Review of coupled-channel scattering results

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based on work with the Hadron Spectrum Collaboration

hadspec.org

Multi-hadron systems from Lattice QCD
February 5-9 2018
INT Seattle

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

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For what's next

Overview of Hadron Spectrum Collaboration (HadSpec) scattering calculations:

m_π = 700 MeV

I=2 ρπ in S, P, D partial waves

See next talk: Antoni Woss

m_π = 391 MeV

I=0, ½, 1, 2 with π, K, η, η’ in S, P, D partial waves

I=½ with Dπ, Dη, D_sK

I=1 πγ → ππ

- L = 16, 20, 24
- 9 publications
- rigorous extractions of S & P wave 2-body resonances ~ up to 3 or 4 body thresholds
- D-wave resonances, typically neglecting (small) 3-body effects

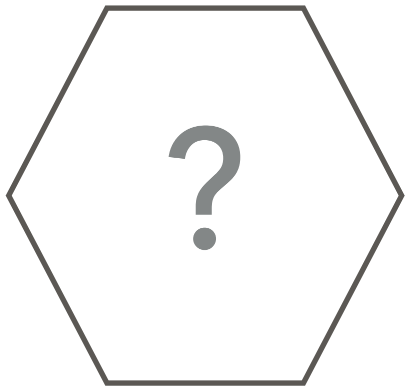
m_π = 236 MeV

I=1 ππ in P, F partial waves

I=0 ππ in S wave

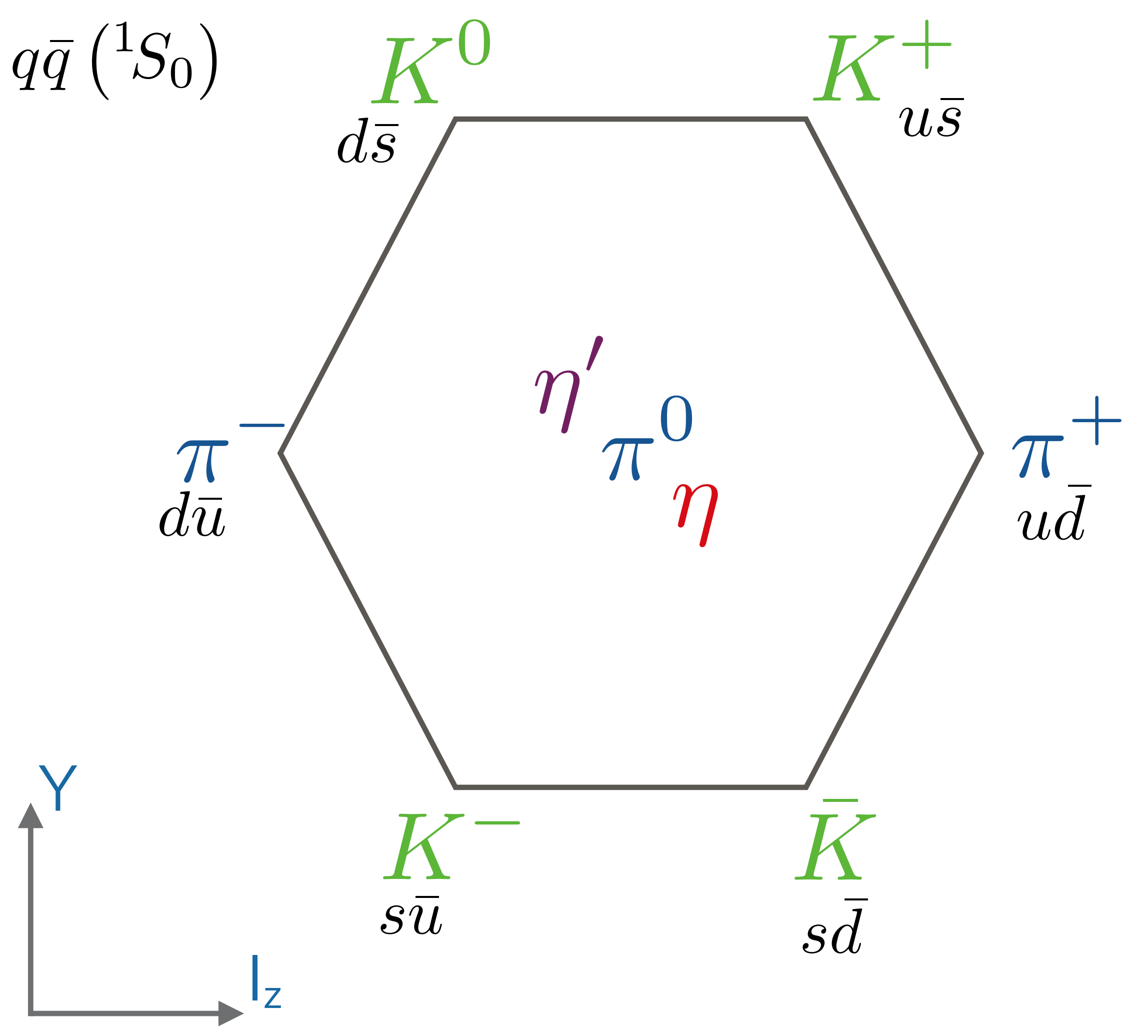
- L=32
- 2 publications
- rigorous extractions of the ρ & σ resonances
- kinematics make it difficult to go higher in energy (3+ body channels open)

N_f=2+1, approximately physical strange quark mass,
anisotropic lattices ~3.5x finer in temporal direction, $a_{spatial}$~0.12 fm

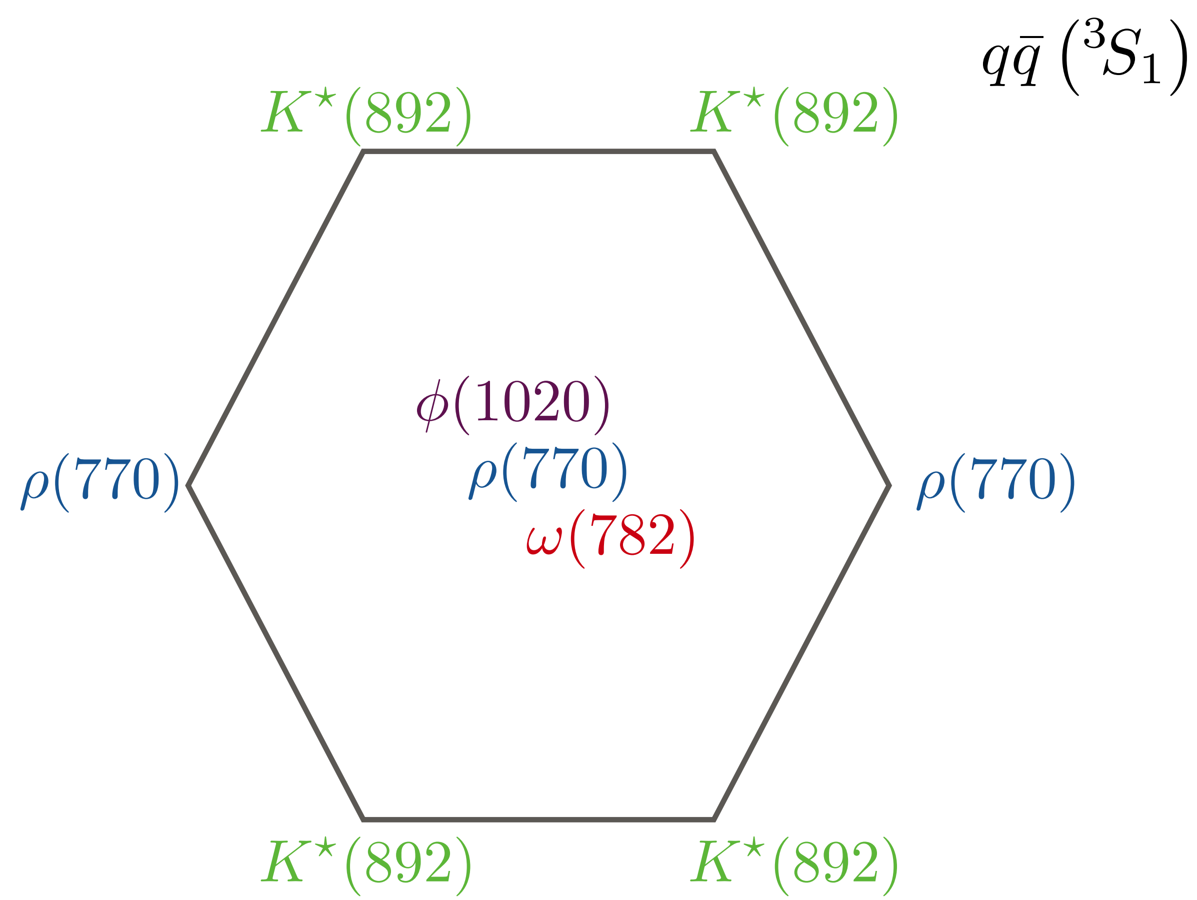


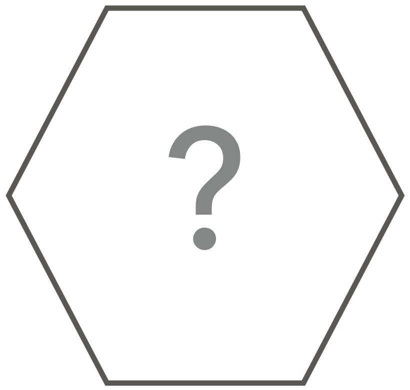
Considering *u,d,s* quarks: $q\bar{q}\,(^{2S+1}L_J)$

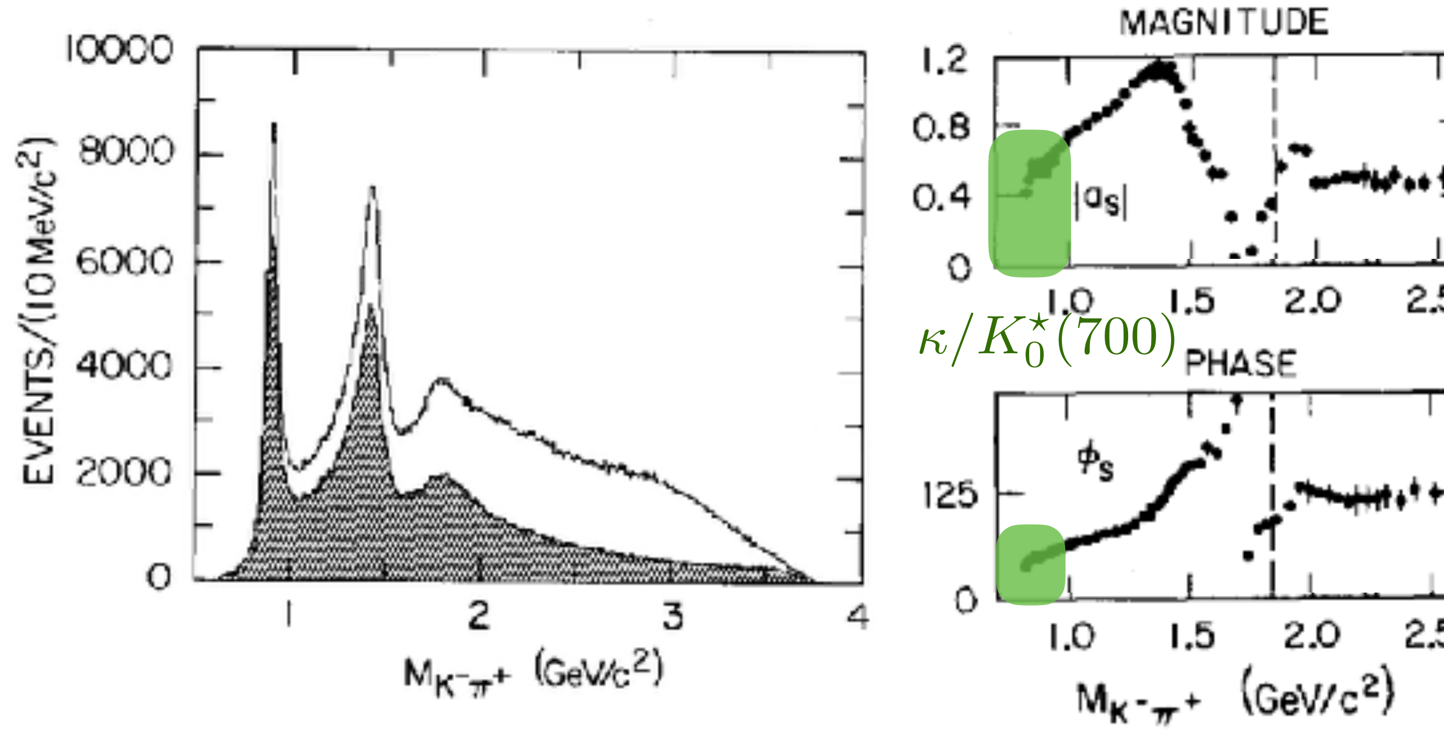
pseudoscalar nonet $J^P=0^-$



vector nonet $J^P=1^-$

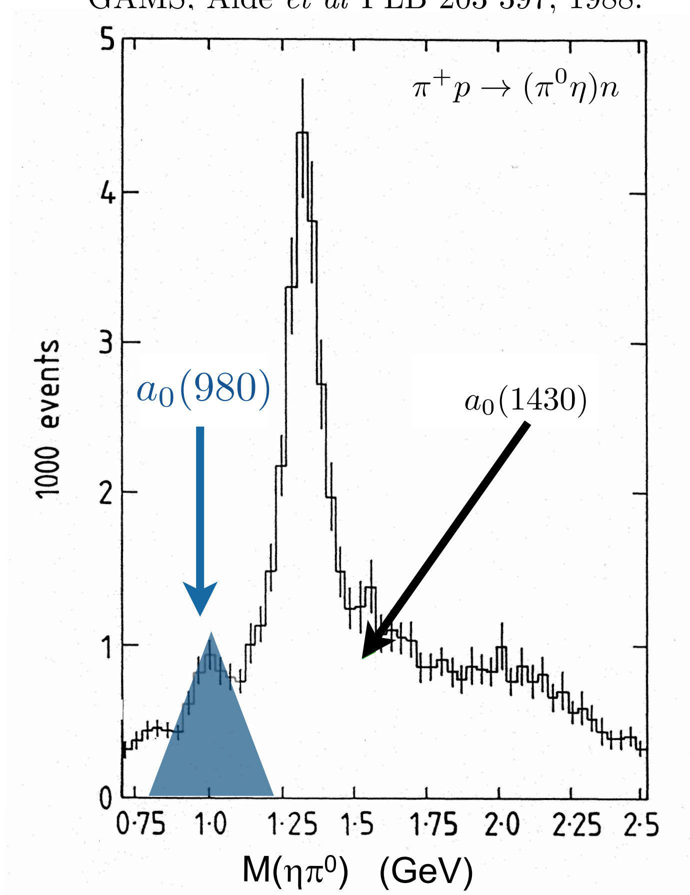


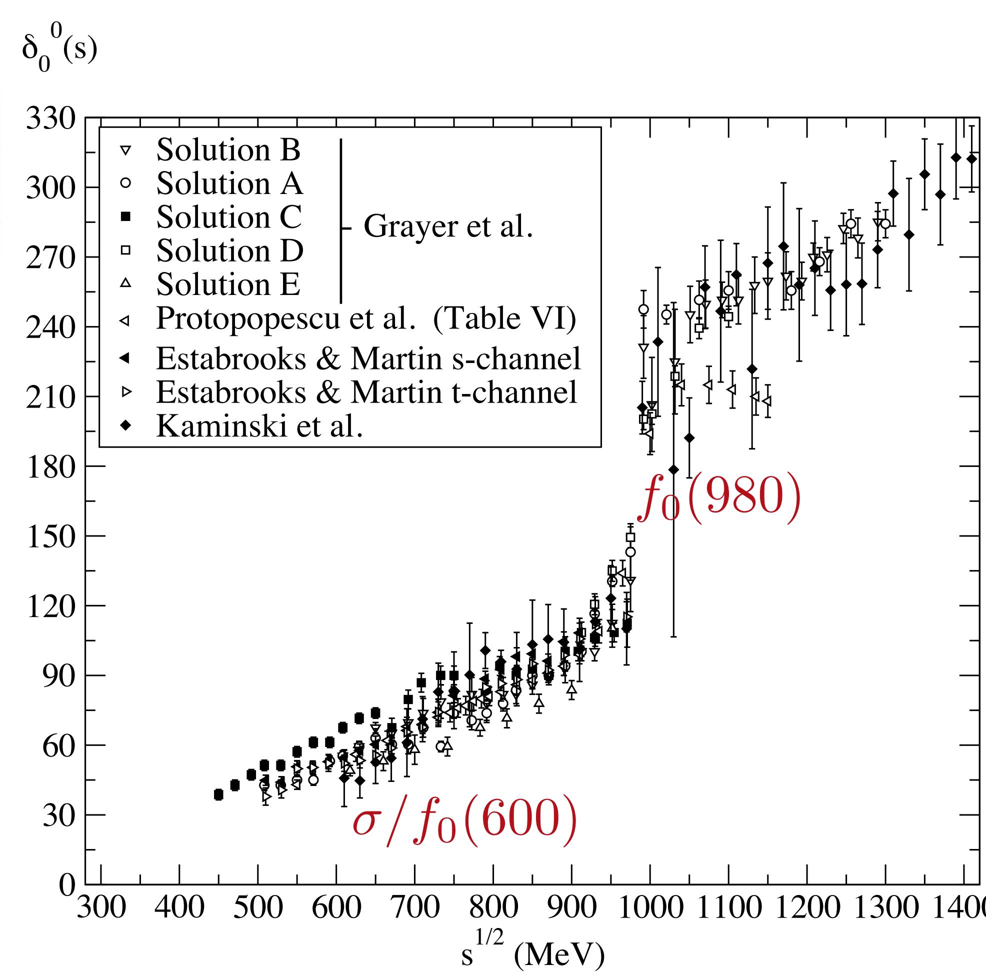


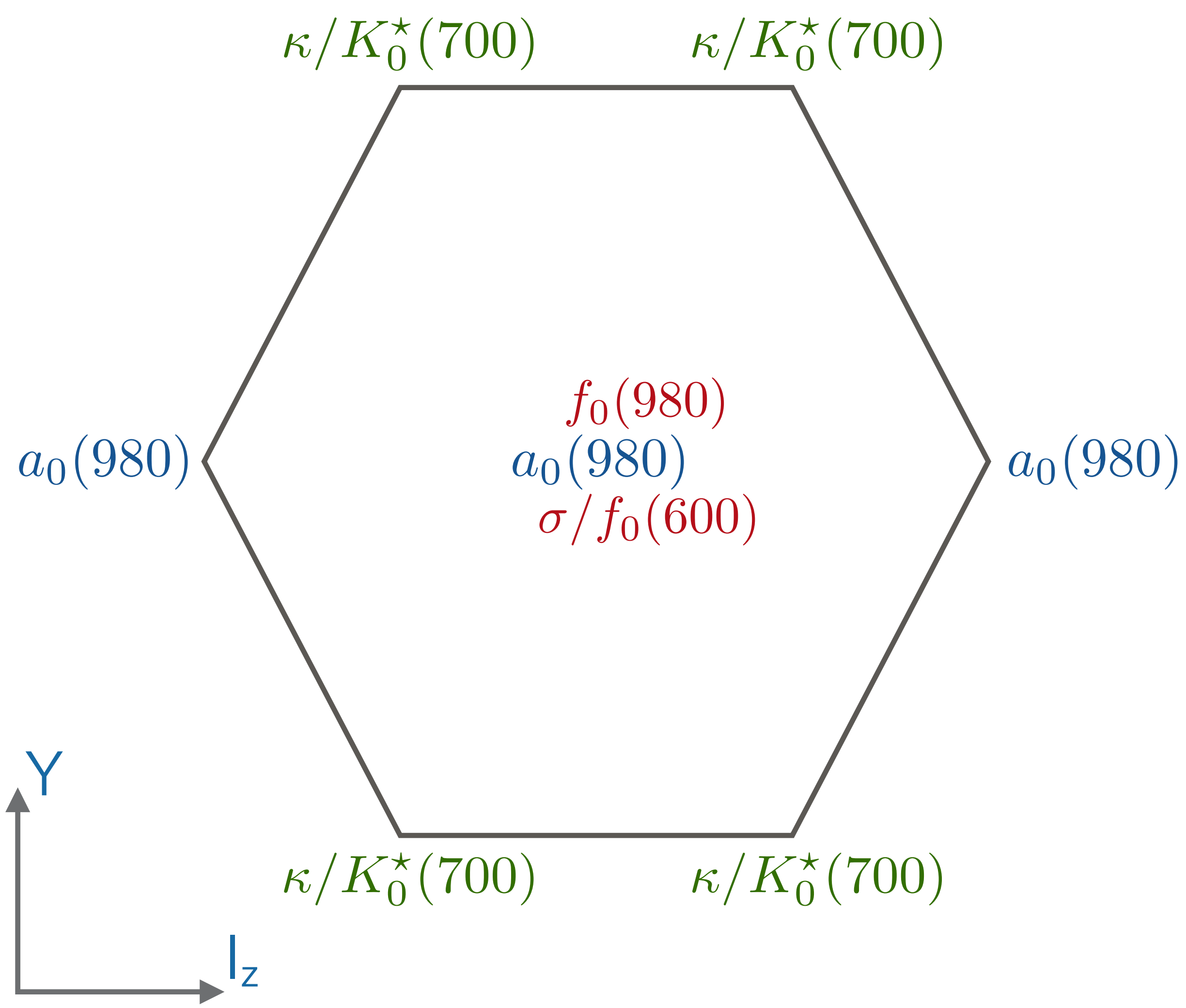


LASS experiment at SLAC $E_K = 11$ GeV

GAMS. Alde *et al* PLB 203 397, 1988.

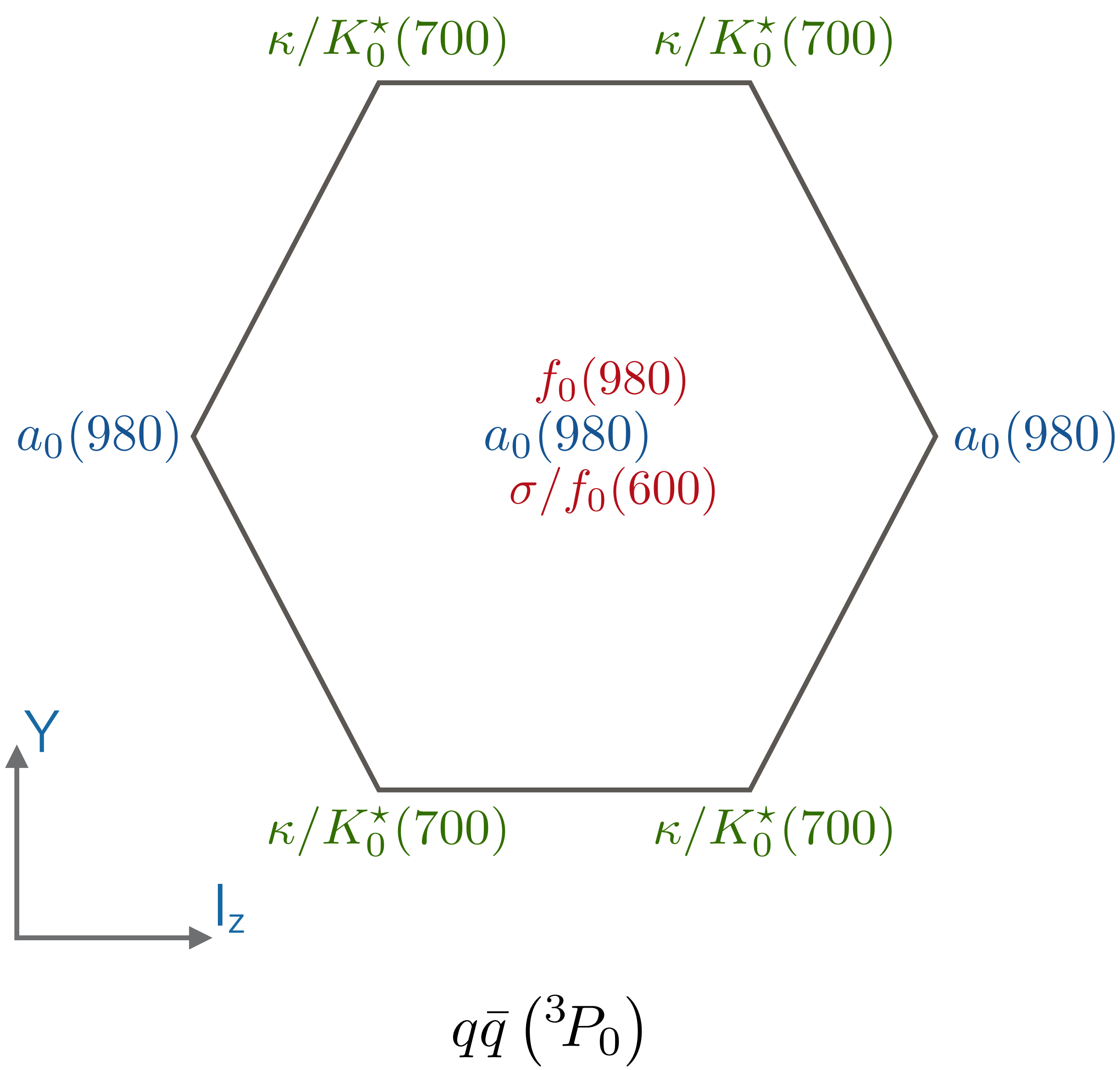


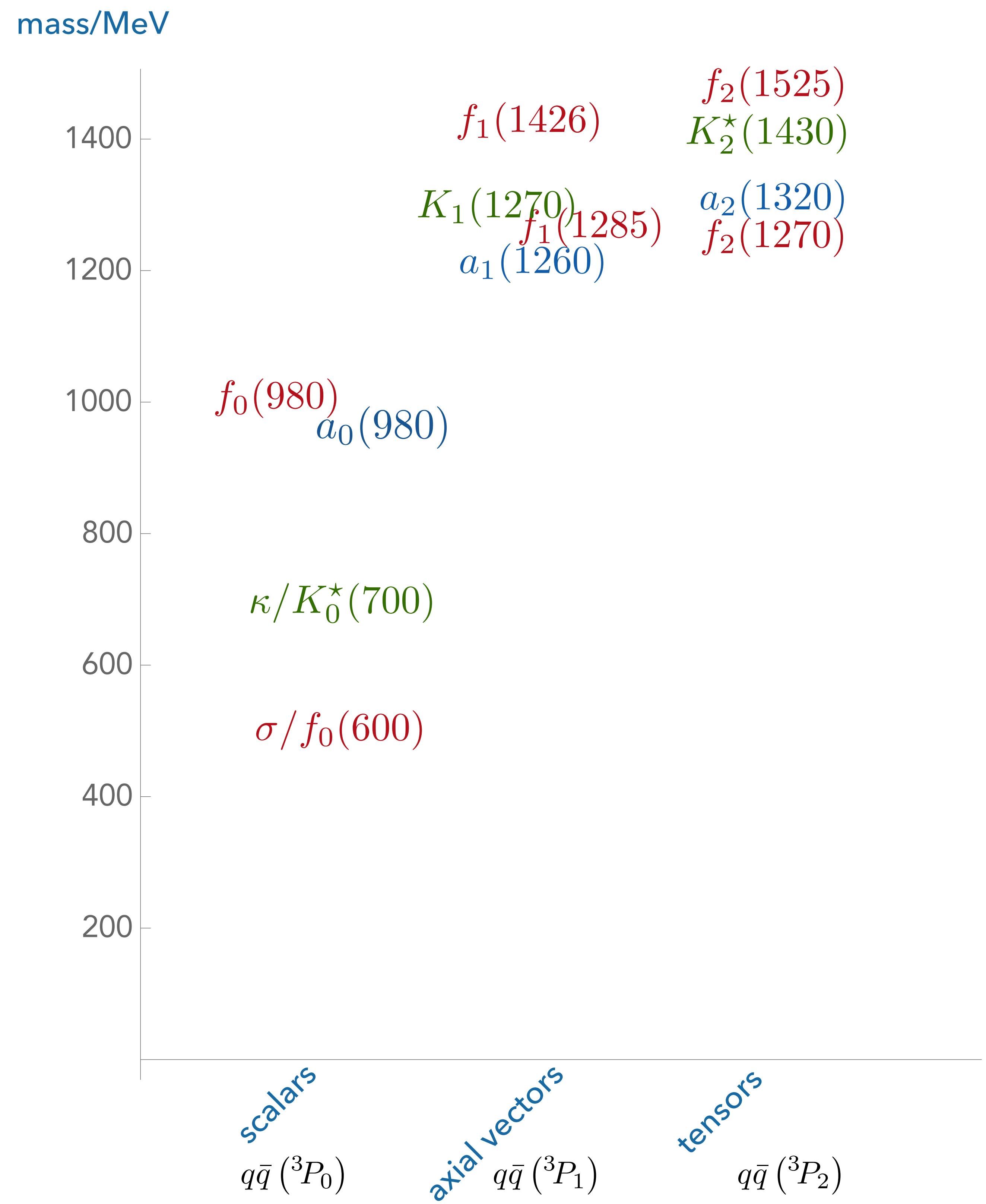




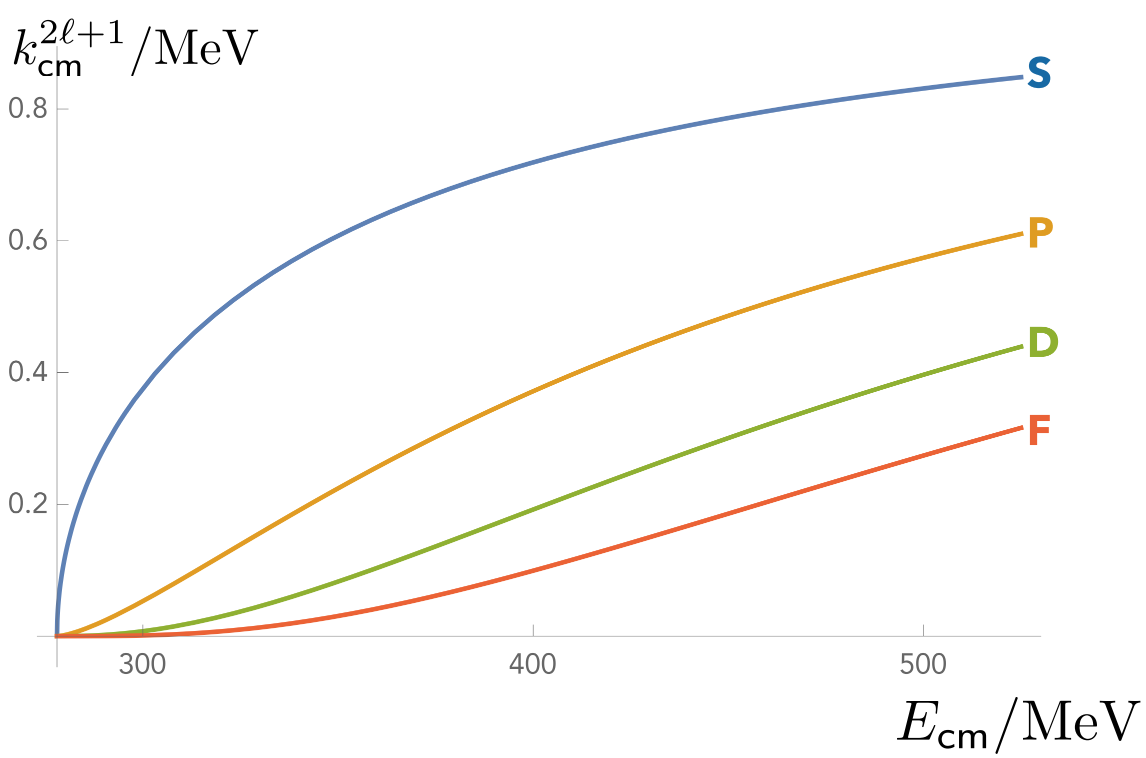
$q\bar{q}\left(^3P_0\right)$

What about the masses?





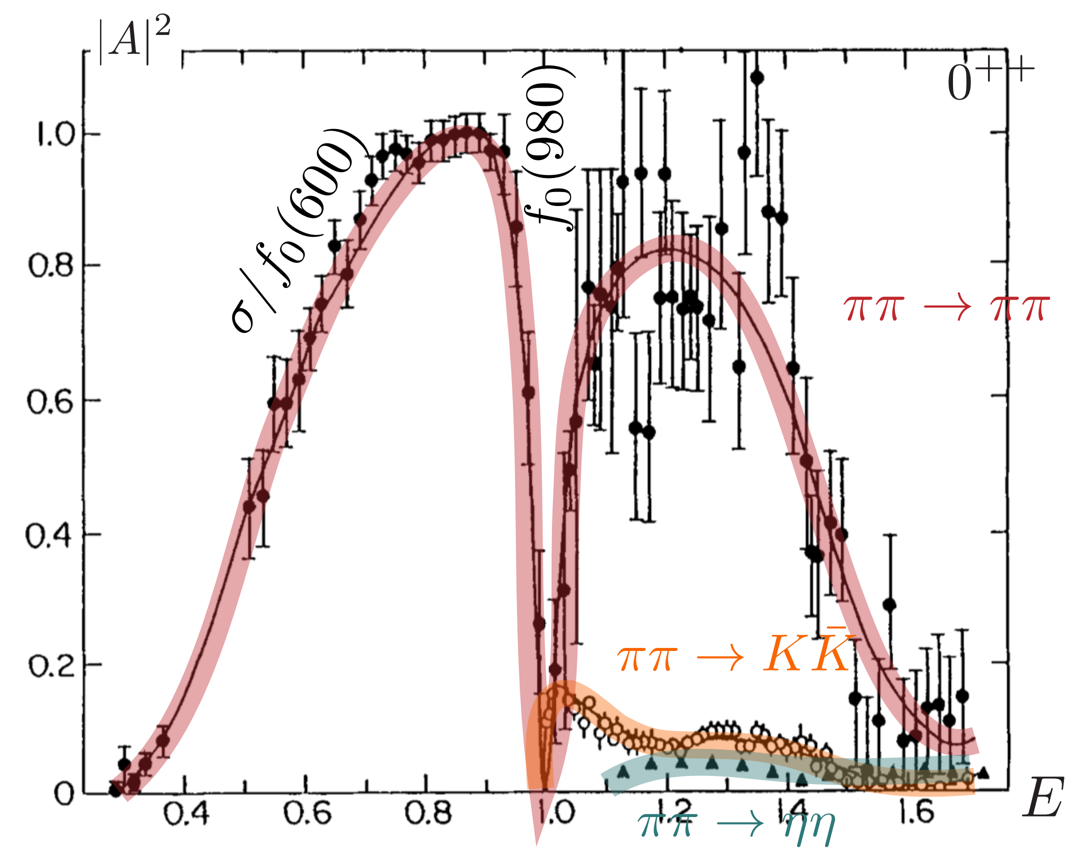
In the scalar sector, amplitudes grow rapidly from threshold:



σ and κ are broad (width ~ mass)

$f_0(980)$ and $a_0(980)$ lie very close to KK threshold

CERN-Munich, ANL, BNL



3 volumes
L=16, 20, 24
anisotropic (3.5 finer spacing in time)
Wilson-Clover

m_π=391MeV

m_K=549MeV

$$\begin{bmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{bmatrix}$$

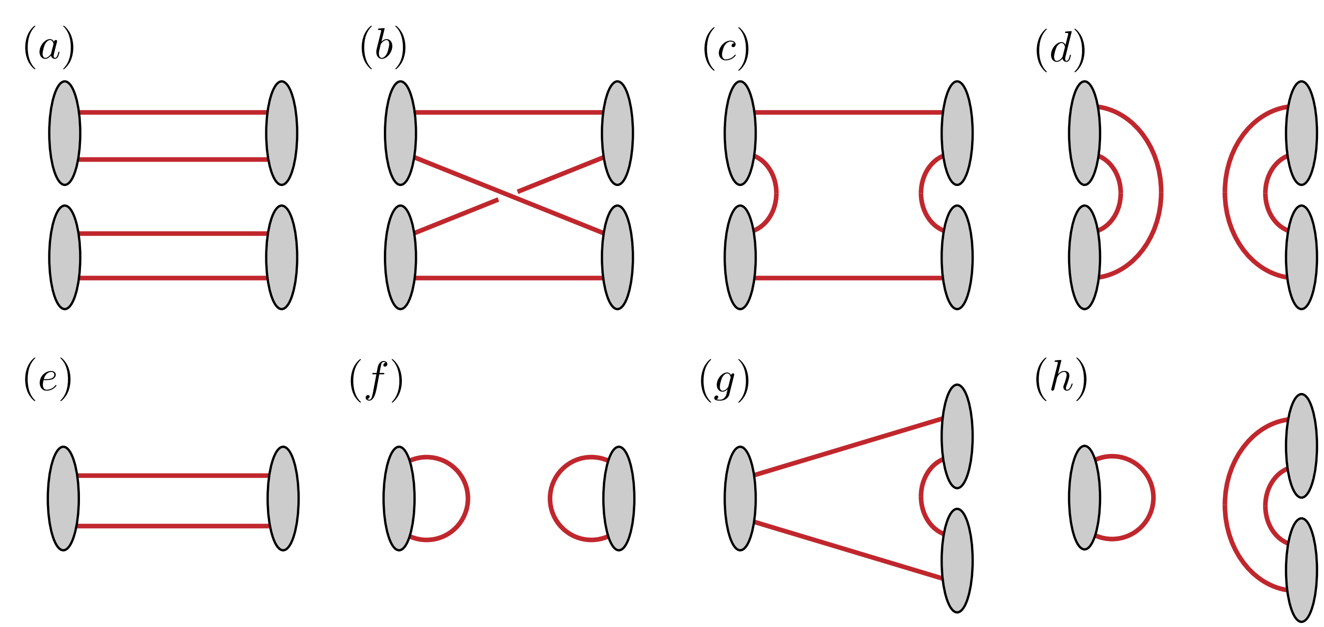
operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$ local qq-like constructions

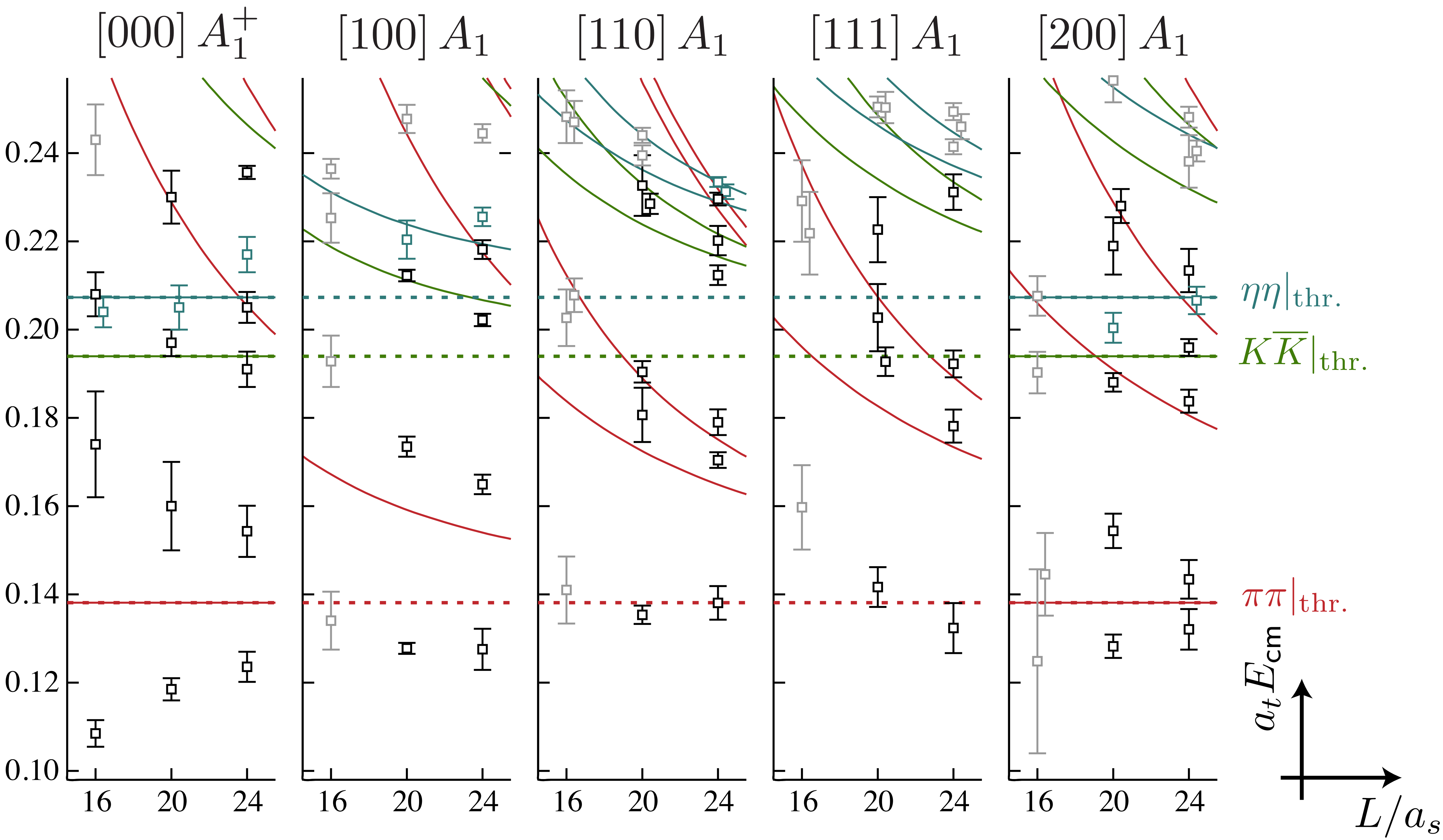
$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$ two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

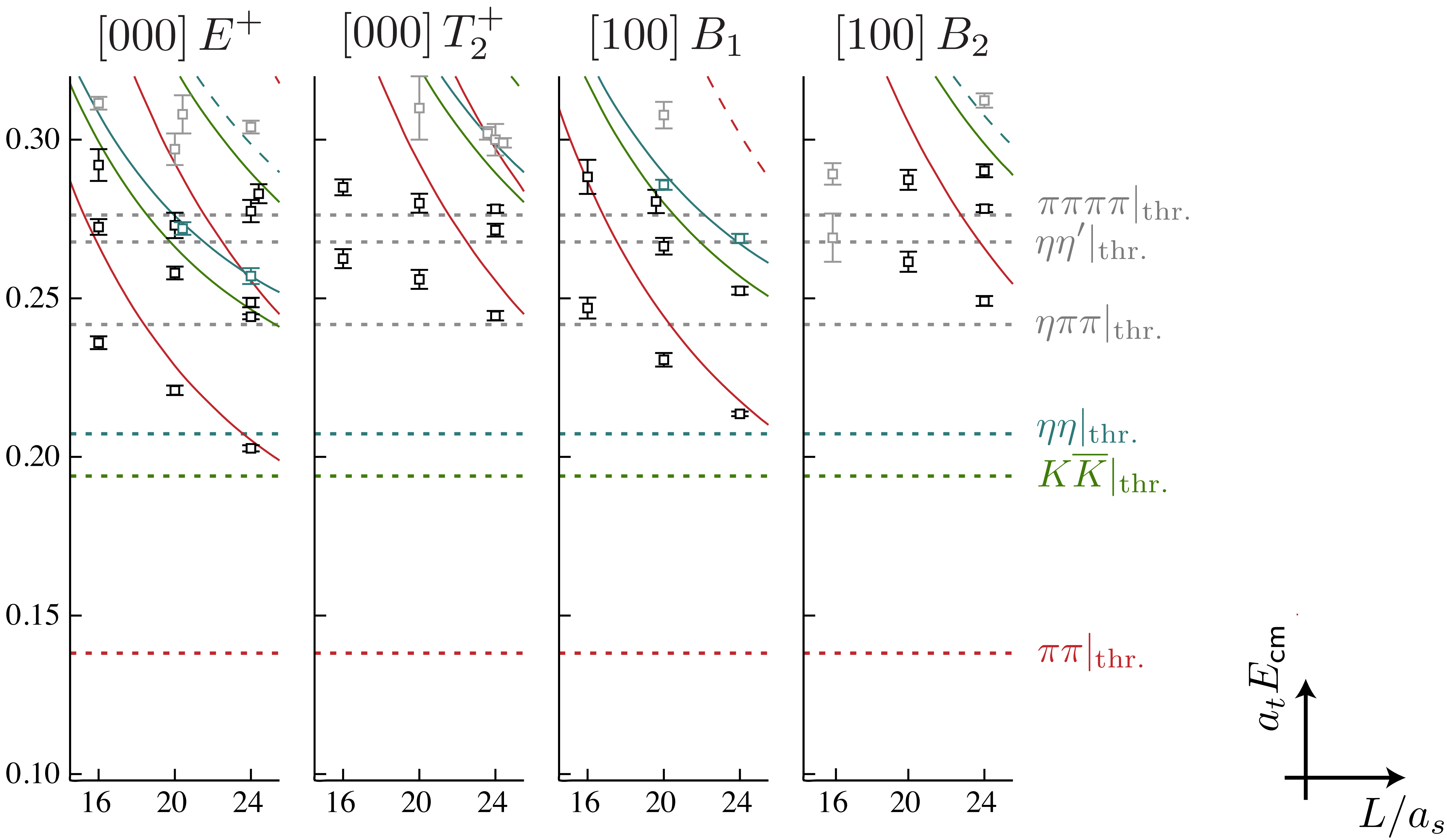
using *distillation* (Peardon *et al* 2009)
many wick contractions, eg just pi-pi & qq operators:



Briceno et al, arXiv:1708.06667

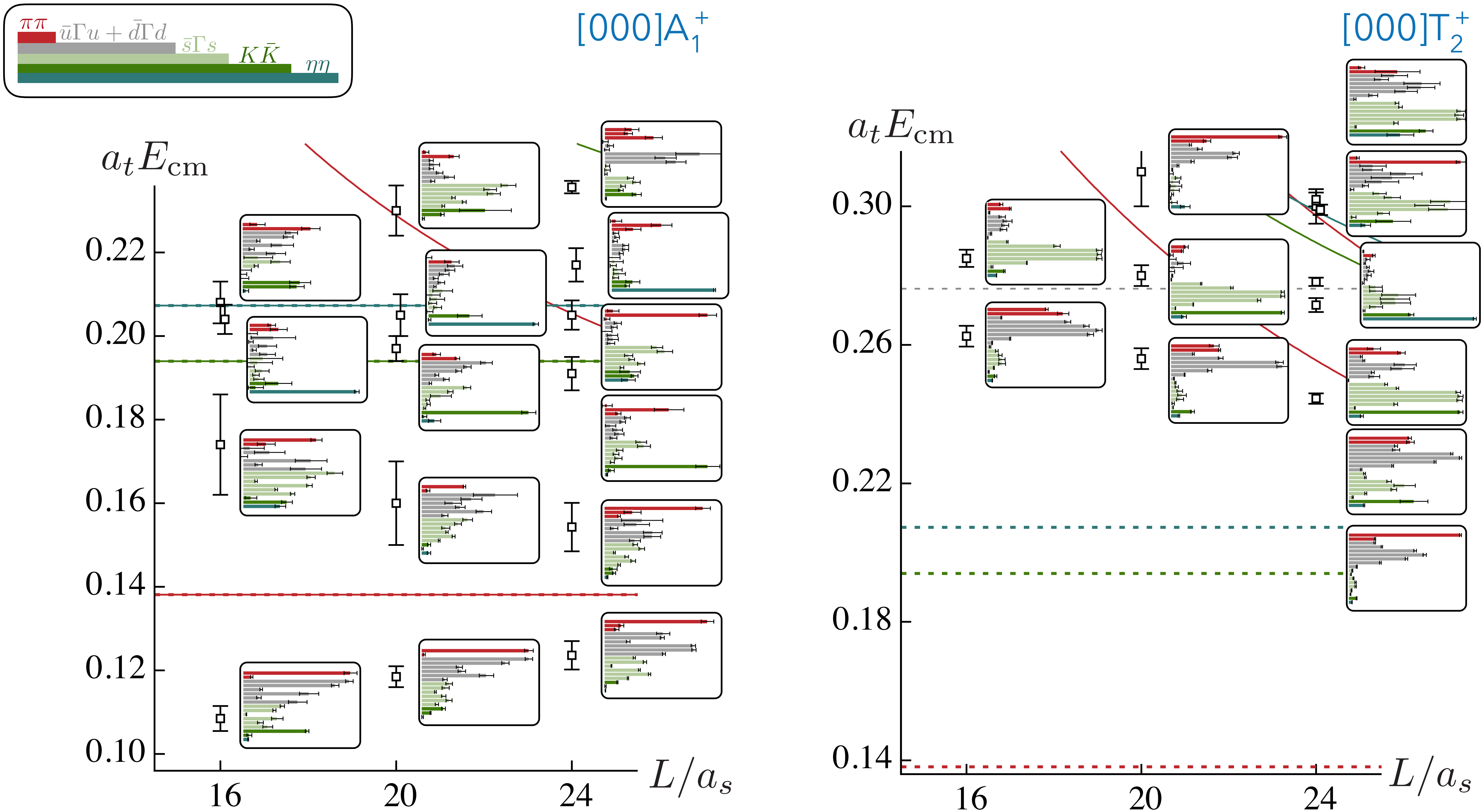


similar types of operators as before: local qq̄ & 2-hadron
conservatively 57 energy levels
dominated by S-wave interactions



conservatively 34 energy levels
dominated by D-wave interactions

Spectra and overlaps



operator overlaps give some intuition

lots of mixing in the scalar sector

- essential to have meson-meson ops even below threshold
- can't always 'read-off' resonance content

recent review by Briceno, Dudek, Young:
arXiv:1706.06223

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space → $\boldsymbol{\rho}$

infinite volume scattering *t*-matrix → $\boldsymbol{t}$

known finite-volume functions → $\boldsymbol{\mathcal{M}}$

Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

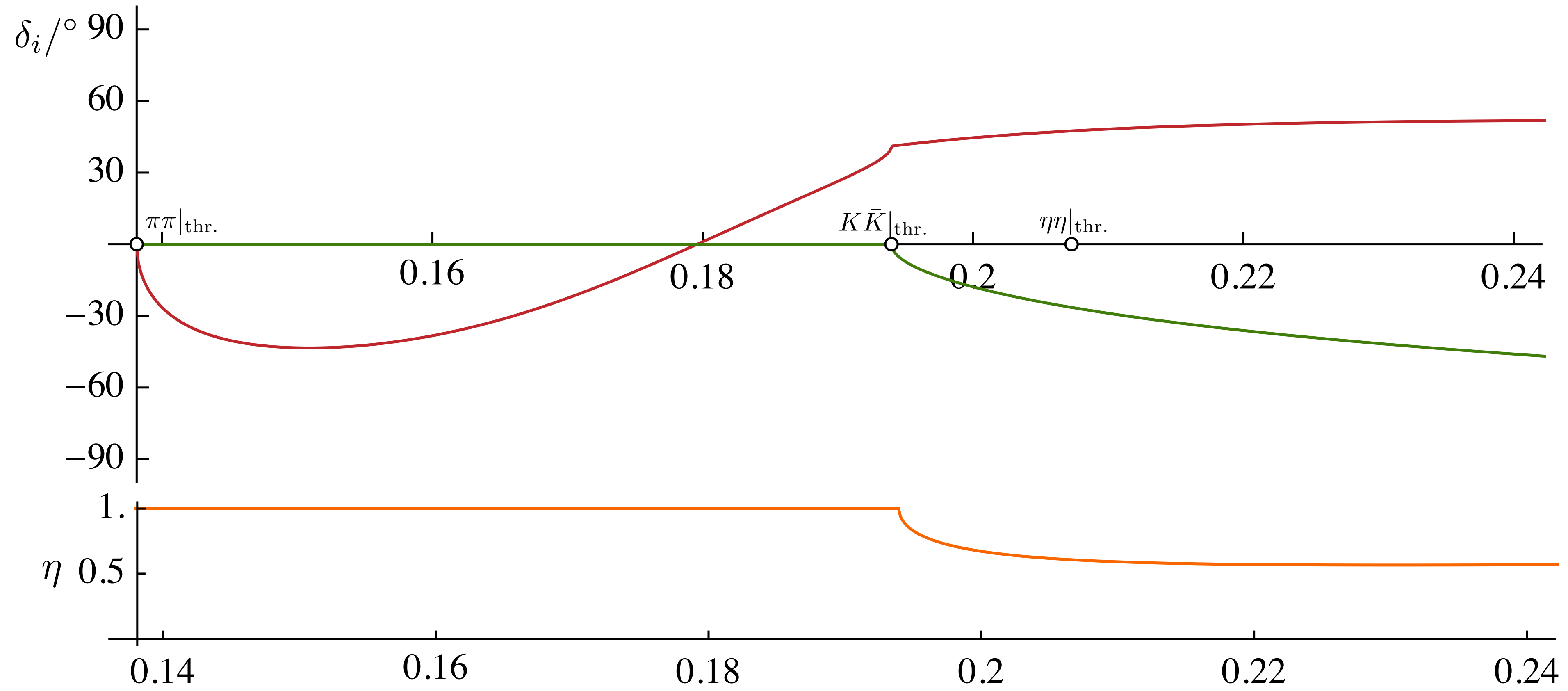
For progress on a general 3-body quantization condition - see other talks

Given some t-matrix...

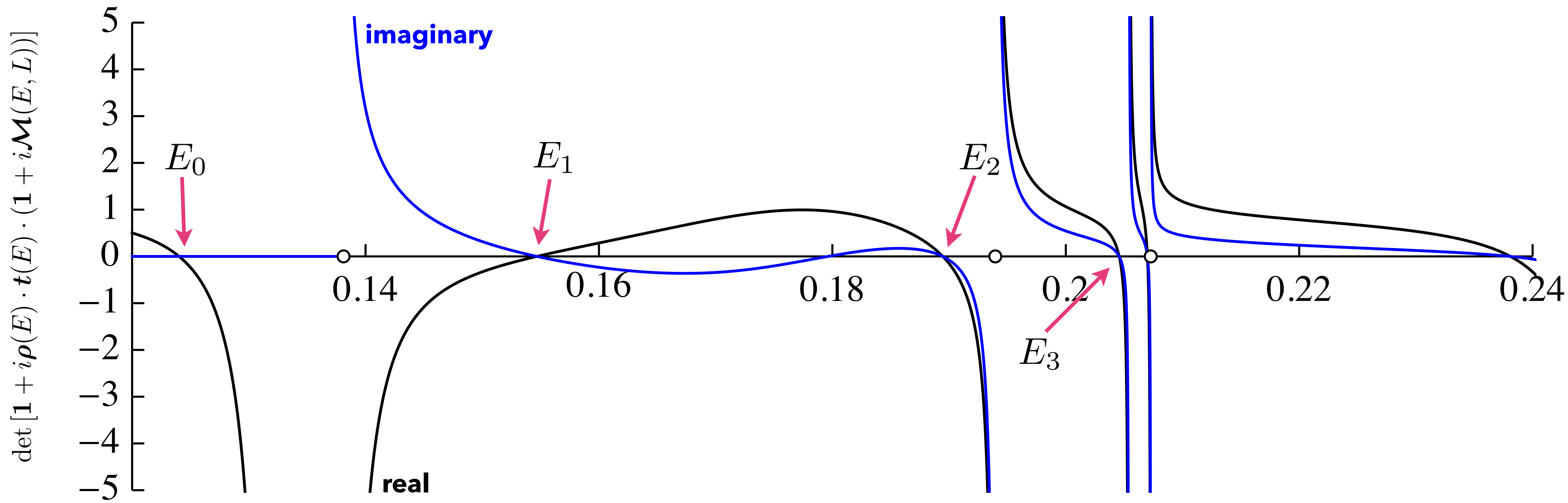
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

$$S_{ii} = \eta e^{2i\delta_i}$$



we can identify the zeros



$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

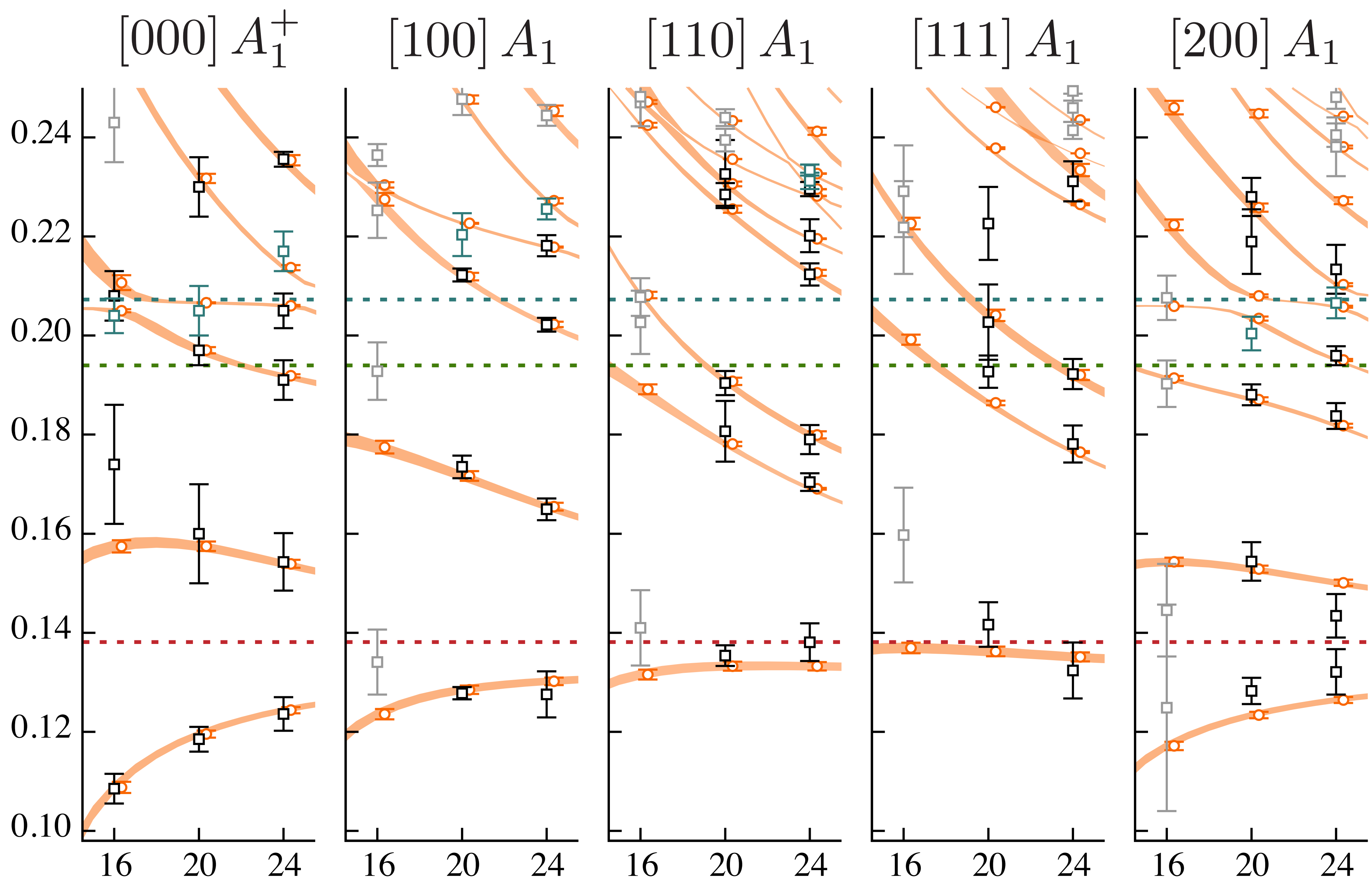
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

An example S-wave spectrum fit

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$



$$\chi^2/N_{\text{dof}} = \frac{44.0}{57-8} = 0.90$$

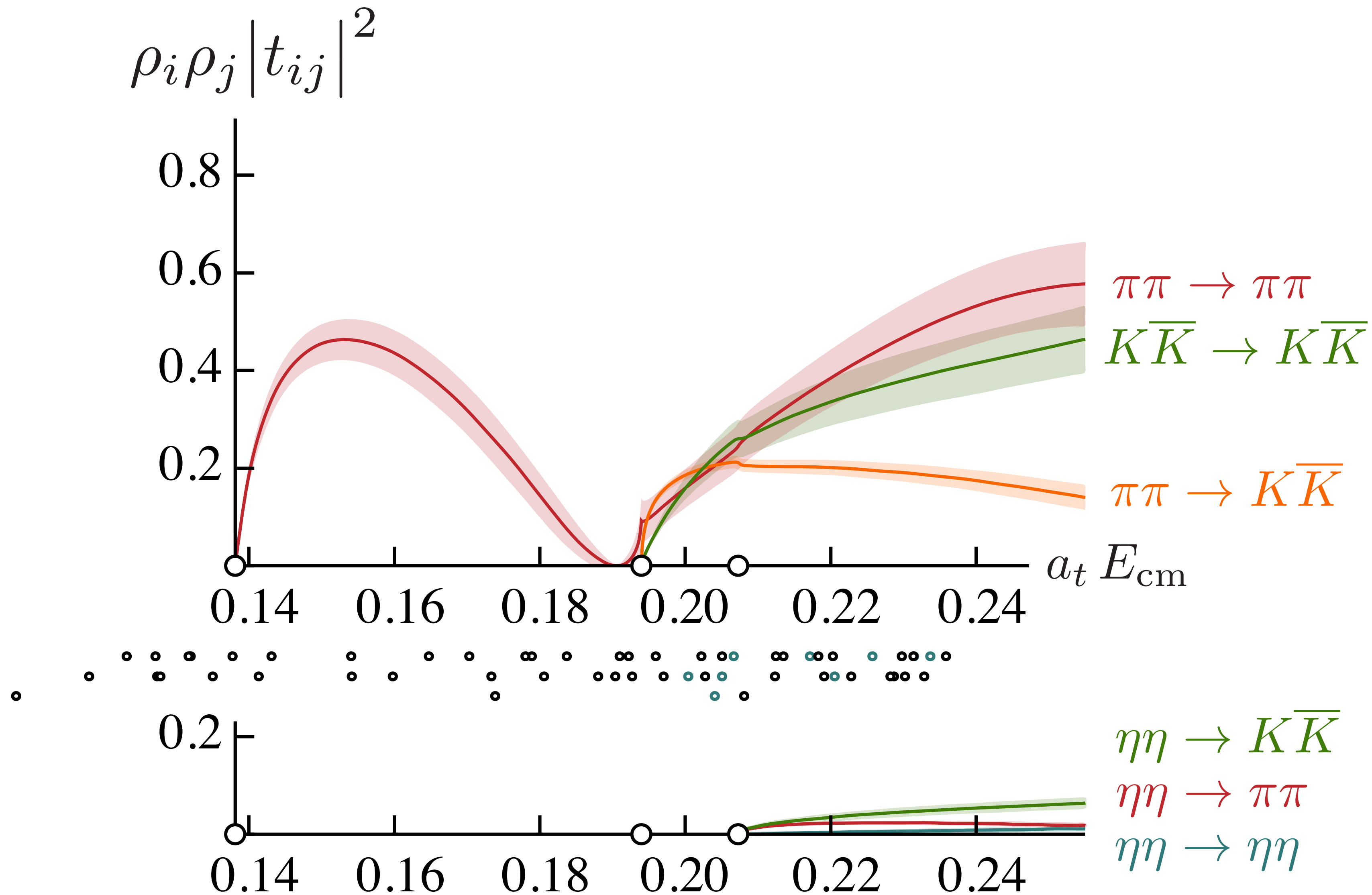
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



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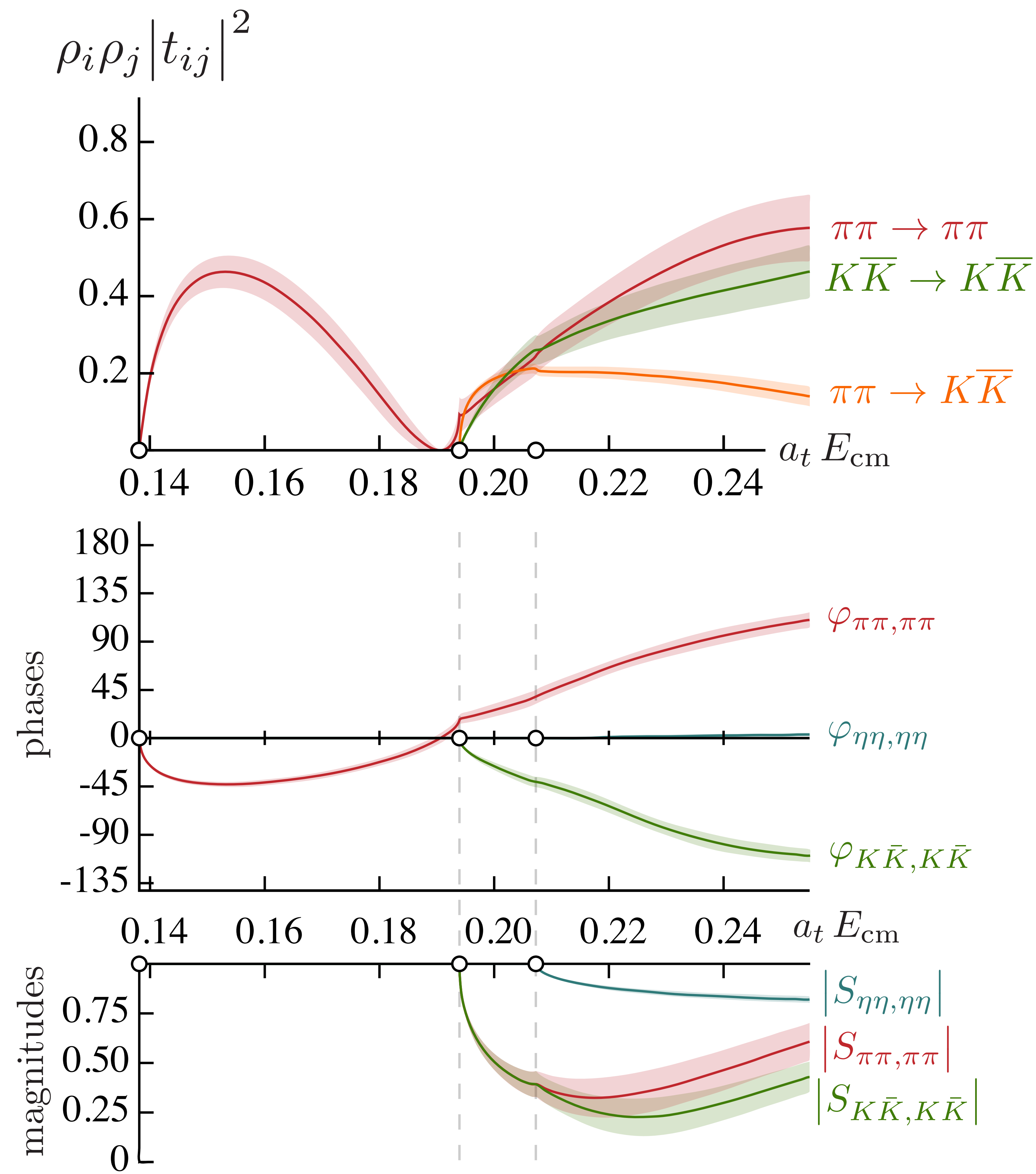
$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

$$S_{ii}(E_{\mathsf{cm}}) = |S_{ii}(E_{\mathsf{cm}})|\, e^{2i\phi_{ii}(E_{\mathsf{cm}})}$$



An example D-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

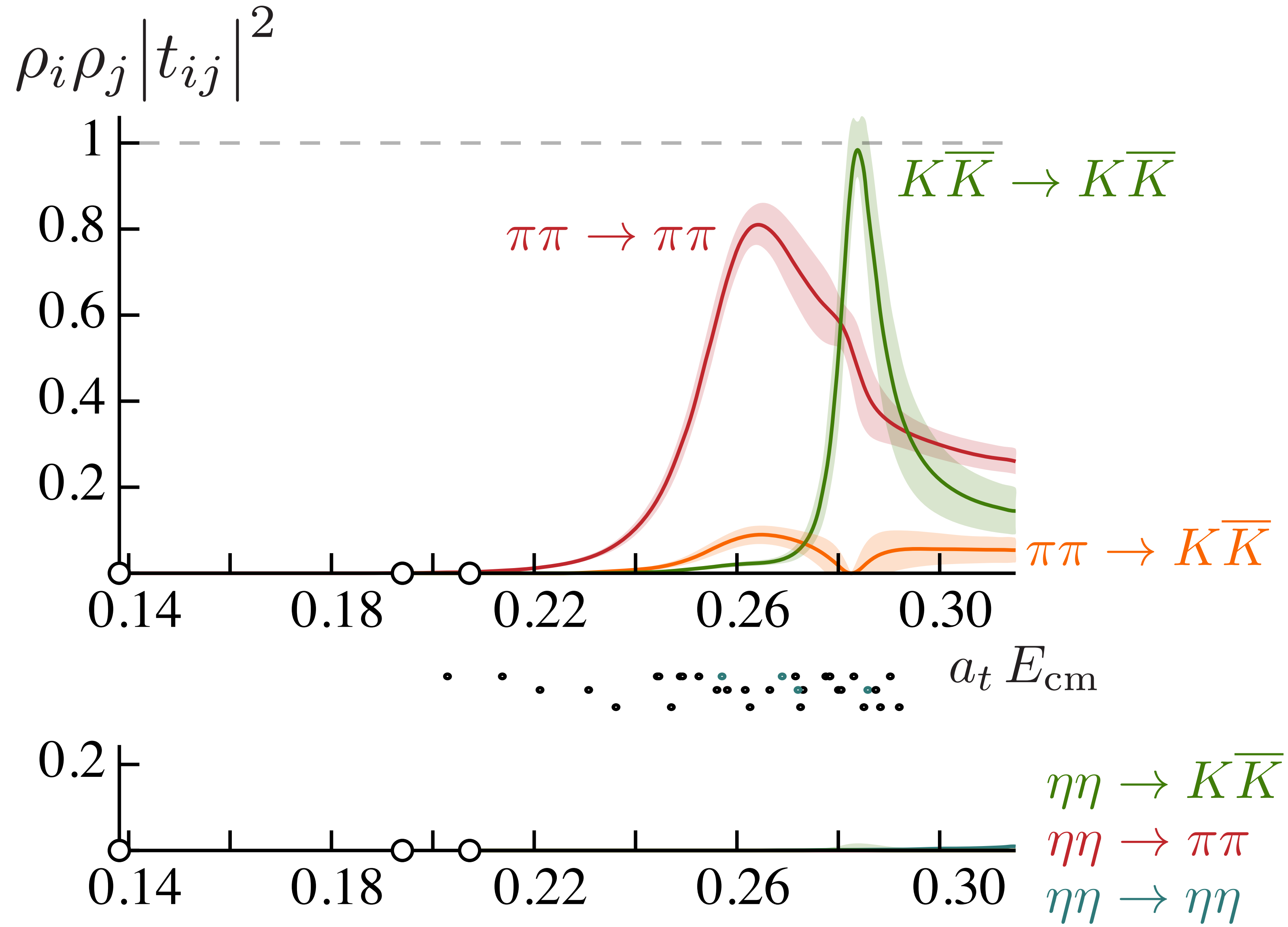
$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

$\gamma_{\eta\eta} \neq 0$

$\gamma_{ij} = 0$ otherwise

$$\chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15$$

34 energy levels



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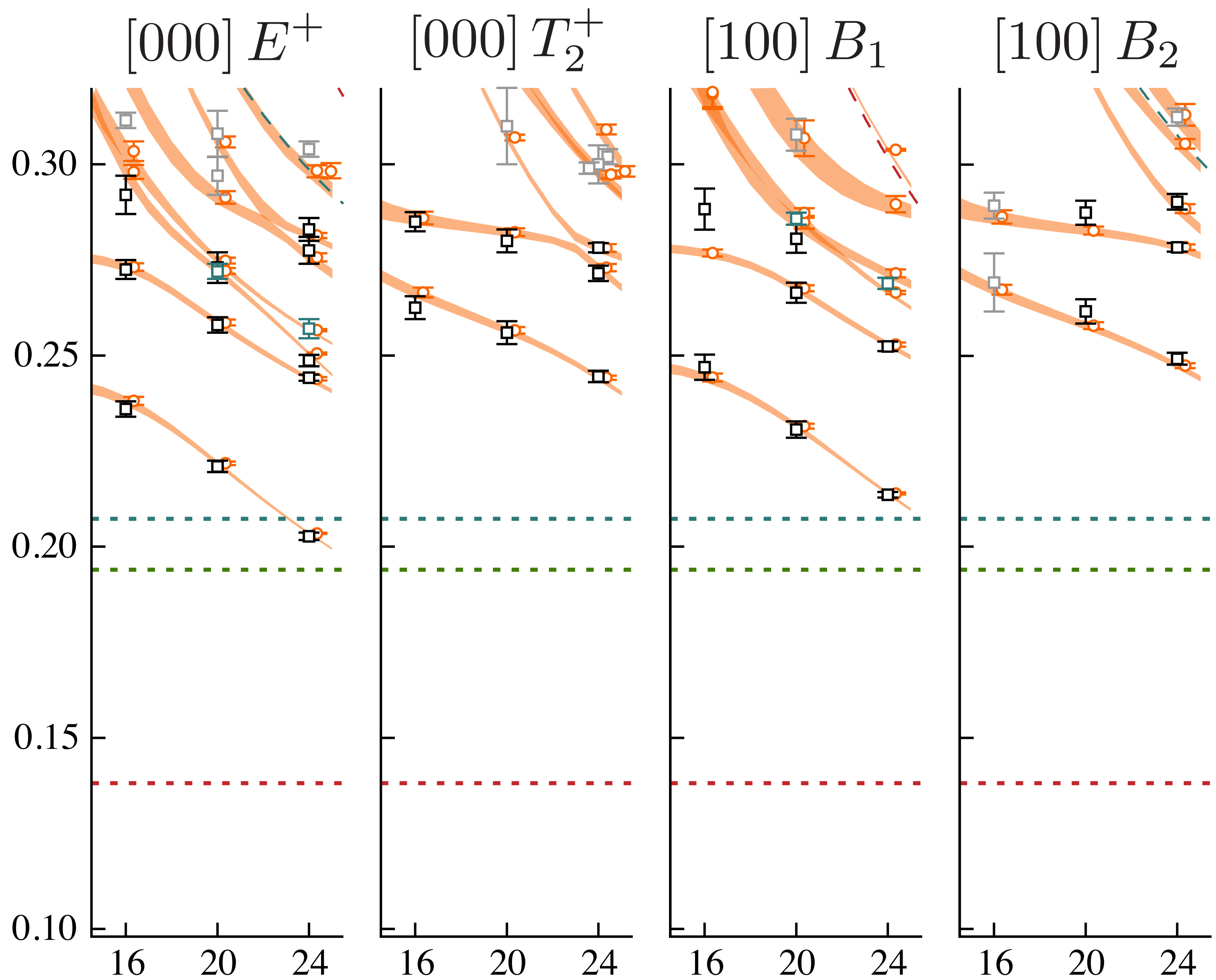

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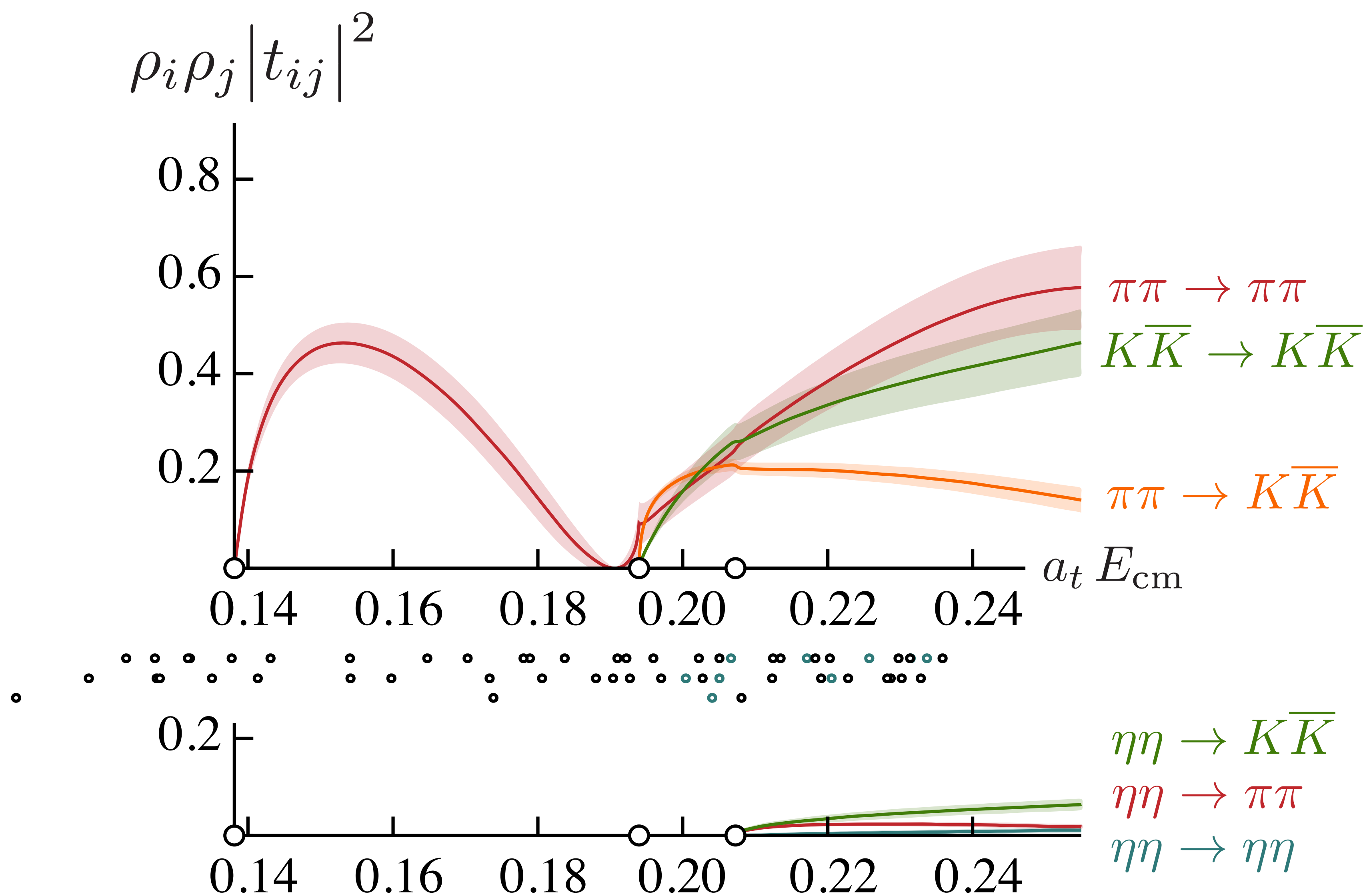
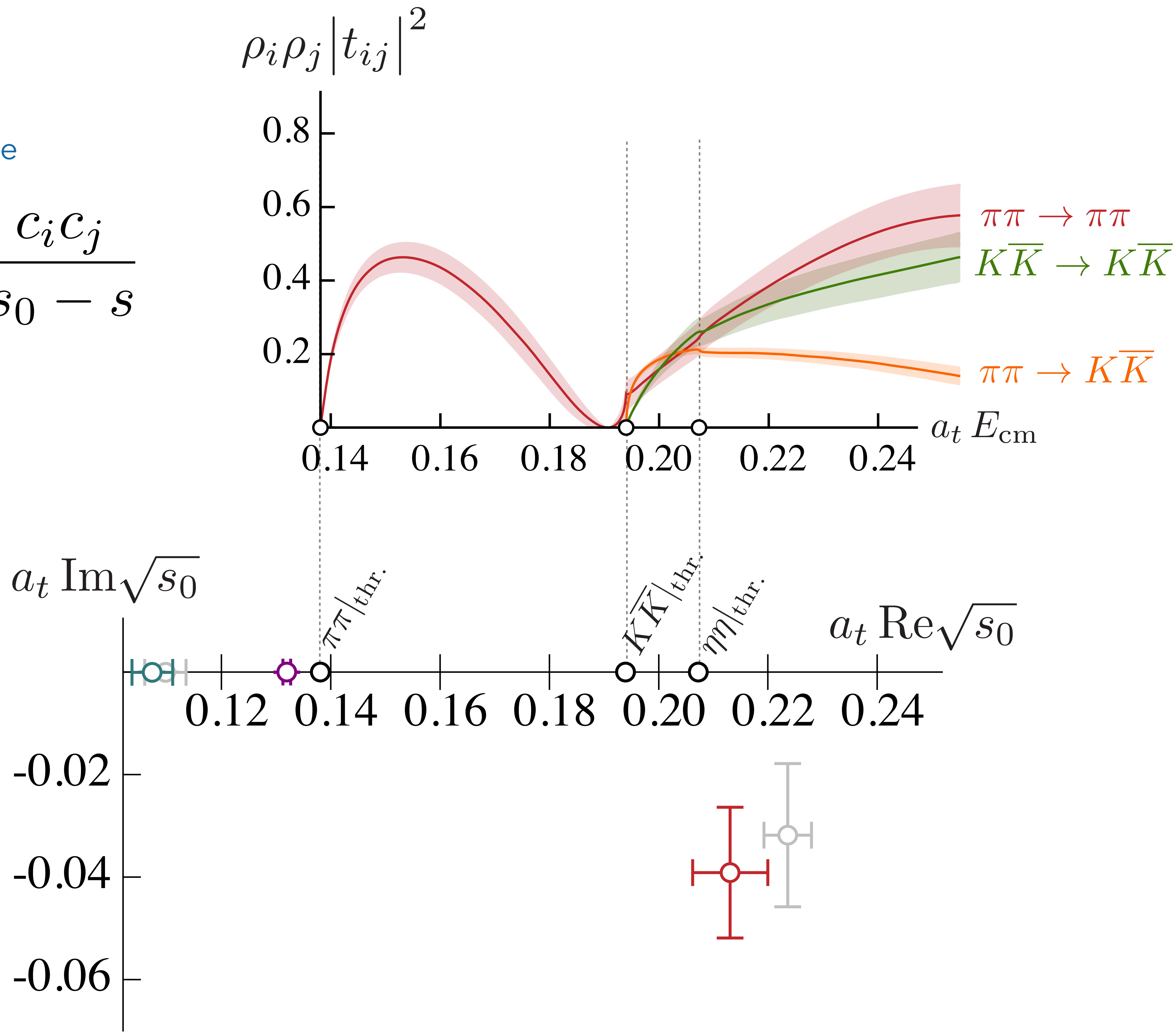
57 energy levels



Near a t-matrix pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

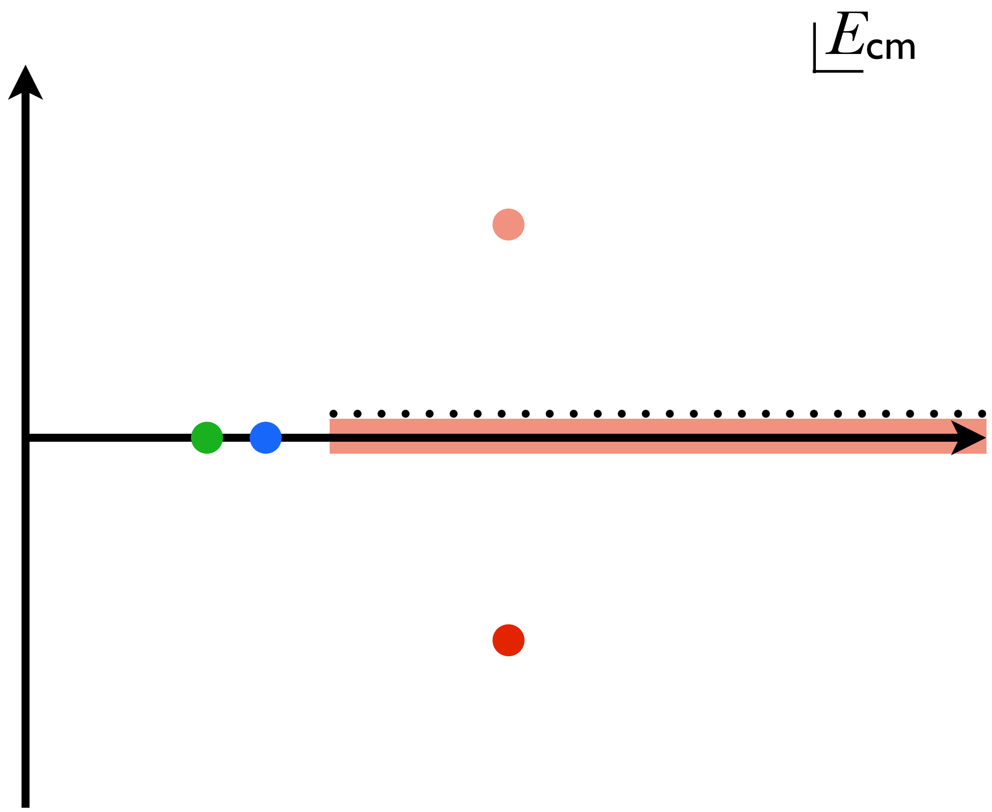


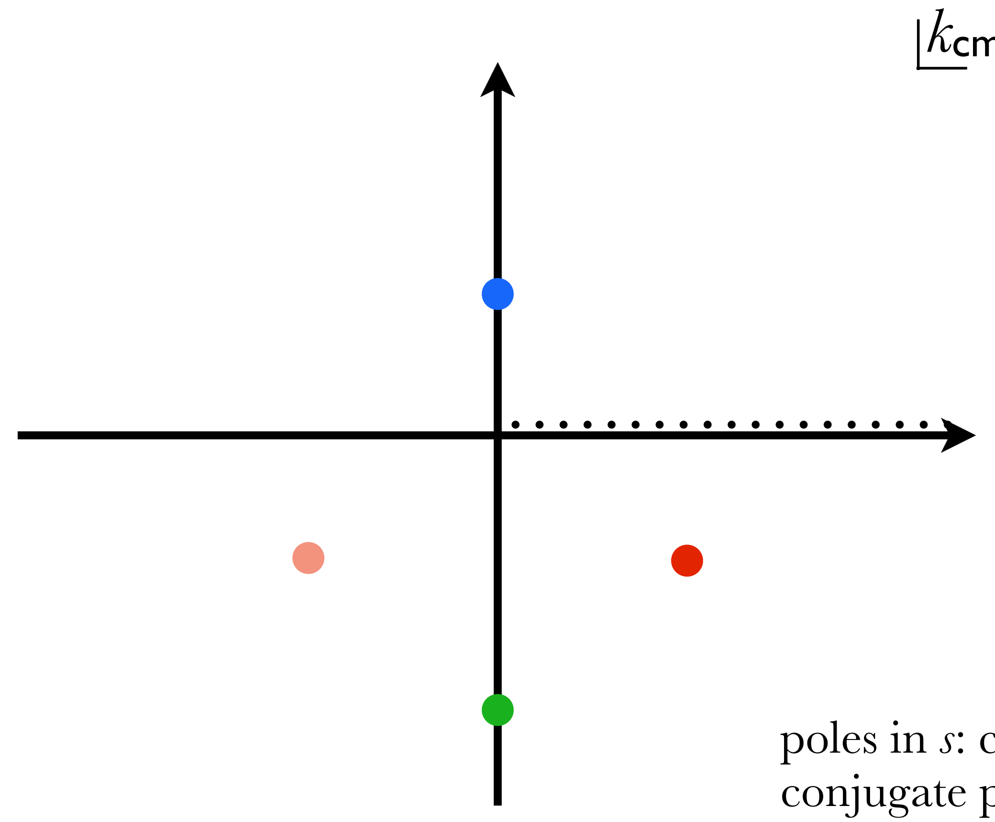
sheet I: bound state
sheet II: resonance

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$k_{cm} = \pm\frac{1}{2}\left(E_{cm}^2 - 4m^2\right)^{\frac{1}{2}}$$

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$





poles in s: complex conjugate pairs in k

Bound state

Resonance

Virtual Bound state

for n-channels, there are 2^n sheets

$$k_{cm} = \pm\frac{1}{2}\left(E_{cm}^2 - 4m^2\right)^{\frac{1}{2}}$$

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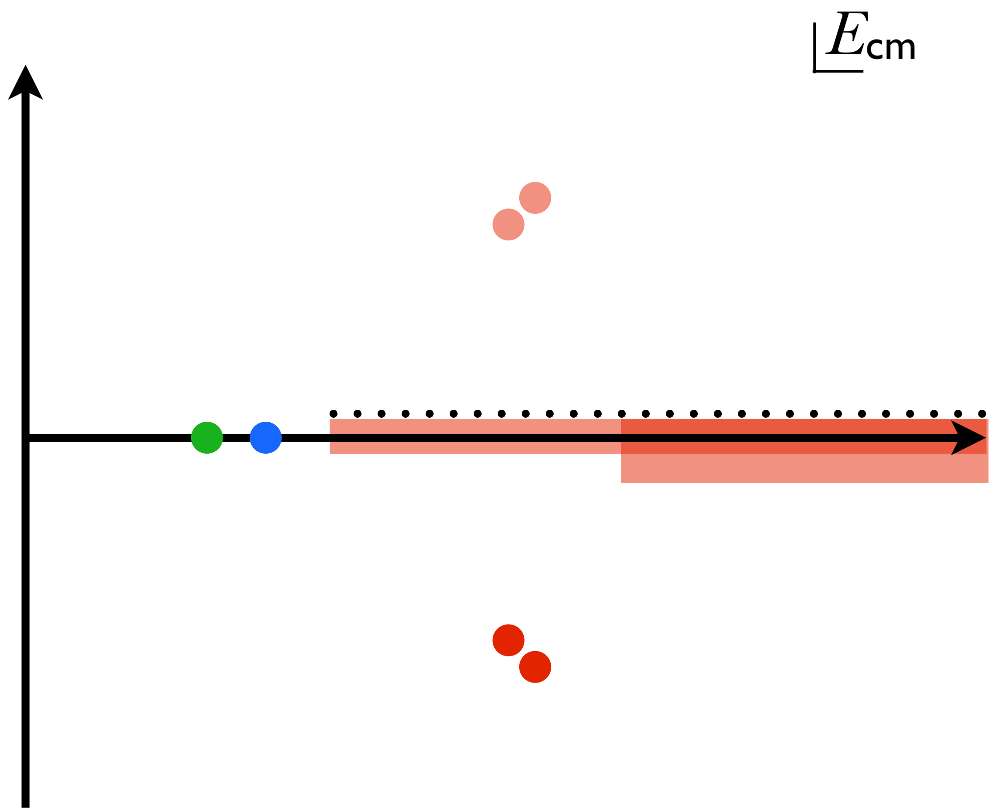

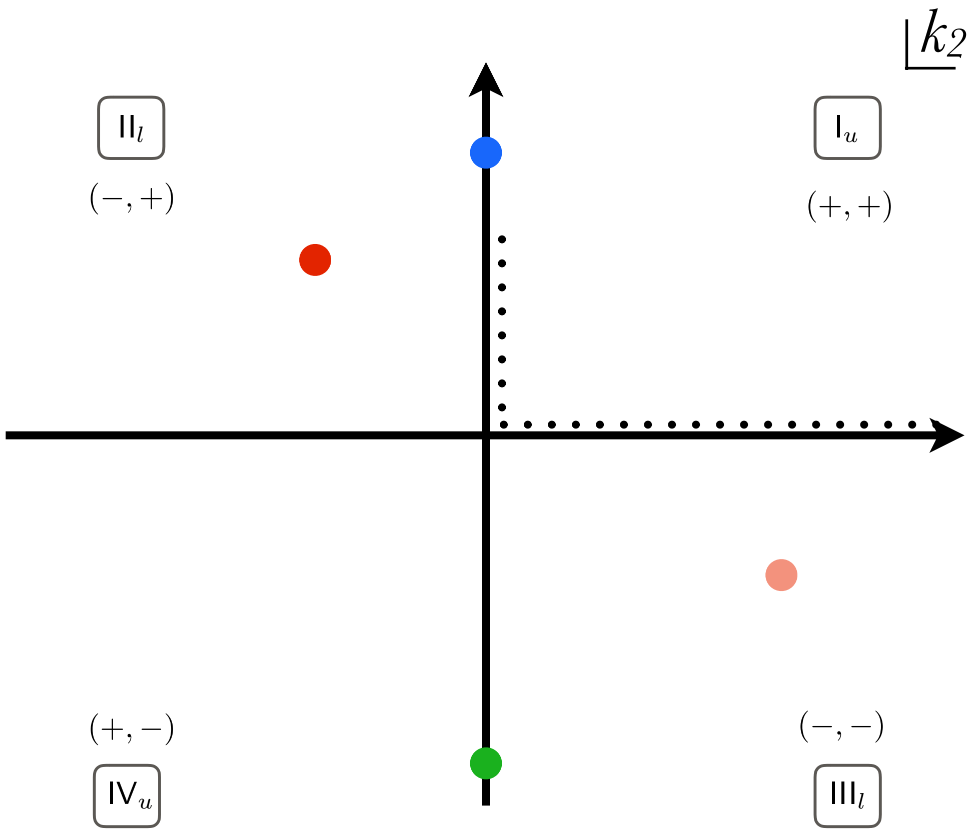

Bound state

Resonances

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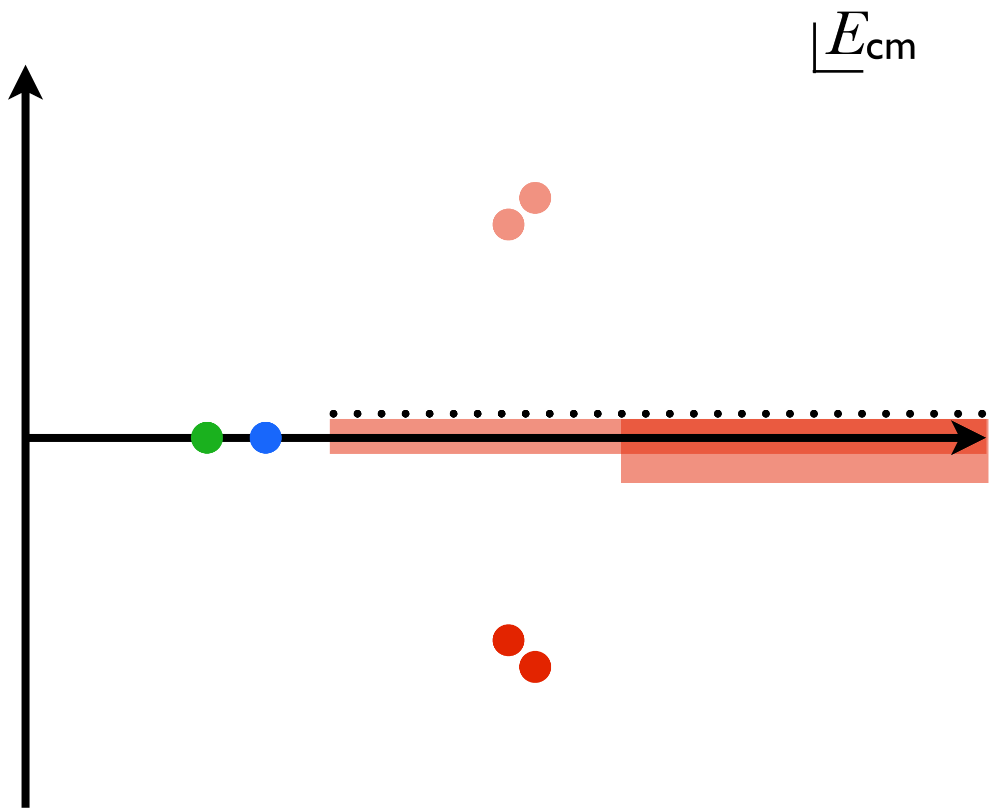
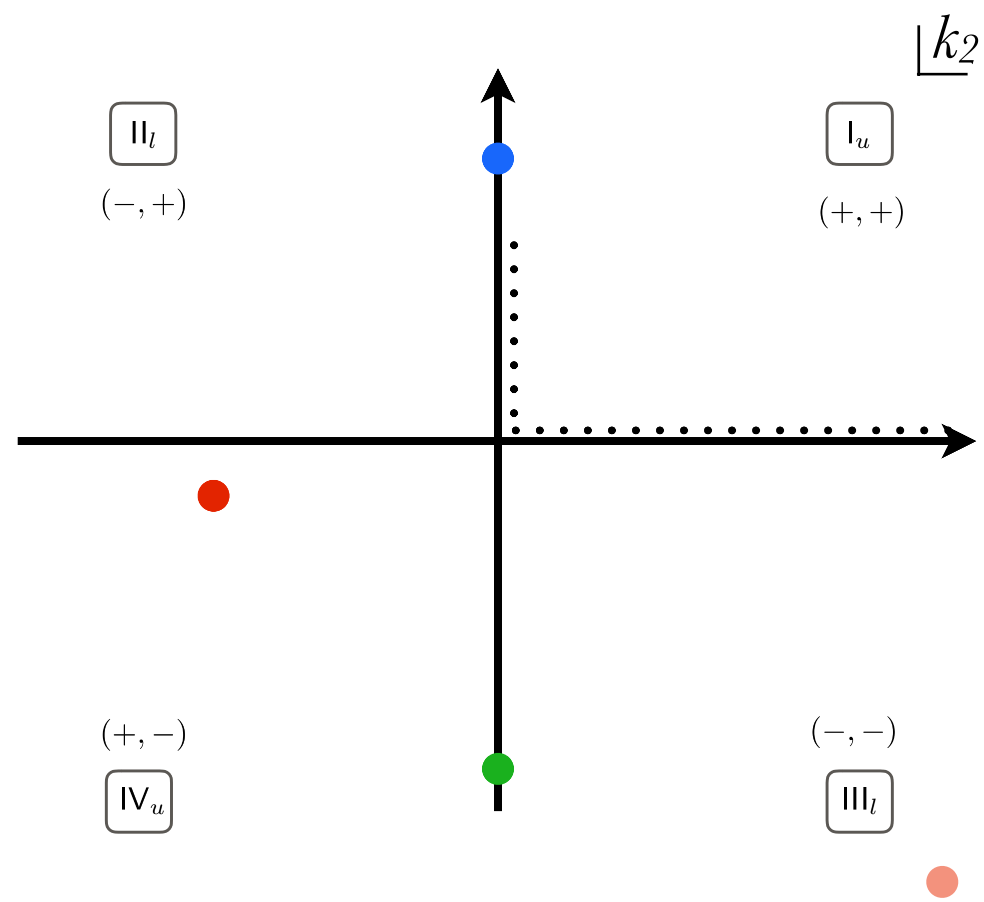




label sheets by signs of Im(k)

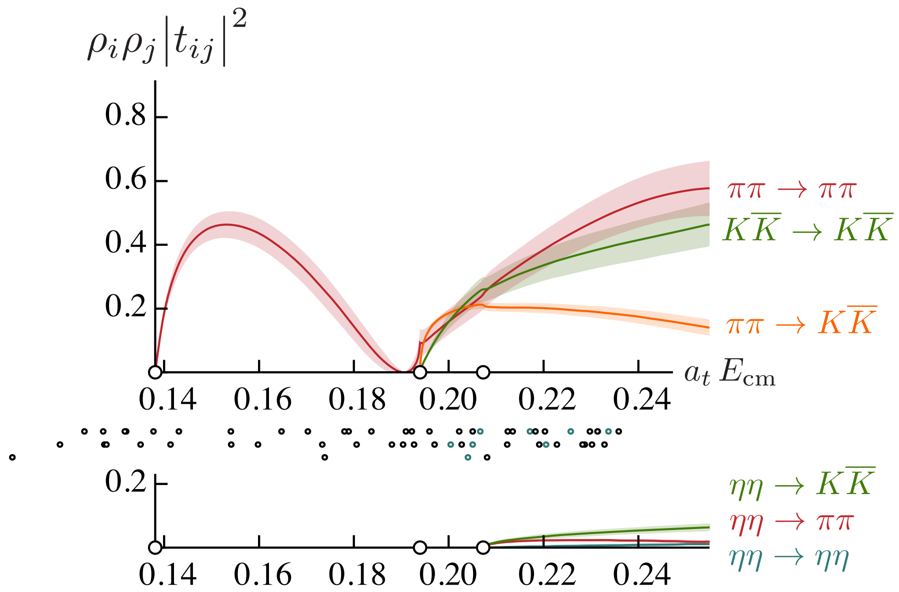
many distributions of pole positions possible

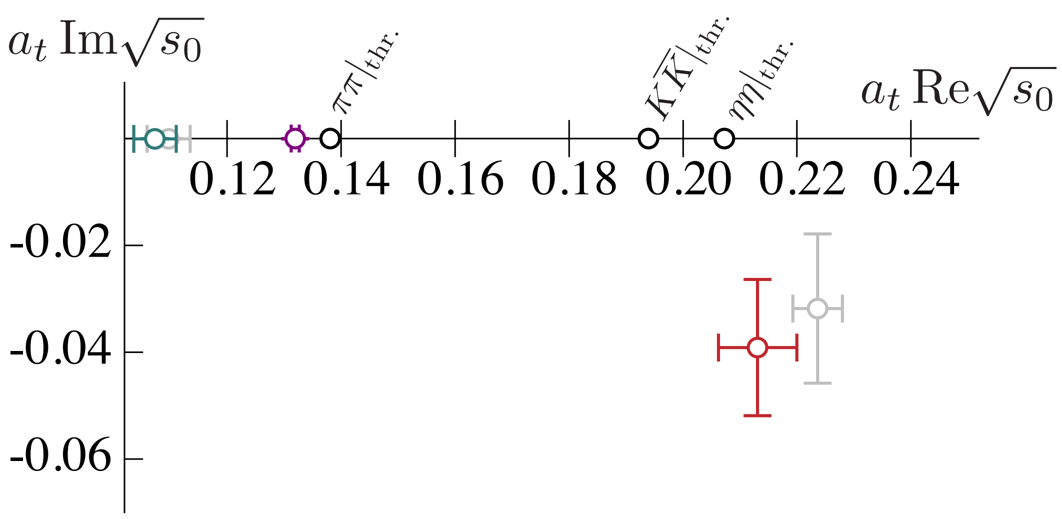
in some cases they can tell us about the composition the state

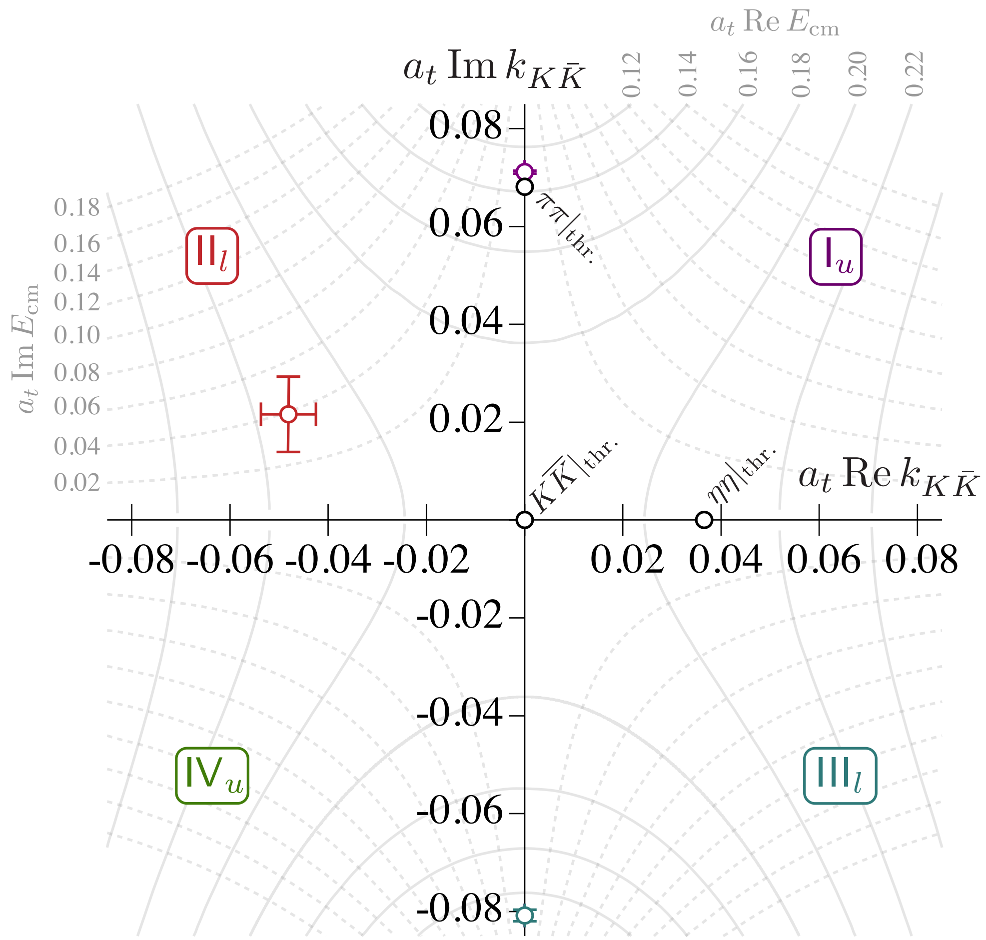
Bound state

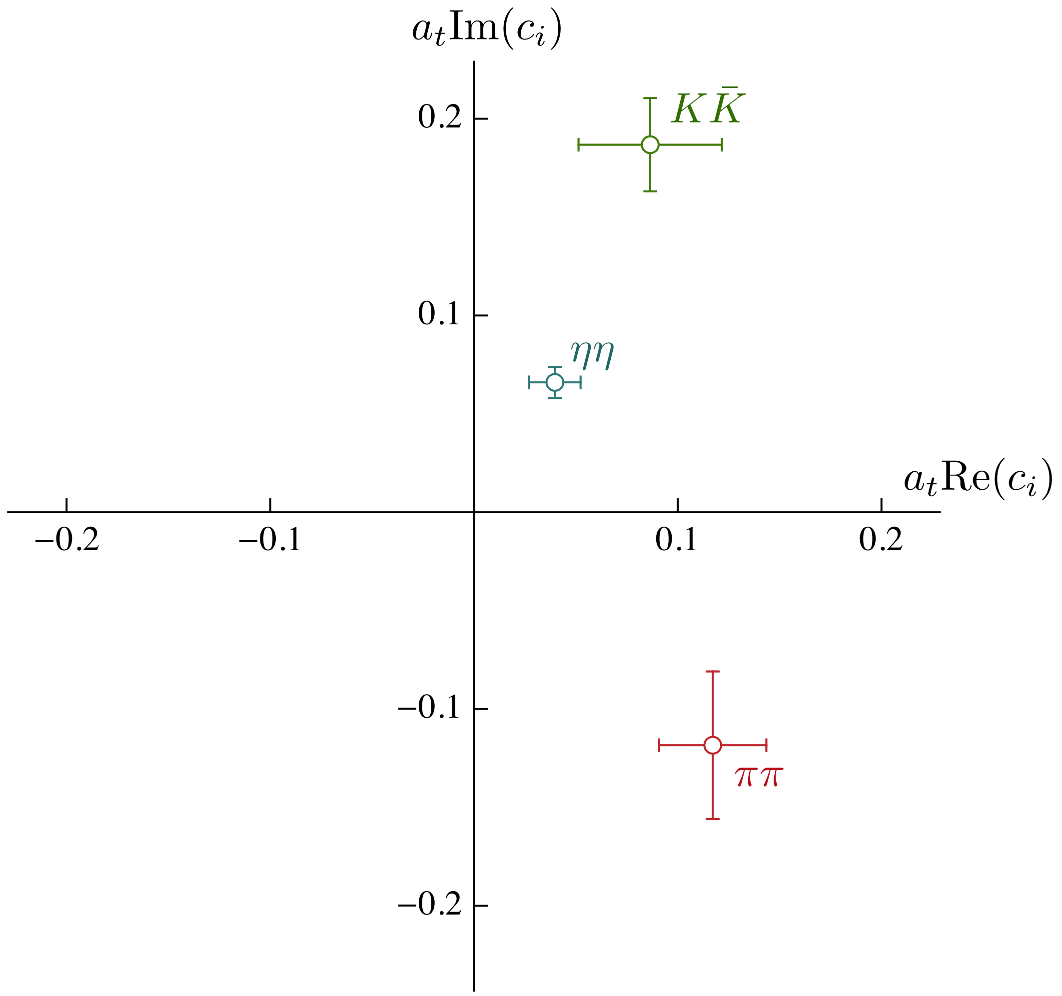
Resonances

Virtual Bound state


$\rho_i \rho_j |t_{ij}|^2$
$\pi\pi \to \pi\pi$
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
$a_t E_{cm}$
$\eta\eta \to K\overline{K}$
$\eta\eta \to \pi\pi$
$\eta\eta \to \eta\eta$

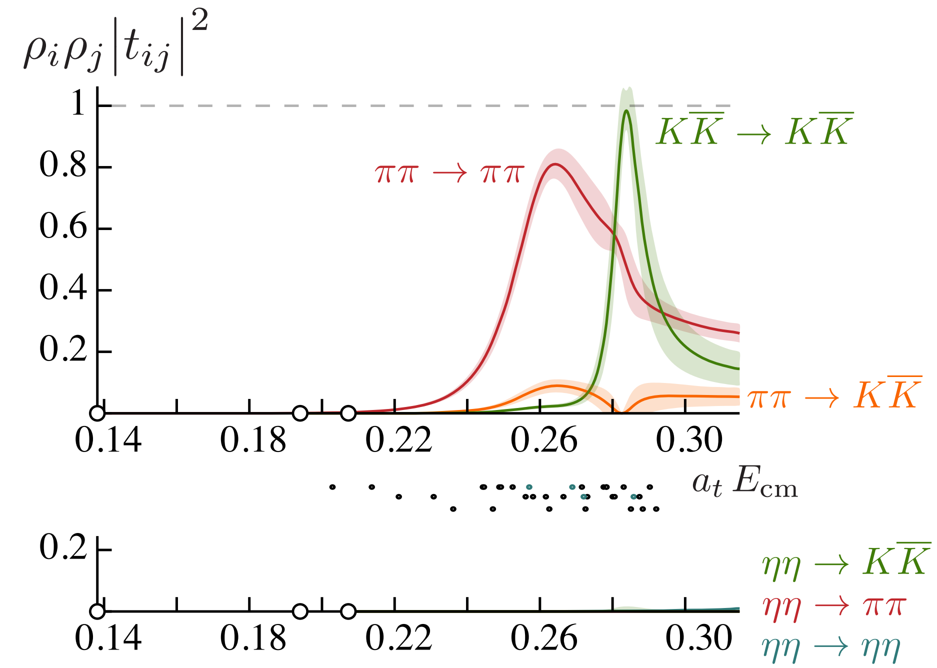

$a_t \operatorname{Im}\sqrt{s_0}$
$a_t \operatorname{Re}\sqrt{s_0}$
$\pi\pi|_{thr.}$
$K\overline{K}|_{thr.}$
$\eta\eta|_{thr.}$

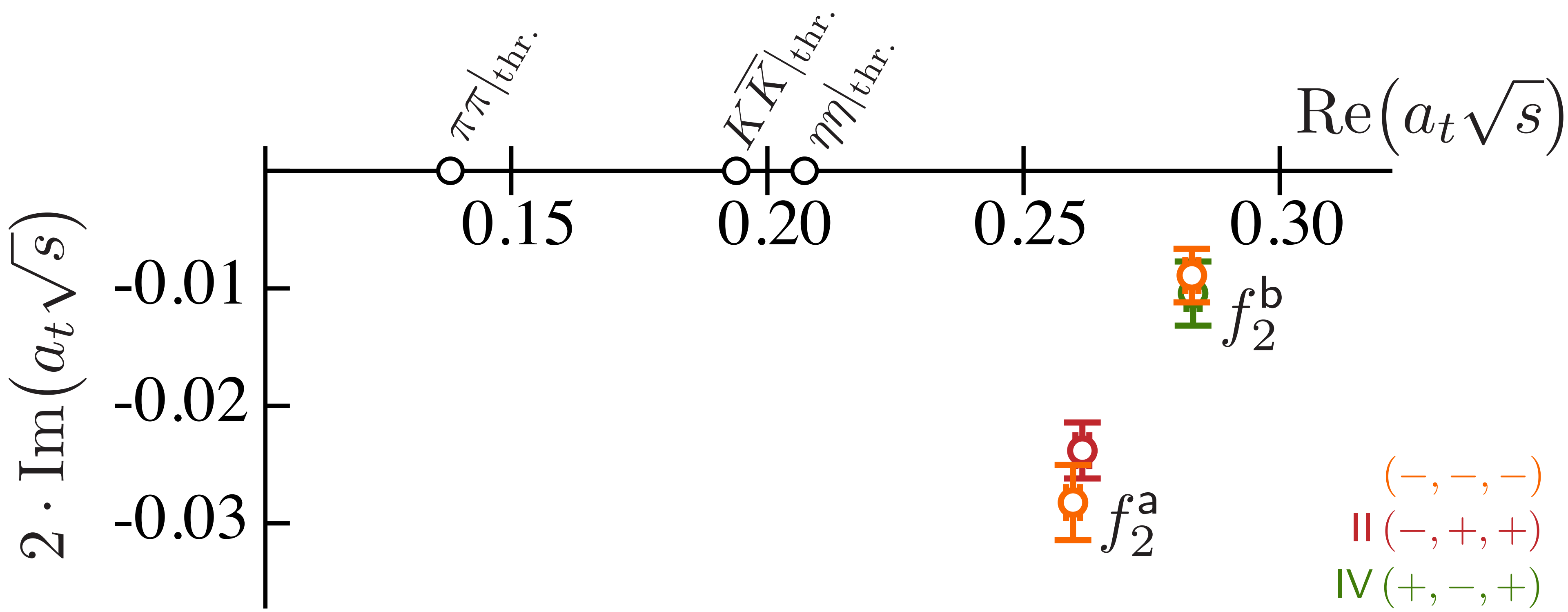

$a_t \operatorname{Im} k_{K\bar{K}}$
$a_t \operatorname{Re} k_{K\bar{K}}$
$a_t \operatorname{Re} E_{cm}$
$a_t \operatorname{Im} E_{cm}$
II_l
I_u
IV_u
III_l
$\pi\pi|_{thr.}$
$K\overline{K}|_{thr.}$
$\eta\eta|_{thr.}$


$a_t\operatorname{Im}(c_i)$
$a_t\operatorname{Re}(c_i)$
$K\bar{K}$
$\eta\eta$
$\pi\pi$

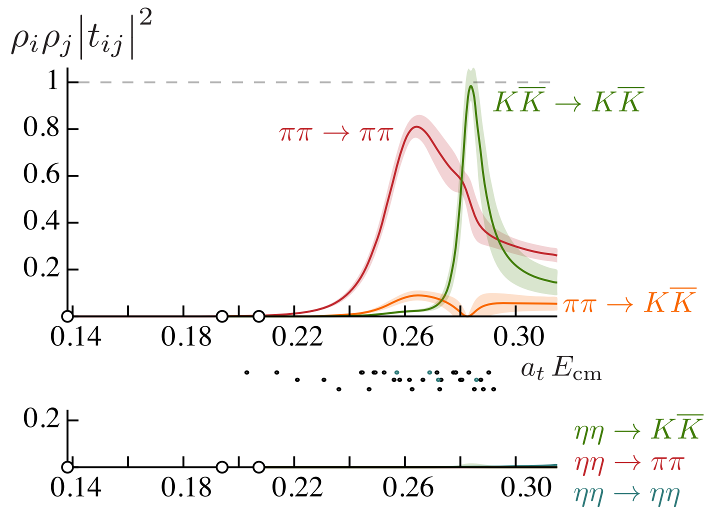
Near a t-matrix pole

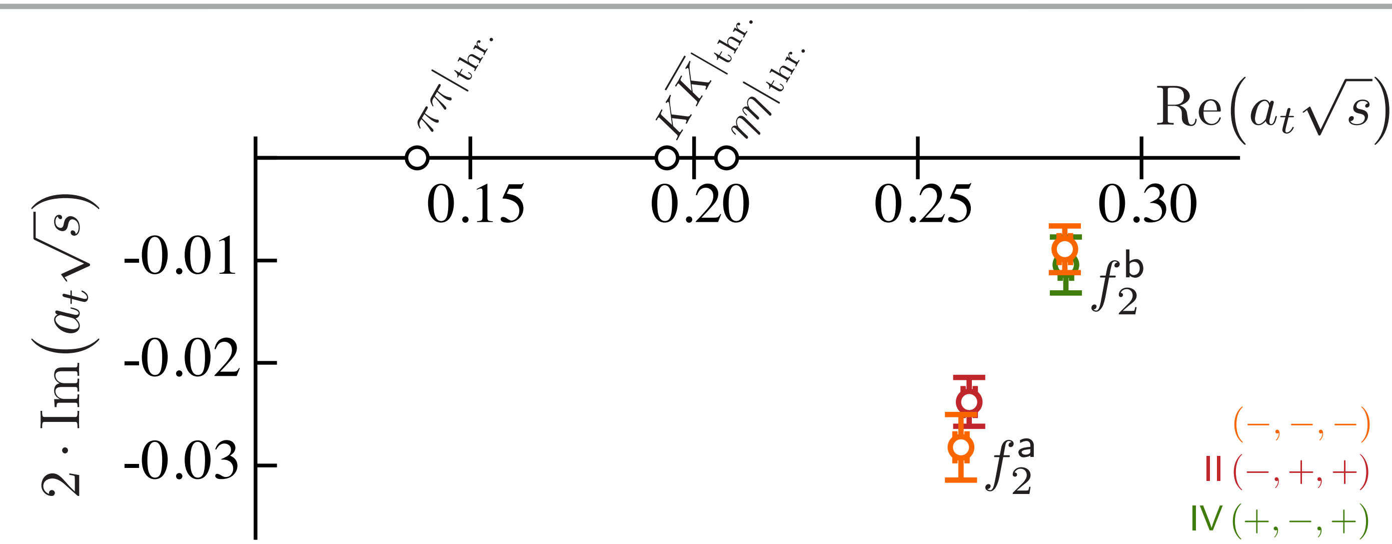
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

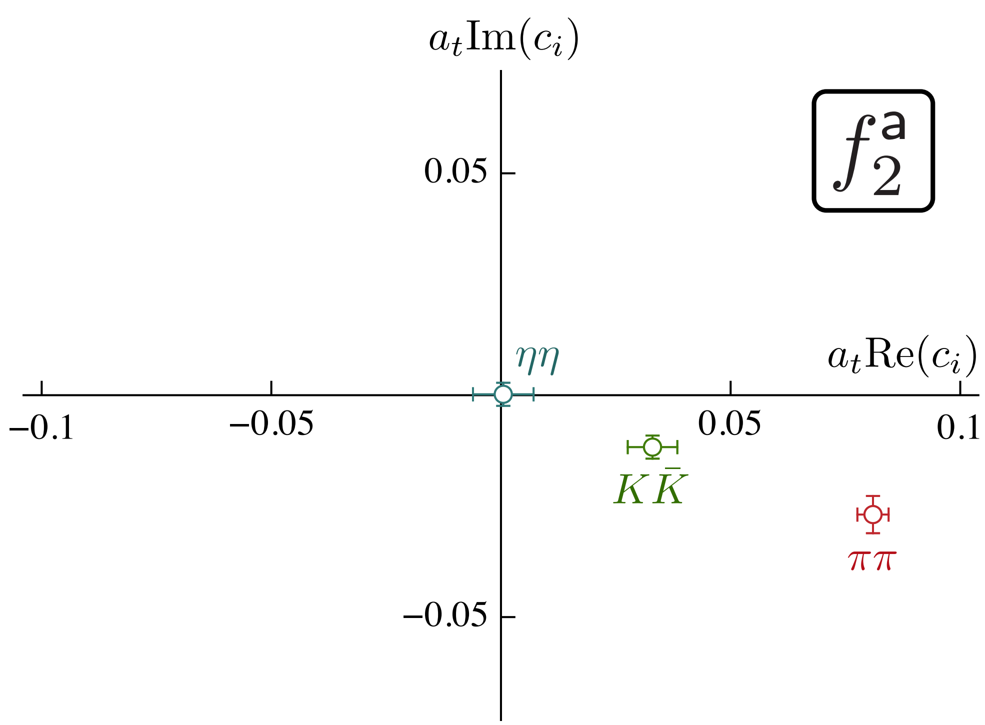


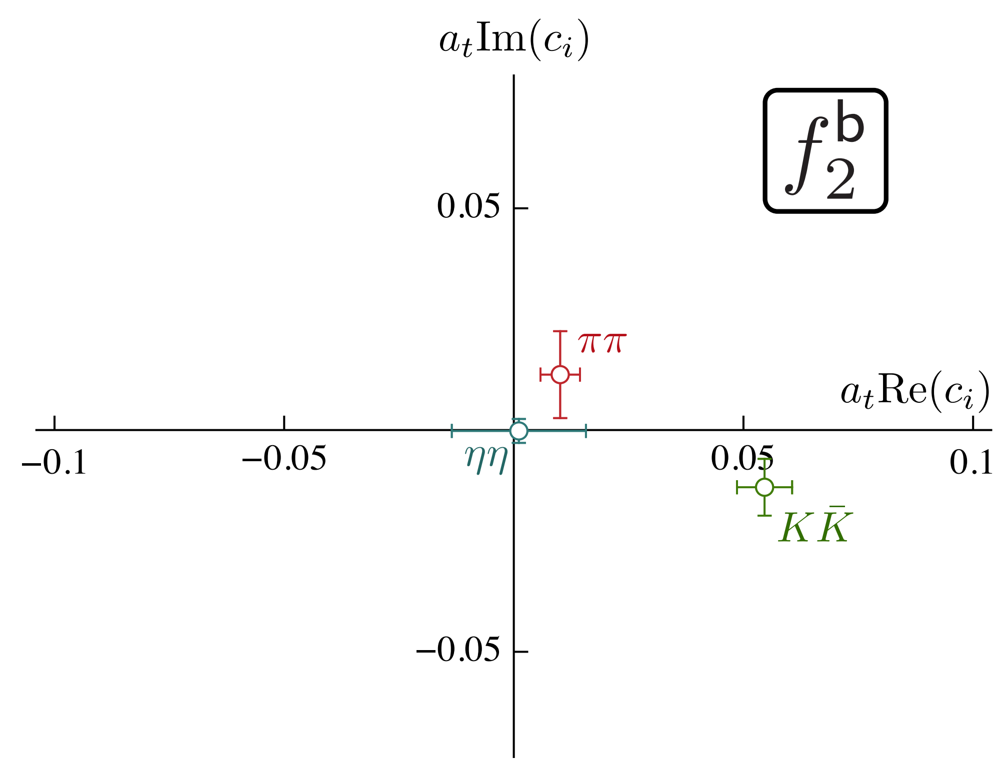


Tensor resonance poles







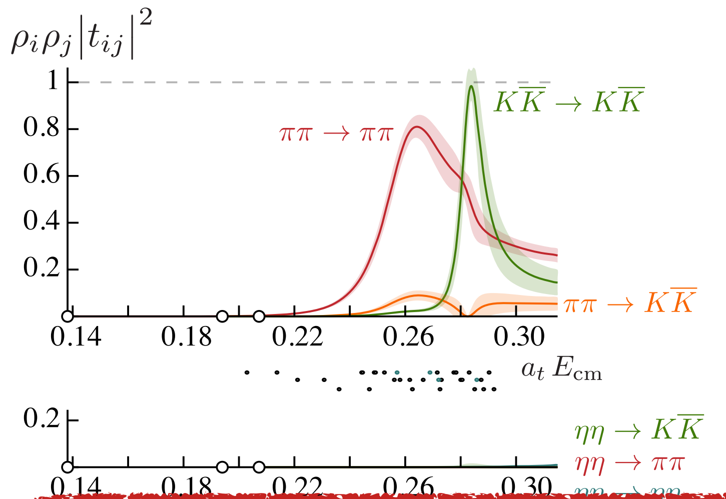


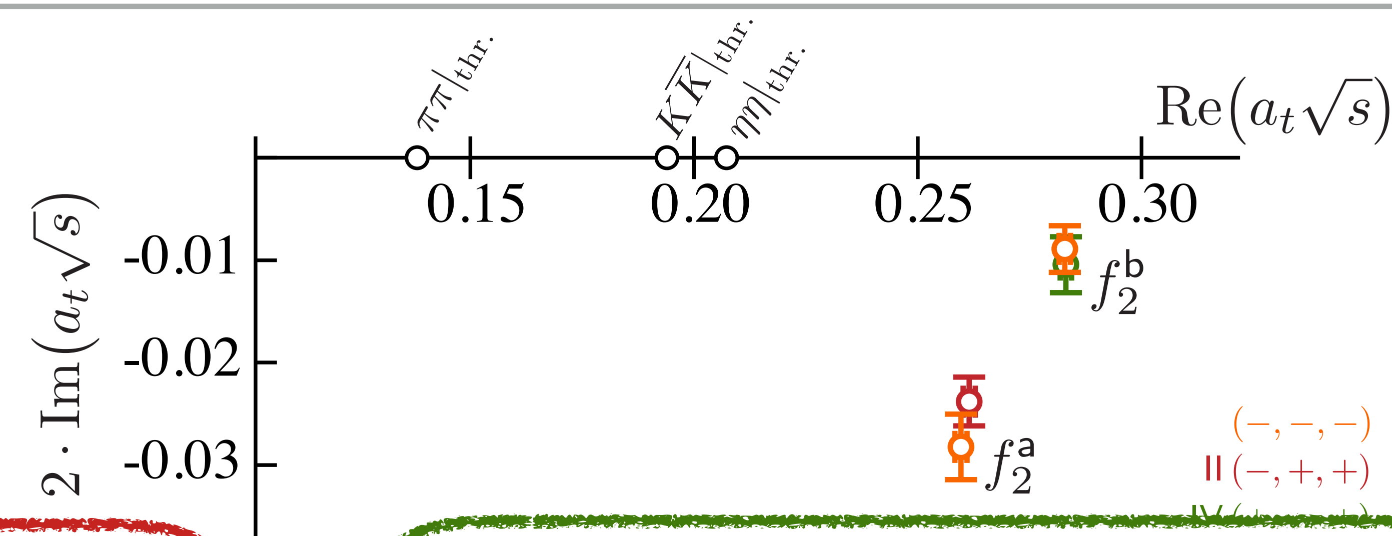
$$f_2^{\rm a}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$$

$$\mathrm{Br}(f_2^{\rm a} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\rm a} \to K\overline{K}) \sim 12\%$$

$$f_2^{\rm b}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)\ \mathrm{MeV}$$

$$\mathrm{Br}(f_2^{\rm b} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\rm b} \to K\overline{K}) \sim 92\%$$





$f_2(1270)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(7.7\ ^{+1.1}_{-3.2})$ %
Γ_3	$K\overline{K}$	$(4.6\ ^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	(2.8 ± 0.4) %

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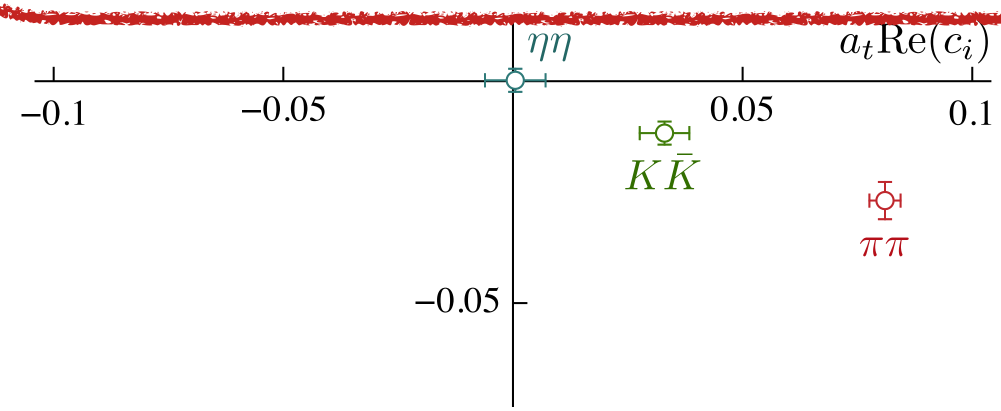
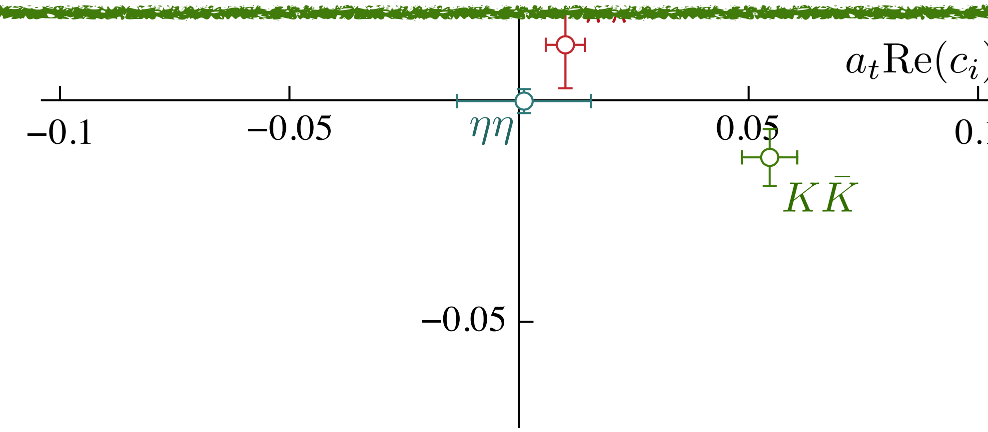
$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	(88.7 ± 2.2) %
Γ_2	$\eta\eta$	(10.4 ± 2.2) %
Γ_3	$\pi\pi$	$(8.2 \pm 1.5) \times 10^{-3}$
Γ_4	$K\overline{K}^*(892)$ + c.c.	



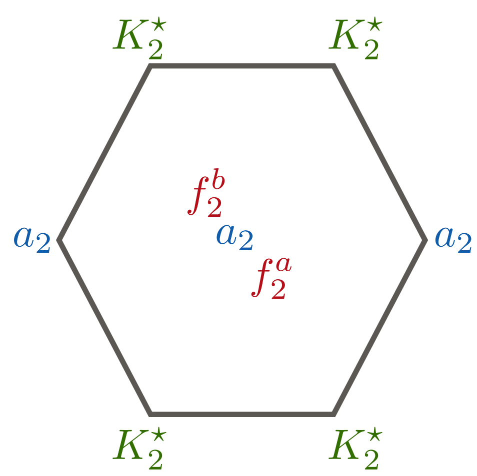


$f_2^a:\quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

$\mathrm{Br}(f_2^a \to \pi\pi) \sim 85\%,\quad \mathrm{Br}(f_2^a \to K\overline{K}) \sim 12\%$

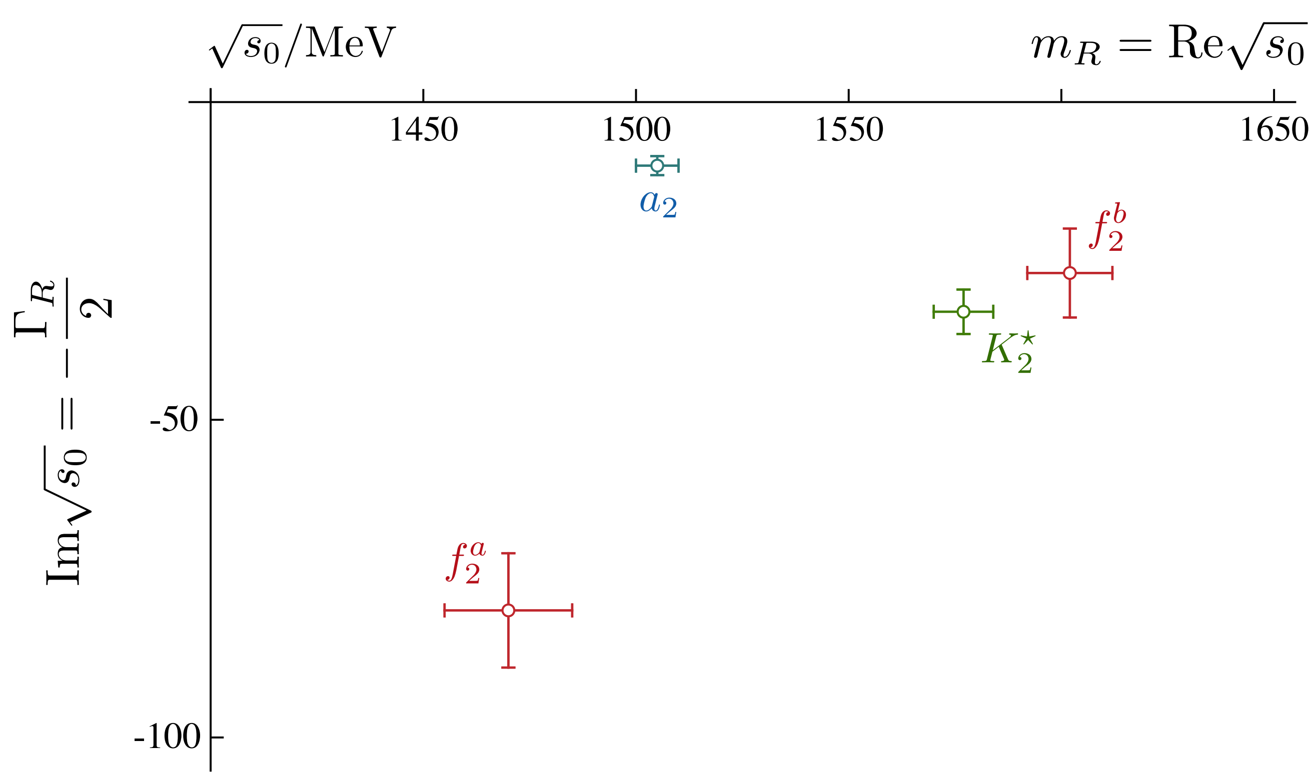
$f_2^b:\quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

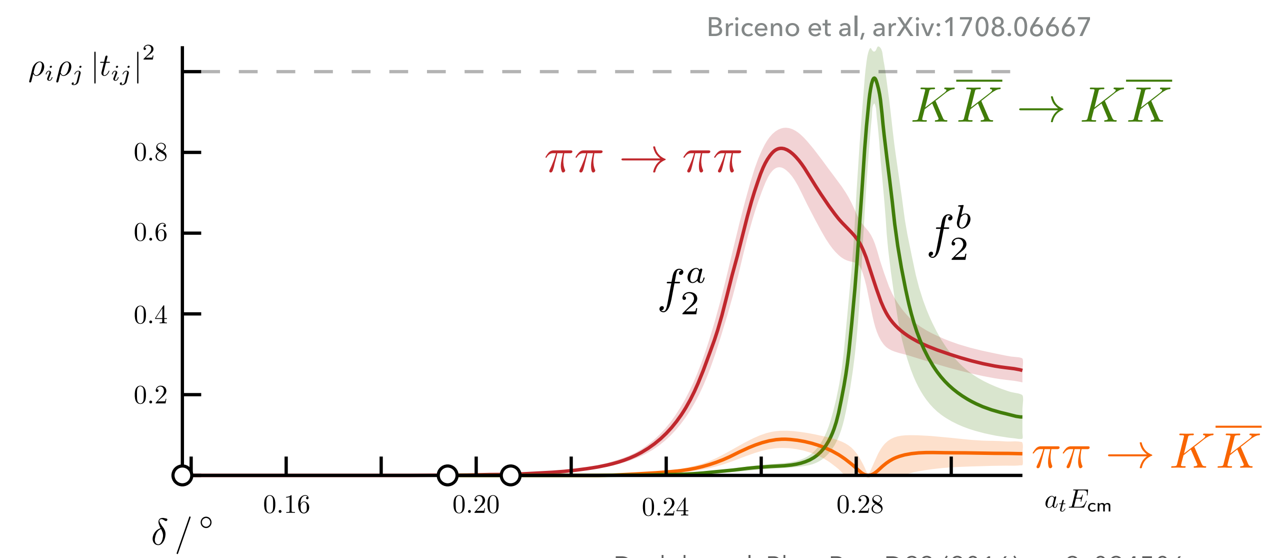
$\mathrm{Br}(f_2^b \to \pi\pi) \sim 8\%,\quad \mathrm{Br}(f_2^b \to K\overline{K}) \sim 92\%$

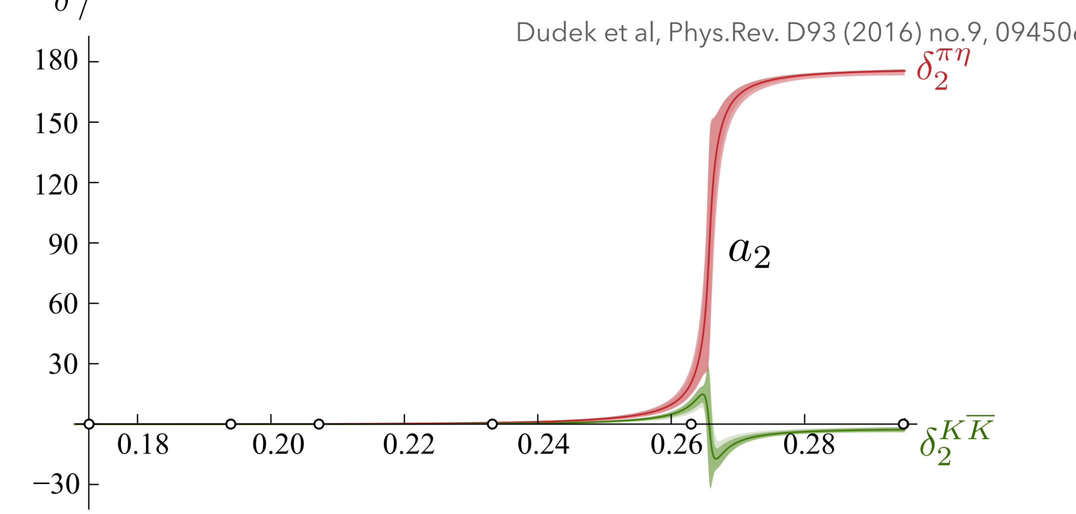


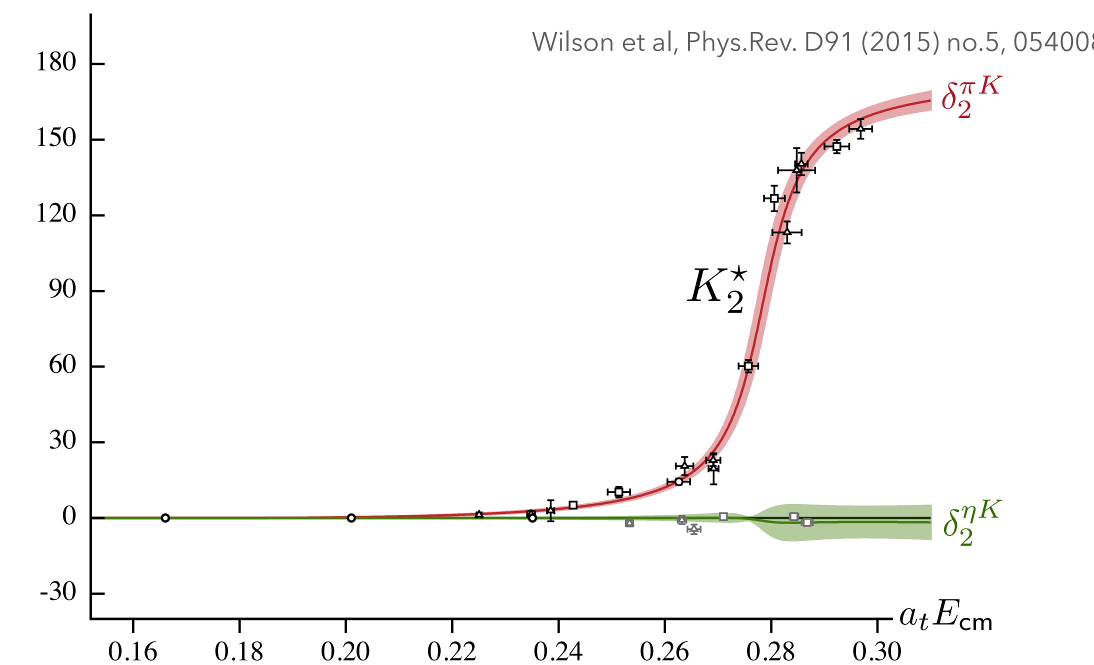
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

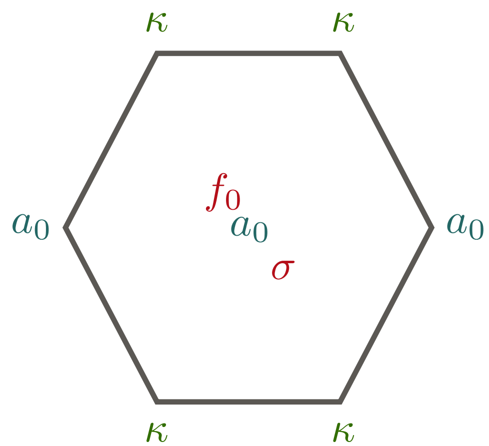
$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$











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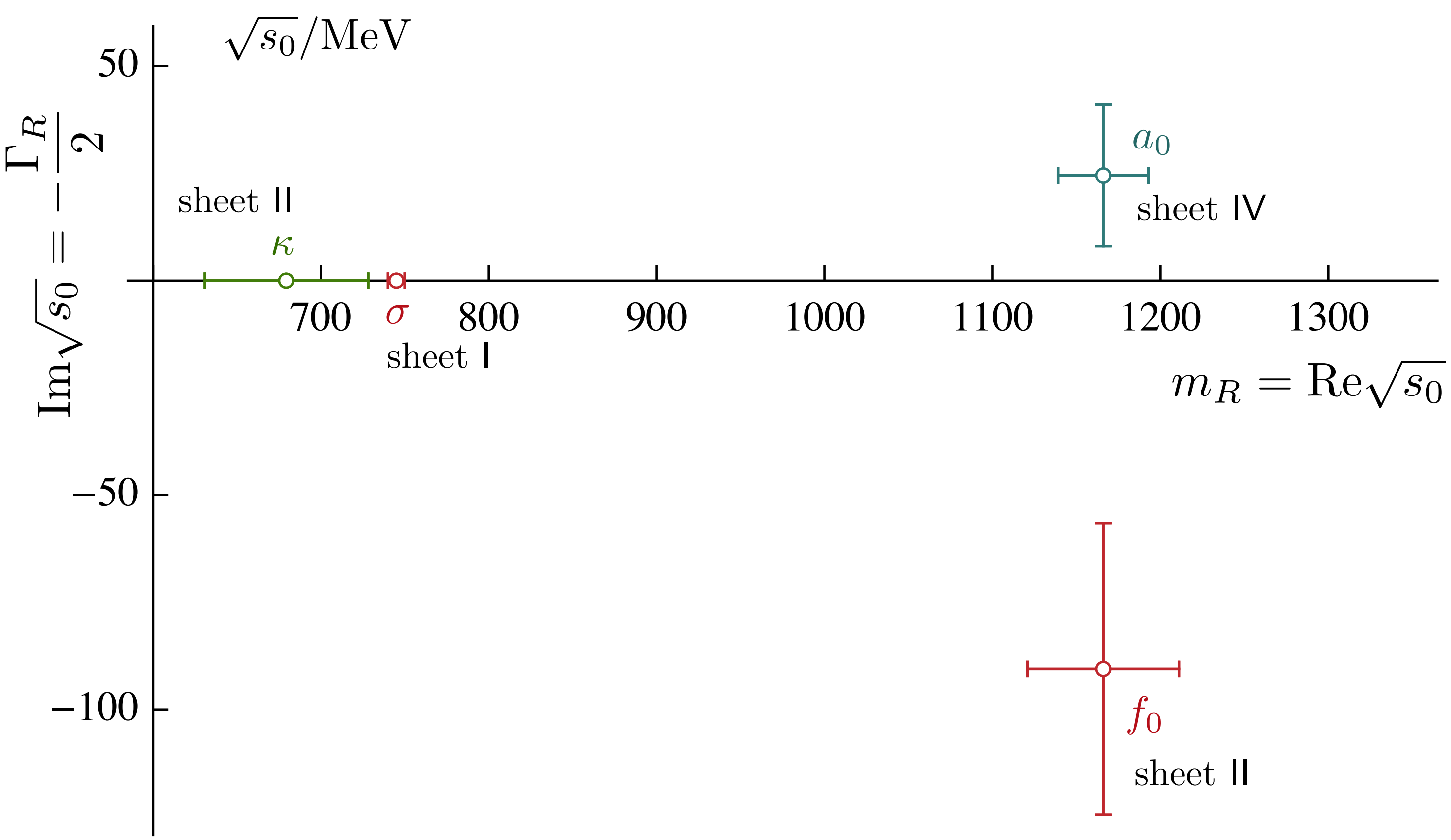
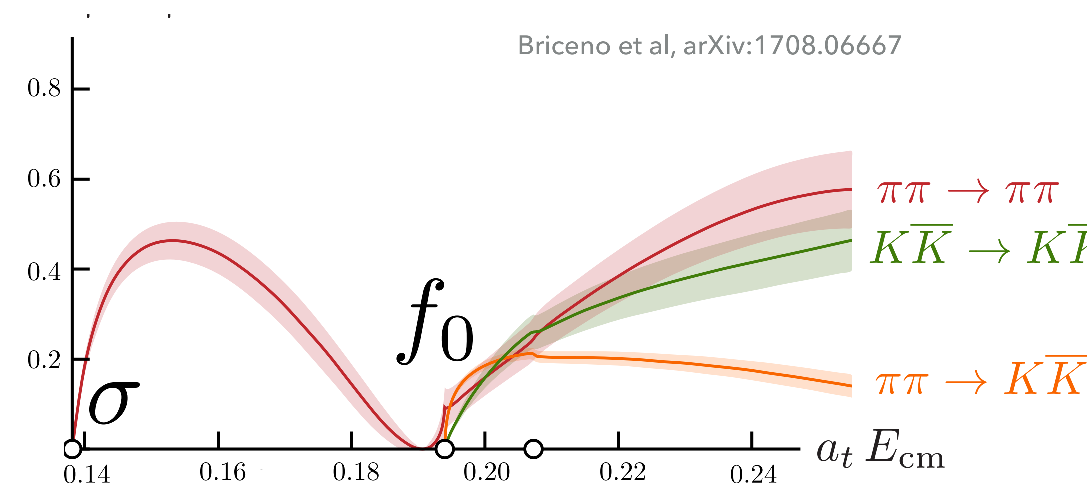
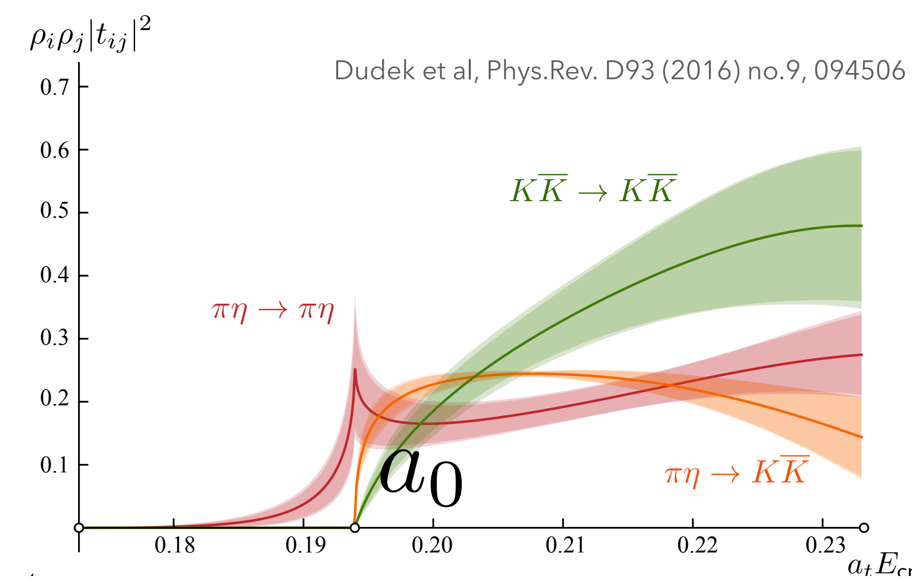
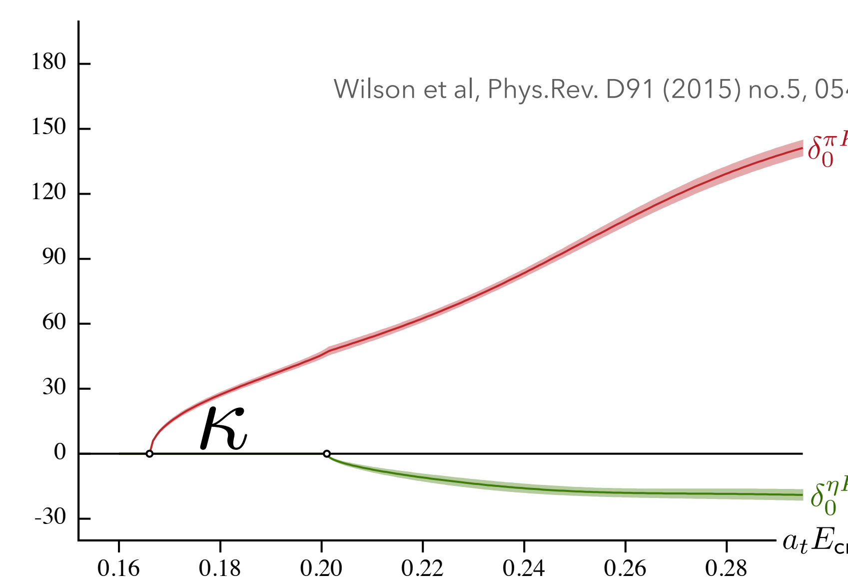






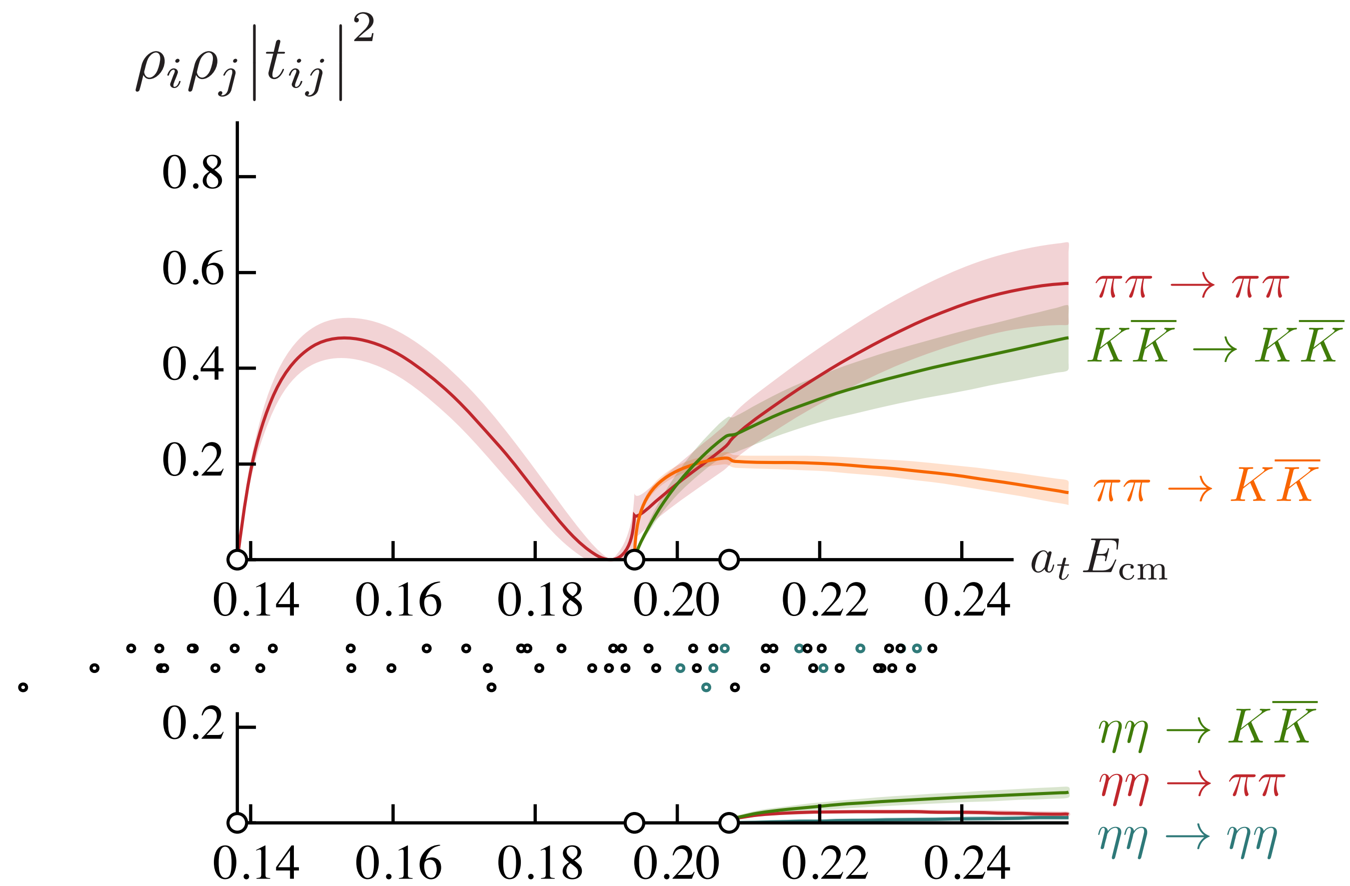


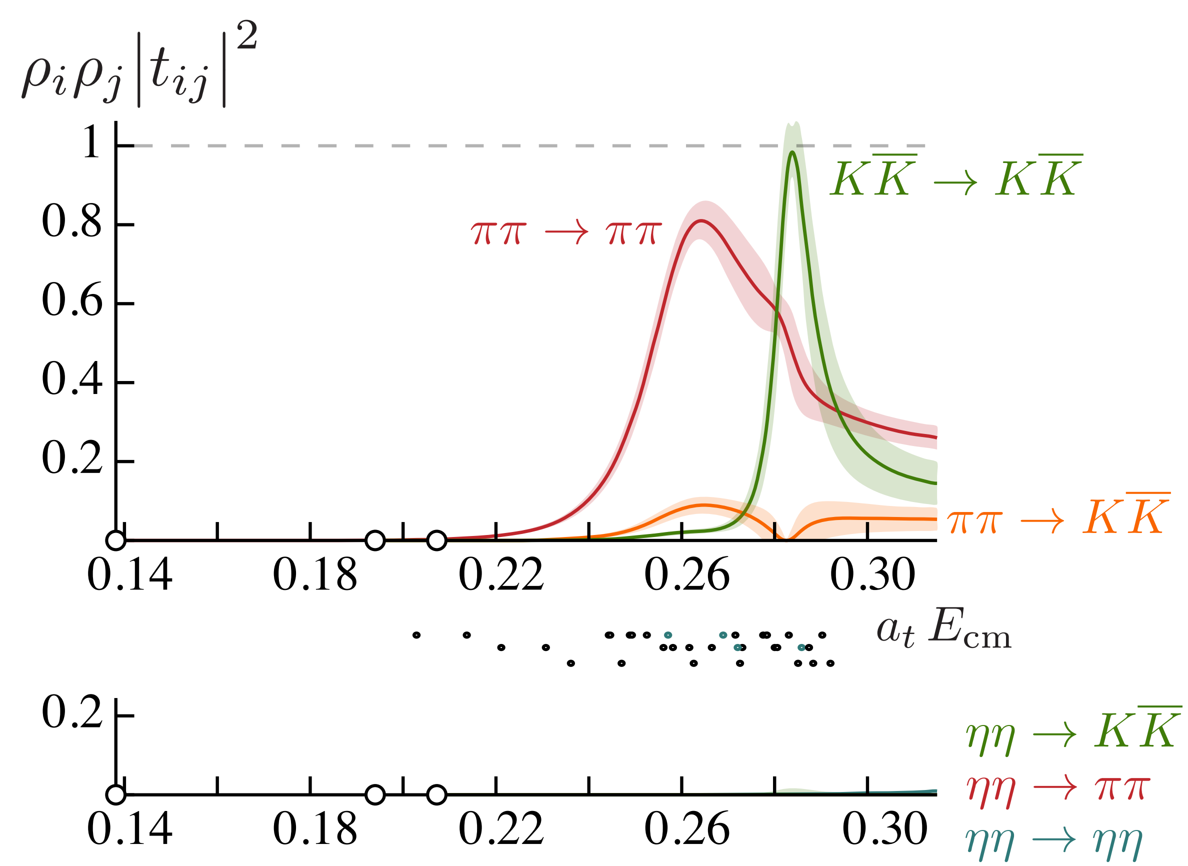
Scattering amplitudes of pairs of pseudo-scalar hadrons can be computed from lattice QCD

Several channels with scalar, vector and tensor resonances have been computed

Control of 3+ body effects needed for
- lighter pion masses
- higher mass resonances

For progress on scattering of particles with spin, see the next talk by Antoni Woss





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Christopher Thomas, Graham Moir, Gavin Cheung, **Antoni Woss** (University of Cambridge)

Nilmani Mathur (Tata Institute)

(**Bold** - authors of one or more of the papers mentioned)