What can we learn from joint EM/GW observations of pulsars?

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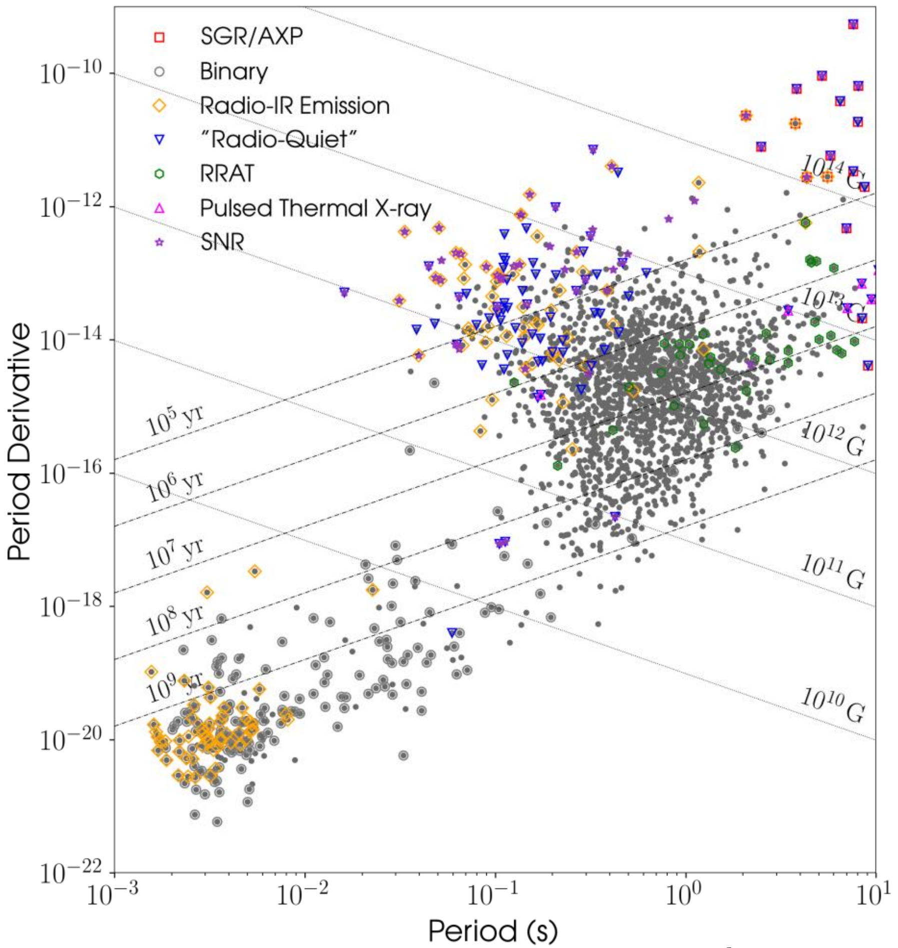
The neutron star Fermi Paradox

- We know of ~2000 pulsars, with ~200 in the GW detector band
- Estimated ~160 000 isolated and ~40 000 binary pulsars in the Galaxy
- A total of $\sim 10^9$ neutron stars in the Galaxy – where are they all?
- Ultimately it's a matter of detector/pipeline sensitivity
- EM+GW observations can help…

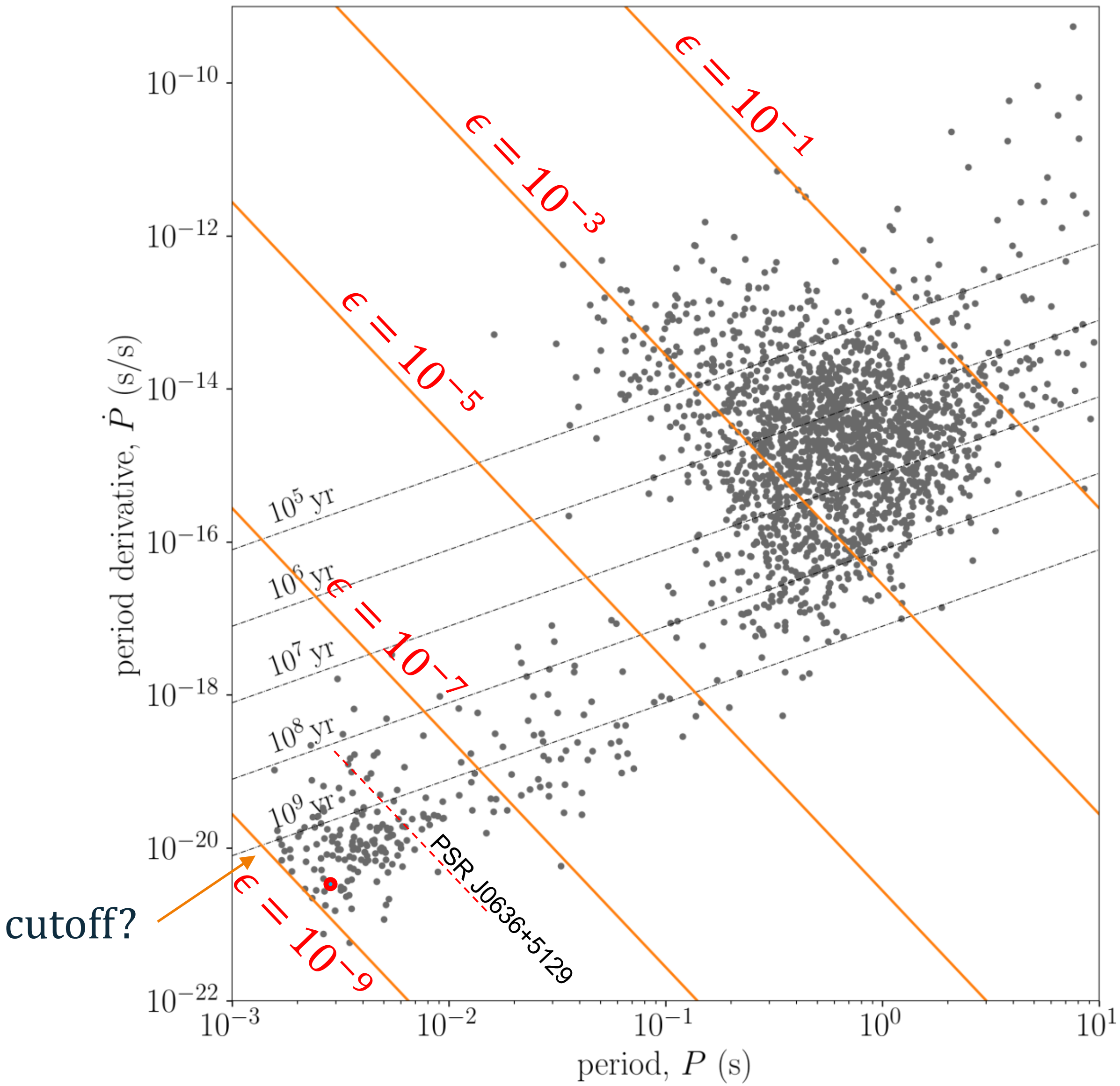
P–Pdot diagram



Matt Pitkin, psrqpy

Gravitars on the P-Pdot diagram

- Lines of constant ellipticity (const. Q_{22}) for pure-GW-braking Gravitars:

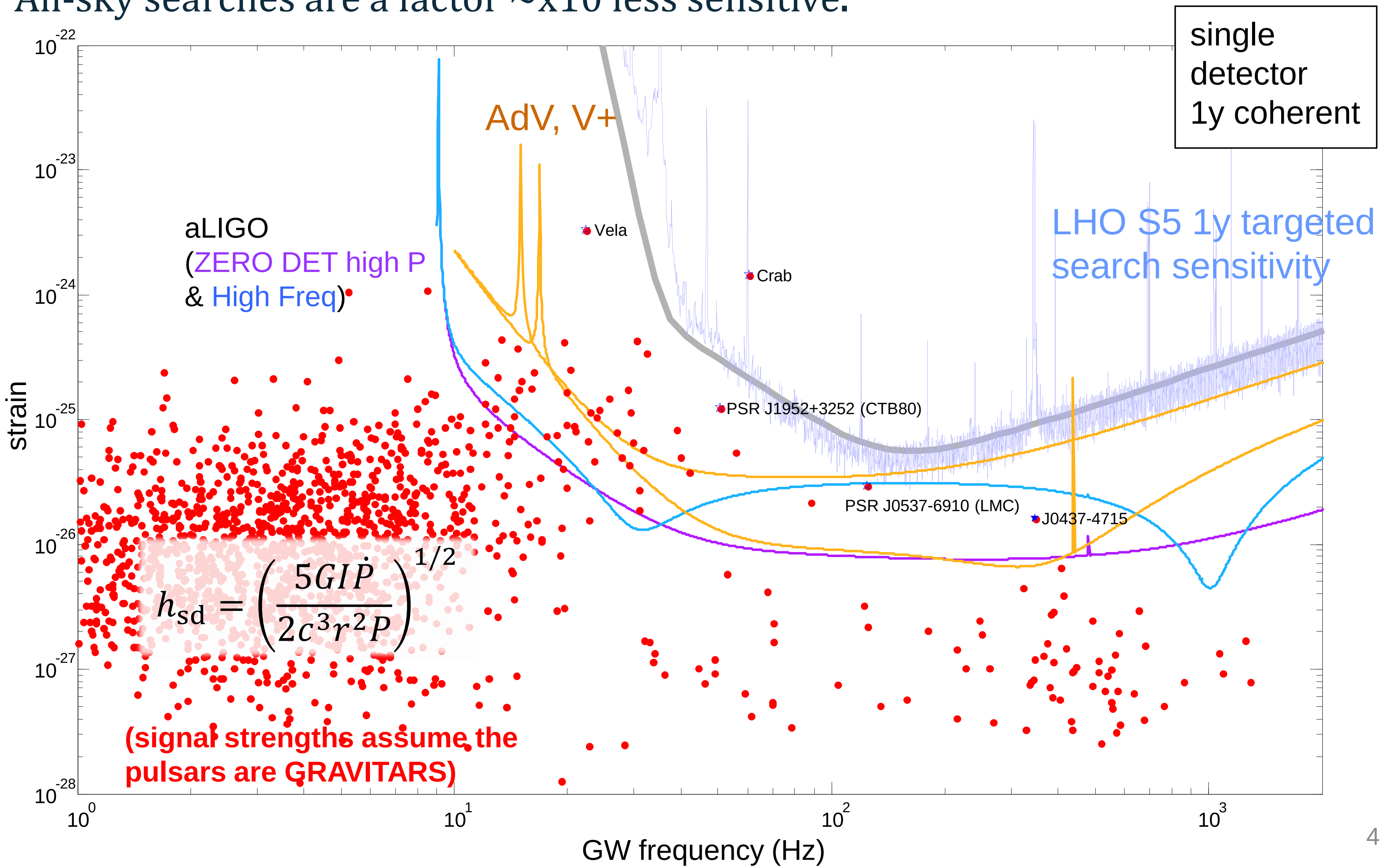


$$\epsilon = \frac{Q_{22}}{I_{zz}}\sqrt{\frac{8\pi}{15}}$$

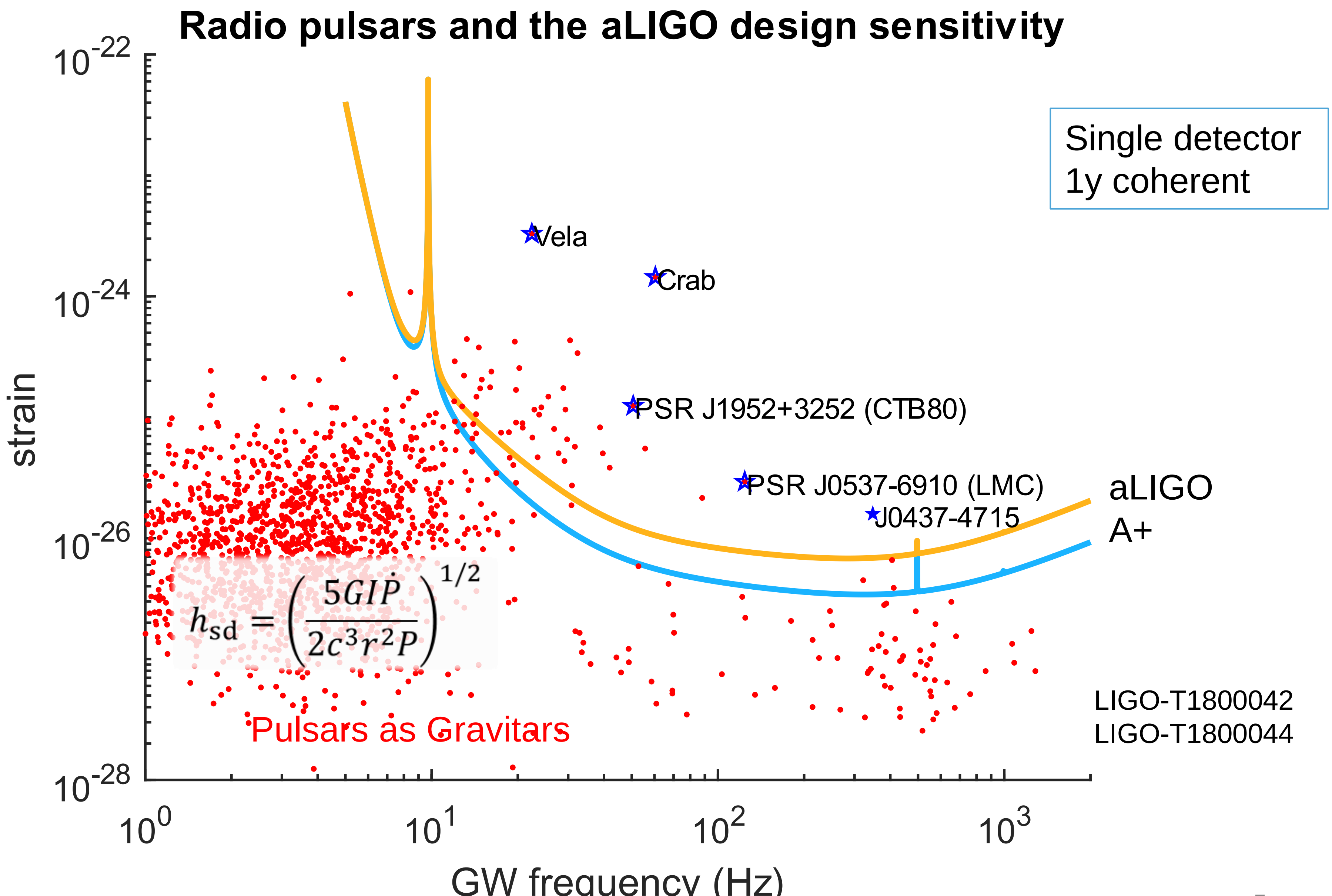
$$I_{zz} = 10^{38}\ \mathrm{kg\ m^2}$$

Overview

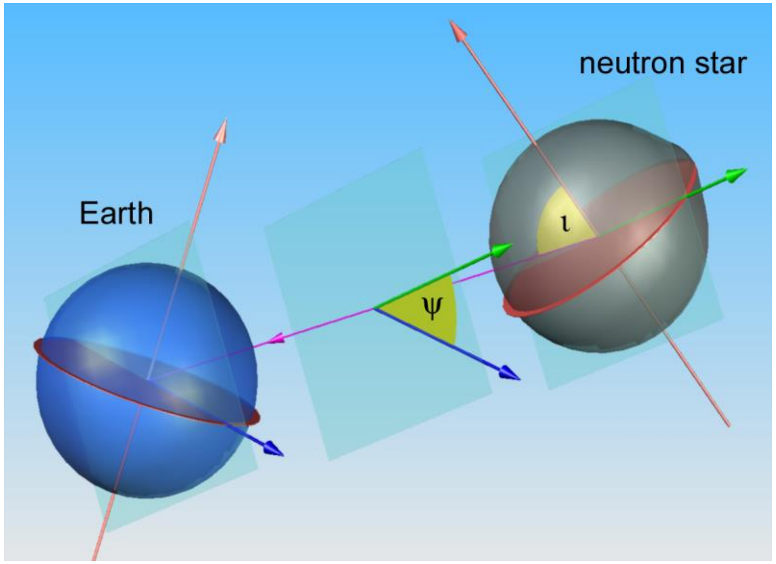
- Targeted search sensitivity (averaged over sky position).
  All-sky searches are a factor ~x10 less sensitive.



Overview


Radio pulsars and the aLIGO design sensitivity
Single detector
1y coherent
Vela
Crab
PSR J1952+3252 (CTB80)
PSR J0537-6910 (LMC)
J0437-4715
aLIGO
A+
$h_{sd} = \left(\frac{5GI\dot{P}}{2c^3 r^2 P} \right)^{1/2}$
Pulsars as Gravitars
LIGO-T1800042
LIGO-T1800044
strain
GW frequency (Hz)
10^{-22}
10^{-24}
10^{-26}
10^{-28}
10^{0}
10^{1}
10^{2}
10^{3}

Reference model: the triaxial ellipsoid



- Seen in the detector as a quasi-sinusoidal strain signal $h(t)$, amplitude-modulated by the diurnal antenna pattern $F_+, F_\times$ of the detector:

$$h(t) = \frac{1}{2} F_+(t,\psi) h_0 (1 + \cos^2 \iota) \cos 2\Phi(t) + F_\times(t,\psi) h_0 \cos \iota \sin 2\Phi(t),$$

- If we know the position and phase evolution of the pulsar from radio observations (see Matt's talk), the only unknowns are:

h_0	the amplitude of the gravitational wave signal	→ Quadrupole, *if you also have the distance*
ψ	the polarisation angle of the signal	The orientation: only informative *with respect to something else!*
ι	the inclination angle of the axis of spin	
$\Phi(0)$	the phase of the GW signal at $t = 0$.	

How can EM observations help in detection?

Sky position and spin

- An all-sky, all-frequency/spindown search for a CW signal is inevitably less sensitive than a targeted search.

- Demonstrate with the F-statistic (a polarisation-insensitive, but sky-position dependent, power spectral density measure):
  - With no signal, it is Chi^2-distributed with 4 degrees of freedom:

$$p(x) = \frac{x}{4} \exp\left(-\frac{x}{2}\right) \qquad (x = \text{"}2F\text{"} > 0)$$

  - False alarm probability (FAP) $= \int_x^\infty p(x)\,\mathrm{d}x = \left(\frac{x}{2} + 1\right) \exp(-x/2)$
  - In a single (targeted) measurement, we therefore need (say) a detection threshold of $x_1 = 9.5$ to give a FAP of 0.05.
  - To give the same *joint* FAP with N measurements we need:

N	Detection threshold x_N
1	9.5
100	20.0
1e6	39.7
1e12	68.4
1e16	87.3

Sky position and spin

- A typical all-sky search might use $\sim 10^{16}$ search templates, so the threshold for detection is $\sim\times$ 10 higher than for a single template ('targeted') search.

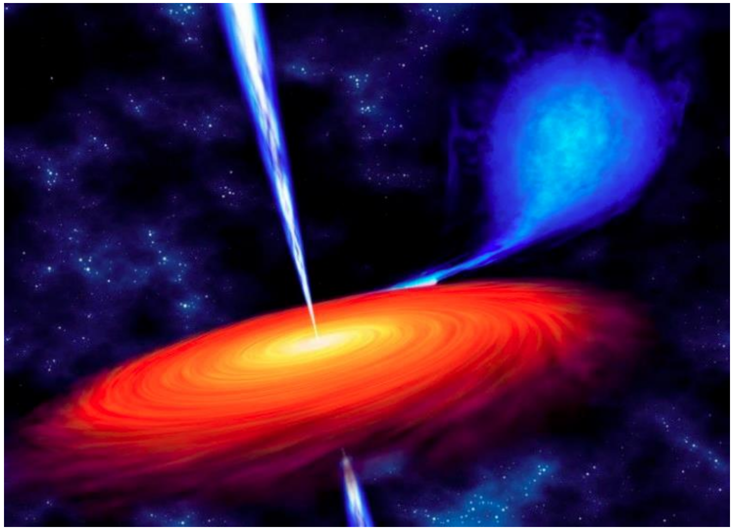
N	x_N
1	9.5
100	20.0
1e6	39.7
1e12	68.4
1e16	87.3

- Conclusion: knowing the sky-position and spin evolution of a neutron star is the equivalent of a $\sim\times$ 10 boost in strain sensitivity – about the same as jumping a generation of detectors (LIGO → aLIGO → 3G)

- (of course, this only boosts the chances of finding a signal from that particular pulsar!)

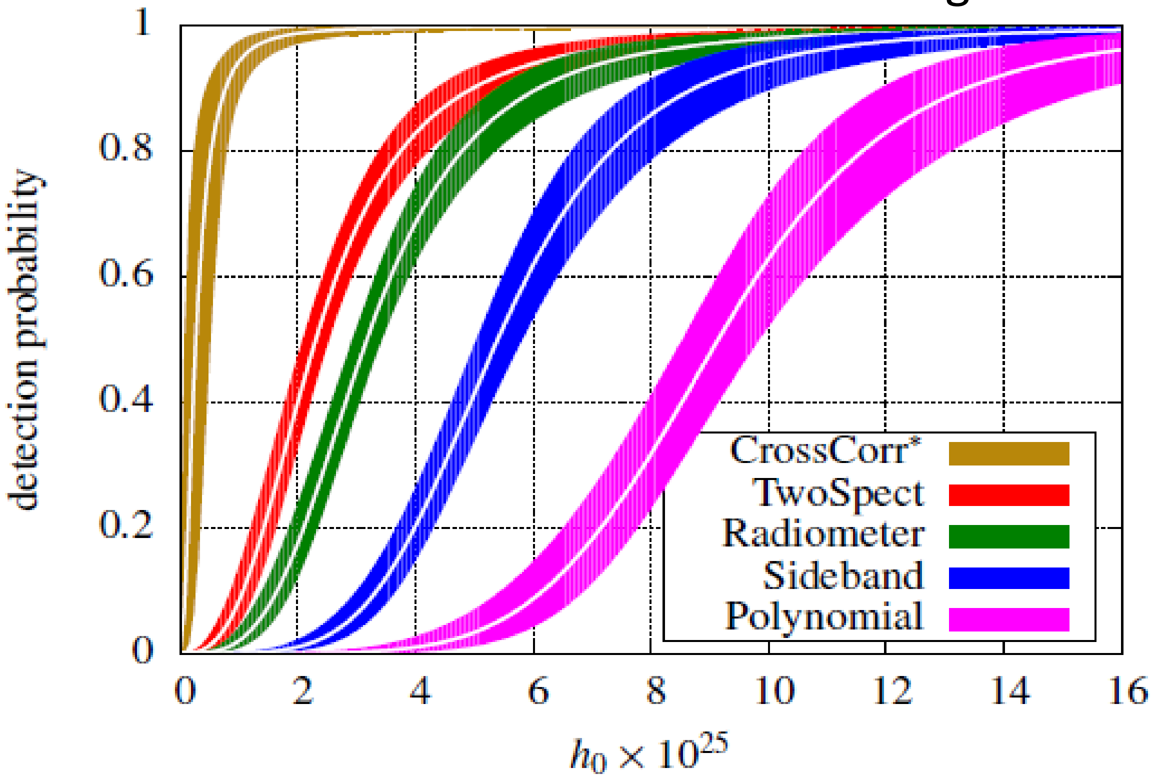
Scorpius -X1

A similar tale: different methods to tackle this system are compared in Messenger et al., *PRD* **92**:023006 (2015)



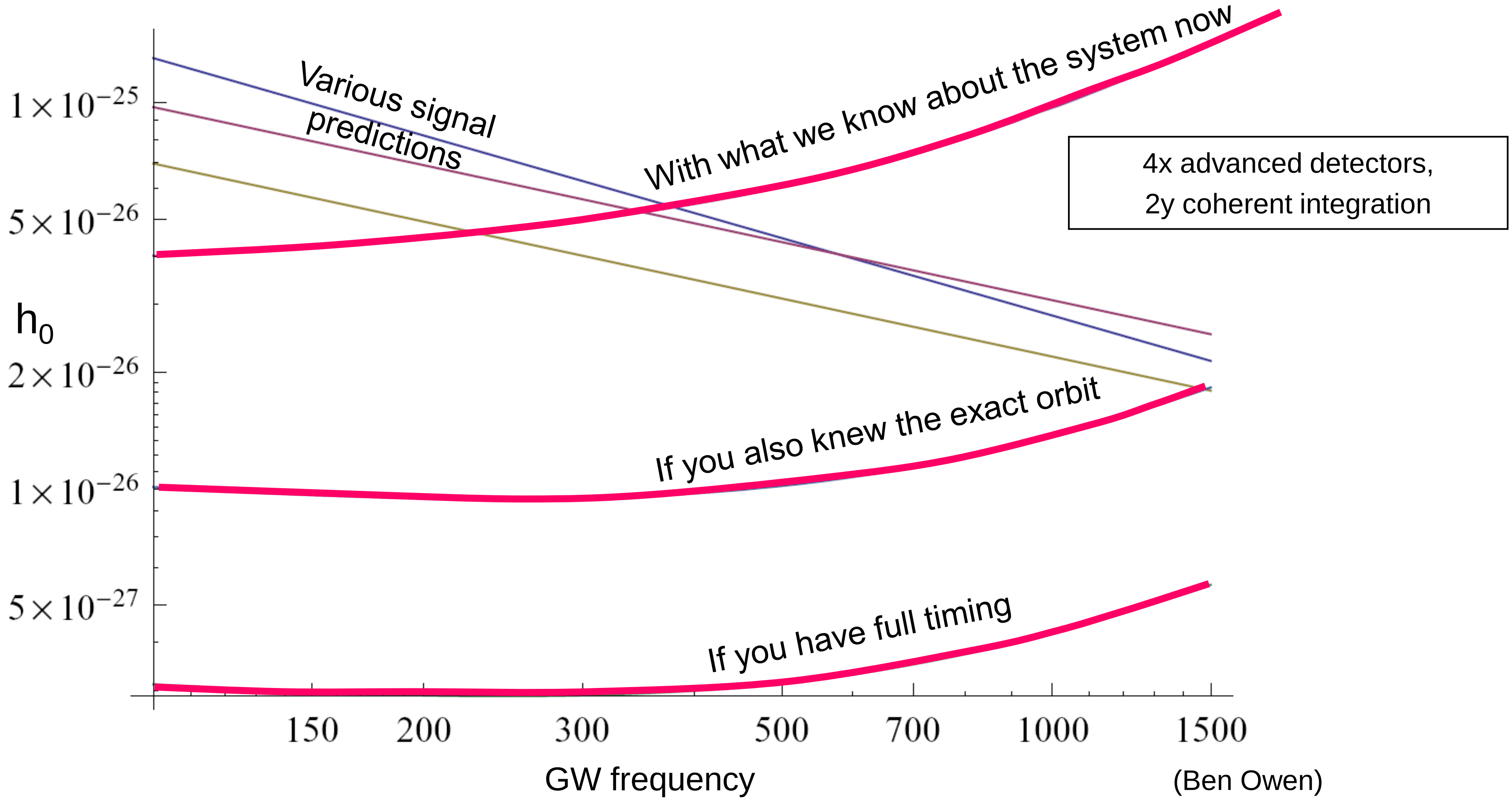
Ralf Schoofs

Sco-X1 Mock Data Challenge



Sco-X1

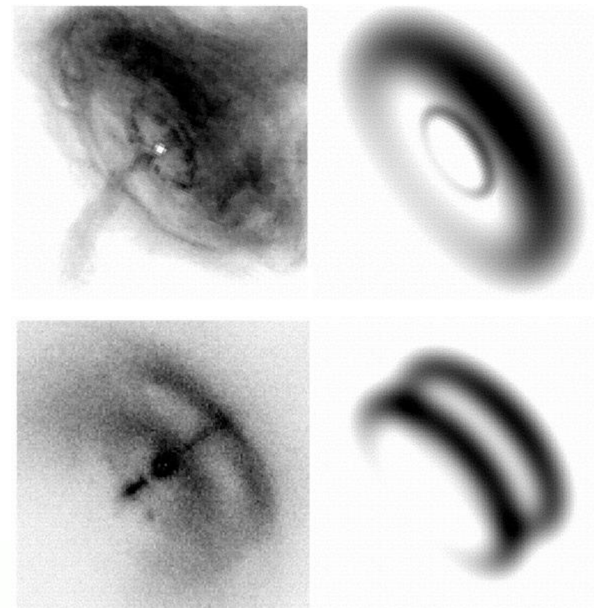
- Similar arguments apply with these more complex systems: the more you know the better you can do. We need EM data!
(See Reinhard's talk on spin wandering)



(Ben Owen)

Spin orientation and direction

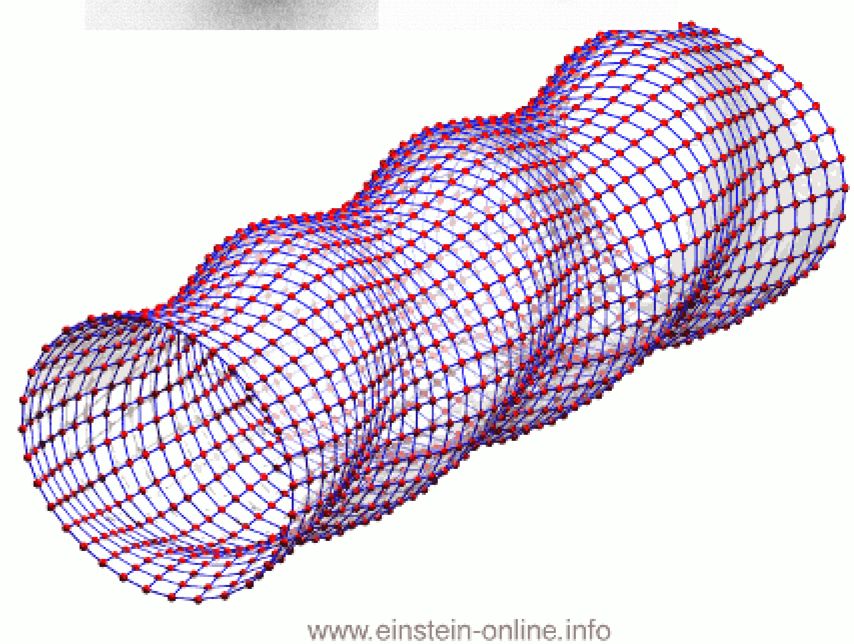
- EM observations of the pulsar wind nebula can constrain the axial orientation of the pulsar (eg, Crab, Vela).



Ng & Romani 2004

- Helps reduce parameter space for GW detection.

- Full GW polarisation would tell us both the axial orientation and the handedness of the spin (not measurable otherwise).



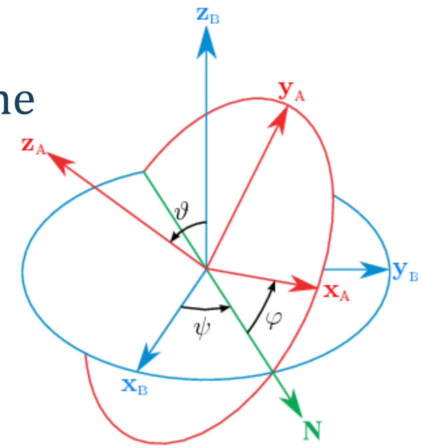
What can we learn about neutron stars with a joint EM/GW detection?

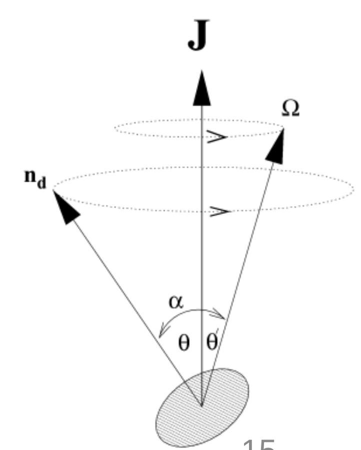
NSs seen in GW and EM

- Some points to note:
  - A single GW-only measurement tells us rather little. We need to know the distance to the source to turn a strain into a quadrupole, and unless the NS is very close (parallax), or has an association, this comes from radio dispersion. However, population studies are possible with *many* GW-only detections.
  - GW signal-to-noise ratios will be low for the foreseeable future – at least 10^4 times fainter than BBH/BNS signals in strain: we won't be able to measure phase on short timescales (weeks) for some time.
  - However, for a given snr the timing precision of GW and EM observations are about the same (e.g., both give ~arcsecond astrometry).

NSs seen in GW and EM

- Synergies:
  - GW signals are ‘spectral lines’, tracking a rotating mass quadrupole, EM signals trace magnetospheric profile and rotation, both with well-defined geometric interpretations. The relative phase of the two gives the physical arrangement of mass and magnetic field.
    - Is matter accreted to, e.g., the magnetic poles?
    - Is there magnetospheric drifting?
  - Many pulsars glitch in radio/X-ray, possibly generating bright GW bursts.
    - Is there a change in the quadrupole orientation in a glitch? (Wednesday’s talks)
    - What is the relationship with pulsar ‘moding’ (e.g., Haskell & Patruno 2017)? Searches for GW glitch signals are ongoing and EM timings are vital here.
  - Combining GW with rotational measurements from EM observations we also get:
    - Spin/r-mode relative frequency measurement ($\sim 4/3$)
    - Identification of free-precession and/or multi-component rotation

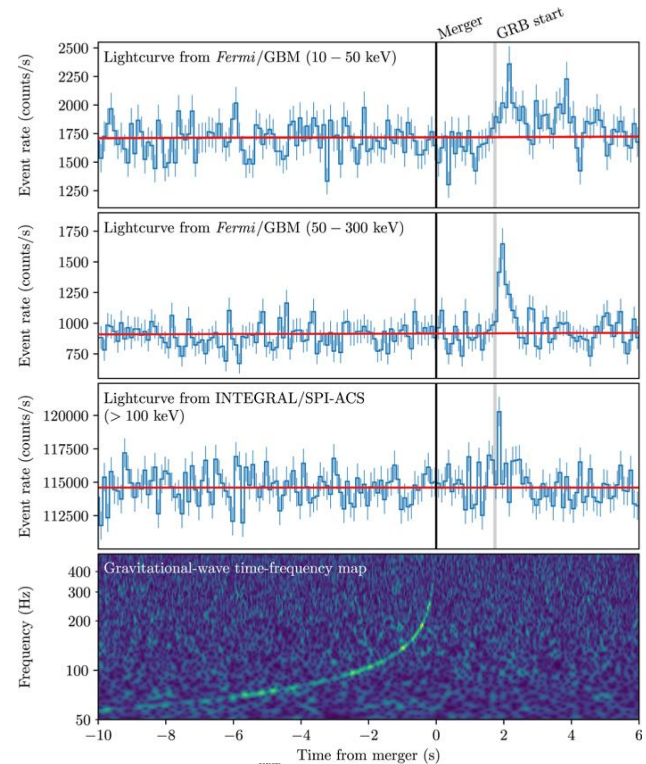




What else would a joint EM/GW detection tell us?

The speed of gravity

- GW sources are some of our best testbeds for GR:
  - BBH GW150914: graviton mass $m_g \leq 1.2 \times 10^{-22}$ eV/c^2
    Abbott et al., PRL. 116, 221101:2016
  - BNS GW170817: coincidence of GW and GRB gives $-3 \times 10^{-15} \leq \frac{\Delta c}{c} \leq +7 \times 10^{-16}$
    Abbott et al., ApJL 848(2):L13(27); 2017
    The precision comes from the joint GW-EM observation.



- GW CW sources can be used to determine the CW propagation speed too, e.g. using the Roemer delay variations over a year ($\sim$ 1 part in 10^6)
  Finn and Romano Phys. Rev D, 88(2), 2013

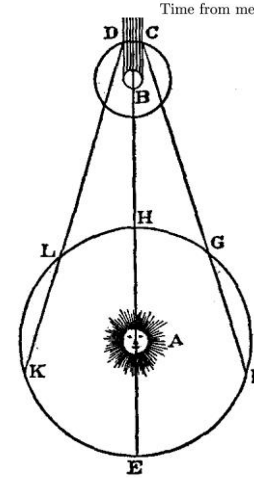


Fig. 70.

Roemer, 1676

The speed of gravity

- Can we do better with CW sources if we have an EM counterpart? Yes!

- Crucial point: the spindown, $\dot{f}$, of the neutron star implies that received frequency is sensitive to the propagation delay. For small differences in the GW and EM wave speeds, δc, the phase difference between the GW and EM signals after time T for a pulsar at distance D is

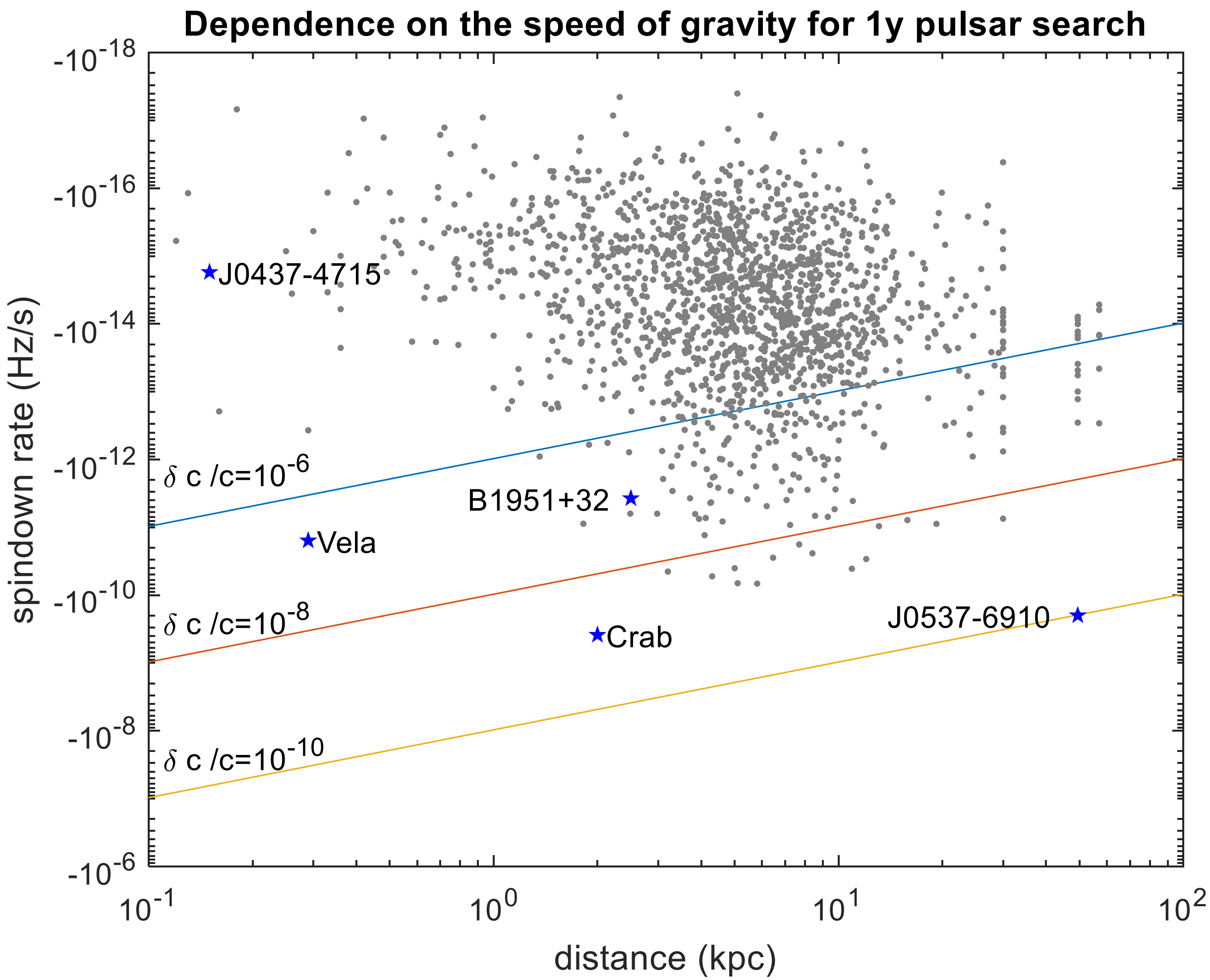
$$\Delta\phi(T) = \dot{f}\, D\, T \frac{\delta c}{c^2}$$

- E.g. for the Crab: $D = 2.2$ kpc, $\dot{f} = 3.77 \times 10^{-10}$ Hz/s, so in 1 year we would have a phase difference of π if

$$\frac{\delta c}{c} \sim 10^{-9}$$

(corollary: a targeted search will fail if $\frac{\delta c}{c}$ is greater than this!)

The speed of gravity



- So, not as good a test of GR as GW170817, but on a different length scale (kpc, rather than Mpc)

EM propagation

- We would expect the GW and the EM signal to travel along the same null geodesic in GR, but only in (i) free space and (ii) a single-ray theory.

- EM signals are strongly dispersed by the interstellar medium. An e$^-$ number density n_e, gives a refractive index , η:

$$\eta \simeq \left(1 - \frac{f_\mathrm{p}^2}{f^2}\right)^{1/2}, \qquad \text{where} \quad f_\mathrm{p} = \frac{1}{2\pi}\left(\frac{n_\mathrm{e} e^2}{\epsilon_0 m_\mathrm{e}}\right)^{1/2},$$

imparting a frequency-dependent delay of

$$\tau_D = 4.15 \times 10^3 \frac{1}{f_\mathrm{MHz}^2} \int_0^D n_{\mathrm{e,cm^{-3}}}(z)\ \mathrm{d}z_\mathrm{pc} \text{ seconds,}$$

but this can largely be compensated using multi-frequency measurements.

- Pulsars also show strong diffractive and refractive scintillation, that cannot be ‘undone’ and affects timing detail.

GW propagation

- GW signals are *very* weakly dispersed by a gas at temperature T and mass density ρ giving a refractive index of

$$\eta \simeq \left(1 - \frac{f_{\mathrm{m}}^2}{f^2}\right)^{1/2}, \qquad \text{where} \quad f_{\mathrm{m}} = \left(\frac{8G\rho}{\pi}\frac{k_{\mathrm{B}}T}{mc^2}\right)^{\frac{1}{2}},$$

Cetoli & Pethick PRD 85, 064036 (2012)

- …so no measurable effects from bulk matter dispersion and scattering, but what about the effect of spacetime curvature on propagation?

- Extreme curvature along the ray path will generate lensing, but more subtle effects are also apparent that will affect GW and EM signals slightly differently:

Shapiro delay

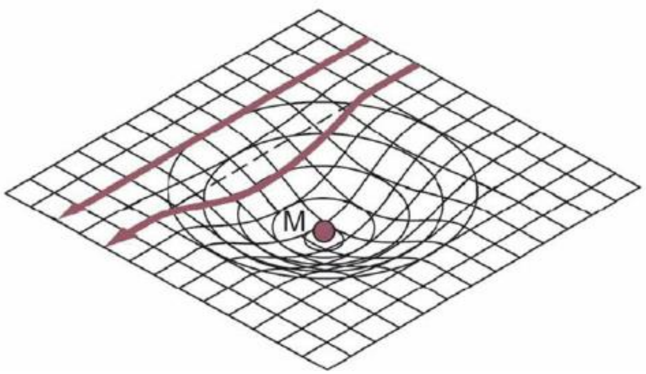
- As noted by Eddington, the effect of GR on (vacuum) light propagation can be reduced to an effective refractive index in a flat spacetime dependent on the local Newtonian potential Φ:

$$\eta = 1 - \frac{2\Phi}{c^2}$$

(Observatory vol. 42, p119-122, 1919)

- Even non-lensed rays will therefore suffer a ('Shapiro') propagation delay in a non-zero potential:

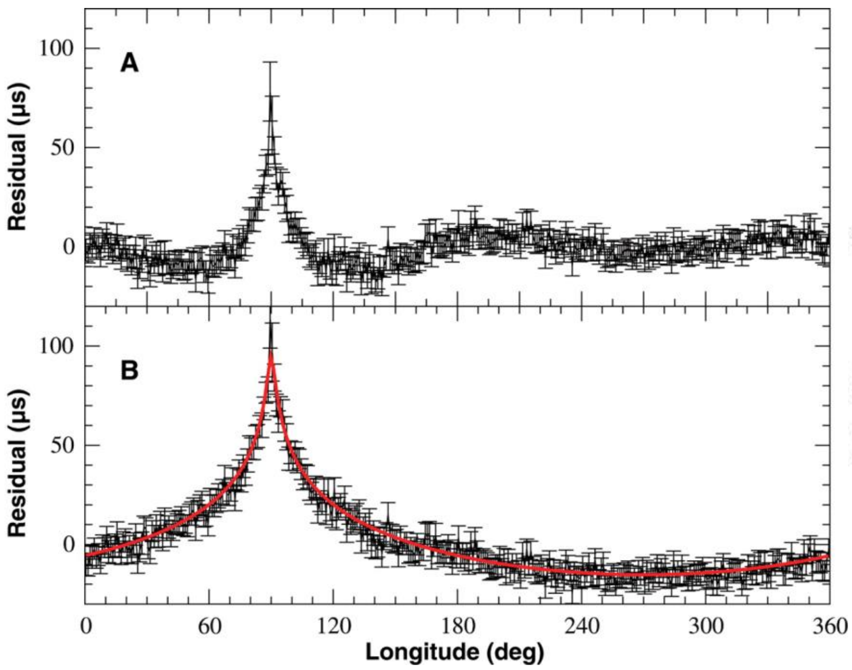
$$\Delta t = -\frac{2}{c^3}\int \Phi \, \mathrm{d}l$$

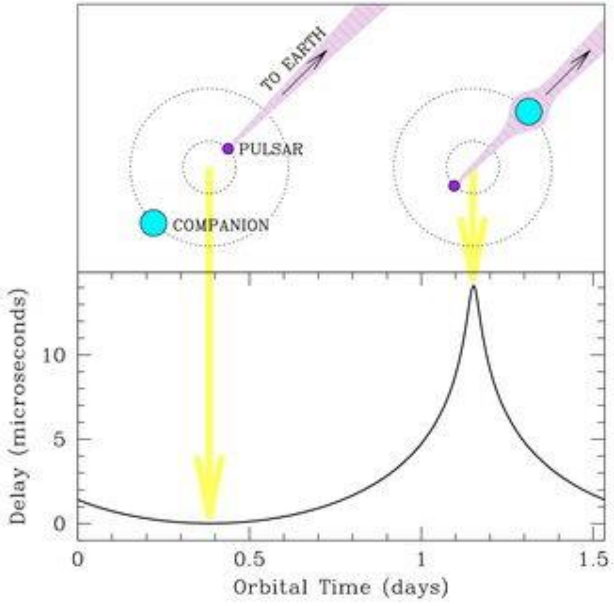


Heinkelmann & Schuh, 2009

Shapiro delay

- Effect is clear in high-inclination pulsar binaries, e.g. the double pulsar PSR J0737−3039



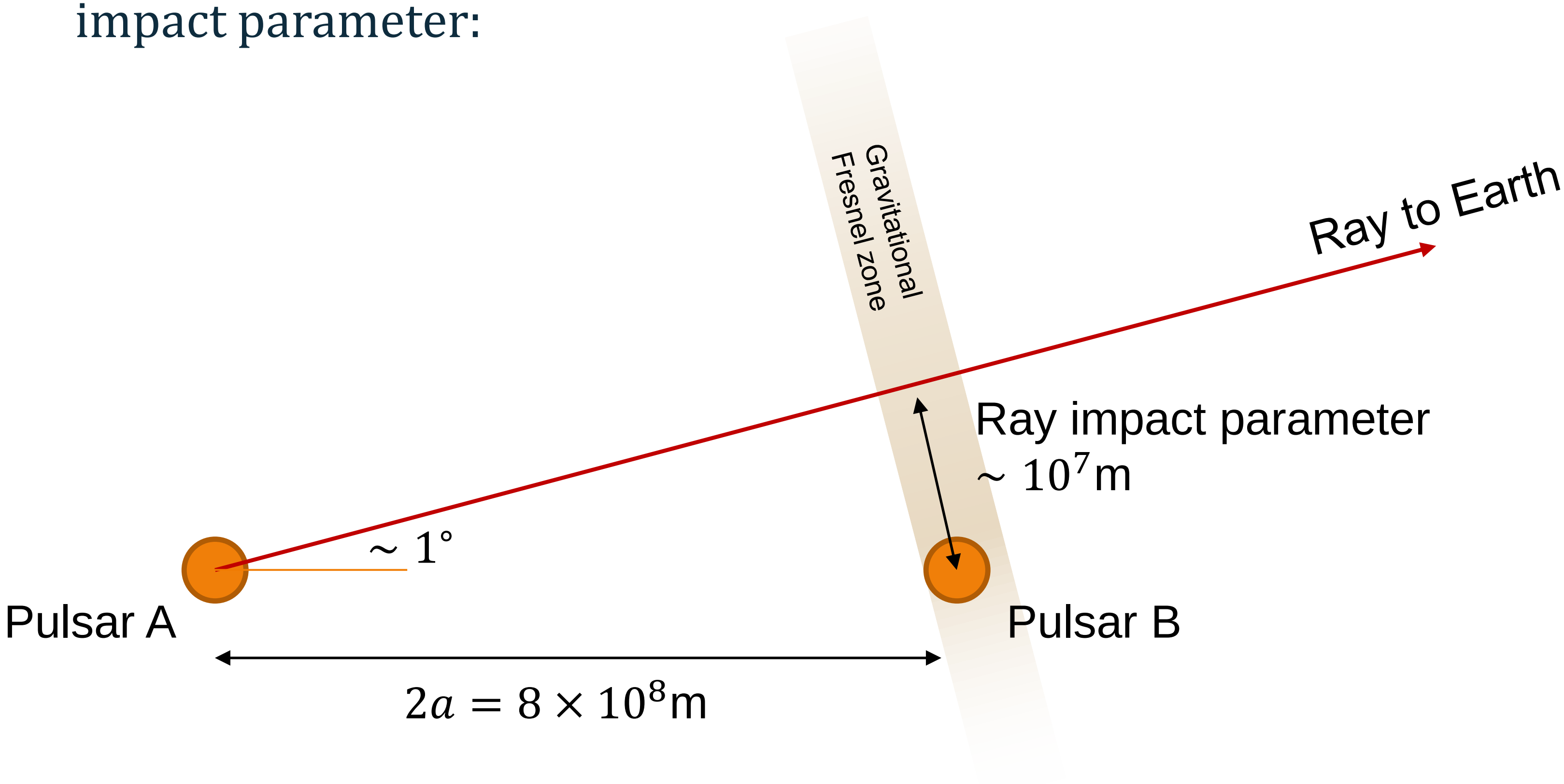


Kramer et al. 2006

- Ray theory is appropriate here ($\lambda \sim 0.2$ m, semimajor axis $a \sim 4 \times 10^8$ m) as variations in delay over the Fresnel scale ($\sim$ 12 km) are small.

Shapiro delay

- What about gravitational waves? Take pulsar A as a source: spin period is 22.7 ms, so $\lambda_g = 3400$ km. The Fresnel zone width is now $\sim \sqrt{2\lambda_g a} = 5 \times 10^7$ m – similar to the ray impact parameter:



- So the GW and EM Shapiro delays will differ markedly.

Shapiro delay

- For a point mass

$$\eta(r) = 1 - \frac{2\Phi}{c^2} = 1 + \frac{2GM}{c^2 r} = 1 + \frac{r_s}{r}$$

  so the (scalar) wave equation becomes

$$\nabla^2 u + \left(1 + \frac{r_s}{r}\right)^2 k_0^2 u = 0$$

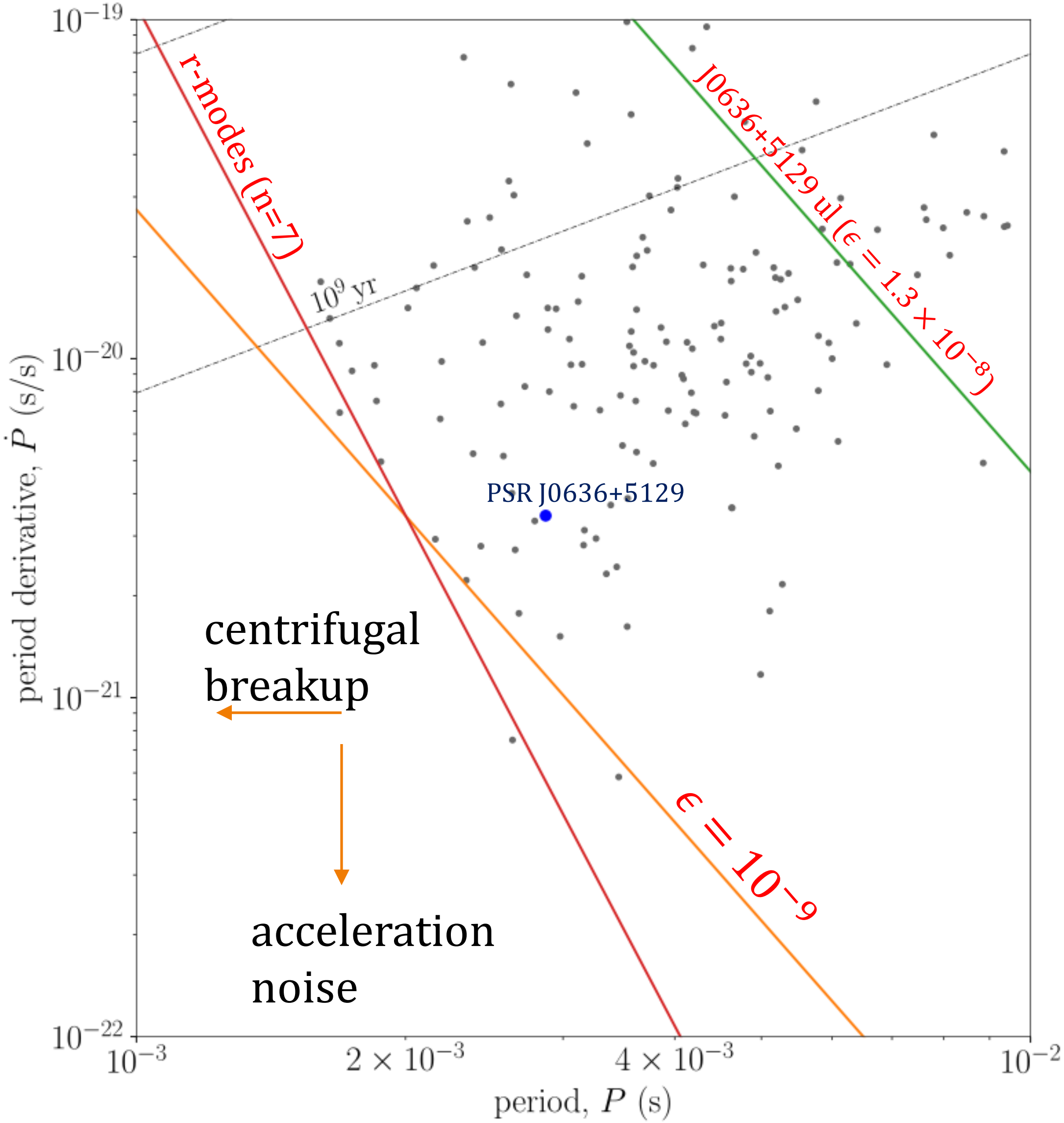
which is equivalent to the (wavefunction) coulomb scattering problem.

- Some good work done on gravitational wave propagation effects by Ryuichi Takahashi (e.g., ApJ 835,103:2017), but much to explore here.

Summary

- The full exploitation, and possibly the detection, of CW signals from neutron stars relies heavily on there being an EM counterpart.
- EM dispersion gives us distance, which is vital for quadrupole measurements and direct EoS constraints.
- GW-EM data together can fully characterise the spin orientation of the pulsar.
- Possibility to explore the relative orientation and shifts of the mass quadrupole and magnetosphere on long timescales.
- Other propagation physics to explore too!

Cutoff?



r-modes (n=7)
J0636+5129 ul ($\epsilon = 1.3 \times 10^{-8}$)
10^9 yr
PSR J0636+5129
centrifugal breakup
acceleration noise
$\epsilon = 10^{-9}$
period derivative, $\dot{P}$ (s/s)
period, P (s)