Introduction to Dense Matter

C. J. Pethick
(U. of Copenhagen and NORDITA)

The Niels Bohr International Academy

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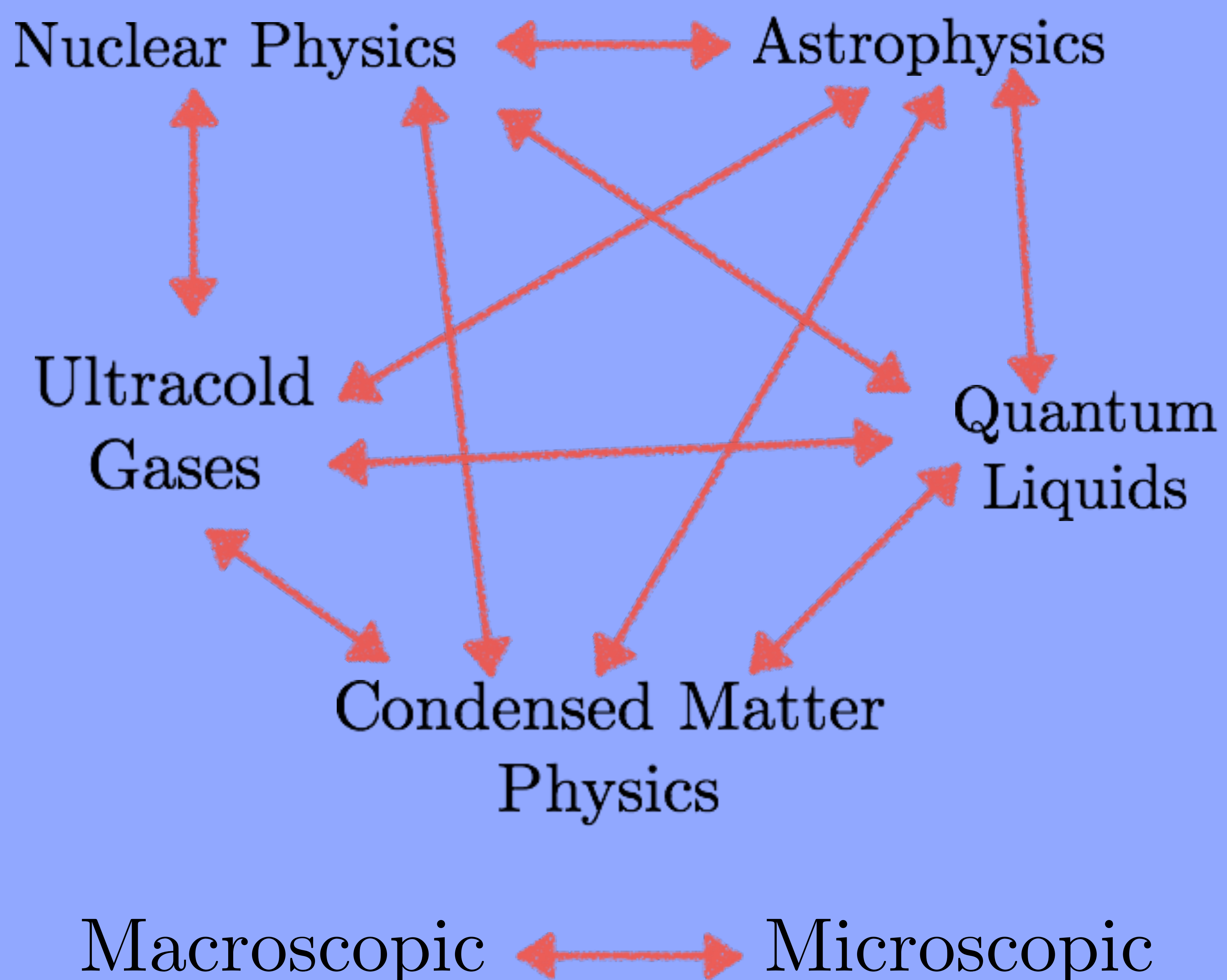
Bottom lines

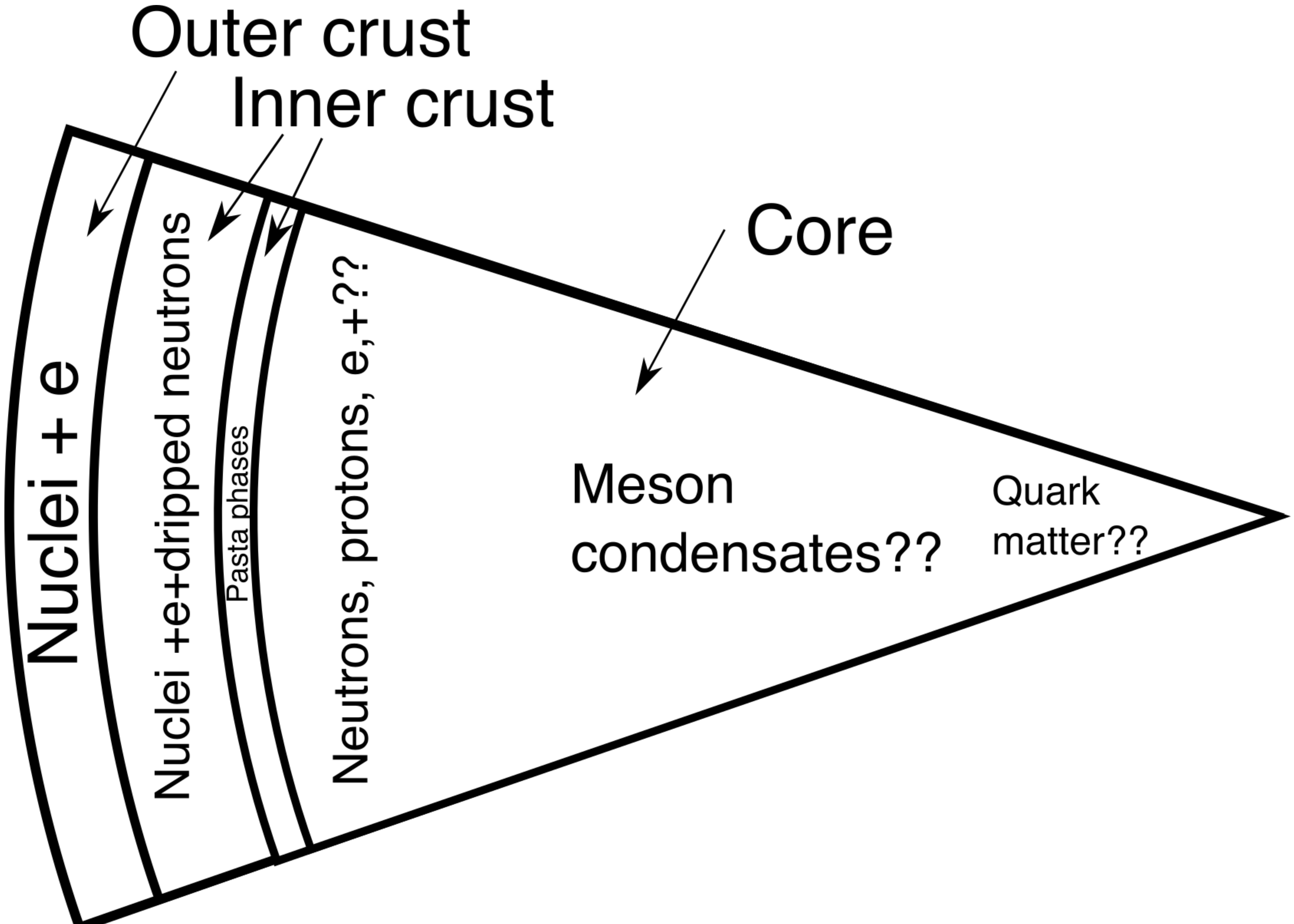
- Exciting time for neutron star studies: new data, progress in theory
- Concentrate on physical principles
- Get inspiration from other systems
- Plenty of work to do on solid phases


Nuclear Physics
Astrophysics
Ultracold
Gases
Quantum
Liquids
Condensed Matter
Physics
Macroscopic
Microscopic

General messages

- No fundamental problems in finding properties of matter below 1-2 times that of nuclear matter.
  Microscopic interactions are well understood from laboratory data.
- Large uncertainties at higher densities due to lack of understanding of basic constituents and interactions.
- Observations, especially mass measurements of neutron stars, provide constraints.

Schematic picture of a neutron star



Electrons

- Weakly interacting except near surface.
  Kinetic energy $\sim (\hbar^2/r_e^2)/2m_e$. Potential energy $\sim e^2/r_e$.
  P.E./K.E $\sim e^2/\hbar v_e$. Electron velocity $v_e \sim \hbar/mr_e$.
  Terrestrial matter: $r_e \sim a_0 = \hbar^2/me^2$.
  Higher densities: Interactions less important.

- Electrons relativistic. $r_e \lesssim \lambda_e$, $\hbar/m_e c = \alpha a_0$, Compton wavelength.
  Fine structure constant $\alpha = e^2/\hbar c \approx 1/137$.
  Screening length $\sim \alpha^{-1/2} r_e \approx 10 r_e$.

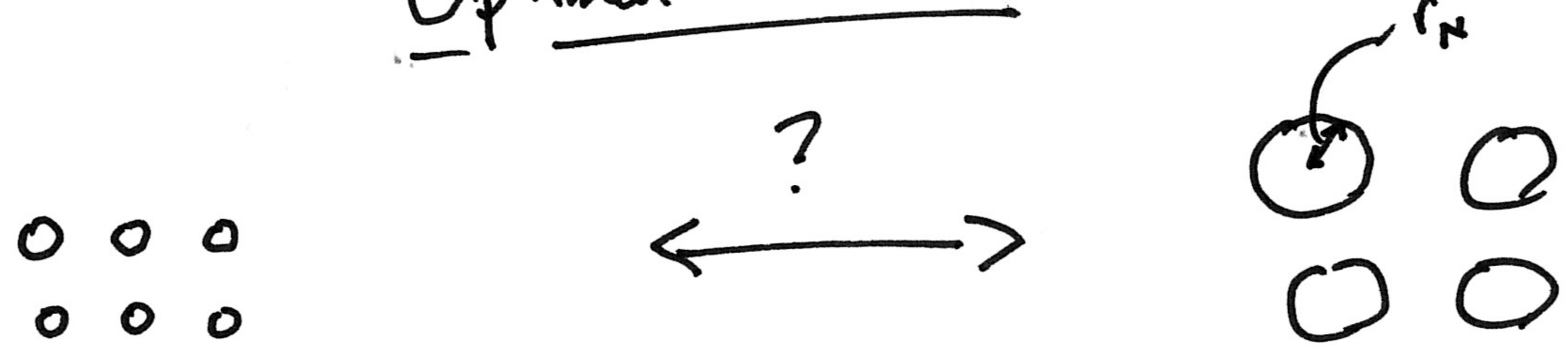
Nuclei

- Matter is cold in neutron stars.
  Nuclear energies $\sim$MeV or more (10^{10}K) than temperature (10^{9}K) or less after one hour.

- Lowest energy nucleus (no electrons for the moment).
  Liquid drop model: bulk, surface and Coulomb energies.

$$E = E_{\rm bulk} + E_{\rm surf} + E_{\rm Coul}$$

Optimal Nucleus



Keep $Z/A = x$ fixed (Z = proton number. A = nucleon (mass) number

Area, surface tension

$$\frac{E_{surf}}{A} \sim \frac{4\pi r_N^2 \, \sigma}{A} \sim \frac{1}{r_N} \qquad \frac{E_{Coul}}{A} \sim \frac{Z^2 e^2}{r_N A} \sim x^2 r_N^2$$

Nuclear radius

Minimize energy/A. Bulk term unaffected.

$$\boxed{E_{surf} = 2 E_{Coul}} \qquad A \simeq \frac{12.5}{x^2}$$

(Terrestrial nuclei. Those around $^{56}_{26}Fe$ most stable.)

Nuclei at Higher Densities

- $\rho \gtrsim 10^6\ g/cm^3$. Electron Fermi energy $\gtrsim 1$ MeV. Important on scale of nuclear energies.

- Electron capture. $p + e^- \to n + \nu_e$

Equilibrium. $\mu_p + \mu_e = \mu_n$ (μ - chemical potential)

Matter becomes more neutron rich.

- Neutron drip. Bulk approx. $E = (-\underset{\sim 16\text{ MeV}}{b_{bulk}} + \underset{\sim 32\text{ MeV}}{b_{symm}}\,\delta^2)A$ $\quad \delta = \frac{N-Z}{N+Z} = 1-2x$ neutron excess

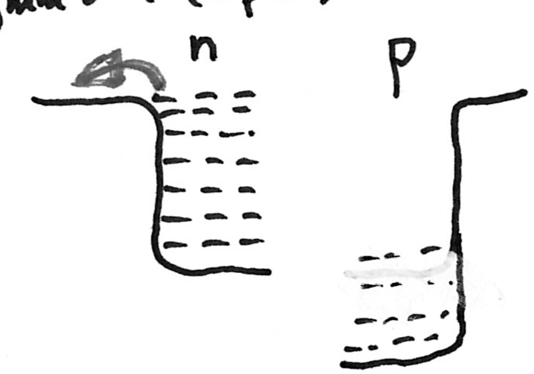
$$\mu_n = -b_{bulk} + 2b_{symm}\delta - b_{symm}\delta^2 \ (+ m_n c^2)$$

$$\mu_n = m_n c^2 \to \delta_{drip} \approx b_{bulk}/2b_{symm} \approx 1/4. \qquad x_{drip} \simeq 3/8 \ (\text{Lowered by Surface \& Coulomb energies})$$

$$\mu_p = -b_{bulk} + 2b_{symm}\delta - b_{symm}\delta^2 + (m_p c^2)$$

$$\mu_n - \mu_p \approx 4 b_{symm}\delta_{drip}$$

$$\approx 2 b_{bulk} \approx \underline{\underline{30\text{ MeV}}}.$$

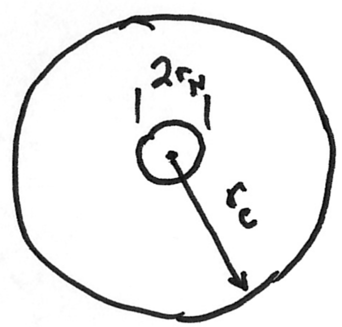


- $\rho_{drip} \simeq 4 \times 10^{11}\ g/cm^3$

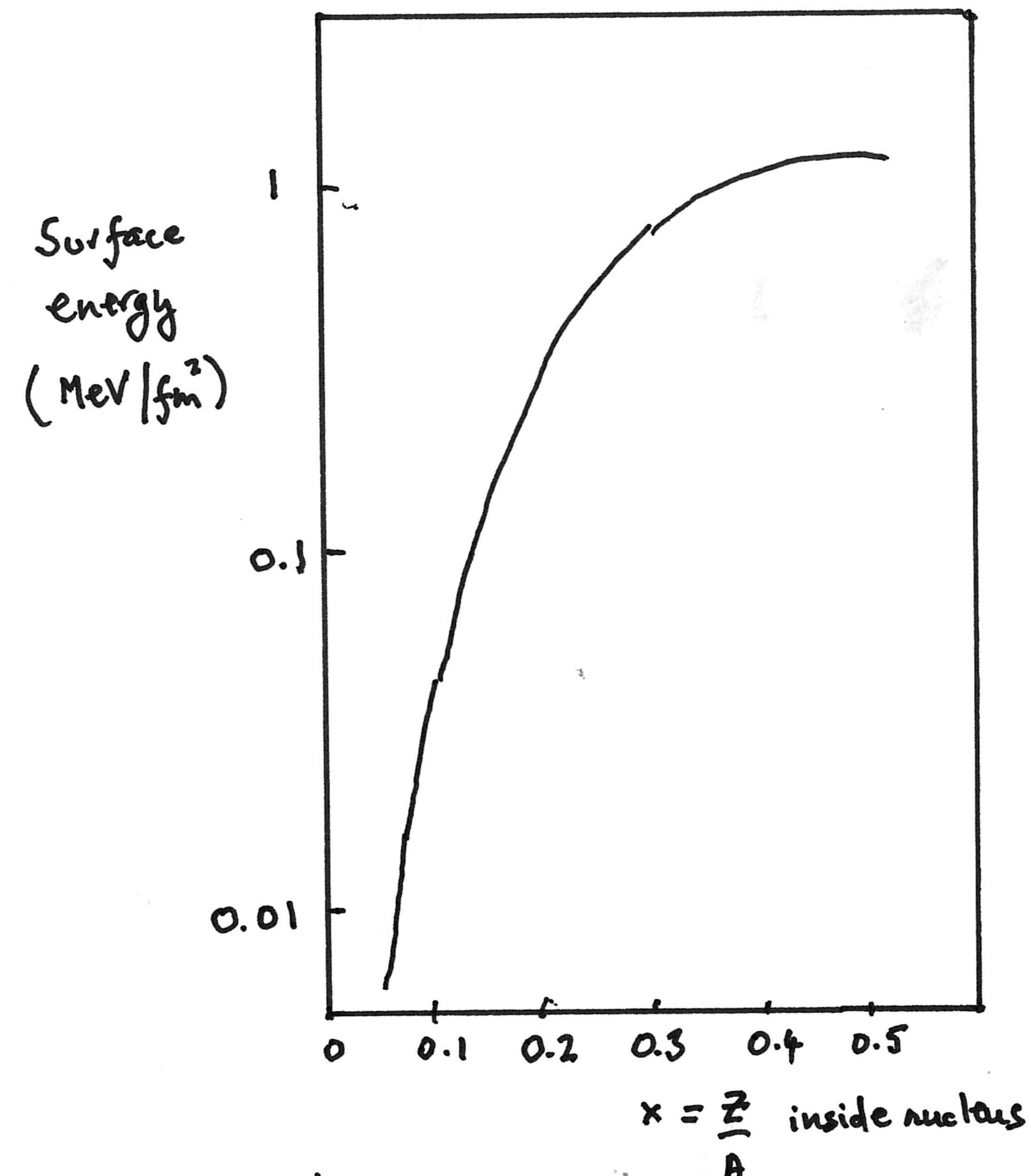
Much below nuclear density!

Effects of higher density

- Surface tension reduced as nuclear matter becomes neutron rich.
- Solid state effects become significant



r_c – radius of sphere containing Z electrons



(Ravenhall, Bennett, CJP PRL 28, 978 (1972))

- Net result: Z remains roughly 40.

Melting

- Classical plasma of point nuclei in a uniform background charge. Dimensionless parameter
$$\Gamma = \frac{Z^2 e^2}{r_\mathrm{c} k_\mathrm{B} T}$$

- At melting $\Gamma_\mathrm{M} \approx 175$.

- Coulomb energy differs little for different crystal structures
fcc $U = -0.895929\ldots Z^2 e^2 / r_\mathrm{c}$
bcc $U = -0.895873\ldots Z^2 e^2 / r_\mathrm{c}$

$$k_\mathrm{B} T_\mathrm{M} = \frac{Z^2 e^2}{\Gamma_\mathrm{M} r_\mathrm{c}} \approx 0.013 Z^{5/3} (nx)^{1/3}\ \mathrm{MeV}$$

Lattice energy

- Electron–nucleus and electron–electron interactions become important as density increases.

- Wigner–Seitz approximation. Replace unit cell by sphere of same volume.

$$\frac{1}{n_{\mathrm{N}}} = \frac{4\pi}{3} r_{\mathrm{c}}^3$$

Lattice energy

Finite size

$$\begin{aligned} E_{\mathrm{Coulomb}} &= \frac{3}{5}\frac{Z^2e^2}{r_{\mathrm{N}}} - \frac{9}{10}\frac{Z^2e^2}{r_{\mathrm{c}}} + \frac{3}{10}\frac{Z^2e^2r_{\mathrm{N}}^2}{r_{\mathrm{c}}^3} \\ &= \frac{3}{5}\frac{Z^2e^2}{r_{\mathrm{N}}}\left[1 - \frac{3}{2}\frac{r_{\mathrm{N}}}{r_{\mathrm{c}}} + \frac{1}{2}\left(\frac{r_{\mathrm{N}}}{r_{\mathrm{c}}}\right)^3\right] \end{aligned}$$

- Vanishes for $r_{\mathrm{N}} = r_{\mathrm{c}}$.

- 15% effect at 1/1000 of nuclear density.

Comments on nuclei

- Up to neutron drip density the equilibrium nuclei are known in the lab.
- At higher densities properties must be estimated from theory.
- Shell effects need to be investigated more.
  Spin-orbit interaction becomes weaker.
  Calculations of neutron drops provide information.

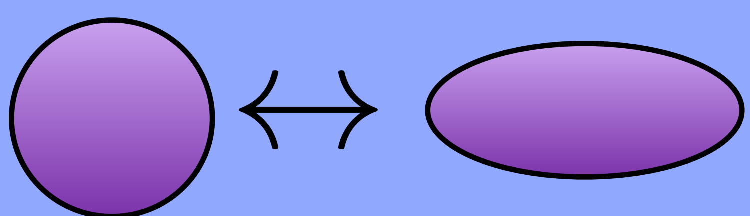
Equilibrium nucleus

- Virial relation still holds, but with the total Coulomb energy, including the lattice contribution.

- Coulomb energy reduced, equilibrium A increases.

Fission instability

- Bohr and Wheeler (1938). Nucleus unstable to quadrupolar distortion if



$$E_{\mathrm{surface}} < \frac{1}{2} E^0_{\mathrm{Coulomb}}$$

E^0_{Coulomb} is the Coulomb energy of an *isolated* nucleus

(Rather insensitive to medium effects.)

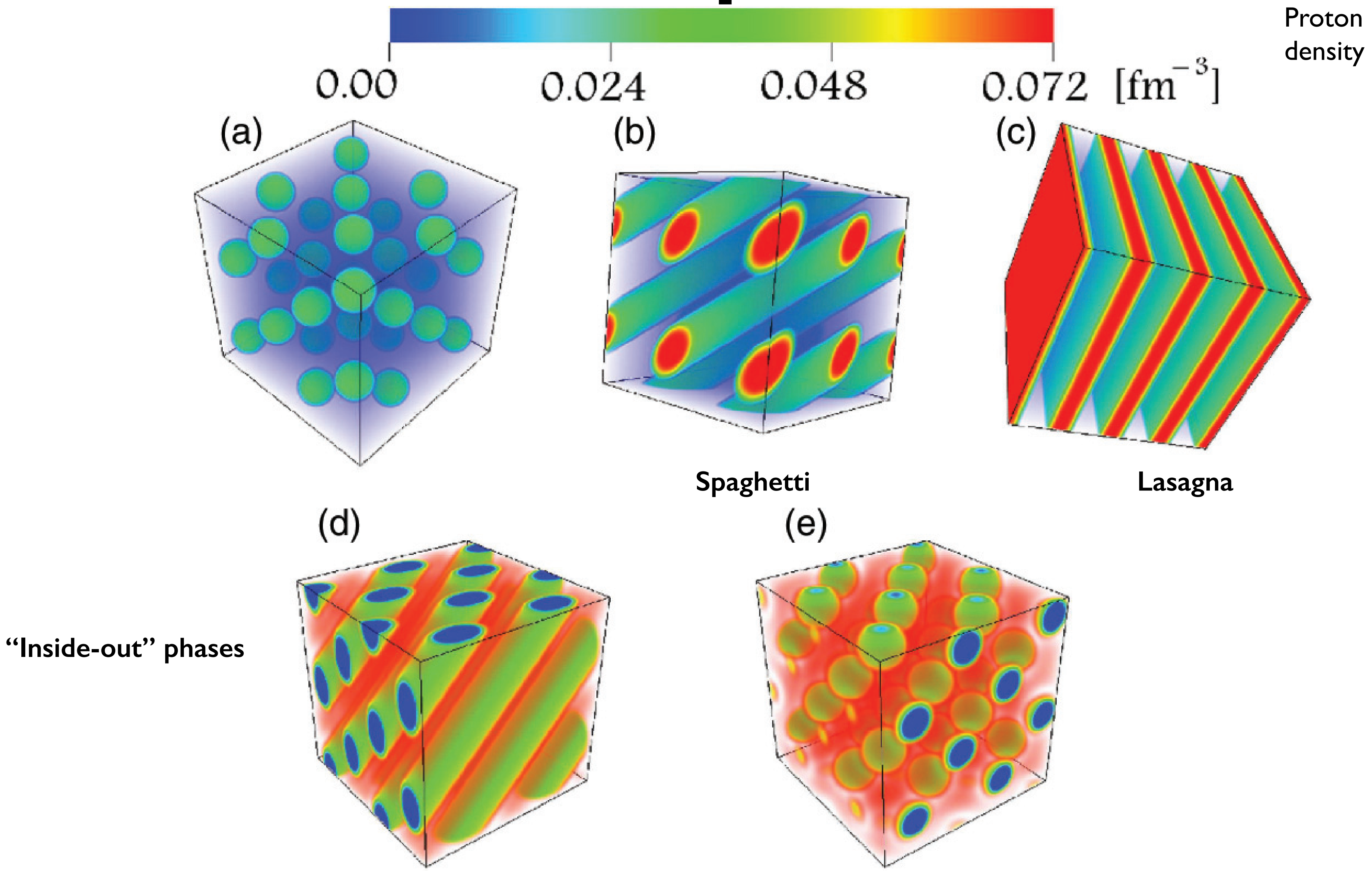
- Equilibrium nucleus unstable if

$$E_{\mathrm{surface}} = 2E_{\mathrm{Coulomb}} = 2E^0_{\mathrm{Coulomb}} \left[1 - \frac{3}{2}\frac{r_{\mathrm{N}}}{r_{\mathrm{c}}} + \frac{1}{2}\left(\frac{r_{\mathrm{N}}}{r_{\mathrm{c}}}\right)^3 \right] = \frac{1}{2} E^0_{\mathrm{Coulomb}}$$

or

$$\frac{r_{\mathrm{N}}}{r_{\mathrm{c}}} \gtrsim \frac{1}{2}$$

Pasta phases



(Image from Okamoto, Minoru et al., Phys.Rev. C **88**, 025801 (2013))

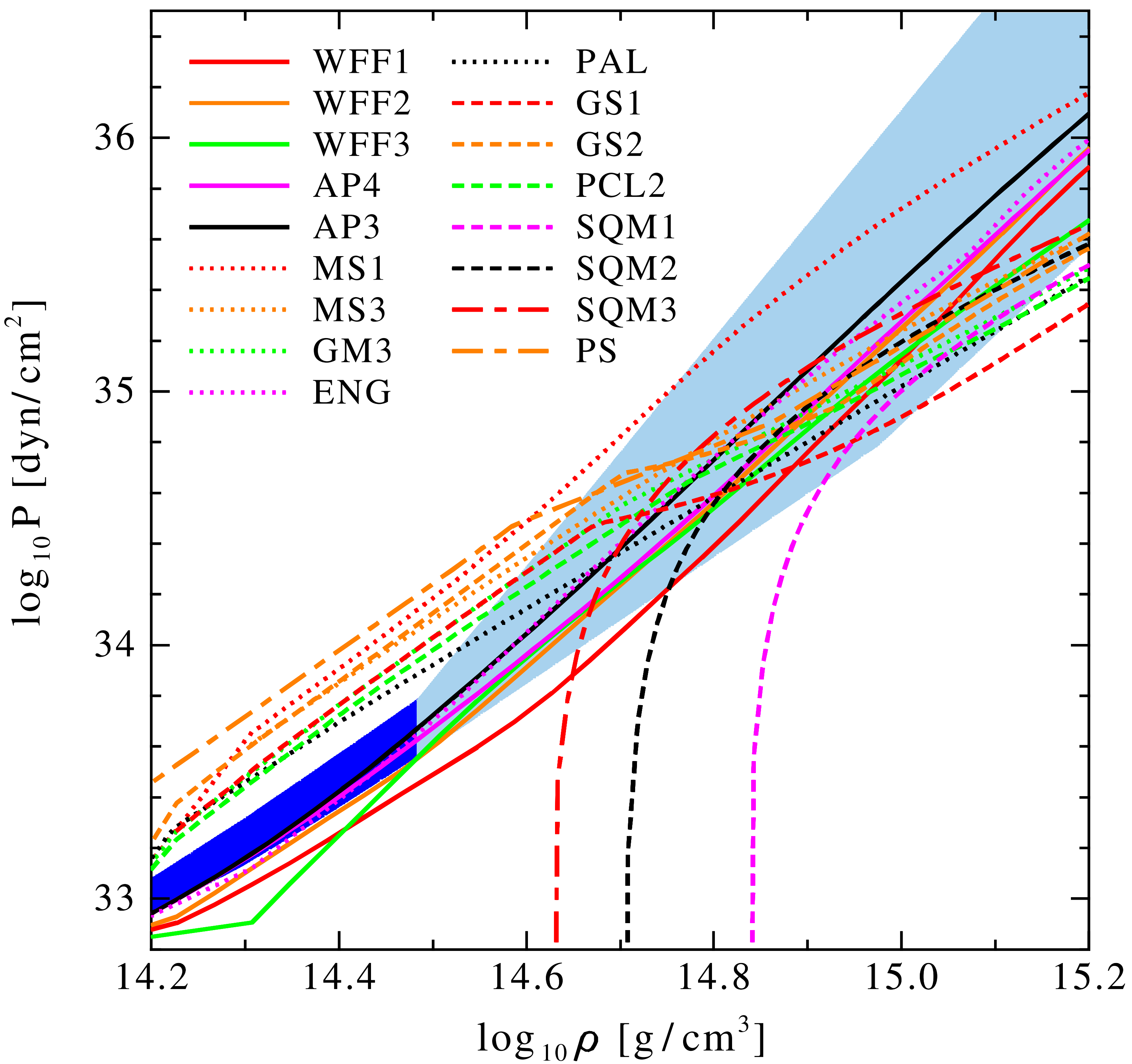
Where does the crust end?

- Start with uniform phase of neutrons, protons and electrons at nuclear density.
- Proton fraction is $\sim 5\%$.
- Reduce density until matter is unstable to creation of density wave. $E = E_0 1/2 V_q \delta n_q^2$. (Actually there are two densities, neutron and proton.)
- Coulomb interaction (and low compressibility of electrons) favors small wavelengths.
- Terms in energy $\propto (\nabla n)^2$ favor large wavelengths.
- Instability density gives upper bound on density at which structure appears. Transition has to be 2nd order on general grounds. ($(\delta n)^3$ term in energy!)
- Include 3rd and 4th order terms. As density is reduced, the most stable state goes through the sequence of pasta phases found from liquid drop ideas.
- Rather general for a number of systems (block copolymers)

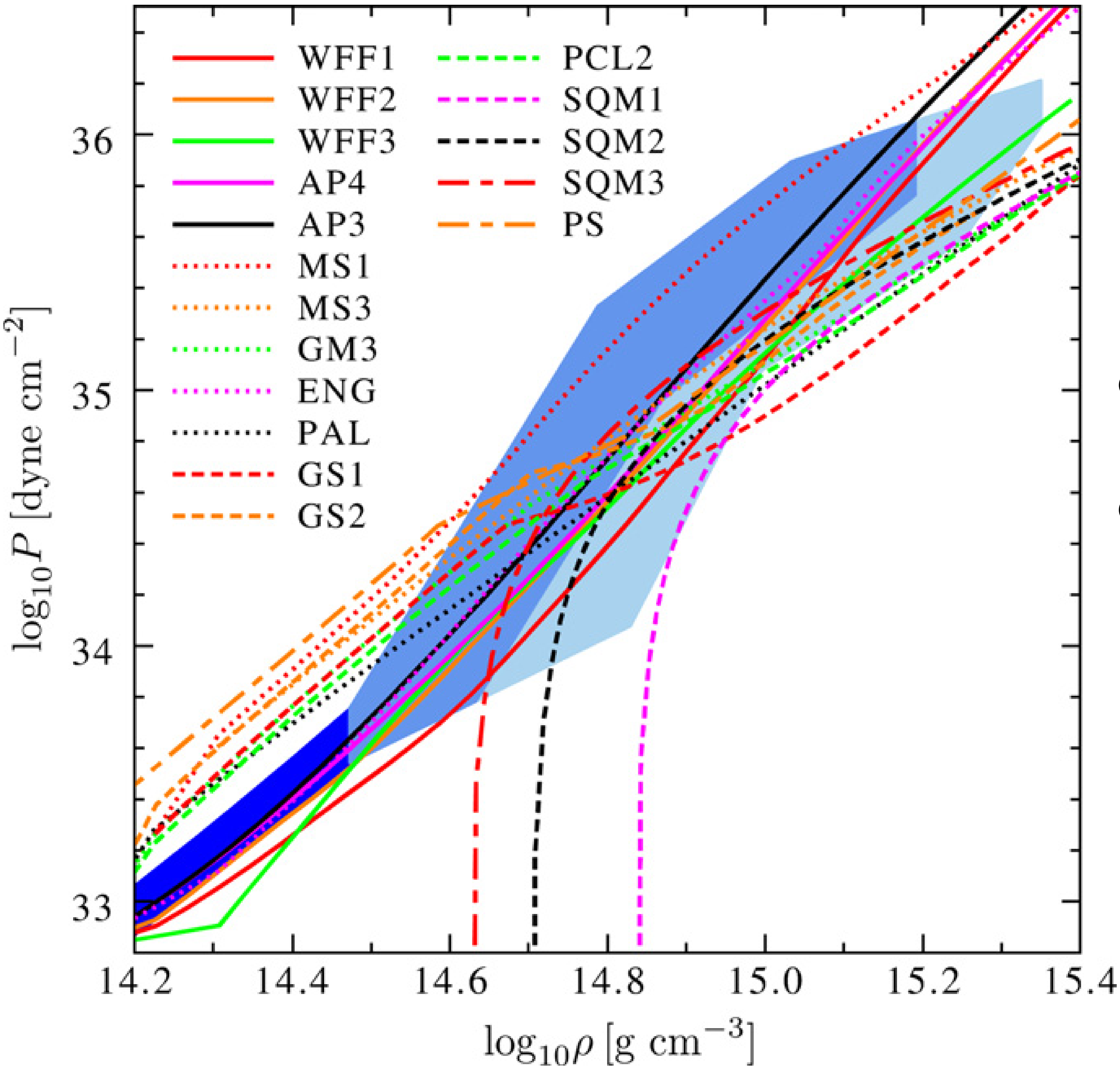
Properties of uniform nuclear and neutron matter

- Solve many-body problem for a specific nucleon-nucleon interaction. Great progress over past few decades due to development of a family of Monte-Carlo methods.
- Interaction obtained by direct fit to N-N scattering data, supplemented by phenomenological 3-body interaction or an effective field theory approach in which one expands the effective interaction between nucleons in powers of the momentum (Weinberg).
- Compare with other models.

Equations of state



Light blue area: constraint provided by existence of a $1.65 M_\odot$ neutron star. (Hebeler, Lattimer, CJP, Schwenk, Phys. Rev. Lett. **105**, 161102 (2010).)



Light blue.
Can support $1.97 M_{\odot}$ star.

Dark blue.
Can support $2.4 M_{\odot}$ star.

Hebeler, Lattimer, CJP, Scwenk, Ap. J 773:11 (2013).

TABLE 1

EQUATIONS OF STATE

Symbol	Reference	Approach	Composition
FP	Friedman & Pandharipande (1981)	Variational	np
PS	Pandharipande & Smith (1975)	Potential	$n\pi^0$
WFF(1–3)	Wiringa, Fiks & Fabrocine (1988)	Variational	np
AP(1–4)	Akmal & Pandharipande (1997)	Variational	np
MS(1–3)	Müller & Serot (1996)	Field theoretical	np
MPA(1–2)......	Müther, Prakash, & Ainsworth (1987)	Dirac-Brueckner HF	np
ENG	Engvik et al. (1996)	Dirac-Brueckner HF	np
PAL(1–6)	Prakash et al. (1988)	Schematic potential	np
GM(1–3)	Glendenning & Moszkowski (1991)	Field theoretical	npH
GS(1–2)	Glendenning & Schaffner-Bielich (1999)	Field theoretical	npK
PCL(1–2)	Prakash, Cooke, & Lattimer (1995)	Field theoretical	npHQ
SQM(1–3)......	Prakash et al. (1995)	Quark matter	Q (u, d, s)

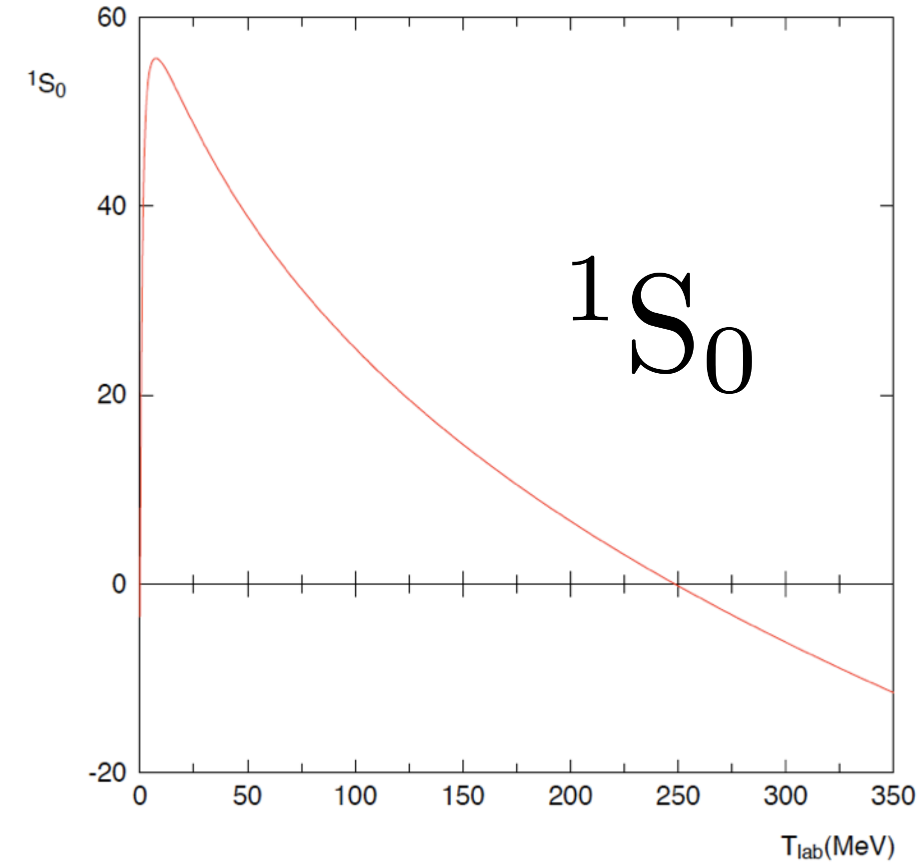
NOTE.—"Approach" refers to the underlying theoretical technique. "Composition" refers to strongly interacting components (n = neutron, p = proton, H = hyperon, K = kaon, Q = quark); all models include leptonic contributions.

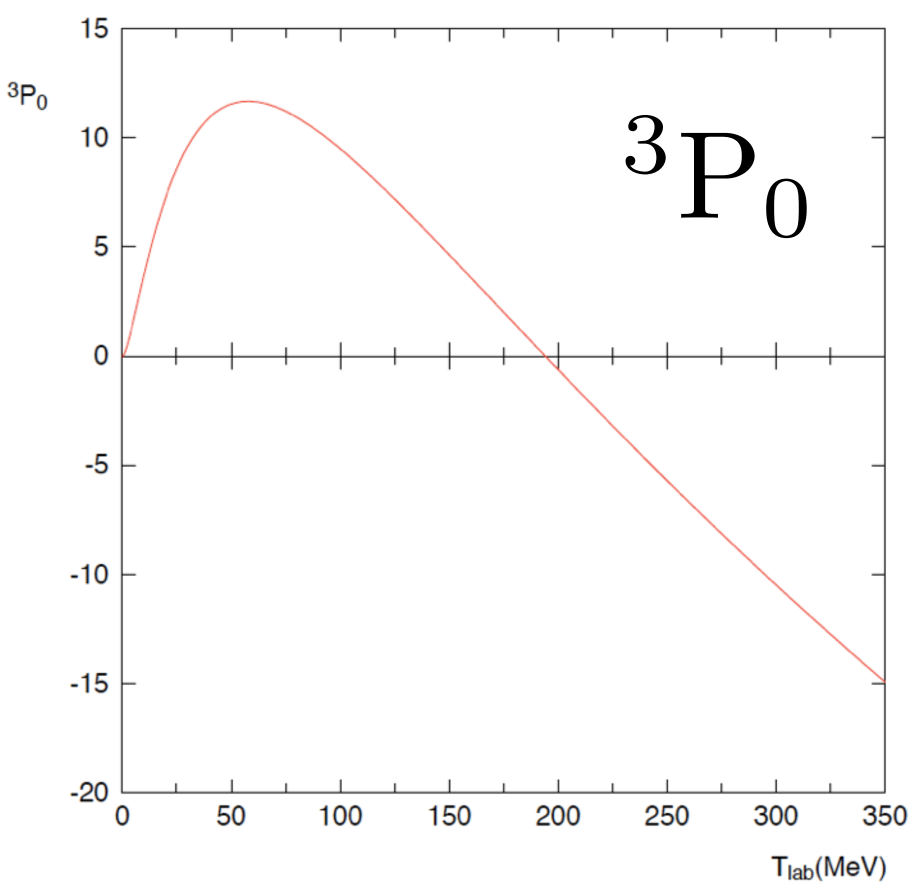
Neutron superfluidity

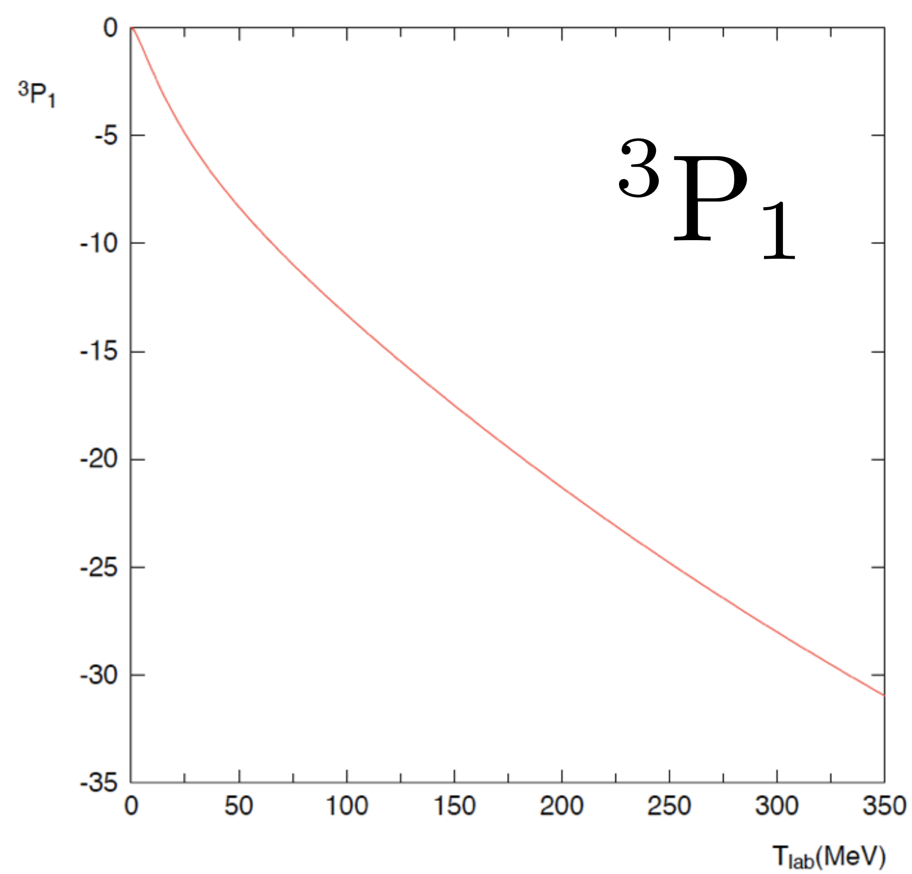
- Phase shifts suggest $^1S_0$ superfluidity (low density) and $^3P_2$-$^3F_2$ (higher density).
- Simplest approach: BCS approximation (mean field).
- Induced interactions (exchange of spin fluctuations) suppress $^1S_0$ gap.
- Inspiration from ultracold atomic gases.
- Reasonable agreement at low densities ($\lesssim n_s/10$).
  (Gor'kov and Melik-Barkhudarov (1961))
- Considerable uncertainties at higher densities.
- Calculations of proton superconductivity more uncertain
  because of the dense neutron medium.

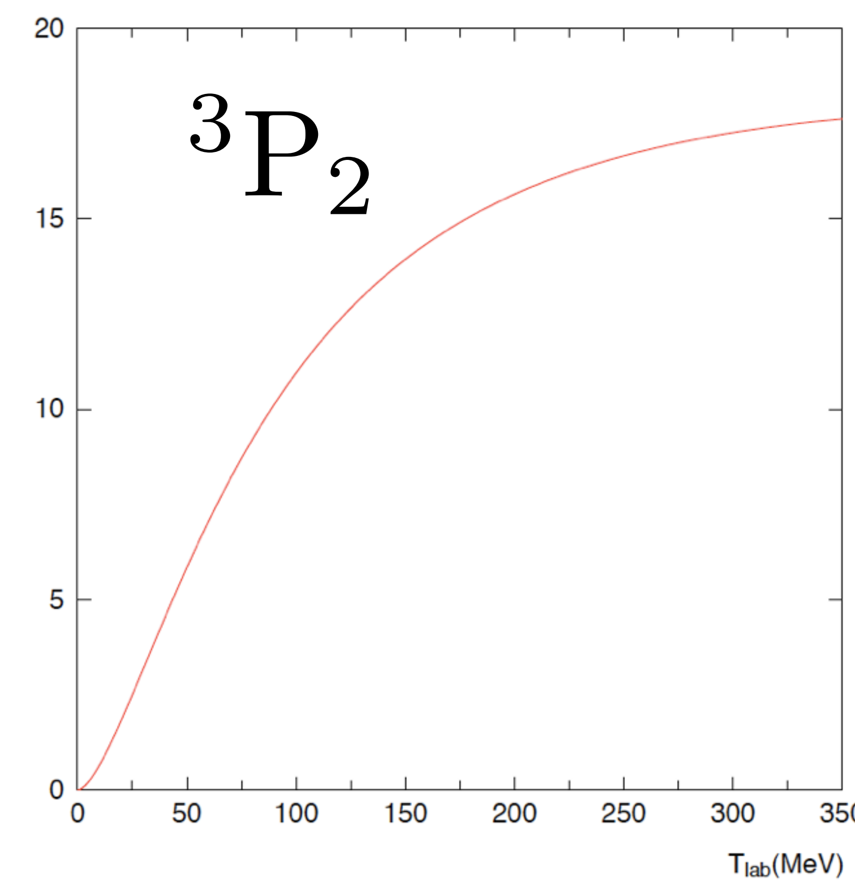
Nucleon-nucleon phase shifts (in degrees)

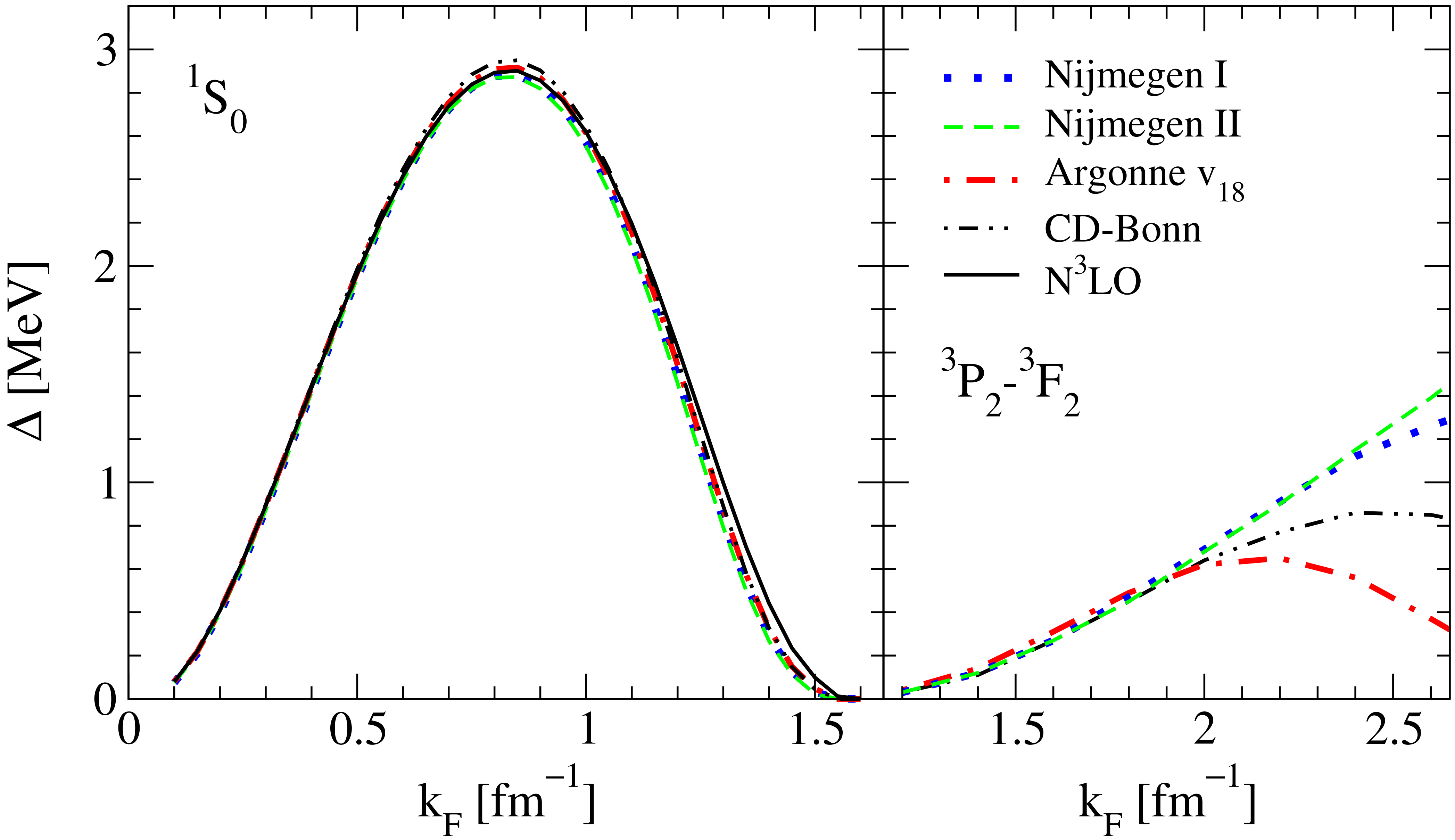
Positive phase shifts correspond to attraction.
(from nn-online.org, Nijmegen)











$^1S_0$
$^3P_2$-$^3F_2$
Nijmegen I
Nijmegen II
Argonne v_{18}
CD-Bonn
N^3LO
Δ [MeV]
k_F [fm^{-1}]
k_F [fm^{-1}]
0
1
2
3
0
0.5
1
1.5
1.5
2
2.5

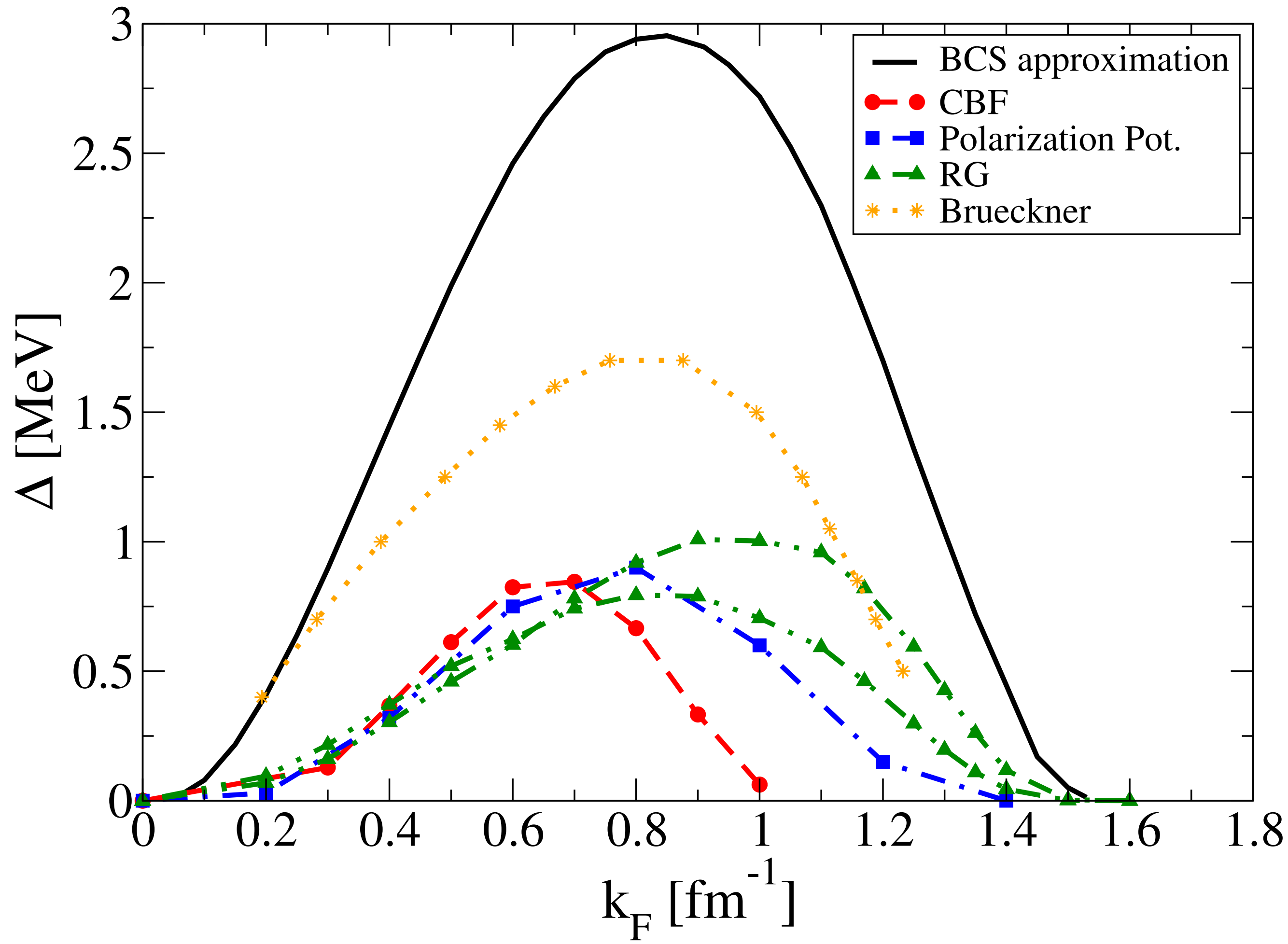
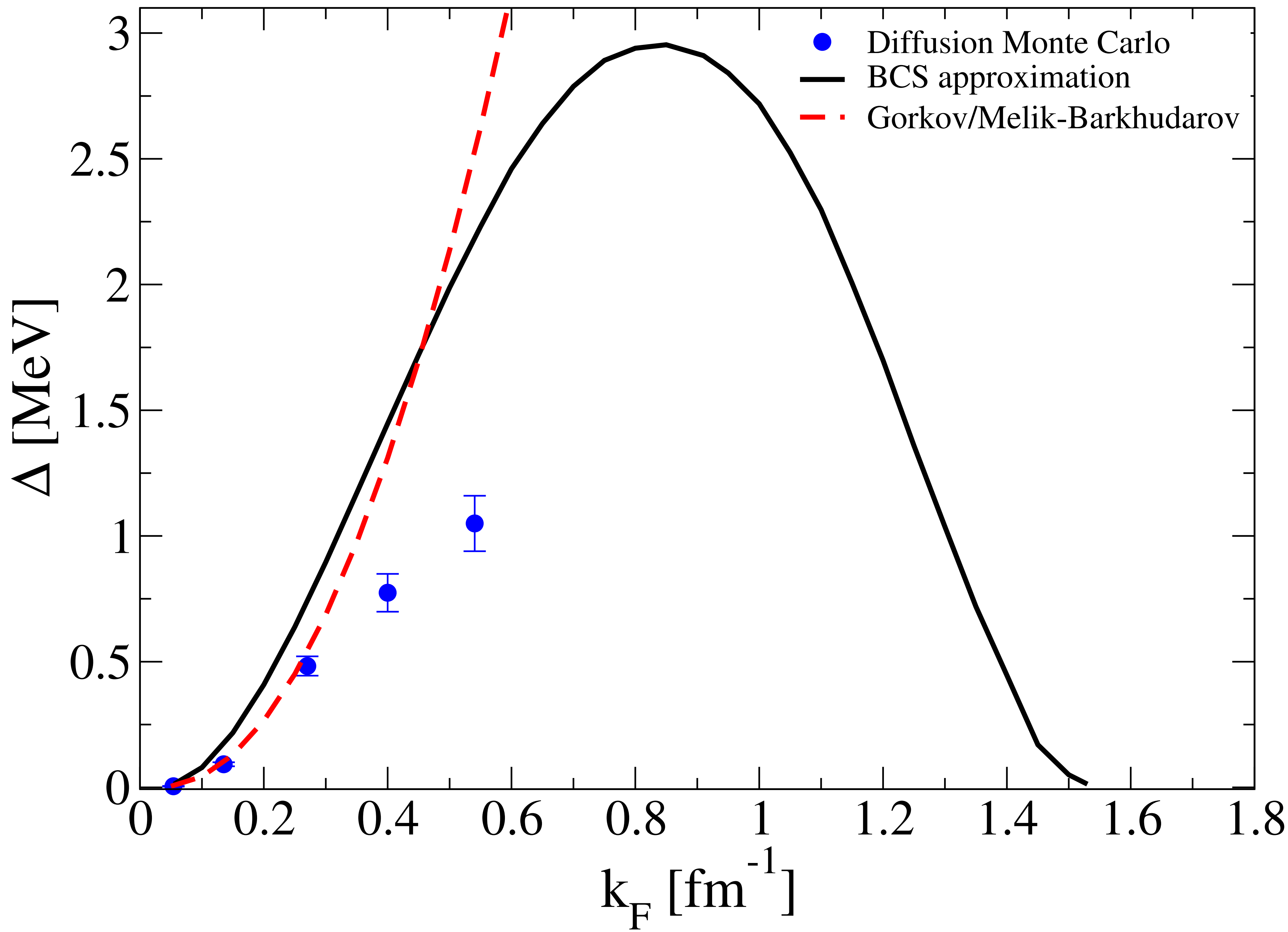


Figure 9: The $^1S_0$ pairing gap Δ at higher densities as a function of Fermi wave number k_F. Results are shown for the BCS approximation (see Fig. 7), for the method of Correlated Basis Functions (CBF) [12], for the polarization potential method, in which induced interactions are calculated in terms of pseudopotentials (Polarization Pot.) [13], for a calculation in which induced interactions in the particle–hole channels are calculated from a renormalization group (RG) approach [42], and for calculations based on Brueckner theory [46].

Nuclear matter density corresponds to $k_{Fn} = 1.68 \text{ fm}^{-1}$

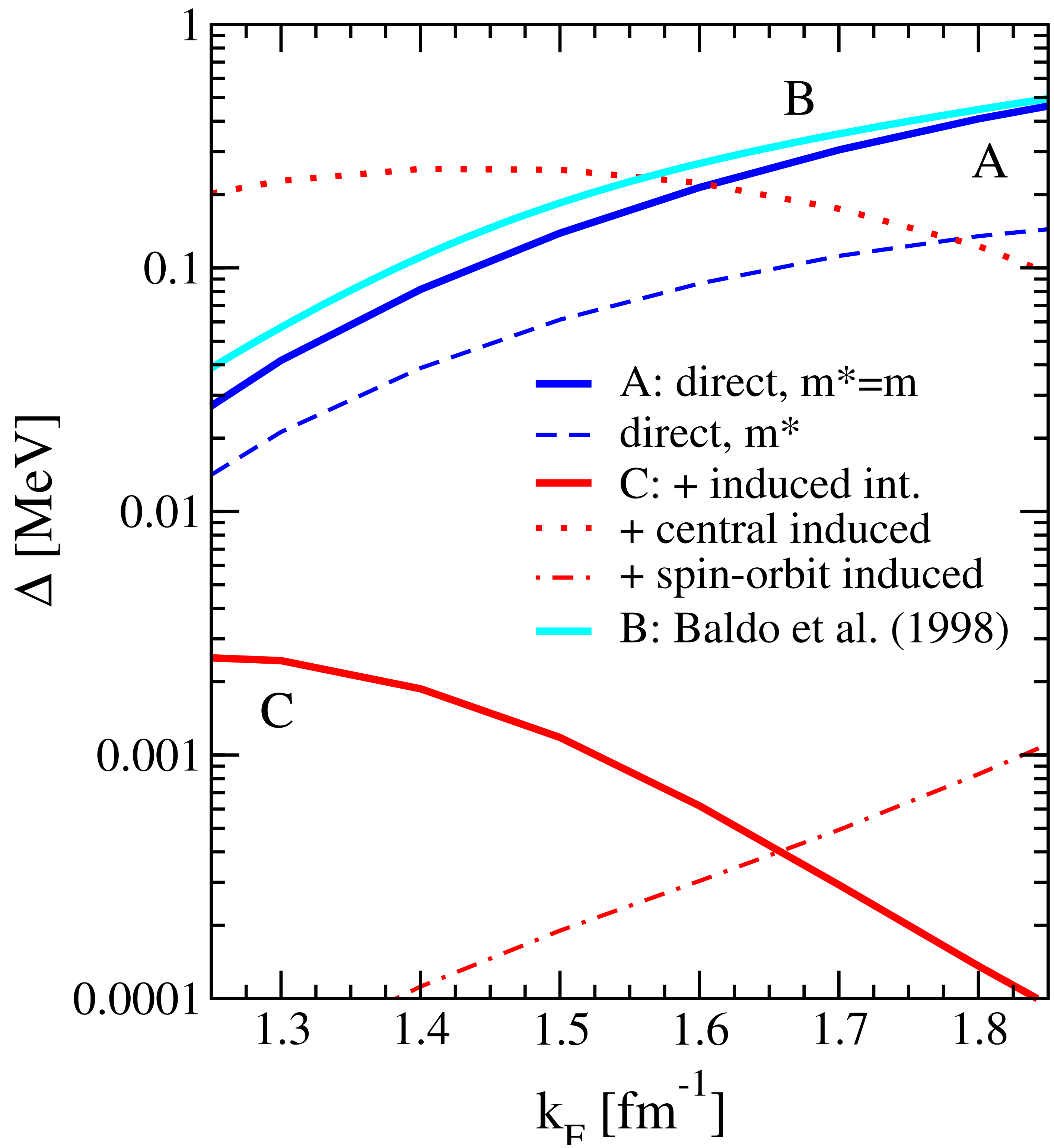
Gezerlis, CJP, and Schwenk, arXiv 1406.6109



Diffusion Monte Carlo
BCS approximation
Gorkov/Melik-Barkhudarov
Δ [MeV]
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
k_F [fm^-1]

$^3P_2$-$^3F_2$ superfluidity

Inclusion of higher-order processes suppresses gaps.
(A. Schwenk and B. L. Friman, Phys. Rev. Lett. **92**, 082501 (2004).)



Neutron superfluid density

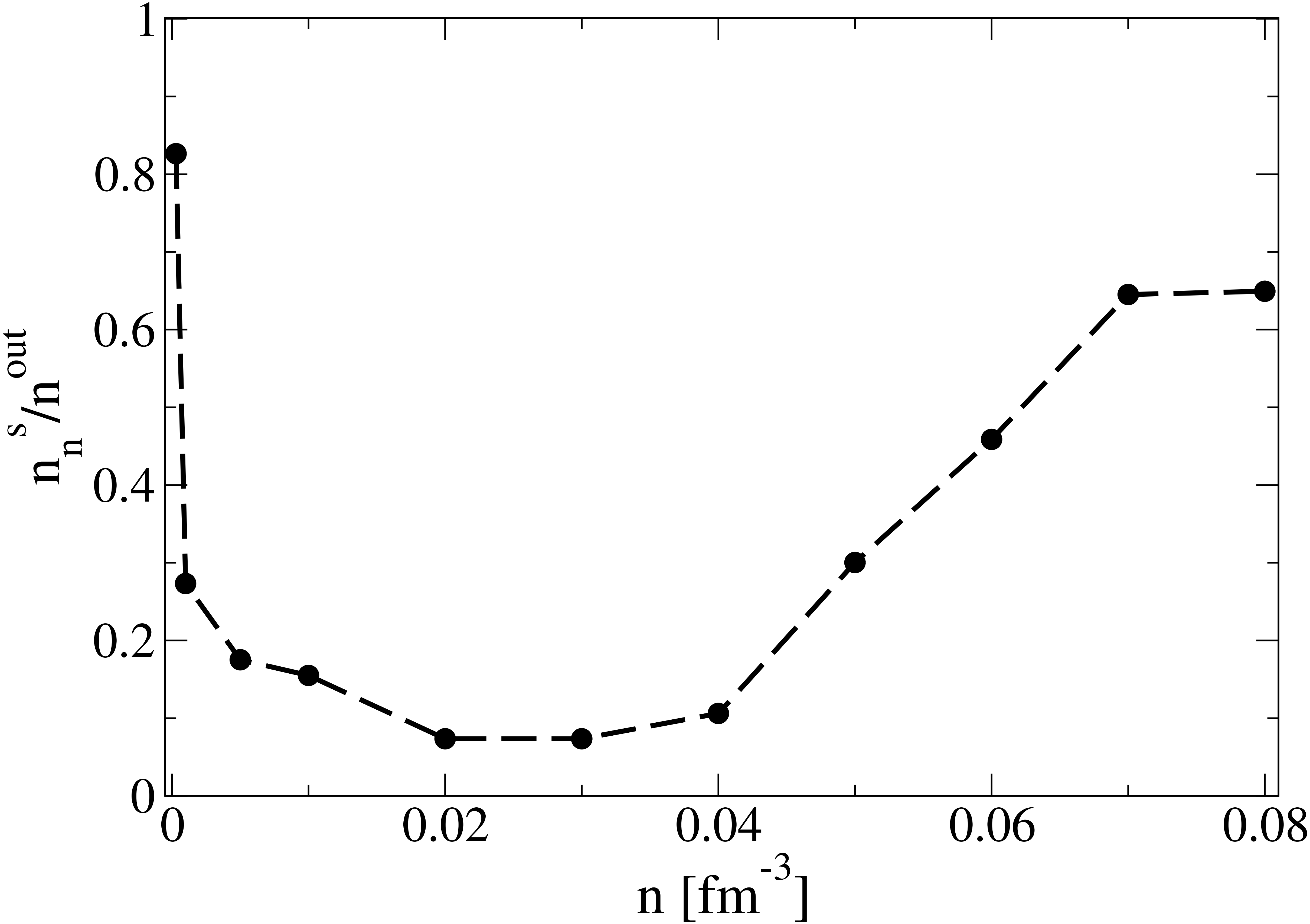
Neutron current density for stationary lattice

$$\mathbf{j}_n = n_n^s \frac{\hbar \nabla \phi_n}{m}$$

$2\phi_n$ – phase of the neutron pair condensate

- Important for glitch models, collective modes, two-fluid hydrodynamics
- Simple estimate: density of neutrons between nuclei
- Band structure calculations suggest strong reduction (Chamel)
- Gives difficulties for glitch models.
  Moment of inertia of superfluid too small
  (Andersson et al. PRL (2012) Chamel PRL (2013)).

Neutron superfluid density



Simple considerations

Scattering of BCS quasiparticles by a spin-independent potential, V.

- Quasiparticles at Fermi momentum are half particles and half holes. Interaction of particle component exactly cancels that of hole component.

- NO SCATTERING OF EXCITATION AT THE FERMI MOMENTUM TO ANOTHER STATE WITH THE SAME ENERGY!

- Quasiparticle energy
  $E_k = \pm\sqrt{\xi_k^2 + \Delta^2}$ where $\xi_k = k^2/2m - \mu$

- Matrix element for scattering of quasiparticle from state $|\mathbf{k}+\rangle$ to state $|\mathbf{k}'+\rangle$

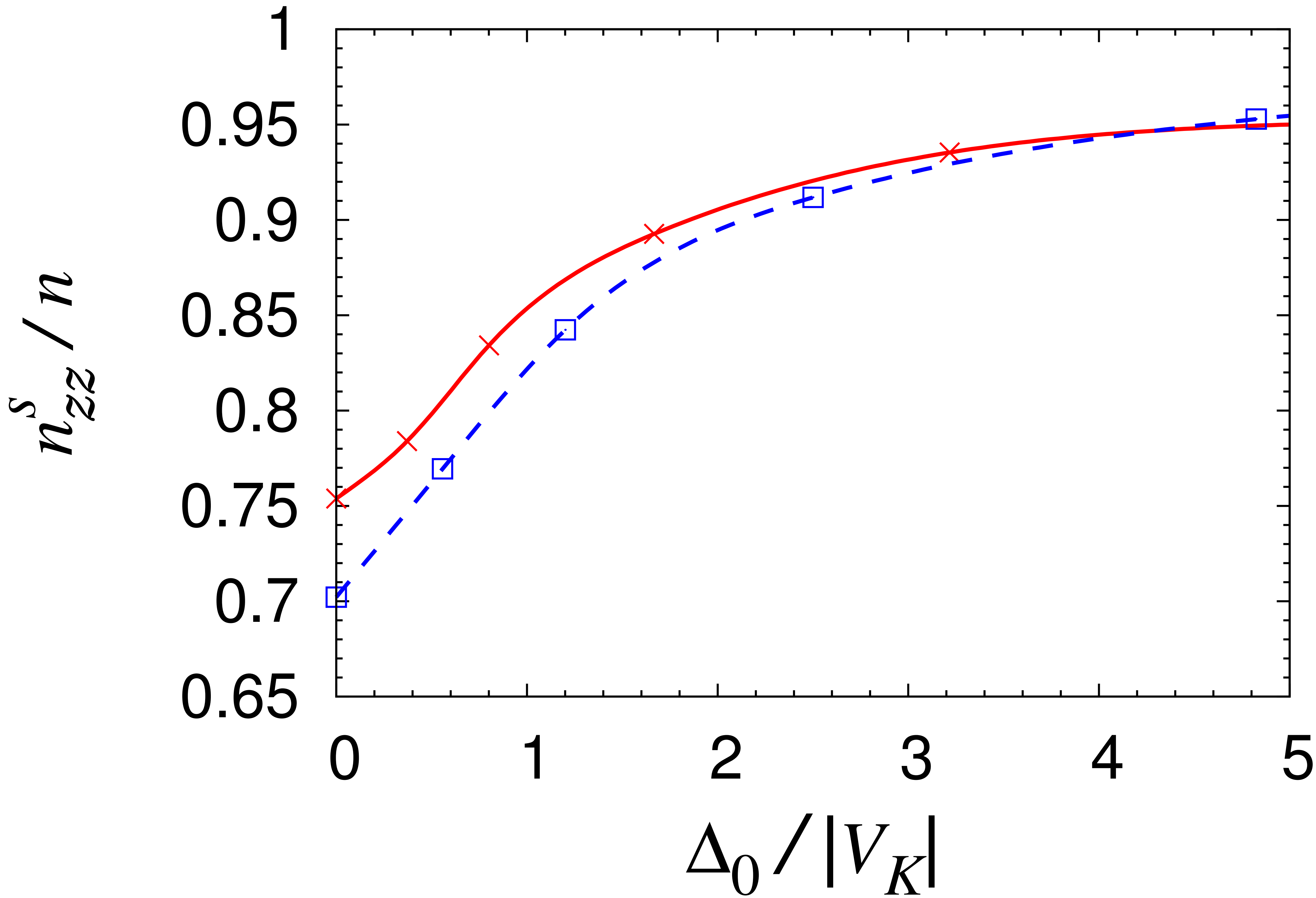
$$(u_{\mathbf{k}}u_{\mathbf{k}'} - v_{\mathbf{k}}v_{\mathbf{k}'})V(\mathbf{k} - \mathbf{k}')$$

where $u_{\mathbf{k}}^2 = (1 + \xi_k/E_k)/2$ and $v_{\mathbf{k}}^2 = (1 - \xi_k/E_k)/2$.

Pairing and band structure

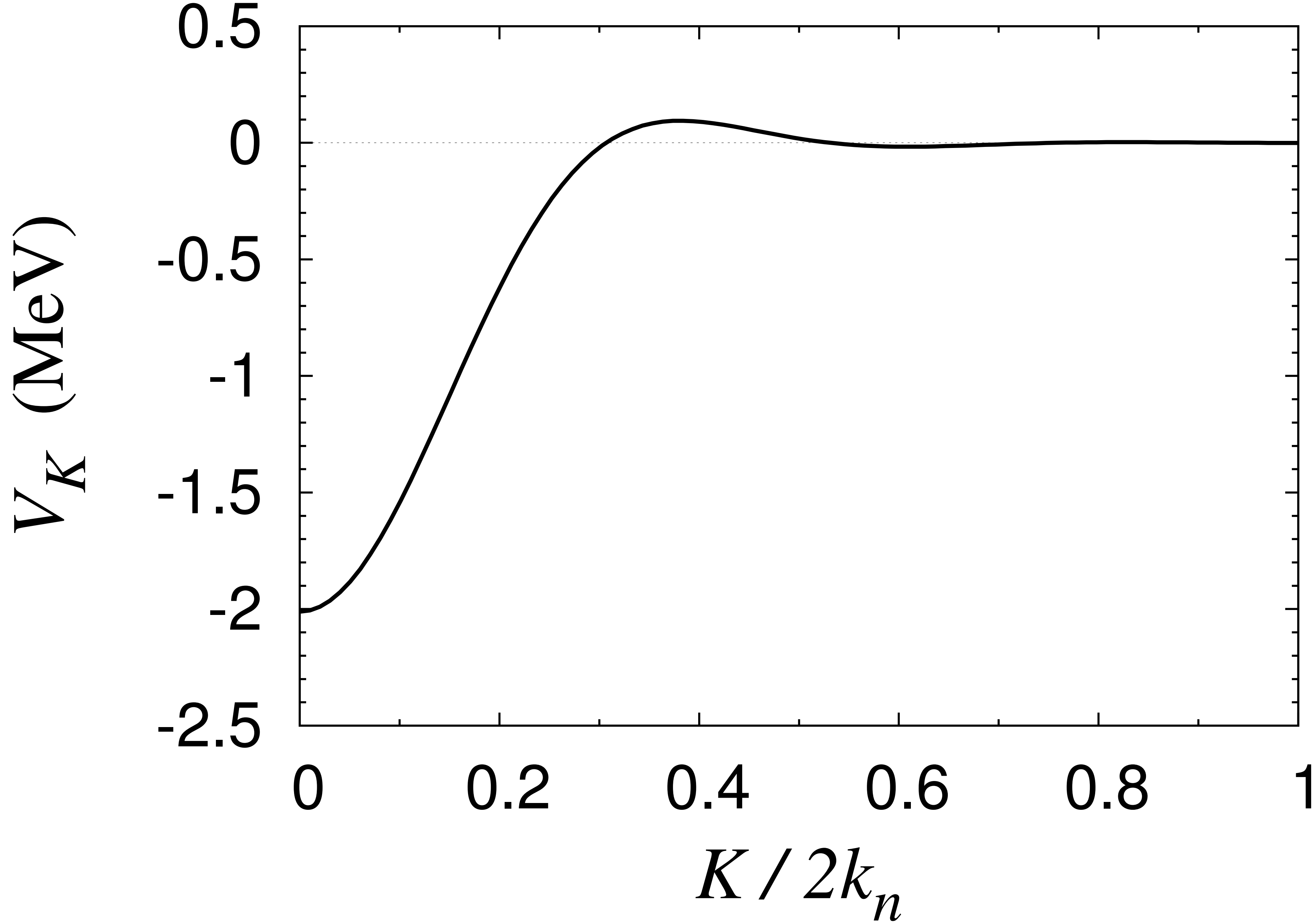
- Watanabe. Fermionic atoms in a periodic potential, with pairing
- One-dimensional and a single Fourier component of the periodic potential
- Band structure effects reduced by pairing. Small if pairing gap is large compared with the periodic potential
- In neutron star inner crust, band gaps are generally larger than the strength of the potential for most Fourier components
- Approximate way of dealing with many Fourier components.
- Conclusion. Band structure can reduce the neutron superfluid density by perhaps 10s of percent but not a factor of 10.
- Glitch models invoking the neutron superfluid in the crust are still viable!

Suppression of band structure effects by pairing



G. Watanabe and CJP, Phys. Rev. Lett. **119**, 062701 (2017).

Fourier transform of potential of nucleus (Chamel)



Pasta phases as liquid crystals

- Structures resemble those of some liquid crystals.
  - Lasagna $\sim$ smectic A.
  - Spaghetti $\sim$ columnar phase.
- Differences
  - Neutrons and protons are superfluid.
  - Two components.
- Structure is very flexible (surface and Coulomb energies compared with bulk energies).
  Two preprints on dynamics. Kobyakov and CJP, arXiv 1803.06254, and Durel and Urban, arXiv 1803.07967.
  Three-fluid model. Two superfluids plus a "normal" component associated with motion of the structure.
- Is there phase coherence between layers in lasagna?
  If not, there can be extra modes since proton motion need not be potential.
  (There could be shear.)
- Clarification needed.

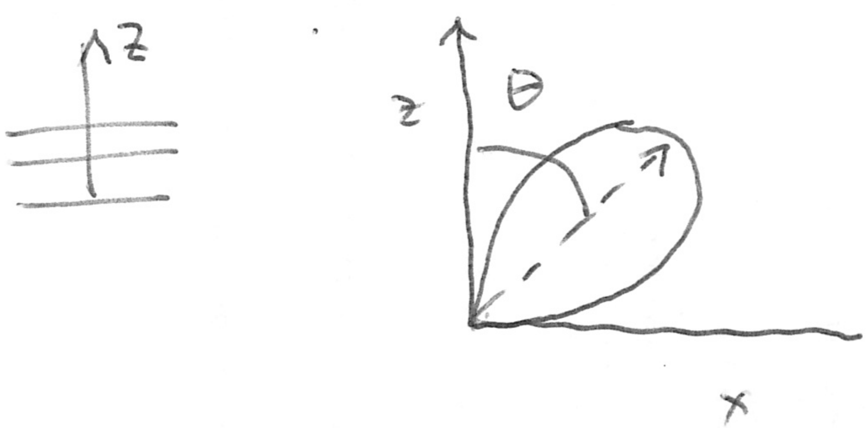
Oscillations

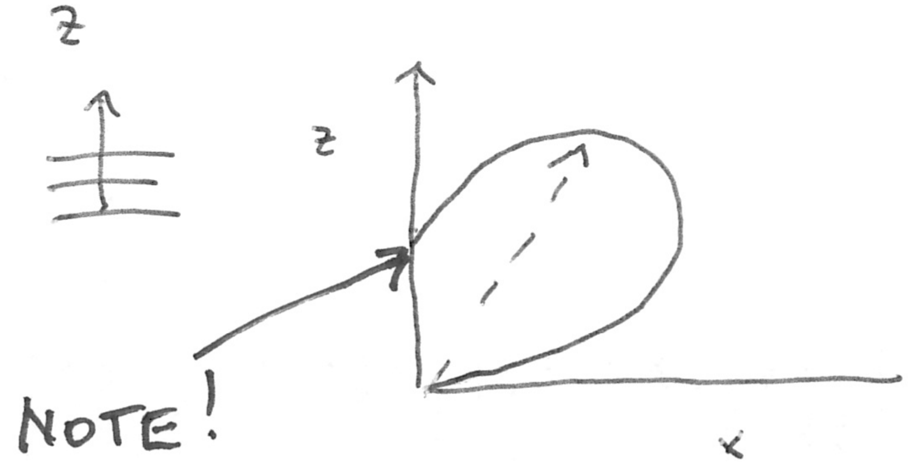
Three-fluid model (D. Kobyakov + CJP 1803.06254)

"Normal fluid". Motion of structure.

Two superfluids. Neutrons + protons. (Electrons move with protons.)

Liquid crystal. Mode velocities in polar plot.





[no "permeation"]

Permeation is frictionless.

Low frequency modes. Lasagna - heat capacity $\propto T^2$ (cf T^3 for 3-d phonons)

Spaghetti and lasagna are not uniform

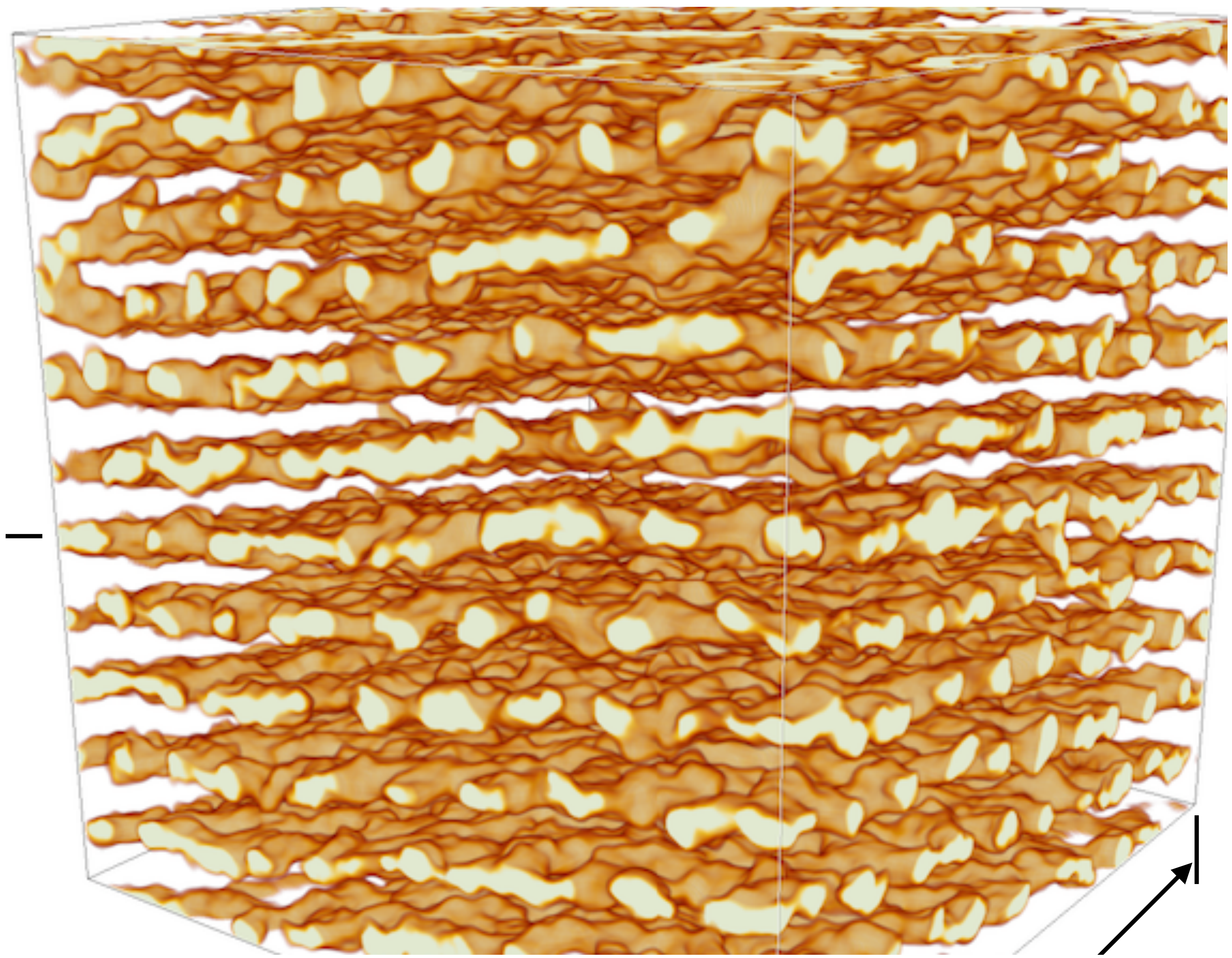
- Shown by numerical microscopic calculations. (Williams, Koonin, Watanabe, Newton, Stone, Horowitz, Caplan,...)

- $\omega^2 \propto Ak_z^2 + Bq_\perp^4$ for uniform lasagna becomes a linear spectrum in all directions when lasagna is modulated.

  Temperature does not destroy long-range order of lasagna.(cf.Landau–Peierls) See also Baym, Friman, Grinstein (Nucl. Phys. B **210**, 193 (1982)) for pion condensates, where similar conclusions apply.

- More general hydrodynamic model needed.

Nuclear Waffles
(Berry, C. Horowitz, …)



Imperfect lasagna

- Cross links, and disorder e.g., “parking garage” structure (Berry, Caplan, Horowitz, Huber, and Schneider, Phys. Rev. C **94**, 055801 (2016).
- Electrical conductivity of electrons reduced. Helps to explain evolution of NS. (Rea, Viganò, and Pons, Nature Physics **9**, 431 (2013))
- But protons are superconducting. If pasta is disordered, the protons could be a GOOD electrical conductor.

Concluding remarks

- Below nuclear density physical problems are well posed.
  Theorists have no excuses!

- Many other topics

  - Microscopic models of matter at supernuclear densities.
  - Neutrino emission in general, and from pasta phases in particular.
  - Elastic properties of polycrystals. Can be extended to pasta phases.
  - Magnetic fields.
  - Flux lines and vortices in pasta.

- Need help from "practical" people, such as metallurgists, polymer scientists, ...
  Some problems that physicists have avoided have been studied in depth because of their importance in the real world.

- Outer part of neutron star is important because many observable phenomena are affected by it.