$\rho\pi$ isospin-2 scattering from lattice QCD

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Outline

Scattering with non-zero intrinsic spin

- Many scattering processes feature hadrons with non-zero intrinsic spin

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- NN scattering, e.g. ${}^3S_1$, ${}^3D_1$ mixing in deuteron
- $a_1 \to \rho\pi$
- $b_1 \to \omega\pi$
- Exotic X, Y, Z states – observed decays into V-Ps and V-V channels

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- Expect no bound-states or resonances – exotic isospin
- Calculations at $SU(3)_F$ point
- Heavy octet pseudoscalars, $m_\pi \sim 700\,\text{MeV}$
- Stable octet vectors, $m_\rho \sim 1020\,\text{MeV}$

Features of vector-pseudoscalar scattering

- Infinite-volume $J^P = \ell \otimes S$ i.e. $\ell \otimes S = |\ell - S| \oplus ... \oplus \ell + S$

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- $S = 1$ and $\ell \geq 1$ – triplets of partial-waves $J = \{\ell - 1, \ell, \ell + 1\}$
- $J^P = 1^+, 2^-, 3^+ ...$ formed from *two* distinct ℓS combinations – dynamical coupling $\{^3S_1, {}^3D_1\}, \{^3P_2, {}^3F_2\}, \{^3D_3, {}^3G_3\}, ...$

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- For $\vec{P} \neq \vec{0}$, relevant symmetry group is $LG(\vec{P})$

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- Different helicity components of vectors subduce into different irreps of $LG(\vec{P} \neq \vec{0})$

Partial-waves in a box at $\vec{P} = \vec{0}$, $\Lambda^P = T_1^+$

Λ^+	T_1^+
$J^+(^3\ell_J)$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
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- Infinite tower of J^P collapse into finitely many Λ^P...
- States in T_1^+ tell us about $^3S_1$, $^3D_1$, $^3D_3$, ... scattering amplitudes

Bridging the infinite and finite volumes

$$\det\left[\mathbf{1} + i\rho(E_{\mathrm{cm}})\,\mathbf{t}(E_{\mathrm{cm}}) \cdot \left(\mathbf{1} + i\overline{\mathcal{M}}(E_{\mathrm{cm}}, L)\right)\right] = 0$$

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Infinite-volume scattering amplitudes $\longleftarrow$ Finite-volume spectra

- Solutions are finite-volume energy levels
- Parameterise **t**-matrix and match model spectra with finite-volume spectra
- Need to calculate finite-volume spectra...

Spectrum determination

- Construct a large matrix of correlation functions – all operators resemble $\rho\pi$ in isospin-2, corresponding to **27** in $\mathrm{SU(3)_F}$ limit

$$C_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) \middle| 0 \right\rangle$$

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- $\rho\pi$ operators – definite Λ and overall momentum $\vec{P}$

$$\rho\pi(\vec{P}) = \sum_{\hat{p}_1,\hat{p}_2} \mathcal{C}(\vec{p}_1,\vec{p}_2;\vec{P})\rho(\vec{p}_1)\pi(\vec{p}_2),$$

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- Let's take T_1^+ as an example...

$\vec{P} = \vec{0}$, T_1^+ spectra

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$[000]\, T_1^+$
$\rho_{[000]}\pi_{[000]}$
$\{2\}\ \rho_{[001]}\pi_{[00\text{-}1]}$
$\{3\}\ \rho_{[011]}\pi_{[0\text{-}1\text{-}1]}$
$\{2\}\ \rho_{[111]}\pi_{[\text{-}1\text{-}1\text{-}1]}$
8 ops.

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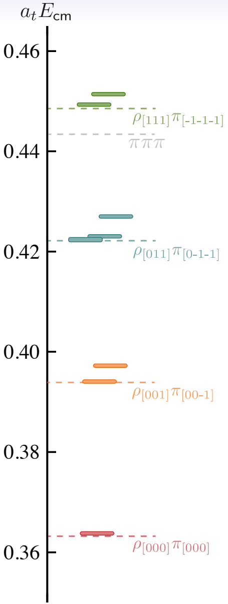
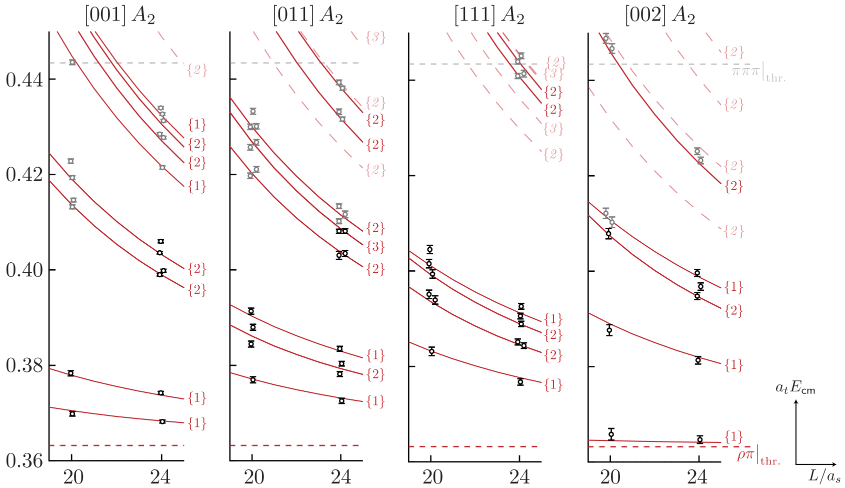
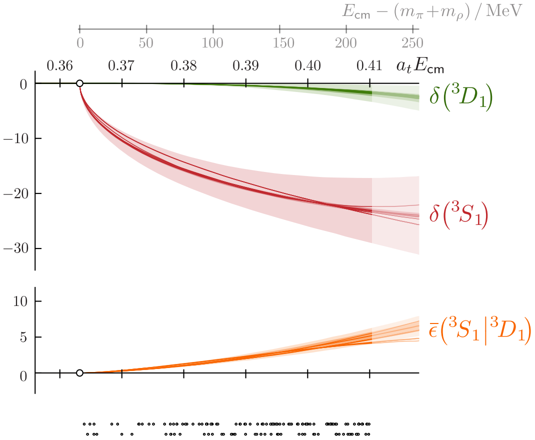


$\vec{P} \neq \vec{0}$ spectra



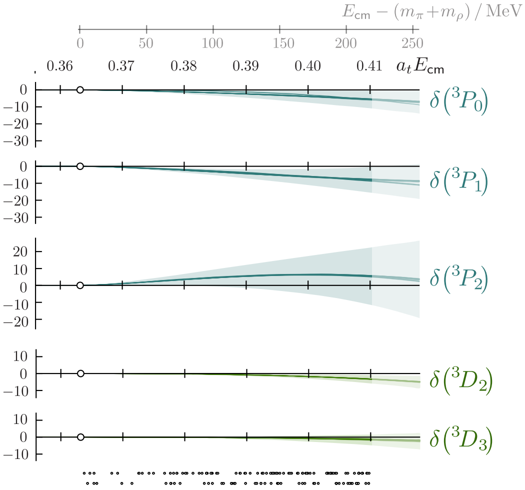
Dynamically coupled $^3S_1$ and $^3D_1$ partial-waves

- Stapp-parameterisation of **S**-matrix in $^3S_1$, $^3D_1$
- Over 140 energy levels as constraints
- Weakly repulsive in $^3S_1$
- $^3D_1$ consistent with zero – hints of weak repulsion
- $\bar{\epsilon}$ significantly non-zero



Phase-shifts for $\ell \leq 2$

- ^3P_J consistent with zero – hints of weak attraction in $^3P_2$ and repulsion in $^3P_{0,1}$
- $^3D_{2,3}$ consistent with zero – hints of weak repulsion



Summary

- A first calculation of $\rho\pi$ in isospin-2 from lattice QCD
- Vector-pseudoscalar scattering including effects of ${}^3S_1$, ${}^3P_{0,1,2}$, ${}^3D_{1,2,3}$-waves
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Thank you for listening!

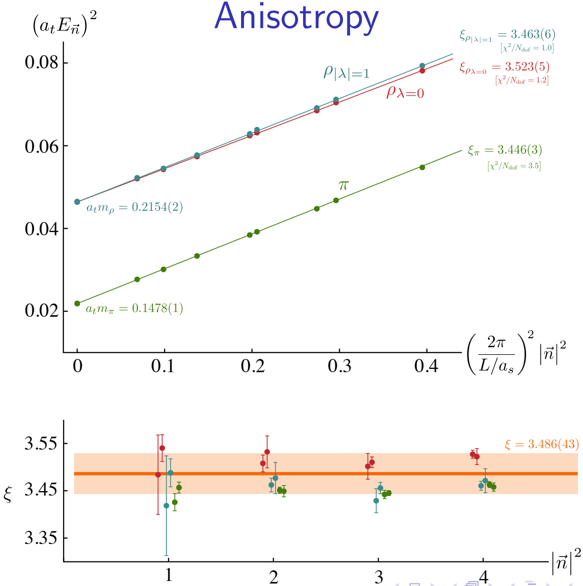
Partial-waves subduction at $\vec{P} = \vec{0}$

Λ^+	A_1^+	A_2^+	T_1^+	E^+	T_2^+
$J^+(^3\ell_J)$			$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$		
				$2^+ \quad (^3D_2)$	$2^+ \quad (^3D_2)$
		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
	...	...	...	...	...

Lattice Details

- Calculations were performed on anisotropic lattices of volumes $(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$ and $24^3 \times 128$, where $a_s = 0.12$ fm and $\xi = a_s/a_t \sim 3.5$
- Gauge fields generated from a tree level Symanzik improved gauge action
- Clover fermion action with $N_f = 3$ degenerate flavours of dynamical quarks tuned to have masses approximately equal to the physical strange quark mass

Anisotropy


$(a_t E_{\vec{n}})^2$
0.08
0.06
0.04
0.02
0
0.1
0.2
0.3
0.4
$\left(\frac{2\pi}{L/a_s}\right)^2 |\vec{n}|^2$
$\rho_{|\lambda|=1}$
$\rho_{\lambda=0}$
π
$\xi_{\rho_{|\lambda|=1}} = 3.463(6)$
$[\chi^2/N_{\rm dof} = 1.0]$
$\xi_{\rho_{\lambda=0}} = 3.523(5)$
$[\chi^2/N_{\rm dof} = 1.2]$
$\xi_\pi = 3.446(3)$
$[\chi^2/N_{\rm dof} = 3.5]$
$a_t m_\rho = 0.2154(2)$
$a_t m_\pi = 0.1478(1)$
ξ
3.55
3.45
3.35
1
2
3
4
$|\vec{n}|^2$
$\xi = 3.486(43)$

S-, t- and K-matrices

- Stapp-parameterisation of **S**-matrix for coupled ${}^3S_1$, ${}^3D_1$. Have $\mathbf{S} = \mathbf{1} + 2i\rho\mathbf{t}$

$$\mathbf{S} = \begin{bmatrix} \cos(2\bar{\epsilon})\exp[2i\,\delta_{^3S_1}] & i\sin(2\bar{\epsilon})\exp[i(\delta_{^3S_1} + \delta_{^3D_1})] \\ i\sin(2\bar{\epsilon})\exp[i(\delta_{^3S_1} + \delta_{^3D_1})] & \cos(2\bar{\epsilon})\exp[2i\,\delta_{^3D_1}] \end{bmatrix},$$

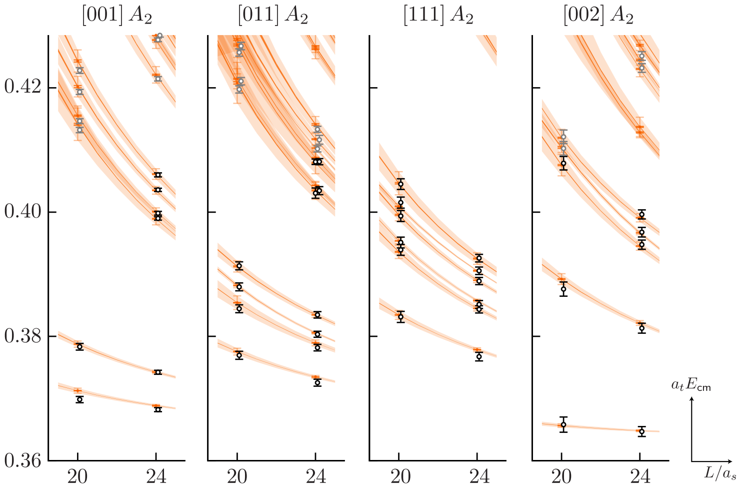
- Express unitarity in terms of a real **K**-matrix

$$\left[t^{-1}(s)\right]_{\ell J,\ell' J} = \frac{1}{(2k_{\mathrm{cm}})^{\ell}}\left[K^{-1}(s)\right]_{\ell J,\ell' J}\frac{1}{(2k_{\mathrm{cm}})^{\ell'}} + \delta_{\ell\ell'}\,I(s)$$

- Form of the **K**-matrix for T_1^+ $\ell \leq 2$

$$\mathbf{K} = \begin{bmatrix} K({}^3S_1|{}^3S_1)(s) & K({}^3S_1|{}^3D_1)(s) & 0 \\ K({}^3S_1|{}^3D_1)(s) & K({}^3D_1|{}^3D_1)(s) & 0 \\ 0 & 0 & K({}^3D_3|{}^3D_3)(s) \end{bmatrix}$$

Model spectra comparison $\vec{P} \neq \vec{0}$



Determining the mixing angle

- $\bar{\epsilon}$ is most sensitive to the off-diagonal parameter in the **K**-matrix, $K(^3S_1|^3D_1)(s) = c_0(^3S_1|^3D_1)$
- The sign of $\bar{\epsilon}$ depends on sign of $c_0(^3S_1|^3D_1)$
- Spectra in $\vec{P} \neq \vec{0}$ irreps sensitive to sign of $\bar{\epsilon}$
- Lattice spectra provide strong constraints on $\bar{\epsilon}$ in energy region considered