Constraining the nuclear EoS by combining nuclear data and GW observations

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A Minimal Nuclear Energy Density Functional

- Bulgac, Forbes, Jin, Perez, Schunk arXiv:1708.08771 (accepted PRC 2018)

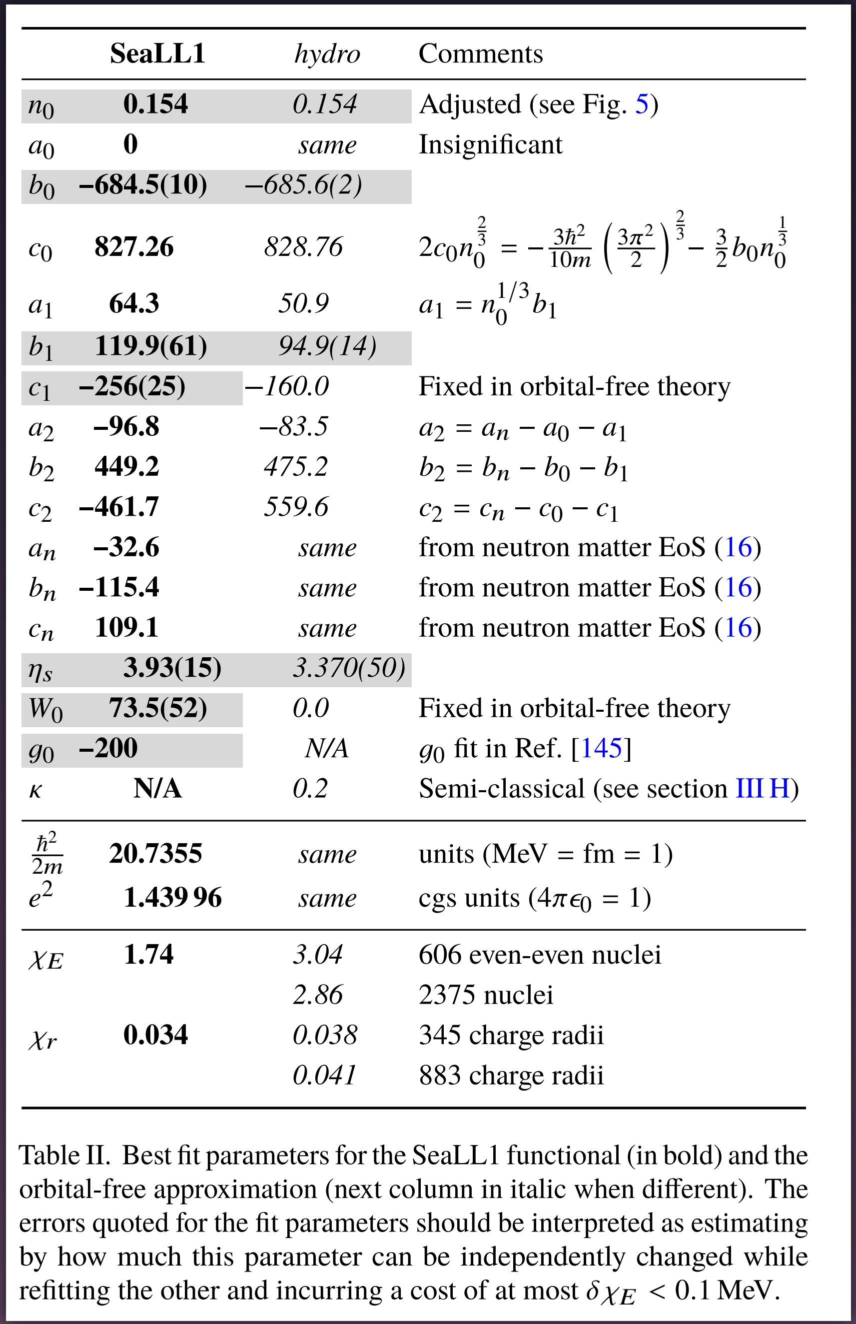
- 6-7 parameters
  - Better accuracy than other NEDF (without beyond mean-field corrections)
    - fewer parameters (others have 13-30)
    - Not fully optimized
  - Separation of dominant from subdominant parameters
  - Masses, Radii, Separation energies, Neutron matter
  - Suitable for dynamics (reasonably fission properties without tuning!)

Inspired by liquid drop

- Inspired by liquid drop formula
- 4-5 parameters:
- Global mass fits (2375 nuclei)

  2.6-2.8MeV compared to to 4-6 parameter Liquid Drop Formula (2.6-3.3MeV)
- Also get 883 charge radii

  0.041fm



	SeaLL1	*hydro*	Comments
n_0	**0.154**	*0.154*	Adjusted (see Fig. 5)
a_0	**0**	*same*	Insignificant
b_0	**–684.5(10)**	*–685.6(2)*	
c_0	**827.26**	*828.76*	$2c_0 n_0^{\frac{2}{3}} = -\frac{3\hbar^2}{10m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} - \frac{3}{2} b_0 n_0^{\frac{1}{3}}$
a_1	**64.3**	*50.9*	$a_1 = n_0^{1/3} b_1$
b_1	**119.9(61)**	*94.9(14)*	
c_1	**–256(25)**	*–160.0*	Fixed in orbital-free theory
a_2	**–96.8**	*–83.5*	$a_2 = a_n - a_0 - a_1$
b_2	**449.2**	*475.2*	$b_2 = b_n - b_0 - b_1$
c_2	**–461.7**	*559.6*	$c_2 = c_n - c_0 - c_1$
a_n	**–32.6**	*same*	from neutron matter EoS (16)
b_n	**–115.4**	*same*	from neutron matter EoS (16)
c_n	**109.1**	*same*	from neutron matter EoS (16)
η_s	**3.93(15)**	*3.370(50)*	
W_0	**73.5(52)**	*0.0*	Fixed in orbital-free theory
g_0	**–200**	*N/A*	g_0 fit in Ref. [145]
κ	**N/A**	*0.2*	Semi-classical (see section III H)
$\frac{\hbar^2}{2m}$	**20.7355**	*same*	units (MeV = fm = 1)
e^2	**1.439 96**	*same*	cgs units ($4\pi\epsilon_0 = 1$)
χ_E	**1.74**	*3.04*	606 even-even nuclei
		2.86	2375 nuclei
χ_r	**0.034**	*0.038*	345 charge radii
		0.041	883 charge radii

Table II. Best fit parameters for the SeaLL1 functional (in bold) and the orbital-free approximation (next column in italic when different). The errors quoted for the fit parameters should be interpreted as estimating by how much this parameter can be independently changed while refitting the other and incurring a cost of at most $\delta\chi_E < 0.1$ MeV.

SeaLL1 NEDF

$$\mathcal{E}[n_n, n_p] = \overbrace{\frac{\hbar^2}{2m}(\tau_n + \tau_p)}^{\text{kinetic}} +$$

$$+ \overbrace{\sum_{j=0}^{2}\left(a_j n^{5/3} + b_j n^2 + c_j n^{7/3}\right)\beta^{2j}}^{\text{homogeneous}} +$$

$$+ \overbrace{\eta_s \sum_{q=n,p} \frac{\hbar^2}{2m}\|\vec{\nabla} n_q\|^2}^{\text{gradient}} + \underbrace{W_0 \vec{J}\cdot\vec{\nabla} n}_{\text{spin-orbit}} +$$

$$+ \underbrace{\sum_{q=n,p} g_{\text{eff}}(\vec{r})|\nu_q(\vec{r})|^2}_{\text{pairing}}$$

$$+ \underbrace{\frac{e^2}{2}\int d^3\vec{r}\,'\frac{n_p(\vec{r})n_p(\vec{r}\,')}{\|\vec{r}-\vec{r}\,'\|} - \frac{3e^2}{4}\left(\frac{n_p(\vec{r})}{3\pi}\right)^{4/3}}_{\text{Coulomb}}$$

SeaLL1 NEDF

$$\mathcal{E}[n_n, n_p] = \boxed{\overbrace{\frac{\hbar^2}{2m}(\tau_n + \tau_p) +}^{\text{kinetic}}}$$

$$+ \overbrace{\sum_{j=0}^{2}\left(a_j n^{5/3} + b_j n^2 + c_j n^{7/3}\right)\beta^{2j}}^{\text{homogeneous}} +$$

$$+ \overbrace{\eta_s \sum_{q=n,p} \frac{\hbar^2}{2m}\|\vec{\nabla} n_q\|^2}^{\text{gradient}} + \underbrace{W_0 \vec{J}\cdot\vec{\nabla} n}_{\text{spin-orbit}} +$$

$$+ \underbrace{\sum_{q=n,p} g_{\text{eff}}(\vec{r})|\nu_q(\vec{r})|^2}_{\text{pairing}}$$

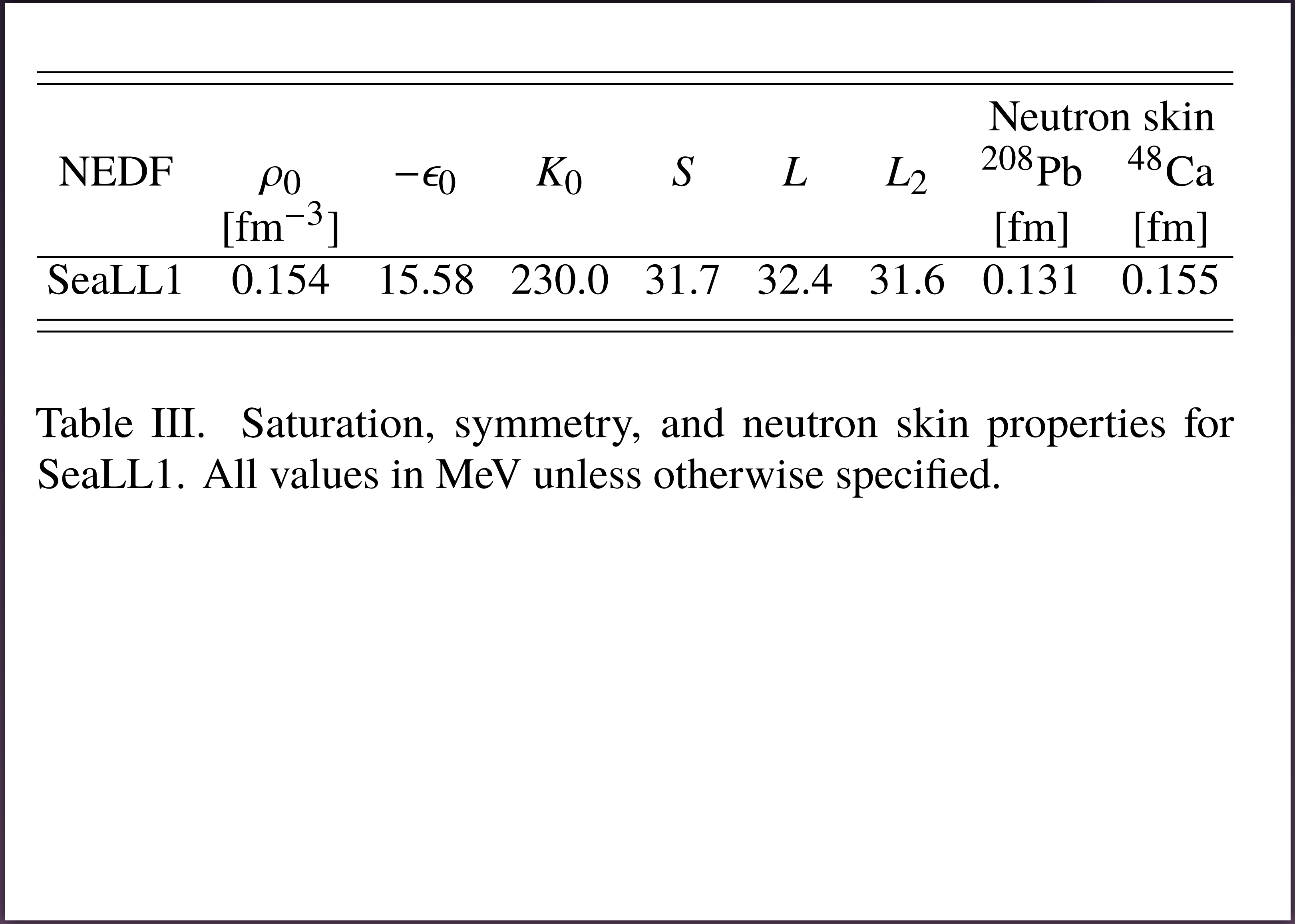
$$+ \underbrace{\frac{e^2}{2}\int d^3\vec{r}'\,\frac{n_p(\vec{r})n_p(\vec{r}')}{\|\vec{r}-\vec{r}'\|} - \frac{3e^2}{4}\left(\frac{n_p(\vec{r})}{3\pi}\right)^{4/3}}_{\text{Coulomb}}$$

- No effective mass
  good level densities
- Orbital-free version from semi-classical expansion
  (2-fluid hydrodynamics)

$$\mathcal{E}[n_n, n_p] = \overbrace{\frac{\hbar^2}{2m}(\tau_n + \tau_p)}^{\text{kinetic}} +$$

$$+ \overbrace{\sum_{j=0}^{2} \left(a_j n^{5/3} + b_j n^2 + c_j n^{7/3}\right) \beta^{2j}}^{\text{homogeneous}} +$$

SeaLL1 NEDF



NEDF	ρ_0 [fm^{-3}]	$-\epsilon_0$	K_0	S	L	L_2	Neutron skin ^{208}Pb [fm]	Neutron skin ^{48}Ca [fm]
SeaLL1	0.154	15.58	230.0	31.7	32.4	31.6	0.131	0.155

Table III. Saturation, symmetry, and neutron skin properties for SeaLL1. All values in MeV unless otherwise specified.

$\beta = (n_n - n_p)/(n_n + n_p)$

Inspired by UFG

PCA reduces to 3 parameters (n_0, ϵ_0, S_2)

K_0 ±25MeV and L_2 not tightly constrained
(one more parameter)

β^4 terms fixed by neutron matter

$$\mathcal{E}[n_n, n_p] = \overbrace{\frac{\hbar^2}{2m}(\tau_n + \tau_p)}^{\text{kinetic}} +$$

$$+ \overbrace{\sum_{j=0}^{2}\left(a_j n^{5/3} + b_j n^2 + c_j n^{7/3}\right)\beta^{2j}}^{\text{homogeneous}} +$$

$$\overbrace{+ \eta_s \sum_{q=n,p}\frac{\hbar^2}{2m}\|\vec{\nabla} n_q\|^2}^{\text{gradient}} + \underbrace{W_0 \vec{J}\cdot\vec{\nabla} n}_{\text{spin-orbit}} +$$

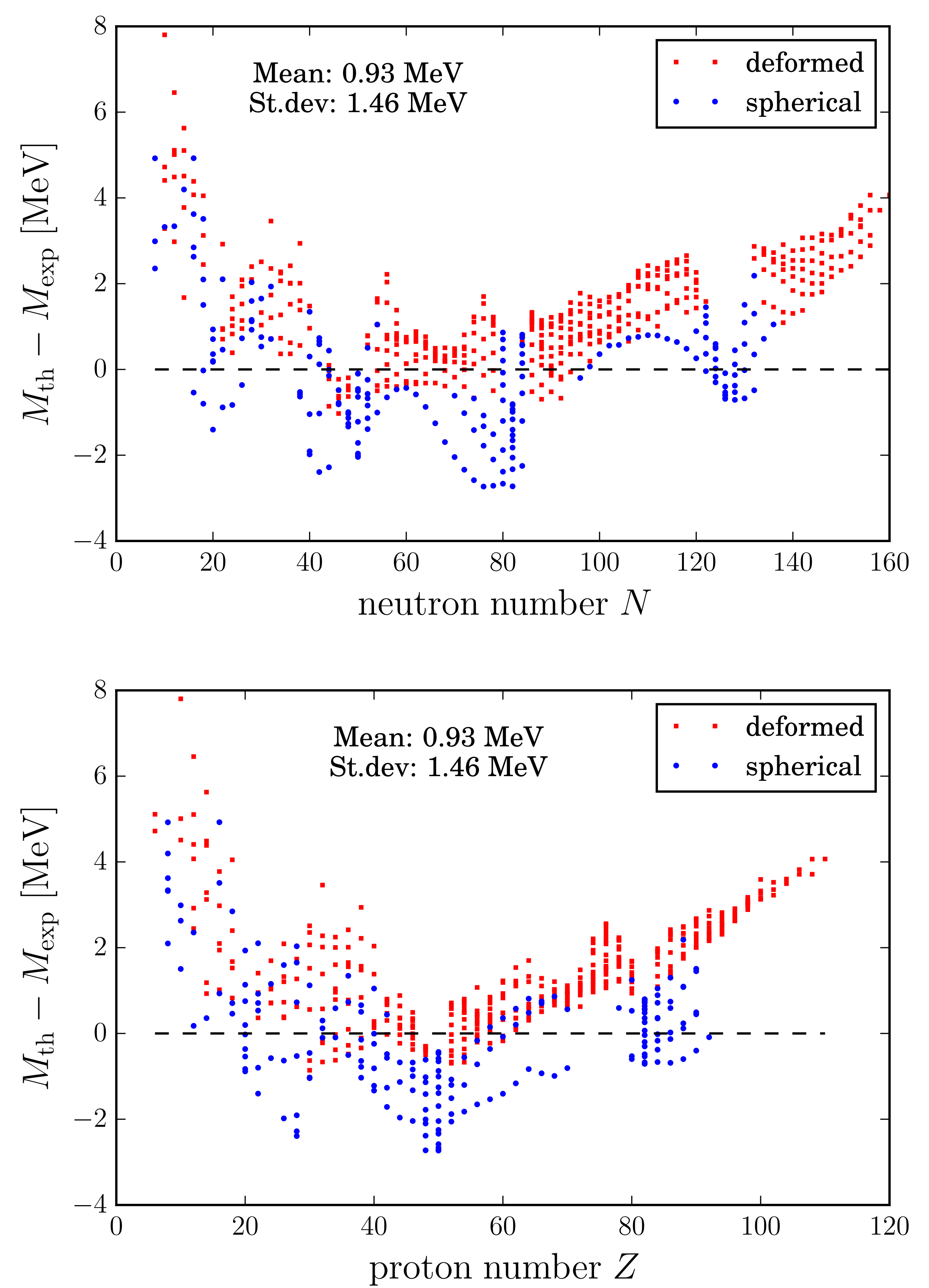
$$\underbrace{+ \sum_{q=n,p} g_{\text{eff}}(\vec{r})|\nu_q(\vec{r})|^2}_{\text{pairing}}$$

$$+ \underbrace{\frac{e^2}{2}\int d^3\vec{r}'\,\frac{n_p(\vec{r})n_p(\vec{r}')}{\|\vec{r}-\vec{r}'\|} - \frac{3e^2}{4}\left(\frac{n_p(\vec{r})}{3\pi}\right)^{4/3}}_{\text{Coulomb}}$$

SeaLL1
NEDF

Adjusts surface tension, diffuseness, pairing etc. Required and constrained by mass/radii fits

Residuals



606 even-even nuclei:

χ_E=1.74MeV (1.46MeV w/o bias)

Fit to 196 spherical nuclei

345 charge-radii

χ_r=0.034fm

No beyond mean-field corrections

Good for self-consistent dynamics

CoM correction for spherical nuclei:

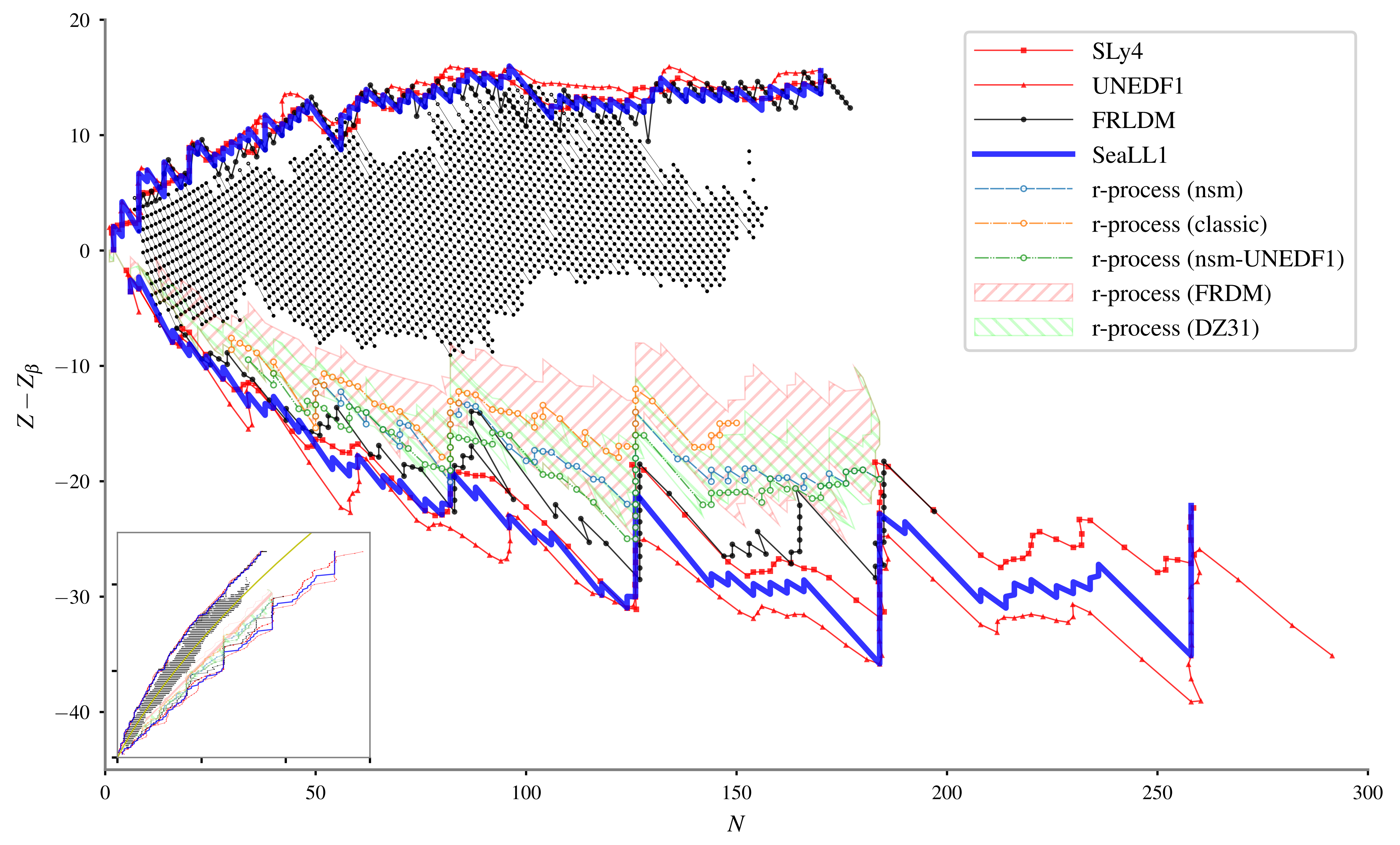
1.54MeV → 0.96MeV

2n (2p) separation energies

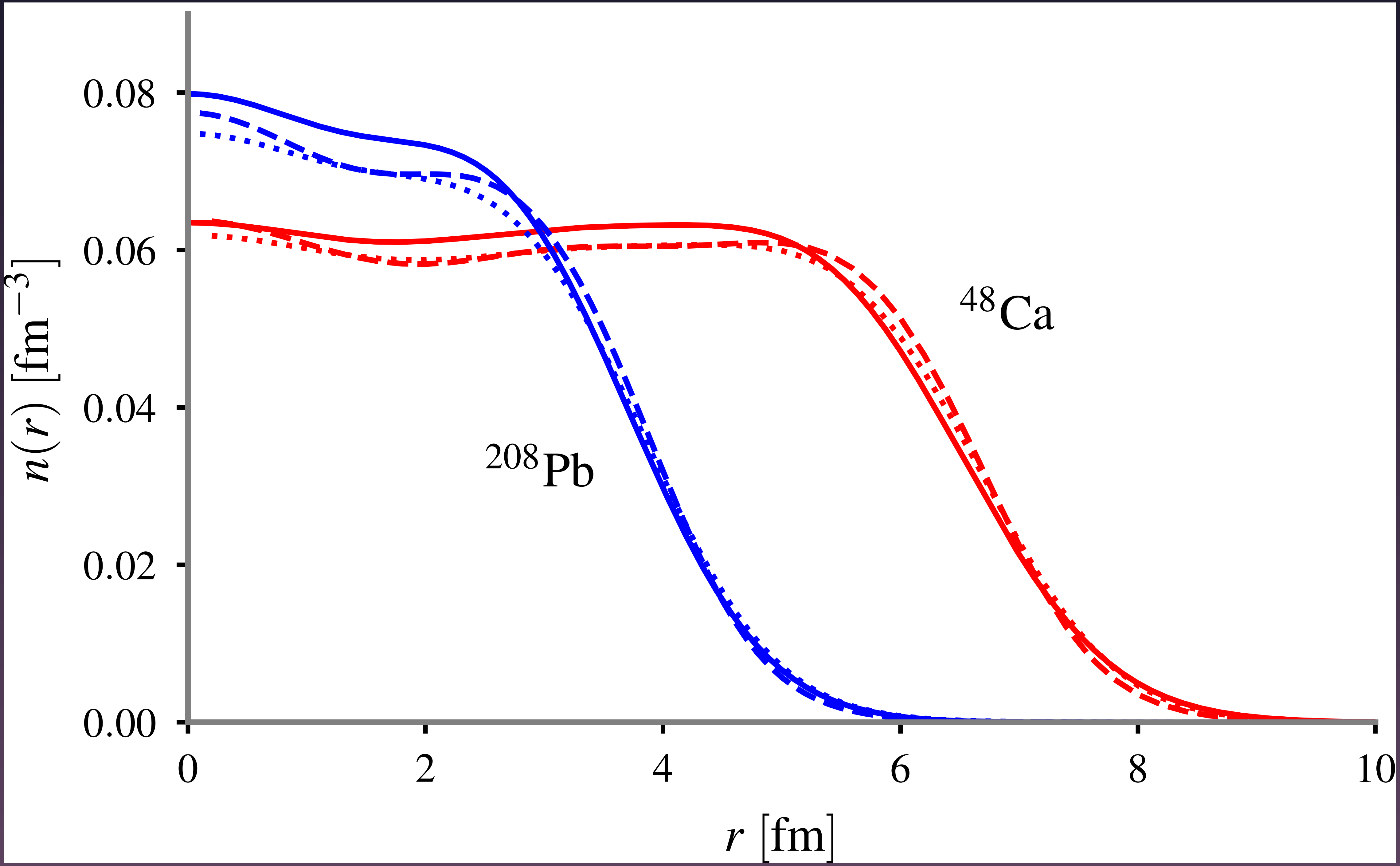
χ_E=0.69MeV (0.59MeV)

Lower than any other NEDF!

Neutron Drip Line

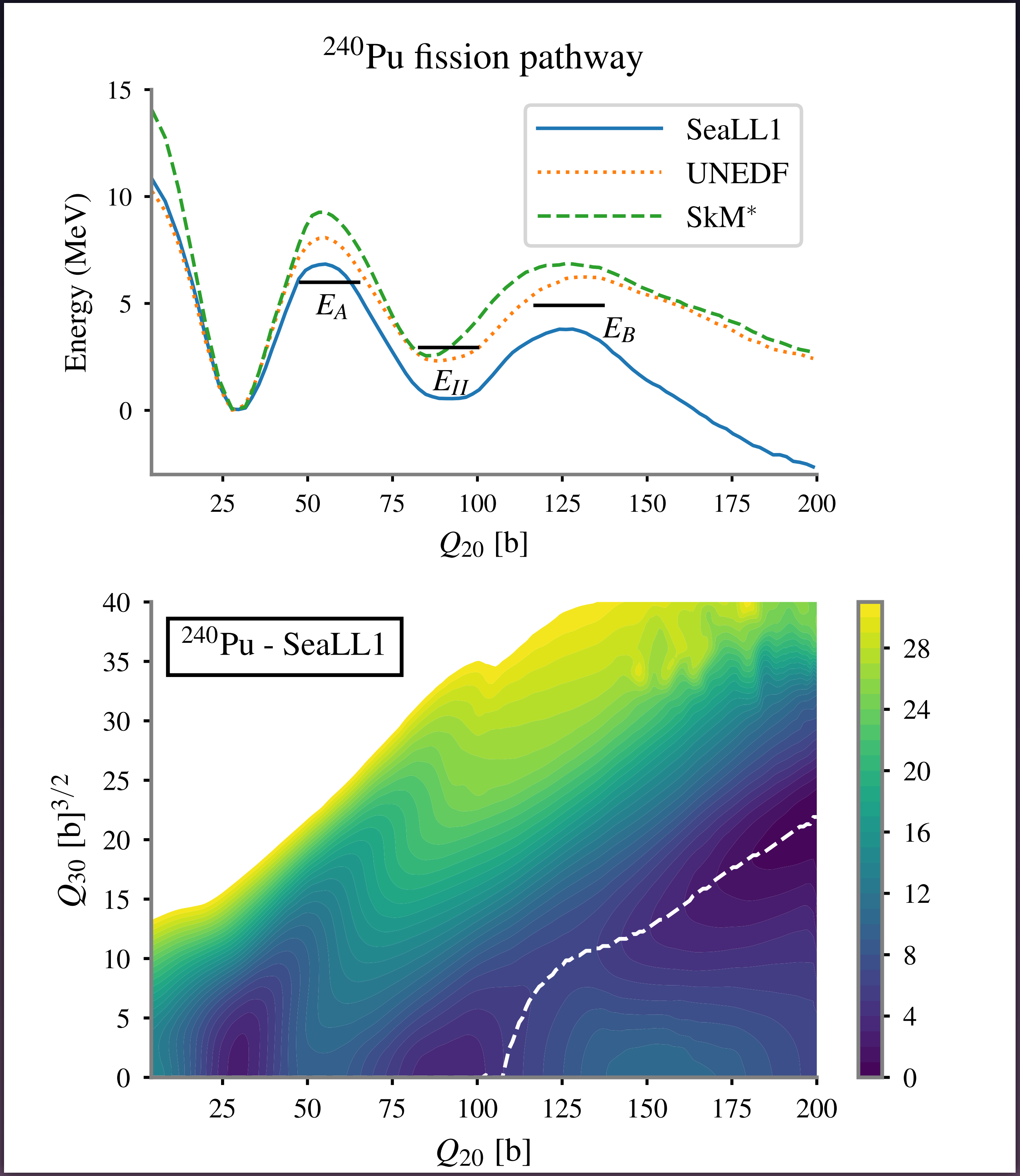


Density Profiles



Solid from De Vries et al. At. Data. Nucl. Data Tables 36, 495 – 536 (1987)

Fission (for Free)



Similar ^{240}Pu fission behaviour as SkM* but no tuning to match fission barriers

Other NEDFs have similar errors in barrier heights

Subdominant and Free Parameters to Tune

- Entrainment terms

  GDR, Thomas-Kuhn-Reiche sum rule, Gamow-Teller resonance

- Gradient terms/density-dependent kinetic terms

  Surface tension and diffuseness, Static dipole susceptibility

  Proton and neutron effective masses

- Isospin-breaking pairing/spin-orbit coupling

  Spin-orbit splittings, Pairing gaps

- Decoupled from primary mass fit parameters

SeaLL1 NEDF

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$$+ \underbrace{\sum_{q=n,p} g_{\text{eff}}(\vec{r})|\nu_q(\vec{r})|^2}_{\text{pairing}}$$

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$\beta = (n_n - n_p)/(n_n + n_p)$

Inspired by UFG

PCA reduces to 3 parameters (n_0, ϵ_0, S_2)

K_0 ±20MeV and L_2 not tightly constrained
(one more parameter)

β^4 terms fixed by neutron matter

Nuclear EoS: $L_2 \neq L$

$$\frac{\mathcal{E}(n_n, n_p)}{n} = \epsilon_0(n) + \epsilon_2(n)\left(\frac{n_n - n_p}{n}\right)^2 + \epsilon_4(n)\left(\frac{n_n - n_p}{n}\right)^4 + \cdots$$

- Isospin expansion about symmetric nuclear matter
- $$\begin{aligned} \epsilon_0(n) &= \varepsilon_0 \qquad\qquad + \tfrac{1}{2}K_0\delta^2 + O(\delta^3) \\ \epsilon_2(n) &= S_2 + L_2 + \tfrac{1}{2}K_0\delta^2 + O(\delta^3) \\ \epsilon_4(n) &= S_4 + L_4 + \tfrac{1}{2}K_0\delta^2 + O(\delta^3) \end{aligned}$$

$$\delta = \frac{n - n_0}{3n_0}, \qquad n = n_n + n_p$$

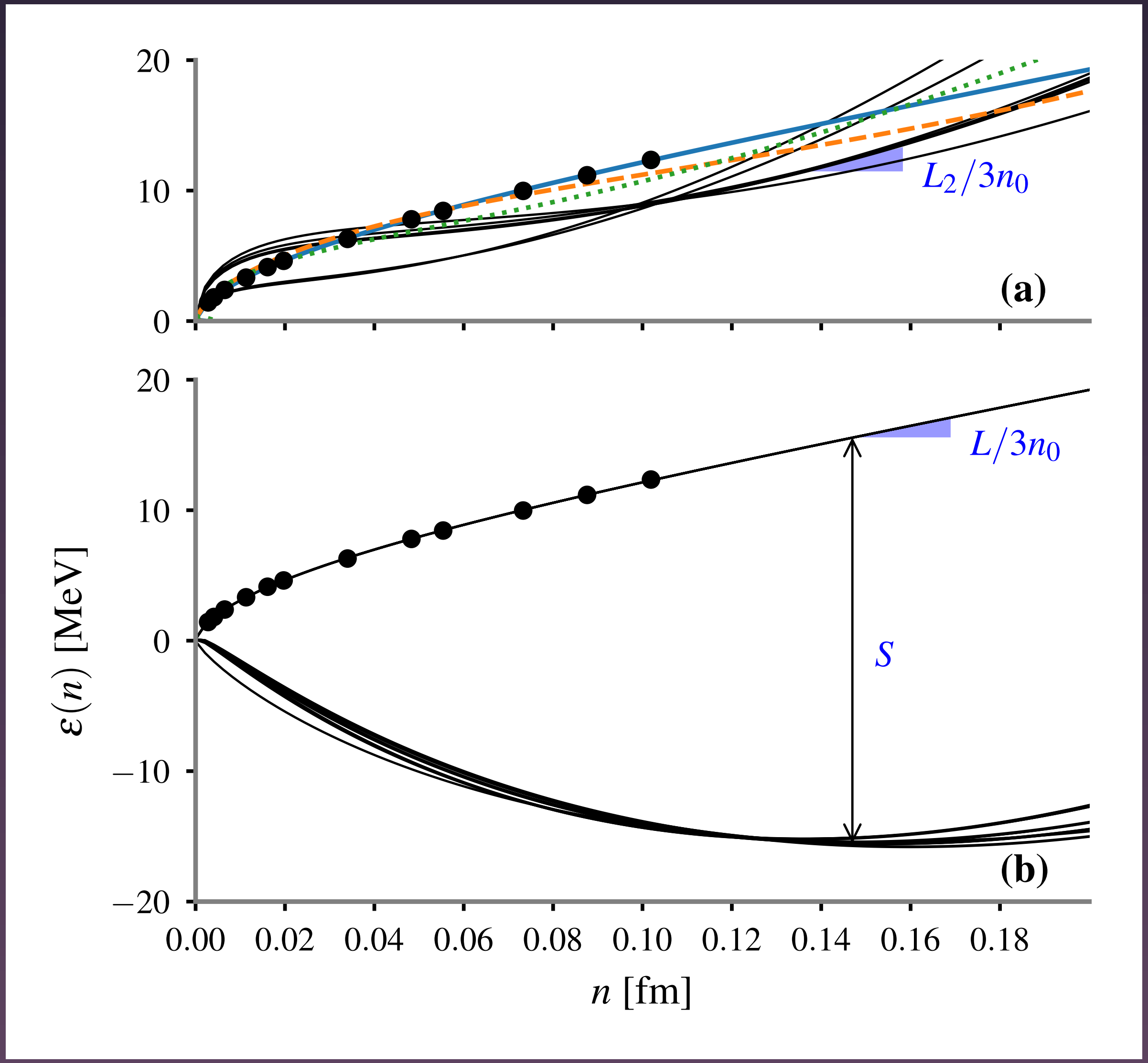
Symmetry Properties

$$\frac{\mathcal{E}(n,0)-\mathcal{E}(n/2,n/2)}{n} = \underbrace{(S_2+S_4+\cdots)}_{S} + \underbrace{(L_2+L_4+\cdots)}_{L}\delta + \cdots$$

- Isospin expansion about symmetric nuclear matter
- $L{=}L_2$ Only if $(n_n - n_p)^4$ terms are insignificant

See also Lattimer NPA 928 (2014) 276

Nuclear EoS: $L_2 \neq L$



- $(n_n - n_p)^2$

  Fit nuclei (masses, radii, skins etc.)

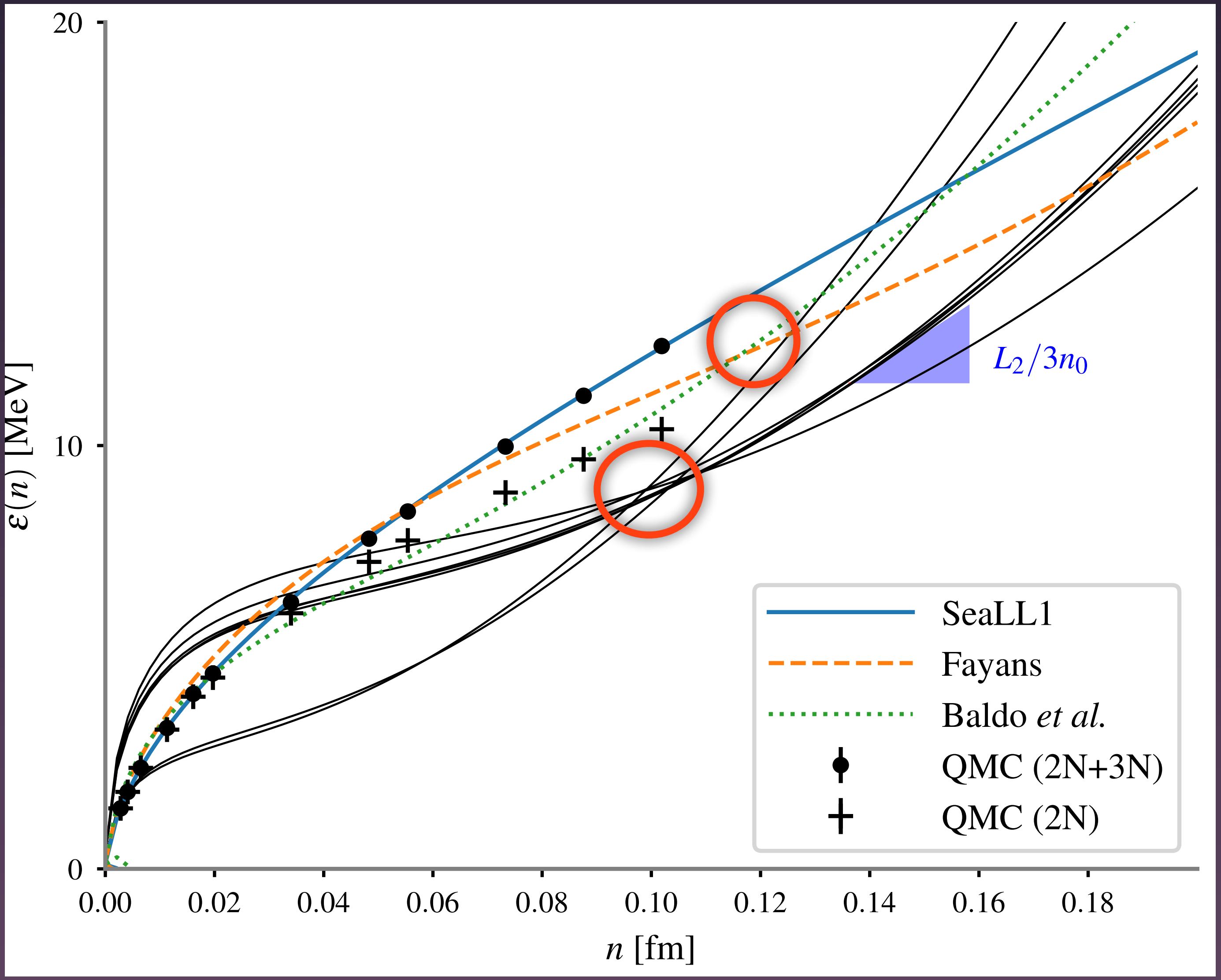
- $(n_n - n_p)^4$

  Seems needed for neutron matter

  Setting these to match neutron matter does not affect quality of fits

Bulgac, Forbes, Jin, Perez, Schunk: 1708.08771 (PRC 2018)

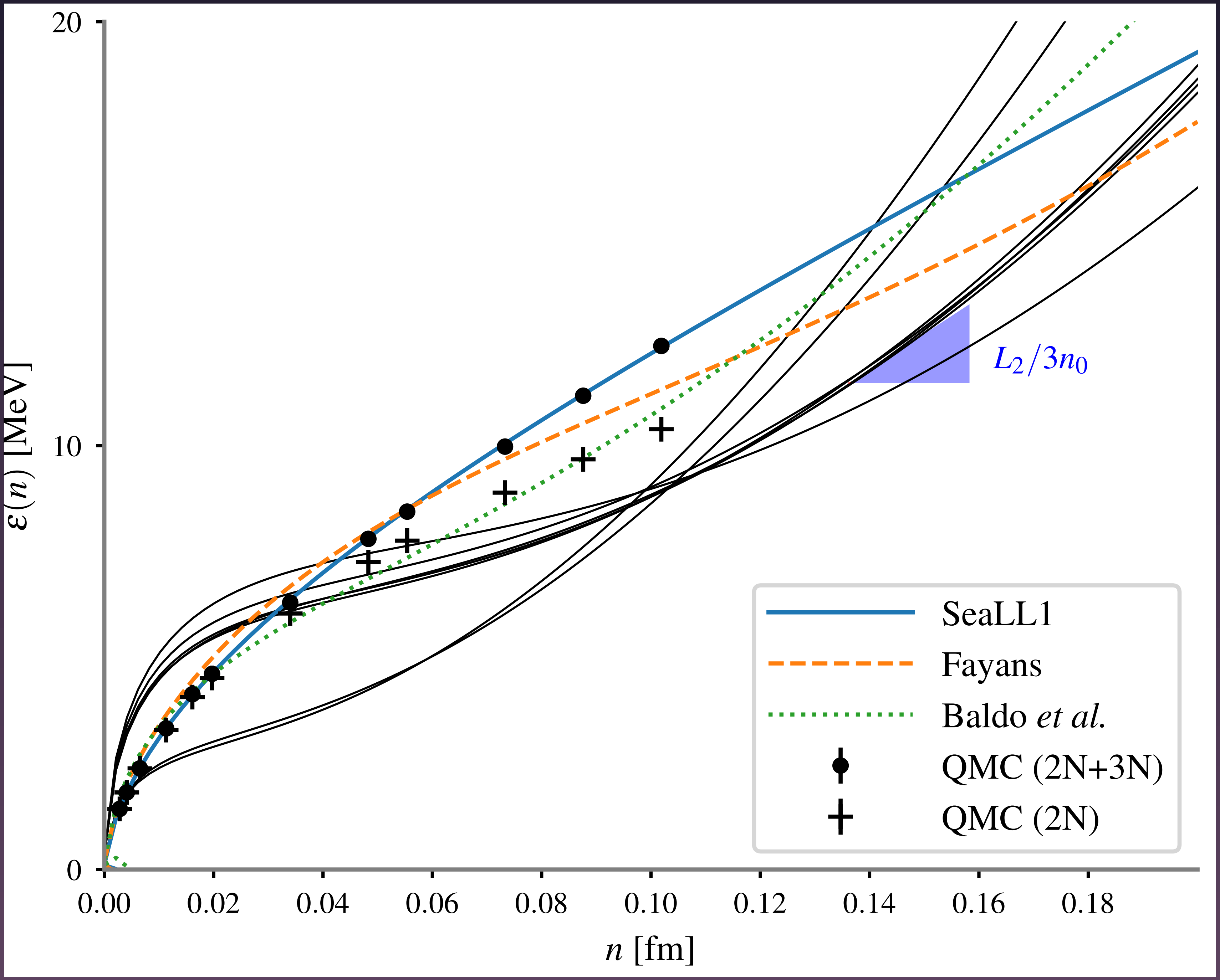
Nuclear EoS: $L_2 \neq L$



- Older functionals fit QMC without 3N
- Hard to fit new QMC with $(n_n - n_p)^2$
- $(n_n - n_p)^4$ fits neutron matter without affecting nuclear fits

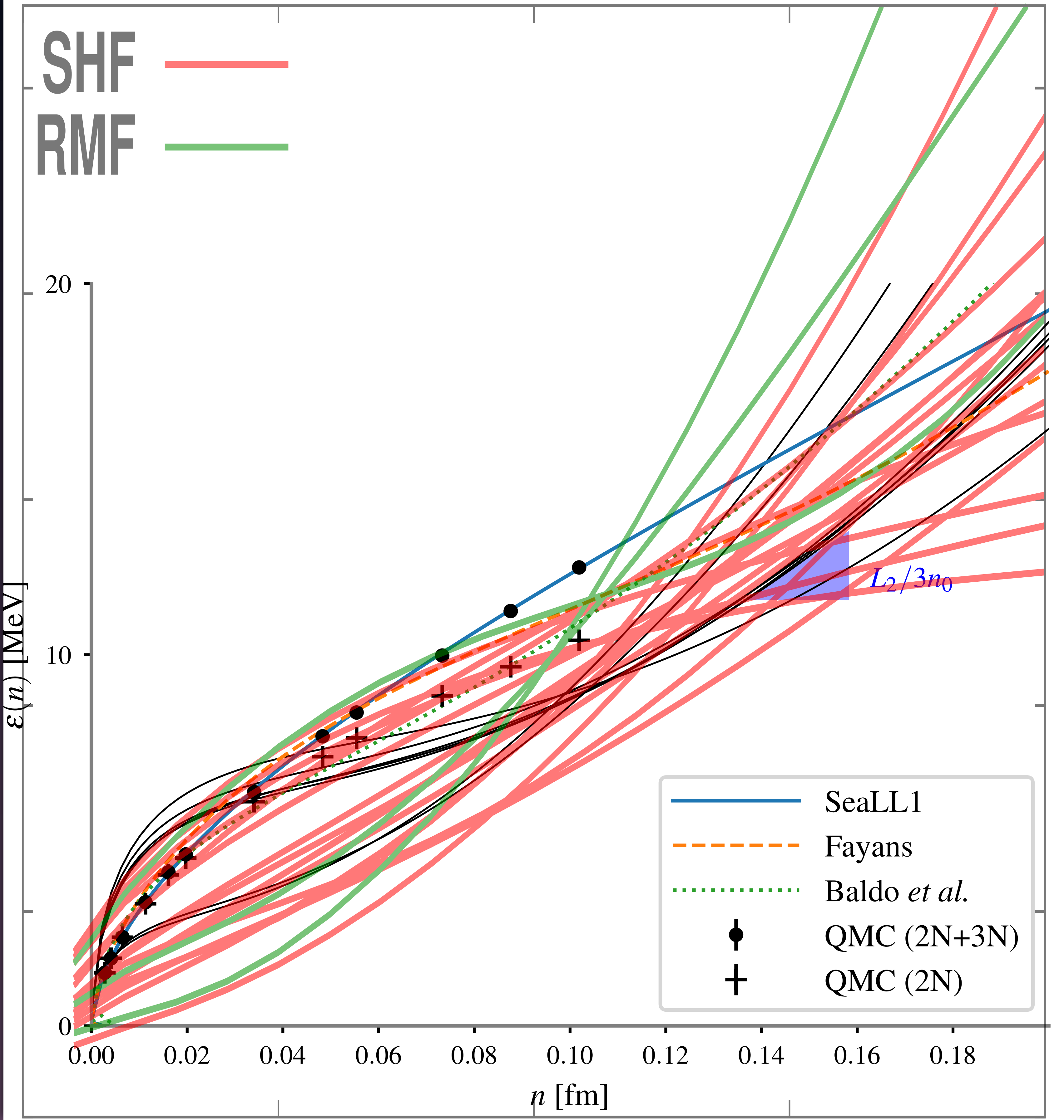
QMC from Wlazłowski, Holt, Moroz, Bulgac, Roche, PRL 113 (2014) 182503 [1403.3753]

$L_2 \neq L$



20
10
0
$\mathcal{E}(n)$ [MeV]
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
n [fm]
$L_2/3n_0$
SeaLL1
Fayans
Baldo *et al.*
QMC (2N+3N)
QMC (2N)

$L_2 \neq L$

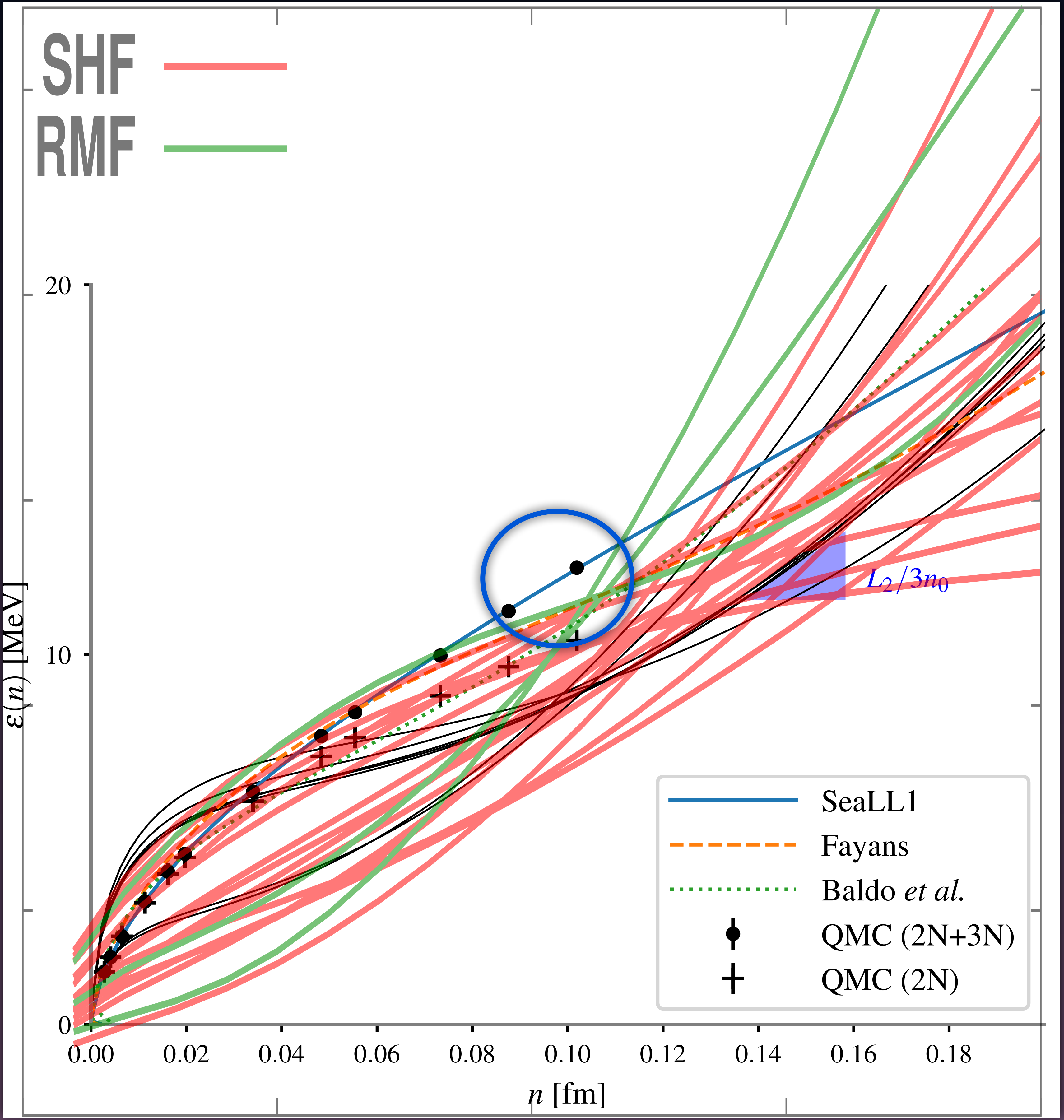


P.-G. Reinhard at ECT*, Trento, Italy, 26-30 January, 2015,
https://sites.google. com/site/ectworkshopns2015/talks

Skyrme and Relativistic Mean Field Theory

Still misses neutron matter

Possible that neutron matter is essentially independent of nuclei?



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$L_2 \neq L$

Skyrme and Relativistic Mean Field Theory

Still misses neutron matter

Possible that neutron matter is essentially independent of nuclei?

Connect Nuclear EoS with Neutron Stars

- Expand from neutron matter
- Constrain with symmetric matter
- Unified EoS

  Compressible Liquid Drop Model (CLDM) matching

  Similar to Fortin et al. PRC 94 (2016) 035804
- Parametrized Core

  Characterize the speed of sound

Connect Nuclear EoS with Neutron Stars

- Outer Crust

  Tabulated Data

- Compressible Liquid Drop Model (CLDM)

  Match outer crust to homogeneous matter

- Homogeneous Matter

  EoS extrapolated from neutron matter

- Core

  Parameterize speed of sound

with Sanjay Reddy (UW), Dake Zhou (UW), Sukanta Bose (WSU)

Parameters

Outer Crust

- None

Compressible Liquid Drop Model (CLDM)

- Match outer crust to homogeneous matter

Homogeneous Matter

- EoS extrapolated from neutron matter

Core

- Parameterize speed of sound

0 Parameters

- Outer Crust
  i.e. Negele-Vautherin
  Augment if needed to ensure convexity
- No parameters

Outer Crust
None

Compressible Liquid Drop Model (CLDM)
Match outer crust to homogeneous matter

Homogeneous Matter
EoS extrapolated from neutron matter

Core
Parameterize speed of sound

2 Parameters

- Unified EoS

  Solve for Wigner-Seitz cell given homogeneous EoS

- Two parameters
  - `C_C`

    Coulomb suppression
  - `sigma_delta`

  Isospin dependence

  Other surface parameter fixed to smoothly match to tabulated outer crust

Lattimer et al. NPA 432 (1985) 646
Steiner PRC 85 (012) 055804

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

Homogeneous Matter

EoS extrapolated from neutron matter

Core

Parameterize speed of sound

13 Parameters

- Pure neutron matter: 4
  a, α, b, β
- Proton polaron: 3
  $\mu_p(\tilde{n}_0), u_p, m^*$
- Symmetric nuclear matter: 6
  n_0, ϵ_0, K_0
  S_2, L_2, K_2

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

Homogeneous Matter

EoS extrapolated from neutron matter

Core

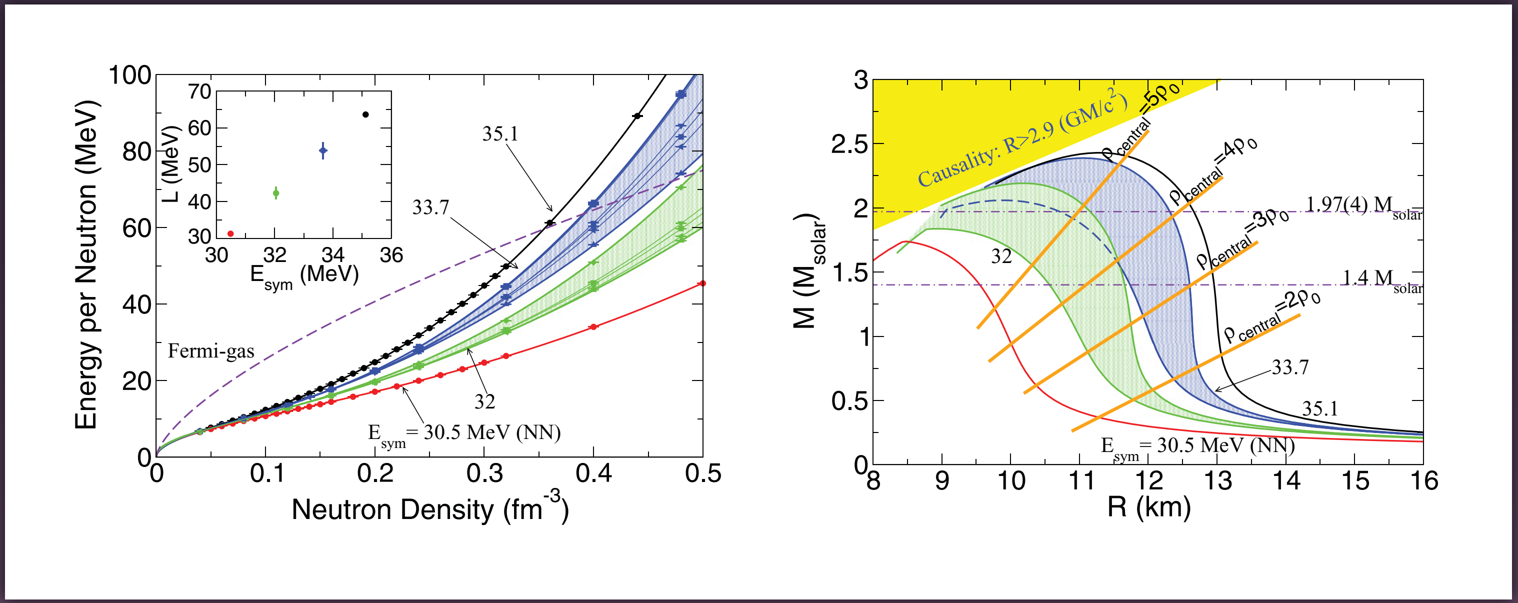
Parameterize speed of sound

4 Parameters

Start from pure Neutron matter

$$E_n(n_n) = \frac{\mathcal{E}_n(n_n)}{n_n} = m_n c^2 + a\left(\frac{n_n}{\bar{n}_0}\right)^{\alpha} + b\left(\frac{n_n}{\bar{n}_0}\right)^{\beta}$$

Generally fits QMC data and trends



Gandolfi et al. PRC 85 (2012) 032801(R)

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

Homogeneous Matter

EoS extrapolated from neutron matter

Core

Parameterize speed of sound

Expand in powers of proton fraction

$$x_p = \frac{n_p}{n_n + n_p}$$

$$E_{np}(n_n, n_p) = (1 - x_p)E_n(n_n) + \\ + x_p\left(m_p c^2 + \Sigma^p(n_B)\right) + \\ + \frac{(2\pi^2)^{2/3}}{2m^*} x_p^{5/3} n_B^{2/3} \\ + x_p^2 f_2(n_B) + x_p^3 f_3(n_B) + \cdots$$

Outer Crust

- None

Compressible Liquid Drop Model (CLDM)

- Match outer crust to homogeneous matter

Homogeneous Matter

- EoS extrapolated from neutron matter

Core

- Parameterize speed of sound

Bulk neutron matter

$$E_{np}(n_n, n_p) = (1 - x_p)E_n(n_n) + $$
$$+ x_p\left(m_p c^2 + \Sigma^p(n_B)\right) +$$
$$+ \frac{(2\pi^2)^{2/3}}{2m^*} x_p^{5/3} n_B^{2/3}$$
$$+ x_p^2 f_2(n_B) + x_p^3 f_3(n_B) + \cdots$$

Outer Crust

- None

Compressible Liquid Drop Model (CLDM)

- Match outer crust to homogeneous matter

Homogeneous Matter

- EoS extrapolated from neutron matter

Core

- Parameterize speed of sound

2 Parameters

Proton polaron

Dilute proton gas in neutron background

QMC for chemical potential

Roggero et al. PRL 112 (2014) 221103

$$E_{np}(n_n, n_p) = (1 - x_p)E_n(n_n) + \\ + x_p\left(m_p c^2 + \Sigma^p(n_B)\right) + \\ + \frac{(2\pi^2)^{2/3}}{2m^*} x_p^{5/3} n_B^{2/3} \\ + x_p^2 f_2(n_B) + x_p^3 f_3(n_B) + \cdots$$

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

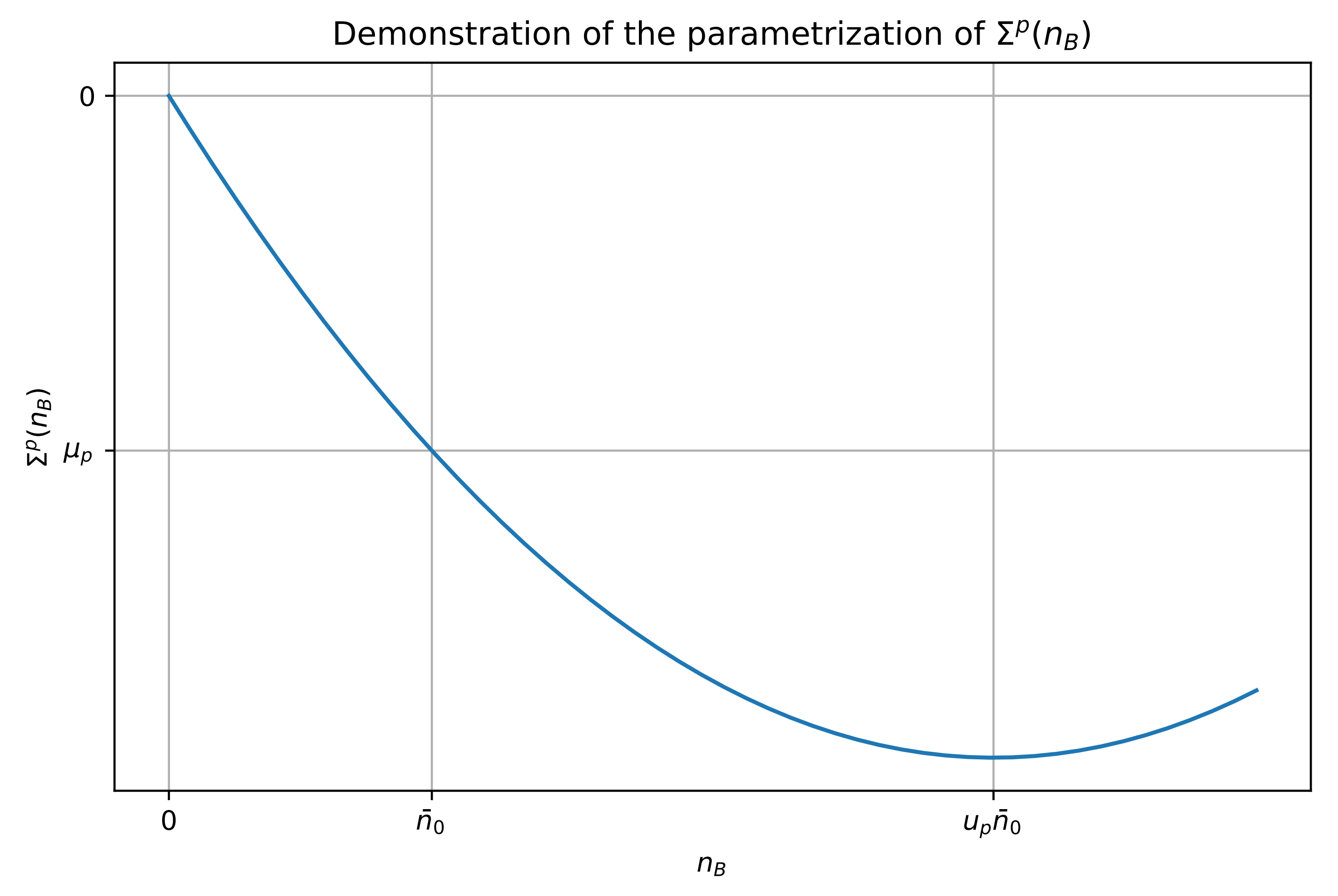
Homogeneous Matter

EoS extrapolated from neutron matter

Core

Parameterize speed of sound

2 Parameters



Polaron Parameters

- $\mu_p(\tilde{n}_0)$
  Chemical potential at saturation
- u_p
  Location of minimum

1 Parameter

Proton gas Fermi energy

- Proton effective mass

$$
\begin{aligned}
E_{np}(n_n, n_p) &= (1 - x_p)E_n(n_n) + \\
&+ x_p\left(m_p c^2 + \Sigma^p(n_B)\right) + \\
&+ \boxed{\frac{(2\pi^2)^{2/3}}{2m^*} x_p^{5/3} n_B^{2/3}} \\
&+ x_p^2 f_2(n_B) + x_p^3 f_3(n_B) + \cdots
\end{aligned}
$$

Outer Crust

- None

Compressible Liquid Drop Model (CLDM)

- Match outer crust to homogeneous matter

Homogeneous Matter

- EoS extrapolated from neutron matter

Core

- Parameterize speed of sound

6 Parameters

Expand and match symmetric nuclear matter

$$\frac{\mathcal{E}(n_n, n_p)}{n} = \epsilon_0(n) + \epsilon_2(n)\left(\frac{n_n - n_p}{n}\right)^2$$

Fix up to $x_p^5 f_5(n_B)$

$$E_{np}(n_n, n_p) = (1 - x_p)E_n(n_n) +$$

$$+ x_p\left(m_p c^2 + \Sigma^p(n_B)\right) +$$

$$+ \frac{(2\pi^2)^{2/3}}{2m^*} x_p^{5/3} n_B^{2/3}$$

$$+ x_p^2 f_2(n_B) + x_p^3 f_3(n_B) + \cdots$$

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

Homogeneous Matter

EoS extrapolated from neutron matter

Core

Parameterize speed of sound

6 Parameters

Expand and match symmetric nuclear matter

$$\frac{\mathcal{E}(n_n, n_p)}{n} = \epsilon_0(n) + \epsilon_2(n)\left(\frac{n_n - n_p}{n}\right)^2$$

Fix up to $x_p^5 f_5(n_B)$

$$\epsilon_0(n) = \varepsilon_0 + \frac{1}{2}K_0\delta^2 + O(\delta^3)$$

$$\epsilon_2(n) = S_2 + L_2 + \frac{1}{2}K_2\delta^2 + O(\delta^3)$$

$$\delta = \frac{n - n_0}{3n_0}, \qquad n = n_n + n_p$$

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

Match outer crust to homogeneous matter

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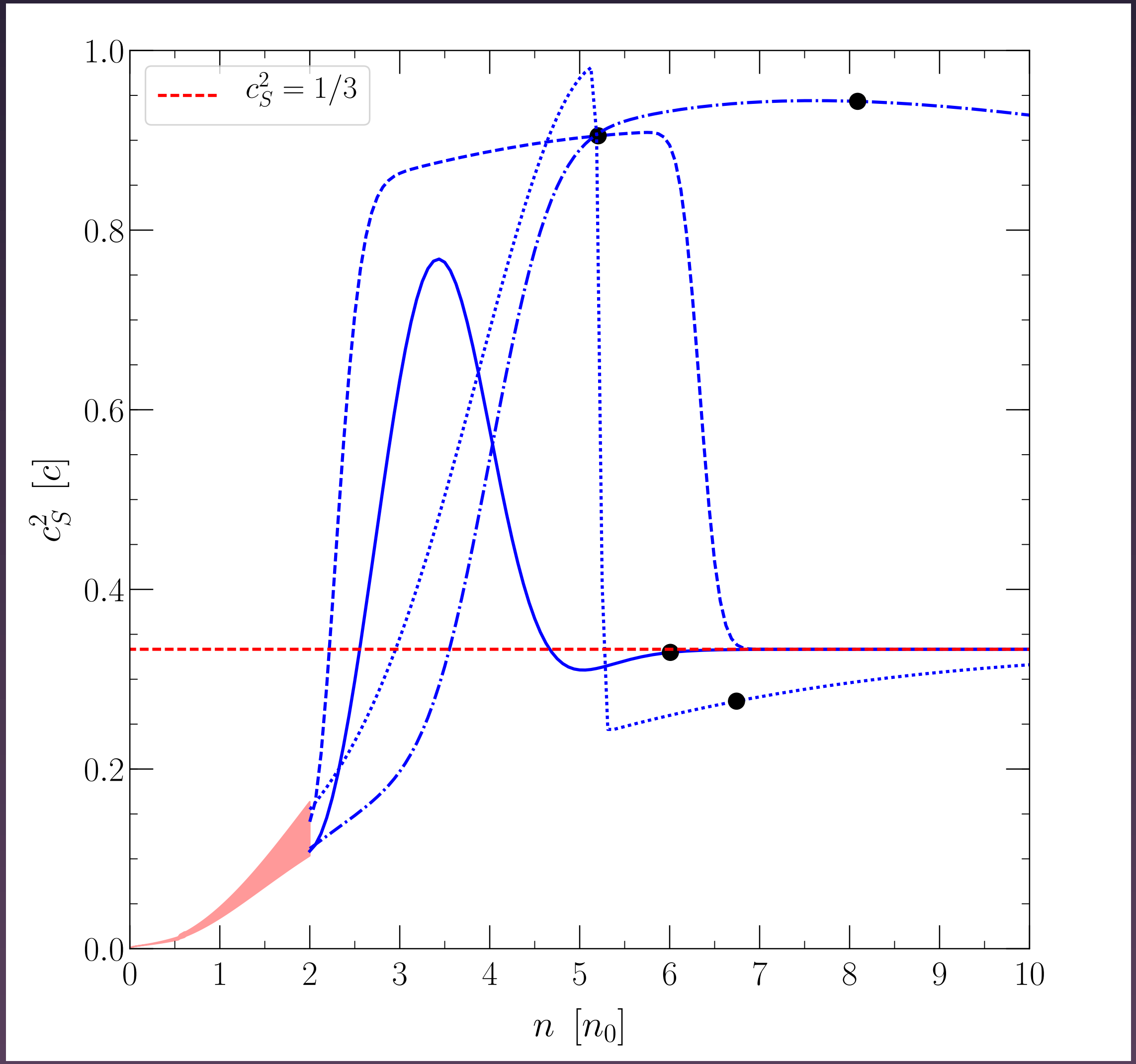
Core

Parameterize speed of sound

3 Parameters

Parametrize speed of sound $C = c_s^2/c^2$



Tews, Carlson, Gandolfi, Reddy [arXiv:1801.01923]

Outer Crust
None

Compressible Liquid Drop Model (CLDM)
Match outer crust to homogeneous matter

Homogeneous Matter
EoS extrapolated from neutron matter

Core
Parameterize speed of sound

3 Parameters

Parametrize speed of sound $C(\mathcal{E}) = c_s^2/c^2$

Quadratic polynomial as function of energy-density
(Easy to invert to get EoS)

- $\mathcal{E}_c$
  Transition from homogeneous matter
- C_{max}
  Maximum of polynomial
- $\mathcal{E}_{\text{max}}$
  Location of maximum

Outer Crust

None

Compressible Liquid Drop Model (CLDM)

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Homogeneous Matter

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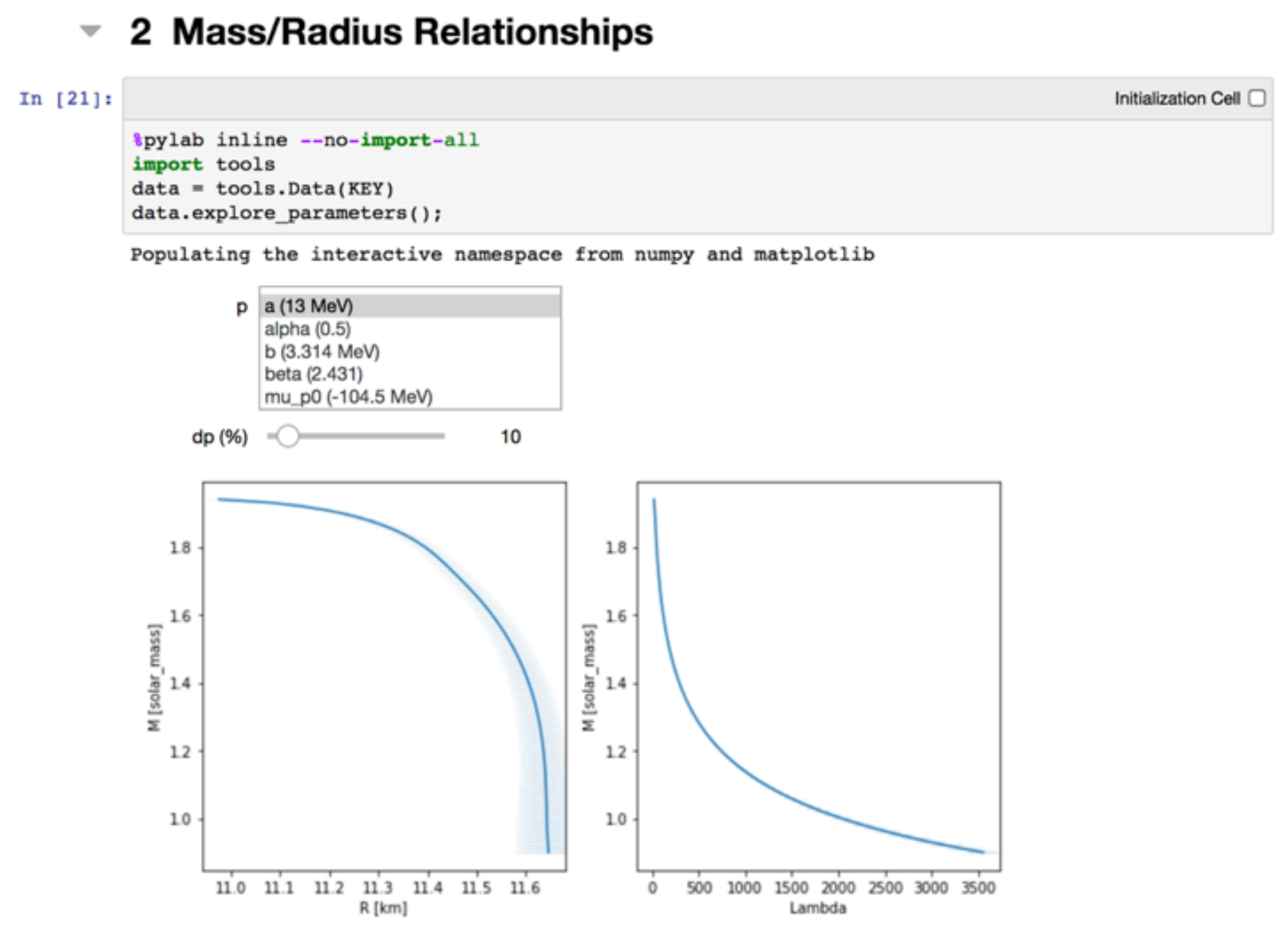
Core

Parameterize speed of sound

Live Demos on Binder (or CoCalc, etc.)

- Reproducible science
- LIGO analysis
- https://github.com/losc-tutorial/LOSC_Event_tutorial
- Parameter Exploration
- https://github.com/mforbes/binder_tov_explorer

Explore Parameters



2 Mass/Radius Relationships

In [21]:

Initialization Cell ☐

```
%pylab inline --no-import-all
import tools
data = tools.Data(KEY)
data.explore_parameters();
```

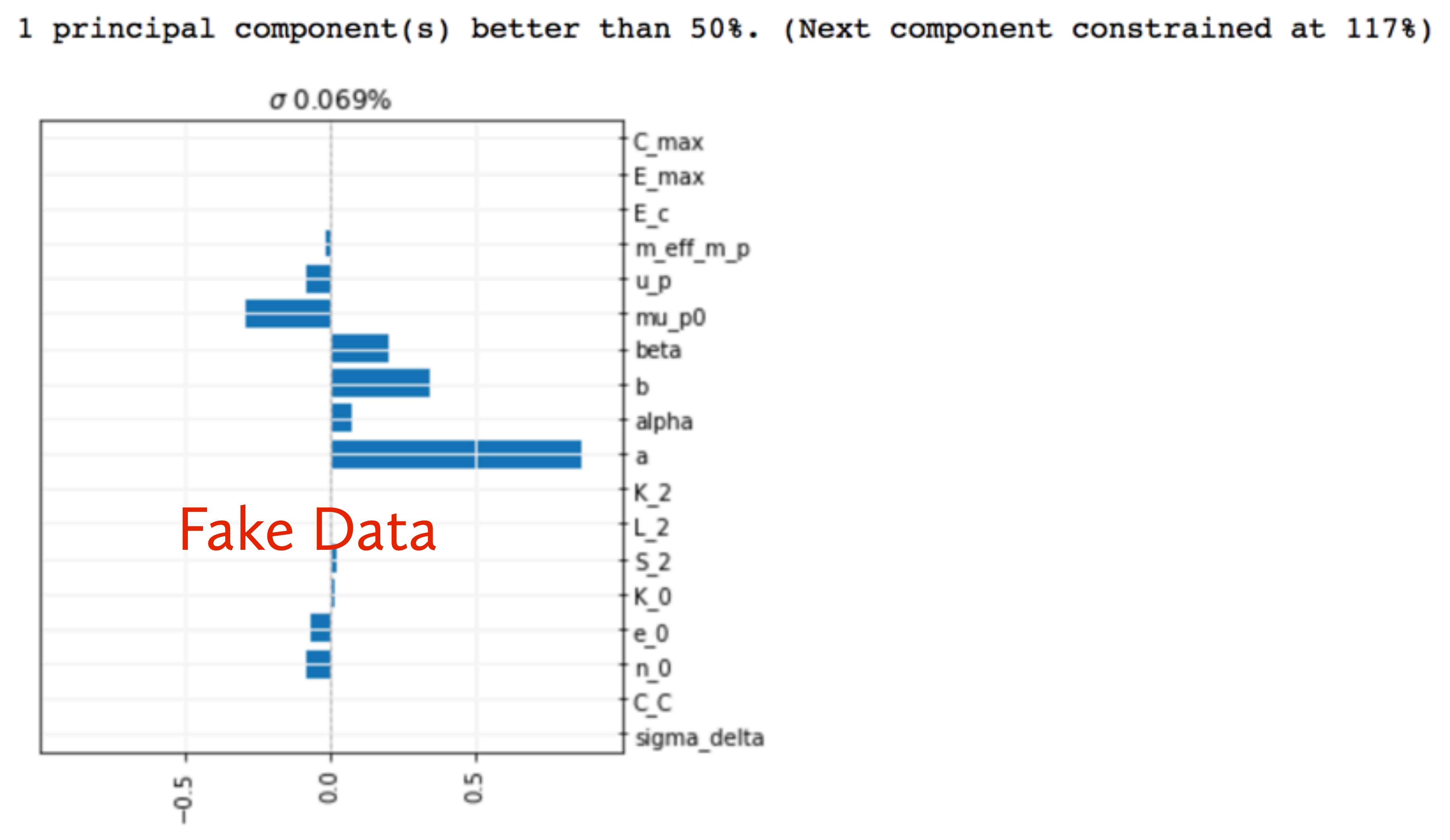
```
Populating the interactive namespace from numpy and matplotlib
```

p
- a (13 MeV)
- alpha (0.5)
- b (3.314 MeV)
- beta (2.431)
- mu_p0 (-104.5 MeV)

dp (%) 10

https://github.com/mforbes/binder_tov_explorer

Principal Components

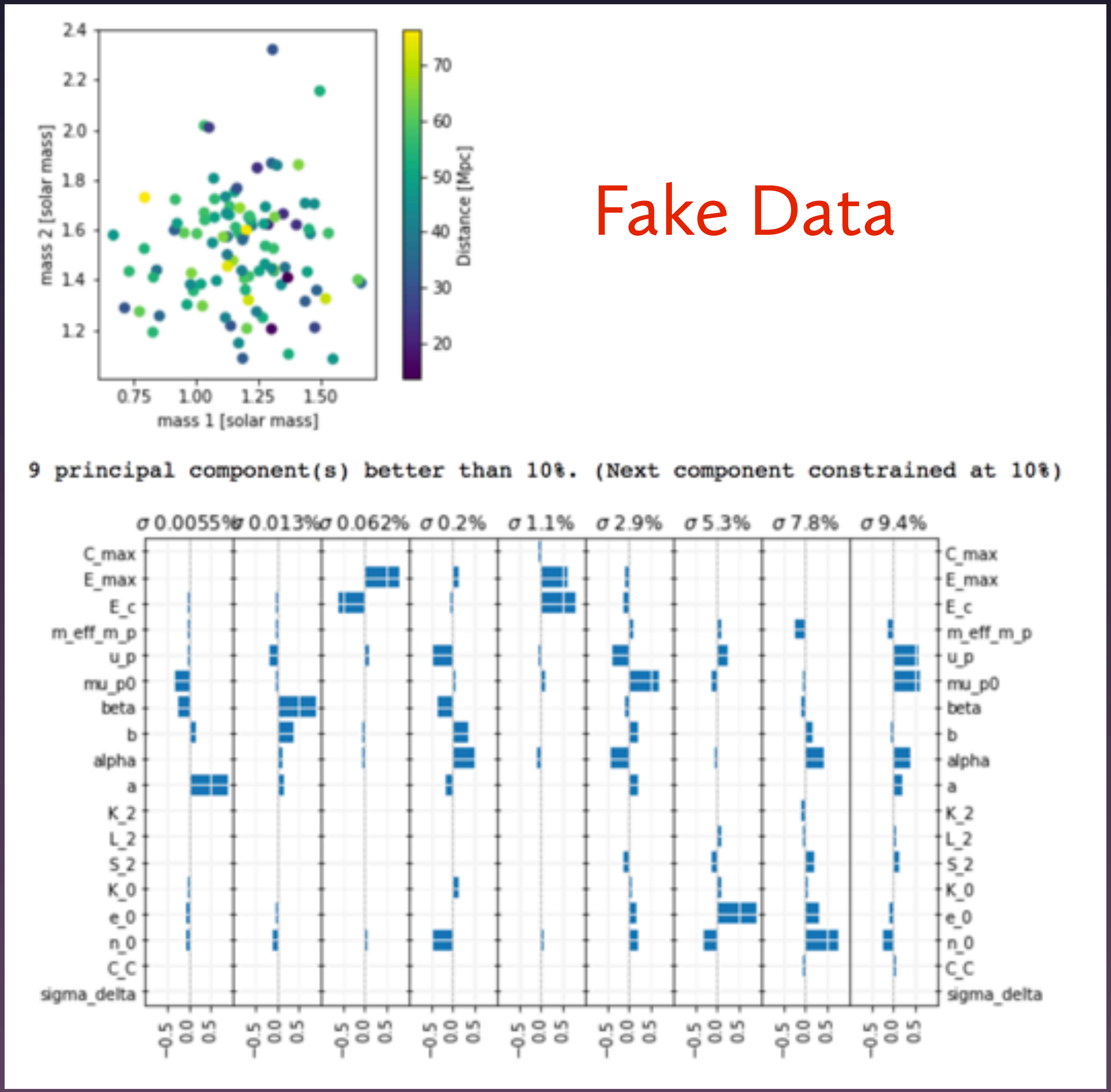


```
1 principal component(s) better than 50%. (Next component constrained at 117%)
```

Fake Data

https://github.com/mforbes/binder_tov_explorer

Principal Components



Fake Data

```
9 principal component(s) better than 10%. (Next component constrained at 10%)
```

https://github.com/mforbes/binder_tov_explorer

Missing? Lots!

- Phase transitions
- Clustering
- Pasta
- Finite T
- Out of equilibrium
- Core?!?

Conclusions

- New Minimal Nuclear Functional SeaLL1
  - Good static and dynamic properties
  - Suggests $L_2 \neq L$
- Parametrize Neutron Star EoS from Neutron Matter
- Explore parameter sensitivity
- Share results and code