Crust breaking strength and durability

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INT Workshop INT-18-71W
Astro-Solids, Dense Matter, and Gravitational Waves
Seattle, April 16 - 20, 2018

Introduction: motivation

Today's key questions:

- What is the minimum, typical, and maximum ellipticity one should expect?

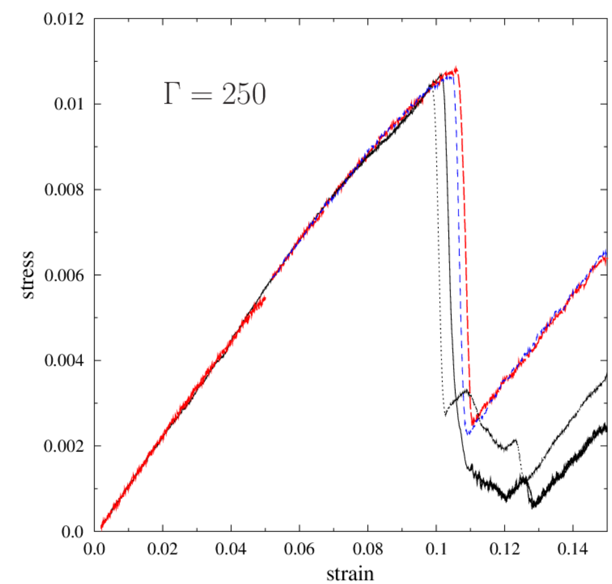
What is the strength (breaking strain) of the crust?

- How does strain evolve in the crust and how does it break?

- What are the implications of observed upper limits on the ellipticity?

- At what point do upper limits become "interesting" (e.g. constrain theory)?

- What are possible mountain building mechanisms (such as asymmetric accretion, temperature gradients, magnetic stress...)?



Typical stress-strain curves
[MD simulations]

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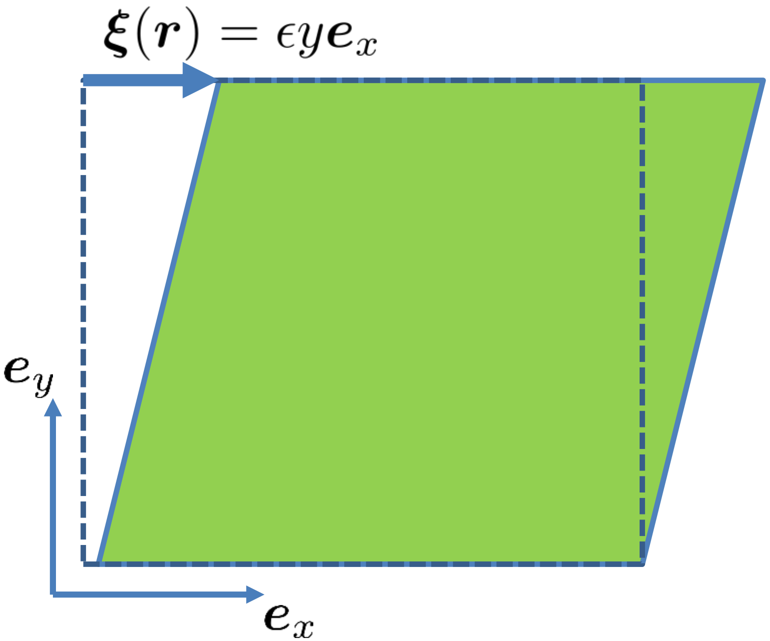
Introduction: Theory of Elasticity



Deformation:

$$\boldsymbol{r} \to \boldsymbol{r} + \boldsymbol{\xi}(\boldsymbol{r})$$

Strain tensor

$$u_{ik} = u_{ki} = \frac{1}{2}\left(\frac{\partial \xi_i}{\partial x_k} + \frac{\partial \xi_k}{\partial x_i}\right)$$

Forces

(to volume element from nearby elements)

$$F_i = \frac{\partial \sigma_{ik}}{\partial x_k}$$

In the crust

$$\sigma_{ik} = -P\delta_{ik} + \Delta\sigma_{ik}$$

EOS

(electrons+neutrons+undeformed lattice)

Elastic part

$\Delta\sigma_{ik}(u_{ik})$

Elastic part of stress tensor

Microphysics: $$\Delta\sigma_{ik} = \Delta\sigma_{ik}(u_{ik}, \ldots)$$

$$\boxed{\Delta\sigma_{ik} \lesssim 10^{-3} P_e}$$

For infinitesimal deformations $$\Delta\sigma_{ik} = \lambda_{iklm} u_{lm}$$

Elastic coefficients (up to 21)

For cubic crystal 3 independent coefficients (2 for isotropic material):

$$c_{11} = \lambda_{xxxx}, \quad c_{12} = \lambda_{xxyy}, \quad c_{44} = \lambda_{xyxy}$$

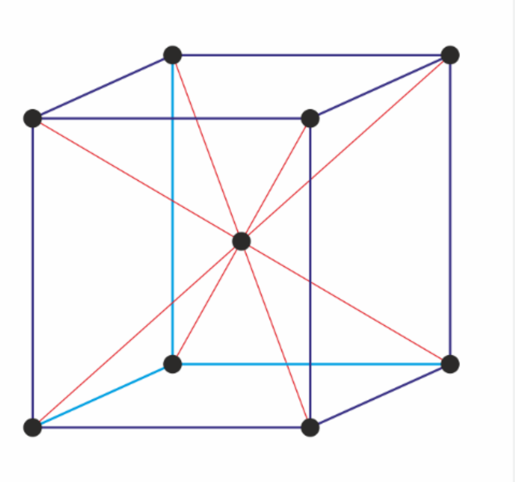
Basic model and scaling

[one component crust – all ions are equal]

$$\Delta\sigma_{ik} = \frac{Z^2 e^2}{a} n\, \Delta\tilde{\sigma}_{ik}\left(u_{ik}, \Gamma = \frac{Z^2 e^2}{aT}, \frac{T}{\hbar\omega_P}, a k_{\mathrm{TF}}, \ldots\right)$$

Point charges, TF screening: $a = (4\pi n/3)^{-1/3}$

Ions form BCC lattice



(Arbitrary) uniform compression/expansion does not lead to breaking
(but can initiate to nuclear reactions)

Beyond scope: Realistic (Jancovici 1962) electron screening [Baiko 2002]
Effects of free neutrons: induced interactions [Kobyakov&Pethick 2016]

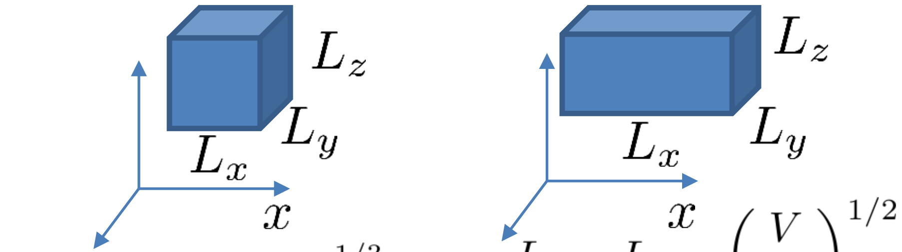
Small deformations: Monocrystal

Fuchs (1936) (neglecting screening):

Very strong anisotropy

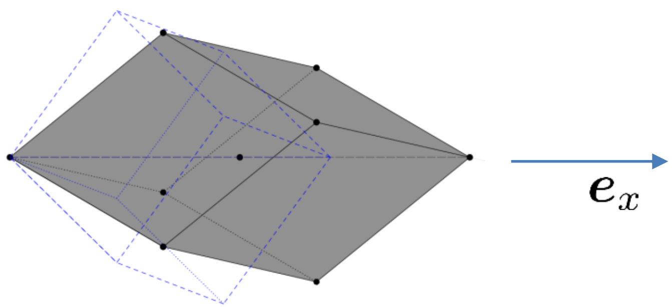
Volume conserving tension:

$$u_{xx} = \epsilon + \frac{3}{2}\epsilon^2$$
$$u_{yy} = u_{zz} = -\frac{1}{2}\epsilon$$



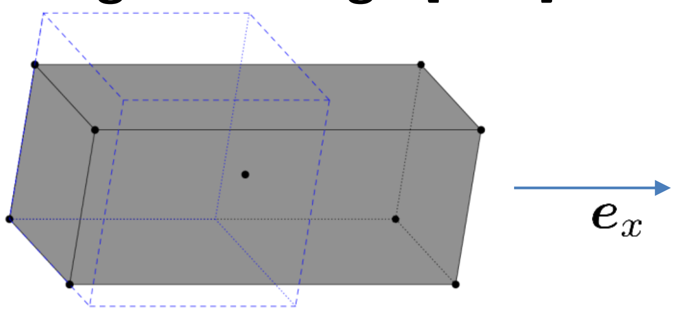
$$L_x = L_y = L_z = V^{1/3}$$
$$L_y = L_z = \left(\frac{V}{L_x}\right)^{1/2}$$

Along cube diagonal [111]



$$\Delta\sigma_{xx} = 2\,c_{44}\epsilon \approx \boxed{0.366\epsilon}\frac{Z^2e^2}{a}n$$
$$\Delta\sigma_{yy} = \Delta\sigma_{zz} = -\frac{1}{2}\Delta\sigma_{xx}$$

Along cube edge [100]

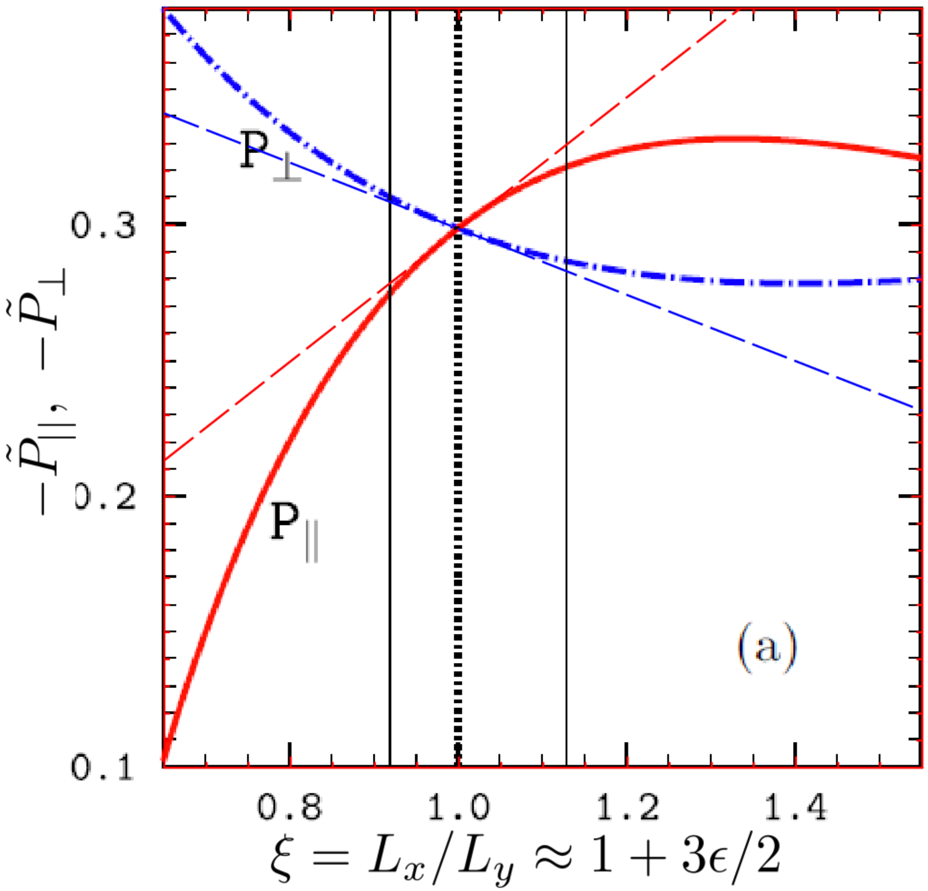


$$\Delta\sigma_{xx} = (c_{12} - c_{11})\epsilon \approx \boxed{0.049\epsilon}\frac{Z^2e^2}{a}n$$
$$\Delta\sigma_{yy} = \Delta\sigma_{zz} = -\frac{1}{2}\Delta\sigma_{xx}$$

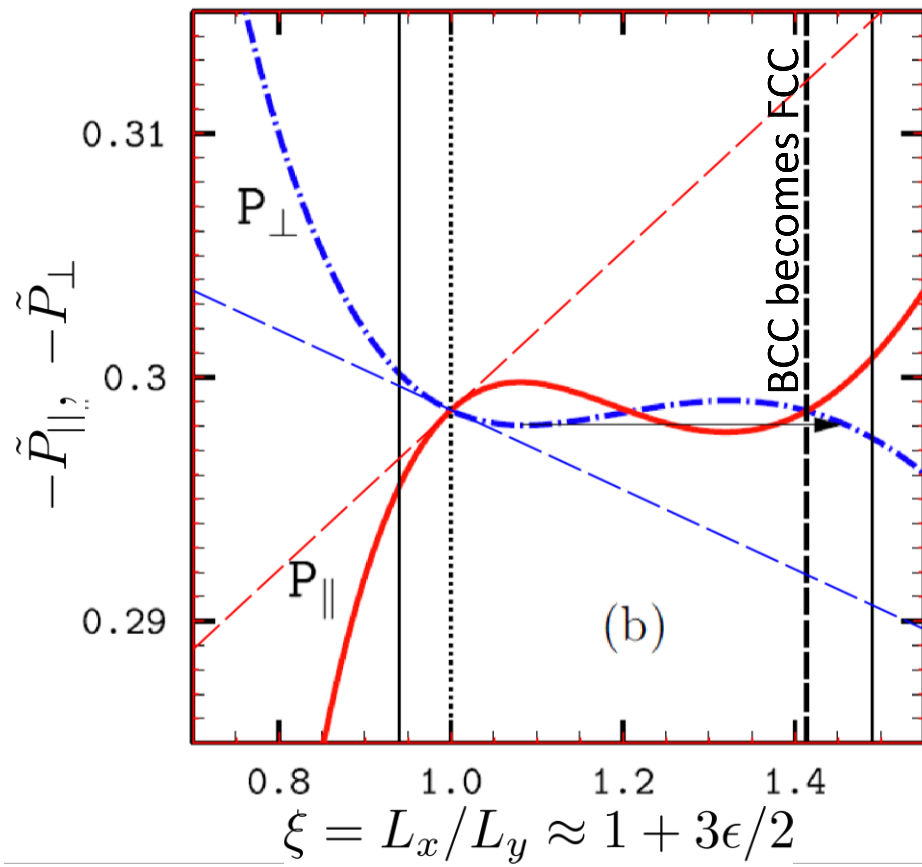
Large deformations: Monocrystall

[Baiko&Kozhberov 2017]

Along cube diagonal [111]



Along cube edge [100]



Stress is strongly nonlinear (at certain directions)

Breaking stress is anisotropic (note the difference in scales)

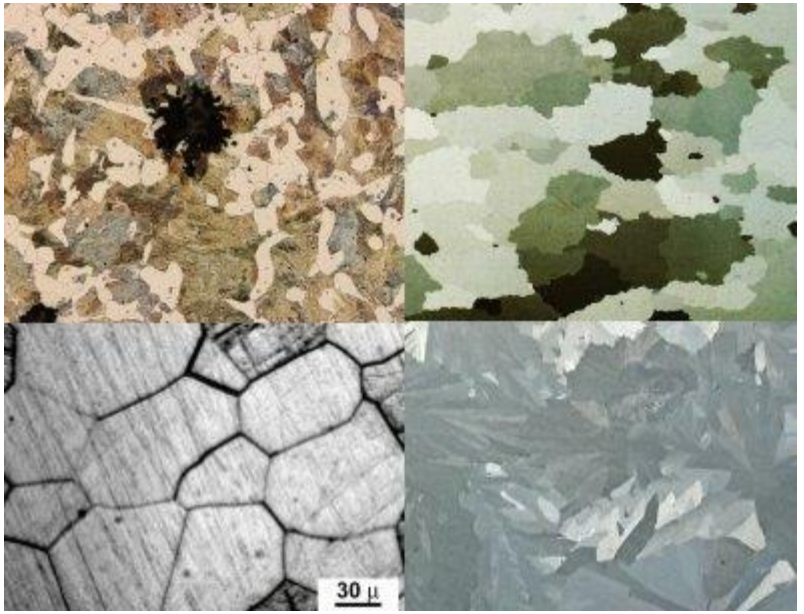
Small deformations: Polycrystal

Polycrystalline matter, made of large number of single crystals, should be isotropic.

$$\Delta\sigma_{ij} = 2\mu_{\mathrm{eff}}\left(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}\right) + K u_{ll}\delta_{ik}$$

Shear modulus

Compression modulus



[From Wikipedia]

Landau&Lifshitz, vol. 7: “The relation between the elastic properties of the whole crystal and those of components crystallites depends on actual form of the latter and the amount of correlation of their mutual orientation.”

Ogata&Ichimaru (1990):
(averaging velocity of shear waves)

$$\mu_{\mathrm{eff}}^{\mathrm{av}} = 0.120\frac{Z^2e^2}{a}n$$

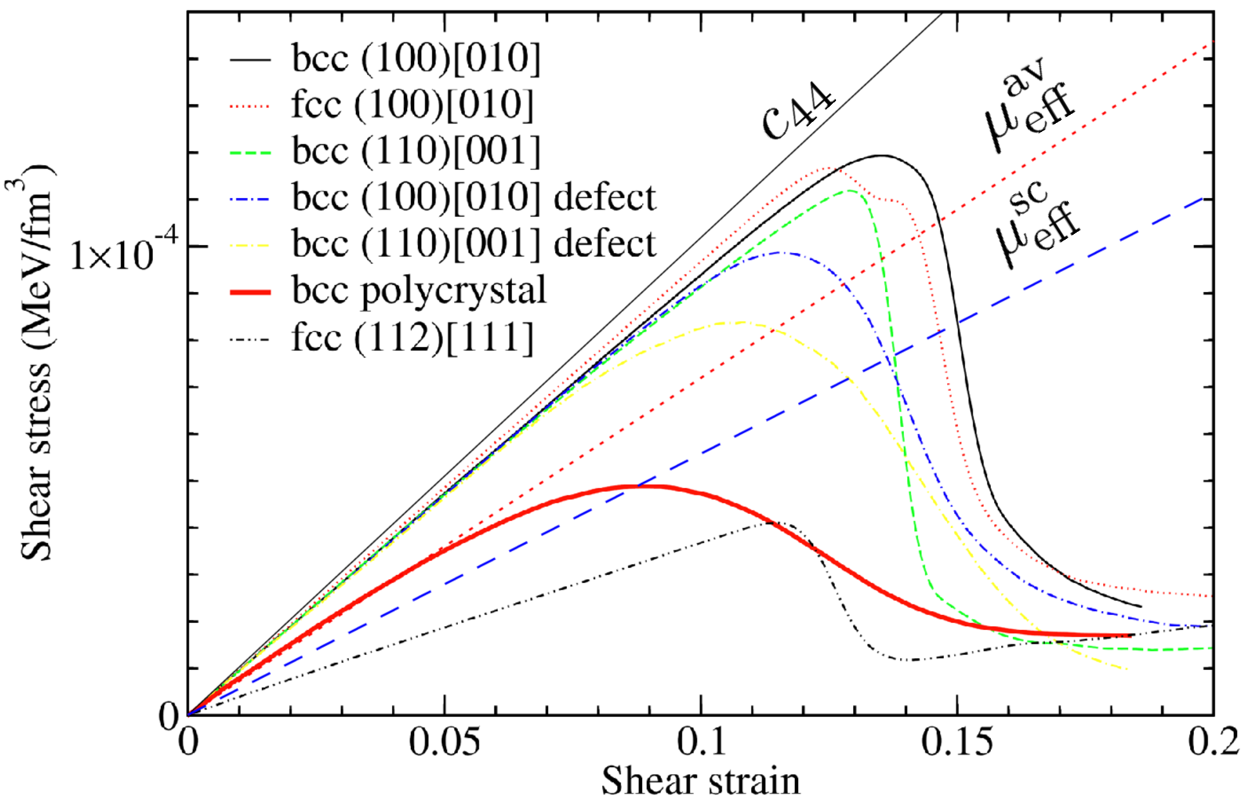
Kobyakov&Pethick (2015):
(self consistent theory by Eshelby 1961)

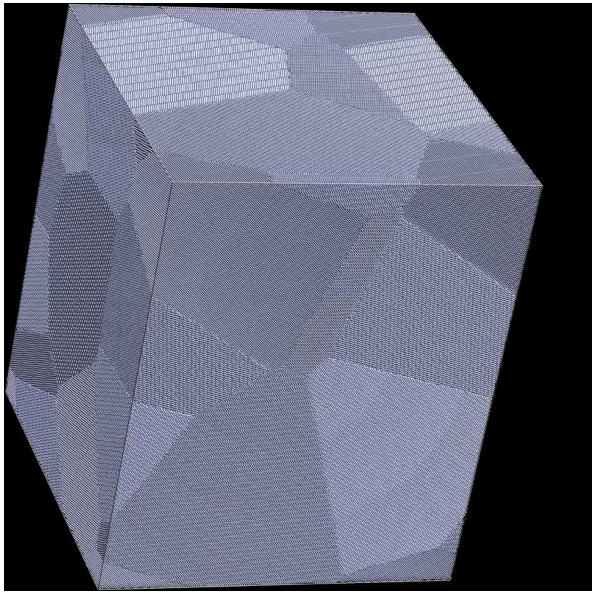
$$\mu_{\mathrm{eff}}^{\mathrm{sc}} = 0.093\frac{Z^2e^2}{a}n$$

28% difference

Small deformations: Polycrystal

Numerical experiment: MD simulations by Horowitz&Kadau (2009) for shear deformations



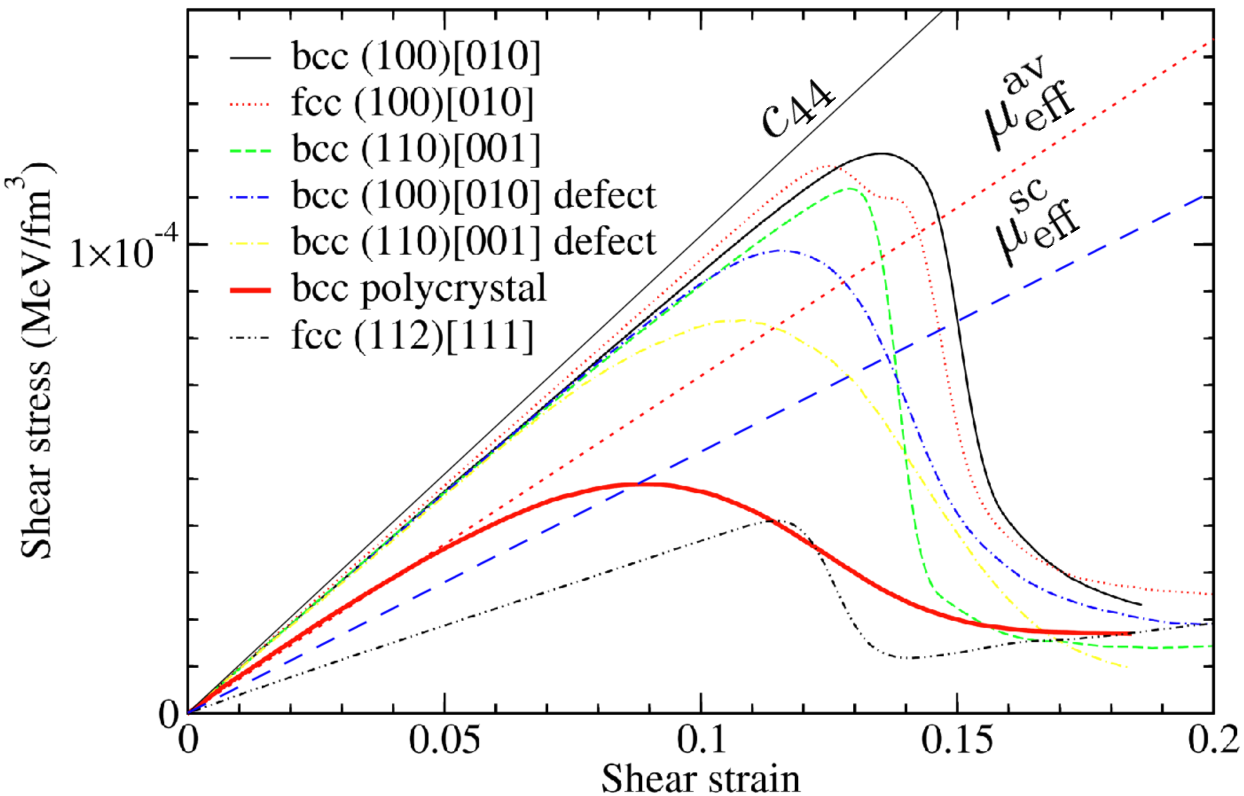


Movie and original figure from Horowitz&Kadau (2009)

Shear stress is almost linear for monocrystal
Effective shear modulus is rather uncertain

Breaking strain&stress

Numerical experiment: MD simulations by Horowitz&Kadau (2009)
[shear deformations]



Fractures and voids does not appear (Horowitz&Kadau, 2009)

Monocrystal

Breaking strain:

$$\epsilon_{\rm b} \sim 0.1$$

Breaking stress:

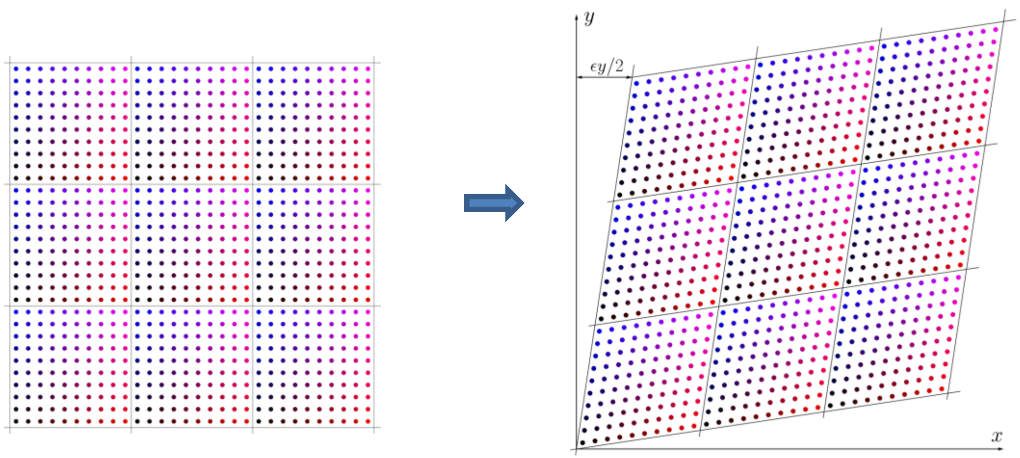
$$\Delta\sigma_{\rm b} \sim c_{44}\epsilon_{\rm b}$$

Polycrystal: breaking strain and stress are moderately reduced

Durability of the crust

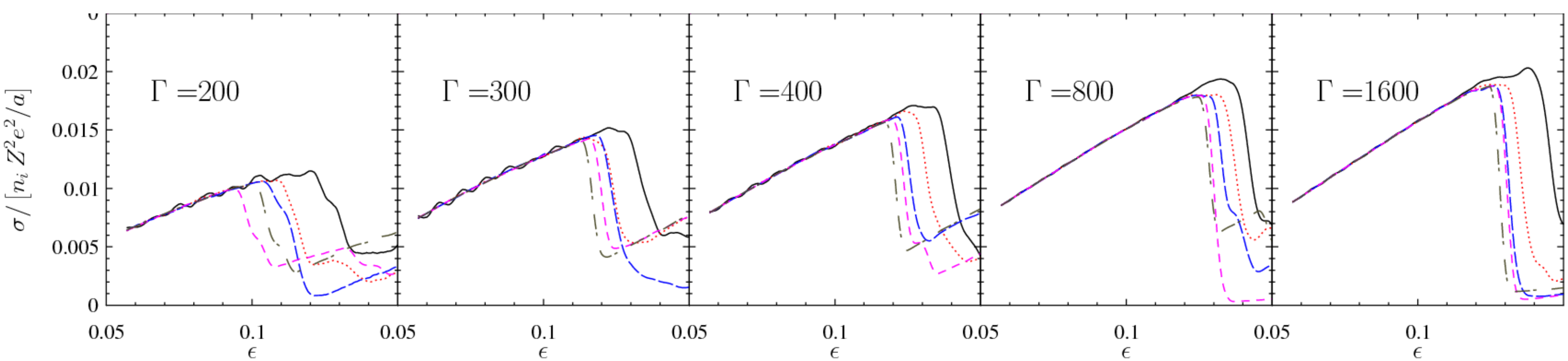
How long crust can support the stress?

MD simulations AC&Horowitz (2010, 2012)



Time-dependent periodic boundary conditions

$$x \rightarrow x + \epsilon y/2$$
$$y \rightarrow y + \epsilon x/2$$
$$z \rightarrow z/(1-\epsilon^2/4)$$
$$\epsilon = vt$$



Breaking stress depends on the temperature and strain rate

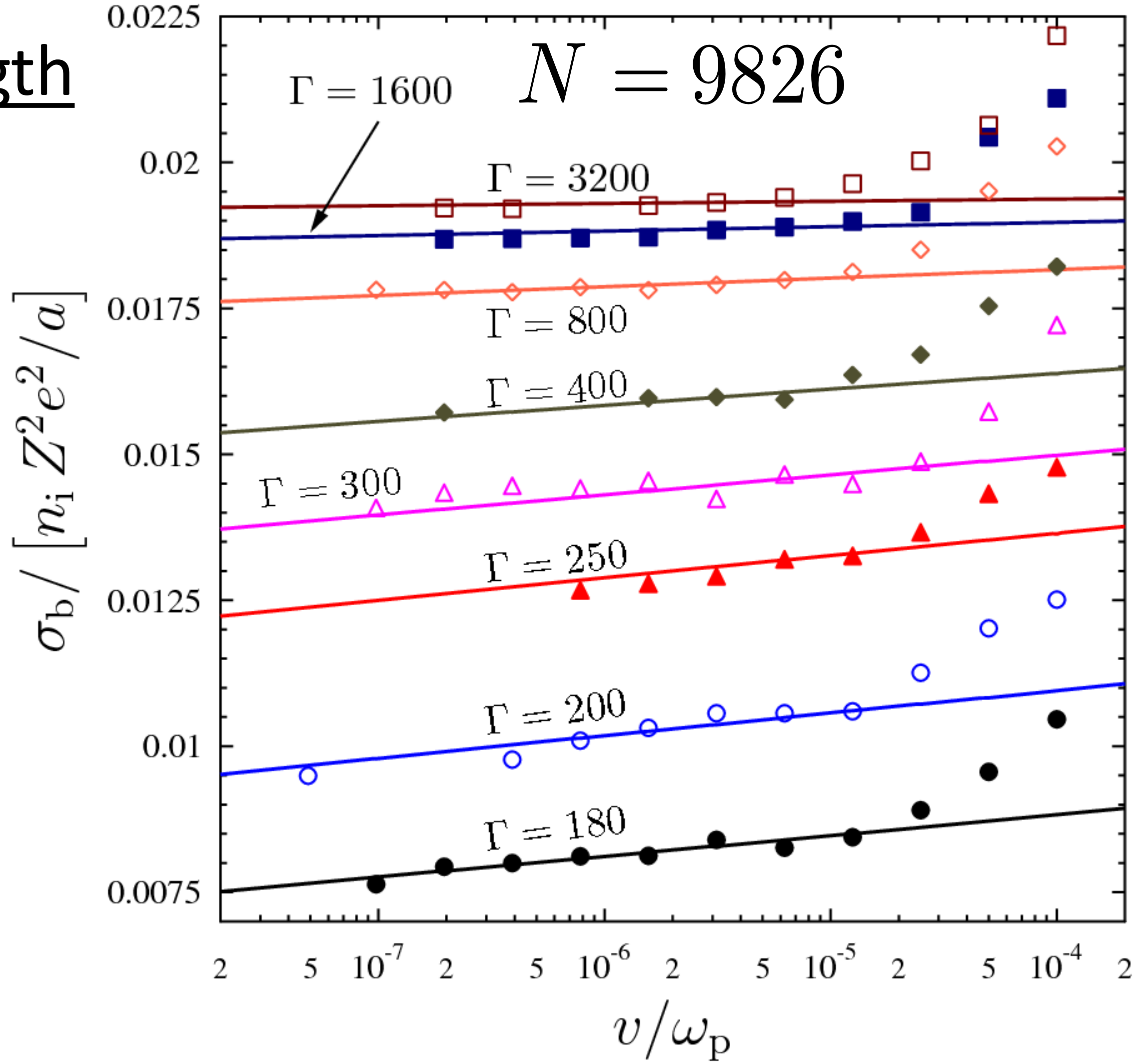
Durability of the crust

Kinetic theory of strength

Zhurkov (1957); Regel et al. (1972)

Stress doesn't cause breaking as it is, but breaking is result of thermal fluctuations. The stress reduces fluctuation energy

$$\tau = \boxed{\tau_0} \exp\left(\frac{\boxed{U} - \sigma \boxed{V}}{T}\right)$$



Parameters to be fited to [numerical] experiment

Figure from AC&Horowitz (2010, 2012)

Stress vs durability

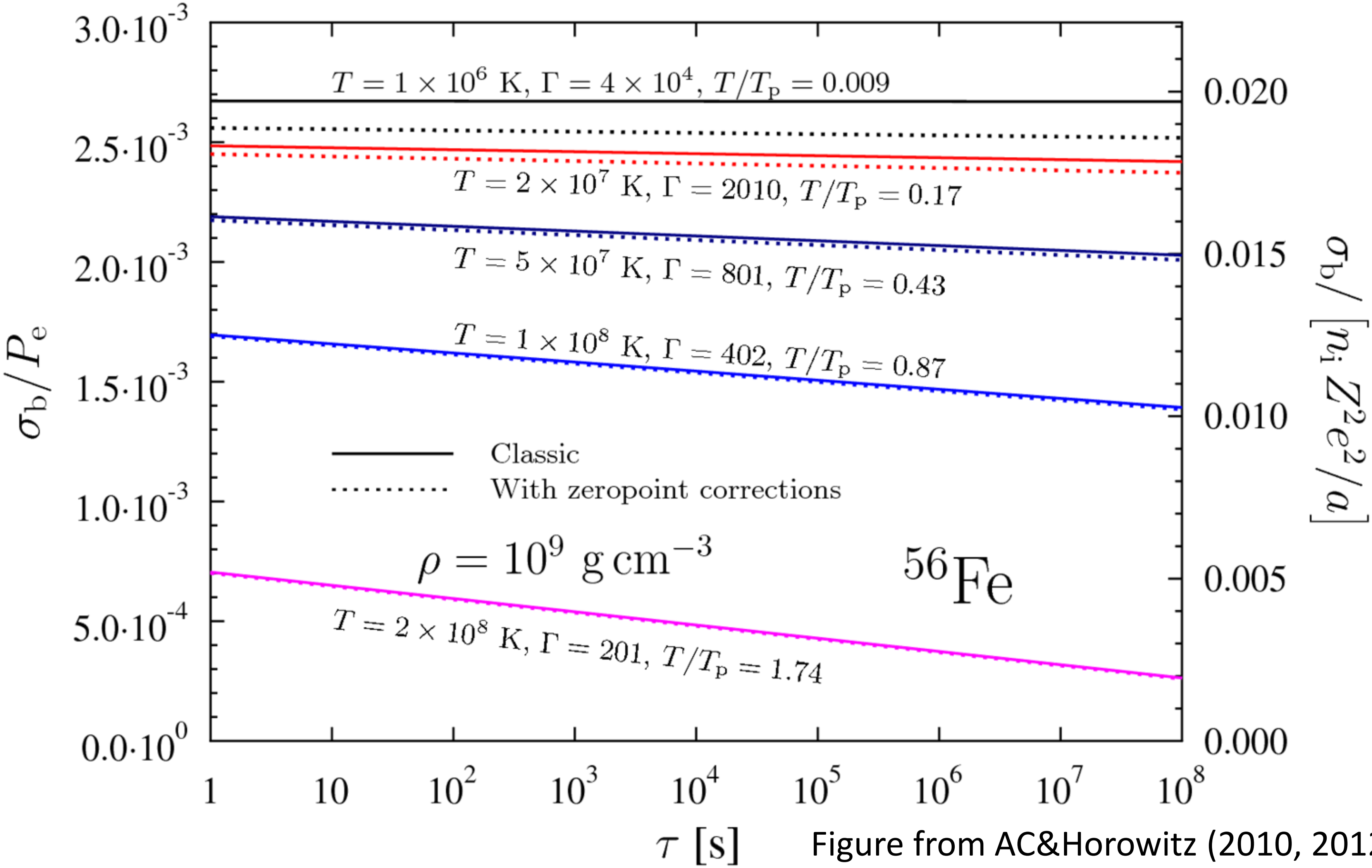


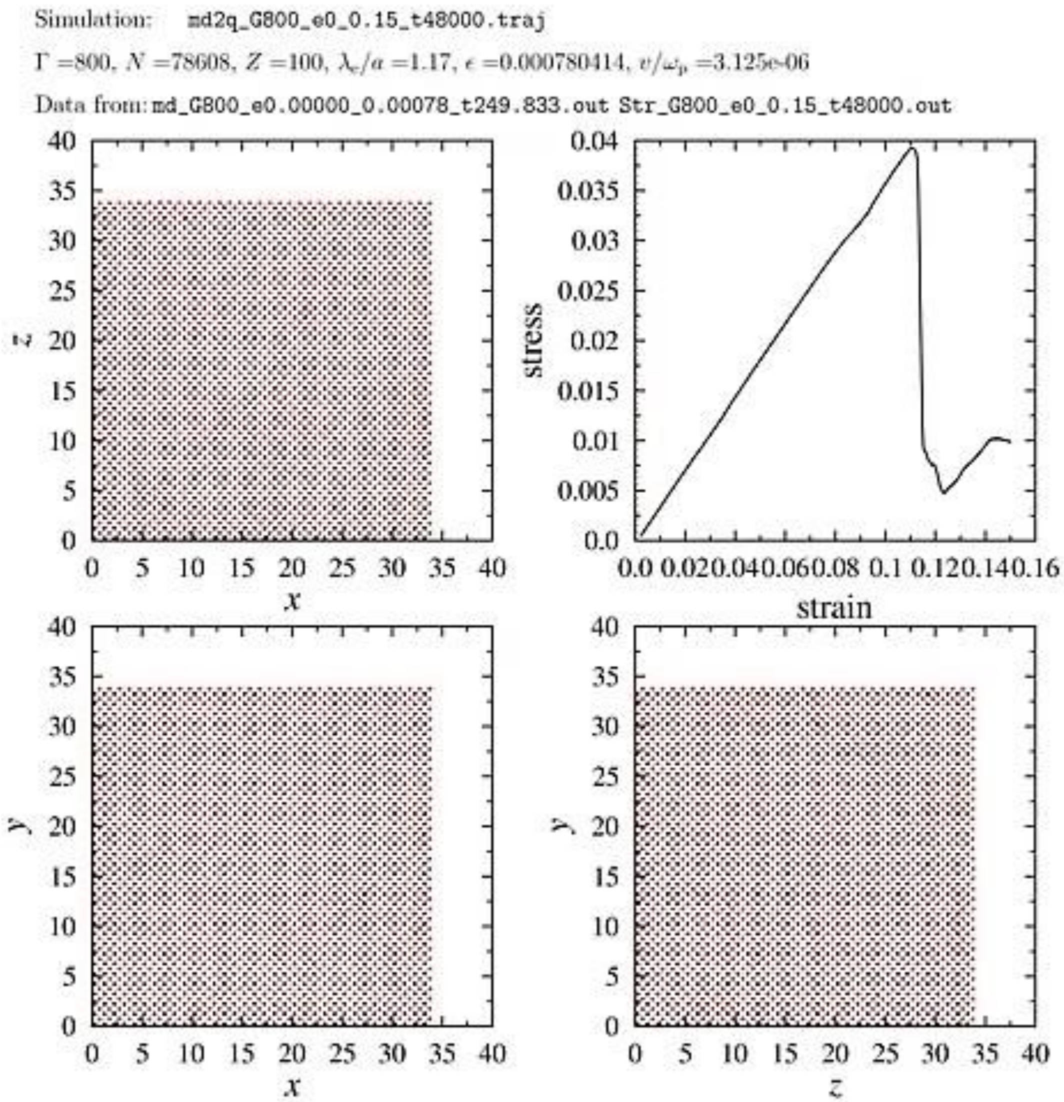
Figure from AC&Horowitz (2010, 2012)

$$\sigma_b \sim 0.01\frac{Z^2e^2}{a}n \sim 10^{-3}P_e$$

Durability at given stress rapidly decrease as temperature increases

How does the crust break?

In MD simulations



In NS

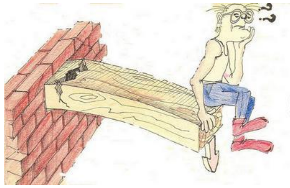
Plastic deformation?

Bingham model

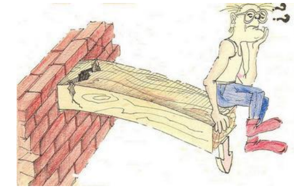
$$v \propto (\sigma - \sigma_b)/\nu$$

Very uncertain

It is worth to catch it by MD simulations



Summary



1. The elastic properties of crystal monocrystals are known. They are anisitropic and nonlinear. Elastic properties of polycrystalline matter are rather uncertain (~28% difference).
2. Uniform compression/expansion does not lead to breaking
3. MD simulations (Horowitz&Kadau, AC&Horowitz, Hoffman&Heyl) suggest that the crust breaks at shear strain ~0.1, corresponding to stress $\Delta\sigma_{\rm b} \sim 0.01 \frac{Z^2 e^2}{a} n \lesssim 10^{-3} P_e$
4. Durability of the crust is a strong function of the stress and decreases with increase of temperature. It can lead to thermoplastic instability (Levin&Beloborodov 2014)
5. The real durability can hardly exceed extrapolation of MD simulations [AC&Horowitz 2010,2012], but I would not be strongly surprised, if in fact it is lower (weakness along edge?)

Short answers to the Bob's question