Non-radial Oscillations of Strange Quark Stars (Crustal Effects)

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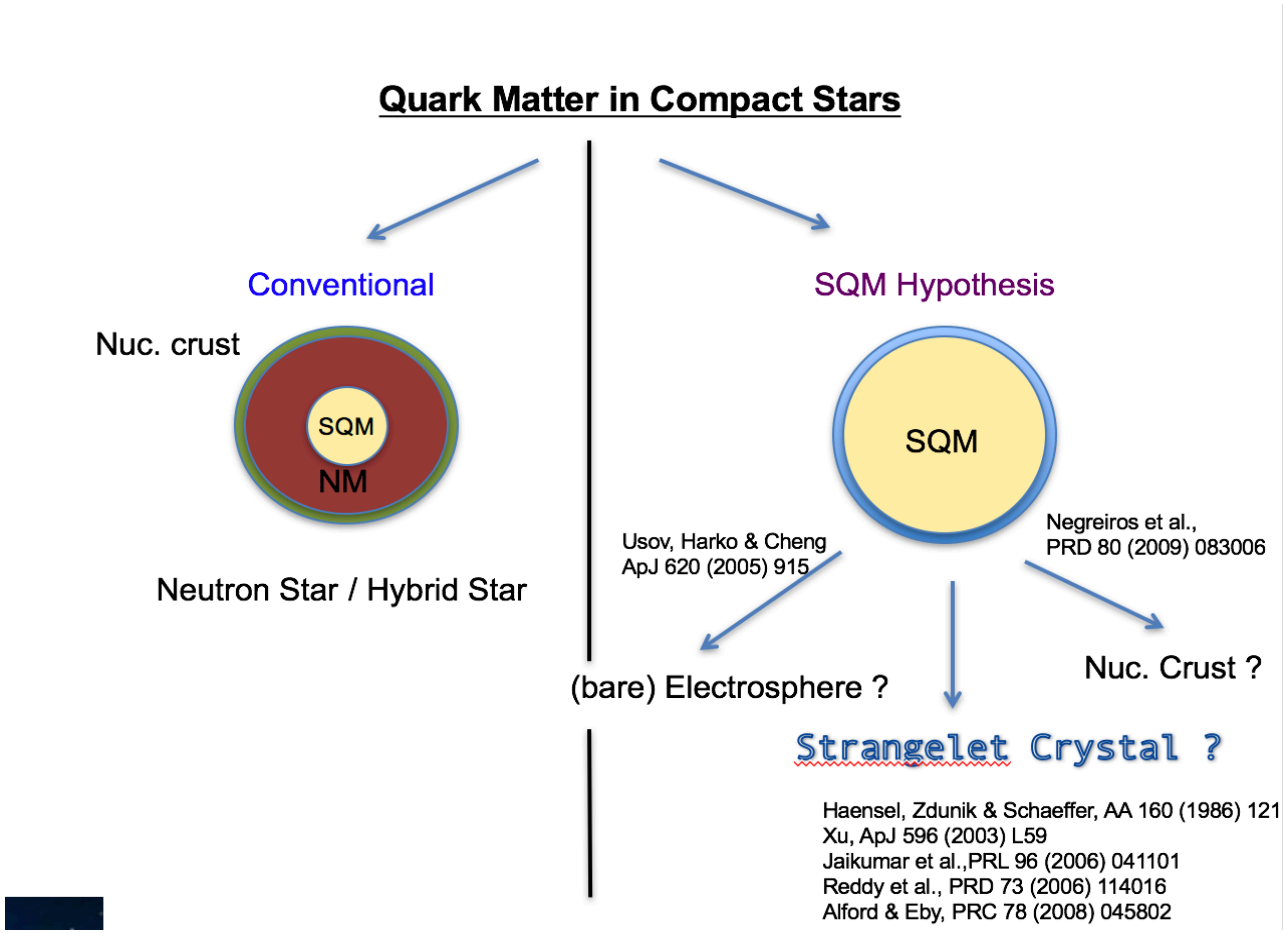
Vasquez-Flores, Hall and Jaikumar, PRC 96, 065803 ('17)

arXiv: 1708.05985 (gr-qc)

Motivation I : Ground State of Strongly Interacting Matter

- Witten: $(E/A)_{uds} < 930$ MeV at zero pressure is possible
- Additional binding due to Color Superconductivity



Motivation II : Gravitational Wave Sources

- Andersson & Kokkotas ('97): Various NS EOS

$$(\sigma_f/\mathrm{kHz}) \approx 0.78 + 1.635 \left(\frac{M}{R^3}\right)^{1/2}$$
$$(\sigma_p/\mathrm{kHz}) \approx \frac{1}{M}\left(1.75 + 5.59\frac{M}{R}\right)$$
$$(\sigma_w/\mathrm{kHz}) \approx \frac{1}{R}\left(20.91 - 9.14\frac{M}{R}\right)$$

- Vasquez-Flores & Lugones ('17): Various SQS EOS

$$(\sigma_f/\mathrm{kHz}) \approx a_1 + a_2 \left(\frac{M}{R^3}\right)^{1/2}$$
$$\left(\frac{M^3\tau_f}{R^4}\right)^{-1} \approx b_1 + b_2 \left(\frac{M}{R}\right)^{1/2} + b_3 \left(\frac{M}{R}\right)$$

Outline

- Computing mode frequencies (different ways)

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- Strangelet Crust
- Results for f, p-mode with Strangelet Crust

Modes (Non-Rotating, Zero-B and Temperature)

- Fluid Displacement (Spheroidal)

$$\xi(r,\theta,\phi,t) = \mathcal{R}\Bigg\{\sum_{lm}\Bigg[\xi_r(r)\,Y_{lm}(\theta,\phi)\hat{r} + \xi_h(r)\left(\frac{\partial Y_{lm}}{\partial\theta}\hat{\theta} + \frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\phi}\hat{\phi}\right)\Bigg]\,\mathrm{e}^{i\omega t}\Bigg\}$$

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- Newtonian for simplicity: (primes = Eulerian perturbations)

$$\begin{aligned}
\text{Continuity}: &\quad \rho' = -\nabla.(\rho_0\xi)\,,\\
\text{Euler}: &\quad \rho_0\,\xi_{tt} = -\nabla\,p' - \rho_0\nabla\phi' - \rho'\nabla\phi_0\,,\\
\text{Poisson}: &\quad \nabla^2\phi' = 4\pi G\rho'\\
\text{Energy}: &\quad p' + \xi.\nabla p_0 = \frac{\Gamma_1 p_0}{\rho_0}(\rho' + \xi.\nabla\rho_0)
\end{aligned}$$

Boundaries

- (Fluid) Center ($r = 0$) : Regularity
  $\implies p', \phi' \sim \mathcal{O}(r^l)\,, \xi_r \sim \mathcal{O}(r^{l-1})$

$$\left(\frac{\rho_{\rm av}}{\rho_c}\right)\frac{\Omega^2 \xi_r}{l} + \frac{p'}{\rho_c g} = 0 \qquad (1)$$

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- (Solid) Interface ($r{=}r_i$) : Traction (T$=\tau.\hat{n}$)) $\implies T_h|_{r=r_c} = 0$

$$\delta\tau = \mu\left(\nabla\xi + (\nabla\xi)^T\right) + \left(\kappa - \frac{2\mu}{3}\right)\mathbb{1}\nabla.\xi \tag{3}$$

Restoring forces

- *(f, p - modes)* : Pressure

$$\sigma^2 \approx k^2 c_l^2 \,; \quad c_l^2 = \Gamma_1 \frac{P}{\rho} + \frac{4}{3}\frac{\mu}{\rho}$$

$$\sigma_{f,p} \sim (1.5 - 8)\mathrm{kHz}$$

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- (*g - modes*): Buoyancy

  $\sigma^2 \approx -Ag$; $\quad A \propto \left[\Gamma_1^{-1} - \Gamma_{\mathrm{eq.}}^{-1}\right]$

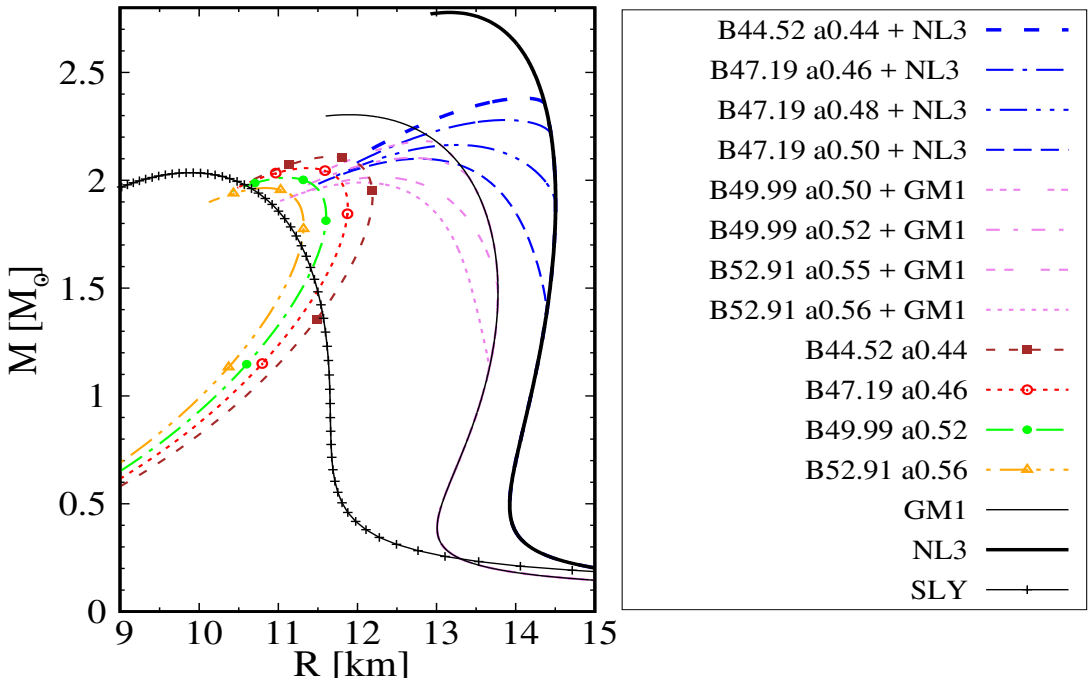
  $\sigma_g \sim (0.1 - 1)$ Hz

- (*s -*): Shear

  $\sigma^2 \approx k^2 c_t^2$; $\quad c_t^2 = \frac{\mu}{\rho}$

  $\sigma_s \sim 100$ Hz ; $\quad \xi_h^c \gg \xi_r^c$

Neutron Stars / Strange Stars - Core EOS



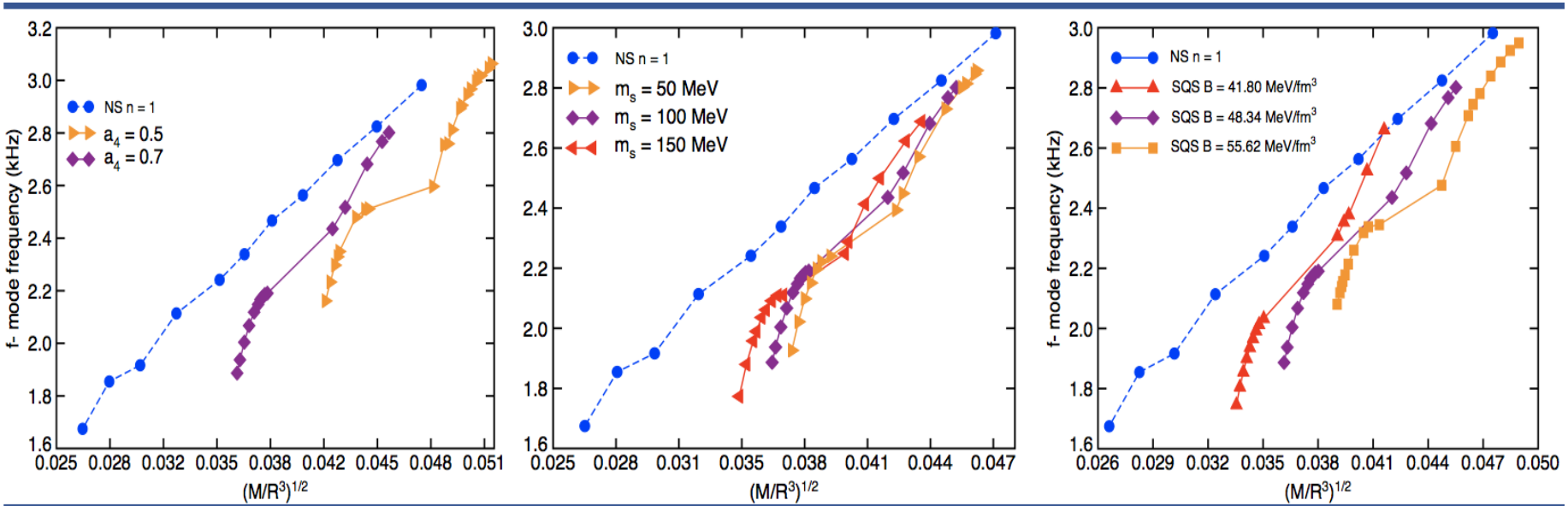
Quark Matter EOS (Bag + a_4)

$$P_{q,\mathrm{core}} = \frac{1}{3}(\epsilon - 4B) - \frac{m_s^2}{3\pi}\sqrt{\frac{\epsilon - B}{a_4}} + \frac{m_s^4}{12\pi^2}\left[2 - \frac{1}{a_4} + 3\ln\left(\frac{8\pi}{3m_s^2}\sqrt{\frac{\epsilon - B}{a_4}}\right)\right]$$

Hadronic EOS (SLy)

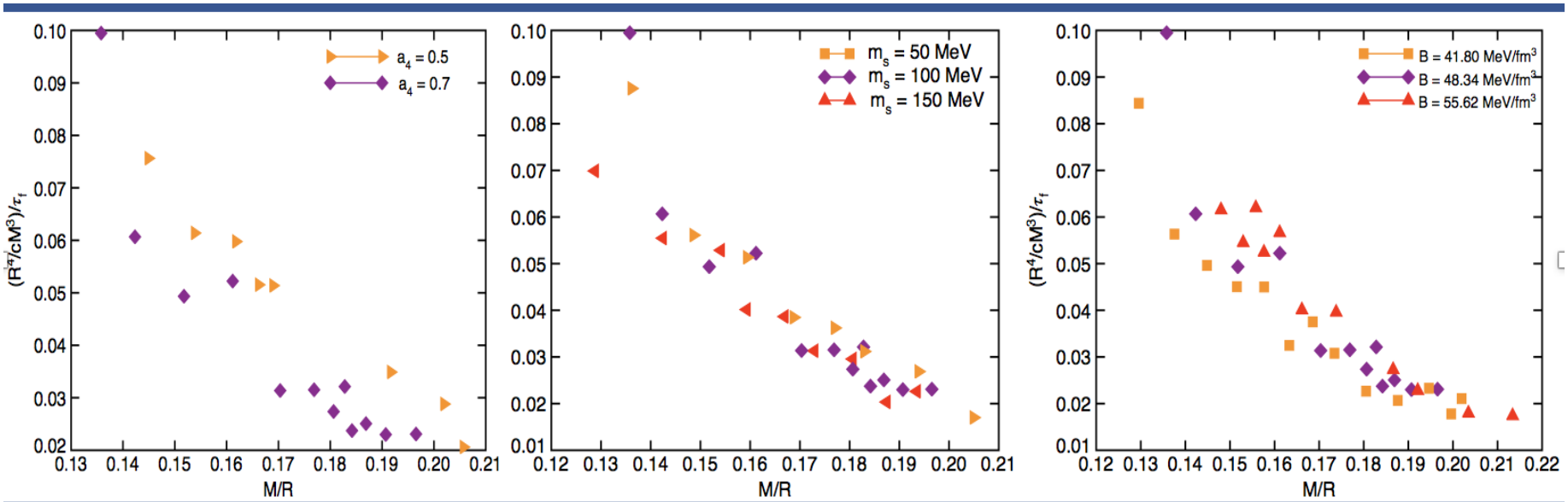
$$10^5 \leq \rho(\mathrm{g/cc}) \leq 10^8\,; \quad \text{BPS}$$
$$10^8 \leq \rho(\mathrm{g/cc}) \leq 5.10^{10}\,; \quad \text{HP}$$
$$5.10^{10} \leq \rho(\mathrm{g/cc}) \leq \rho_c\,; \quad \text{SLy}$$

f-mode Frequency (GR)



QCD Interaction Parameter | Strange Quark Mass | Bag Constant

f-mode Damping time (GR)



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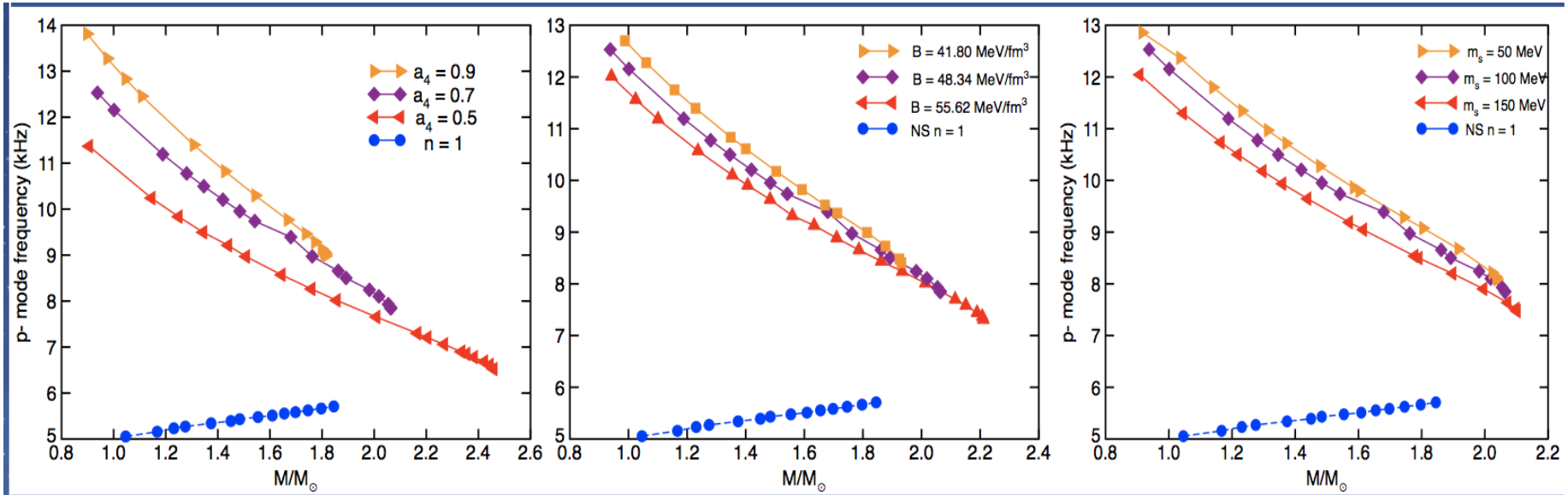


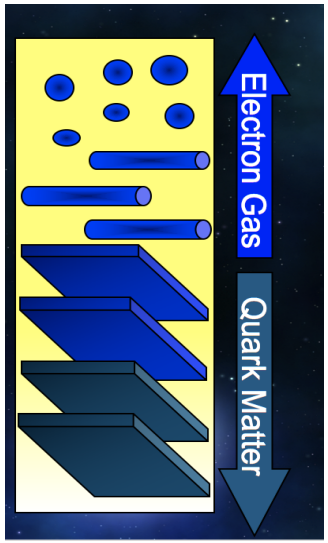
The Strangelet Crust

Globally neutral mixed phase of quarks and electrons*

$$P_{\text{strangelet}} = P_{\text{outside}} \implies P_q(\mu_q, \mu_e) = 0$$

$$P_{\text{crust}}(r) = \frac{\tilde{\mu}_e(r)^4}{12\pi^2}$$
$$\epsilon_{\text{crust}}(r) = f(r)\epsilon_0$$

($\tilde{\mu}_e, f, \epsilon_0$ calculated from Quark matter EOS)



*Jaikumar, Steiner, Reddy: PRL96, 041101 ('06)
Alford, Rajagopal, Reddy, Steiner: PRD73, 114016 ('06)
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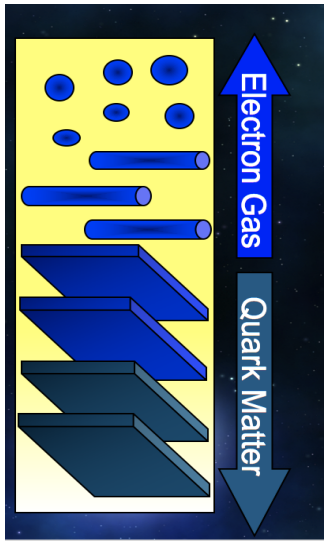

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▶

μ_{crit}(MeV)	m_s(MeV)	Z/A	R_*(fm)	ΔR(m)	Shear (keV/fm^3)
280	100	0.039	8.32	23.01	1.04
290	150	0.079	7.91	94.44	87.5
300	200	0.125	7.87	236.66	1699

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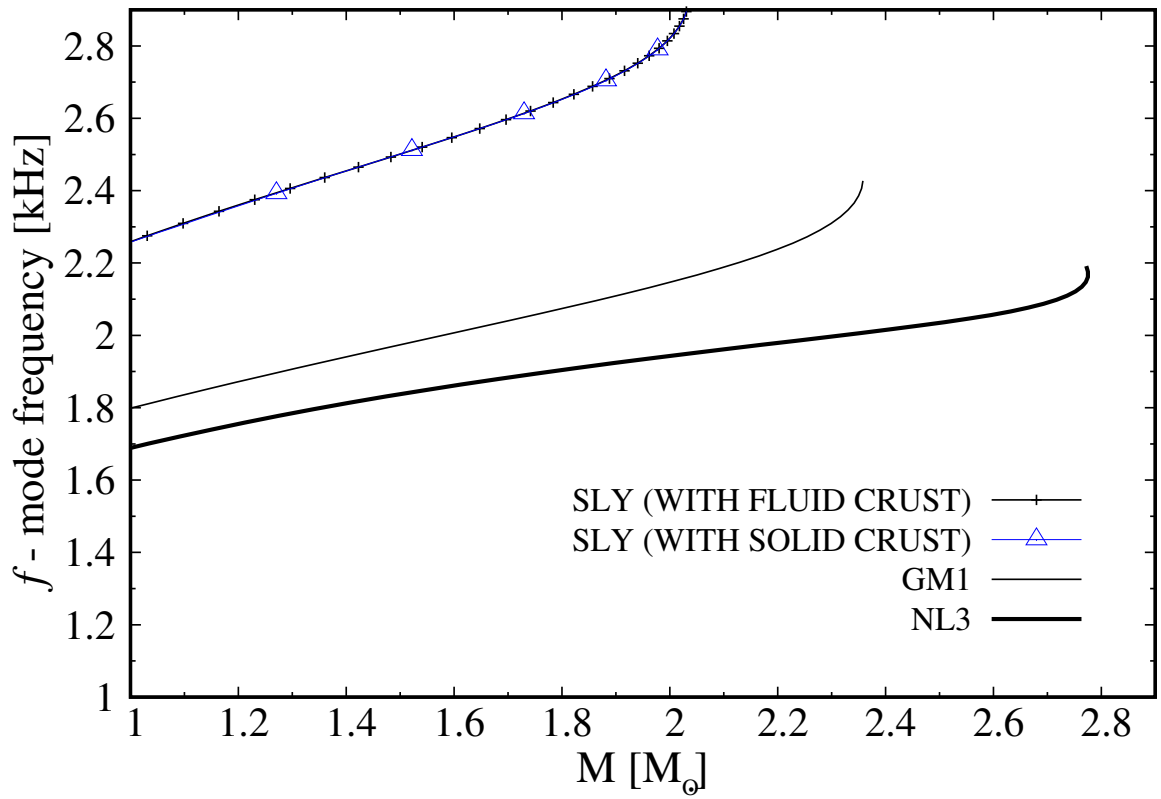
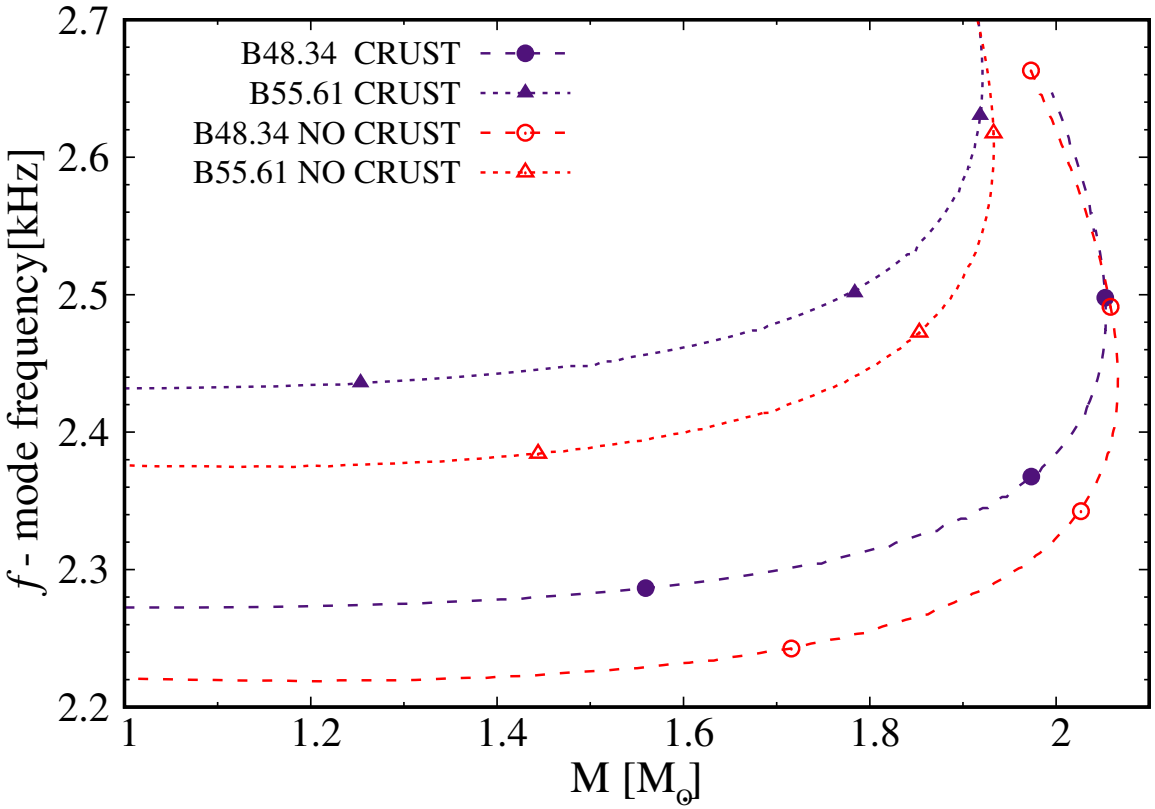
f-mode : Neutron Stars with Crust



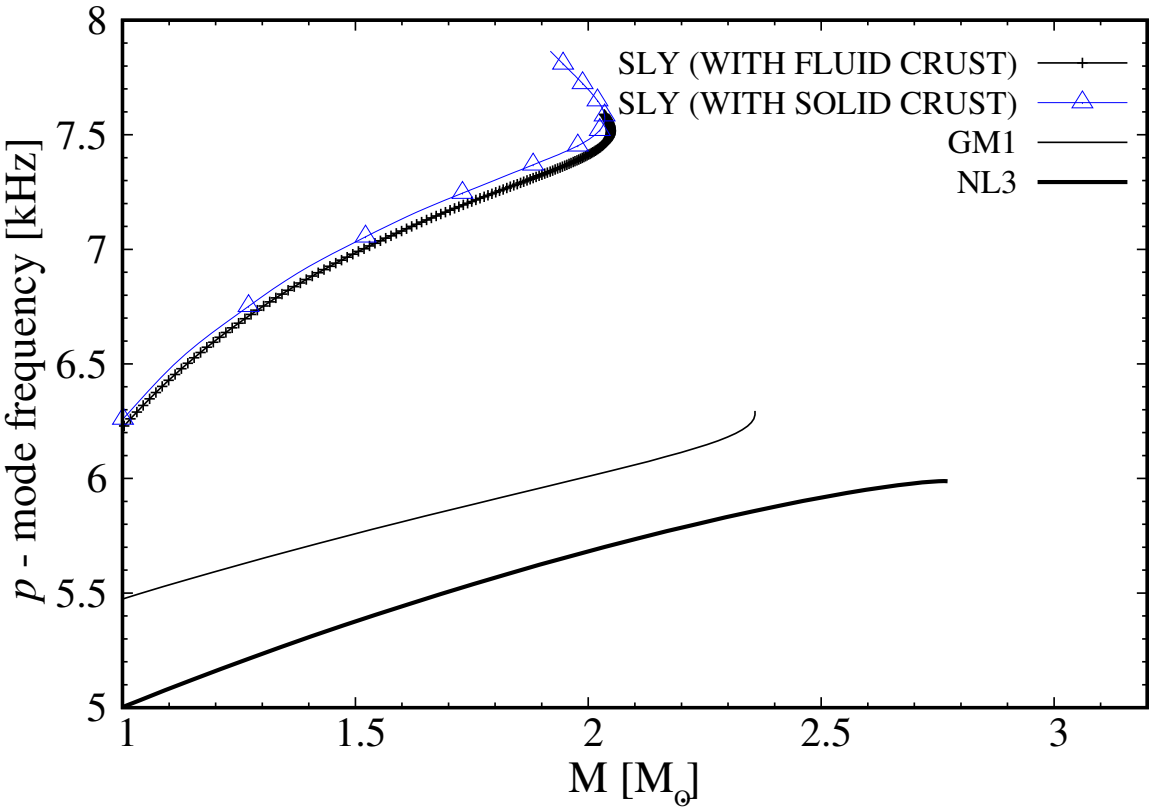
- No discernible impact of crust on f-mode frequency (σ_f)
- $2.45 \leq \sigma_f(\mathrm{kHz}) \leq 2.90$ for $1.4 \leq M/M_\odot \leq 2.0$
- In range of LIGO, cryogenic resonant detectors (MINI-GRAIL, Schenberg).

f-mode : Strange stars with Strangelet Crust



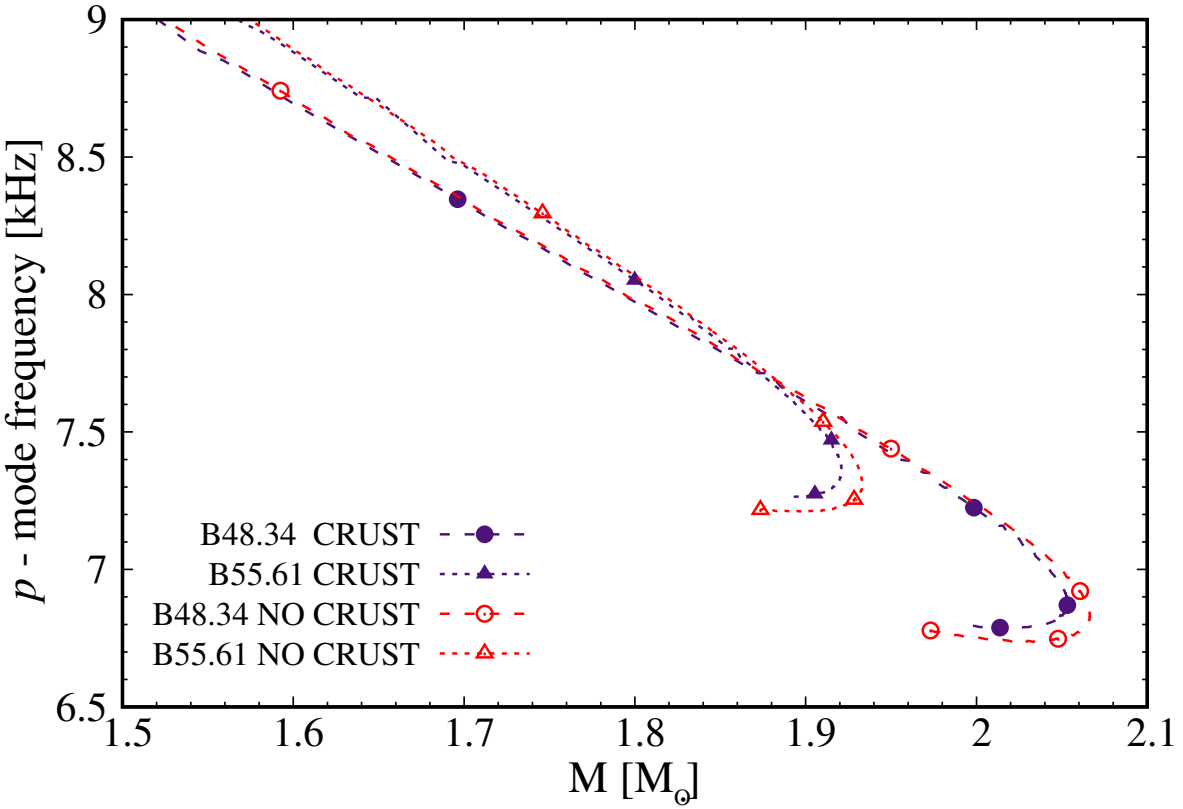
- σ_f approximately constant until $1.5 M_\odot$
- Strangelet crust *increases* σ_f by ~ 200 Hz
- Effect persists if strangelet crust$\rightarrow$ thick hadronic crust

p-mode : Neutron Stars



- $6.0 \leq \sigma_p(\mathrm{kHz}) \leq 7.5$ for SLy EOS
- Hadronic crust *slightly increases* σ_p (measure of μ/κ)

p-mode : Strange Stars



- σ_p trend with increasing mass opposite to hadronic stars
- Strangelet crust does not discernibly change σ_p ($\mu_{\text{strangelet}} \ll \mu_{nuclear}$)

Discussion

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  i-mode couples to tidal fields, achieves pre-merger resonance
  strangelet crust may shatter easier than nuclear crust.
  (Tsang et al., PRL 108, 011102 (2012))

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- Ideas : Strangelet Dwarfs, Thorne-Żytkow Objects?

Detectability of f,p-Modes

Andersson & Kokottas ('97):

$$h_{\rm eff} \sim 10^{-21} \left(\frac{E}{10^{-6} M_\odot c^2} \right)^{1/2} \left(\frac{2{\rm kHz}}{\sigma_f} \right)^{1/2} \left(\frac{50{\rm kpc}}{r} \right)$$

Local Group : $\sim 10^{-6}\%$ mass energy
Virgo cluster : $\sim 1\%$ mass energy

Relativistic Extension: Cowling Approximation

Yoshida & Lee, A&A 395, 201 ('02)

- Stress tensor for elastic solid

$$
\begin{aligned}
T_{\alpha\beta} &= \rho u_\alpha u_\beta + p q_{\alpha\beta} - 2\mu\Sigma_{\alpha\beta} \\
q_{\alpha\beta} &= g_{\alpha\beta} - u_\alpha u_\beta / c^2 \\
\Sigma_{\alpha\beta} &= e^\nu \int_0^t \sigma_{\alpha\beta} dt'
\end{aligned}
$$

- Relativistic Euler equation

$$\sum_\alpha (\rho + P) u^\alpha u_{\beta;\alpha} = \partial_\beta P - \sum_\alpha u_\beta u^\alpha (\partial_\alpha P) - \sum_\alpha 2\mu \Sigma^\alpha_{\beta;\alpha}$$

- Boundary conditions are physically the same