

# Free precession & magnetic stars

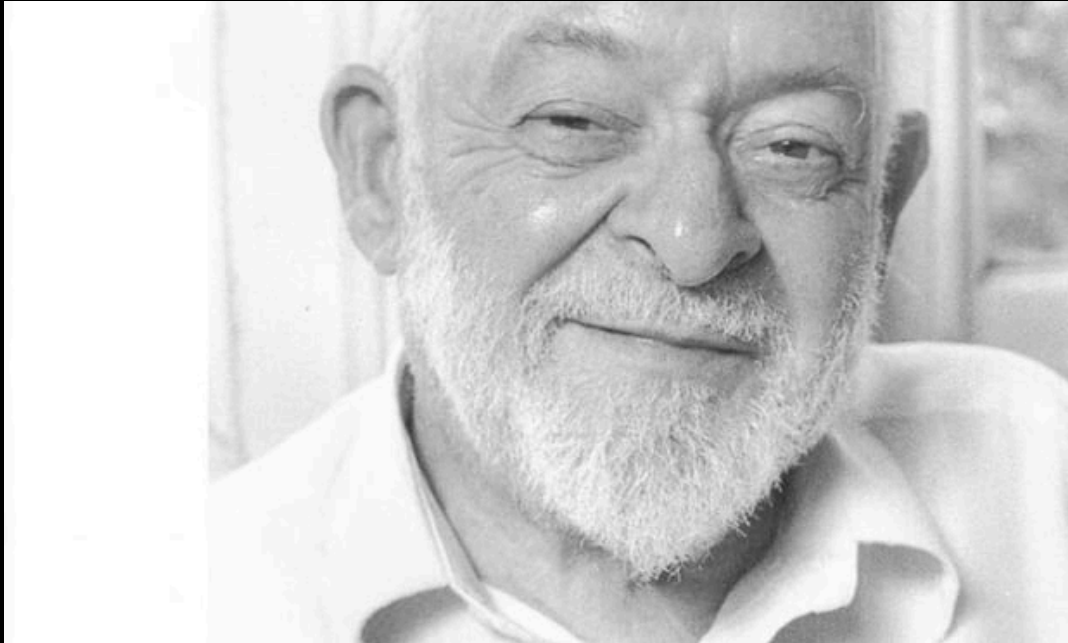
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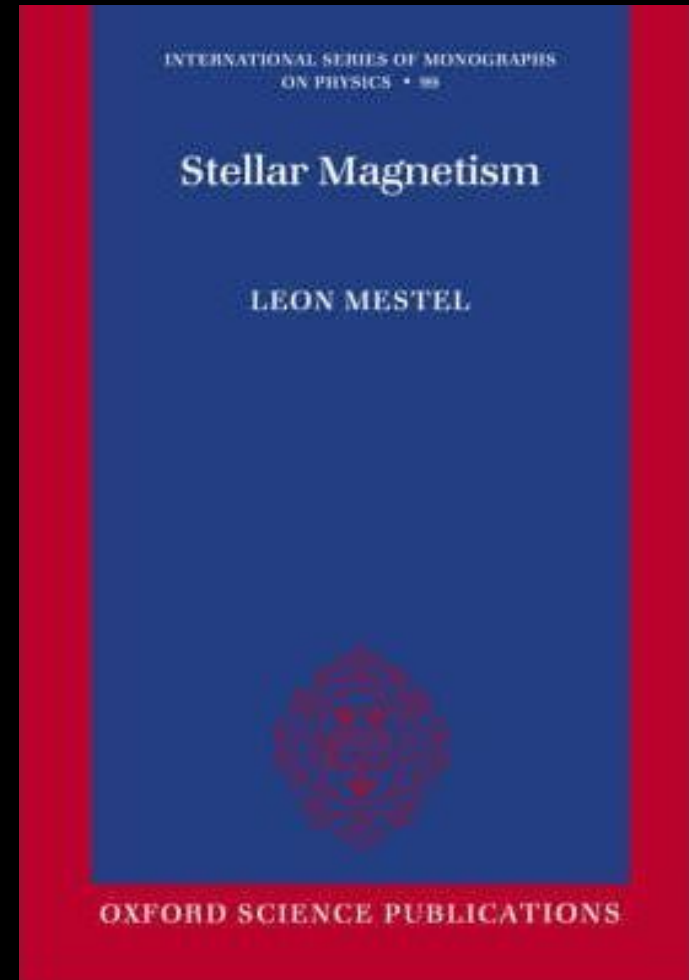
Figure credit: National Science Foundation/LIGO/Sonoma State University/A. Simonnet.

# Leon Mestel: 1927 – 2017

Pioneer in the modelling of stellar magnetic fields.



Developed model for precessing  
*magnetic star.*



# Outline

- What is free precession?
- Why is it interesting?
- Does it happen?
- The magnetic precession problem

# What is free precession?

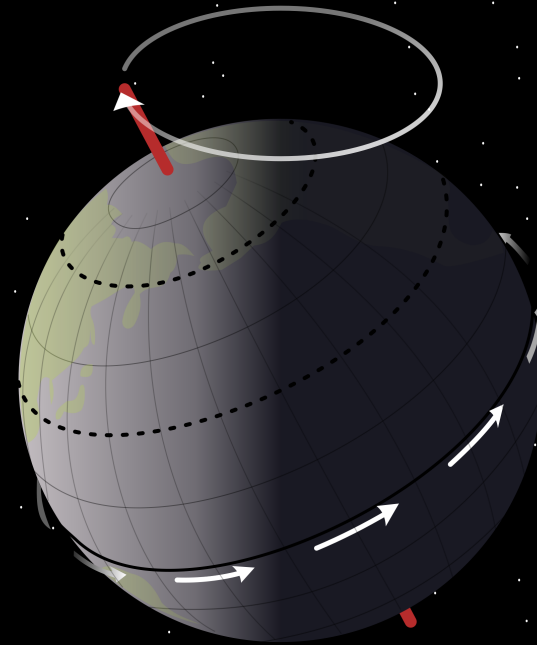
Most general motion of a rigid body.

For biaxial body, motion consists of two superimposed rotations.

Precession period depends upon stellar deformation:

$$\frac{P}{P_{fp}} \sim \frac{\Delta I}{I}$$

Observed for Earth: “Chandler wobble”,  $P_{fp} \approx 435$  days.



Why is precession interesting?

# 1: Constraining stellar structure

For neutron stars, pinned vorticity adds to precession frequency:

$$\frac{P}{P_{fp}} = \frac{\Delta I}{I} + \frac{I_{pin}}{I}$$

$\Delta I$  sourced by crustal and/or magnetic strain.

$I_{pin}$  sourced by pinned neutron superfluid.

$\Rightarrow$  measurement of  $P_{fp}$  gives info on stellar structure.

Longevity of excited precession also probes interior.

## 2: Potential GW source

Precessing biaxial star emits at two harmonics, ( $f$ ,  $2f$ ).

For small wobble angle, lower harmonic dominant:

$$h \sim \frac{1}{r} f^2 \Delta I \theta$$

Detectability limited not only by finite shear modulus and breaking strain, but also dissipation (DIJ & Andersson 2002).

Does free precession occur?

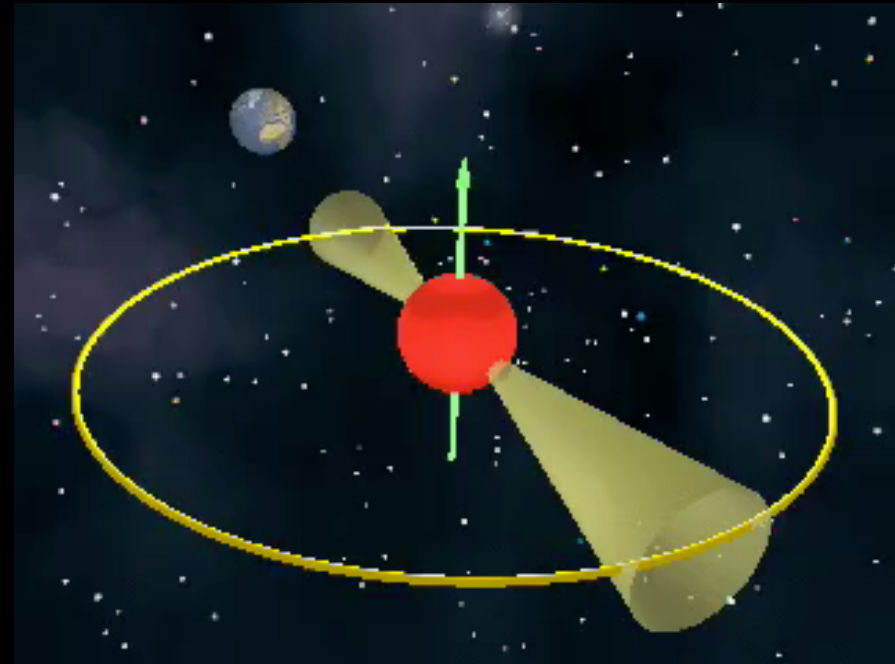


# Effect of precession on radio pulsations

Precession will leave imprint on radio data.

Could affect timing, pulse shape and polarization.

Modulations occur on (long) free precession period.



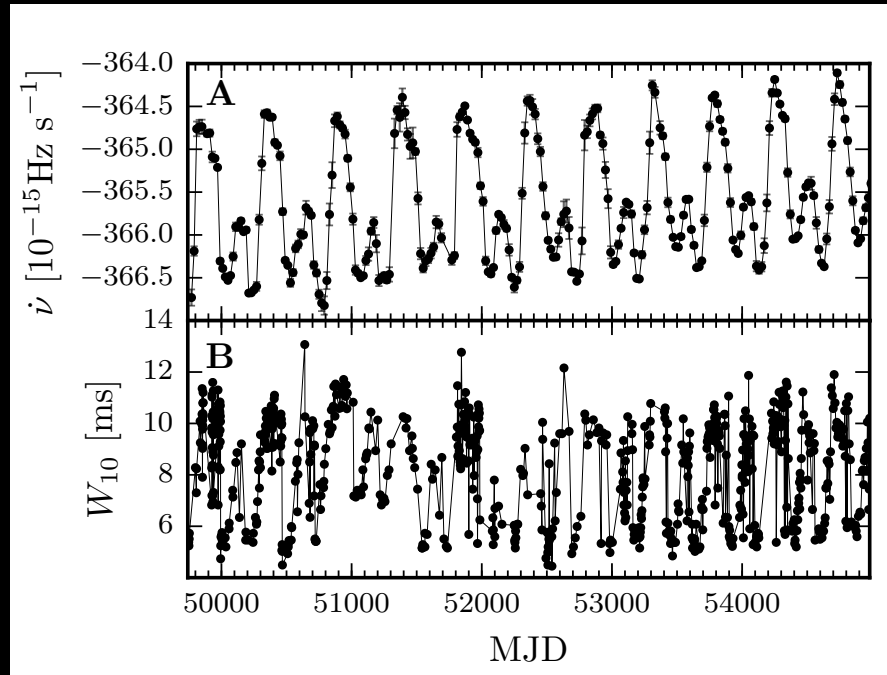
Animation: Jodrell Bank

# Pulsar precession?

Quasi-periodicities seen in a handful of pulsars (Lyne+ 2010).

Cleanest candidate B1828-11: correlated timing and pulse shape variations.

Initially interpreted as small angle free precession (Stairs+ '00, DIJ & Andersson '01, Link & Epstein '01)



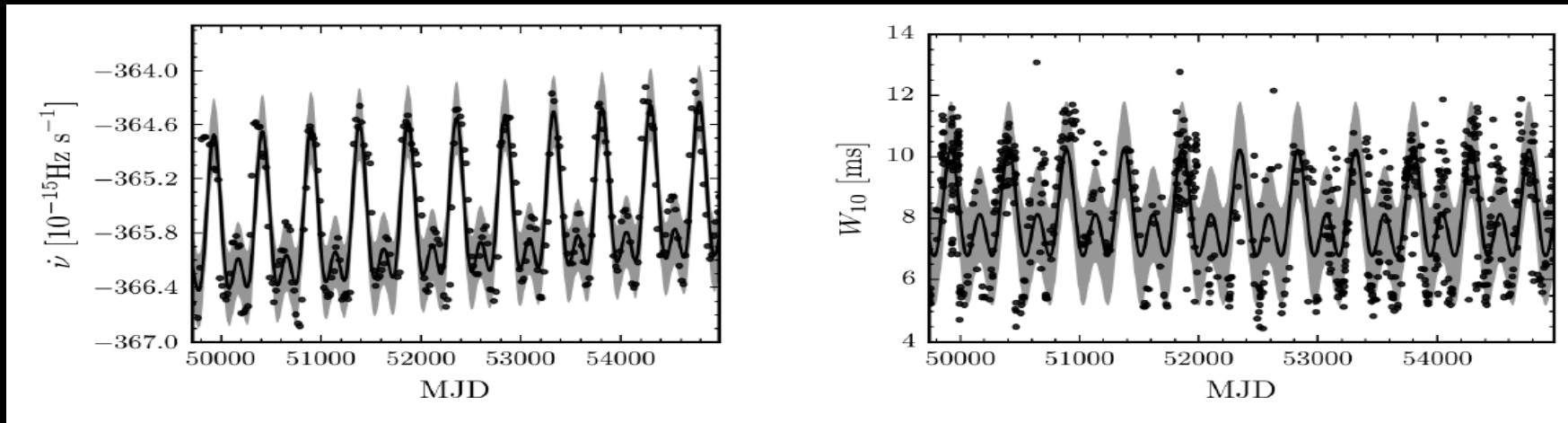
Data courtesy of Lyne+ (2010)



# Precession v Switching

Carried out a Bayesian comparison between precession and switching model (Ashton, DIJ & Prix, 2015).

Found precession was significantly better fit (odds ratio  $10^{2.7 \pm 0.5}$ ).



Selected posterior parameters:

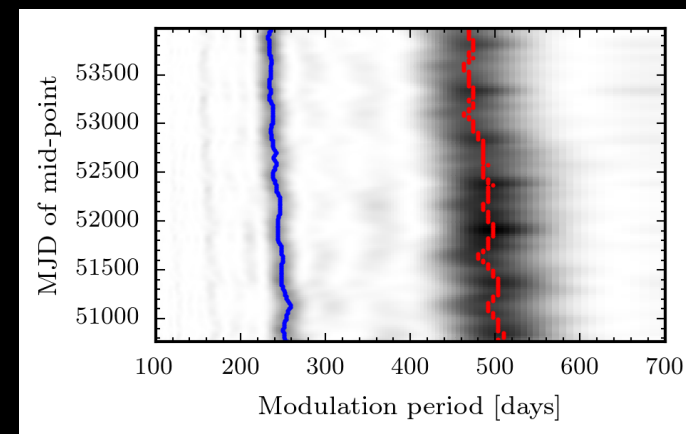
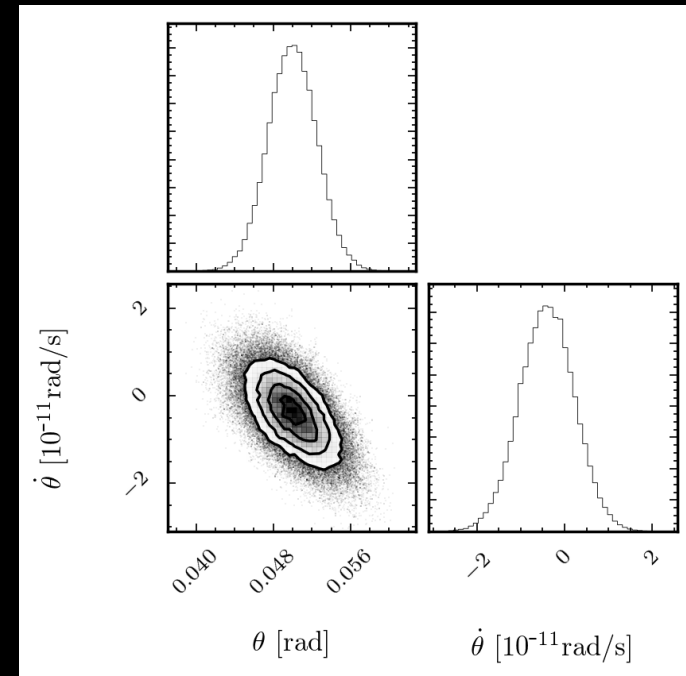
$$\frac{P}{P_{fp}} \sim 10^{-8}, \quad \theta \sim 2.8^\circ, \quad \chi \sim 88.9^\circ$$

# Further analysis...

Refined model to allow for changing  $\theta$  and  $P_{fp}$  (Ashton, DIJ & Prix 2017).

Found no evidence for decay of wobble angle  $\theta$ ...

...but found  $P_{fp}$  to be decreasing on timescale  $\sim 200$  years!



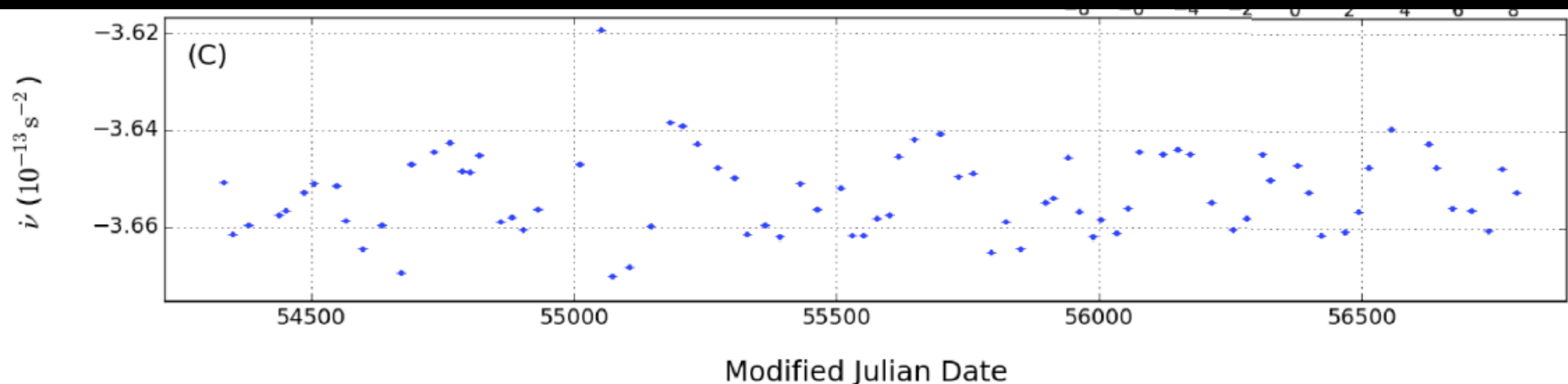
# ...and a glitch

B1828-11 underwent a glitch in 2009.

DIJ, Ashton & Prix (2017) derived consistency requirements between glitch and precession.

Expect  $P_{fp}$  to increase significantly after glitch.

Data suggests otherwise (Brook+ 2016). Problematic!



# Magnetic precession problem

Expect even a purely fluid star to be able to precess, if magnetically deformed.

Relevant to:

1. Main sequence stars
2. Newly-formed neutron stars
  - Core collapse supernovae
  - Binary coalescence

Interesting, as may be important for:

1. Gravitational wave emission (especially the “millisecond magnetar” scenario; Paul Lasky’s talk).
2. Setting the pulsar spin-dipole inclination angle.

# Basic picture

Magnetic field distorts star, giving ellipticity:

$$\epsilon_B \sim \frac{B^2 R^3}{GM^2/R} \sim 10^{-6} \left( \frac{B}{10^{15} \text{ G}} \right)^2$$

This then sets the precession period:

$$P_{\text{fp}} \sim \frac{P}{\epsilon_B} = 10^6 \text{ seconds} \left( \frac{P}{1 \text{ second}} \right) \left( \frac{10^{15} \text{ G}}{B} \right)^2$$

Precession consists of rapid spin of magnetic axis about fixed angular momentum axis...

...plus slow superimposed rotation about magnetic axis.

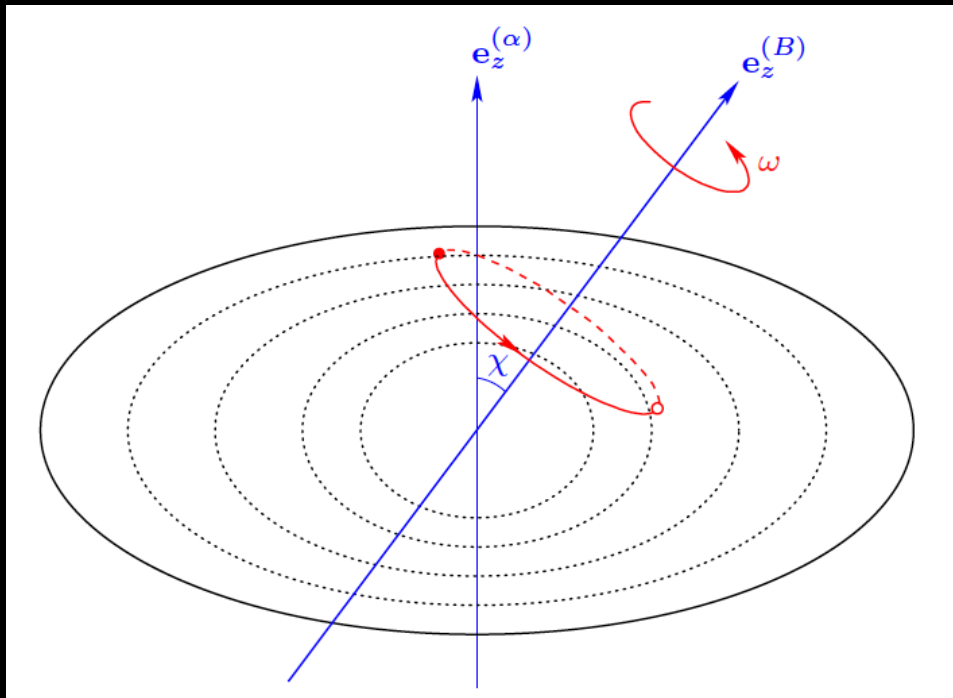


# Non-rigid response

Spin axis traces out a cone of half-angle  $\chi$  as viewed from rotating star.

But spin axis defines centrifugal bulge, of size:

$$\epsilon_{\Omega} \sim \frac{I\Omega^2}{GM^2/R} \sim 0.1 \left( \frac{f}{\text{kHz}} \right)^2$$



$\Rightarrow$  have density wave propagating around star

# Non-rigid response: “xi-motions”

Time-varying density perturbation will induce displacement motion in the stellar fluid, Mestel’s “xi-motions”.

Crucial for calculating dissipation rates (e.g. due to shear & bulk viscosities), and hence evolution in wobble angle.

Must satisfy continuity equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}), \quad \vec{v} = \frac{\partial \vec{\xi}}{\partial t}$$

But this isn’t sufficient to compute  $\xi$ .

# Solution: Lander & DIJ 2017

Take full set of PDEs, and exploit smallness of  $\epsilon_\Omega$  and  $\epsilon_B$ :

$$\rho \frac{d\vec{v}}{dt} = -\nabla P - \rho \nabla \Phi + \frac{1}{4\pi} (\nabla \times \vec{B}) \times \vec{B},$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}),$$

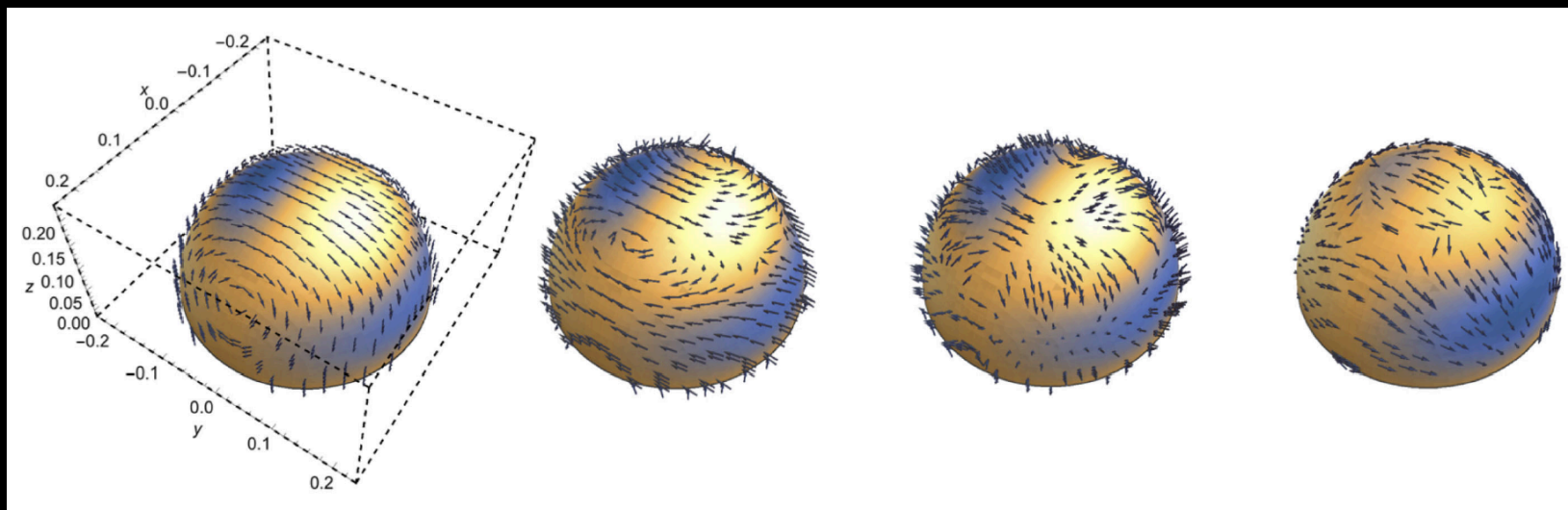
$$\nabla^2 \Phi = 4\pi G \rho,$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}),$$

$$P = P(\rho),$$

$$\nabla \cdot \vec{B} = 0.$$

- Zeroth order: spherical star
- Order  $\epsilon_\Omega$ : rotational deformed star
- Order  $\epsilon_B$ : magnetically deformed star
- Order  $\epsilon_\Omega \epsilon_B$ : xi-motions!



# Application: newly born NSs

Imagine NS born with some non-zero inclination angle.

Set of coupled ODEs:

Temperature:  $\frac{dT}{dt} = -\text{neutrino cooling}$

Spin frequency:  $\frac{d\Omega}{dt} = -EM \text{ spindown}$

Inclination angle:  $\frac{d\chi}{dt} = \pm \text{viscous dissipation} - EM \text{ alignment}$

Oblate ( $\epsilon_B > 0$ ): alignment

Prolate ( $\epsilon_B < 0$ ): counteralignment,  
aka "Mestel-Jones spin-flip"

# Spin-flip with EM torque

For prolate star, get competition between bulk viscosity and EM torque.

Bulk viscosity frequency dependent (Lindblom & Owen 2002):

$$\zeta \sim \frac{\tau}{1 + (\omega\tau)^2}$$

$\tau \sim T^{-6}$ , microphysical reaction rate

$\omega \sim f\epsilon_B \sim fB^2$ , precession frequency

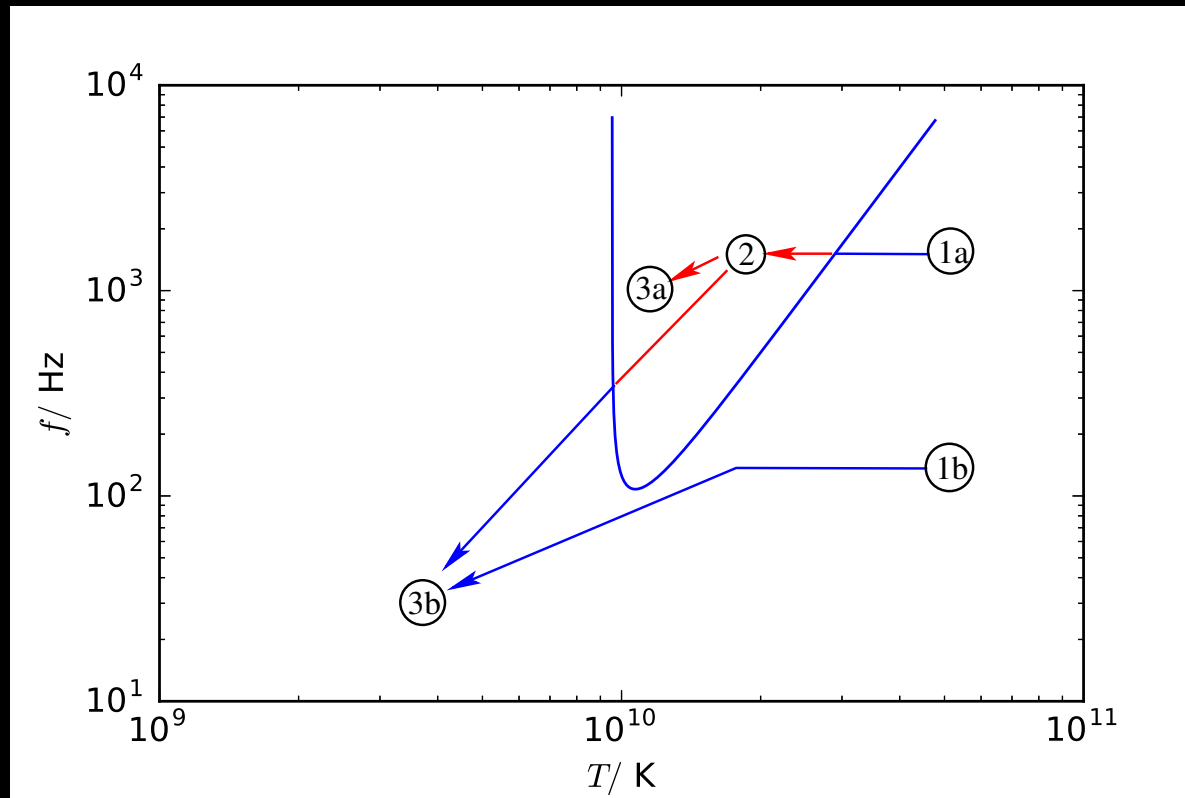


Complex interplay

EM torque  $\sim f^3 B^2$

# “Orthogonalisation” curve

Can use back-of-the-envelope formulae to estimate curve in  $(f, T)$  plane above which spin-flip active:

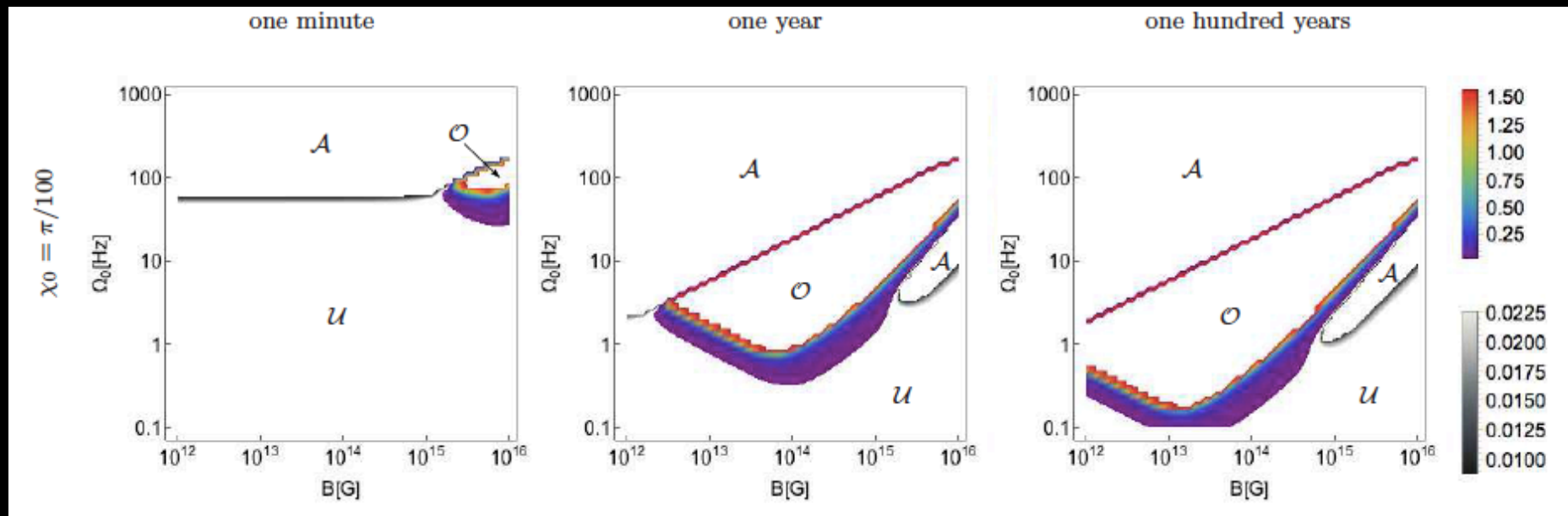


$$B = 10^{16} \text{ G}$$

# Time evolutions

Time evolutions of coupled ODEs show rich structure.

In early stages of interpreting results (Lander & DIJ 2018 in prep).



# Summary

- Free precession potentially unique probe of NS structure...
- ...and possible source of gravitational waves.
- Not clear if active in observed pulsar population.
- Magnetic precession may be important in early life of NS.