Solid quark matter

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Outline

I Where can solid quark matter occur?
Hybrid Stars or Strange Stars

II Hybrid stars
Crystalline color-superconducting quark matter ("LOFF")
Mixed phase ("pasta") at nuclear-quark boundary
Chiral density wave / DGR / quarkyonic chiral spiral [not in this talk]

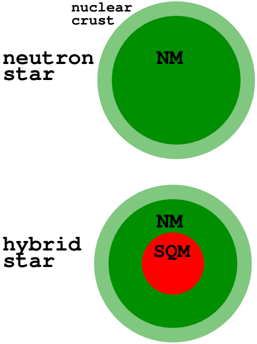
III Strange Stars
Strangelet crystal crust (another mixed phase)

IV Where next?

Quark matter in compact stars

Conventional scenario

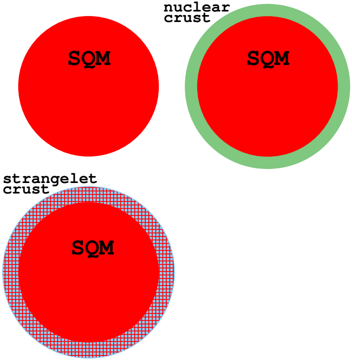
Neutron/hybrid star



Strange Matter Hypothesis

(Bodmer 1971; Witten 1984; Farhi, Jaffe 1984)

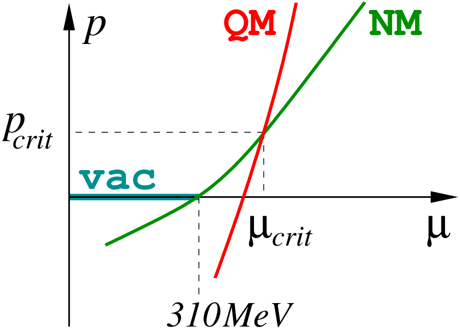
Strange star



I. Two scenarios for quark matter

Conventional scenario

Vac→NM→QM



Nuclear→quark matter transition
at high pressure, ($\mu_{\rm crit}$, $p_{\rm crit}$)

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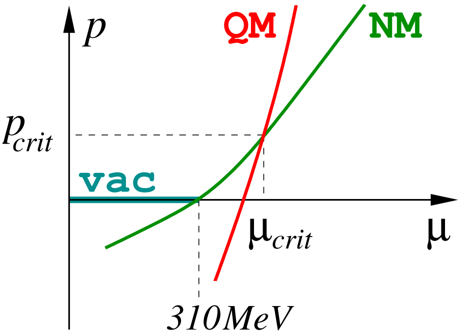
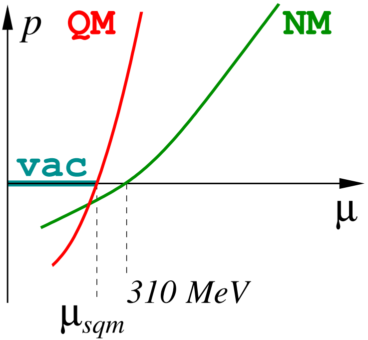


Nuclear→quark matter transition at high pressure, ($\mu_{\rm crit}$, $p_{\rm crit}$)

Strange Matter Hypothesis

Vac→QM

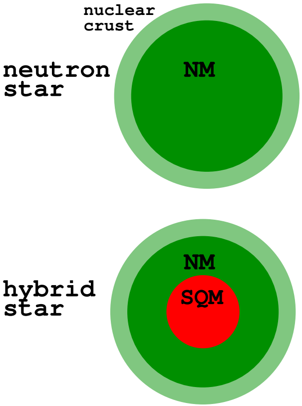


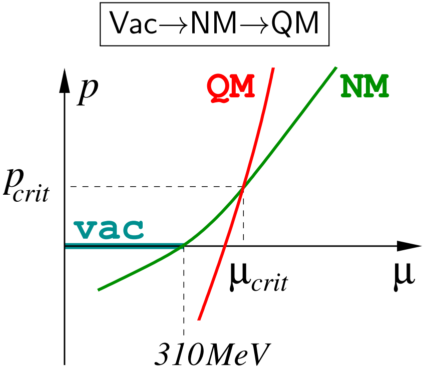
Vacuum→quark matter transition at $\mu = \mu_{\rm sqm}$, $p = 0$.

Strange quark matter (SQM) is the favored phase down to $p = 0$.

II. Conventional hypothesis

Transition from nuclear matter to quark matter occurs at high pressure.
Compact stars have nuclear crust/mantle, possible quark matter core.





Nuclear→quark matter
at high pressure, ($\mu_{\rm crit}$, $p_{\rm crit}$)

Color superconducting phases

Attractive QCD interaction $\Rightarrow$ Cooper pairing of quarks.

We expect pairing between *different flavors*.

Quark Cooper pair: $\langle q^\alpha_{ia} q^\beta_{jb} \rangle$

color $\alpha, \beta = r, g, b$
flavor $i, j = u, d, s$
spin $a, b = \uparrow, \downarrow$

Each possible BCS pairing pattern P is an 18×18 color-flavor-spin matrix

$$\langle q^\alpha_{ia} q^\beta_{jb} \rangle_{1PI} = \Delta_P \, P^{\alpha\beta}_{ij\,ab}$$

The attractive channel is:

	space symmetric	[s-wave pairing]
	color antisymmetric	[most attractive]
	spin antisymmetric	[isotropic]
$\Rightarrow$	flavor antisymmetric	

Initially we will assume the most symmetric case, where all three flavors are massless.

Color-flavor-locked (“CFL”) quark pairing

Equal number of colors and flavors gives a special pairing pattern, color-flavor locked quark matter
(Alford, Rajagopal, Wilczek, `hep-ph/9804403`)

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

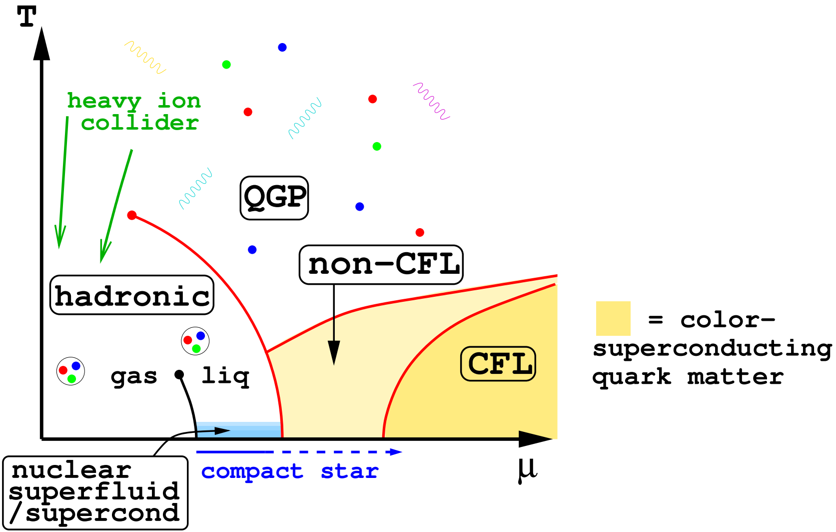
The real world: M_s and neutrality

In the real world there are three factors that combine to oppose pairing between different flavors.

1. Strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. Neutrality requirement. Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. Weak interaction equilibration. In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

These factors favor different densities of u, d, s which *obstructs* pairing between different flavors.

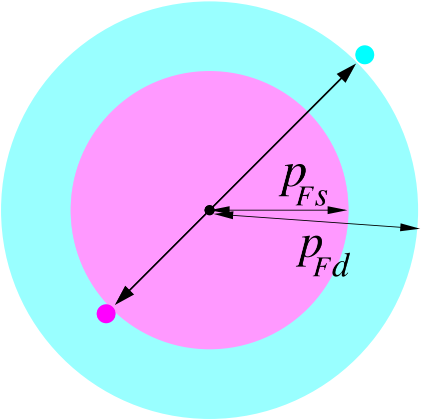
Conjectured QCD phase diagram

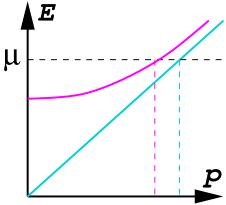


heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Mismatched Fermi surfaces vs. Cooper pairing



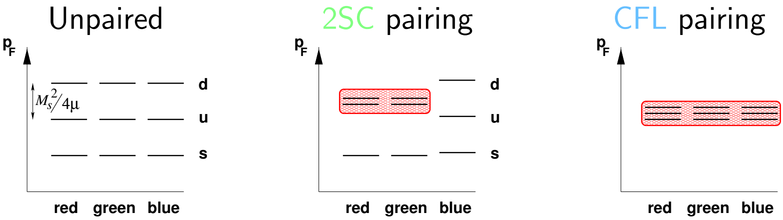


s and *d* quarks near their Fermi surfaces cannot have equal and opposite momenta.

The strange quark mass is the cause of the mismatch.

$$p_{Fd} - p_{Fs} \approx p_{Fd} - p_{Fu} \approx \frac{M_s^2}{4\mu}$$

Cooper pairing vs. the strange quark mass



CFL: Color-flavor-locked phase, favored at the highest densities.

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N}\epsilon_{ijN}$$

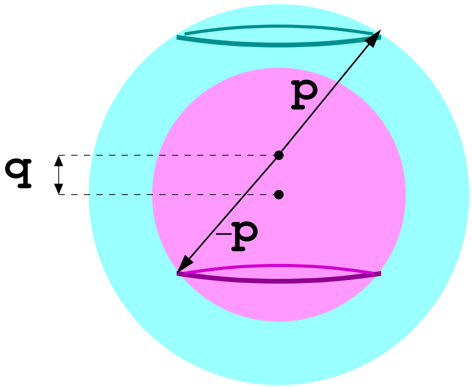
2SC: Two-flavor pairing phase. May occur at intermediate densities.

$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3}\epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: CFL with kaon condensation (CFL-K^0),
crystalline phase (LOFF), p-wave "meson" condensates,
single-flavor pairing (color-spin locking, ∼liq ^{3}He-B).

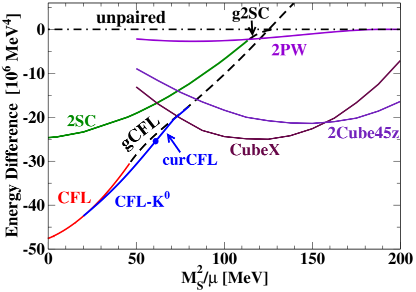
Crystalline (LOFF) superconductivity

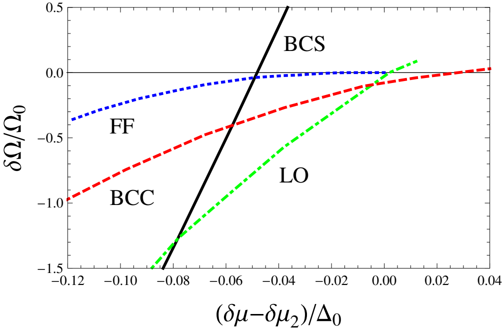
When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.



Every quark pair in the condensate has the same nonzero total momentum $2\mathbf{q}$ (single plane wave).

Free energy comparison of phases





Ginzburg-Landau theory*
of LOFF condensate

(Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`)

Diagonalization of mean field NJL model

(Cao, He, Zhuang, `arXiv:1502.03392`)

* Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

Properties of the LOFF crystal

Shear modulus of LOFF crystal is much greater than regular nuclear crust:

$$\nu_{QM} = 2.47\,\mathrm{MeV\,fm^{-3}} \left(\frac{\Delta}{10\,\mathrm{MeV}}\right)^2 \left(\frac{\mu}{400\,\mathrm{MeV}}\right)^2$$

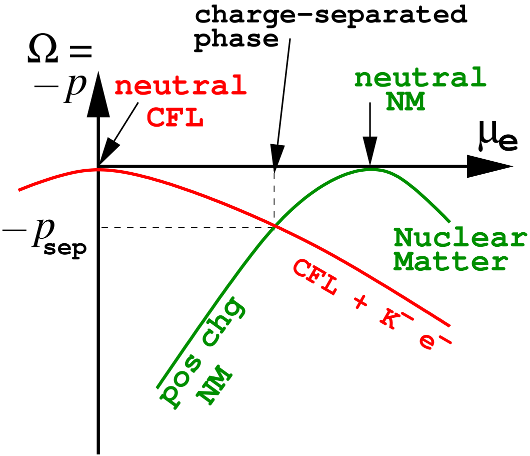
compare: $$\nu_{NM} = c\frac{n_{\mathrm{ion}}(Ze)^2}{a} \sim 0.1 \text{ to } 20\,\mathrm{keV\,fm^{-3}}$$

(Mannarelli, Rajagopal, Sharma, `hep-ph/0702021`)

Pinning force of CFL sf vortex to LOFF crystal is “comparable to that on neutron superfluid vortices in a conventional neutron star crust”.

Mixed phase at nuclear/quark interface

If the surface tension is low enough, there will be charge separation at the CFL/nuclear interface.



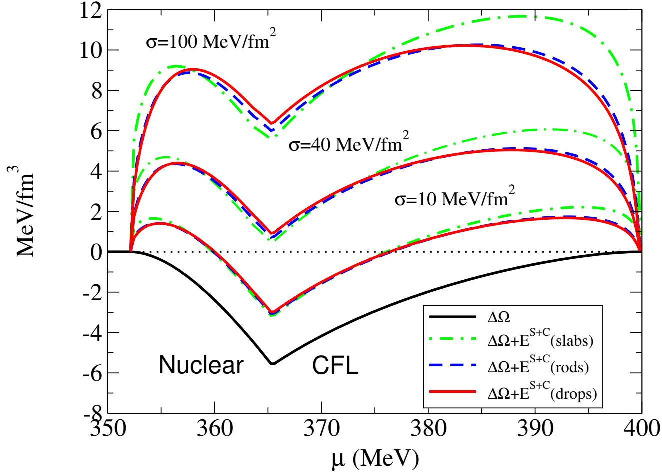
charge density $\rho = \dfrac{d\Omega}{d\mu_e}$

Neutral nuclear matter and *neutral* CFL quark matter can coexist at zero pressure.

But if they have different electrostatic potentials μ_e then $p_{\rm sep} > 0$ and it is preferable* to form a charge-separated phase with intermediate μ_e.

* unless surface costs are too high, e.g. surface tension, electrostatic energy

Mixed phase vs. surface tension

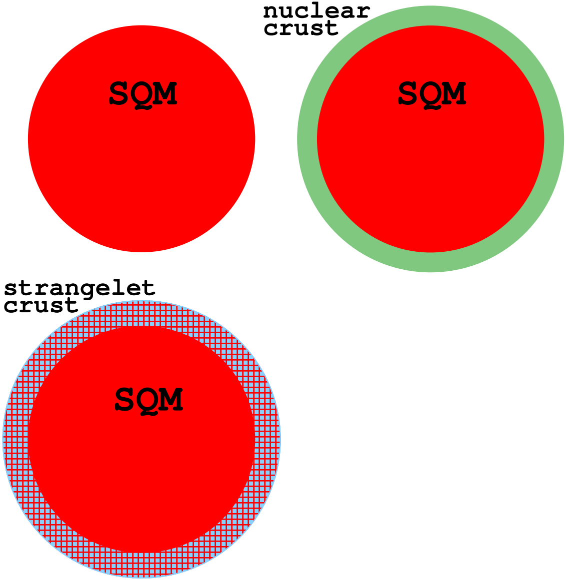


(Alford, Rajagopal, Reddy, Wilczek, `hep-ph/0105009`)

Shear modulus for mixed phase with *unpaired* quark matter:

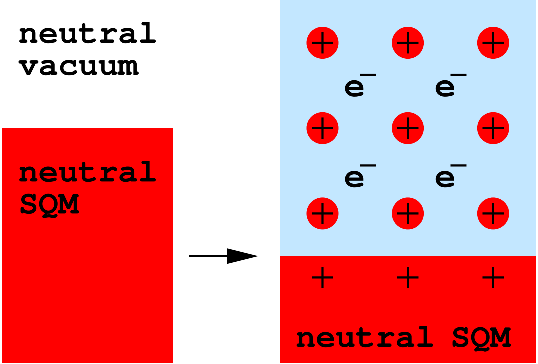
$\nu_{\text{mix}} \sim 10\,\text{keV fm}^{-3}$ $(2 \times 10^{31}\ \text{erg/cm}^3)$ (Owen, `astro-ph/0503399`)

III. Strange Matter Hypothesis



Strangelet crystal crust

At zero pressure, if its surface tension is low enough, strange matter, like nuclear matter, will undergo charge separation and evaporation in to charged droplets.



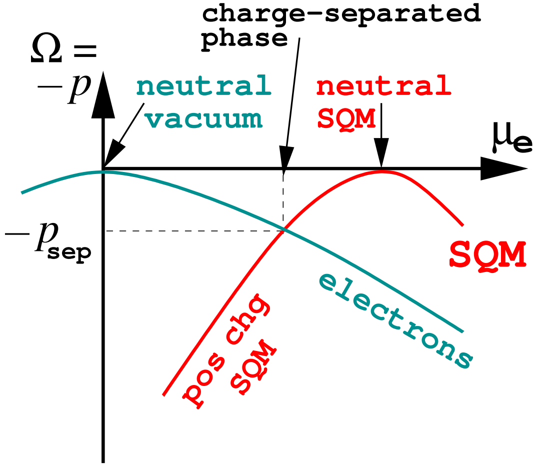
$\sigma_{\rm crit} \lesssim 10\,\text{MeV fm}^{-2}$

Crust thickness
$\Delta R \lesssim 1\,\text{km}$

(Alford, Eby, `arXiv:0808.0671`)

(Jaikumar, Reddy, Steiner, `nucl-th/0507055`)

Charge separation again



charge density $\rho = \frac{d\Omega}{d\mu_e}$

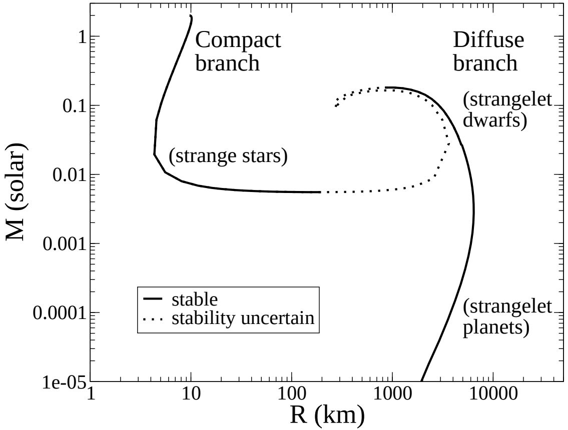
Neutral quark matter and *neutral* vacuum can coexist at zero pressure.

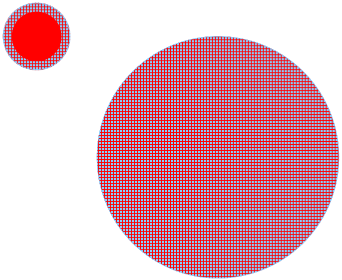
But if they have different electrostatic potentials μ_e then $p_{\rm sep} > 0$ and it is preferable* to form a charge-separated phase with intermediate μ_e.

∗ unless surface costs are too high, e.g. surface tension, electrostatic energy

Strange quark matter objects

Similar to nuclear matter objects, if surface tension is low enough.





(Alford, Han `arXiv:1111.3937`)

Properties of the strangelet crust

It is qualitatively similar to the crust on a neutron star, except that the nuclei become strangelets with a much lower charge to mass (Z/A) ratio.

- Thickness: could be $\sim 10^2$ to 10^3 m,
  but very sensitive to strange matter parameters (surface tension, charge susceptibility, etc).
- Thermal conductivity $\kappa \sim 300\,\mathrm{MeV}^2$ at $T \sim 0.1\,\mathrm{MeV}$, comparable to nuclear crusts
- Shear modulus: similar to nuclear crust?

  (Reddy and Watts, `astro-ph/0609364`)

  see talk by Jaikumar

IV. The future

What microscopic properties of solid phases are important?

- Shear modulus, breaking strain
- Thermal conductivity
- Melting temperature
- ... ?

LOFF phases in hybrid stars:

- Favored crystal structure as a function of density
- Pinning of superfluid vortices (glitches)

Mixed nuclear-quark phase in hybrid stars:

- Mixing of other quark matter (or nuclear matter) phases
- Favored pasta form as a fn of density (depends on surface tension)

Strangelet crystal crust on strange quark stars:

- Could neutron stars really be quark stars with crust ?
  Cooling after accretion, X-ray burst oscillations
  Strangelet pollution problem