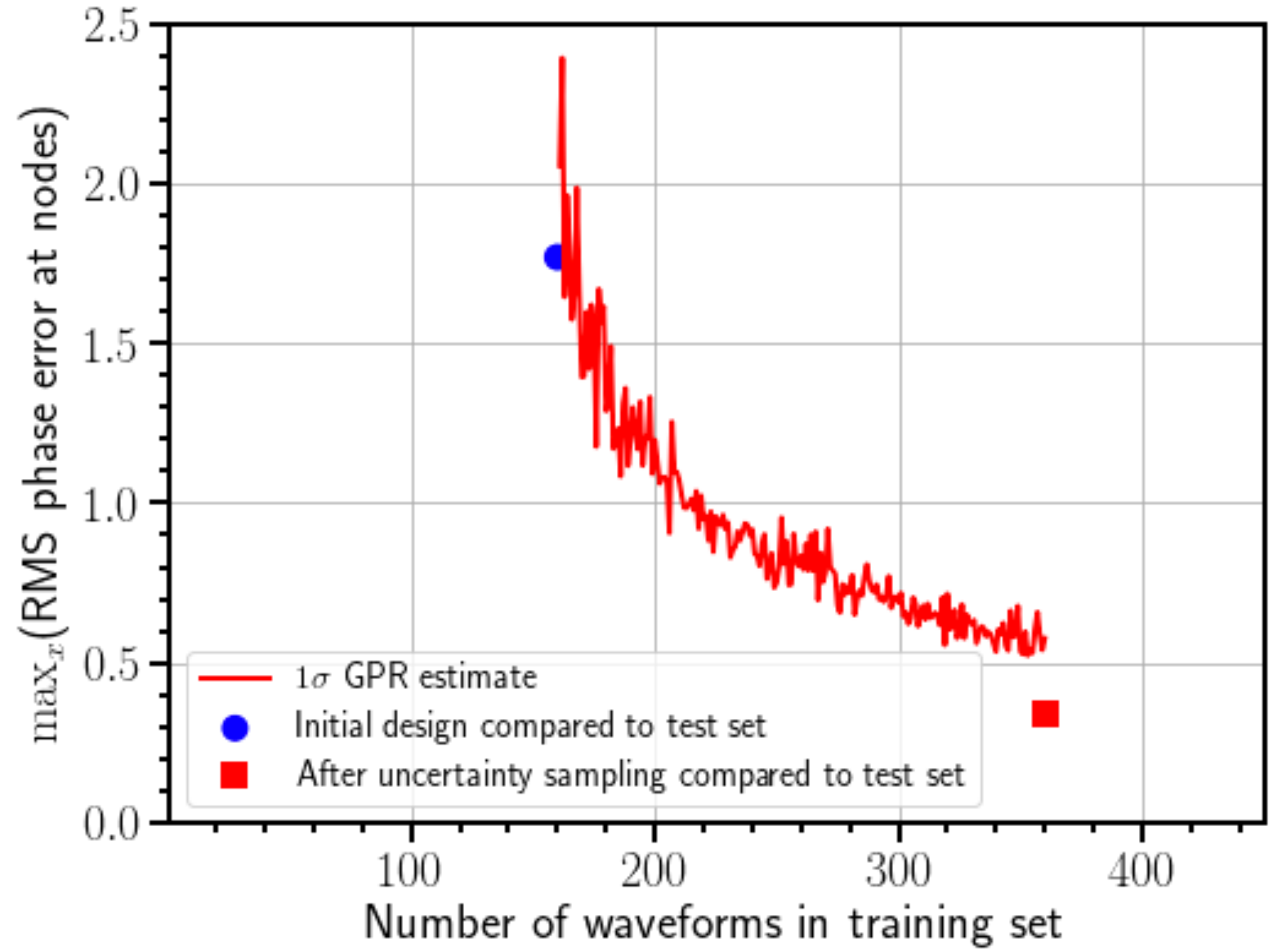




Updating surrogate with active learning

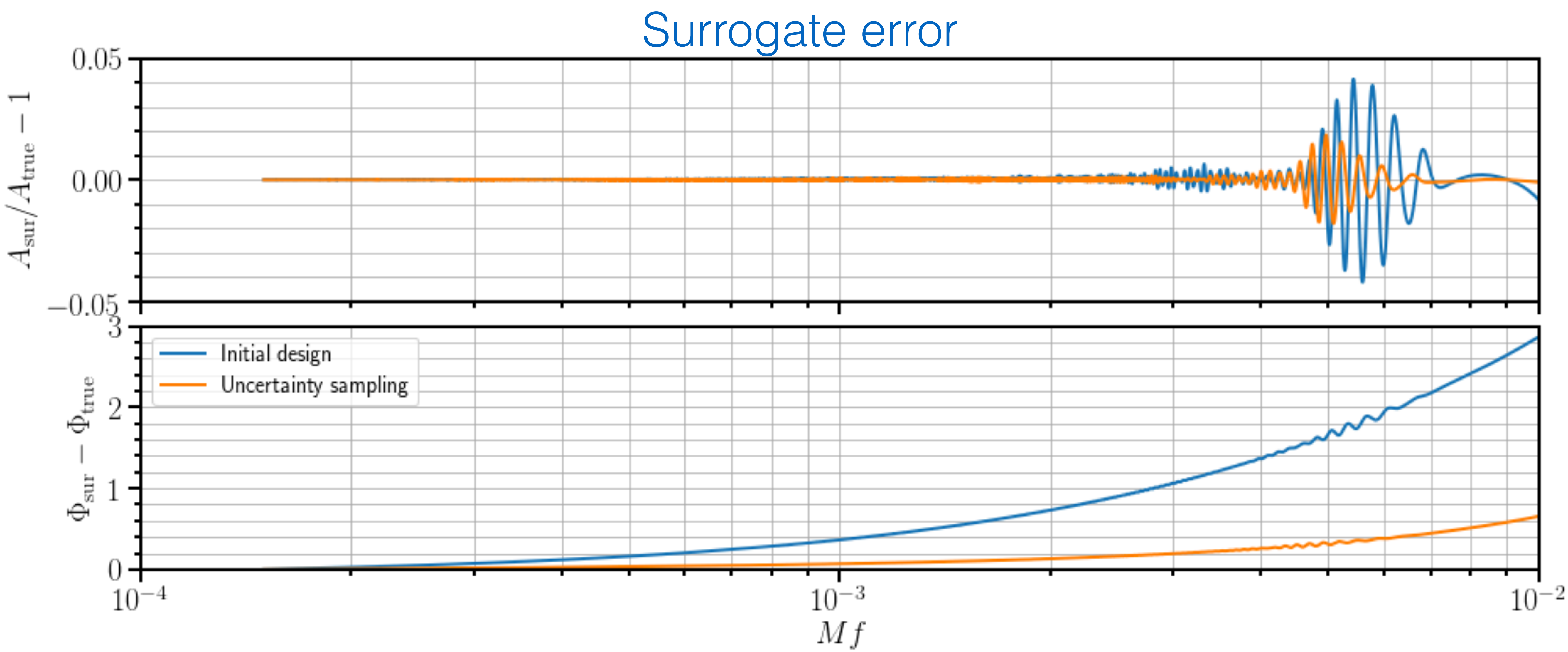
- GPR provides inexpensive estimate of surrogate model error
  1. Iteratively choose 200 new points that maximize the GPR error
  2. Generate those 200 new waveforms
  3. Reconstruct the surrogate model





Updating surrogate with active learning

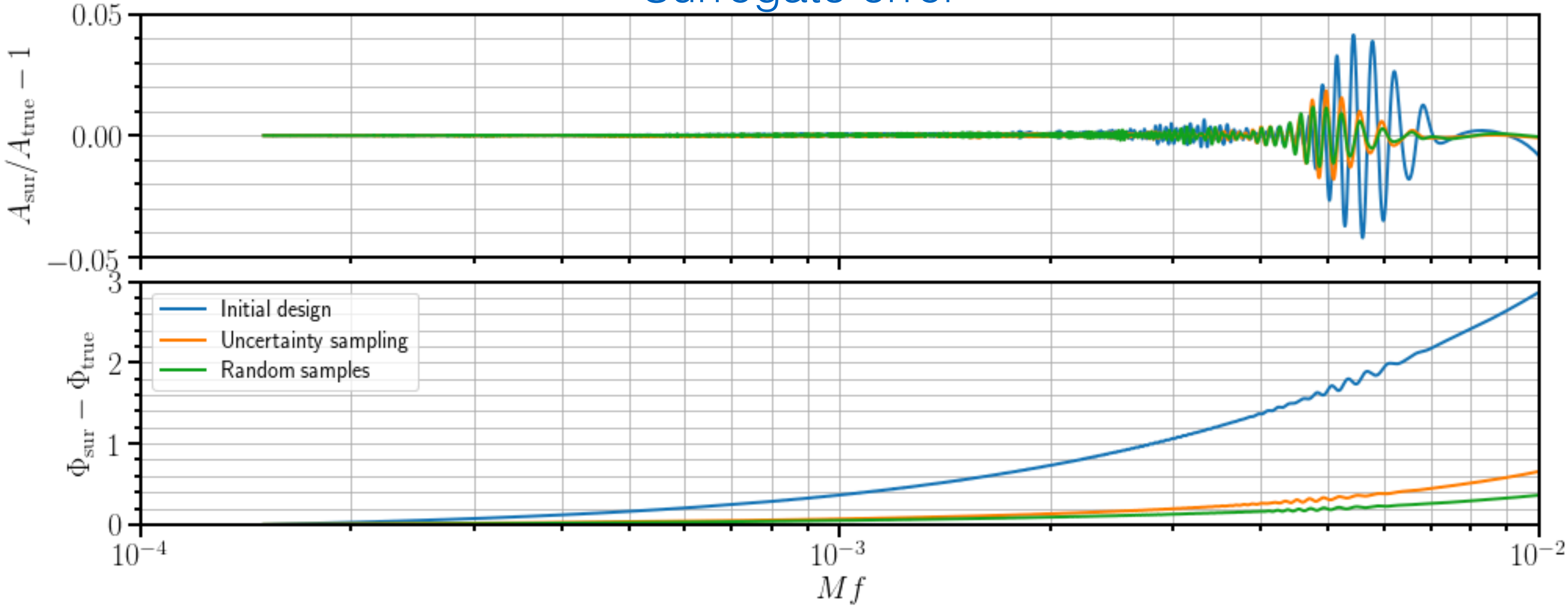
- Compared surrogate to 1000 test-set waveforms
- Maximum phase error of 0.6 rad (5x improvement)



Updating with random training set

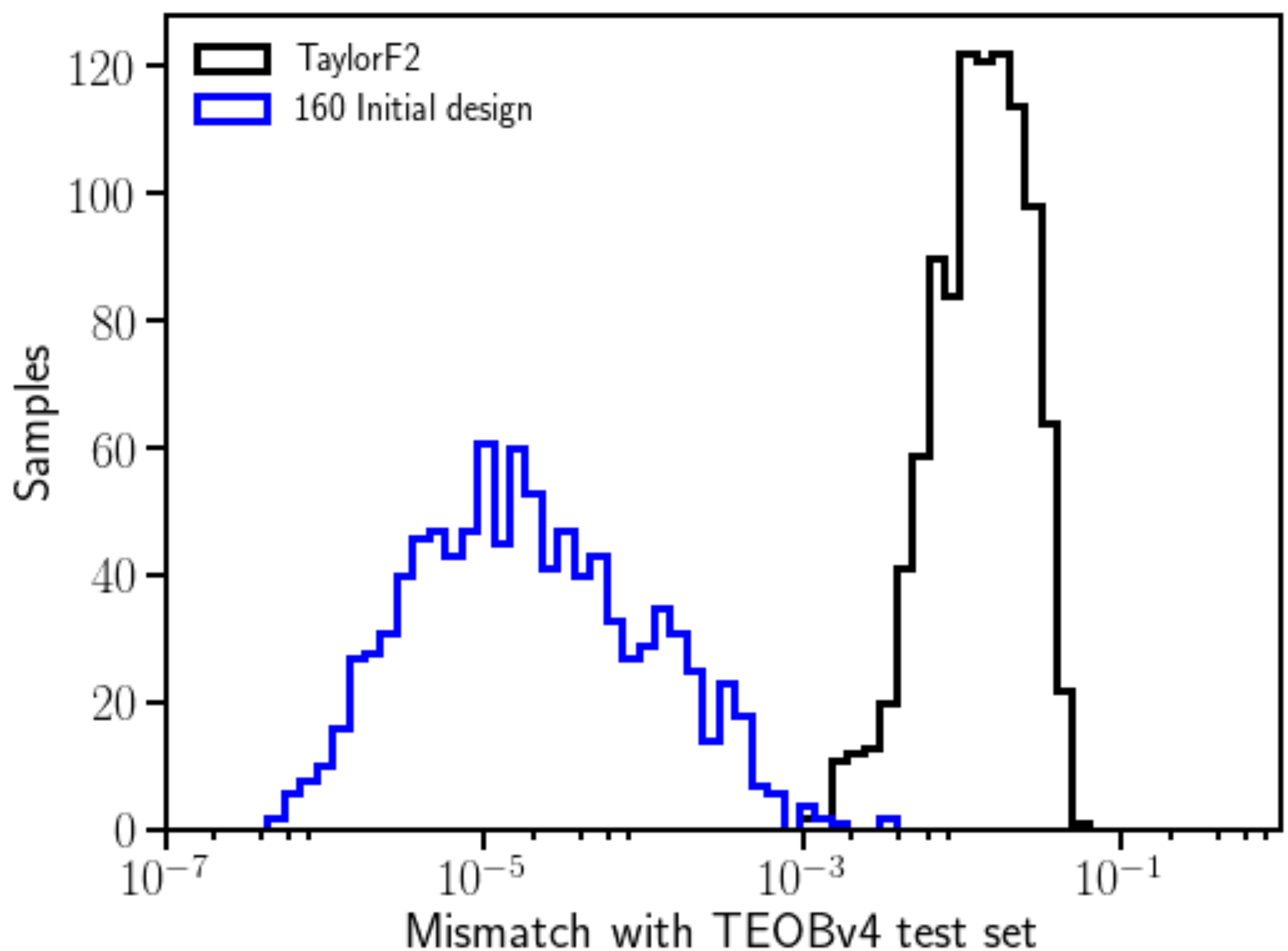
- Add an additional set of 1000 randomly sampled waveforms
- Maximum phase error of 0.4rad compared to test set

Surrogate error



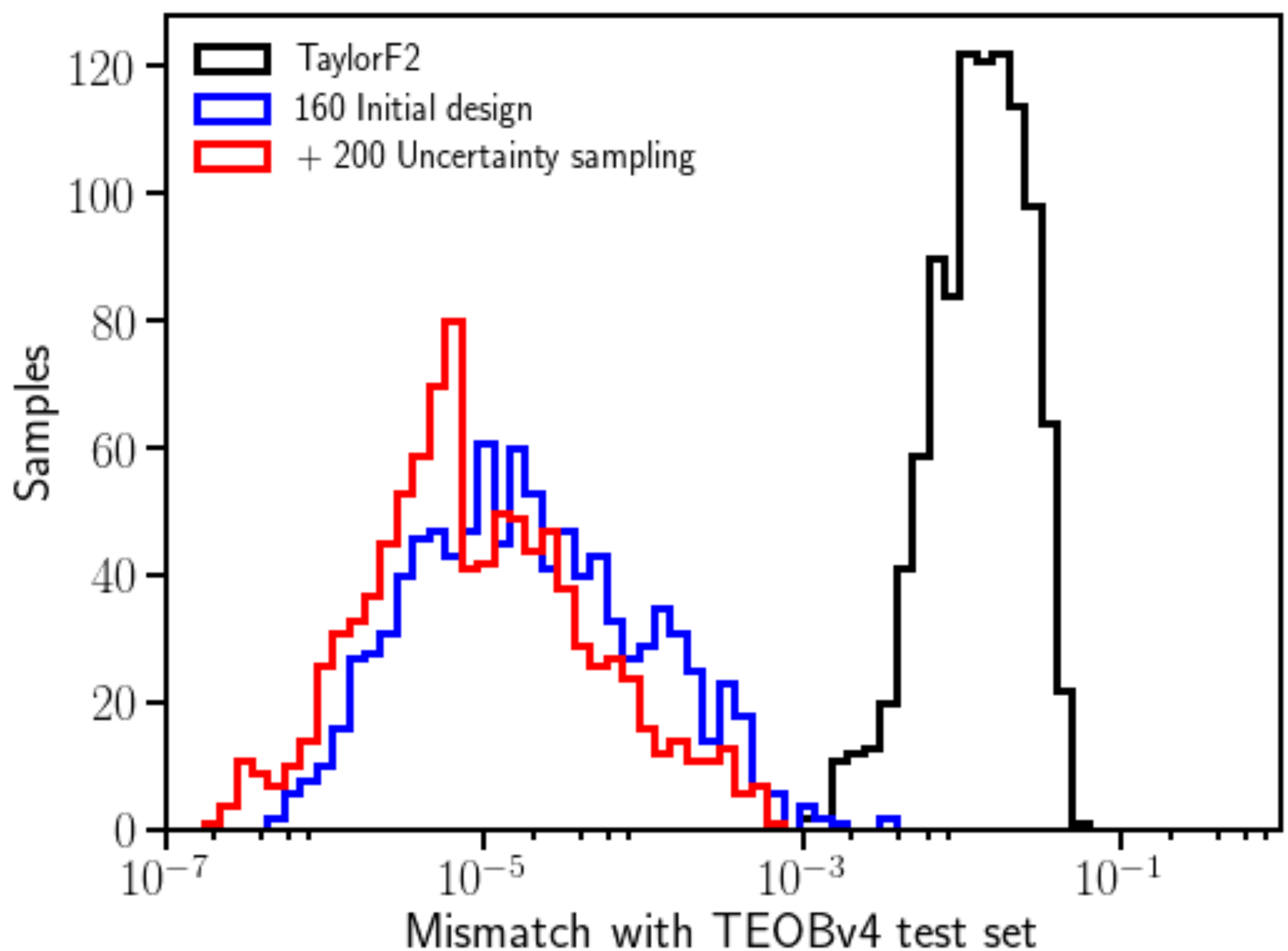
Accuracy of surrogate

- Estimate final error with 2nd test set of 1000 waveforms
- Mismatch is the loss in SNR resulting from surrogate error



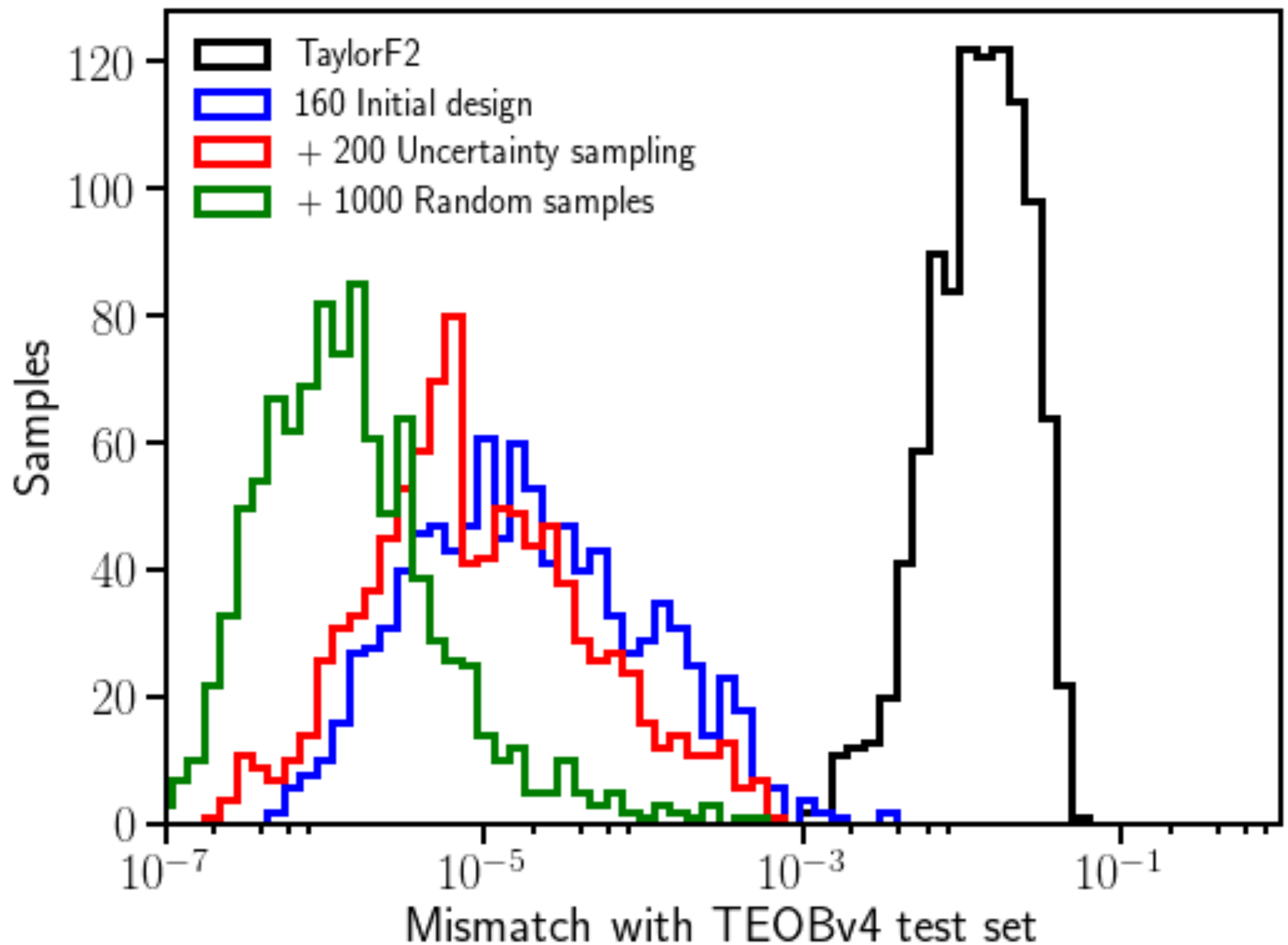
Accuracy of surrogate

- Estimate final error with 2nd test set of 1000 waveforms
- Mismatch is the loss in SNR resulting from surrogate error



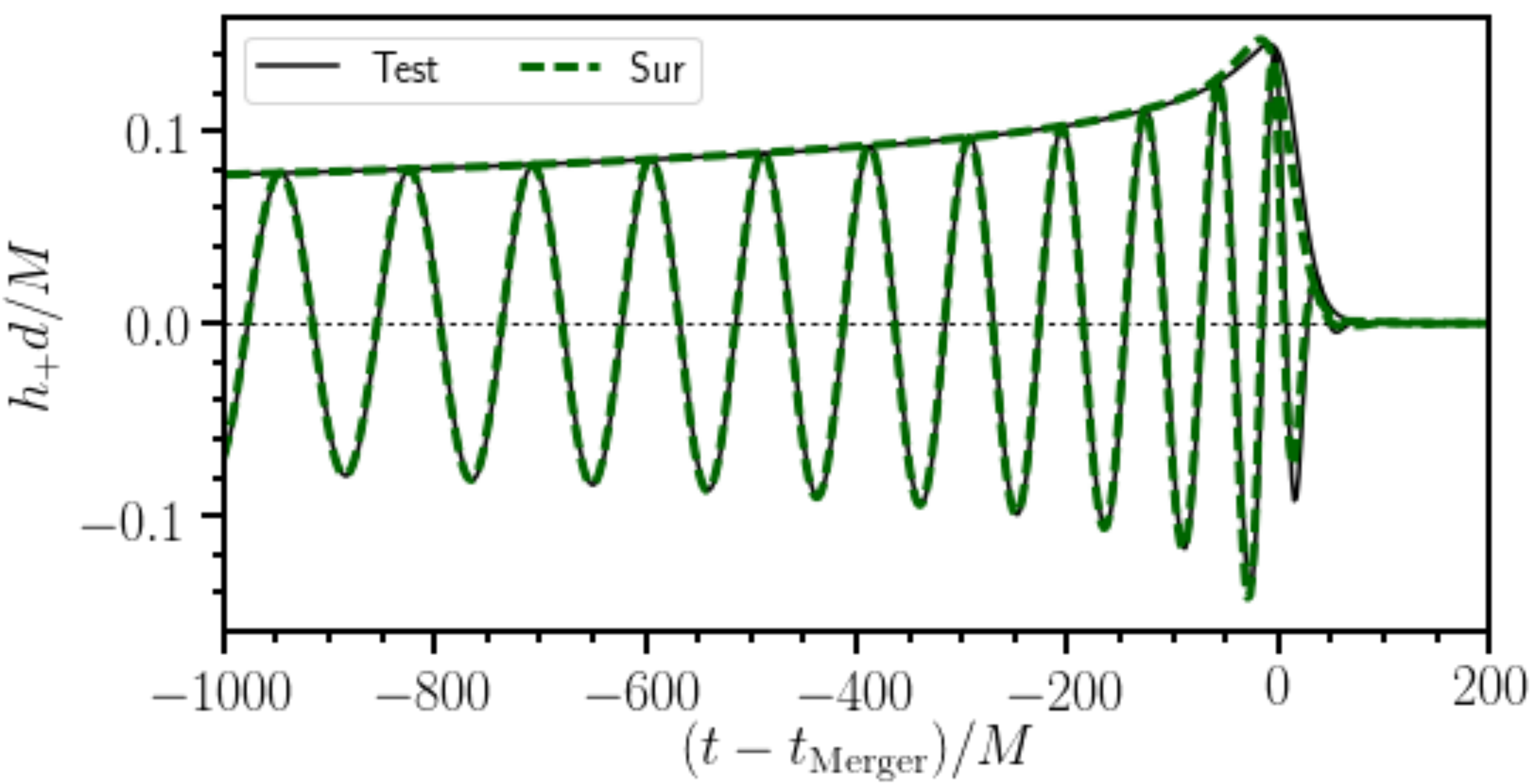
Accuracy of surrogate

- Estimate final error with 2nd test set of 1000 waveforms
- Mismatch is the loss in SNR resulting from surrogate error
  - Maximum mismatch: $< 10^{-3}$



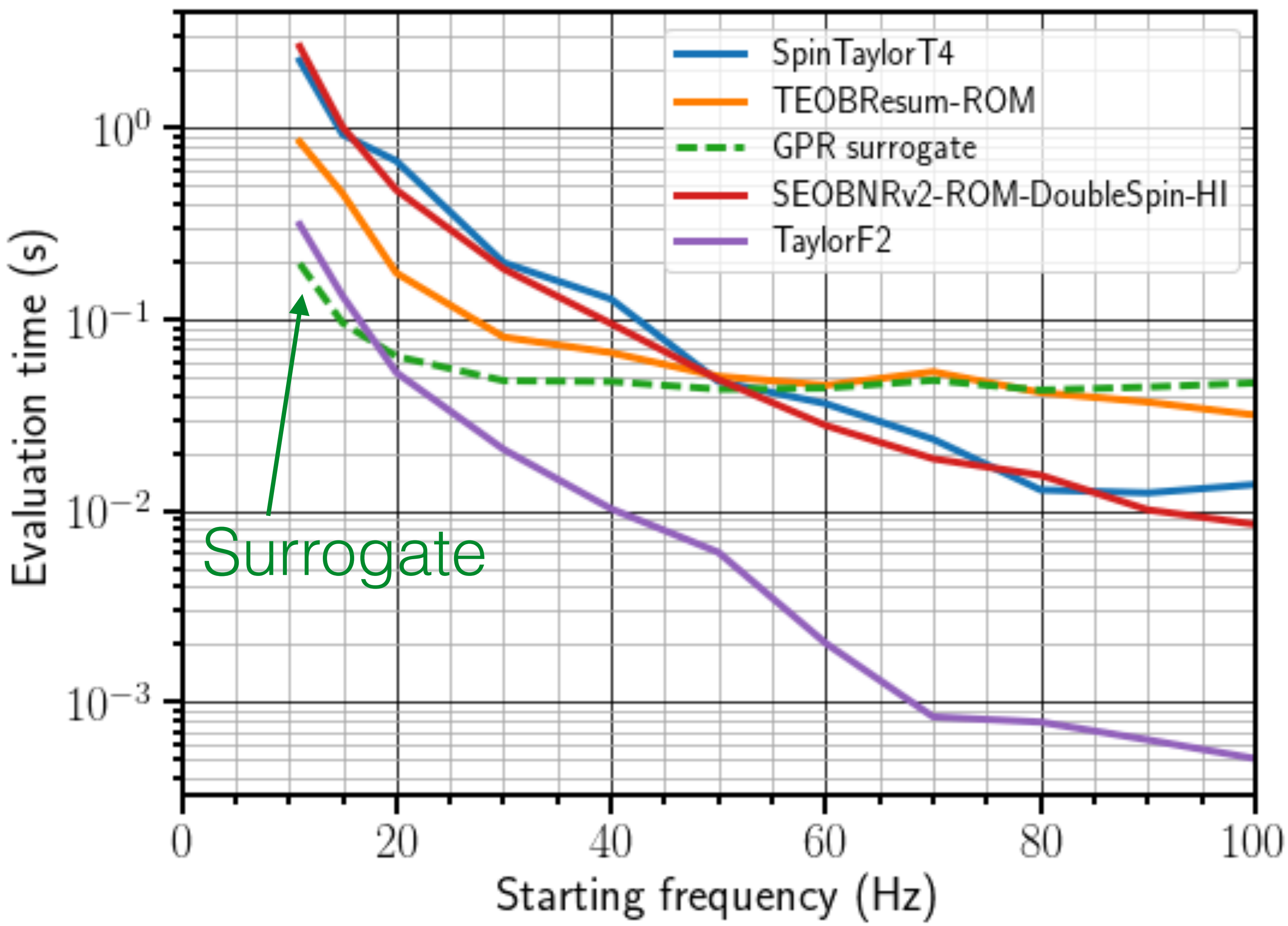
Accuracy of surrogate

- Can compare to original time-domain waveforms with IFFT
- Surrogate with largest mismatch:



Speed of surrogate

- Spin-tidal-EOB code takes 10 minutes—1 day
- GPR Surrogate:
    - Fixed cost of ~50 ms for computing surrogate
    - Additional cost for resampling to desired frequencies (interpolation)
    - Faster than all waveforms in LALSuite below ~18 Hz



Future work

- Parameter estimation with GW170817 (1-2 months)
- Generate surrogate for alternative aligned-spin model: TEOBResumS
- Use for phase evolution of phenomenological precessing models

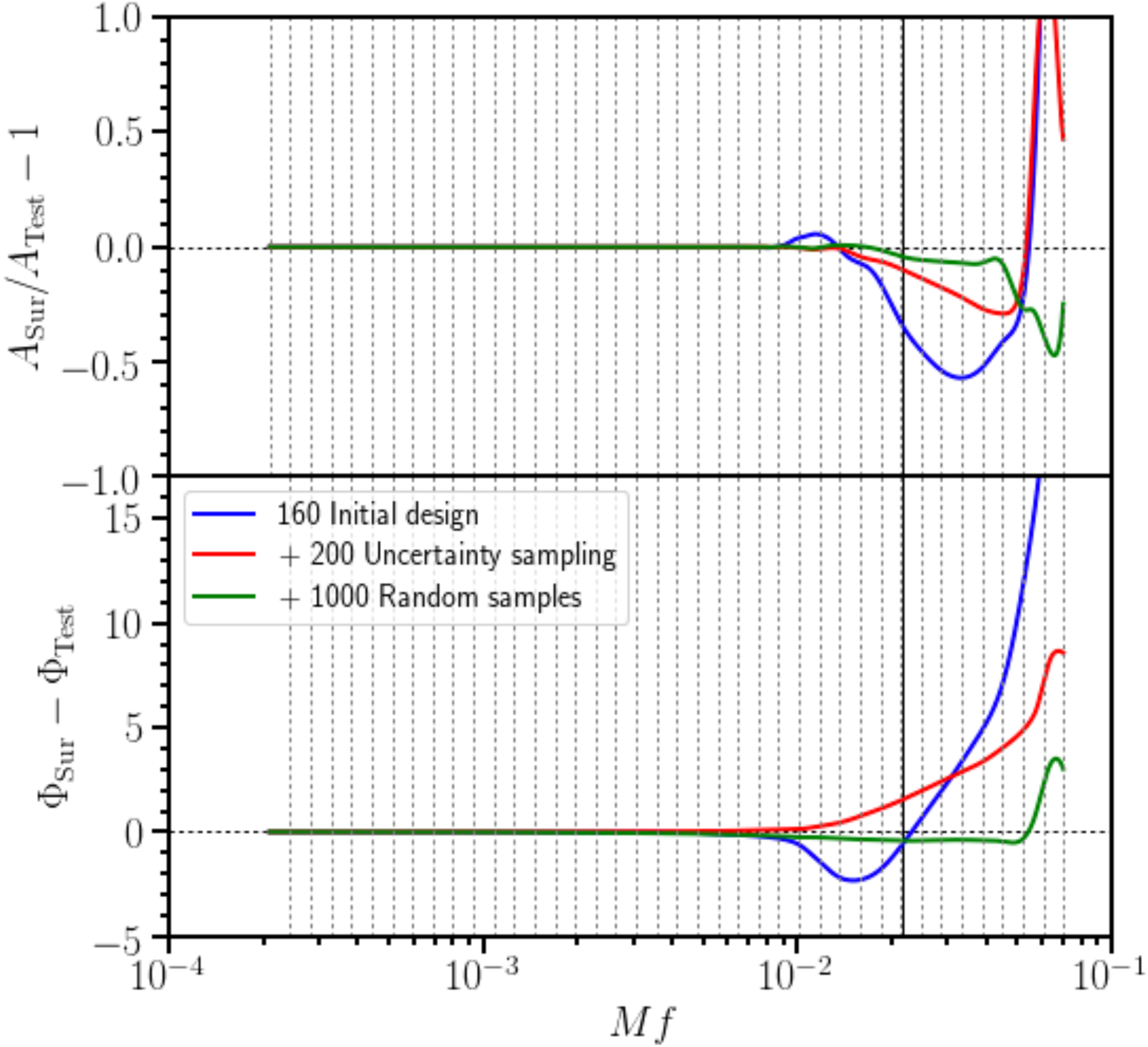
Conclusions

- BNS inspiral observations can be used to measure tidal parameters
- But, they require accurate and fast waveform models
- Surrogate of spin-tidal-EOB model can make this possible, and preliminary model should be available within a month

Thank you

Extra slides

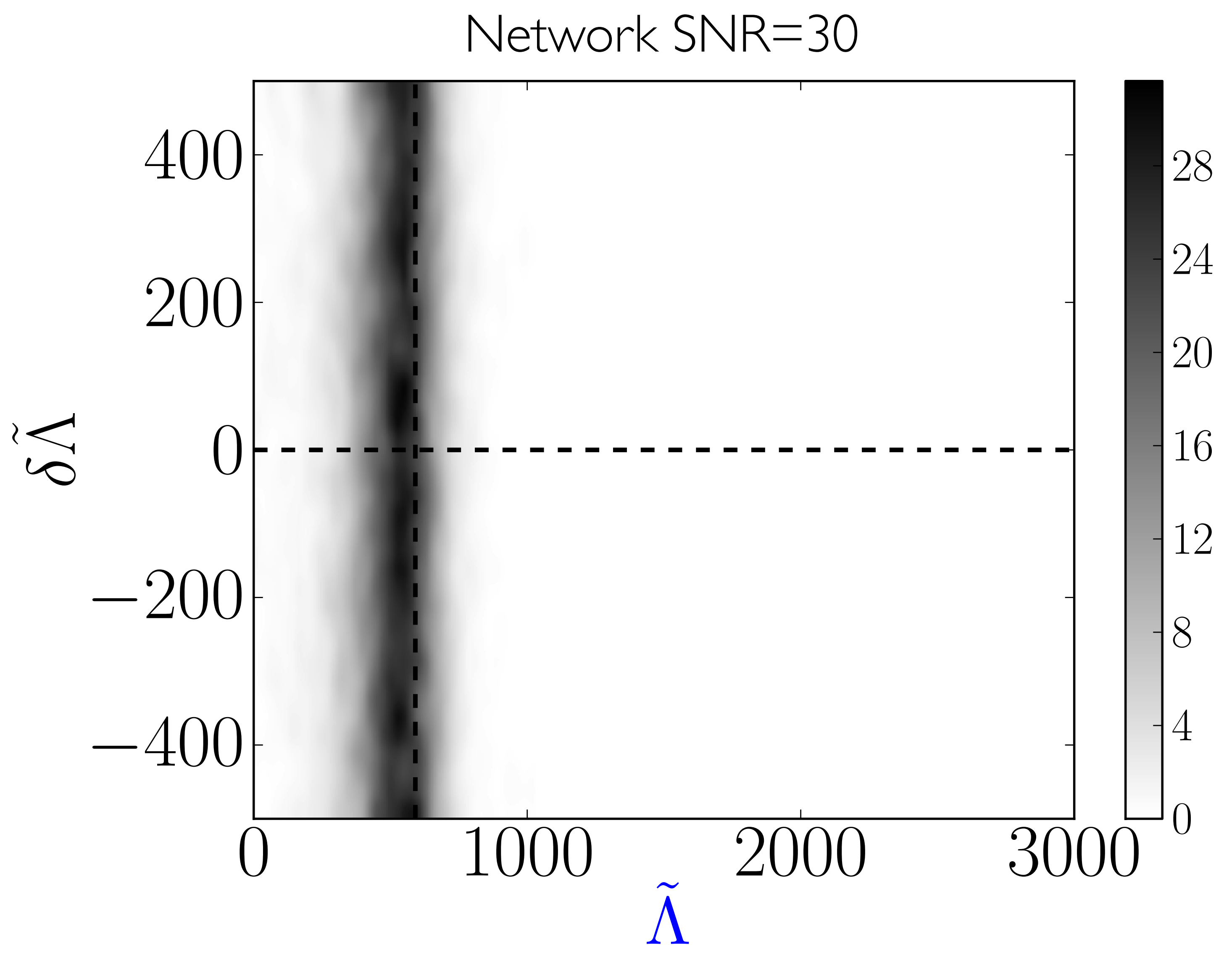
Error in surrogate



Measuring matter effects with BNS inspiral

- Can measure a linear combination of the tidal parameters $\tilde{\Lambda}$

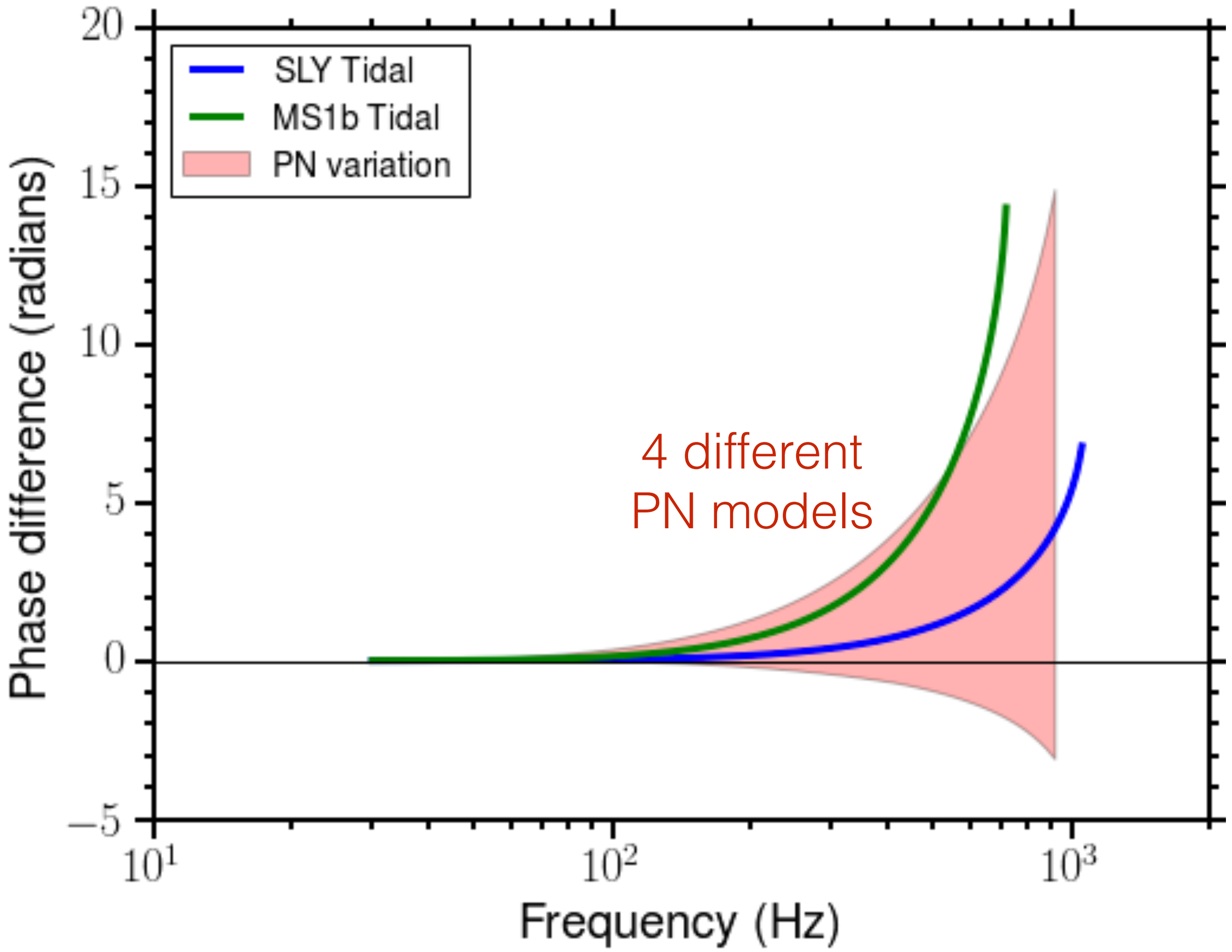
$$\tilde{\Lambda} = \frac{8}{13}\left[(1+7\eta-31\eta^2)(\Lambda_1+\Lambda_2)+\sqrt{1-4\eta}(1+9\eta-11\eta^2)(\Lambda_1-\Lambda_2)\right]$$



L. Wade et al. PRD 89, 103012 (2014)

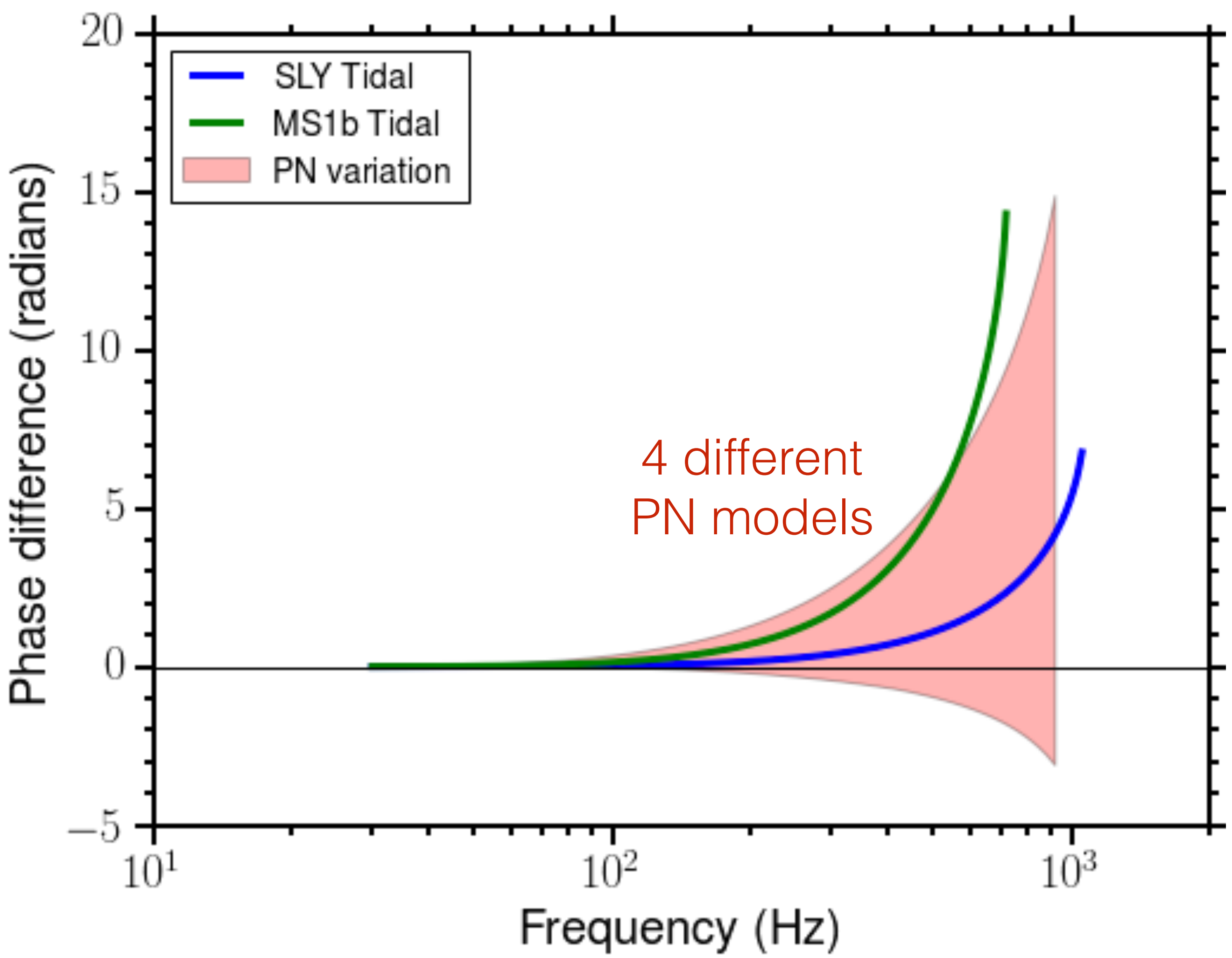
The problem

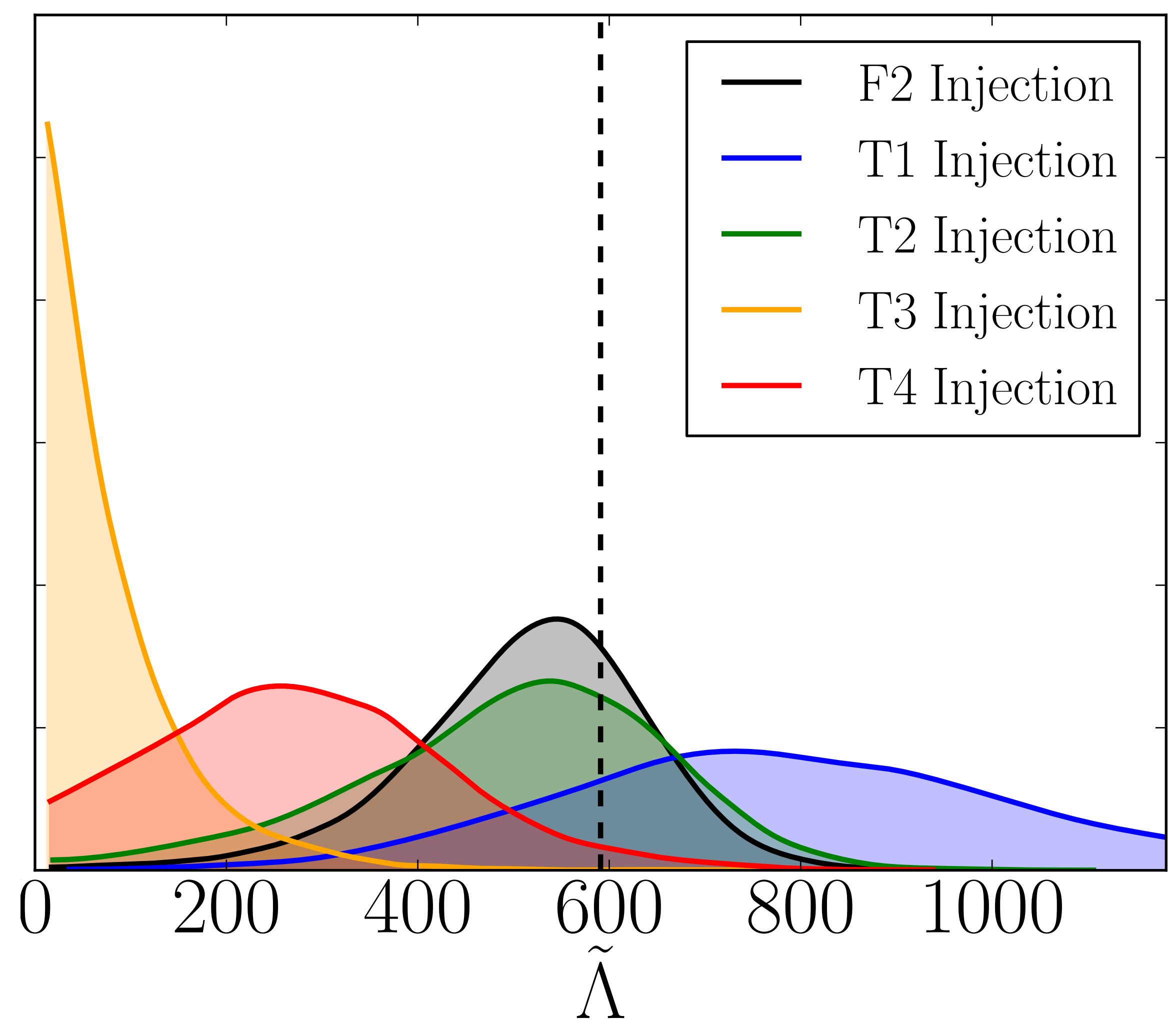
- Waveforms fast enough for Bayesian parameter estimation are too inaccurate (10^7 samples at 0.1s each)



The problem

- Waveforms fast enough for Bayesian parameter estimation are too inaccurate (10^7 samples at 0.1s each)

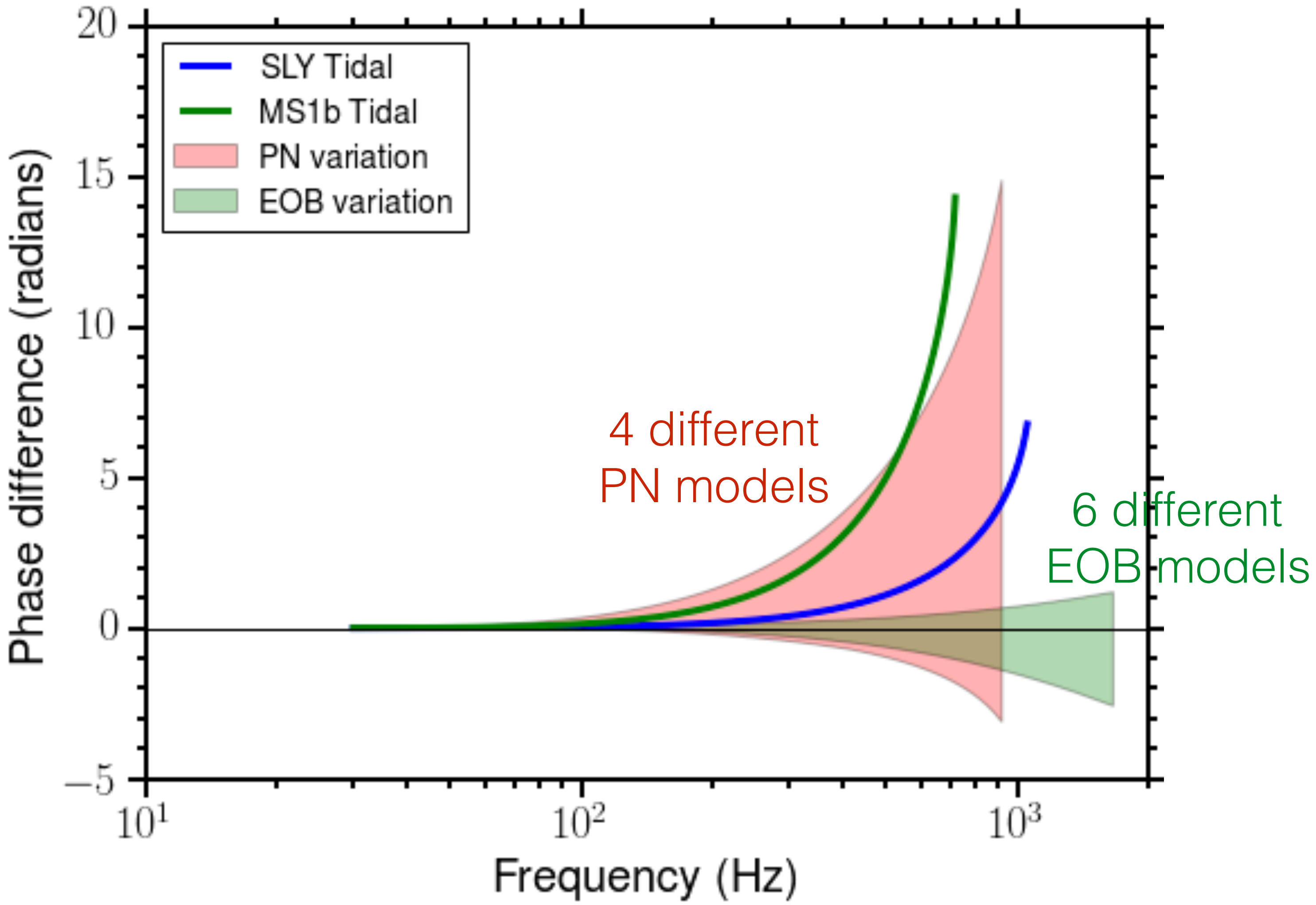


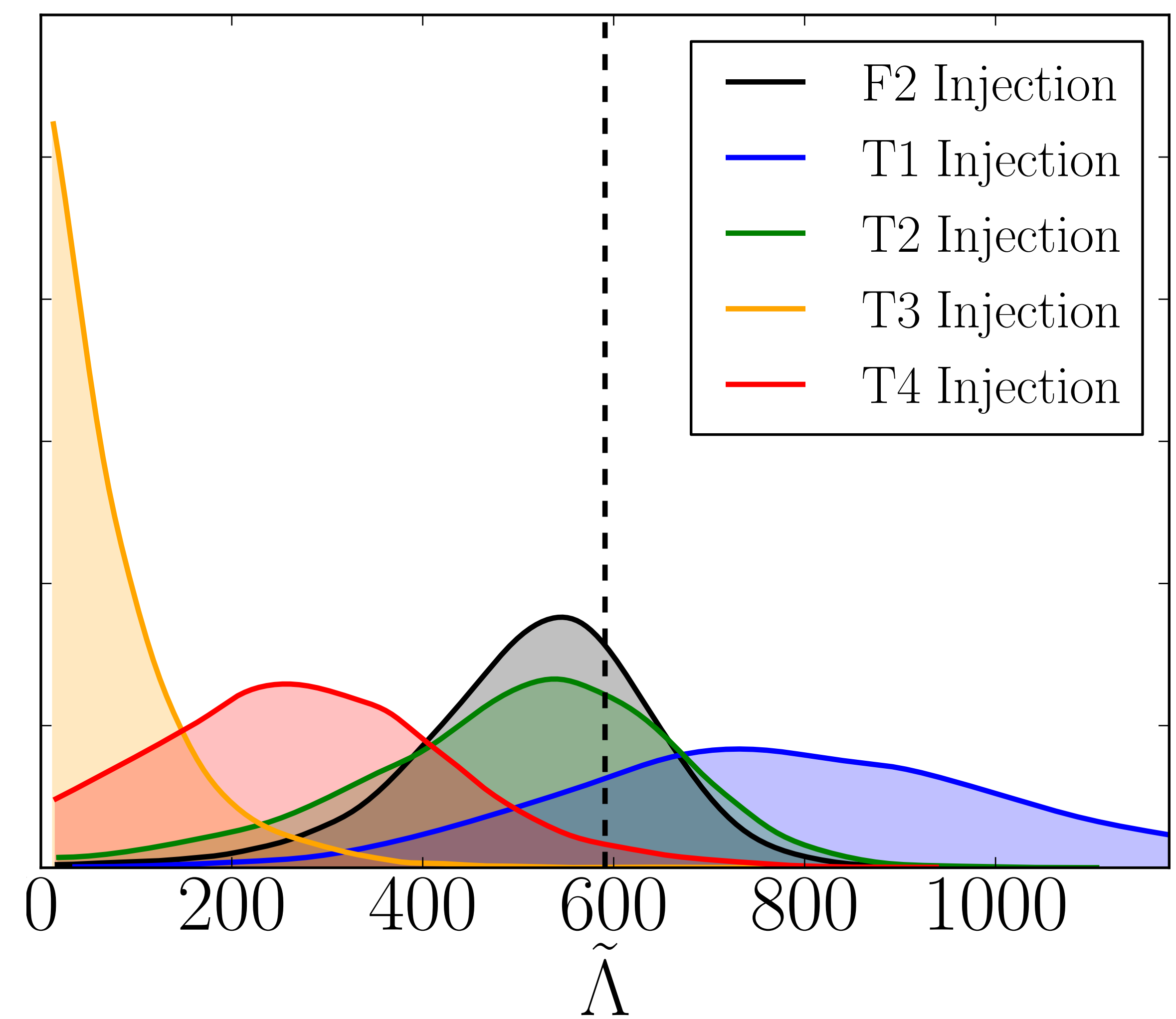


L. Wade et al. PRD 89, 103012 (2014)

The problem

- Waveforms fast enough for Bayesian parameter estimation are too inaccurate (10^7 samples at 0.1s each)
- Accurate waveforms are too slow (10 minutes—1 day each)

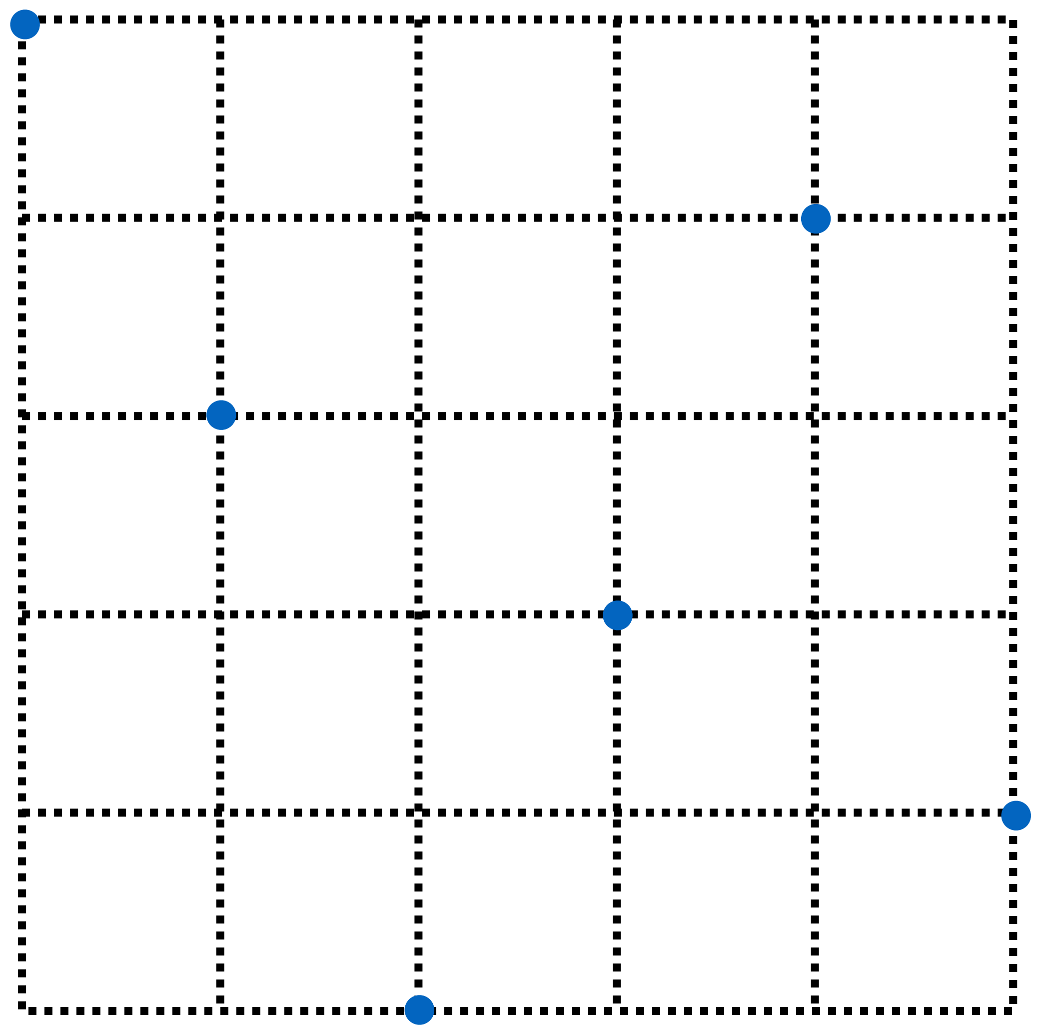




L. Wade et al. PRD 89, 103012 (2014)

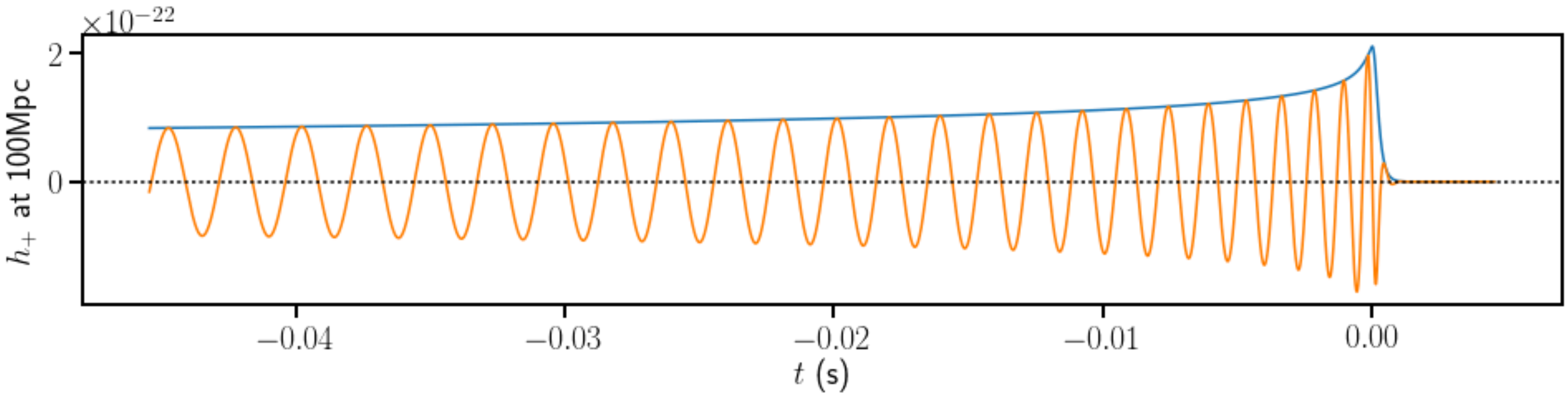
Latin hypercube design

- Begin with space-filling latin hypercube design (LHD)
  - Samples each parameter uniformly
  - Noncollapsing (subspace also a LHD)
    - Doesn't waste samples if waveform depends weekly on a parameter



Spin-tidal-EOB waveform model

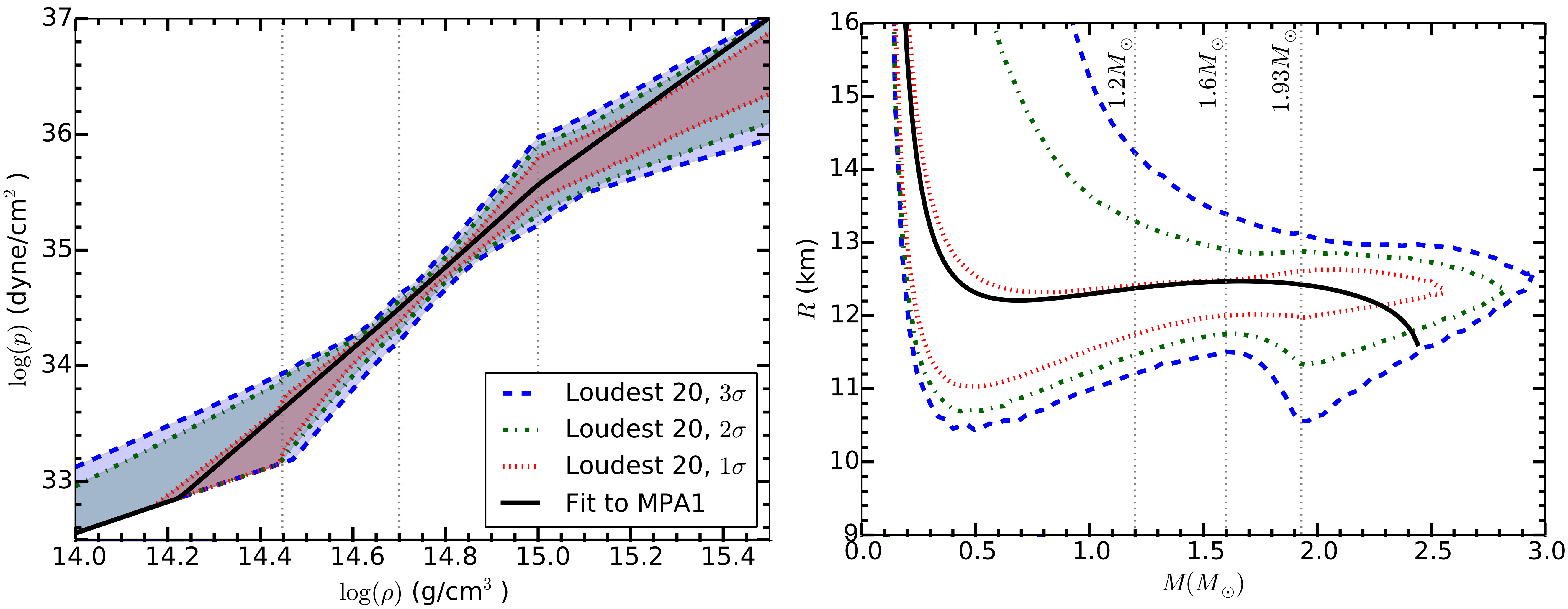
- Effective-one-body model with aligned spin and dynamic tides
  T. Hinderer et al. PRL 116, 181101 (2016)
    - $\ell = 2, 3$ tidal parameters
    - $\ell = 2, 3$ f-mode frequencies
    - End is tapered to agree with numerical BNS simulations
    - 12-dimensional parameter space (1 mass, 1 spin, 4 matter parameters per NS)



Measuring the EOS with BNS inspiral

- Can measure a linear combination of the tidal parameters $\tilde{\Lambda}$
- Stacking observations allows us to directly measure a parameterized EOS

40 detected events at design sensitivity



B. Lackey, L. Wade. PRD 91, 043002 (2015)