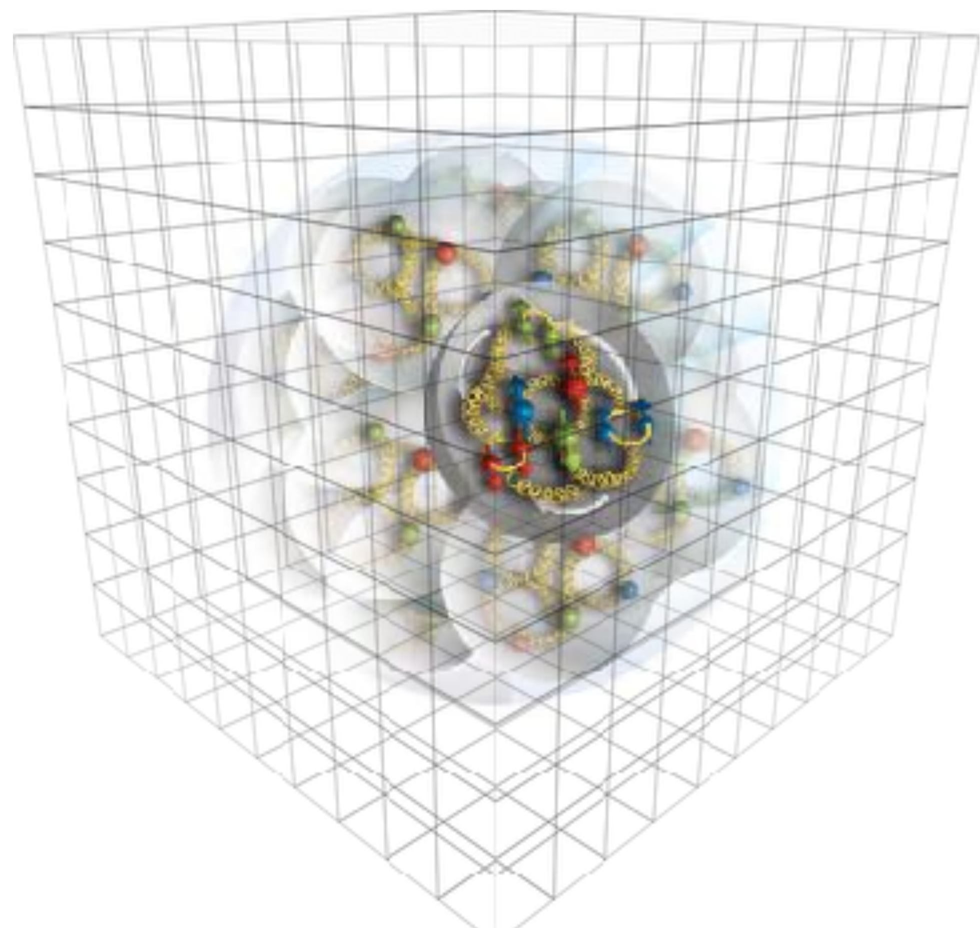
Nuclear Matrix Elements NPLQCD



Phiala Shanahan
William and Mary
Thomas Jefferson National Accelerator Facility

Unphysical nuclei

- Nuclei with A<5
- QCD with unphysical quark masses

  $m_\pi \sim 800$ MeV, $m_N \sim 1{,}600$ MeV

  $m_\pi \sim 450$ MeV, $m_N \sim 1{,}200$ MeV

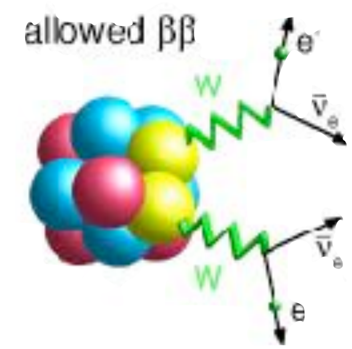
- Nuclear structure: magnetic moments, polarisabilities

  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

- First nuclear reaction: np→dγ
  [PRL **115**, 132001 (2015)]

- Proton-proton fusion and tritium β-decay
  [PRL **119**, 062002 (2017)]

- Double β-decay
  [PRL **119**, 062003 (2017),
  PRD **96**, 054505 (2017)]



- Gluon structure of light nuclei
  [PRD **96** 094512 (2017)]

- Scalar, axial and tensor MEs
  [arXiv:1712.03221]

NPLQCD

Unphysical nuclei

- Nuclei with A<5
- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

- Nuclear structure: magnetic moments, polarisabilities

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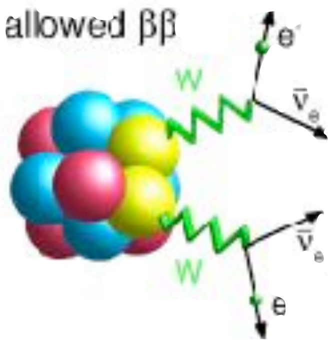
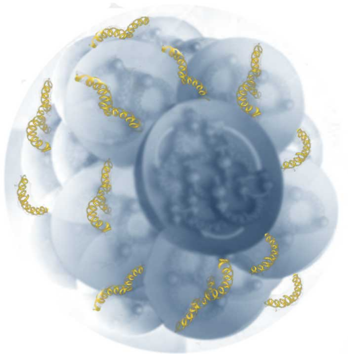


- Gluon structure of light nuclei

  [PRD **96** 094512 (2017)]



- Scalar, axial and tensor MEs

  [arXiv:1712.03221]

NPLQCD

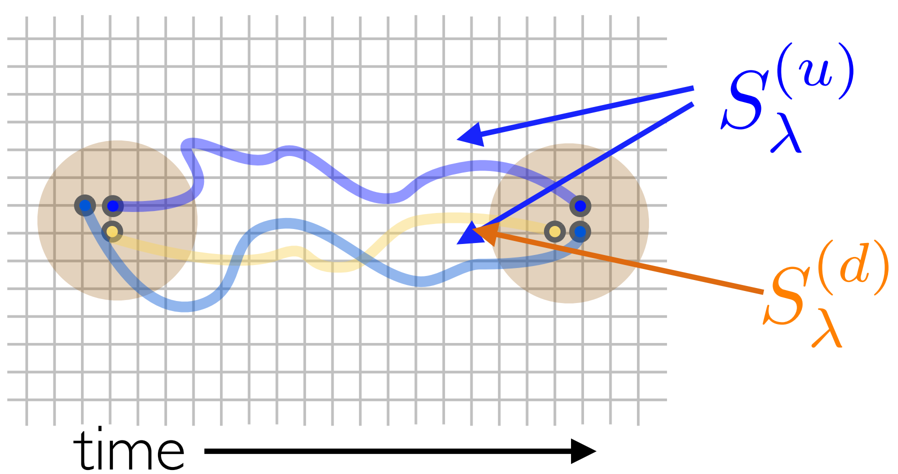
Axial background field

Will's talk: fixed magnetic field ➡ moments, polarisabilities
Axial MEs: fixed axial background field ➡ axial charges, other matrix elts.

Construct correlation functions from propagators modified in axial field

compound propagator — constant

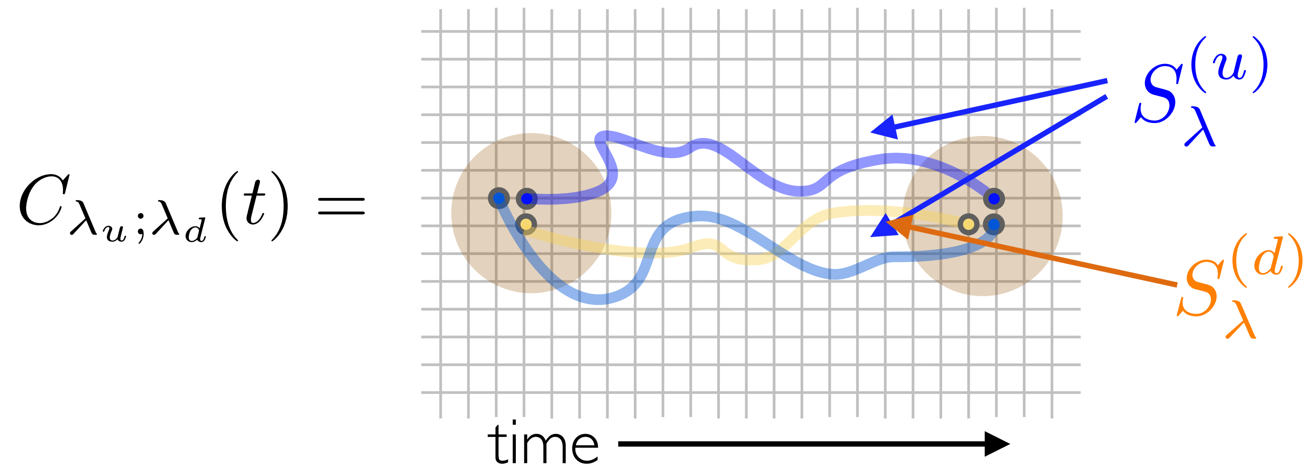
$$S^{(q)}_{\lambda}(x,y) = S^{(q)}(x,y) + \lambda_q \int dz\, S^{(q)}(x,z)\,\gamma_3\gamma_5\, S^{(q)}(z,y)$$

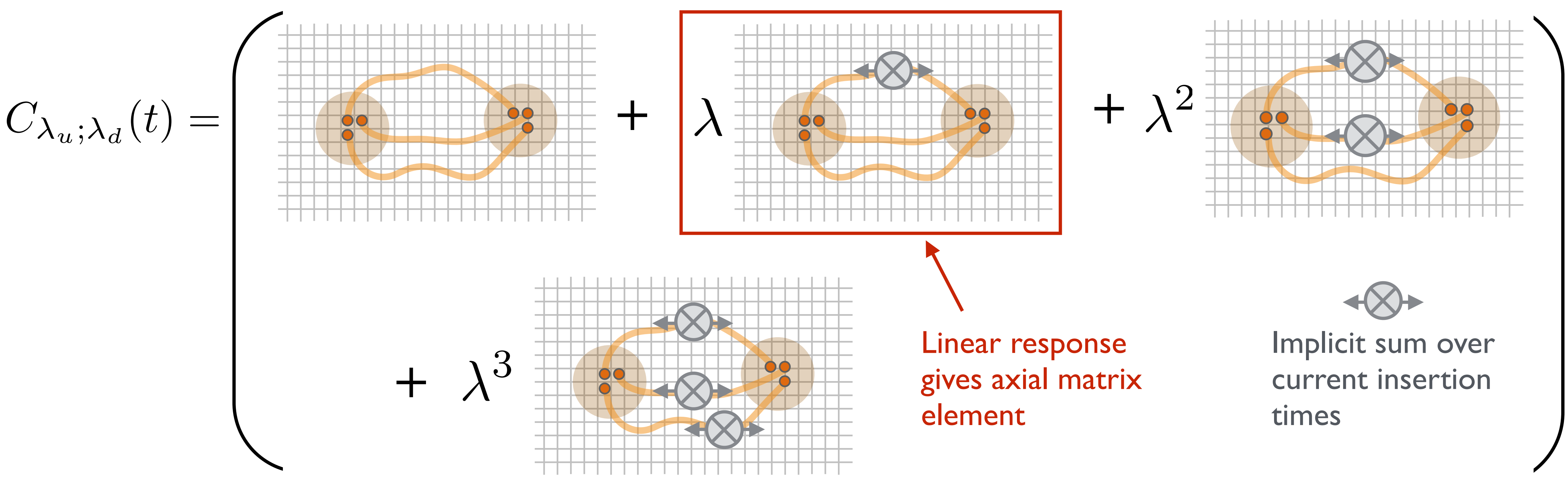
$C_{\lambda_u;\lambda_d}(t) =$ 

Linear response ⬌ axial matrix element

[NPLQCD Nucl. Phys. **A743**, 170 (2004)]

Axial background field

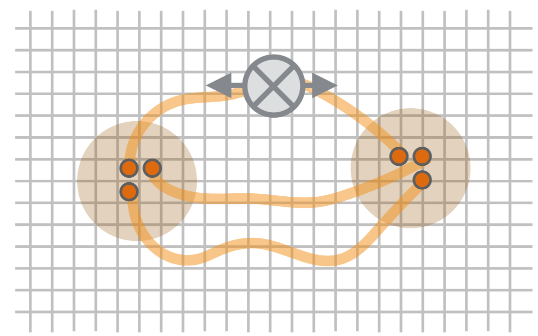


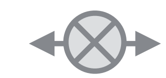


Axial background field

Example: determination of the proton axial charge

$$C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} =$$





Implicit sum over current insertion times

Time difference isolates matrix element part

$$\begin{aligned} C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} &= \sum_{\tau=0}^{t} \langle 0|\chi^\dagger(t) J(\tau) \chi(0)|0\rangle \\ &= \ldots \\ &= Z_0 e^{-M_p t}\left[C + t\,\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})\right] \end{aligned}$$

Irrelevant constants

Excited states

Matrix element

$$\left(C_{\lambda_u;\lambda_d}(t+1) - C_{\lambda_u;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t}\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})$$

Proton axial charge

- Extract matrix element through linear response of correlators to the background field
- Form ratios to cancel leading time-dependence

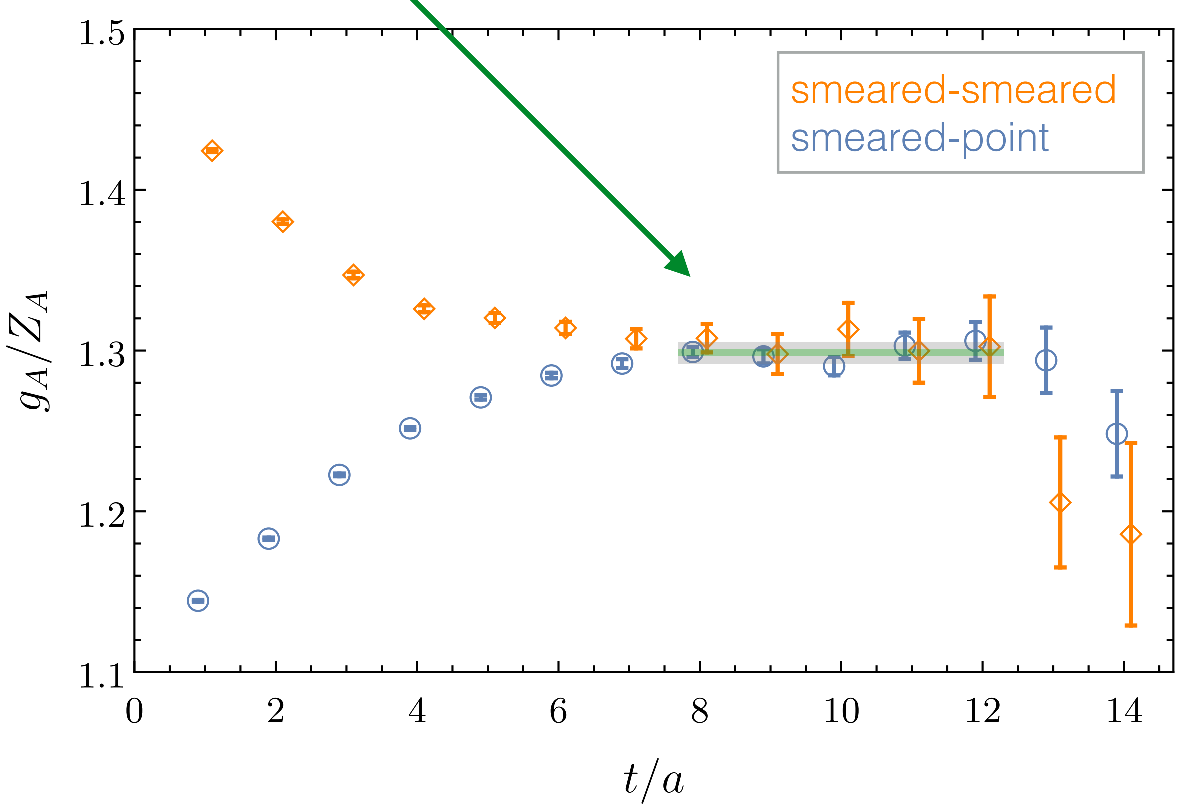
$$R_p(t) = \frac{\left(C^{(p)}_{\lambda_u;\lambda_d=0}(t) - C^{(p)}_{\lambda_u=0;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)}}{C^{(p)}_{\lambda_u=0;\lambda_d=0}(t)}$$

At late times:

$$R_p(t+1) - R_p(t) \overset{t\to\infty}{\longrightarrow} \frac{g_A}{Z_A}$$

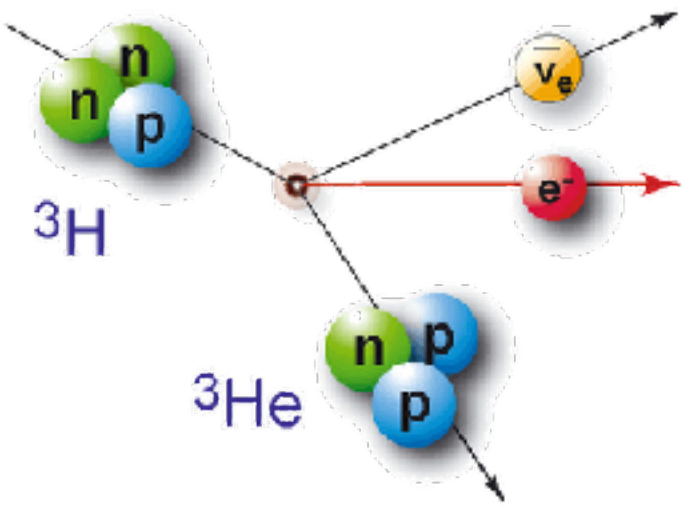
- Matrix element revealed through "effective matrix elt. plot"

constant fit to plateau region



Tritium β-decay

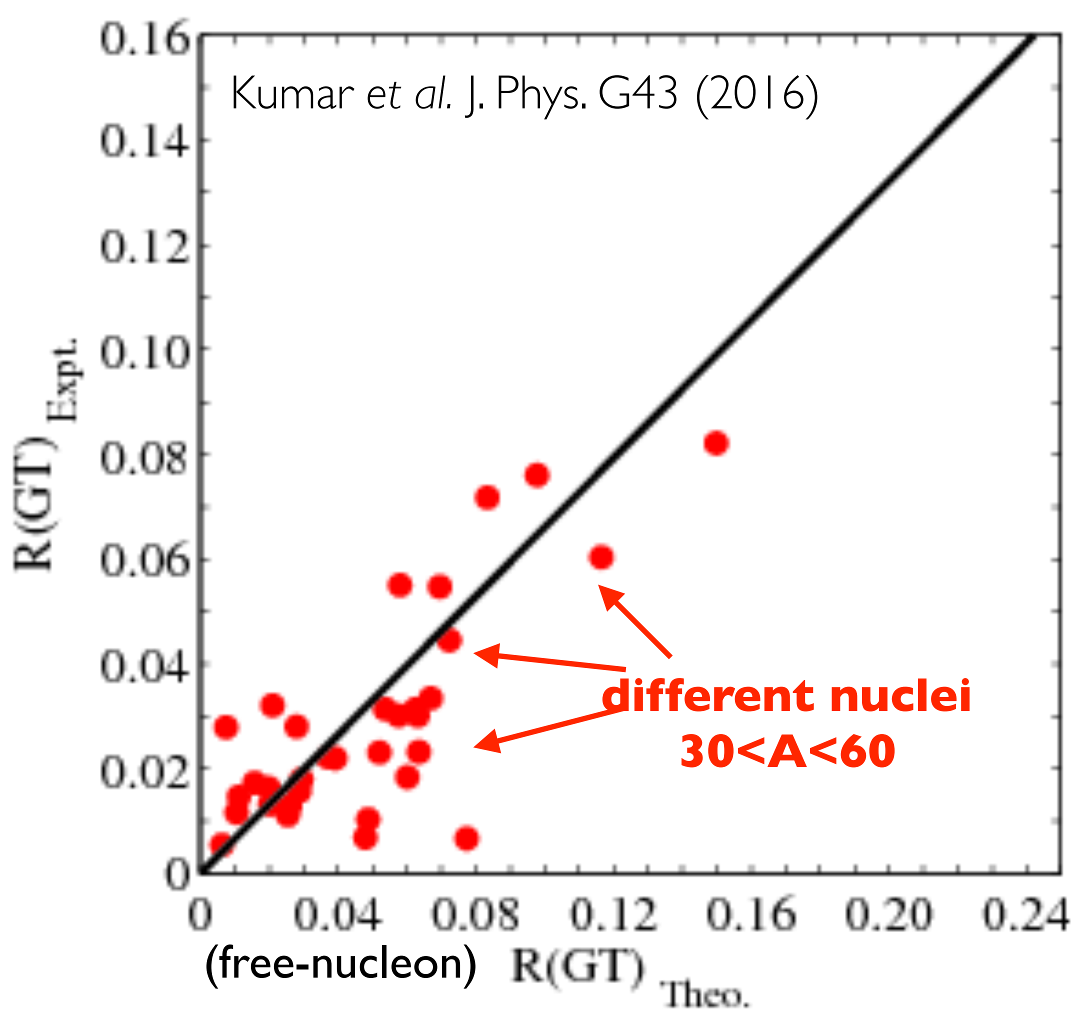
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle$ ➡ better predictions for decay rates of larger nuclei

Calculate

$$g_A \langle \mathbf{GT} \rangle = \langle ^3\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^- \mathbf{q} | ^3\mathrm{H} \rangle$$



Tritium β-decay

$$\frac{(1+\delta_R) f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V \, g_A^2 \langle \mathbf{GT} \rangle^2}$$

half-life

vector ME

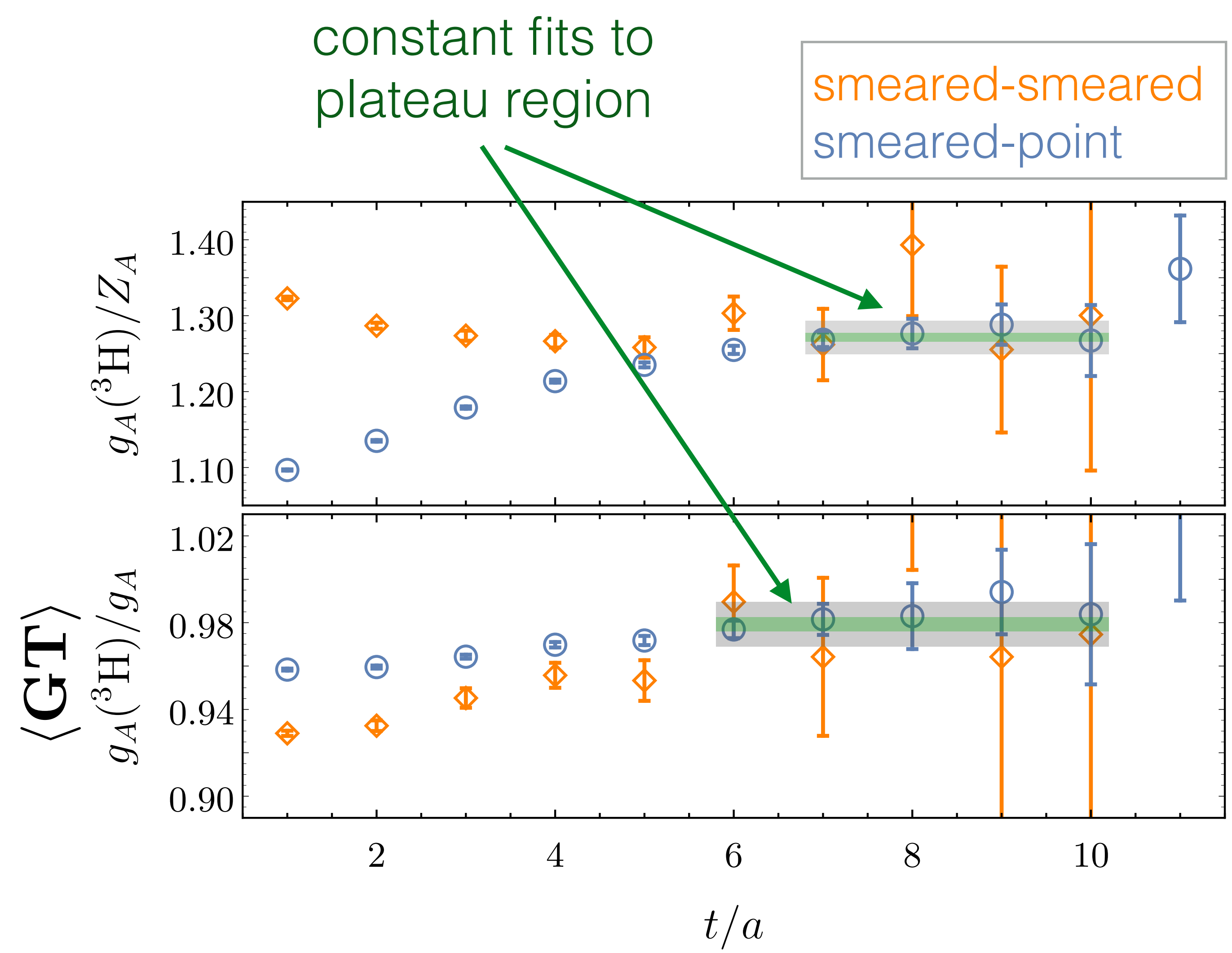
axial ME

known from theory or expt.

- Form ratios of compound correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle$$

- Ground state ME revealed through "effective ME plot"



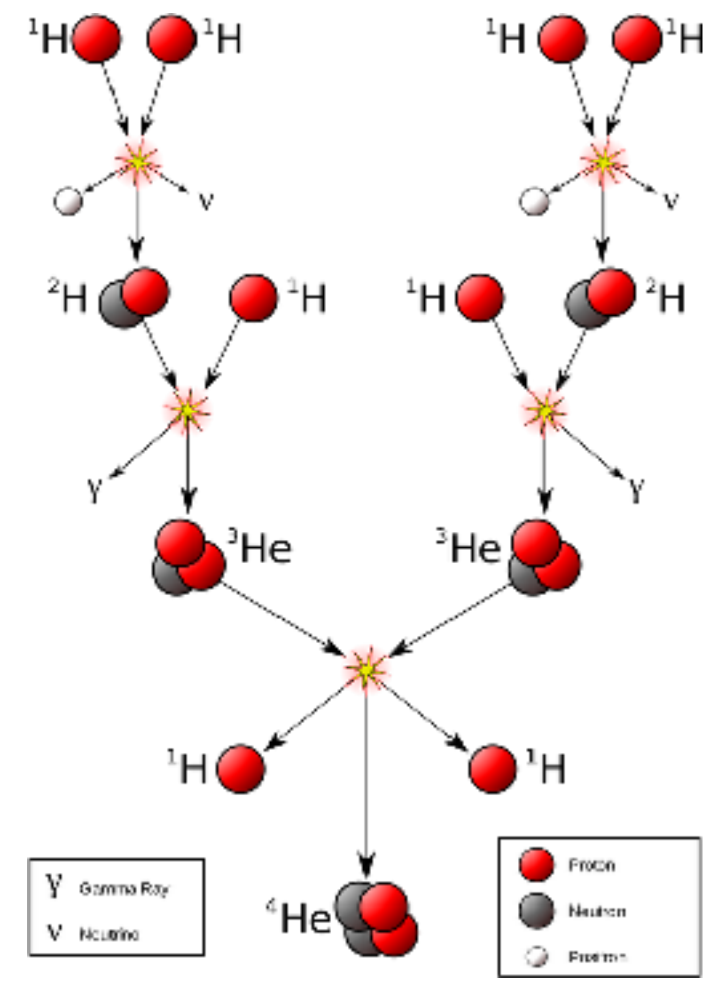
Proton-proton fusion

- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate $\langle d;3|A_3^3|pp\rangle$

$\rightarrow L_{1,A},\ \ell_{1,A},\ \overline{L}_{1,A},\ldots$

$pp \to de^+\nu$ cross-section



- Related to:
  - Neutrino breakup reaction (SNO)
  - Muon capture reaction (MuSun)

pp

$$p+p \to {}^2\mathrm{H}+e^+ + \nu_e$$
$${}^2\mathrm{H}+p \to {}^3\mathrm{He}$$
$${}^3\mathrm{He}+{}^3\mathrm{He} \to {}^4\mathrm{He}+2p$$

Proton-proton fusion

- Form ratios of compound correlators to cancel leading time-dependence

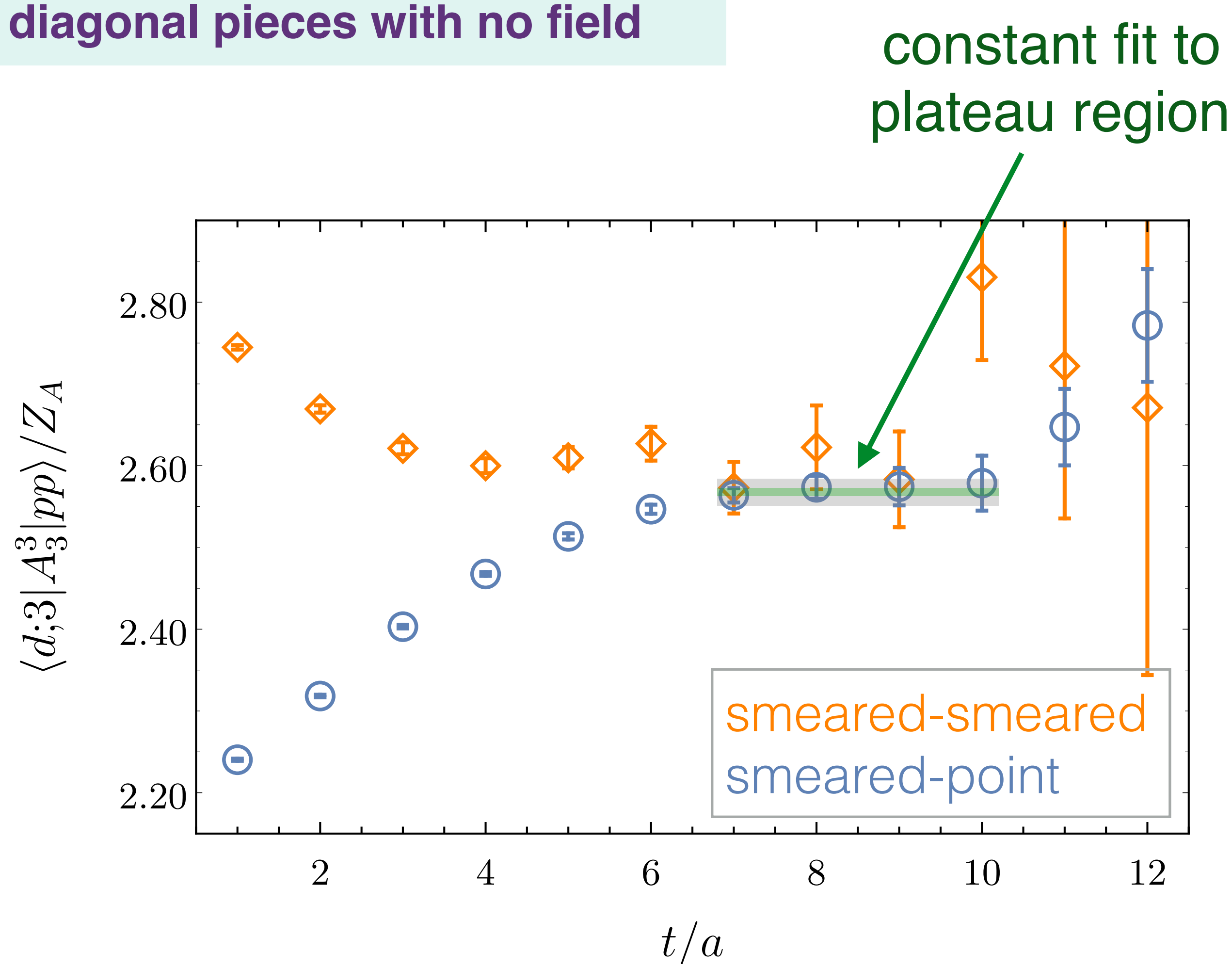
transition pieces linear in Λ

$$R_{^3S_1,^1S_0}(t) = \frac{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)\,C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}$$

diagonal pieces with no field

- Fit a constant to the 'effective matrix element plot' at late times

$$R_{^3S_1,^1S_0}(t+1) - R_{^3S_1,^1S_0}(t) \xrightarrow{t\to\infty} \frac{\langle ^3S_1; J_z=0|A_3^3|^1S_0; I_z=0\rangle}{Z_A} = \frac{\langle d;3|A_3^3|pp\rangle}{Z_A}$$



Proton-proton fusion

Want to relate lattice QCD ME to

- LECs of EFTs
- pp-fusion cross section

- Finite-volume quantisation condition: relate $\langle d;3|A_3^3|pp\rangle$ to scale-indep. LECs
  - Pionless EFT: $\overline{L}_{1,A}$
  - Dibaryon formalism: $\overline{\ell}_{1,A}$
- Define a new related quantity, $L_{1,A}^{sd-2b}$, which should have mild pion-mass dependence (remove effective range terms in $\overline{L}_{1,A}$)
- Extrapolate $L_{1,A}^{sd-2b}$ to the physical point

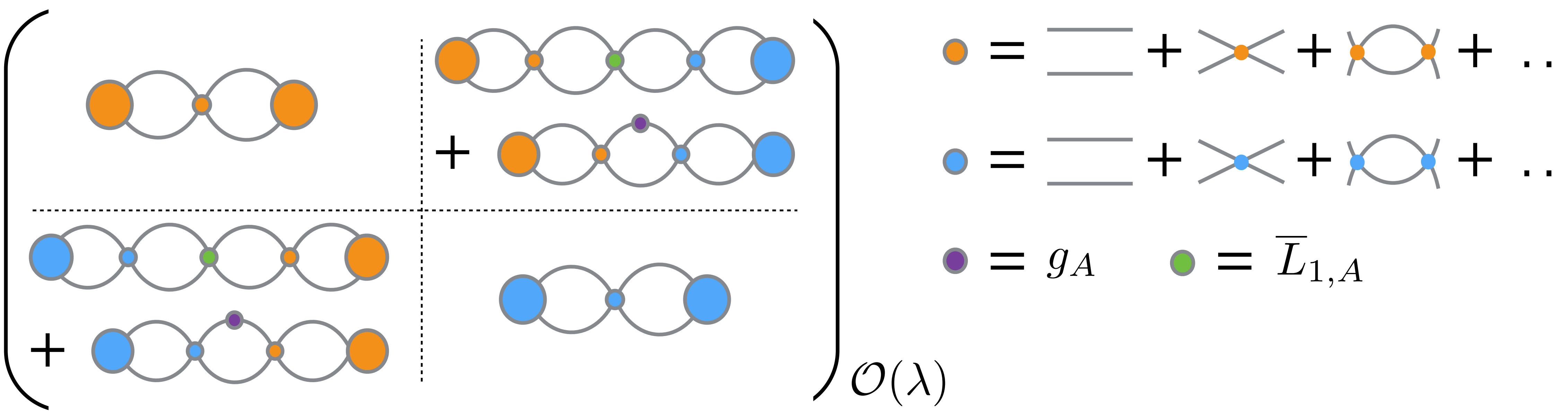
➡ Prediction for $\overline{L}_{1,A}$, $\overline{\ell}_{1,A}$ at the physical point

➡ Prediction for physical cross-section

Detmold and Savage, Nucl. Phys. A743, 170 (2004).

Finite-volume quantisation

- Axial field splits degeneracy of the nucleon doublet
- ${}^3S_1$ and ${}^1S_0$ channels mix
- Construct 2x2 inverse scattering amplitude matrix in background field



- Continuum integrals from bubble diagrams → discrete sums
- Det = 0 ↔ poles of scattering amplitude ↔ eigenenergies

Briceno, Davoudi ,Phys.Rev. D88 (2013) 094507

Finite-volume quantisation

- Det of inverse scattering matrix = 0 ⟷ eigenenergies are solutions of

$$\left[p\cot\delta^{^3S_1} + \delta G_0^V(p;L)\right]\left[p\cot\delta^{^1S_0} + \delta G0^V(p;L)\right] = \left[W_3 g_A M \overline{L}_{1,A} - W_3 g_A G_1^V(p;L)\right]^2$$

two-body LEC

from effective range expansion

finite-volume sums

weak coupling

→ Matrix element related to LEC

$$|\delta E^{^3S_1 - ^1S_0}|/W_3 = |\langle ^3S_1 | A_3^3 | ^1S_0\rangle| = Z_d^2(4 g_A \gamma \overline{L}_{1,A} + 2 g_A)$$

- Define combination that characterises two-nucleon contribution
  Expect mild pion-mass dependence → can extrapolate

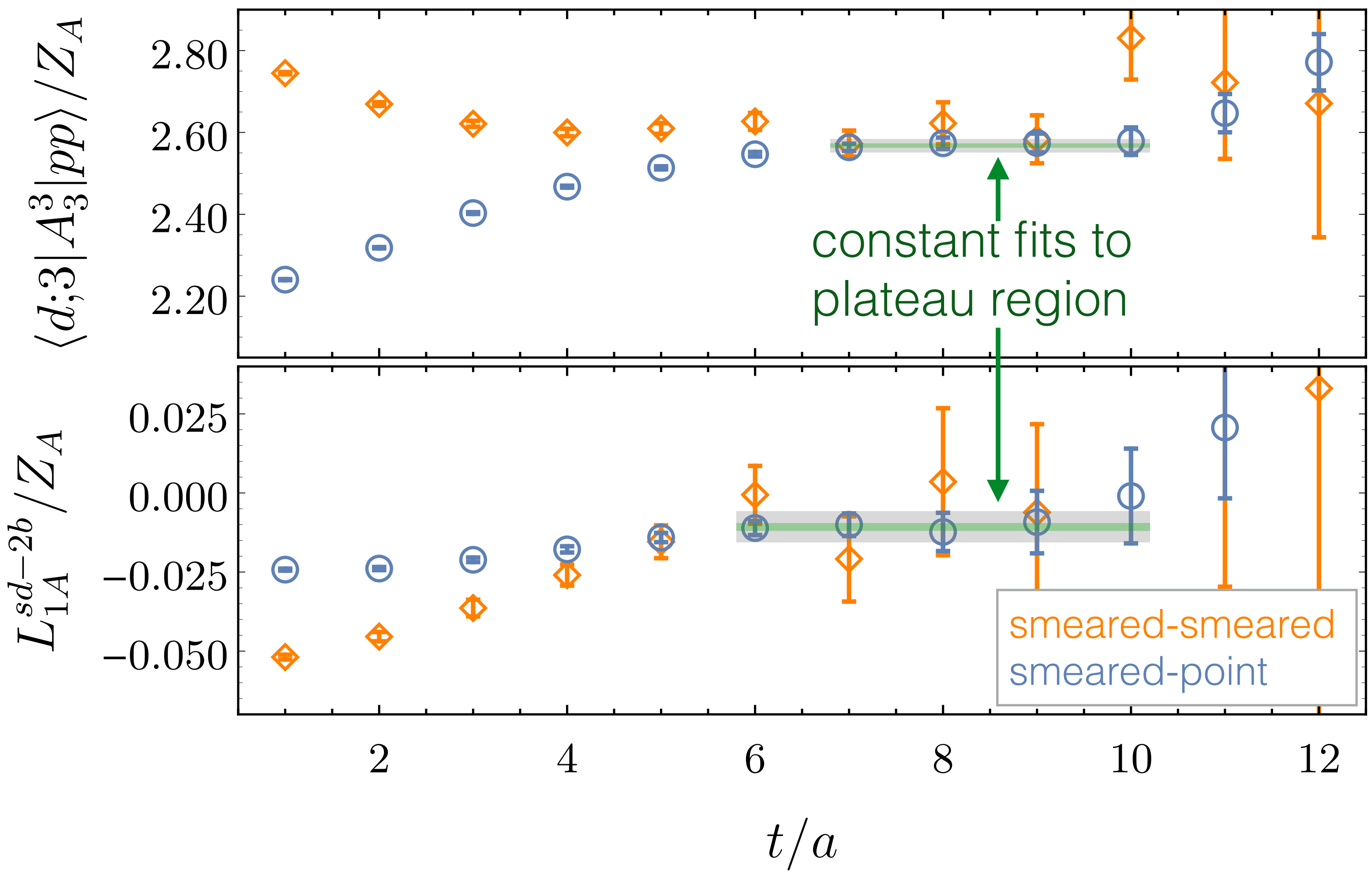
experience from $np \to d\gamma$

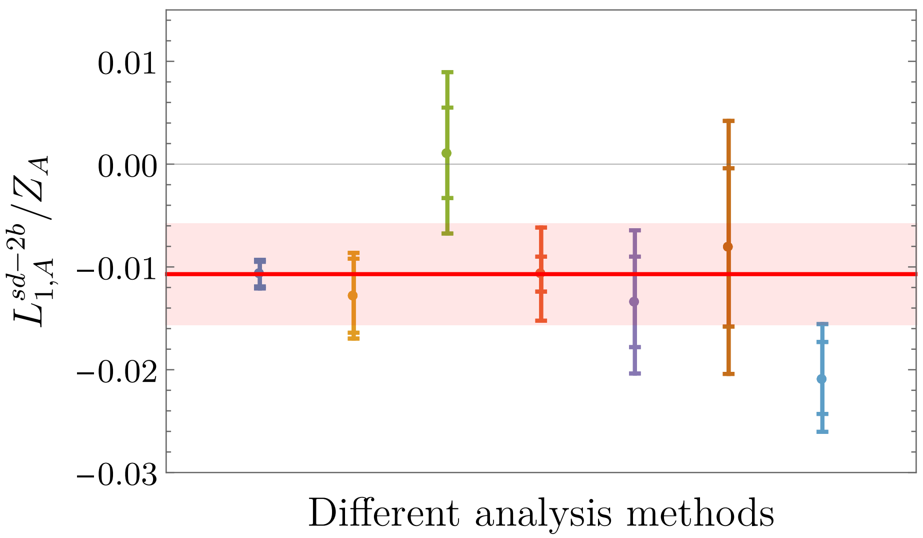
$$L_{1,A}^{sd-2b} \equiv (\langle d;3|A_3^3|pp\rangle - 2g_A)/2$$

$$Z_d = 1/\sqrt{1-\rho\gamma}$$

Briceno, Davoudi ,Phys.Rev. D88 (2013) 094507

Proton-proton fusion





$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.0107(12)(49)$$

Extrapolate, predict physical cross-section

Proton-proton fusion

Low-energy cross section for $pp \to de^+\nu$ dictated by the matrix element

$$\left|\langle d; j \,|A_k^-|\, pp\rangle\right| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\, \Lambda(p)\, \delta_{jk}$$

Relate $\Lambda(0)$ to extrapolated LEC using EFT

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}}\{e^\chi - \gamma a_{pp}[1 - \chi e^\chi \Gamma(0,\chi)] + \frac{1}{2}\gamma^2 a_{pp}\sqrt{r_1\rho}\} - \frac{1}{2g_A}\gamma a_{pp}\sqrt{1-\gamma\rho}\, \boxed{L_{1,A}^{sd-2b}}$$

extrapolated lattice value

C_η	Sommerfield factor
γ	Deuteron binding mtm
$r_1,\ \rho$	Effective ranges
a_{pp}	pp scattering length
$\Gamma(0,\chi)$	Incomplete gamma func.
	$\chi = \alpha M_p/\gamma$

N^2LO $\not\pi$ EFT with effective range contributions resummed using the dibaryon approach

Butler and Chen, Phys. Lett. B520, 87 (2001)
Detmold and Savage, Nucl. Phys. A743, 170 (2004).

Proton-proton fusion

Physical cross-section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

statistical

systematic
- fitting
- analysis
- uncertainties of phys. mass inputs

quark mass extrap. (50% additive)

Can also extract

$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

renormalisation scale $\mu = m_\pi$

higher-order $\not{\pi}$ EFT corrections (power-counting)

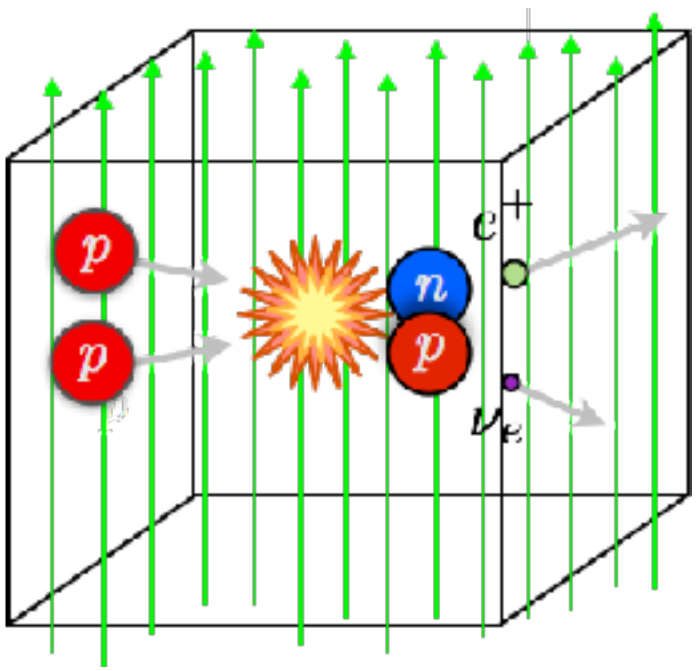
Proton-proton fusion

- Fusion cross section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

$$\Lambda(0) = 2.652(2)$$ (models/EFT)

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)



- Relevant counter-term in EFT

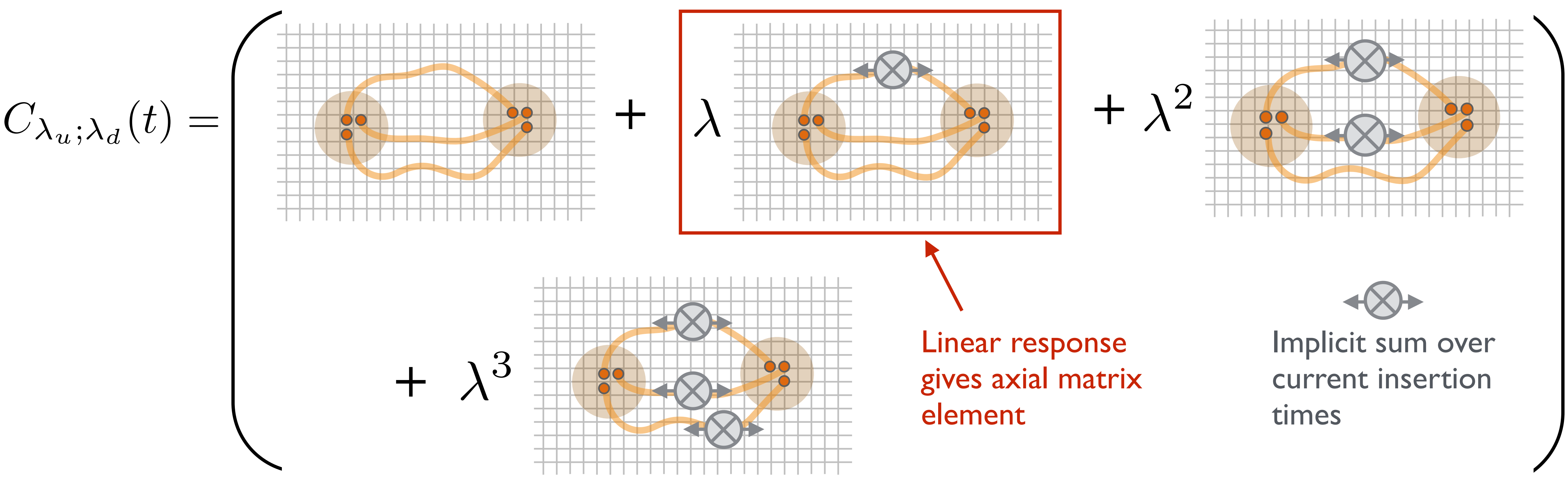
$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

$$L_{1,A} = 3.6(5.5)\ \mathrm{fm}^3$$ (reactor expts.)

M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

Higher-order insertions

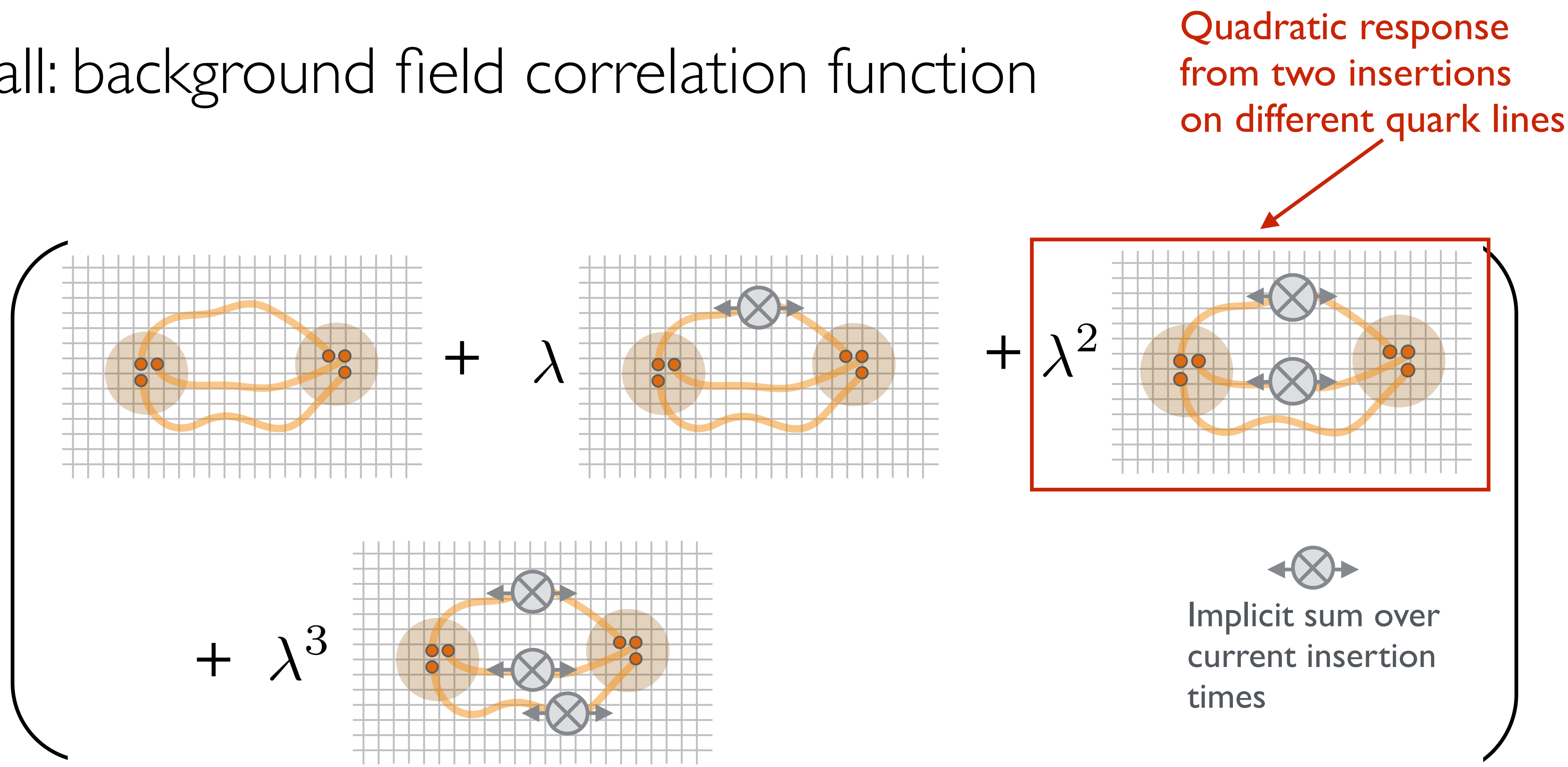
- Can access terms with more current insertions from same calculations
- Recall: background field correlation function



Higher-order insertions

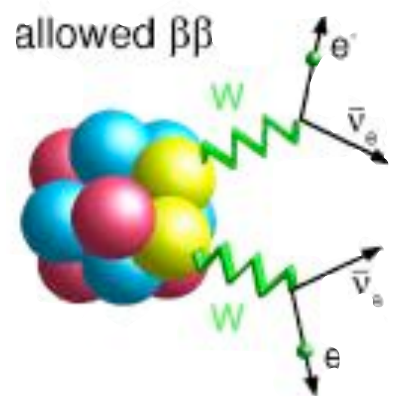
- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

$$C_{\lambda_u;\lambda_d}(t) =$$

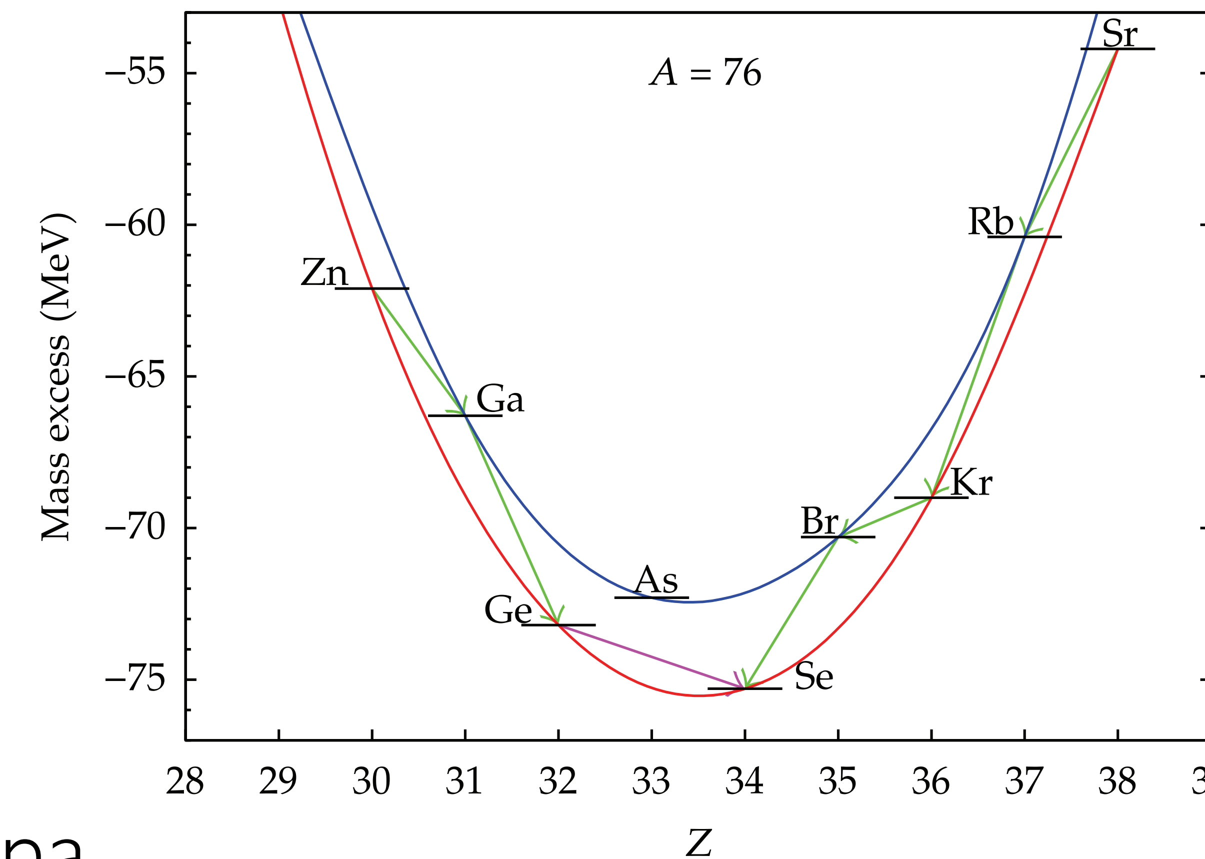


Double β-decay

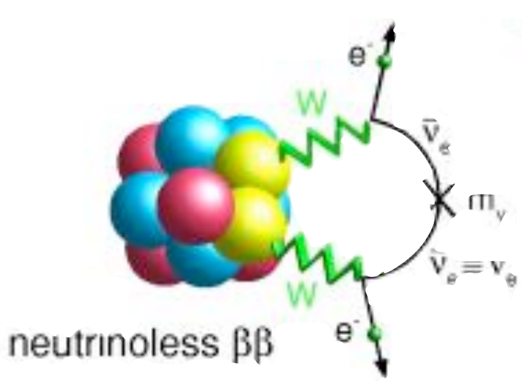
- Certain nuclei allow observable ββ decay



$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19}$ y



- If neutrinos are massive Majorana fermions 0νββ decay is possible



$T_{1/2}^{0\nu\beta\beta} > 10^{25}$ y

Calculate two-current nuclear matrix elements → dictate half-life

Second order weak interactions

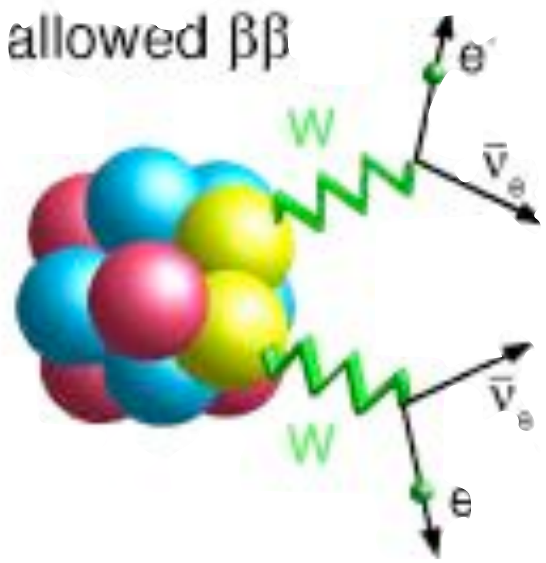
PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)

- Background axial field to second order

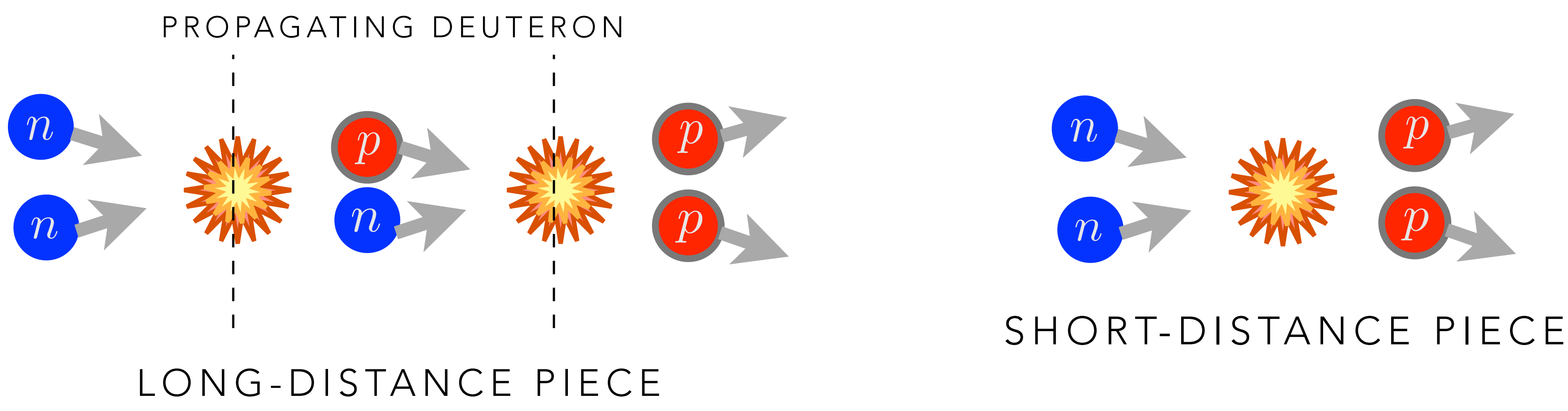
➡ nn→pp transition matrix element

$$M^{2\nu}_{GT} = 6 \int d^4x d^4y \langle pp | T \left[J_3^+(x) J_3^+(y) \right] | nn \rangle$$

many technical complications

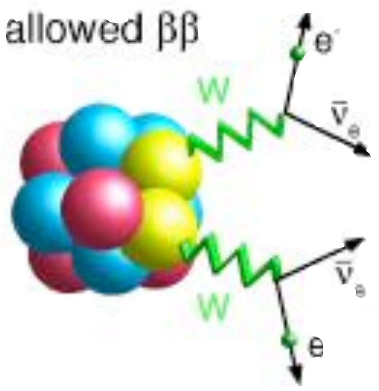


- Non-negligible deviation from long distance deuteron intermediate state contribution



Second order weak interactions

PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)



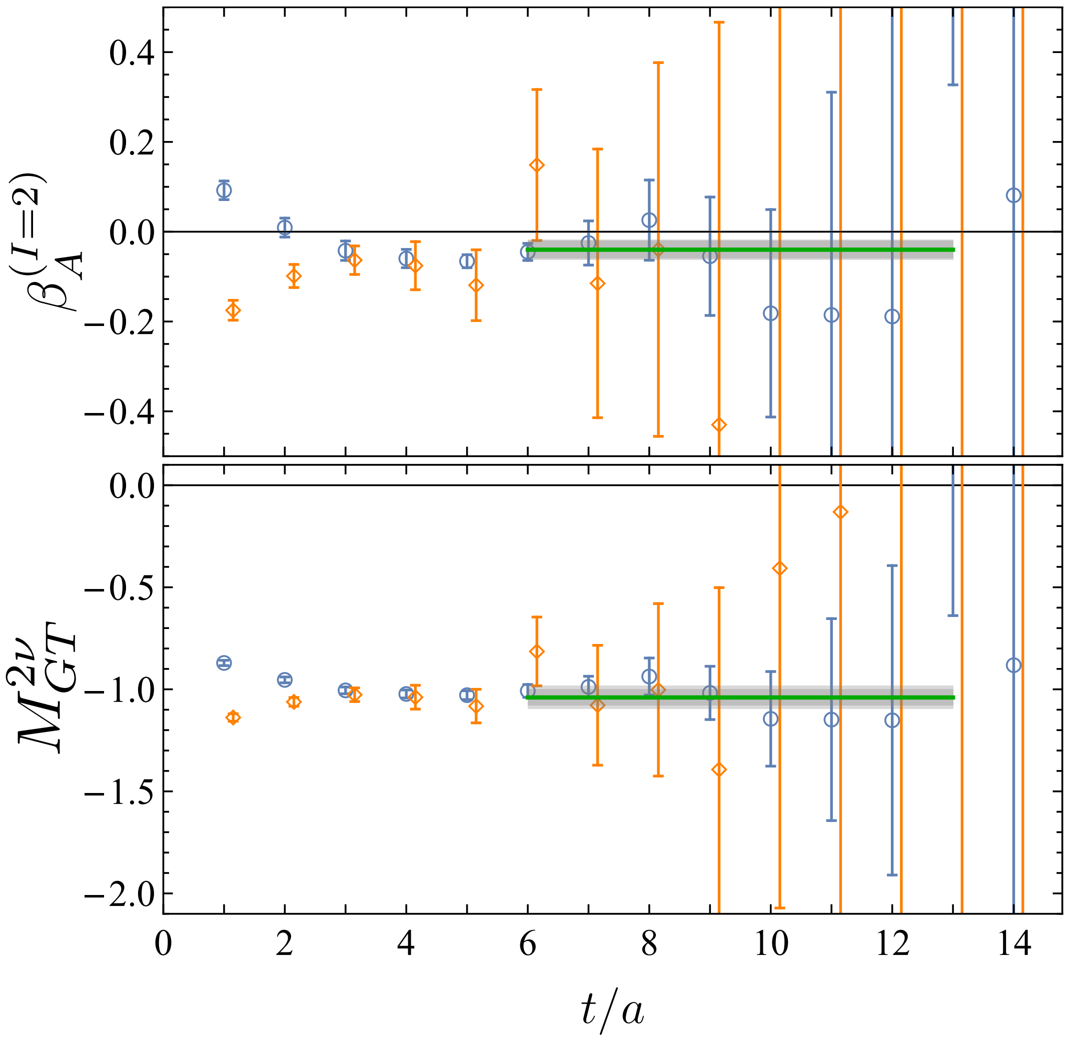
- Non-negligible deviation from long distance deuteron intermediate state contribution

Isotensor axial polarisability

$$M_{GT}^{2\nu} = -\frac{|M_{pp\to d}|^2}{E_{pp}-E_d} + \beta_A^{(I=2)}$$

➡ Quenching of g_A in nuclei is insufficient!

- TBD: connect to EFT for larger systems



Gluon structure of nuclei

How does the gluon structure of a nucleon change in a nucleus?

Ratio of structure function F_2 per nucleon for iron and deuterium

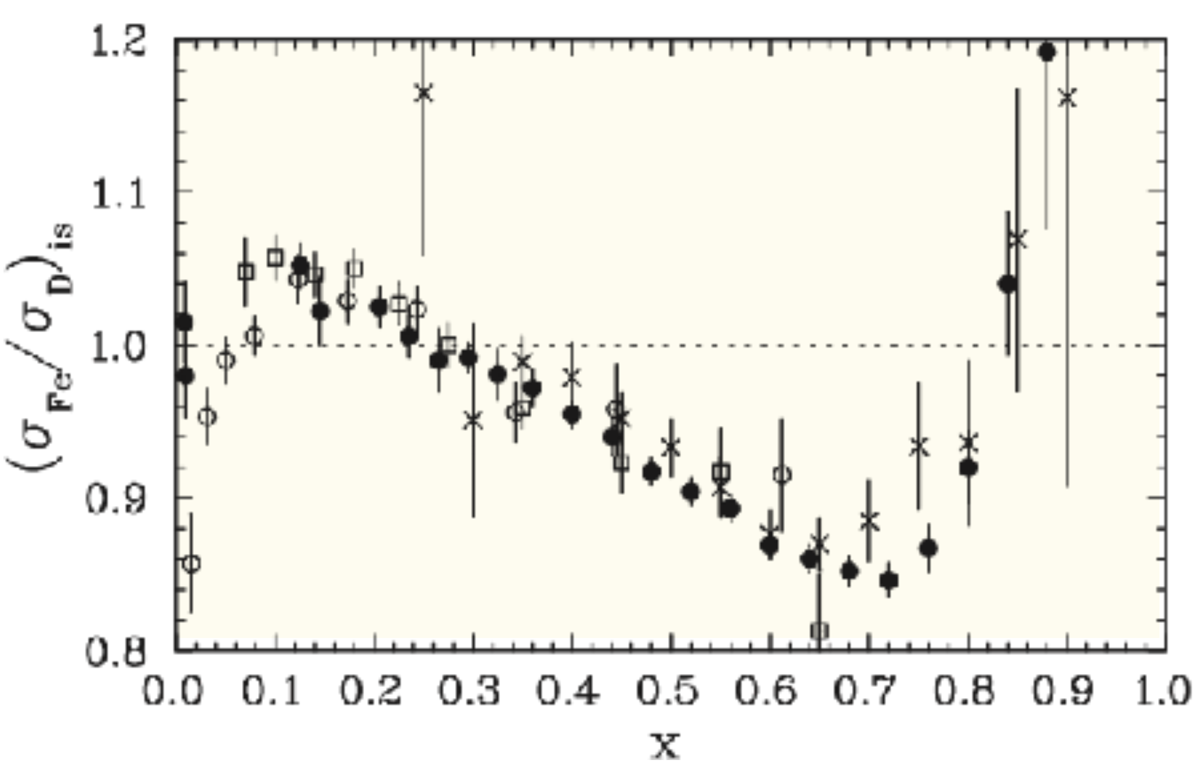
$$F_2(x, Q^2) = \sum_{q=u,d,s...} x\, e_q^2\, [q(x, Q^2) + \overline{q}(x, Q^2)]$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?



Longitudinal momentum fraction

Nuclear glue, m_π ~450 MeV

Look for **nuclear (EMC) effects** in the first moments of the spin-independent gluon structure function

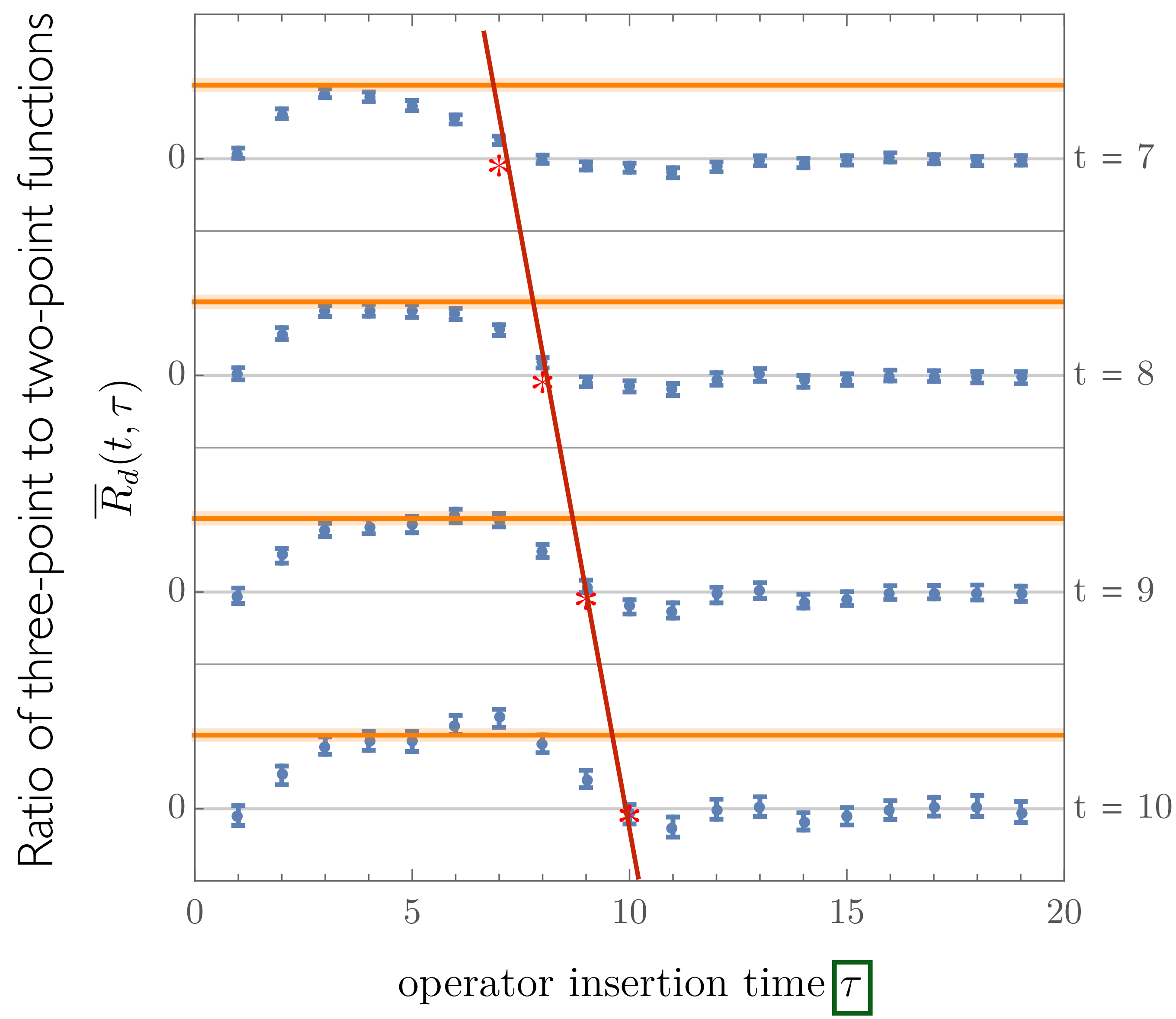
Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

PRD96 094512 (2017)

Deuteron gluon momentum fraction

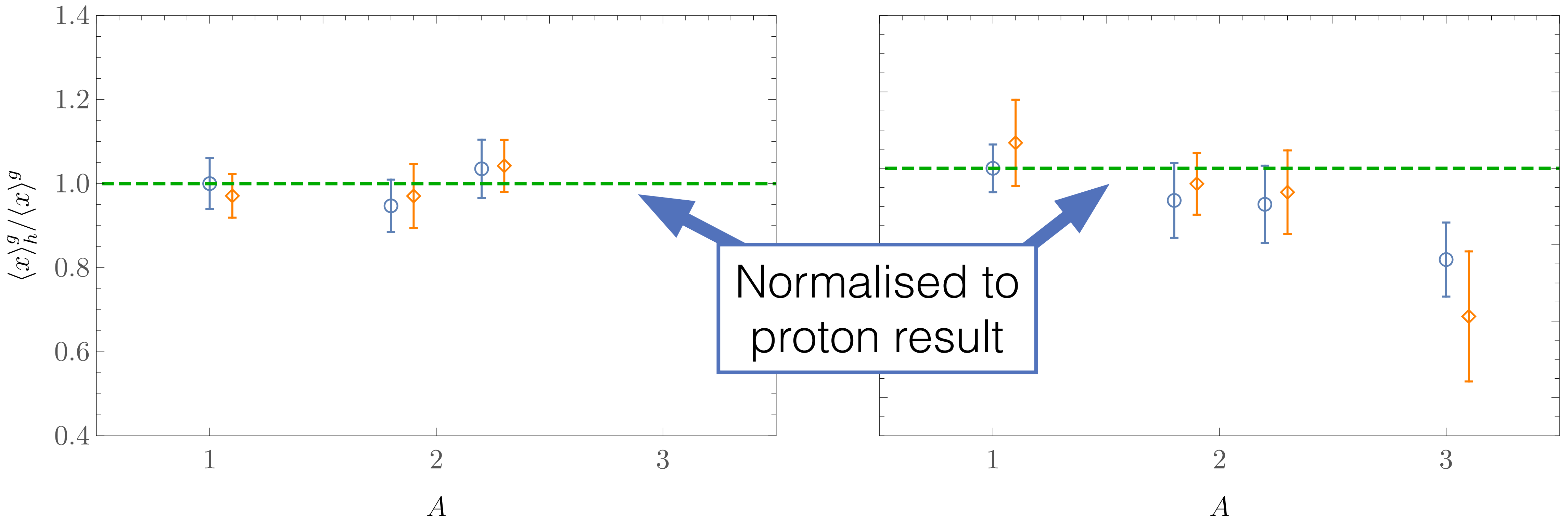
Ratio ∝ matrix element for $0 \ll \tau \ll t$



Gluon momentum fraction

PRD96 094512 (2017)

- Matrix elements of the **Spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators



m_π ~450 MeV

m_π ~800 MeV

Gluonic Transversity

Double helicity flip structure function Δ(x,Q²)

Jaffe and Manohar, "Nuclear Gluonometry" Phys. Lett. B223 (1989) 218

- **Hadrons:** Gluonic Transversity (parton model interpretation)

$$\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \mathrm{Tr} Q^2 \, x^2 \int_x^1 \frac{dy}{y^3} \left[g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2) \right]$$

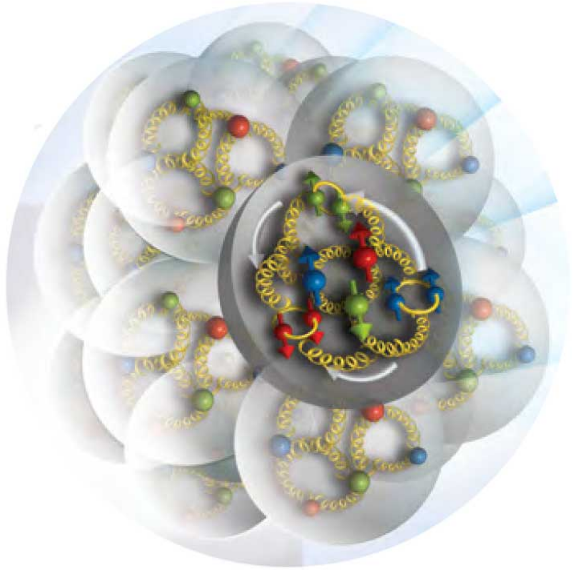
$g_{\hat{x},\hat{y}}(y, Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p | \mathcal{O} | p \rangle = 0$$

$$\langle N, Z | \mathcal{O} | N, Z \rangle \neq 0$$



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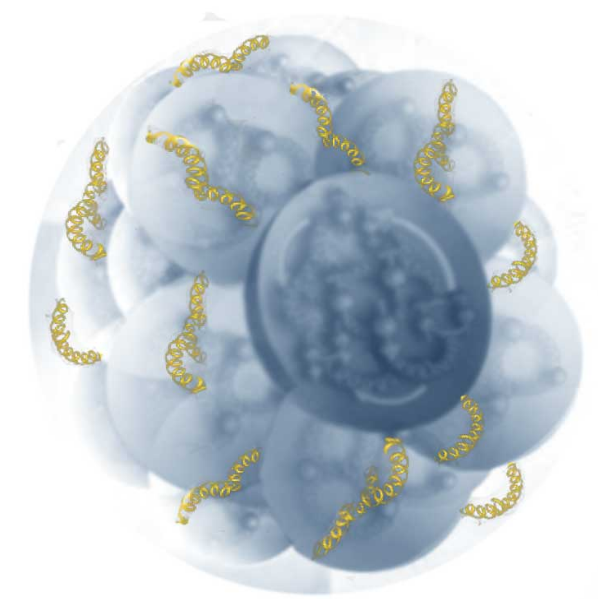
- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



Non-nucleonic glue in deuteron

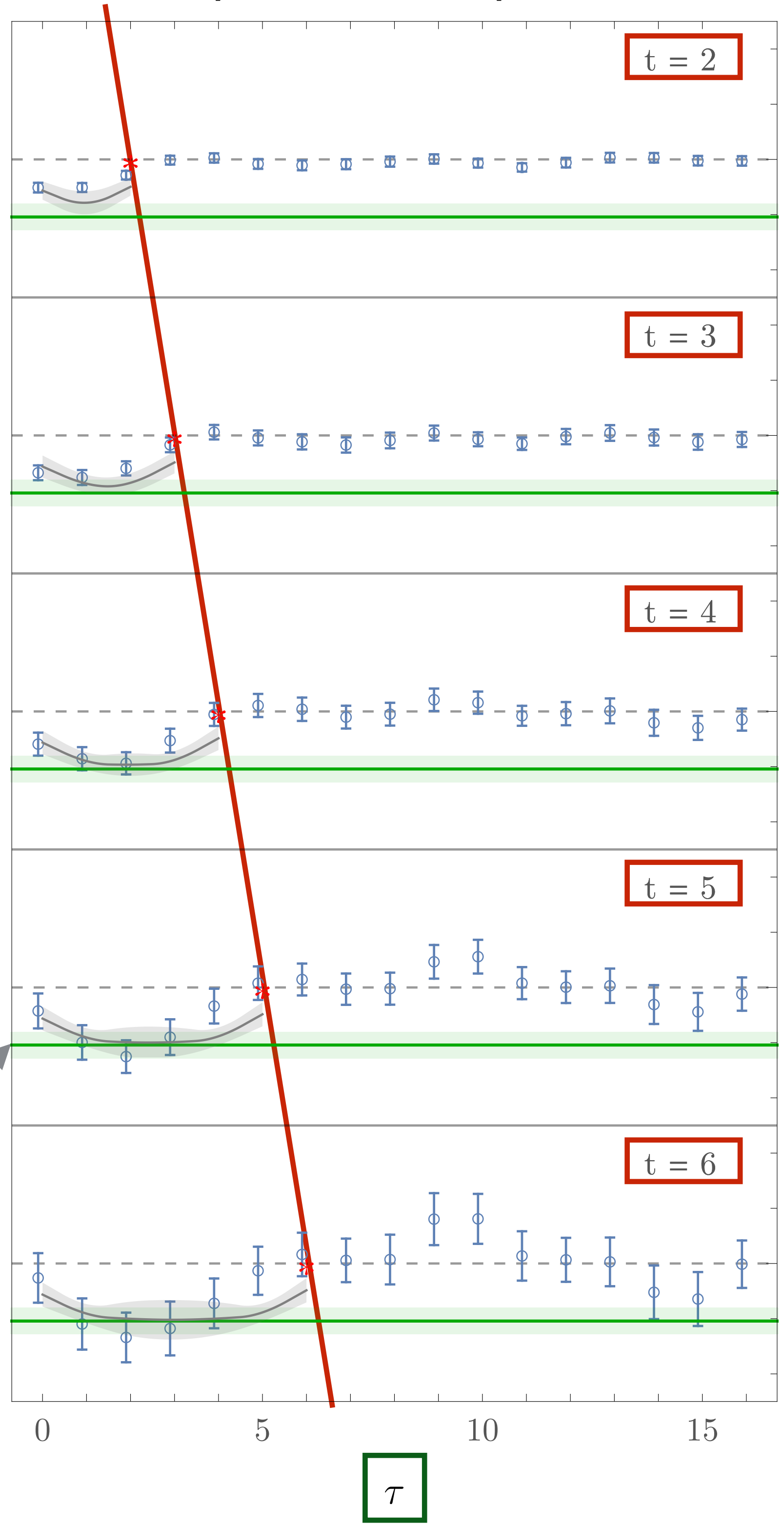
PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron, m_π ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- **Hypothesis of no signal ruled out to better than one part in 10^7**
- Magnitude relative to momentum fraction as expected from large-N_c

Ratio $\propto$ matrix element for $0 \ll \tau \ll t$

Ratio of 3pt and 2pt functions



Scalar & tensor nuclear MEs

- Axial, scalar, tensor charges of light nuclei A<4, at unphysical value of the quark masses m_{π} ~800 MeV
  - Complete flavour-decomposition including strange quarks

Scalar

- Possible DM interaction is through scalar exchange
- Direct detection depends on nuclear matrix element

Tensor

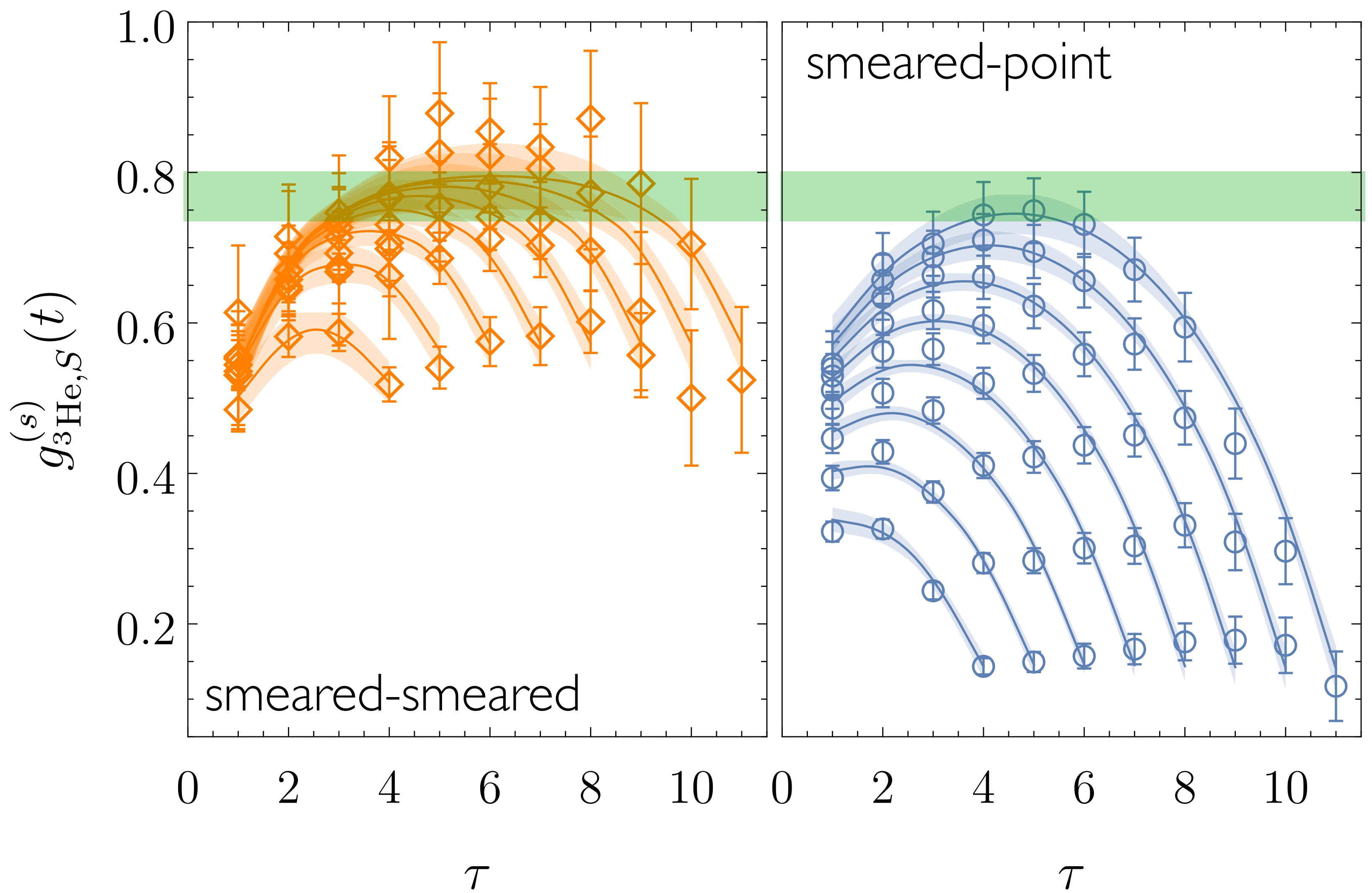
- Quark electric dipole moment (EDM) contributions to the EDMs of light nuclei
- Input for searches for nuclear EDMs as evidence for BSM CP violation

arXiv:1712.03221

Strange matrix elements

- Complete flavour-decomposition including strange quarks
  - Disconnected contributions estimated stochastically
    [Arjun Gambhir, LLNL & LBNL]

Strange scalar ME Helium 3



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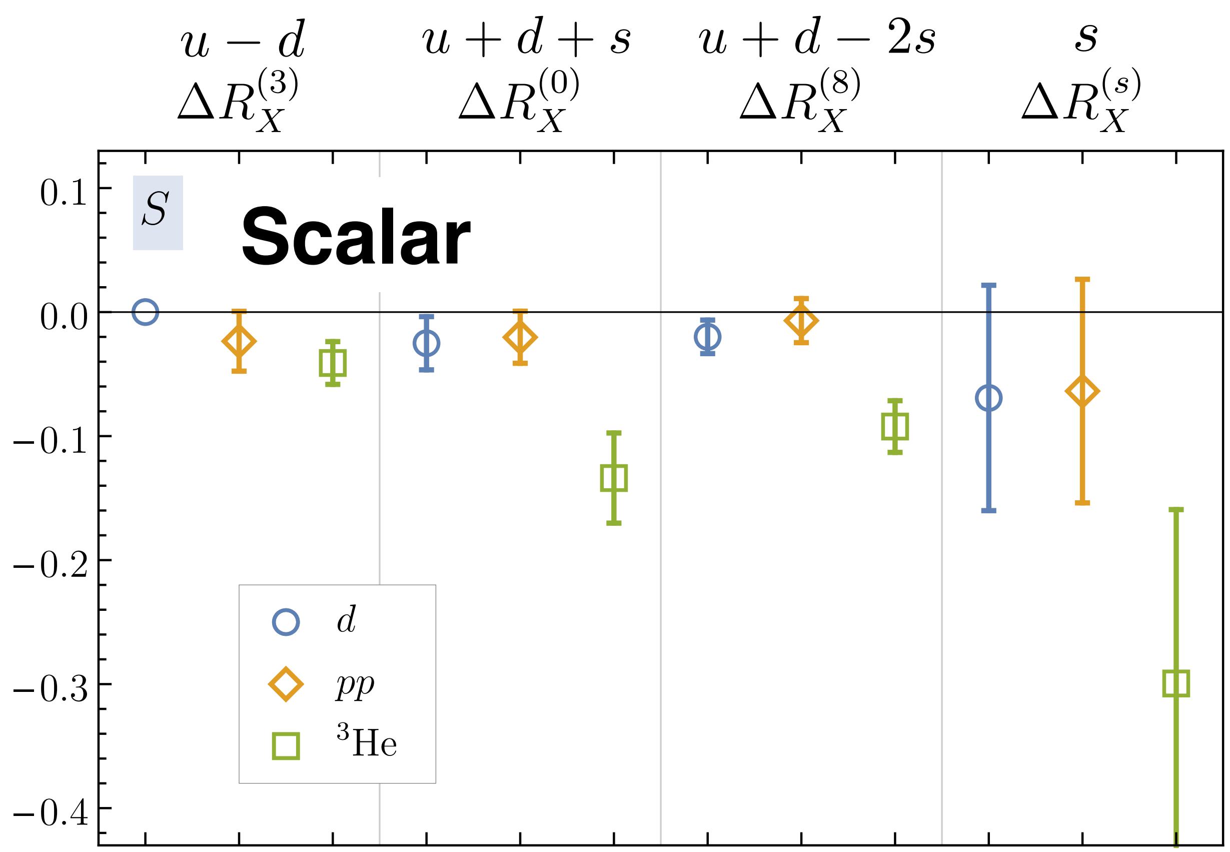
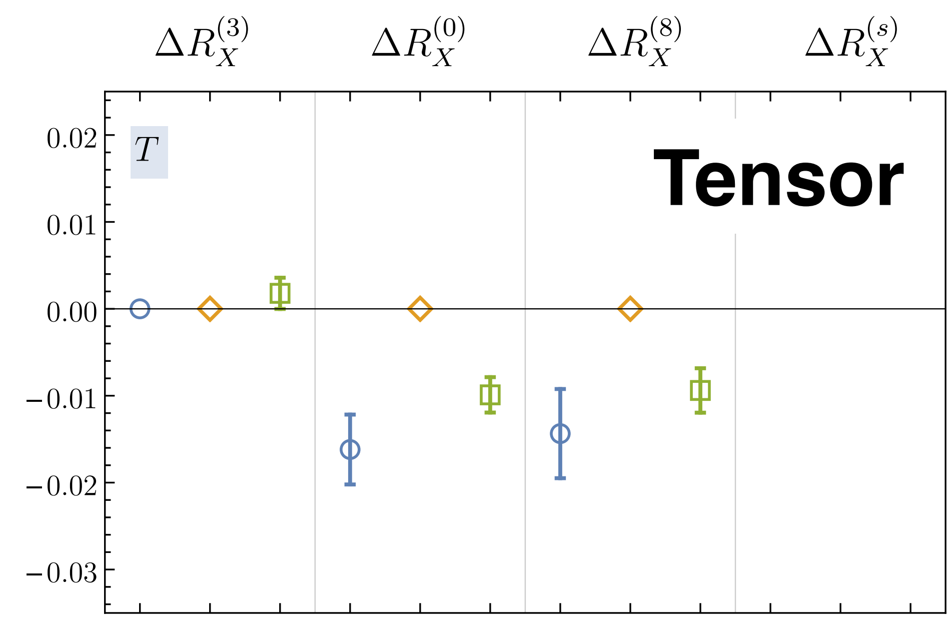
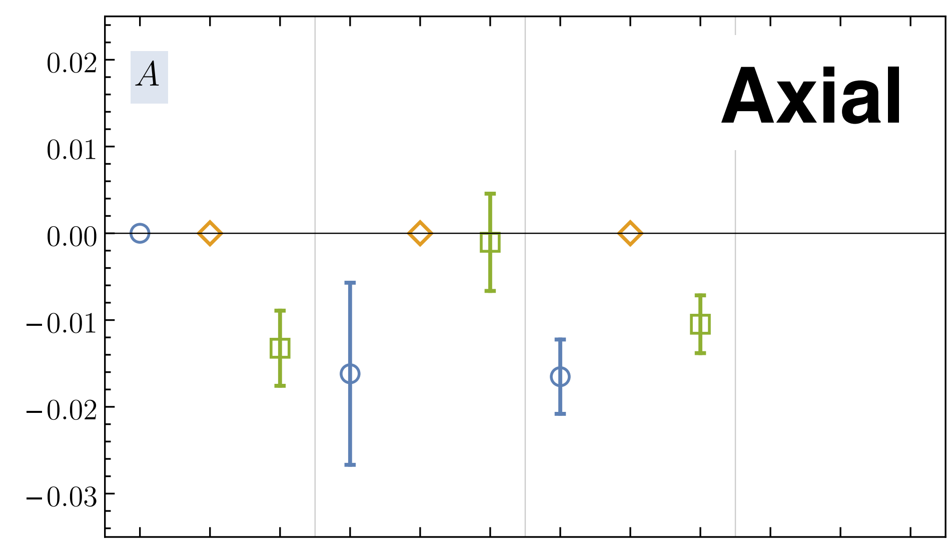


ME / Nucleon ME − naive expectation



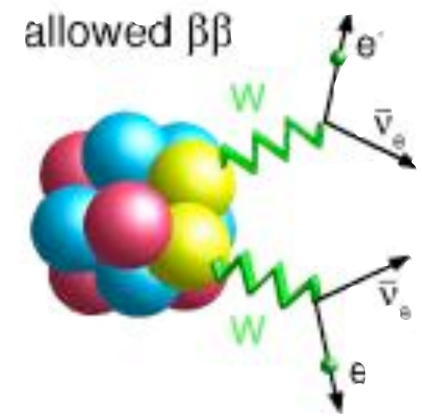


- Naive expectation determined by baryon#, isospin, spin
- O(10%) nuclear effects in the scalar charges
- Nuclear modifications scale with magnitude of corresponding charge (i.e., baryon# for scalar, spin for tensor, axial)

arXiv:1712.03221

Nuclear MEs from LQCD

- Nuclear matrix elements important to experimental programs e.g,
  - Neutrino breakup reaction (SNO)
  - Muon capture reaction (MuSun)
  - Double-beta decay
  - Electron-Ion Collider
  - Nuclear electric dipole moments
  - Dark matter direct detection



- Current state-of-the-art: significant systematics but phenomenologically interesting at current precision

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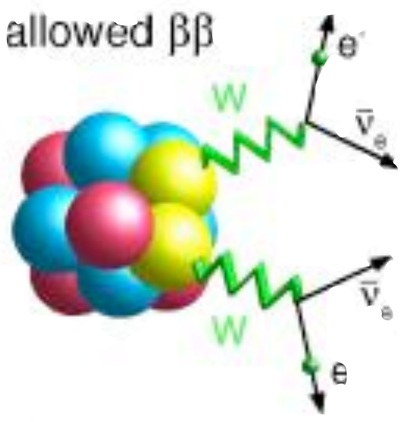

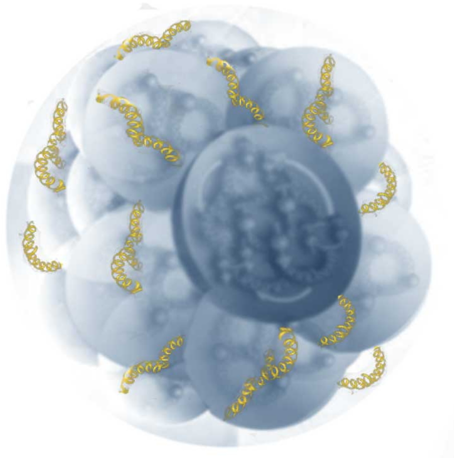

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