Nuclear Matrix Elements NPLQCD

Phiala Shanahan

William and Mary

Thomas Jefferson National Accelerator Facility

Unphysical nuclei

- Nuclei with A<5
- QCD with unphysical quark masses m_{π} ~800 MeV, m_N~1,600 MeV

 m_{π} ~450 MeV, m_N~1,200 MeV

- Proton-proton fusion and tritium β -decay [PRL **119**, 062002 (2017)]
- Double β-decay [PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)]

- Nuclear structure: magnetic moments, polarisabilities [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]
- First nuclear reaction: np→dγ [PRL **115**, 132001 (2015)]
- Gluon structure of light nuclei [PRD **96** 094512 (2017)]
- Scalar, axial and tensor MEs [arXiv:1712.03221]

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Axial background field

Will's talk: fixed magnetic field moments, polarisabilities Axial MEs: fixed axial background field **and axial charges**, other matrix elts.

Construct correlation functions from propagators modified in axial field

Axial background field

Axial background field

Example: determination of the proton axial charge

Time difference isolates matrix element part

$$
C_{\lambda_u;\lambda_d}(t) \Big|_{\mathcal{O}(\lambda)} = \sum_{\tau=0}^t \langle 0 | \chi^\dagger(t) J(\tau) \chi(0) | 0 \rangle
$$

\n= ...
\n
$$
= Z_0 e^{-M_p t} \left[C + t \langle p | A_3^{(u)}(0) | p \rangle + \mathcal{O}(e^{-\delta t}) \right]
$$

\n
$$
(C_{\lambda_u;\lambda_d}(t+1) - C_{\lambda_u;\lambda_d}(t)) \Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t} \langle p | A_3^{(u)}(0) | p \rangle + \mathcal{O}(e^{-\delta t})
$$

\nMatrix element

Proton axial charge

- Extract matrix element through linear response of correlators to the background field
- Form ratios to cancel leading time-dependence

$$
R_p(t) = \frac{\left(C_{\lambda_u;\lambda_d=0}^{(p)}(t) - C_{\lambda_u=0;\lambda_d}^{(p)}(t)\right)\Big|_{\mathcal{O}(\lambda)}}{C_{\lambda_u=0;\lambda_d=0}^{(p)}(t)}
$$

At late times:

$$
R_p(t+1) - R_p(t) \stackrel{t \to \infty}{\longrightarrow} \frac{g_A}{Z_A}
$$

Matrix element revealed through "effective matrix elt. plot"

Tritium β-decay

Simplest semileptonic weak decay of a nuclear system

- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- **Understand multi-body contributions** to $\langle \mathbf{GT} \rangle$ \longrightarrow better predictions for decay rates of larger nuclei

Calculate $g_A \langle \mathbf{GT} \rangle = \langle ^3\mathrm{He} | \overline{\mathbf{q}} \gamma_\mathbf{k} \gamma_5 \tau^- \mathbf{q} | ^3\mathrm{H} \rangle$

Tritium β-decay *The GT Matrix Element for Tritium -decay:* The half-life of tritium, *t*1*/*2, is related to the F and GT matrix

Form ratios of compound correlators to cancel leading time-dependence:

$$
\frac{\overline{R}_{\rm^{3}H}(t)}{\overline{R}_{p}(t)} \stackrel{t\rightarrow\infty}{\longrightarrow} \frac{g_{A}(^{3}{\rm H})}{g_{A}} = \langle \mathbf{GT} \rangle
$$

Ground state ME revealed through "effective ME plot"

Proton-proton fusion

- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate
$$
\langle d; 3|A_3^3|pp\rangle
$$

\n $L_{1,A}, \ell_{1,A}, \overline{L}_{1,A}, \dots$
\n $pp \rightarrow de^+ \nu$ cross-section

Related to: Neutrino breakup reaction (SNO)

Muon capture reaction (MuSun)

 $\boldsymbol{p}\boldsymbol{p}$ \rightarrow ²H + e⁺ + ν_e $p+p$ $^{2}H + p$ \rightarrow ³He ${}^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + 2p$

Proton-proton fusion The low-energy cross section for *pp* ! *de*⁺⌫ is dictated (*I^z* = 0) component of the ³ *S*¹ (¹ *S*0) channel, *A^u* ³ = *u*35*u*, and *c*2*,*³ are irrelevant terms. Calculations of the axial matrix element at three or more values of *^u*

• Form ratios of compound correlators to cancel leading time-dependence construction is the Sommerfeld of the Sommerfeld of the Sommerfeld of the Sommerfeld of the Sommerfeld factor i A similar procedure yields the term that is linear in *^d* compound correlators to cancel reading t

Fit a constant to the 'effective matrix element *plot'* at late times $\overline{}$ $\overline{\$

³ ⇤(*p*) *jk,* (7)

⌘ *gAC*⌘

$$
R_{^{3}S_{1},^{1}S_{0}}(t+1) - R_{^{3}S_{1},^{1}S_{0}}(t)
$$
\n
$$
\xrightarrow{t \to \infty} \frac{\langle ^{3}S_{1}; J_{z} = 0 | A_{3}^{3} | ^{1}S_{0}; I_{z} = 0 \rangle}{Z_{A}}
$$
\n
$$
= \frac{\langle d; 3 | A_{3}^{3} | pp \rangle}{Z_{A}}
$$

Proton-proton fusion

Want to relate lattice QCD ME to

- LECs of EFTs
- pp-fusion cross section
- Finite-volume quantisation condition: relate $\langle d; 3|A_3^3|pp\rangle$ to scale-indep. LECs
	- Pionless EFT: $\overline{L}_{1,A}$
	- Dibaryon formalism: $\bar{\ell}_{1,A}$
- Define a new related quantity, $L_{1,A}^{sd-2b}$, which should have mild pion-mass dependence (remove effective range terms in $\overline{L}_{1,A}$)
- Extrapolate $L_{1,A}^{sd-2b}$ to the physical point
	- Prediction for $L_{1,A}$, $\ell_{1,A}$ at the physical point
	- Prediction for physical cross-section

Finite-volume quantisation

- Axial field splits degeneracy of the nucleon doublet
- 3S_1 and 1S_0 channels mix
- Construct 2x2 inverse scattering amplitude matrix in background field

Continuum integrals from bubble diagrams \rightarrow discrete sums $Det = 0$ poles of scattering amplitude \leftarrow eigenenergies

Finite-volume quantisation *^L*˜1*,A* ⌘ *^L*1*,A* ⁺ 1 ^p*r*1*r*3*.* (8)

Det of inverse scattering matrix = 0 \leftarrow eigenenergies are solutions of the nucleon conditions of the nucleon formalism, I find the nucleon formalism, I find the following relations of the following relations of the following relations of the following relations of the following r .
H ring $\frac{1}{2}$ matriy : \overline{a} 1 *k*² *p*² *eigene* $\overline{6}$ et of inverse scattering matrix = 0 \rightarrow eigenenergies dianuple in the set of \mathcal{L} *p* . (10) \overline{a} .LV
- $\overline{\text{in}}$ L $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$ $(2, 3)$ |
|*k*
2 *p*
2 *p*
2 **p** $\frac{1}{2}$ ة
≀ \det of inverse scattering matrix $= 0$ $\int_{0}^{2\pi} f(x) \, dx$

Matrix element related to LEC theory gives where

$$
|\delta E^{^3S_1-^1S_0}|/W_3=|\langle ^3S_1\,|A_3^3|^1S_0\rangle|=Z_d^2(4g_A\gamma\overline{L}_{1,A}+2g_A)
$$

• Define combination that characterises two-nucleon contribution Expect mild pion-mass dependence estrapolate residue of the deuteron propagator at the pole (which in this limit is the same as that for 5. *Degenerate deeply bound two-nucleon states as in the m*⇡ = 800 *MeV world*: The simplest tine combination that characterises two-nucleon contribution between the two channels in presence of the weak interactions, and degenerate perturbations, and degenerate per

3*|*

 \rightarrow

$$
L_{1,A}^{sd-2b} \equiv (\langle d; 3|A_3^3|pp\rangle - 2g_A)/2
$$

$$
Z_d = 1/2
$$

$$
Z_d=1/\sqrt{1-\rho\gamma}
$$

Briceno, Davoudi ,Phys.Rev. D88 (2013) 094507 *^d gA*(*L*1*,A* + up to exponentially small corrections that scale as *^eL/L*, where ⁼ *ip* is the binding

Proton-proton fusion

Extrapolate, $L_{1,A}^{sd-2b}$ $= -0.0107(12)(49)$ predict physical \rightarrow Z_A cross-section

Proton-proton fusion distance two-nucleon axial-vector operator, with coe cient *L*1*,A* [4], is expected to give the leading contribution I ⌦ *d*; *j A k* \overline{a} n ' ³ ⇤(*p*) *jk,* (7)

Low-energy cross section for $pp \rightarrow de^+\nu$ dictated by the matrix element The low-energy cross section for *pp* ! *de*+⌫ is dictated *^k*(*x*) is the axial current with isospin and spin ross section for $pp \rightarrow de^+\nu$ dictated by the index, *C*⌘ is the Sommerfeld factor and is the deuteron binding momentum. The quantity ⇤(*p*) has been calcu-

$$
\left|\left\langle d;j\left|A_{k}^{-}\right|pp\right\rangle \right|\equiv g_{A}C_{\eta}\sqrt{\frac{32\pi}{\gamma^{3}}}\,\Lambda(p)\,\delta_{jk}
$$

Relate $\Lambda(0)$ to extrapolated LEC using EFT index, α is the some α is the deuteron and α renormalization-scale independent short-distance quan- $\Lambda(0)$ to extrapolated LEC using EFT

 $\mathbf{1}$

$$
\Lambda(0) = \frac{1}{\sqrt{1 - \gamma \rho}} \{ e^{\chi} - \gamma a_{pp} [1 - \chi e^{\chi} \Gamma(0, \chi)] +
$$

$$
\frac{1}{2} \gamma^2 a_{pp} \sqrt{r_1 \rho} \} - \frac{1}{2g_A} \gamma a_{pp} \sqrt{1 - \gamma \rho} L_{1, A}^{sd - 2b}
$$
extrapolated lattice value

tity *Lsd*2*^b* $\left| \begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array} \right|$ C_{η} Sommerfield factor Deuteron binding mtm r_1, ρ Effective ranges a_{pp} pp scattering length $\Gamma(0,\chi)$ Incomplete gamma func. γ $\chi = \alpha M_p / \gamma$

 N^2 LO $\cancel{\pi}$ EFT with effective range contributions \mathbb{R}^{mtm} scattering parameters and contributions and \mathbb{R}^{mtm} corrections: \Box **Frace** *y* En time shootive range continuations resummed using the dibaryon approach tering length is *app*, *r*¹ and ⇢ are the e↵ective ranges in N^2 LO $\cancel{\pi}$ EFT with effective range contributions

{e app[1 *e*(0*,*)] + *Butler and Chen, Phys. Lett. B520, 87 (2001)* $\frac{1}{2}$ **incomplete gamma function. A determination of** *L***
2004). A determination of** *L***
2004).** 7 (2001)
343 179 (3004) (2001) .

Proton-proton fusion

Physical cross-section dictated by

Proton-proton fusion

(models/EFT)

• Fusion cross section dictated by

 $\Lambda(0) = 2.6585(6)(72)(25)$

 $\Lambda(0) = 2.652(2)$

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

• Relevant counter-term in EFT

 $L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)$ fm³

at a renormalization scale *µ* = *m*⇡. The uncertainties $L_{1,A} = 3.6(5.5)~\mathrm{fm}^3$ (reactor expts.)

M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

Quadratic response from two insertions on different quark lines

Double β-decay

- Certain nuclei allow observable ββ decay is energetically Neutrino-less double beta decay
- forbidden or hindered by large *J* difference. $T_{1/2}^{2\nu\beta\beta}\gtrsim10^{19}$ y Neutrino-less double beta decay

If neutrinos are massive Majorana fermions 0vββ decay is possible fermions 0**v**ββ decay is possible *ignalially arrows* \overline{a}

of even *Z*—even *N* nuclei with respect to odd *Z*—odd *N* nuclei. For odd *A* nuclei there is a single mass parabolate two-current and the range 1018–1022 y. The range 1018–1022 years of the neutrinoless double-life beta decay "0*νββ*#, proposed by Furry \$4%after the Majorana theory of the neutrino \$5%. The 45 nuclear matrix elements

Second order weak interactions *Two-neutrino -Decay:* The focus of this Letter is on $\mathbf{f}_{\mathbf{f}}$ for $\mathbf{f}_{\mathbf{f}}$ axial-current interactions enters iractions in *^L*(1) ⁼ *g^A N†W^a* ³ 3⌧ *^aN*

given by PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

Background axial field to second order $\frac{1}{2}$ $\frac{1}{2}$ where *Q* = *Enn Epp*, *G*2⌫(*Q*) is a known phase-space

 2 decay of the dinucleon system. The dinucleon system system system system system. The dinucleon system is α

In → pp transition matrix element
\n
$$
M_{GT}^{2\nu} = 6 \int d^4x d^4y \langle pp|T[J_3^+(x)J_3^+(y)]|nn\rangle
$$

\nmany technical complications

2

Non-negligible deviation from long distance deuteron intermediate state contribution *I*3 = 1 decrease control decrease to determine 11101180101011 iermediale state contribution ron

Second order weak interactions decond order we excited states in the ³ *S*¹ and ¹ *S*⁰ channels, respectively. 150

 $\frac{1}{2}$ PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

> Non-negligible deviation from long distance deuteron intermediate state contribution 2*C*(*nn*) 0;0 (*t*) it is straightforward warranteer in strip $\overline{}$ *te* co **cont** $\overline{1}$ lOr

$$
M^{2\nu}_{GT}=-\frac{|M_{pp\to d}|^2}{E_{pp}-E_d}+\beta^{(I=2)}_A
$$

• Quenching of g_A in nuclei is $\frac{d}{dx}$ = $\frac{d}{dx}$ \sim cient of the axial polarisability of the axial p \blacktriangleright Quenching

⁺ *^c* ⁺ *d e^t* ⁺ *^O*(*e*)*,*

Isotensor axial polarisability

TBD: connect to EFT for larger systems the deuteron-pole contribution to give a quantity that σ late times,

Gluon structure of nuclei

How does the gluon structure of a nucleon change in a nucleus?

Ratio of structure function F_2 per nucleon for iron and deuterium

$$
F_2(x, Q^2) = \sum_{q=u,d,s...} x e_q^2 [q(x, Q^2) + \overline{q}(x, Q^2)]
$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?

Nuclear glue, m_{π} ~450 MeV

Look for nuclear (EMC) effects in the first moments of the spin-independent gluon structure function

Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

Deuteron gluon momentum fraction

Ratio \propto matrix element for $\frac{1}{2}$ *C*3(*t,* ⌧) Ratio α matrix element
for $0 \ll \overline{\tau} \ll \overline{t}$

PRD96 094512 (2017)

Gluon momentum fraction

PRD96 094512 (2017)

- Matrix elements of the Spin-independent gluon operator in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

Double Helicity Flip Gluon Structure Function: (*x, Q*²) Gluonic Transversity ² *, n* = 2*,* ⁴*,* ⁶ *...,* (11) where *Aⁿ* is renormalized at the scale *µ*² = *Q*², and \mathbf{r} 450 meV and the strange quark mass is such that the strange quark mass is such that the strange \sim resulting mass of the is 1040(3) MeV.

Double helicity flip structure function $\Delta(x,Q^2)$ Parto and Manohar "N e and Manohar, "Nuclear *^Mn*(*Q*²) = ^Z ¹ r *dxxⁿ*¹(*x, Q*²) (12) ad Manobar "Nuclear Gluonom" Jaffe and Manohar,"Nuclear Gluonometry" Phys. Lett. B223 (1989) 218

Double Helicity Flip Gluon Structure Function: (*x, Q*²)

• Hadrons: Gluonic Transversity (parton model interpretation) For a target in the infinite momentum frame polarized in the *x*ˆ direction trons: Gluonic Transversity (parton mod its momentum (defined to be in the ˆ*z* direction), *Oµ*⌫*µ*1*µ*²

$$
\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 x^2 \int_x^1 \frac{dy}{y^3} \left[g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2) \right]
$$

 $g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly linearly polarised in \hat{x} , \hat{y} direction 'Exotic' Glue in the Nucleus in the Nucleu G_{μ} Ω^2) · probability of finding a gluon with momer $\frac{1}{2}$ α dimension under renormalization (this operator mixes α intraction y linearly four-

Gluons aligned rather than perpendicular to it in the transverse planet rather than perpendicular to it is in the transverse planet rather than perpendicular to it is in the transverse planet rather than α a **lai:** Exotic Cluo

gluons not associated with individual nucleons in nucleus and $\frac{1}{2}$ in the Nucleus $\frac{1}{2}$ \mathcal{P}_1 is the Nucleus Shanahan (MIT) Exotic Glue in the Nucleus July 8, 2016 8 \mathcal{P}_2 transverselverselverselselselselsel
Transverselsel

CICI: EXEC Guide
$$
\langle p|O|p\rangle = 0
$$

\nons not associated individual nucleus $\langle N, Z|O|N, Z\rangle \neq 0$

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Transverselsel

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$$
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$$

\nons not associated

\nindividual nucleus

\n $\langle N, Z|O|N, Z\rangle \neq 0$

Non-nucleonic glue in deuteron

First moment of gluon transversity distribution in the deuteron, $m_{\pi} \sim 800$ MeV

- First evidence for non-nucleonic gluon contributions to nuclear structure
- Hypothesis of no signal ruled out to better than one part in $10⁷$
- Magnitude relative to momentum fraction as expected from large- N_c

Ratio \propto matrix element for $0 \ll T \ll t$ $\sqrt{R_{\text{at}}^2}$ *^C*2(*t*) / *^A*2*,* ⁰ ⌧ ⌧ ⌧ *^t ^C*3(*t,* ⌧)

PRD96 094512 (2017) Ratio of 3pt and 2pt functions

Scalar & tensor nuclear MEs

- Axial, scalar, tensor charges of light nuclei A<4, at unphysical value of the quark masses $m_{\pi} \sim 800$ MeV
	- Complete flavour-decomposition including strange quarks

Scalar

- **Possible DM** interaction is through scalar exchange
- **Direct detection** depends on nuclear matrix element

Tensor

- Quark electric dipole moment (EDM) contributions to the EDMs of light nuclei
- Input for searches for \Box nuclear EDMs as evidence for BSM CP violation

arXiv:1712.03221

Strange matrix elements

Complete flavour-decomposition including strange quarks

Disconnected contributions estimated stochastically [Arjun Gambhir, LLNL & LBNL]

arXiv:1712.03221

 τ

Scalar & tensor nuclear MEs -0.3

- Naive expectation determined by baryon#, isospin, spin 0.02
- O(10%) nuclear effects in the scalar charges 0.00
- Nuclear modifications scale with magnitude of corresponding charge (i.e., baryon# for scalar, spin for tensor, axial) -0.01

0.02 arXiv:1712.03221

Nuclear MEs from LQCD

- Nuclear matrix elements important to experimental programs e.g,
	- Neutrino breakup reaction (SNO)
	- Muon capture reaction (MuSun)
	- Double-beta decay
	- Electron-Ion Collider
	- Nuclear electric dipole moments
	- Dark matter direct detection
- Current state-of-the-art: significant systematics but phenomenologically interesting at current precision

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