Superfluid Density of Neutrons in the Inner Crust of Neutron Stars:

New Life for Pulsar Glitch Models

GW & C. J. Pethick, PRL **119**, 062701 (2017).

Gentaro Watanabe
(Zhejiang Univ.)

Introduction

Superfluid density

Landau's two-fluid description of supefluids:

normal component: $n^n(T)$ $v_n(T)$

superfluid component: $n^s(T)$ $v_s(T)$

Superfluid density *n*^{s} : Density which contributes to SF flow.

Response of current to the phase twist of SF order parameter.

$$\Delta(\mathbf{r}) = \tilde{\Delta}(\mathbf{r}) e^{2i\mathbf{Q}\cdot\mathbf{r}}$$

energy density

$$\boxed{n^s_{ij} = m \frac{\partial \mathcal{E}(n, \mathbf{Q})}{\partial Q_i \partial Q_j}}$$

(inverse of *m**)

mom. / particle of bulk flow

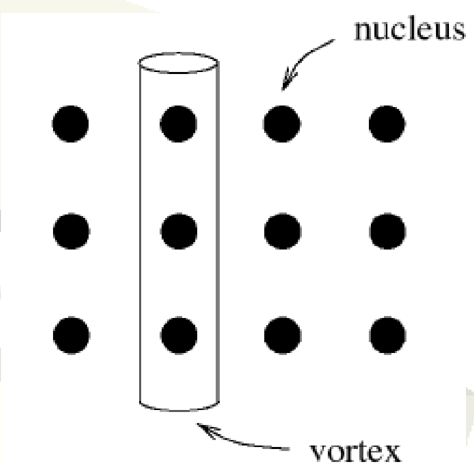
Glitch model based on n-superfluidity

2 fluids in NS crust

dripped neutrons: superfluid → quantized vortices

nuclei (form a lattice):
- normal fluid → pinning center of vortices
- charged → coupled to a EM field

decelerated by pulsar emission



Glitch model based on n-superfluidity

pinning/unpinning of vortices of n-superfluid

n-sf: angular mom. reservoir

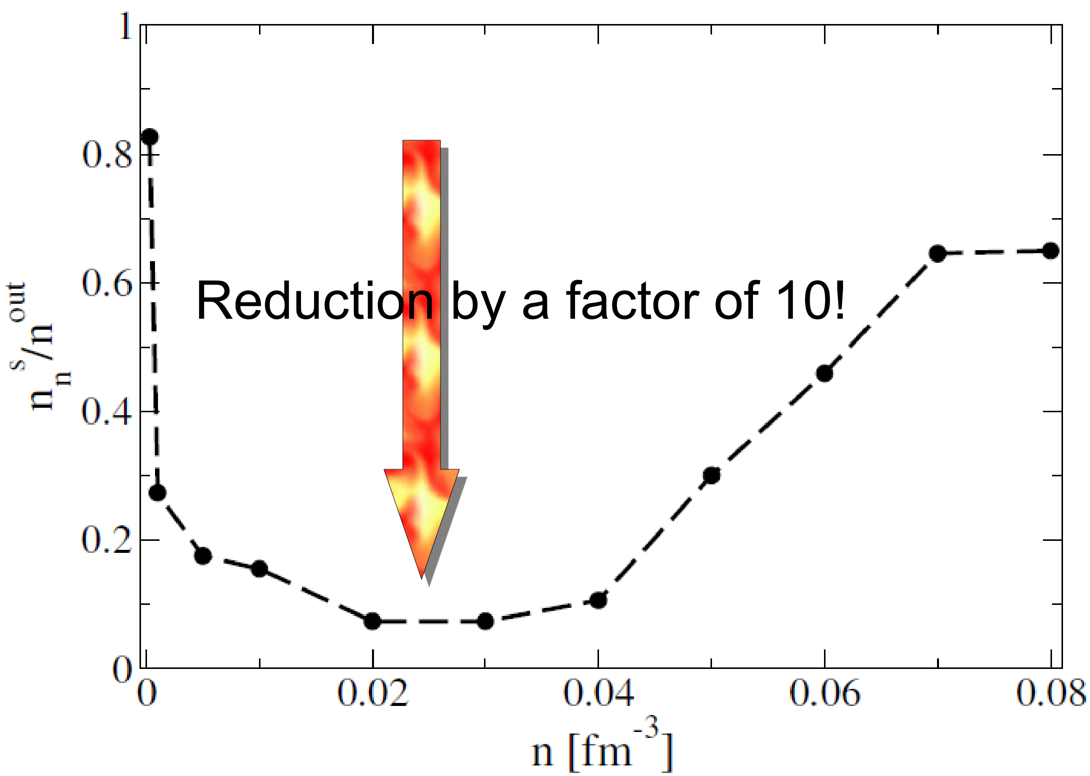
Crisis of the glitch models

Chamel, PRC **85**, 035801 (2012)

Band calculation without pairing.

(HF with nuclear interaction of Skyrme type)

Good approx. if the effect of pairing is weak.



Superfluid density:

(inverse of *m**)

energy density

$$\boxed{n^s_{ij} = m \frac{\partial \mathcal{E}(n, \mathbf{Q})}{\partial Q_i \partial Q_j}}$$

mom. / particle of bulk flow

Insufficient superfluid density to explain glitches!

Mom. of inertial of n-superfluid is too small.

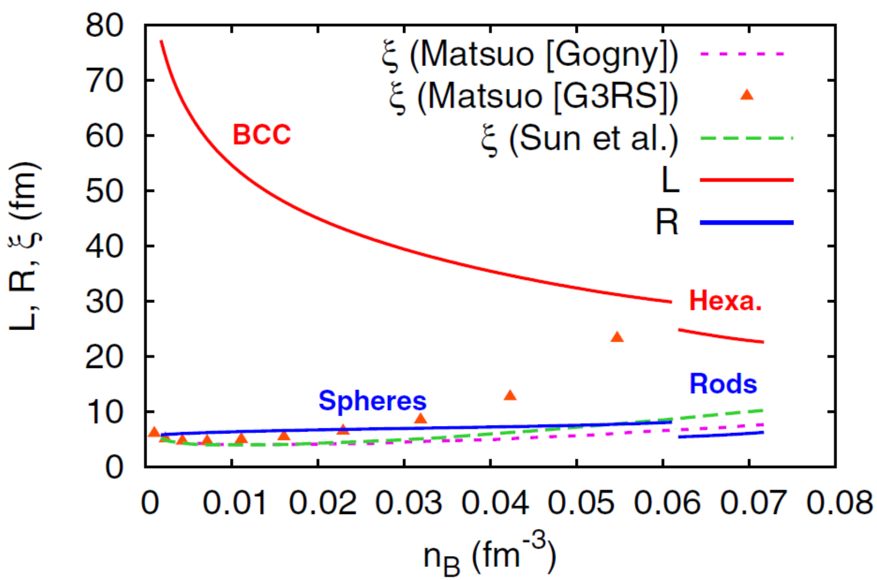
Andersson *et al*., PRL (2012); Chamel, PRL (2013); Delsate *et al*., PRD (2016).

SF density of neutrons in the inner crust of NSs:

New life for pulsar glitch models

GW & Pethick, PRL **119**, 062701 (2017).

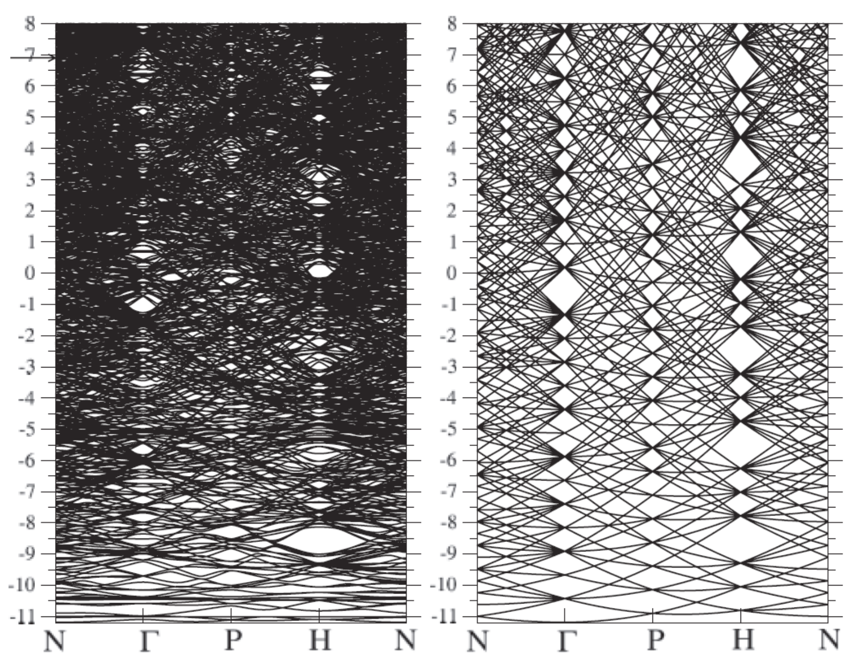
Difficulty of the problem



Martin & Urban, PRC (2016)

ξ_{BCS} ~ R (nuclear radius)

Hydrodyn. theory is invalid.



Chamel, PRC (2012)

Need to look at the band structure in detail.

# of neutrons / nucleus >> 1

neutrons occupy ~500 bands

Take-home messages

Both “gaps” are important!

(band gap) vs (pairing gap)

Pairing drastically reduces the effects of band gap when

$|\Delta| \gtrsim$ (lattice potential height)

$\boxed{|\Delta|/(\text{band gap})}$ matters even though $|\Delta|/E_F \ll 1$

Superfluid density may be large enough to account for glithces.

New life for glitch models!

GW & Pethick, PRL **119**, 062701 (2017).

Poor man's analysis

Scattering of quasiparticles by spin-indep. pot.:

$$H_{\text{int}} = \sum_{\mathbf{k},\mathbf{k}',\sigma} V(|\mathbf{k}-\mathbf{k}'|) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k}',\sigma}$$

$$a^\dagger_{\mathbf{k},\sigma} = u_k \alpha^\dagger_{\mathbf{k},\sigma} + \sigma v_k \alpha_{-\mathbf{k},-\sigma}$$

fermion quasiparticle

$$\langle \mathbf{k}'\sigma | H_{\text{int}} | \mathbf{k}\sigma \rangle = \underbrace{(u_k u_{k'} - v_k v_{k'})}_{\text{coh. factor}} V(|\mathbf{k}-\mathbf{k}'|)$$

$$|\mathbf{k}\sigma\rangle = \alpha^\dagger_{\mathbf{k},\sigma}|0\rangle$$

On the Fermi surface ($k = [2m\mu]^{1/2}$), $u_k = v_k = 1/\sqrt{2}$

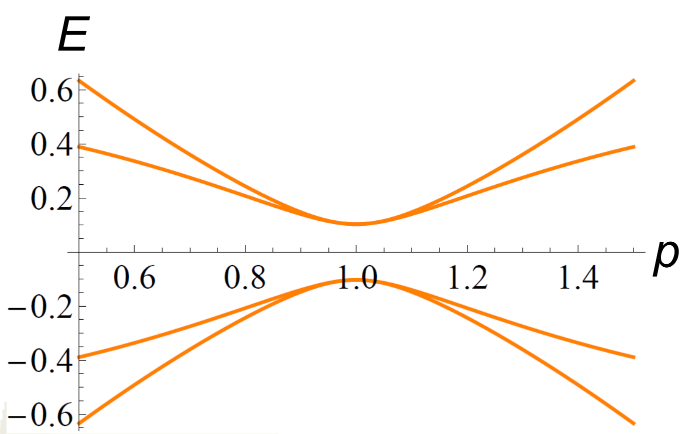
$$\langle \mathbf{k}'\sigma | H_{\text{int}} | \mathbf{k}\sigma \rangle = 0$$

No net scattering on Fermi surface.

∵ Potential for particles and holes are equal and opposite.

Simple analysis by 2-band model

$$\begin{array}{rl} & \begin{array}{cccc} \text{particle } (p) & \text{hole } (p) & \text{particle } (p\text{-}K) & \text{hole } (p\text{-}K) \end{array} \\ H = & \begin{pmatrix} \frac{p^2}{2} - \frac{1}{2} & \Delta & V & 0 \\ \Delta & -\left(\frac{p^2}{2} - \frac{1}{2}\right) & 0 & -V \\ V & 0 & \frac{1}{2}(p-K)^2 - \frac{1}{2} & \Delta \\ 0 & -V & \Delta & -\left[\frac{1}{2}(p-K)^2 - \frac{1}{2}\right] \end{pmatrix} \end{array}$$



p : quasimom. of a quasiparticle (in units of p_F)

K : reciprocal lattice vector (in units of p_F)

V : strength of the lattice pot. (in units of $2E_F$)

Δ : pairing gap (in units of $2E_F$)

Nested case: $K = 2$

Eigenvalues @ $p=p_F$: $\pm\sqrt{\Delta^2 + V^2}$ (doubly degenerate)

$|\Delta|/|V| \gtrsim 1$ Pairing effect is important even if $|\Delta|/E_F \ll 1$

Bogoliubov-de Gennes approach

Basic equation (BdG eq.):

$$\begin{pmatrix} \tilde{H}'_{\mathbf{Q}}(\mathbf{r}) & \tilde{\Delta}(\mathbf{r}) \\ \tilde{\Delta}^*(\mathbf{r}) & -\tilde{H}'_{-\mathbf{Q}}(\mathbf{r}) \end{pmatrix} \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix} = \epsilon_i \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix}$$

Q : quasimom. per particle of superflow

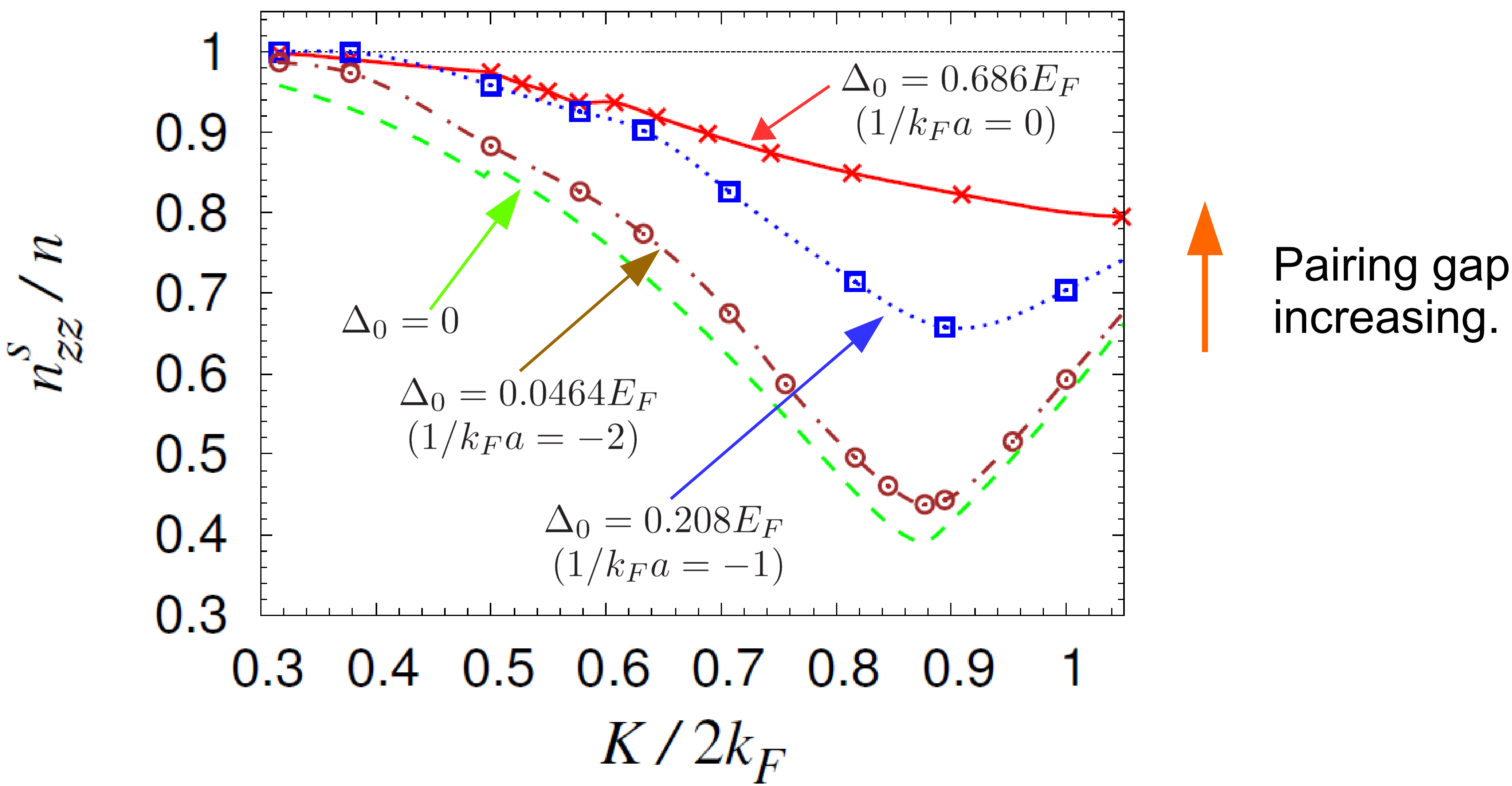
$$\tilde{H}'_{\mathbf{Q}}(\mathbf{r}) = \frac{1}{2m}(-\mathrm{i}\boldsymbol{\nabla} + \mathbf{Q} + \mathbf{k})^2 + V_{\mathrm{ext}}(\mathbf{r}) - \mu$$

1D sinusoidal pot.

$$V_{\mathrm{ext}}(\mathbf{r}) = V_K(e^{\mathrm{i}Kz} + e^{-\mathrm{i}Kz})$$

Calculate $$n^s_{ij} = m\frac{\partial \mathcal{E}(n, \mathbf{Q})}{\partial Q_i \partial Q_j}$$

Effects of the pairing gap (1)

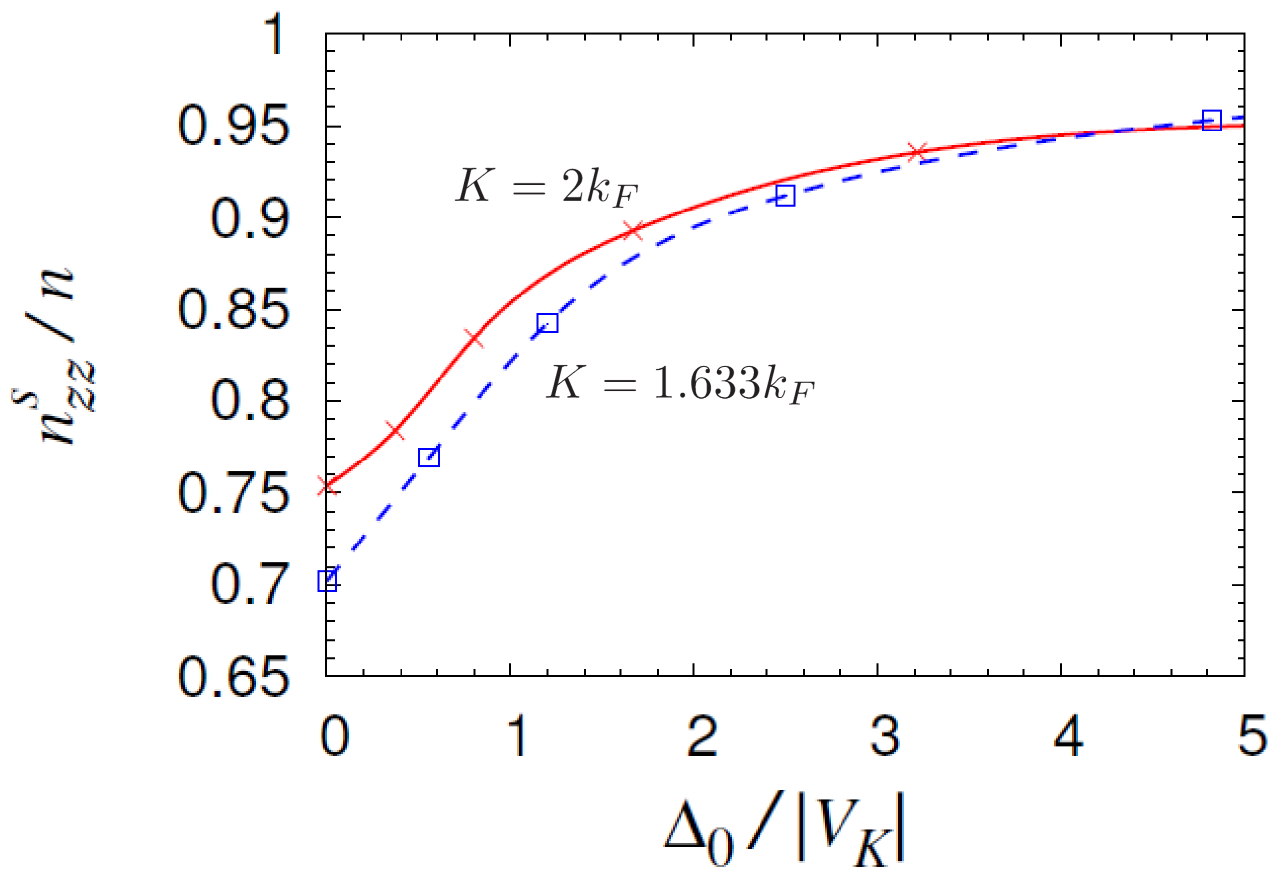


$$V_K = 0.25(K/2k_F)^2 E_F$$

Reduction of n^s due to band gap is suppressed by paring gap.

Effects of the pairing gap (2)

Suppression of band gap effect by pairing



Approximate fit: $n^s_{zz}(\Delta_0) = n - \dfrac{n - n^s_{zz}(0)}{\boxed{\sqrt{1 + (\Delta_0/|V_K|)^2}}}$

suppression factor

Application to NS crusts

<u>Obstacles:</u>

- Lattice pot. in NS crusts has many Fourier components.
- Neutrons occupy ~ 500 bands.
- 3D lattice: average over the orientation of lattice is needed.

→ Direct BdG approach is formidable.

Take a shortcut!

Superfluid density in NS crusts

Assumption: pairs of RLVs {**K**$_i$, -**K**$_i$} contribute to n^s independently.

$$n^s_{ij}(\mathbf{K}) = \underbrace{n^s_{zz}\hat{K}_i\hat{K}_j}_{\text{longitudinal}} + \underbrace{n[\delta_{ij} - \hat{K}_i\hat{K}_j]}_{\text{transverse}}$$

$$\frac{n^s}{n^o_n} \approx \exp\left\{-\frac{2n^o_n}{n_N}\int_0^1 x^2 dx \boxed{\left[1 - \frac{n^s_{zz}(K, V_K, \Delta)}{n}\right]}\right\}$$

effect of lattice pot.

$$1 - \frac{n^s_{zz}(K, V_K, \Delta)}{n} \approx \boxed{\left[1 - \frac{n^s(K, V_K, \Delta = 0)}{n}\right]} \underline{\frac{1}{\sqrt{1 + (\Delta/|V_K|)^2}}}$$

$x \equiv K/2k_n$

$$\approx \underbrace{\boxed{(1 + 3.5x)}\boxed{\frac{|V_K|}{E_F}}}_{\text{approximate fit}}$$

K-dep.

effect of pairing gap

form factor of lattice pot.

(V_K-dep.)

Superfluid density in NS crusts

Focus on the case where the reduction of n^s is largest.

In Chamel (2012): Avr. density $n = 0.03\ \mathrm{fm}^{-3}$

neutron Fermi energy $E_F^o = 16.4$ MeV

$$\Delta \approx 1 - 1.5\ \mathrm{MeV}$$

No pairing limit ($\Delta = 0$): $n^s/n_n^o \simeq 0.20$ (cf. Chamel's result ~ 0.1)

Δ = 1MeV: $n^s/n_n^o \simeq 0.64$

Δ = 1.5MeV: $n^s/n_n^o \simeq 0.71$ Only 29% reduction!

Superfluid density is large enough.

→ Glitch models based on superfluidity are still tenable!

Summary & conclusion

Study of neutron superfluid density in neutron star crusts

- Both pairing gap and band gap are imporant.

$|\Delta|/(\text{band gap})$ matters rather than $|\Delta|/E_F$

- Effects of the band gap is suppressed in NS crusts.

$n^s/n_n^o \sim 0.1$ → $\simeq 0.7$

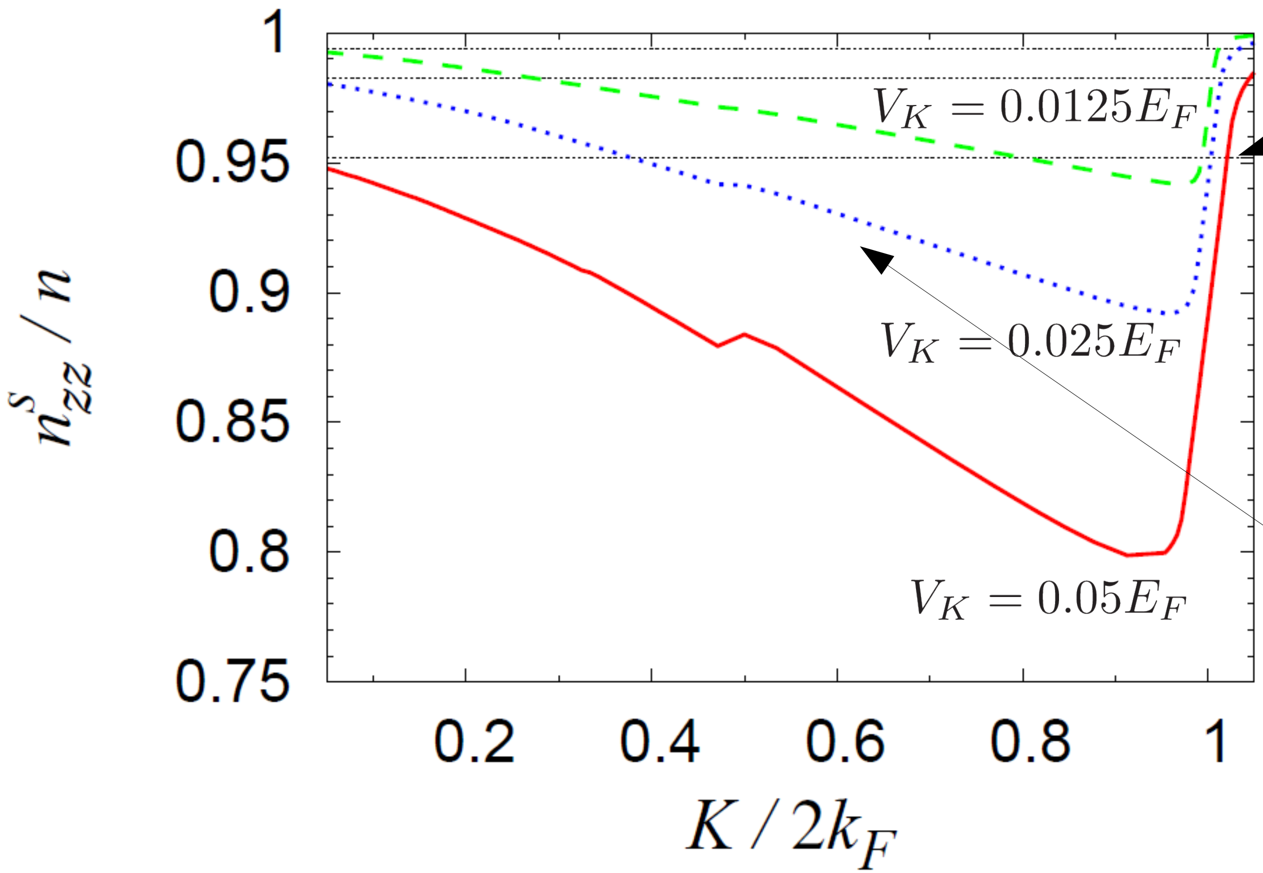
No pairing   Pairing included

Pulsar glitch models get new life!

GW & Pethick, PRL **119**, 062701 (2017).

K and V_K dependence in normal limit

Normal limit: $\Delta = 0$; for sinusoidal pot.



Little effect of lattice for $K > 2k_F$

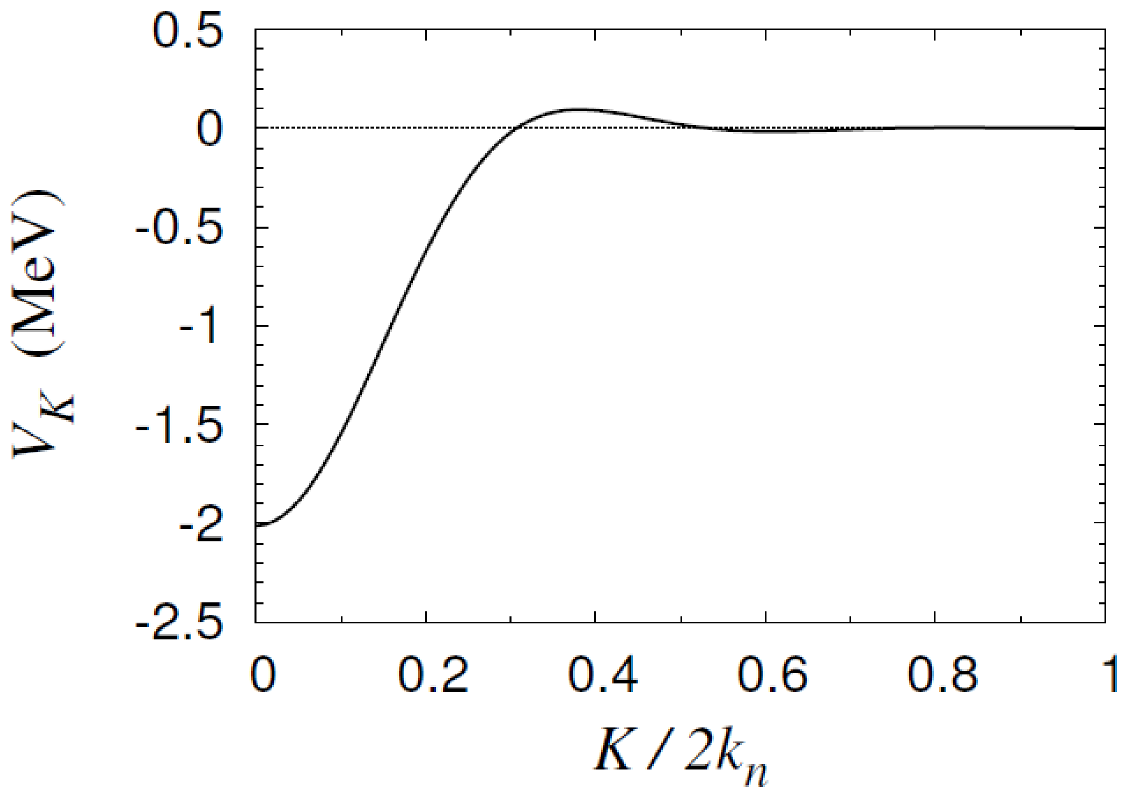
Scattering with mom. transfer > $2k_F$ is kinematically forbiden.

Almost linear wrt $K/2k_F$

Approximate fit: $1 - n^s_{zz}(K, V_K, \Delta = 0)/n = \left(1 + 3.5\frac{K}{2k_F}\right)\frac{|V_K|}{E_F}$

Form factor of lattice pot. in NS crusts

Fourier transform of MF pot. in Chamel's calculation.



Reciprocal lattice vectors (RLVs)

bcc lattice → fcc in reciprocal space

Min: $\mathbf{K} = \frac{4\pi}{d}(\pm 1, \pm 1, 0)$ etc. (12 RLVs)

$$K_{\rm min}/2k_n \simeq 0.12$$

2nd: $\mathbf{K} = \frac{4\pi}{d}(\pm 2, 0, 0)$ etc. (6 RLVs)

$$K = \sqrt{2} K_{\rm min}$$

3rd: $\mathbf{K} = \frac{4\pi}{d}(\pm 1, \pm 1, \pm 2)$ etc. (24 RLVs)

$$K = \sqrt{3} K_{\rm min}$$

$|V_K|$ decreases rapidly with K.

$$K/2k_n \gtrsim 0.15 \Rightarrow |V_K| \lesssim \Delta$$

$$K/2k_n \gtrsim 0.25 \Rightarrow |V_K| \ll \Delta$$

Superfluid density in NS crusts (1)

Assumption: pairs of RLVs {$\mathbf{K}_i$, -$\mathbf{K}_i$} contribute to n^s independently.

$$n^s_{ij}(\mathbf{K}) = n^s_{zz}\hat{K}_i\hat{K}_j + n[\delta_{ij} - \hat{K}_i\hat{K}_j]$$

longitudinal transverse

avr. over orientation for cubic symmetry

$$\frac{n^s}{n} = 1 - \frac{1}{3}[1 - n^s_{zz}(K, V_K, \Delta)/n]$$

contribution from many RLVs in crusts

Only one of $\mathbf{K}_i$ & -$\mathbf{K}_i$ is included.

$$\frac{n^s}{n^o_n} = \prod_{\mathbf{K}_i}{}' \left\{1 - \frac{1}{3}\left[1 - \frac{n^s_{zz}(K, V_K, \Delta)}{n}\right]\right\}$$

$$\approx \exp\left\{-\frac{1}{6}\sum_{\mathbf{K}_i}\left[1 - \frac{n^s_{zz}(K, V_K, \Delta)}{n}\right]\right\}$$

sum → integral

$$\frac{n^s}{n^o_n} \approx \exp\left\{-\frac{2n^o_n}{n_N}\int_0^1 x^2 dx \left[1 - \frac{n^s_{zz}(K, V_K, \Delta)}{n}\right]\right\}$$

density of nuclei

$$x \equiv K/2k_n$$