Equation of State Constraints from GW170817

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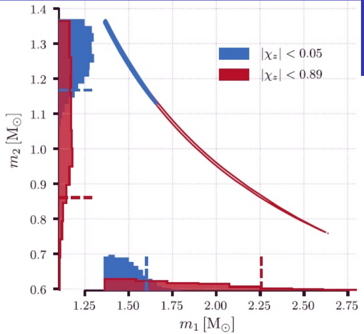
GW170817 Source Properties



90% confidence intervals from LVC

$$D = 40^{+8}_{-14}\ \mathrm{Mpc}$$

Chirp mass

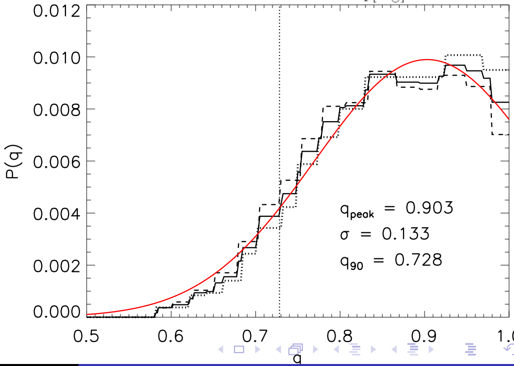
$$\mathcal{M} = 1.188^{+0.004}_{-0.002} M_\odot$$

$$m_1 = 1.42^{+0.18}_{-0.06} M_\odot$$

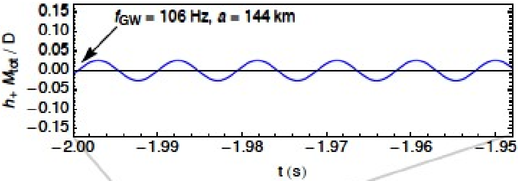
$$m_2 = 1.29^{+0.07}_{-0.13} M_\odot$$

$$q = \frac{m_2}{m_1} = 0.90^{+0.10}_{-0.17}$$

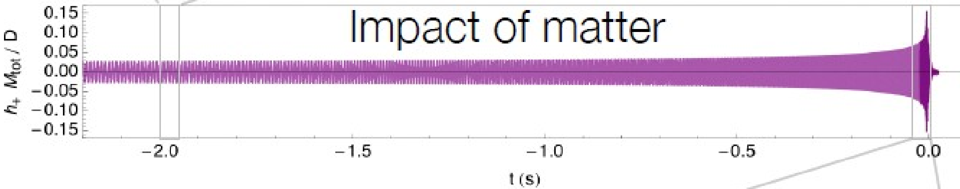
The binary tidal deformability $\bar{\Lambda} < 800$



Impact of matter



Hard to modify inspiral: transfer of ~10^{46} erg at ~100 Hz modifies phase by 10^{-3} radians (Crust shattering, Tsang et al 1110.0467)

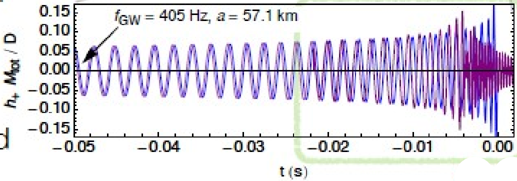


Tidal interactions lead to accumulated phase shift at higher frequencies.

$$\delta\Phi_t = -\frac{117}{256}\frac{(1+q)^4}{q^2}\left(\frac{\pi f_{GW} G\mathcal{M}}{c^3}\right)^{5/3}\bar{\Lambda}$$

For the final coalescence, numerical simulations are required

credit: Jocelyn Read



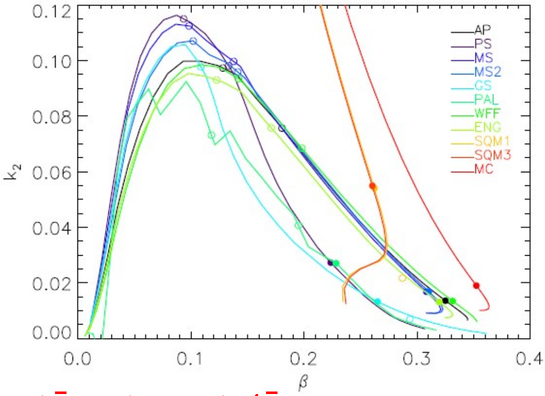
Tidal Deformability

Tidal deformability λ is the ratio between the induced dipole moment Q_{ij} and the external tidal field E_{ij}, $Q_{ij} \equiv -\lambda E_{ij}$.

k_2 is the dimensionless Love number. It is convenient to work with the dimensionless

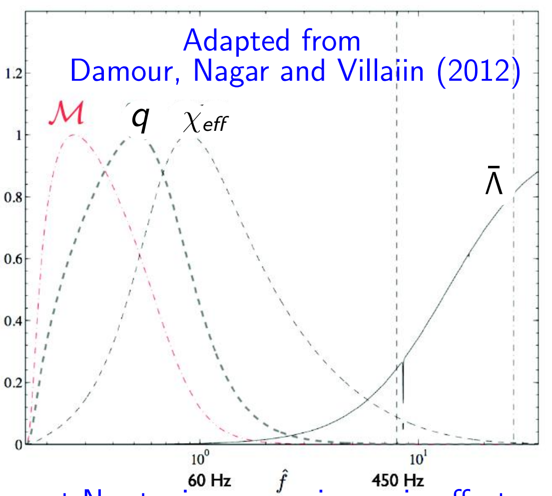
$$\Lambda = \frac{\lambda c^{10}}{G^4 m^5} \equiv \frac{2}{3} k_2 \left(\frac{Rc^2}{Gm}\right)^5$$

For a binary neutron star, the relevant quantity is $(q = m_2/m_1 \leq 1)$



$$\bar{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\bar{\lambda}_1 + (12 + q)q^4 \bar{\lambda}_2}{(1 + q)^5}.$$

When We Know What



In a post-Newtonian expansion, spin effects can be characterized by a single parameter χ_{eff}.
For spins aligned with $\vec{L}$, spin effects act oppositely to tides.

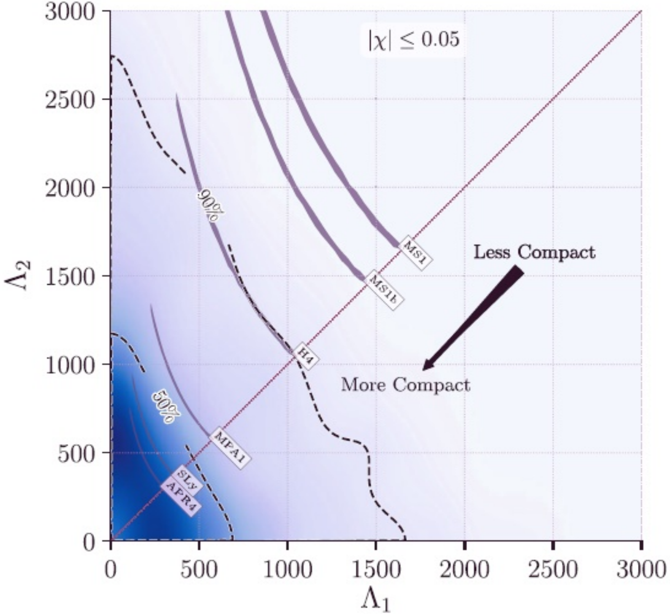
LIGO/VIRGO Parameter Determination

Although there are 11 free wave-form parameters to the lowest post- Newtonian order that includes finite-size effects, LVC used a model with 13 parameters to fit the waveform:

Extrinsic:
- Sky location (2)
- Distance (1)
- Inclination (1)
- Coalescence time (1)
- Coalescence phase (1)
- Polarization (1)

Intrinsic:
- Component masses (2)
- Spin parameters (2)
- Tidal parameters (2)

GW170817 Tidal Deformability Constraints



LIGO/VIRGO (2017)

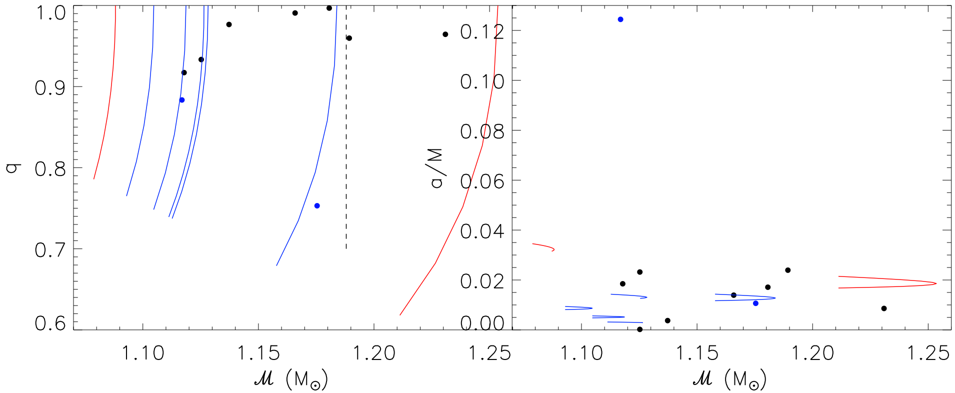
A Re-Analysis of GW170817

- Take advantage of the precisely-known electromagnetic source position (Soares-Santos et al., 2017).
- Use existing knowledge of the H_0 and the redshift of NGC 4993 to fix the distance (Cantiello et al., 2017).
- Utilize the assumption that both neutron stars have the same equation of state by determining and using a correlation among m_1, m_2, Λ_1 and Λ_2.
- The baseline model thus has 9 instead of 13 parameters.
- Explore the effects of varying the mass and deformability parameter priors.
- Explore the effects of neglecting component spins.

Properties of Observed DNS

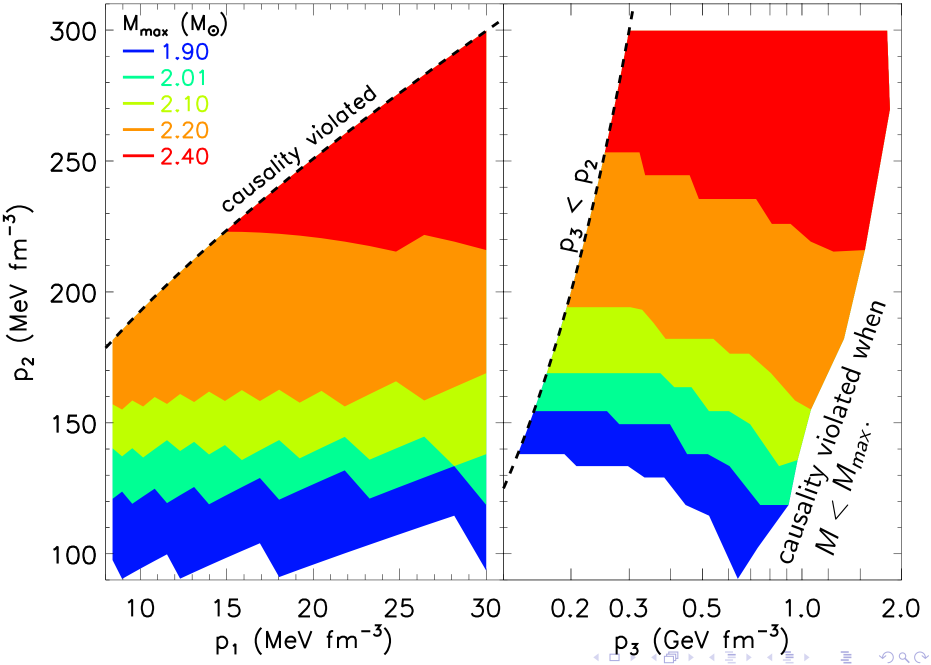
DNS with only an upper limit to m_p

DNS with $\tau_{GW} = \infty$

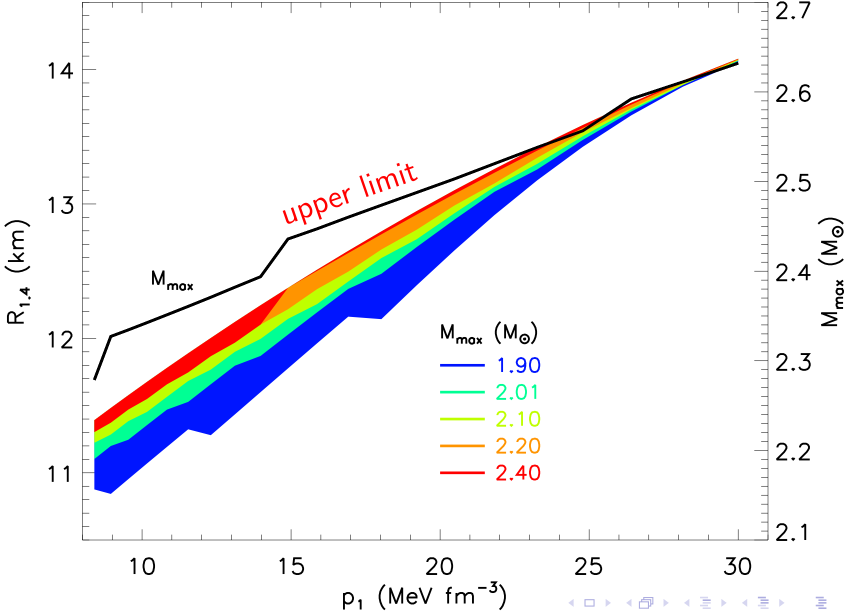


Piecewise Polytropic Equations of State

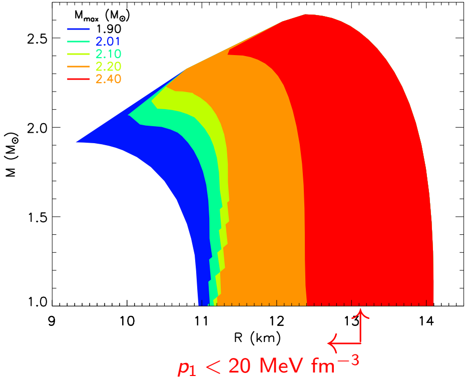
- For many reasons, it's believed neutron stars have hadronic crusts; the EOS is well-determined below $n_0 \sim 0.5 n_s$.
- $n_0 = n_s/2.7$, $p_0 = 0.2177$ MeV fm^{-3}, $\varepsilon_0 = 56.24$ MeV fm^{-3}.
- Read et al. found that $M - R$ is well-approximated with an EOS above n_0 containing as few as 3 polytropic segments.
- Read et al. found optimal upper boundaries (n_1, n_2, and $n_3 = 1.85 n_s$, $3.7 n_s$, and $7.4 n_s$) globally fit wide varieties of hadronic EOSs, leaving just 3 EOS parameters: p_1, p_2, and p_3.
- Neutron matter theory, nuclear experiment, and the unitary gas suggest that 8.4 MeV fm$^{-3} < p_1 < 20$ MeV fm^{-3}, but we extend the upper limit to 30 MeV fm^{-3}. These limits imply $32 < S_v/\mathrm{MeV} < 38$ and $39 < L/\mathrm{MeV} < 85$.
- The parameters p_2 and p_3 are limited from above by causality and below by a maximum mass $1.9 M_\odot < M_{max} < 2.4 M_\odot$.
- The parameters p_1, p_2 and p_3 are uniformly sampled.

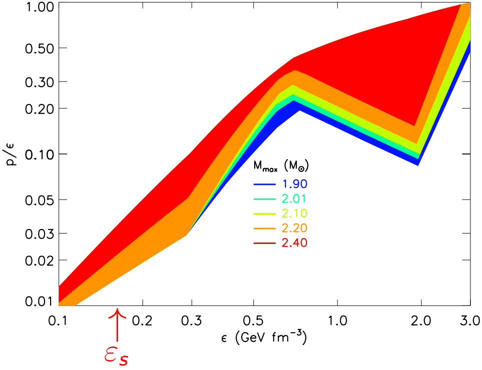

M_{max} ($M_\odot$)
1.90
2.01
2.10
2.20
2.40
causality violated
$p_3 < p_2$
causality violated when $M < M_{max}$.
p_2 (MeV fm^{-3})
p_1 (MeV fm^{-3})
p_3 (GeV fm^{-3})
300
250
200
150
100
10
15
20
25
30
0.2
0.3
0.5
1.0
2.0

The Radius-Pressure-M_{max} Correlations

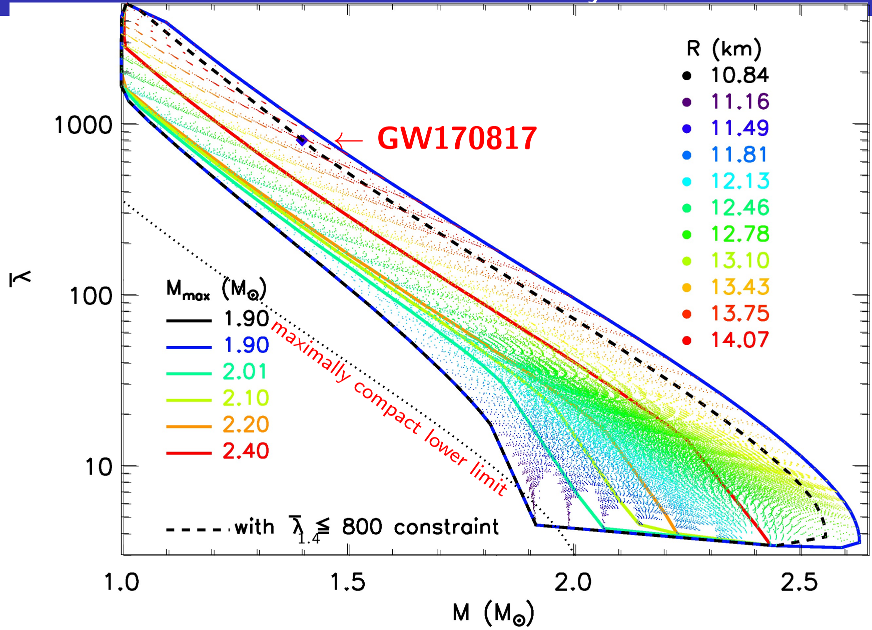


$M - R$ and EOS Constraints

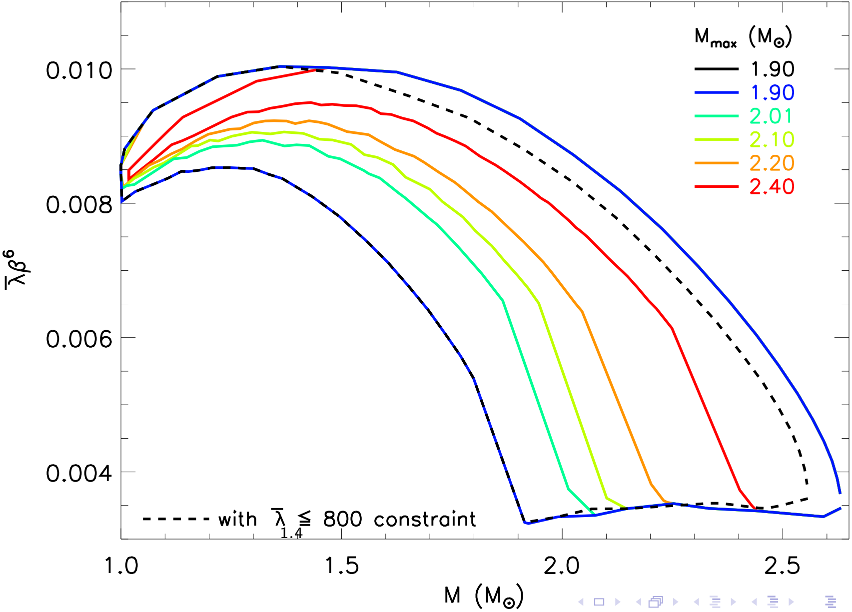




Dimensionless Tidal Deformability



Dimensionless Tidal Deformability



$\Lambda \propto \beta^{-6}$ and $R_1 = R_2$ Correlations

In the GW170817 mass range, $1.1 < m/M_\odot < 1.6$, $k_2 \propto \beta^{-1}$, and $\Lambda \simeq a\beta^{-6}$. Piecewise polytropes give $a = 0.0093 \pm 0.0007$.

Furthermore, in this mass range, R is insensitive to m. For $M_{max} \gtrsim 2M_\odot$, $<\Delta R> = -0.07$ km and $\sqrt{<(\Delta R)^2>} = 0.11$ km, where $\Delta R = R_{1.6} - R_{1.1}$.

With the assumptions $\Lambda = a\beta^{-6}$ and $R(m) = R_{1.4}$, one finds

$$\Lambda_2 = q^{-6}\Lambda_1,$$

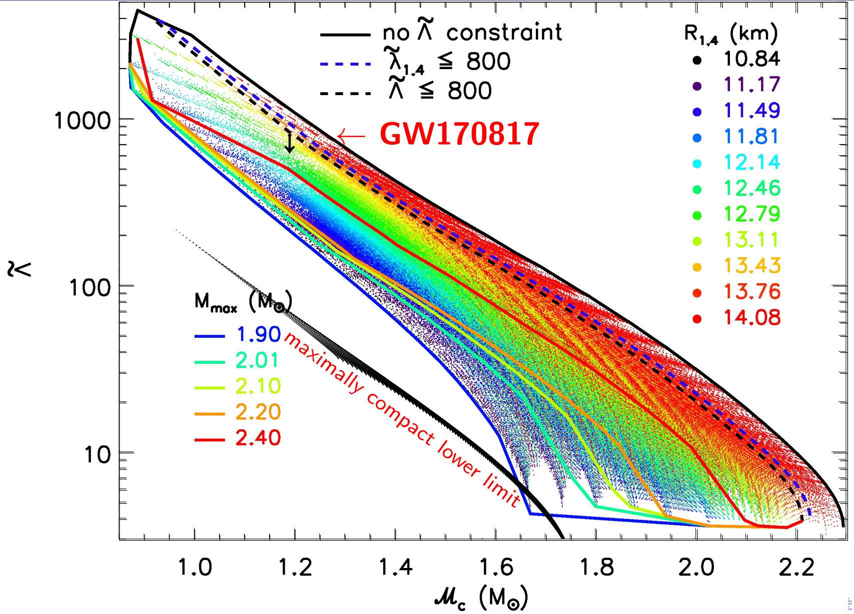
$$\bar{\Lambda} = \frac{16a}{13}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \frac{q^{8/5}}{(1+q)^{26/5}}(12 - 11q + 12q^2),$$

which is remarkably insensitive to q:

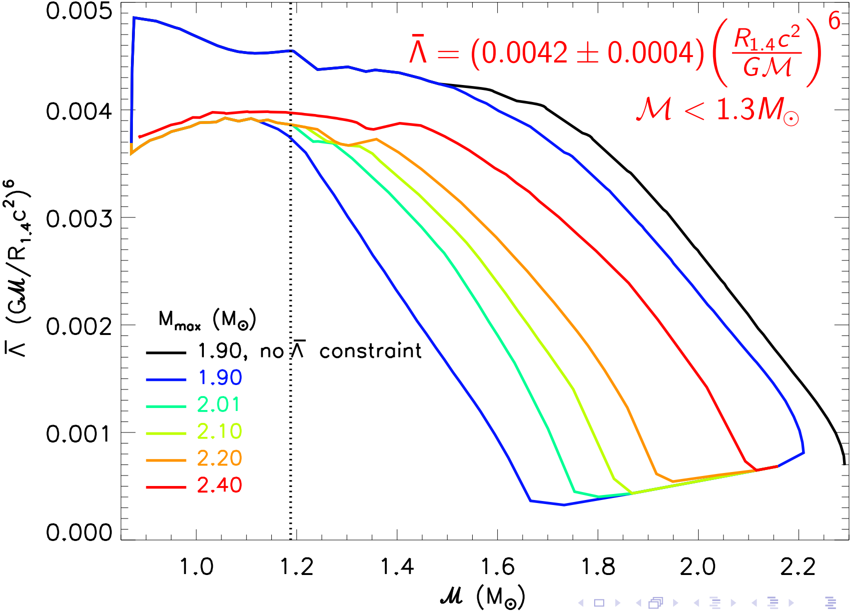
$$\frac{\partial\bar{\Lambda}}{\partial q} = \frac{16a}{65}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \frac{(1-q)q^{3/5}}{(1+q)^{31/5}}(96 - 263q + 96q^2)$$

vanishes when $q = 1$. $\bar{\Lambda}(q = 0.75)/\bar{\Lambda}(q = 1) = 1.02$.

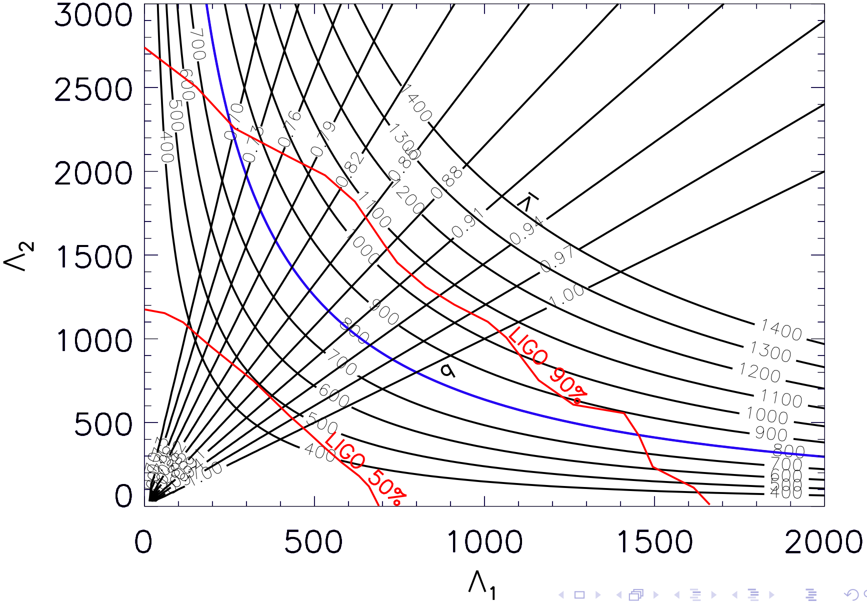
Dimensionless Binary Tidal Deformability



Dimensionless Binary Tidal Deformability

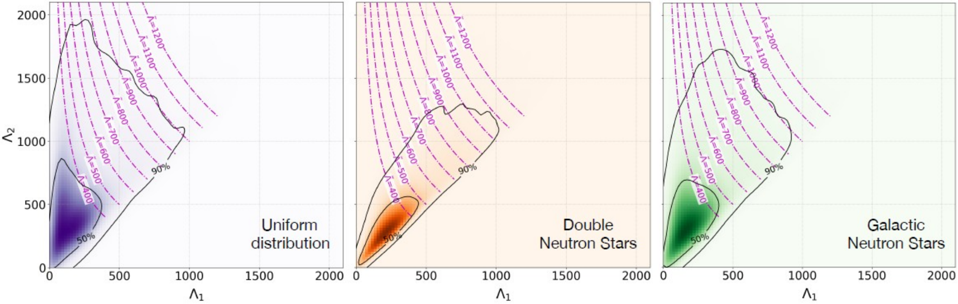


Tidal Deformabilities with Correlations



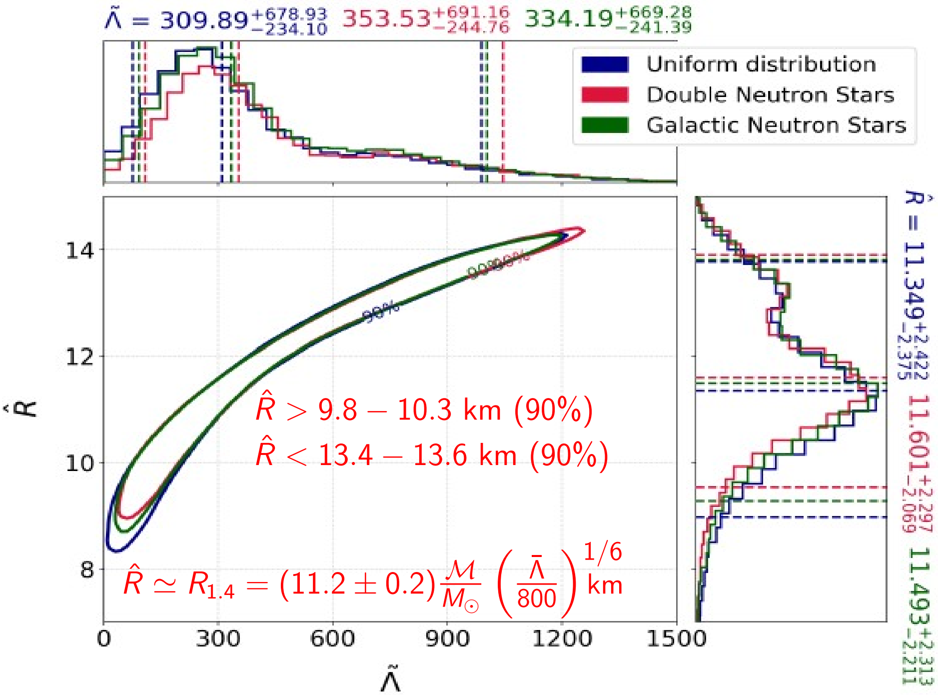
GW170817 Baseline Results for $\Lambda_1 - \Lambda_2$

Uncertainties reflect 90% credible intervals



Uniform distribution	Double Neutron Stars	Galactic Neutron Stars
$\bar{\Lambda} = 310^{+679}_{-234}$	$\bar{\Lambda} = 354^{+691}_{-245}$	$\bar{\Lambda} = 334^{+670}_{-241}$
$\bar{\Lambda} < 825$ (90%)	$\bar{\Lambda} < 852$ (90%)	$\bar{\Lambda} < 888$ (90%)
$\bar{\Lambda} > 125$ (90%)	$\bar{\Lambda} > 170$ (90%)	$\bar{\Lambda} > 140$ (90%)
$\mathcal{B} = 250$	$\mathcal{B} = 110$	$\mathcal{B} = 97$

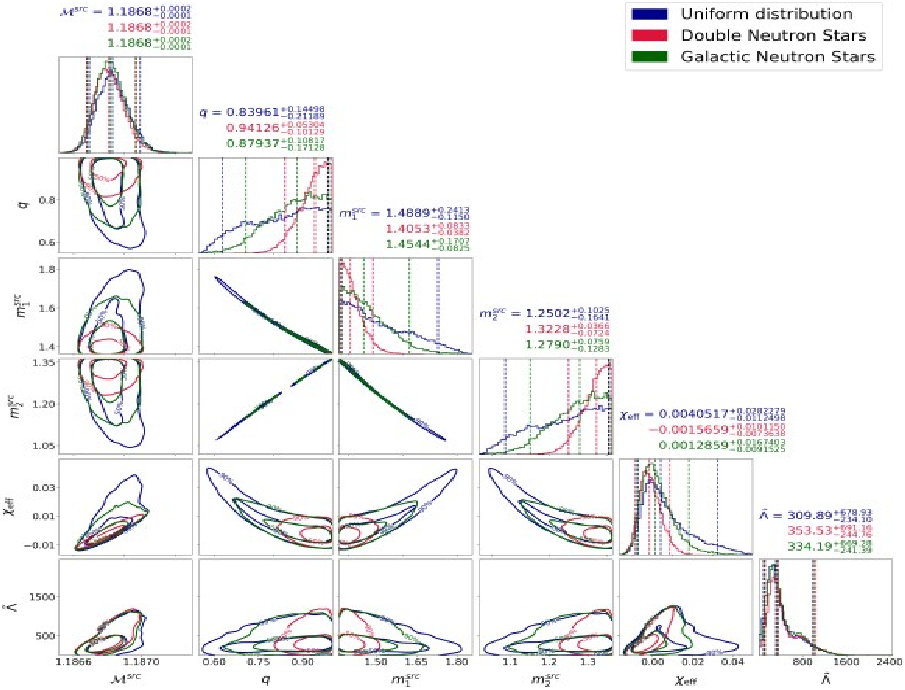
$\mathcal{B}$ is Bayes Factor relative to runs with uncorrelated Λ's



$$\hat{R} > 9.8 - 10.3 \text{ km } (90\%)$$

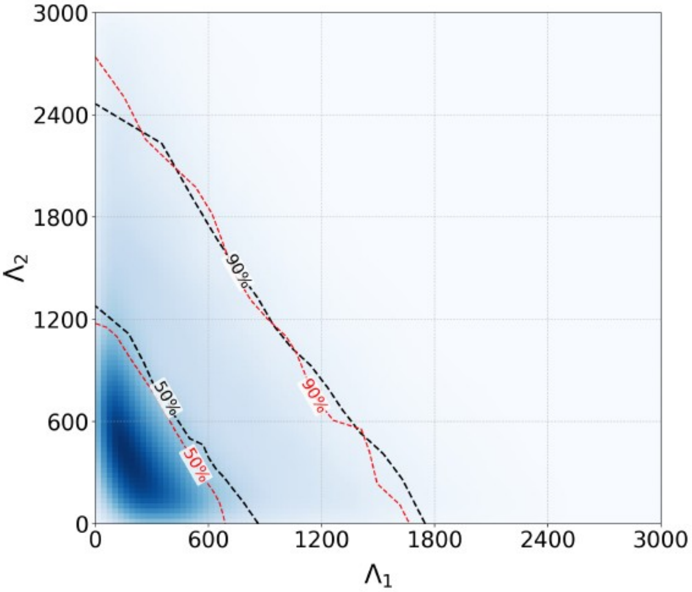
$$\hat{R} < 13.4 - 13.6 \text{ km } (90\%)$$

$$\hat{R} \simeq R_{1.4} = (11.2 \pm 0.2)\frac{\mathcal{M}}{M_\odot}\left(\frac{\bar{\Lambda}}{800}\right)^{1/6} \text{km}$$



Uniform distribution
Double Neutron Stars
Galactic Neutron Stars
$\mathcal{M}^{src}$ = 1.1868
1.1868
1.1868
q = 0.83961
0.94126
0.87937
m_1^{src} = 1.4889
1.4053
1.4544
m_2^{src} = 1.2502
1.3228
1.2790
χ_{eff} = 0.0040517
−0.0015659
0.0012859
$\tilde{\Lambda}$ = 309.89
353.53
334.19
$\mathcal{M}^{src}$
q
m_1^{src}
m_2^{src}
χ_{eff}
$\tilde{\Lambda}$

Comparison with LVC Assuming Uncorrelated Λ's



Conclusions from GW170817

- A constraint on $\bar{\Lambda}$ corresponds to a constraint on the neutron star radius in the GW170817 mass range:
$$\hat{R} \simeq (3.69 \pm 0.04)\bar{\Lambda}^{1/6}(\mathcal{M}/M_\odot) \text{ km}, \quad \frac{d\hat{R}}{d\bar{\Lambda}} \simeq 0.0022 \text{ km}.$$
- This correlation is insensitive to q which is poorly-determined. It's also reasonably valid if one star is a hybrid star.
- The LVC quoted constraint $\bar{\Lambda} < 700 - 800$ is not justified by their uncorrelated $\Lambda_1 - \Lambda_2$ results, which instead imply $\bar{\Lambda} < 1100$. The bias introduced by not correlating Λ_1 and Λ_2 is about $+250$, even when considering hybrid (twin) stars.
- A non-zero $\bar{\Lambda}$ is favored relative to its neglect; correlated $\Lambda_1 - \Lambda_2$ is favored relative to uncorrelated $\Lambda_1 - \Lambda_2$.
- Zero spin priors are favored relative to their inclusion. Spins increase the mean value of $\bar{\Lambda}$ and increase its uncertainty.
- The lower limit to $\bar{\Lambda}$ we obtain is above the causally-mandated minimum of about 65 for $\mathcal{M} = 1.19M_\odot$ and $M_{max} > 2M_\odot$.