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Introduction to the dispersive approach and successes in the mesonic sector

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Multi-Hadron Systems from Lattice QCD
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π and K interactions are interesting

- QCD Goldstone Bosons. Test chiral symmetry breaking

- Lightest non-strange and strange resonances seen there.

  Particularly scalar mesons

- Final products in almost all hadronic interactions

  Their re-**scattering** is essential in many hadronic processes

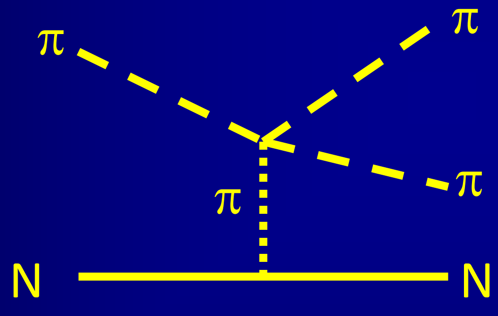
ππ and πK SCATTERING data are poor

- π and K are unstable. Still, beams can be made.

But NOT luminous enough for **ππ and πK** collisions: **Indirect measurements**

1) From meson-Nucleon scattering

Chew-Low Extrapolation (see Gribov's book Sect. 2.6.2)



Initial state not well defined, model dependent off-shell extrapolations (OPE, absorption, A_2 exchange...).
Needs Meson- N-partial wave extraction. Problems with phase shift ambiguities, etc...

As a consequence… VERY LARGE SYSTEMATIC UNCERTAINTIES

ππ scattering data. Example: CERN Munich Experiment

SYSTEMATIC uncertainties larger than STATISTICAL

Nuclear Physics B75 (1974) 189–245. North-Holland Publishing Company

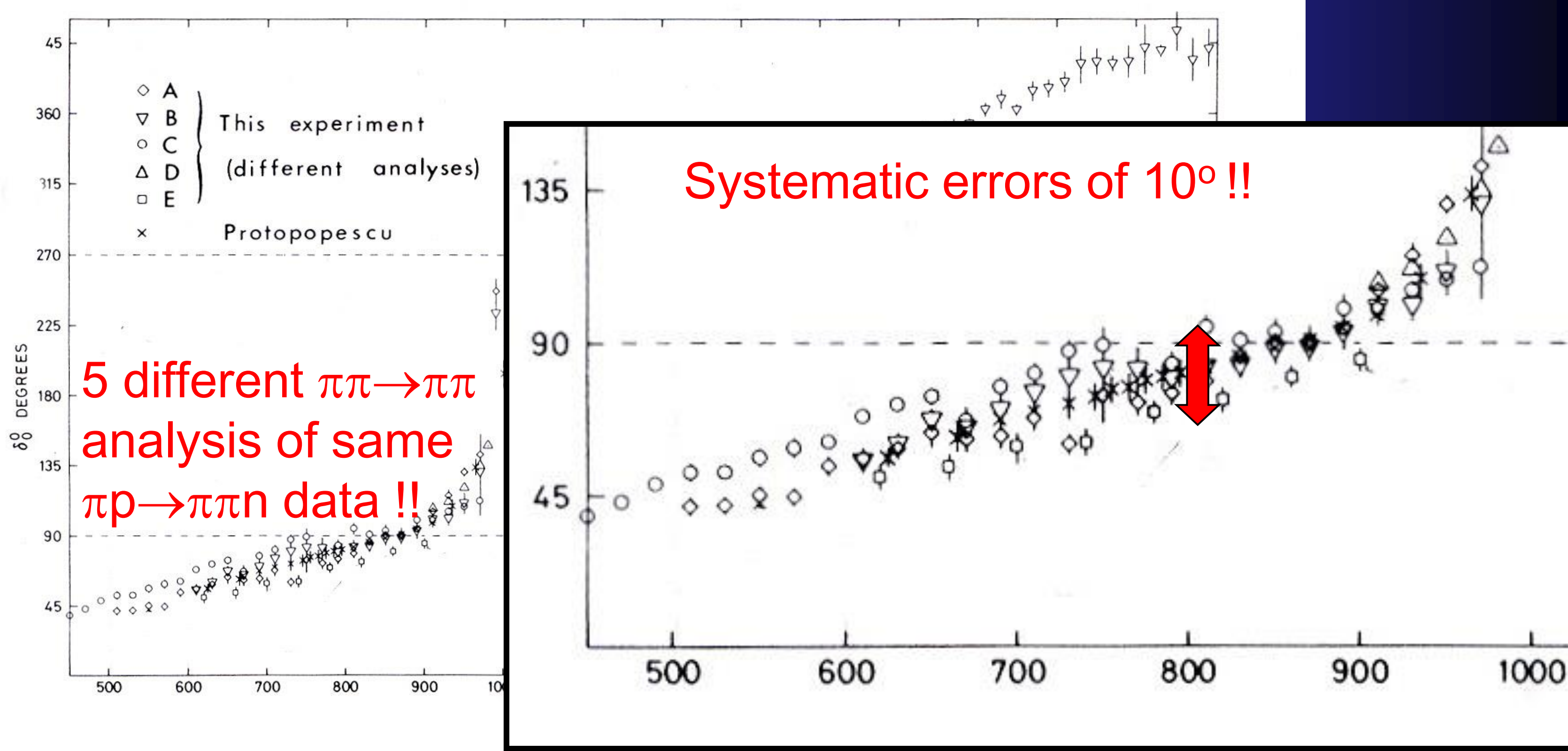


Fig. 31. The ππ scattering phase shift δ_0^0 for spin 0 and isospin 0 as determined by various analyses of our 17 GeV/c $\pi^+\pi^-$ data (A,B,C,D,E), compared with the previous results by Protopopescu et al. [28]: (A) Analysis based on pole extrapolations of the moments (subsect. 4.7.1. (A)). (B) Analysis at each $m_{\pi\pi}$ with $\pi^+\pi^-$ amplitudes assumed to be nucleon-spin and ππ-spin coherent, involving a parametrization to describe the $m_{\pi\pi}$ dependence (subsect. 4.7.2. (B)). (C) Analysis at each $m_{\pi\pi}$ with $\pi^+\pi^-$ amplitudes assumed to be nucleon-spin coherent and using absorption corrections (subsect. 4.7.3. (C)). (D) Analysis with a constant K-matrix fit using $\pi^+\pi^-$ and K^+K^-n data simultaneously (subsect. 4.7.4 (D)). (E) Analysis with $\pi^+\pi^-$ amplitudes assumed to be ππ-spin coherent and using the ρ-meson line shape (subsect. 4.7.5 (E)).

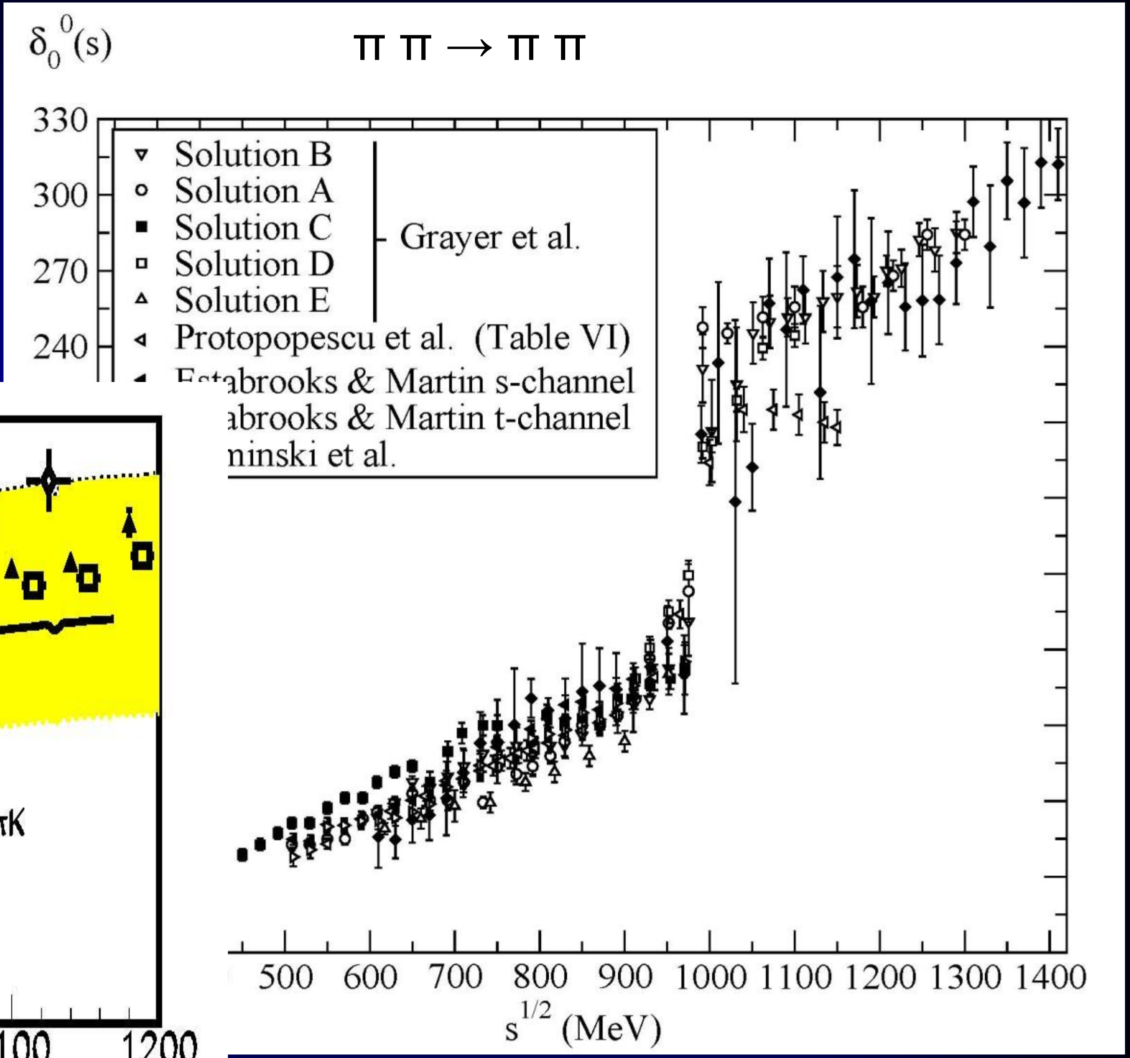
237

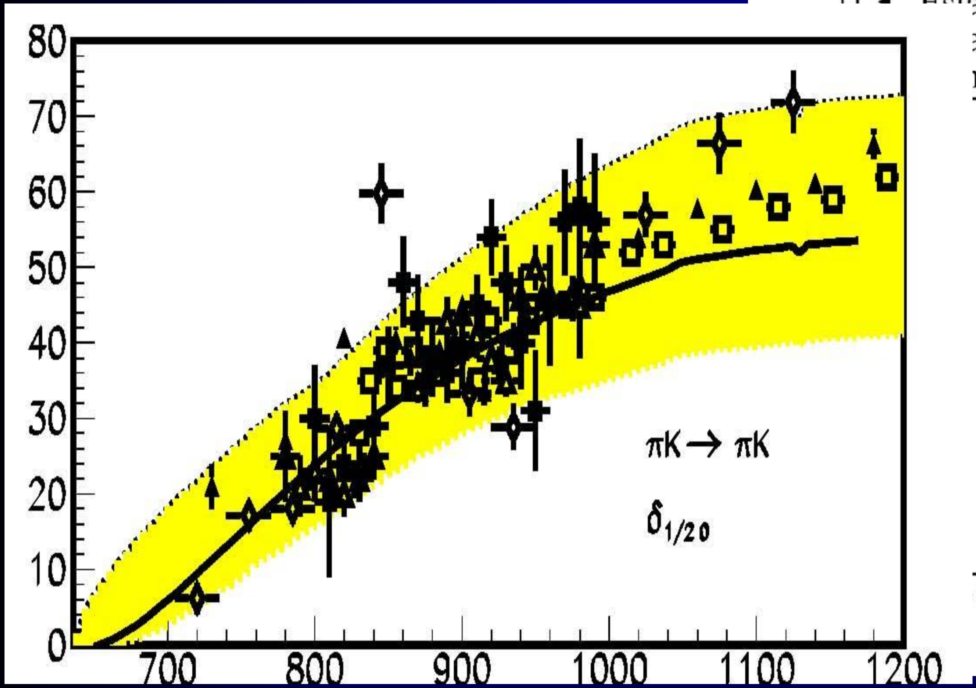
Motivation **ππ and πK SCATTERING data are often in conflict**

First problem:

CONFLICTING DATA SETS

From meson- Nucleon





This talk: show how useful DISPERSION RELATIONS

and ANALYTICITY can be

Motivation — $\pi\pi$ and πK SCATTERING data are bad

- π and K are unstable. Still, beams can be made.

  But NOT luminous enough for **$\pi\pi$ and πK** collisions: **Indirect measurements**

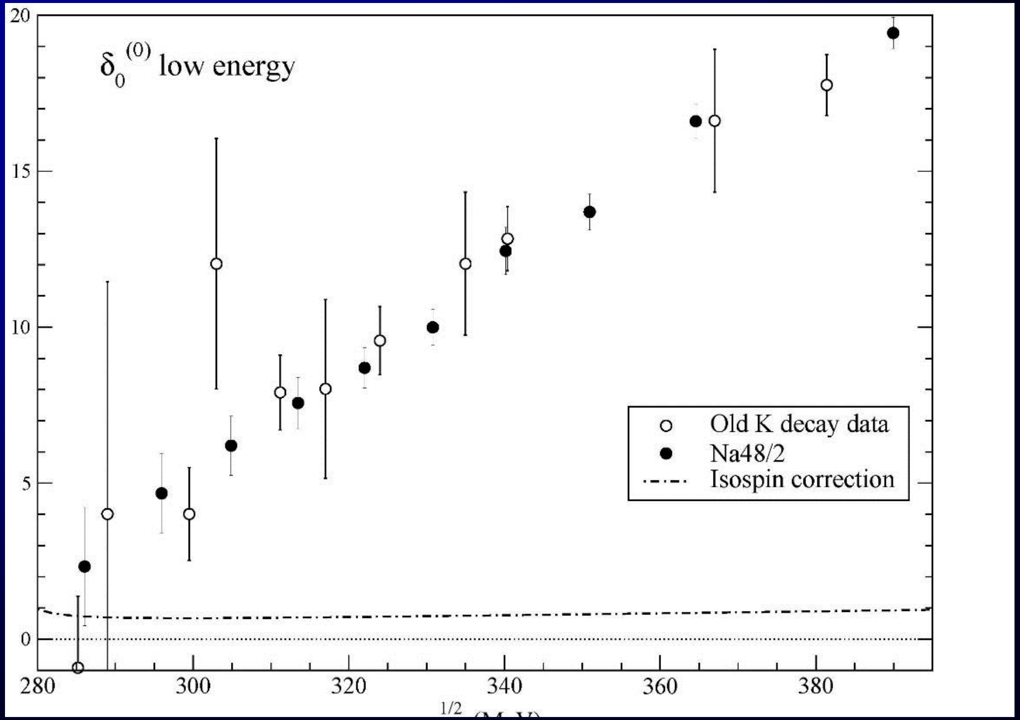
2) The only good data :From $K\rightarrow\pi\pi e\nu$ ("K_{l4} decays")

Geneva-Saclay (77), E865 (01), **NA48/2 (2010)**

Pions on-shell.

Very precise

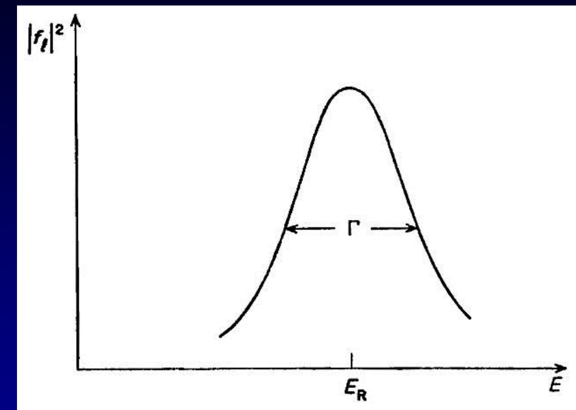
BUT Limited:
only $\pi\pi\rightarrow\pi\pi$
only δ_{00}-δ_{11}.
only $E<M_K$



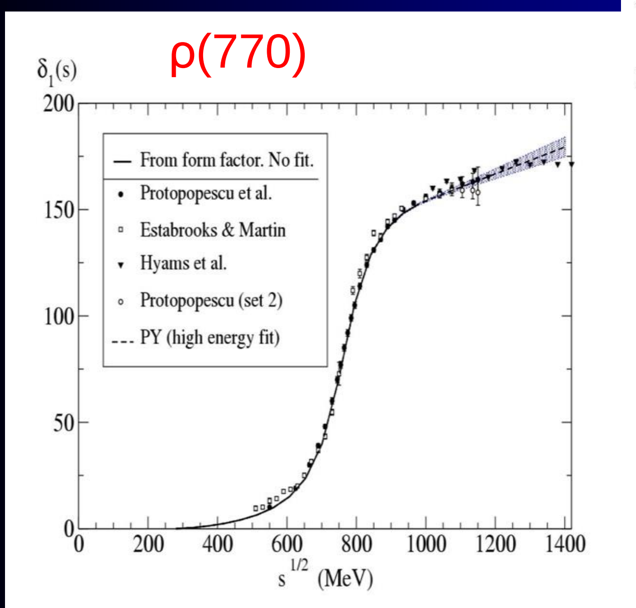
Motivation: Resonances in meson-meson scattering

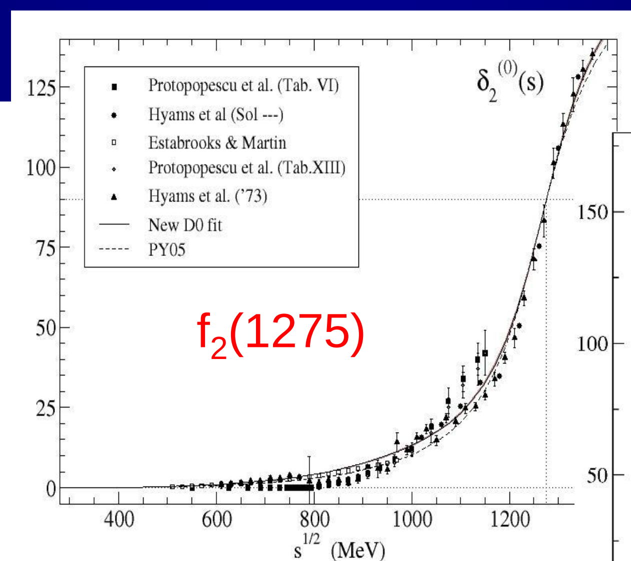
Usually, they are described by Breit-Wigner shapes

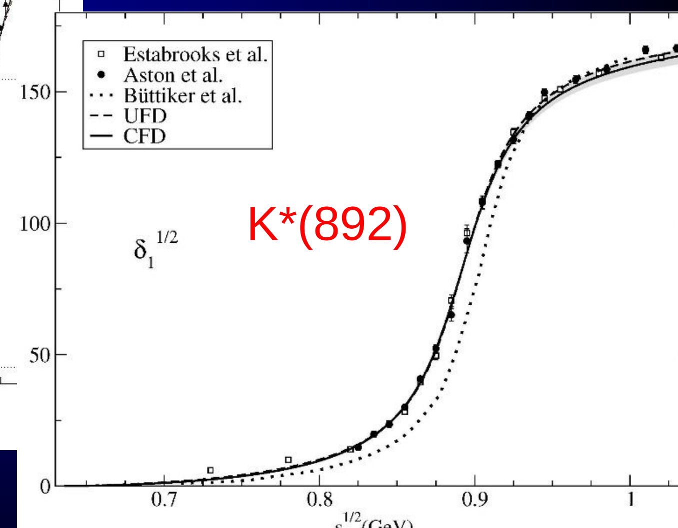
$$\sim \frac{M\,\Gamma(s)}{M^2 - s - iM\,\Gamma(s)}$$



Which in the elastic case produce a typical phase shift rapid increase from 0 to 180 degrees that we have already found several times

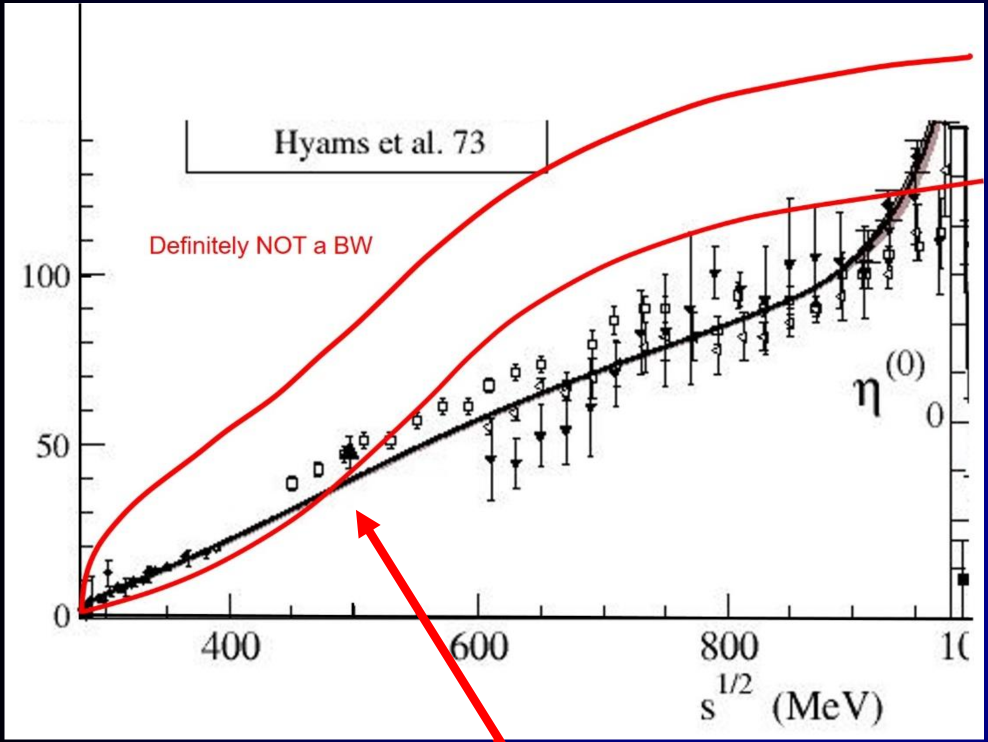


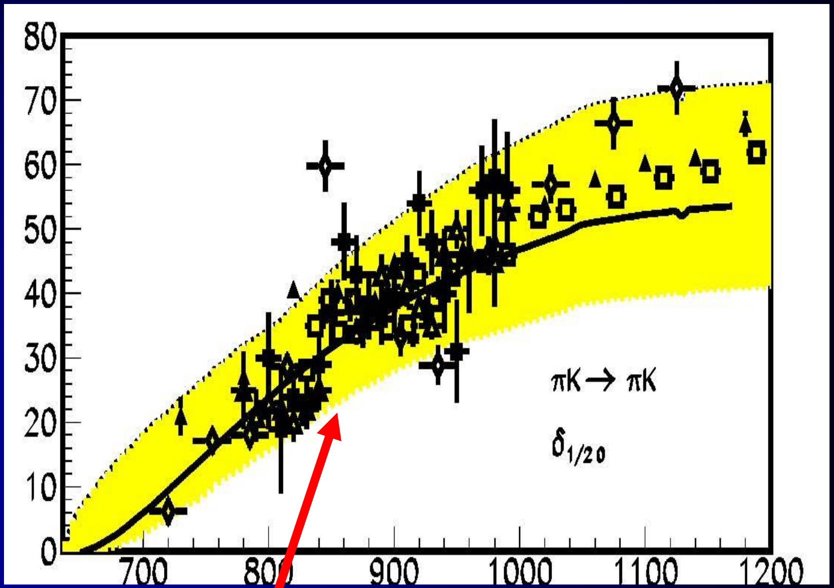




These are easily identified…

Breit-Wigner shapes are easily recognizable…



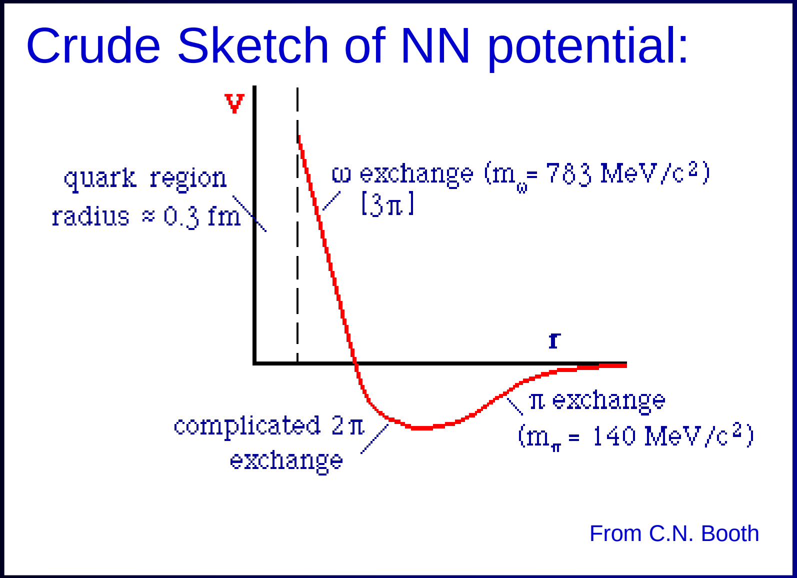


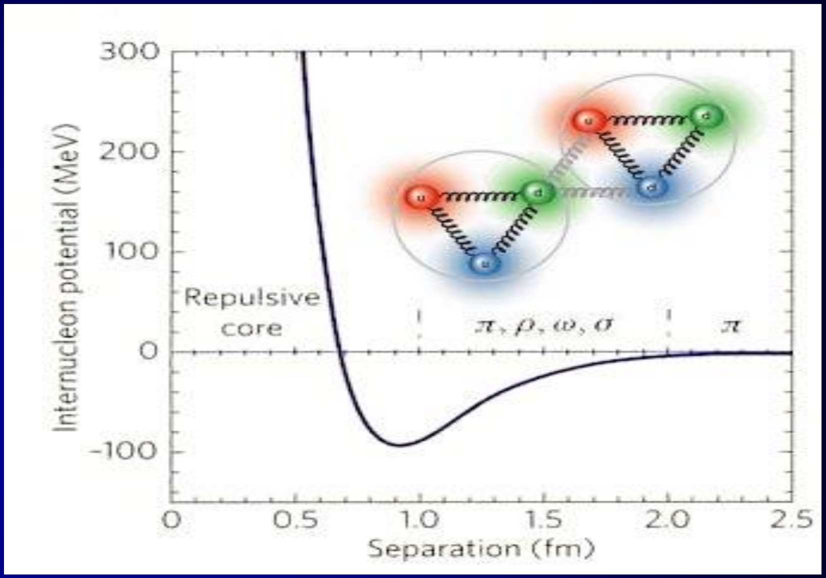
But do you see resonances there?

Nevertheless there is a resonance (a pole) on each graph: the σ/f_0(500) and the κ/K_0*(800) light scalars

Motivation: The $f_0(500)$ or σ, half a century around

- I=0, J=0 ππ exchange very important for nucleon-nucleon **attraction!!**





Scalar-isoscalar field already proposed by Johnson & Teller in 1955

Name given by Schwinger in 1957. Multiplet of isospin.

Soon interpreted within "Linear sigma model" (Gell-Mann) or Nambu Jona Lasinio - like models, in the 60's.

Motivation: Light scalars

- The f_0's have the vacuum quantum numbers.

  Relevant for spontaneous chiral symmetry breaking.

- Glueballs: Feature of non-abelian QCD nature
  The lightest one expected with these quantum numbers
  If κ exists σ almost discarded as glueball (also by lattice)

- Why lesser role in the saturation of ChPT parameters?

- SU(3) classification. How many multiplets? Inverted hierarchy?

  If too many states one might be glueball

- Non ordinary mesons? Tetraquarks, molecules, mixing…

First of all it is relevant to settle their existence, mass and width

Since the 60's-70's many MODELS in conflict

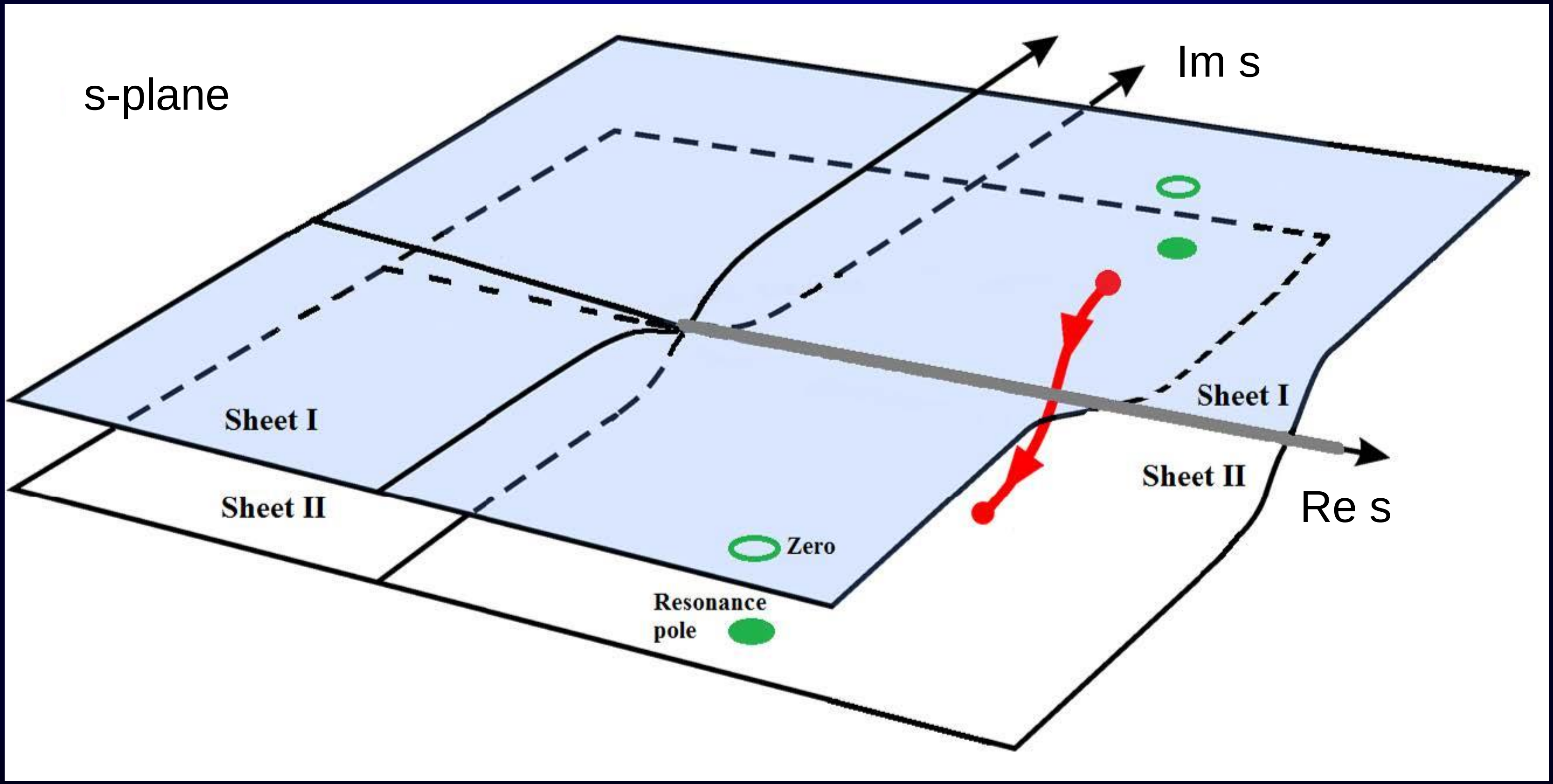
Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their pole positions and residues *

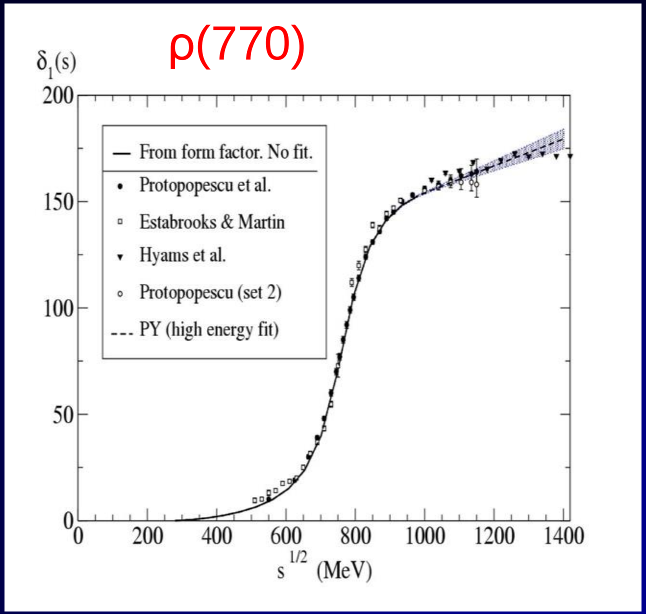
$\sqrt{s_{pole}} \approx M - i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



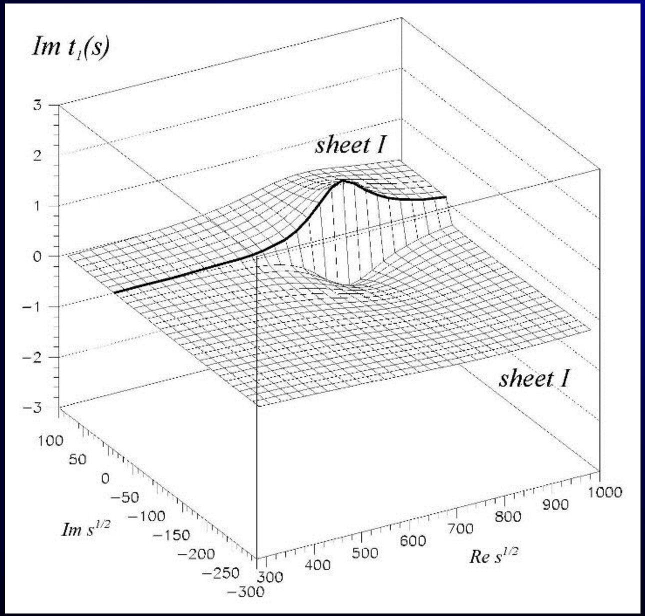
Resonances as poles

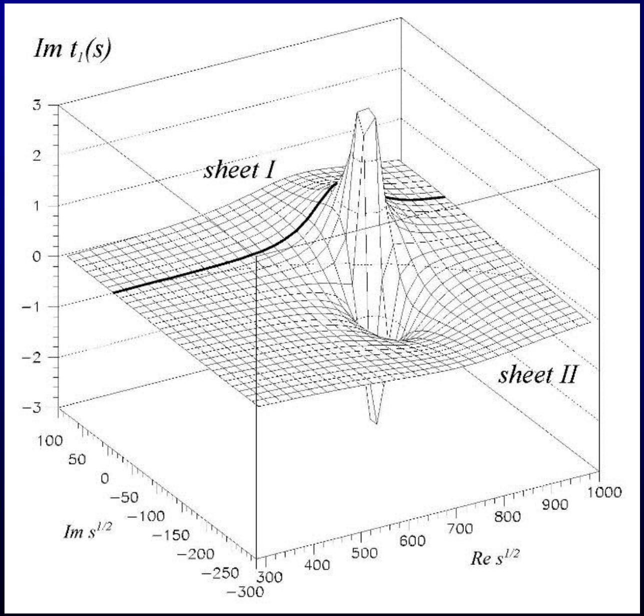
Example:
the ρ(770)



It is a paradigmatic example of a relatively narrow and isolated resonance in an elastic channel

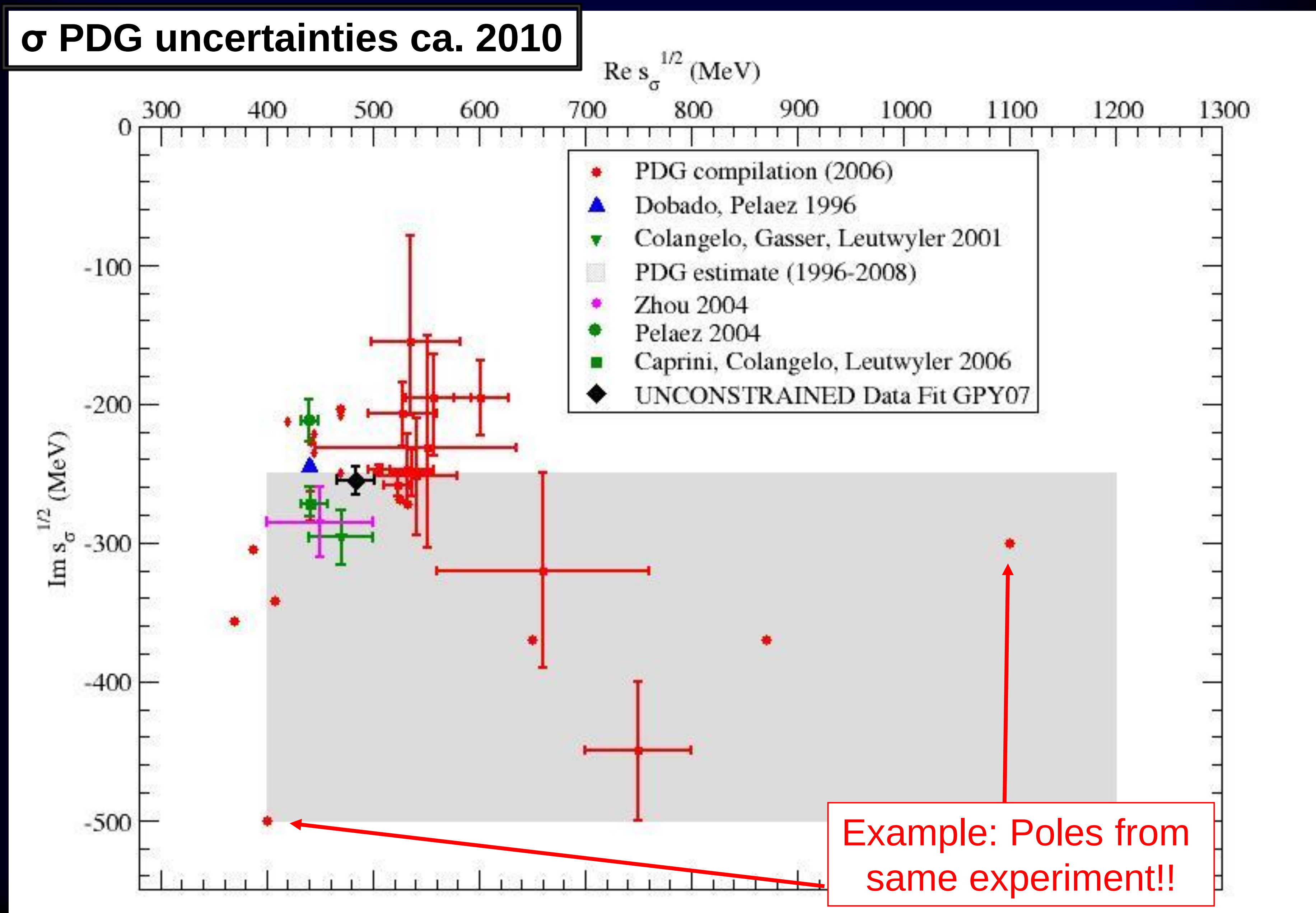
But things are not always that simple…





Actually, the use of naive theoretical tools also adds to the confusion

(Breit-Wigners, Isobars, K-matrices…)



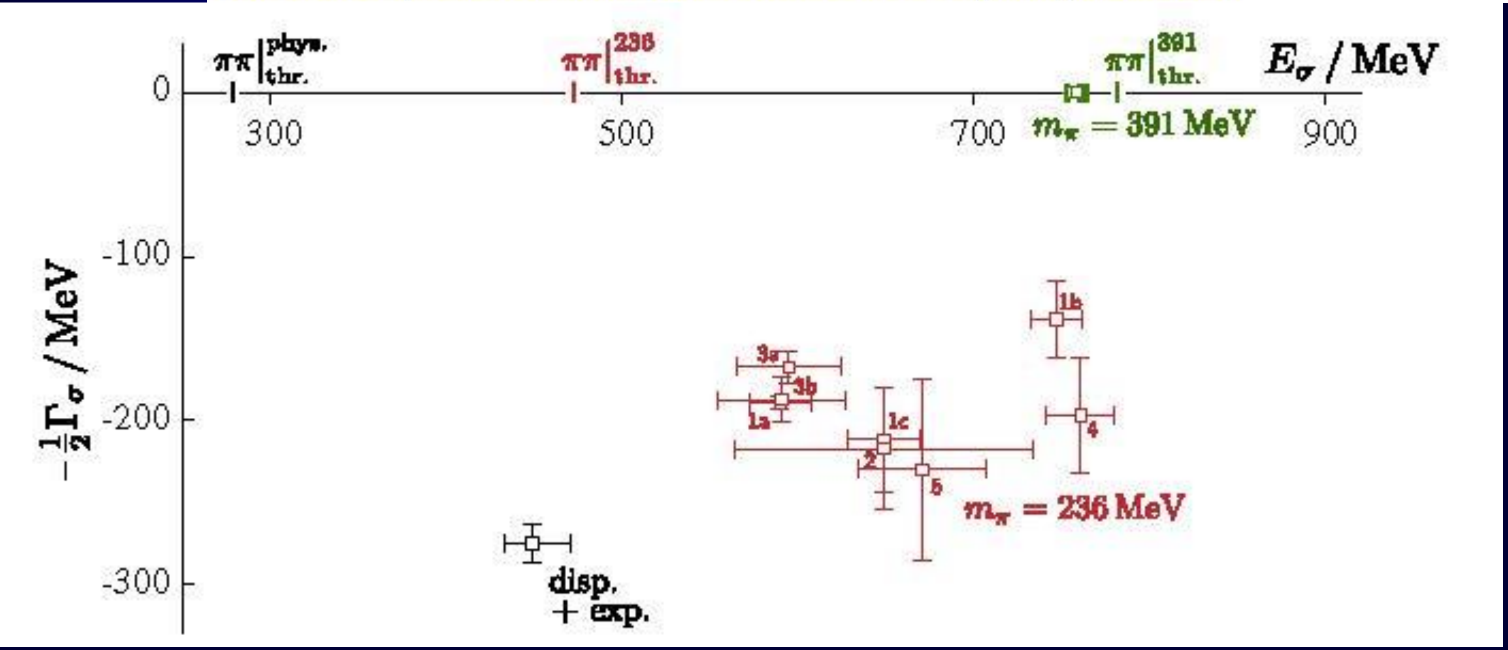
- LATTICE, similar but much smaller problem

PRL **118,** 022002 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

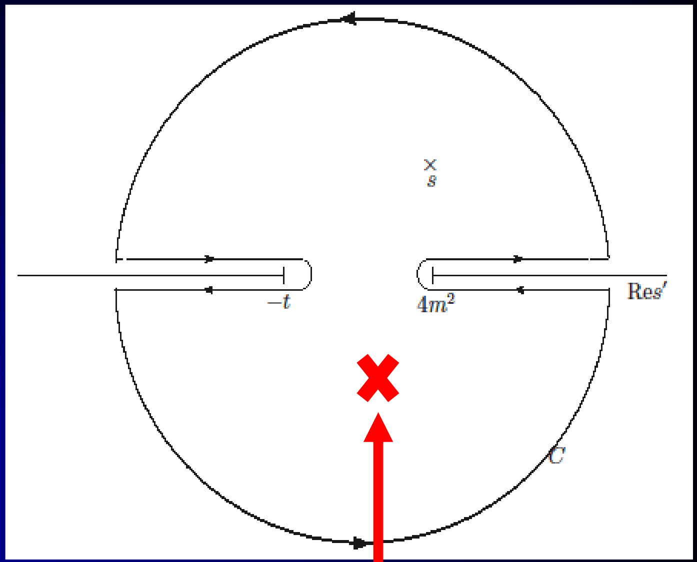
Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

functions are computed including all required quark propagation diagrams, and from these the discrete spectrum of states in the finite volume defined by the lattice boundary is extracted. From the volume dependence of the spectrum, we obtain the S-wave phase shift up to the $K\bar{K}$ threshold. Calculations are performed at two values of the u, d quark mass corresponding to $m_\pi = 236, 391$ MeV, and the resulting amplitudes are described in terms of a σ meson which evolves from a bound state below the $\pi\pi$ threshold at the heavier quark mass to a broad resonance at the lighter quark mass.



- Analytic continuation slightly model dependent. Maybe Roy-like eqs could help.

CAUSALITY:
Amplitudes T(s,t) are ANALYTIC in complex s plane but for cuts for thresholds.
Crossing implies left cut from u-channel threshold



Cauchy Theorem determines T(s,t) at ANY s, from an INTEGRAL on the contour

If T->0 fast enough at high s, curved part vanishes

$$T(s,t) = \int_{th}^{\infty} \frac{Im\,T(s',t)}{s-s'} ds' + LC$$

Otherwise
(subtractio
Left cut u

We can calculate t(s)
anywhere we want using the same integral expression

Good for:

1) Calculating T(s,t) where there is not data

2) Constraining data analysis

3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane without extra assumptions

So, we need to get rid of ONE VARIABLE
to write CAUCHY THEOREM in terms of the other one

1) Fix one variable in terms of the other
(fixed-t, hyperbolic relations…)

2) Integrate one variable and keep the other
(partial wave dispersión relations)

Single variable Dispersion Relations (DR)

- **1) Fixed-t Dispersion Relations** (or fixed-s) for $T(s,t_0)$

  Simple analytic structure in s-plane, simple derivation and use

  Left cut: With crossing may be rewritten in terms of physical region

  Most popular: $t_0=0$, **FORWARD DISPERSION RELATIONS** (FDRs).
  (Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

  One equation per amplitude.
  High Energy part very well known since Forward Amplitude~ Total cross section

  Positivity in the integrand contributions, good for precision.

  Calculated up to 1400 MeV (ππ) or 1.7 GeV (πK)

  Not practical for unphysical sheets

Forward dispersion relations for π π.

Complete isospin set of 3 forward dispersion relations for :

- Two s-u symmetric amplitudes. $F_{0+} \equiv \pi^0\pi^+ \to \pi^0\pi^+$, $F_{00} \equiv \pi^0\pi^0 \to \pi^0\pi^0$
  ONE SUBTRACTION
  Only depend on two isospin states. Positivity of imaginary part

$$\mathrm{Re}\,F(s) - \mathrm{Re}\,F(4M_\pi^2) = \frac{s(s-4M_\pi^2)}{\pi} PP \int_{4M_\pi^2}^{\infty} ds' \frac{(2s'-4M_\pi^2)\,\mathrm{Im}\,F(s')}{s'(s'-s)(s'-4M_\pi^2)(s'+s-4M_\pi^2)}$$

Additional sum rules SRJ, SRK if evaluated at $s=2M_\pi^2$ (Adler Zeros),

- The $I_t=1$ s-u antisymmetric amplitude

$$\mathrm{Re}\,F(s) = \frac{(2s-4M_\pi^2)}{\pi} PP \int_{4M_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,F(s')}{(s'-s)(s'+s-4M_\pi^2)}$$

At threshold is the Olsson sum rule

Partial-wave Dispersion Relations

Analytic structure complicated if unequal masses (Circular cuts)

Left cut: With crossing may be rewritten in terms of physical region.

But then different partial waves coupled.

In practice, limited to a finite energy.

But **good** and simple **for** elastic **resonance poles**

For elastic partial waves the second Riemann sheet is easy to obtain.

Due to elastic unitarity:

$$S^{II}(s) = \frac{1}{S^{I}(s)}$$

Recalling $S(s) = 1 + 2i\sigma\, t(s), \quad \sigma(s) = \frac{k}{2\sqrt{s}}$

The second sheet is then:

$$\boldsymbol{t^{II}(s) = \frac{t^{I}(s)}{1 + 2i\sigma\, t^{I}(s)}}$$

Looking for resonance poles
is nothing but looking for a zero in that denominator
on the first Riemann sheet accesible with the pw DR

The real improvement: Analyticity (and Effective Lagrangians)

Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,…

Uses Chiral Perturbation Theory amplitudes inside dispersion relation.

Relatively simple, although different levels of rigour. Generates all scalars

Crossing (left cut) approximated… , not so good for precisión but good for connecting with QCD

Roy-like equations.

70's Roy, Basdevant, Pennington, Petersen…

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski, JRP…

Left cut implemented with precision . Use data on all waves + high energy .

Optional: ChPT predictions for subtraction constants

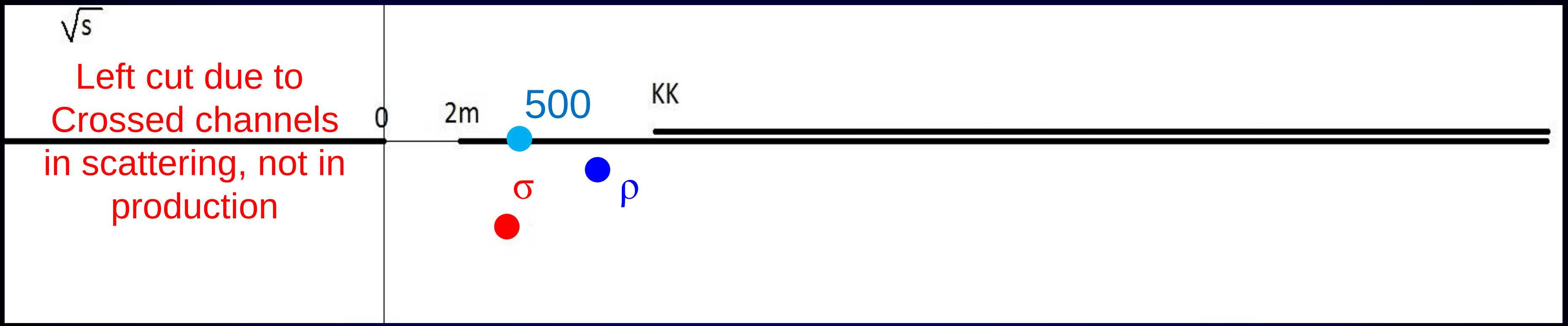
The most precise and model independent pole determinations

f_0(500) and K_0*(800) existence, mass and width

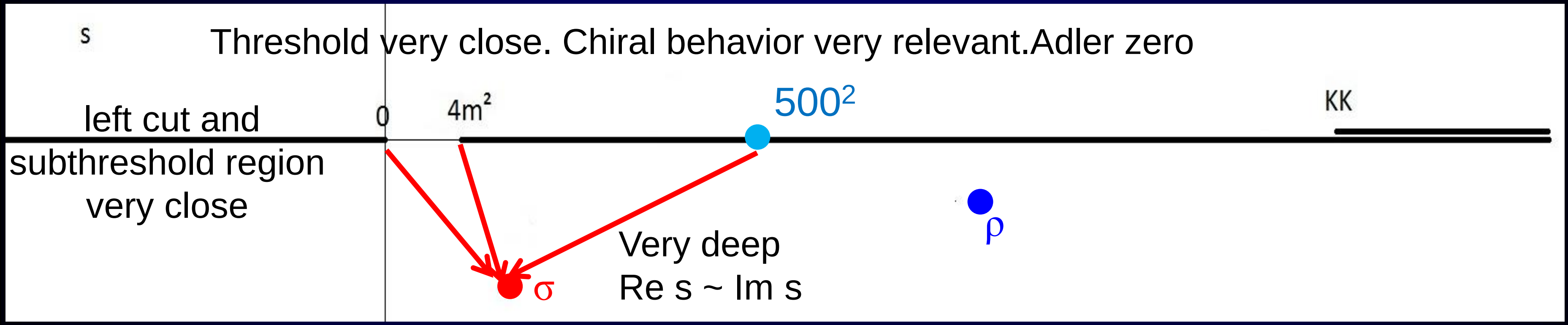
firmly established with precision

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

It is somewhat misleading to think of analyticity in terms of $\sqrt{s}$



Since the partial wave is analytic in *s* ….



For the σ and κ a good control of the left cut and threshold region is important.
This is why Roy-like equations are so relevant for precise pole determinations.

1) Choose the number of subtractions (2=Roy, 1=GKPY)

2) Write fixed-t dispersion relations and project them in partial waves. Limited to s≤ 68 m_π^2 ~ O(1.1) GeV (More complicated extensions exist)

3) Use $s \leftrightarrow u$ crossing symmetry to re-write:

- left cut in terms of partial wave expansions of the other channels. But crossed channels are also ππ→ππ. Coupled equations.
- Subtraction terms

4) Truncate for low energy and low pw. The rest is input (driving terms)

Complications for πK→πK (Roy-Steiner Eqs). Also for πN and γγ→ππ)

2) Different masses. Better use "hyperbolic" Dispersion Relations for larger applicability domain.

3) Crossing involves other processes (ππ→KK). More equations coupled.

Both are coupled channel equations for the infinite partial waves:

I=isospin 0,1,2 , ℓ =angular momentum 0,1,2….

$$\mathrm{Re}\, t_\ell^{(I)}(s) = ST_\ell^{(I)}(s) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{1} PP \int_{4M_\pi^2}^{s\,\max} ds'\, K_{\ell\ell'}^{II'}(s')\, \mathrm{Im}\, t_\ell^{(I)}(s') + DT_\ell^{(I)}(s)$$

Partial wave on real axis	SUBTRACTION TERMS (polynomials)	KERNEL TERMS known	DRIVING TERMS (truncation) Higher waves and High energy
	ROY:2nd order	More energy suppressed	Very small
	GKPY:1st order	Less energy suppressed	small

“OUT” =? “IN (from our data parametrizations)”

Two strategies

- SOLVE equations: (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern…)

  S and P wave solution for Roy or GKPY equations unique at low energy if high-energy, higher waves and scattering lengths known. (in isospin limit)

  NO scattering DATA used at low energies ($\sqrt{s} \leq 0.8 \sim 1\ GeV$)

  Good if interested in low energy scattering and do not trust data.

  Uses ChPT input for threshold parameters

I guess this is NOT what you would like to do with your lattice data

- Impose Dispersion Relations on fits to data. (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

  Use any functional form and fit to DATA imposing DR within uncertainties.

  Also needs input on other waves and high energy.

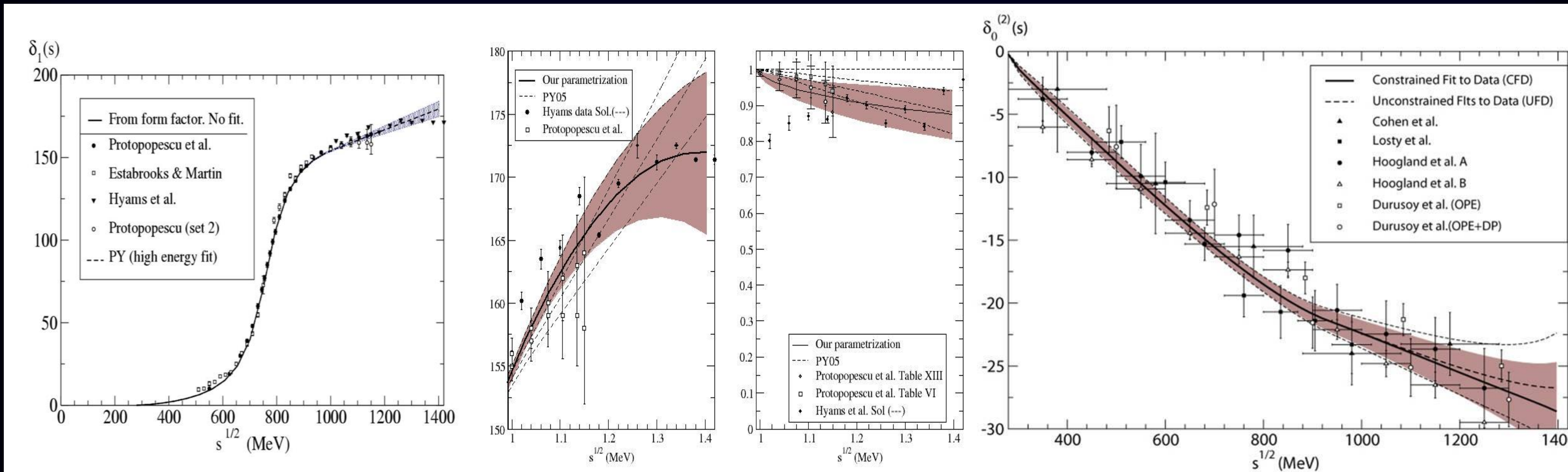
  (But you can use physical inspiration for clever choices of parameterizations)

Our series of works: 2005-2011

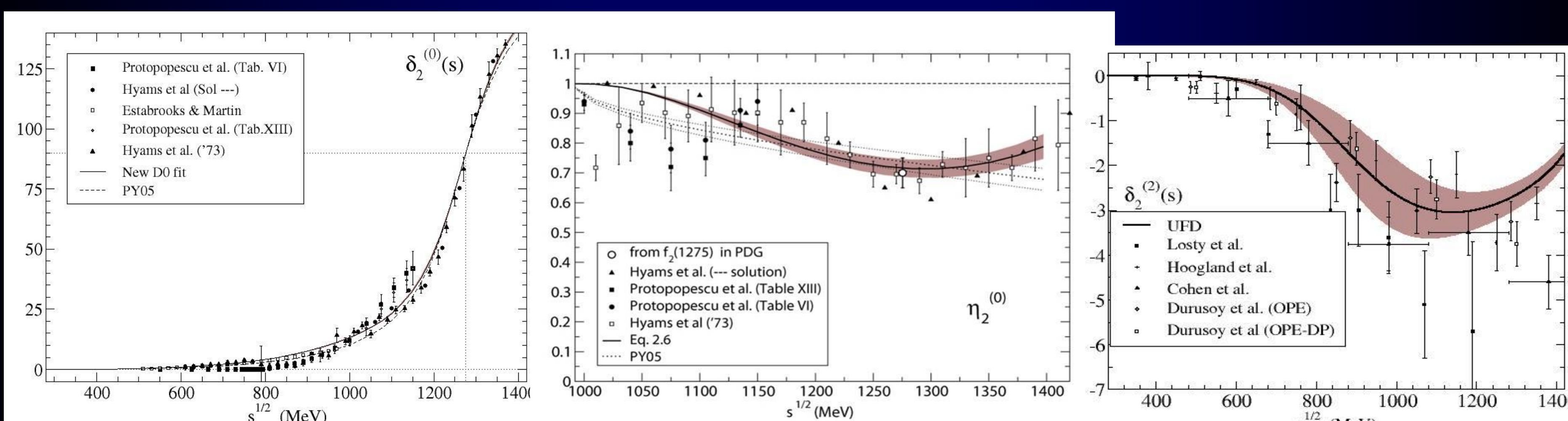
R. Kaminski, JRP, F.J. Yndurán Eur.Phys.J.A31:479-484,2007. PRD74:014001,2006
JRP ,F.J. Yndurán. PRD71, 074016 (2005) , PRD69,114001 (2004),
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Yndurán, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

Simple UNconstrained Fits to Data: P wave, IJ=11



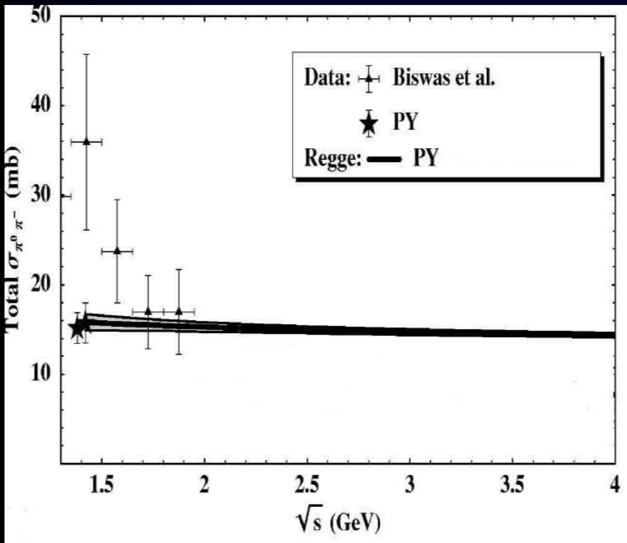
Simple fits easy to write down for phase shifts and inelasticities
For P,S2,D0,D2,anf F waves

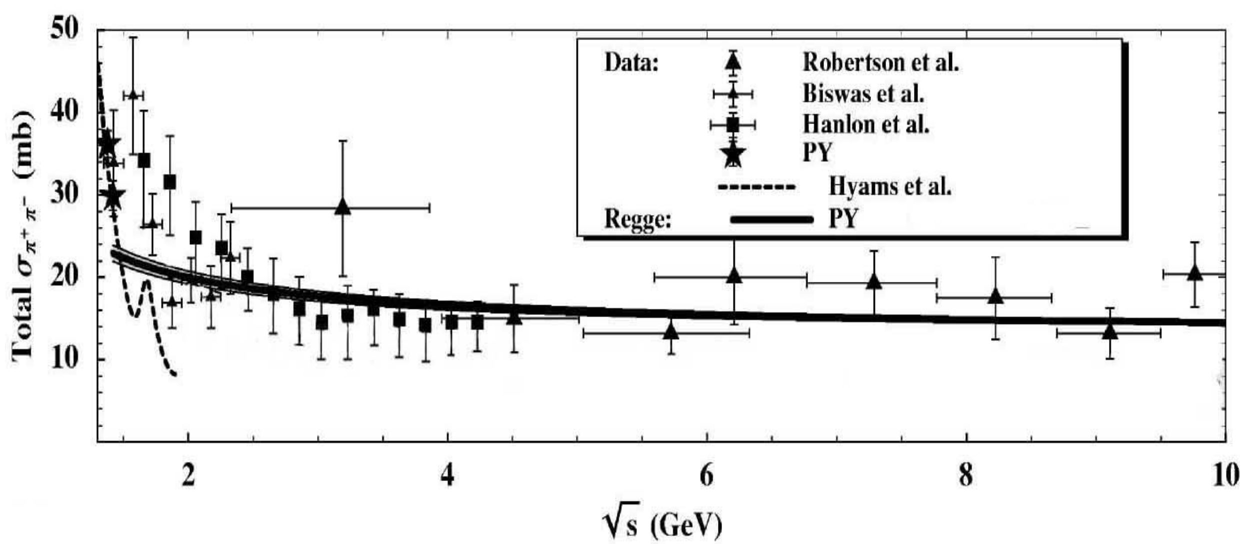


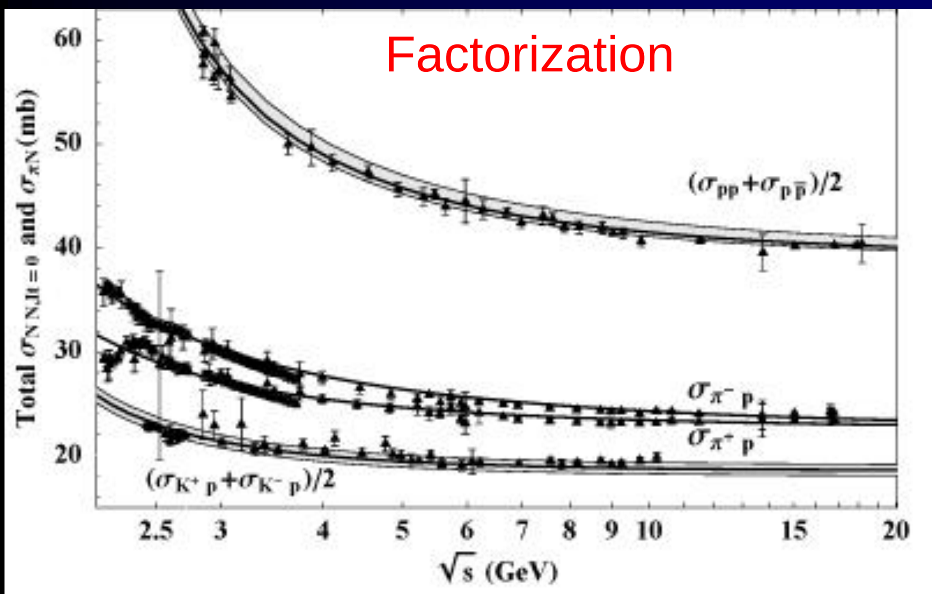
UNconstrained Fits for High energies

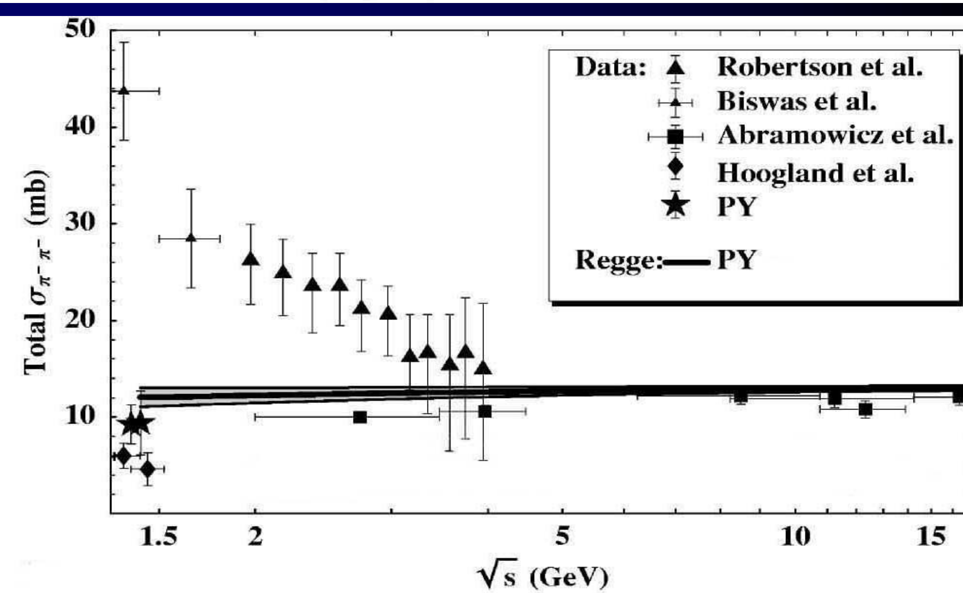
For simplicity we use **Regge parametrizations** of data

JRP, F.J.Ynduráin. PRD69,114001 (2004)





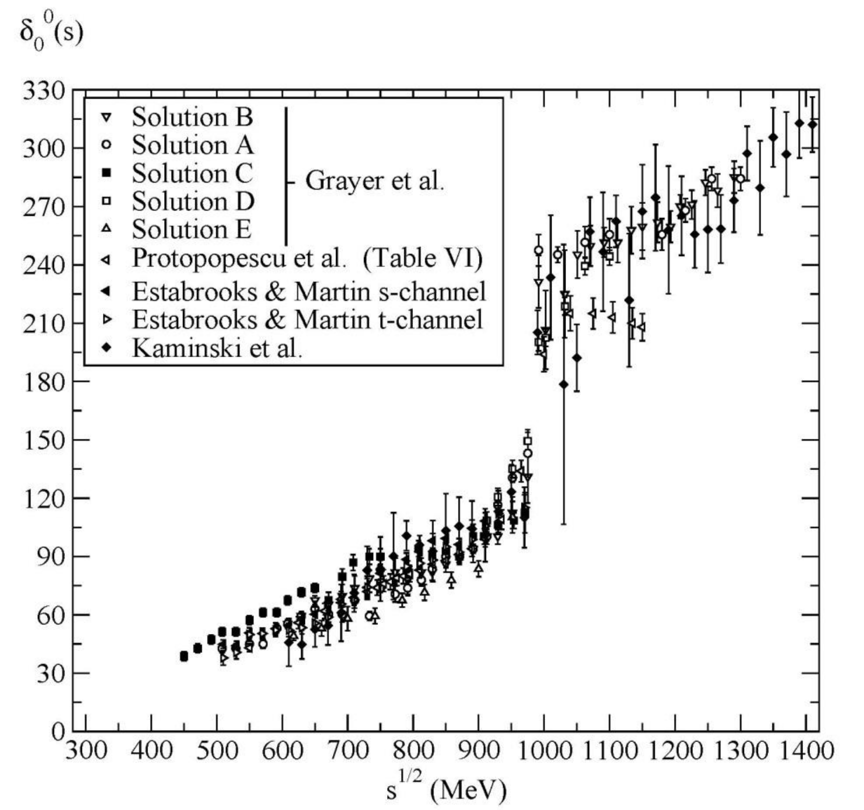


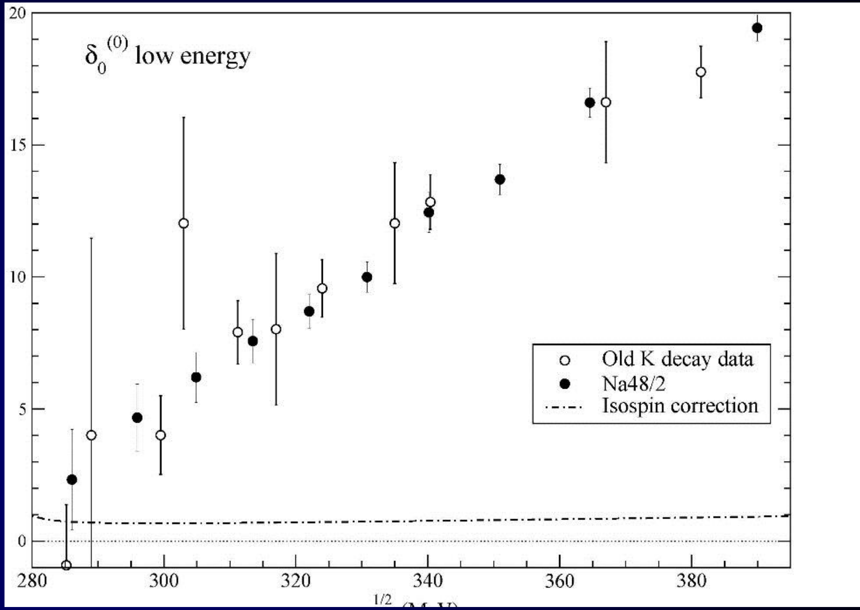


To be discussed later…

The complicated wave is the S0 wave (IJ=00)

We have already seen the data is a mess…. Only Kl4 reliable



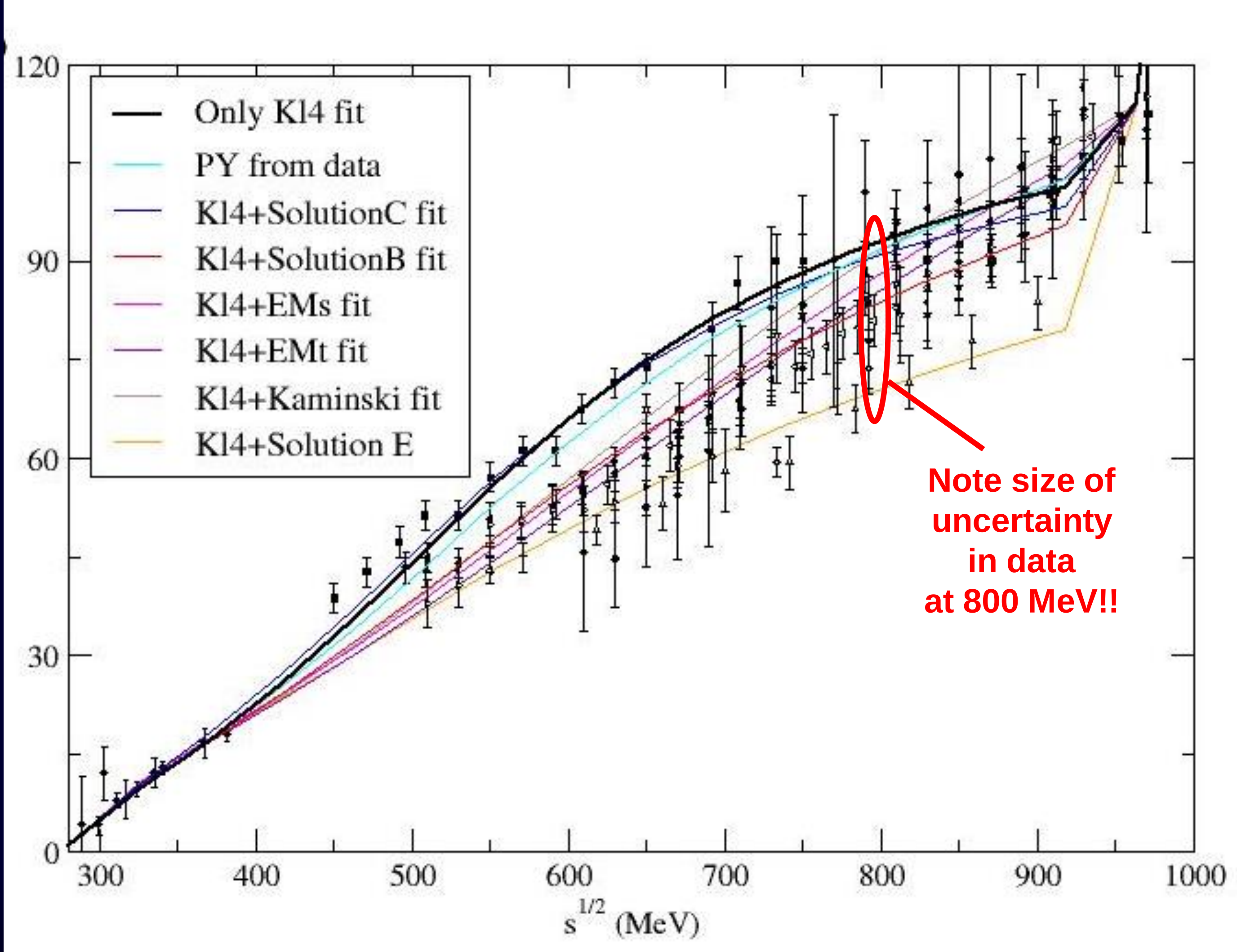


Always include Kl4, but two possibilities:

- Average data
- Fit individual sets

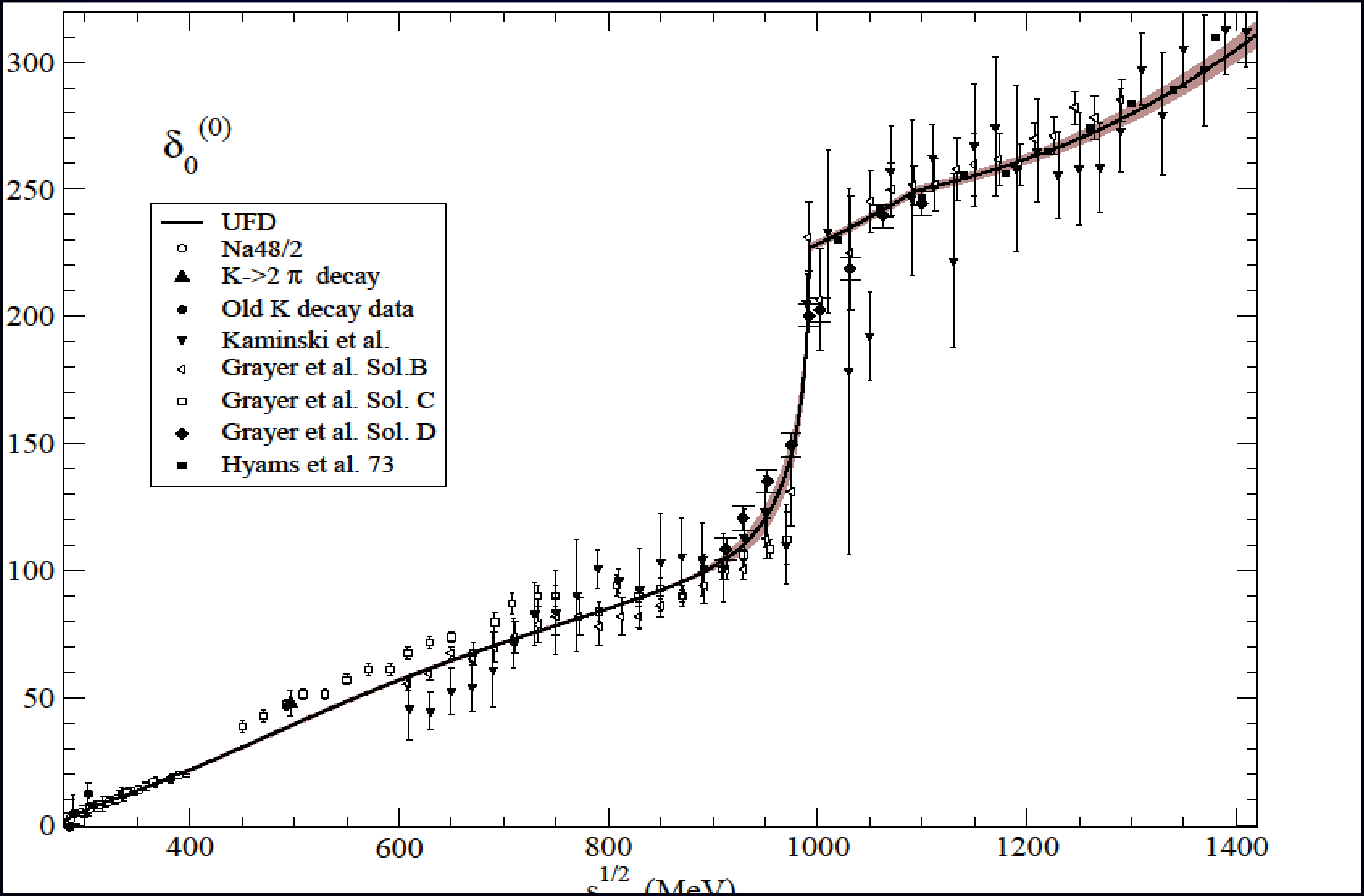
The S0 wave. Different sets

Fits to different sets including also K_{l4} data



S0 wave: UNconstrained fit to data (UFD).

Global fit, averaging all sets where they roughly coincide

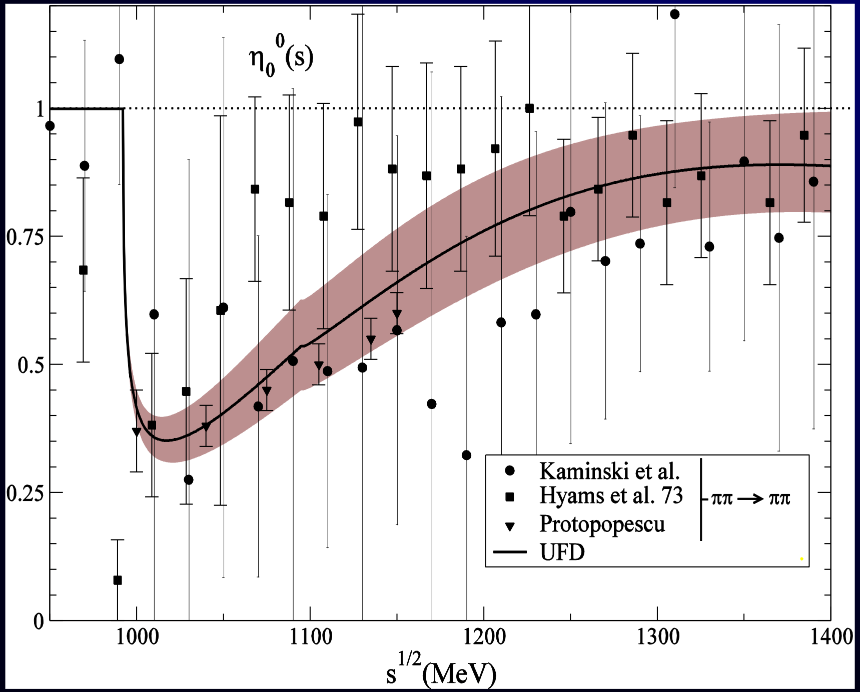


S0 wave: DIP vs NO DIP inelasticity scenarios

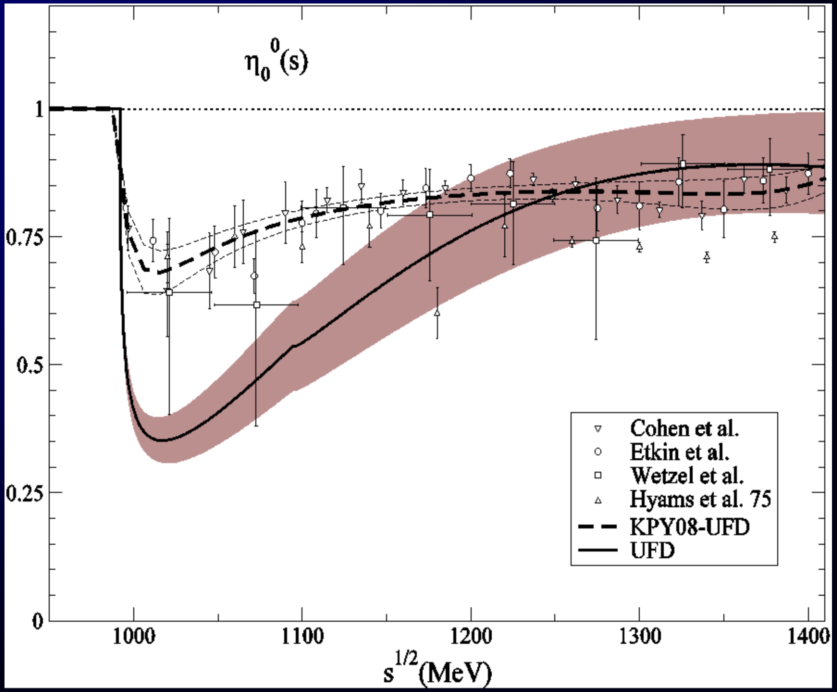
Longstanding controversy for inelasticity : (Pennington, Bugg, Zou, Achasov….)

There are inconsistent data sets for the inelasticity above 1 GeV near the $f_0(980)$ region

Some prefer a “dip” structure…



... whereas others do not



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R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

↓

Check Dispersion Relations

How well the Dispersion Relations are satisfied by unconstrained fits

Every 25 MeV we look at the difference between both sides of the DR divided by the uncertainty

We define an averaged χ^2 over these points, that we call d^2

d^2 close to 1 means that the relation is well satisfied

$d^2 >> 1$ means the data set is inconsistent with the relation.

This is **NOT a fit** to the relation, just a check of the fits!!.

How well the Dispersion Relations are satisfied by unconstrained fits

Only TWO FDRs involve the S0 wave
The 00 FDR is very sensitive

Data sets	$\bar{d}^2$ for $T^{I_t=1}$	$\bar{d}^2$ for T^{00}
Global fit from [176]	0.3	3.5
K_{e4}+ Grayer et al. B	1.0	2.7
K_{e4}+ Grayer et al. C	0.4	1.0
K_{e4}+ Grayer et al. E	2.1	0.5
K_{e4}+ Kaminski et al.	0.3	5.0
K_{e4}+ Grayer et al. A	2.0	7.9
K_{e4}+ EM, s channel	1.0	9.1
K_{e4}+ EM, t channel	1.2	10.1
K_{e4}+ Protopopescu et al. VI	1.2	5.8
K_{e4}+ Protopopescu et al. XII	1.2	6.3
K_{e4}+ Protopopescu et al. VIII	1.8	4.2

Other sets, not so badly. Do not discad them but ROOM FOR IMPROVEMENT

Some S0 data sets are very incompatible with FDR below 900 MeV
Considered clearly inconsistent and discarded

Lessons:

Dispersion Relations can be useful to discard conflicting data sets
Despite nice-looking fits, analytic properties WRONG.
Careful with extrapolations to complex plane

Our series of works: 2005-2011

R. Kaminski, JRP, F.J. Ynduráin Eur.Phys.J.A31:479-484,2007. PRD74:014001,2006
JRP ,F.J. Ynduráin. PRD71, 074016 (2005) , PRD69,114001 (2004),
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

↓

Check Dispersion Relations

Some sets discarded
Others, room for improvement

↓

Impose FDRs, Roy & GKPY Eqs
on data fits
"**Constrained Data Fits CDF**"
Describe data and are consistent with Dispersion relations

Imposing FDR's, Roy Eqs., GKPY Eqs. and crossing sum rules

To improve our fits, we can IMPOSE FDR's, Roy Eqs. GKPY Eqs. and some SRs

We obtain CONSTRAINED FITS TO DATA (CFD) by minimizing:

$$\chi^2 = W\left\{\overline{d_{00}}^2 + \overline{d_{0+}}^2 + \overline{d_{It=1}}^2 + \overline{d_{S0_{Roy}}}^2 + \overline{d_{S2_{Roy}}}^2 + \overline{d_{P_{Roy}}}^2 + \overline{d_{S0_{GKPY}}}^2 + \overline{d_{S2_{GKPY}}}^2 + \overline{d_{P_{GKPY}}}^2\right\}$$

3 FDR's — 3 Roy Eqs. — 3 GKPY Eqs.

$$+ \overline{d_{SR_J}}^2 + \overline{d_{SR_K}}^2 + \sum_k \frac{(p_k - p_k^{exp})^2}{\delta p_k^2}$$

Sum Rules for crossing — Parameters of the unconstrained data fits

W roughly counts the number of effective degrees of freedom
(sometimes we add weight on certain energy regions)

Imposing FDRs and Sum rules

After imposing FDRs and SRs
The resulting fits differ by less than ~1σ -1.5 σ from original unconstrained fits

Data Fits Constrained with FDR	$\bar{d}^2$ for $T^{I_t=1}$	$\bar{d}^2$ for T^{00}	K
Global fit from [176]	0.4	0.66	1.6σ
K_{e4}+ Grayer et al. C	0.37	0.32	1.5σ
K_{e4}+ Grayer et al. B	0.37	0.83	4.0σ
K_{e4}+ Grayer et al. E	0.6	0.09	6.0σ
K_{e4}+ Kaminski et al.	0.43	1.08	4.5σ

Remarkable improvement in 00 FDR

But some sets cannot be made to satisfy SR: DISCARDED

Fit C included within uncertainties of “Global Fit”.

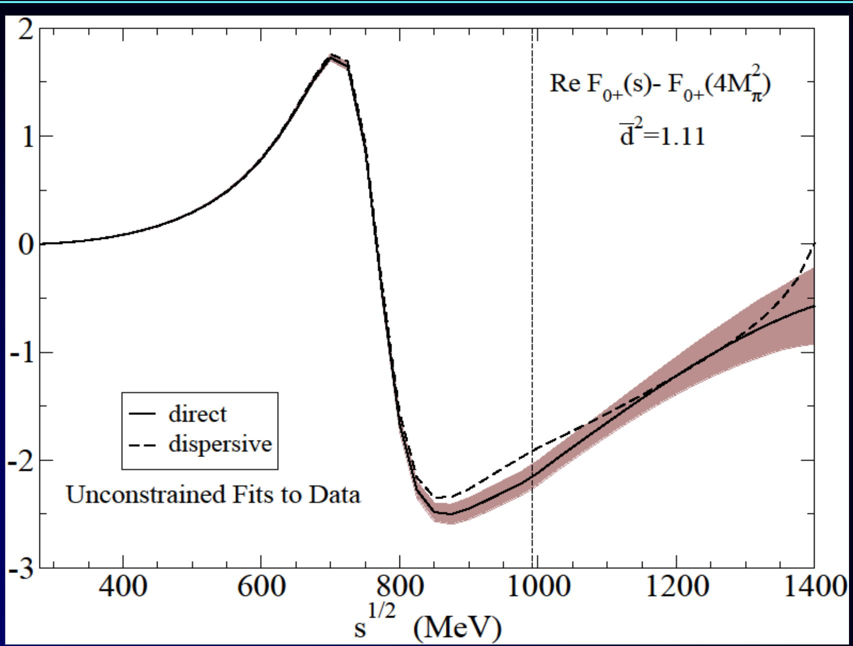
So **we keep the “Global Fit”**

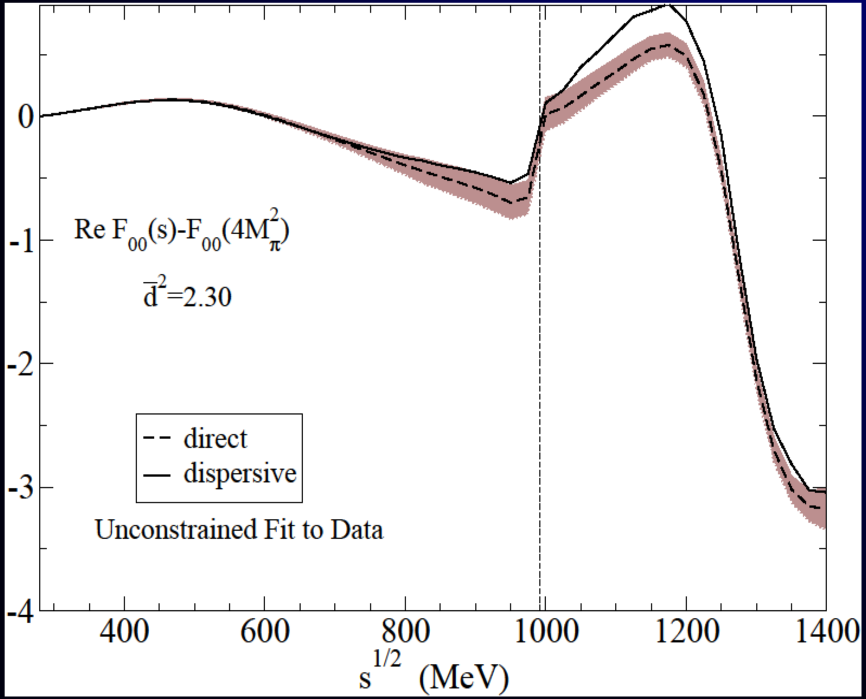
Forward Dispersion Relations for UNCONSTRAINED fits

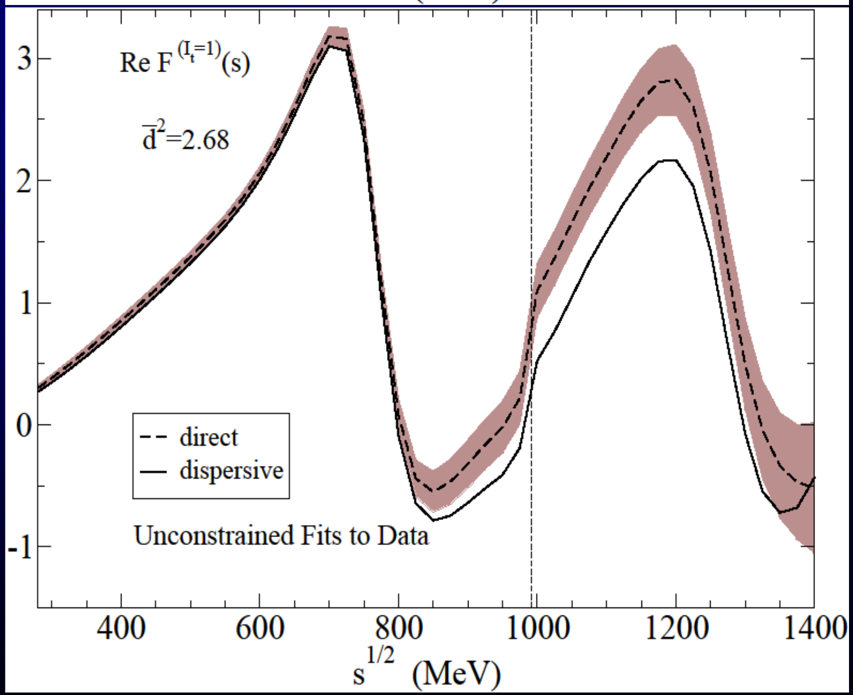
FDRs averaged d^2

	<932MeV	<1400MeV
$\pi^0\pi^0$	0.31	2.13
$\pi^0\pi^+$	1.03	1.11
$I_t=1$	1.62	2.69

NOT GOOD! In the intermediate region. Need improvement





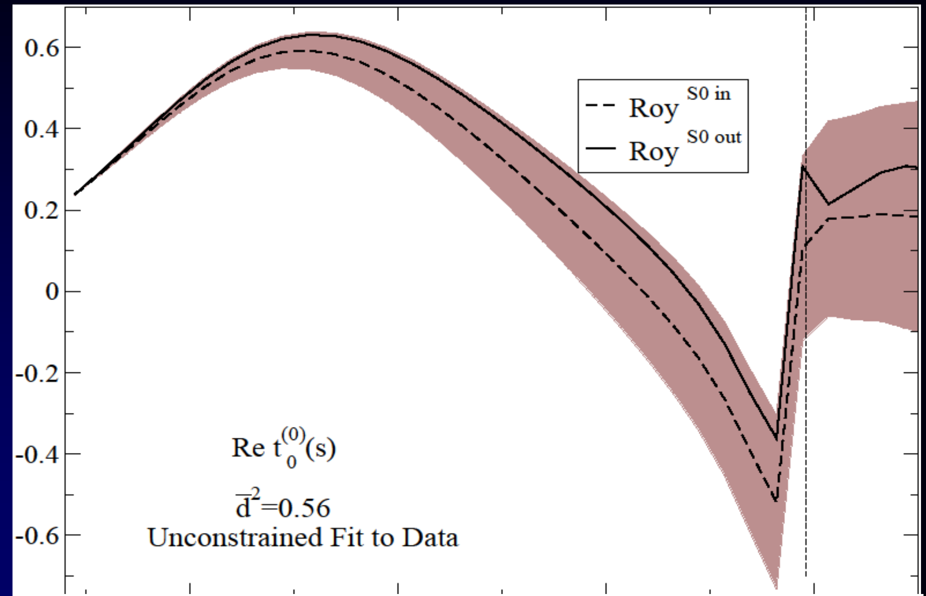


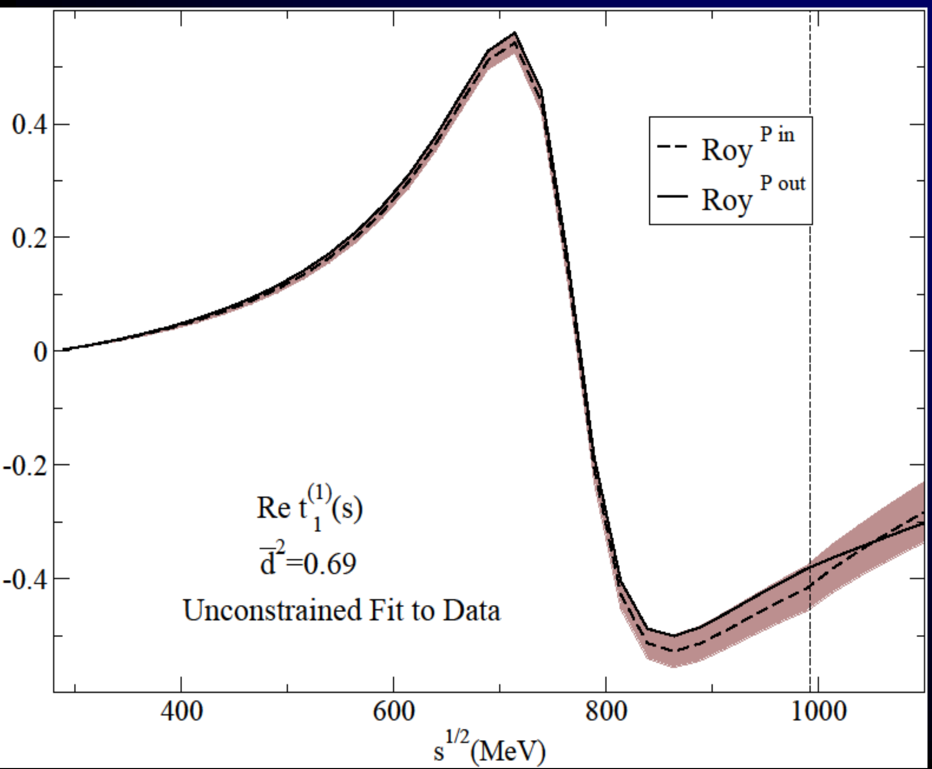
Roy Eqs. for UNCONSTRAINED fits

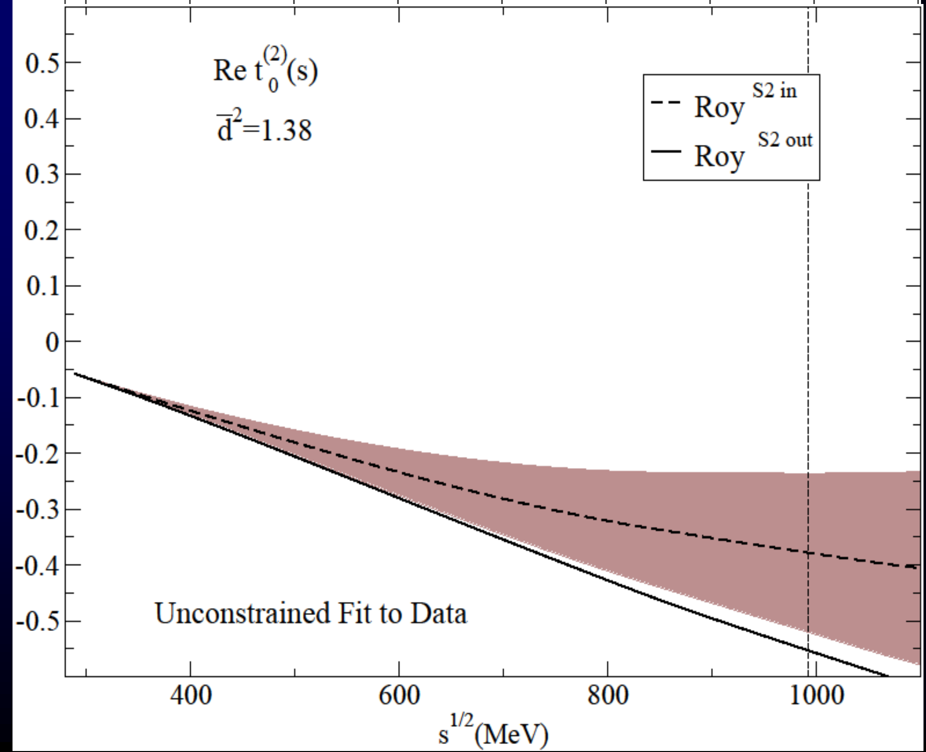
Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.64	0.56
P wave	0.79	0.69
S2 wave	1.35	1.37

GOOD! But room for improvement





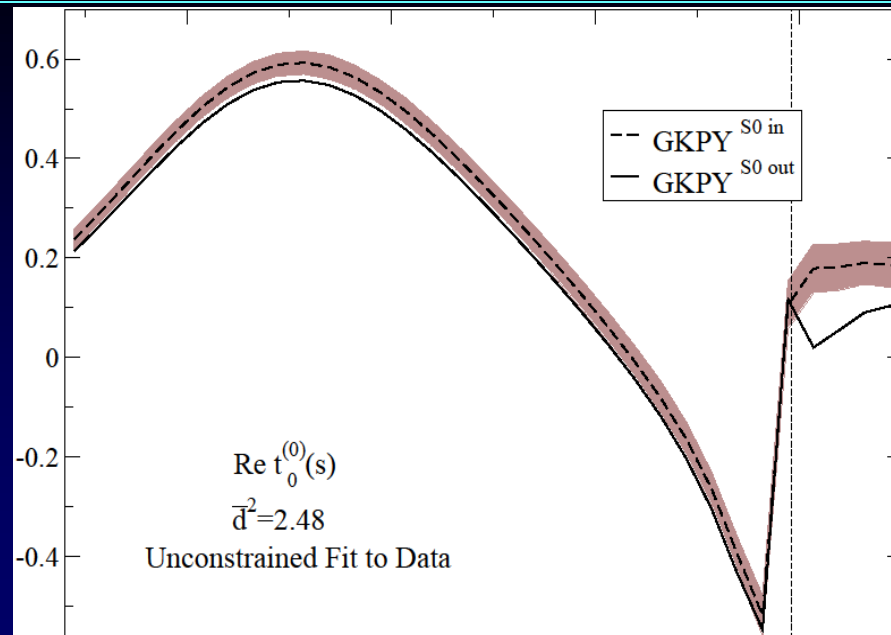


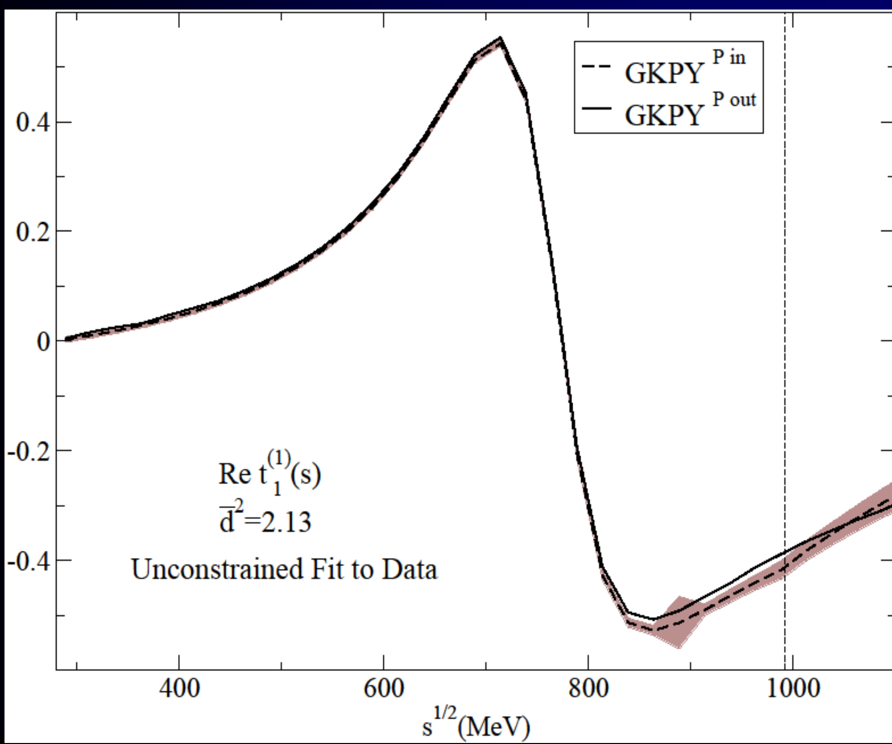
GKPY Eqs. for UNCONSTRAINED fits

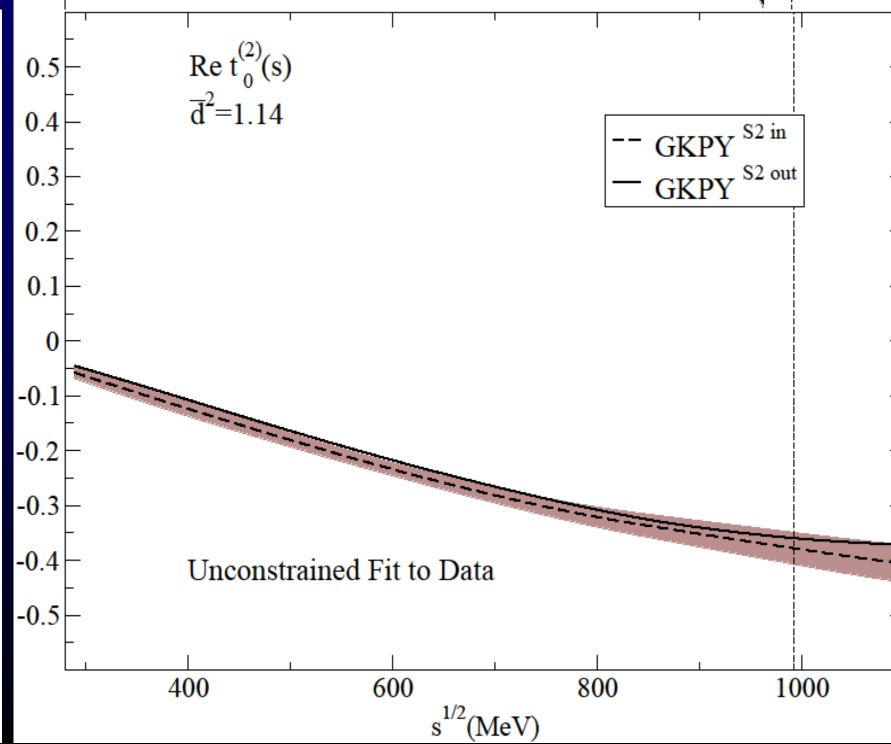
Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	1.78	2.42
P wave	2.44	2.13
S2 wave	1.19	1.14

Pretty bad. GKPY Eqs are much stricter
Lots of room for improvement





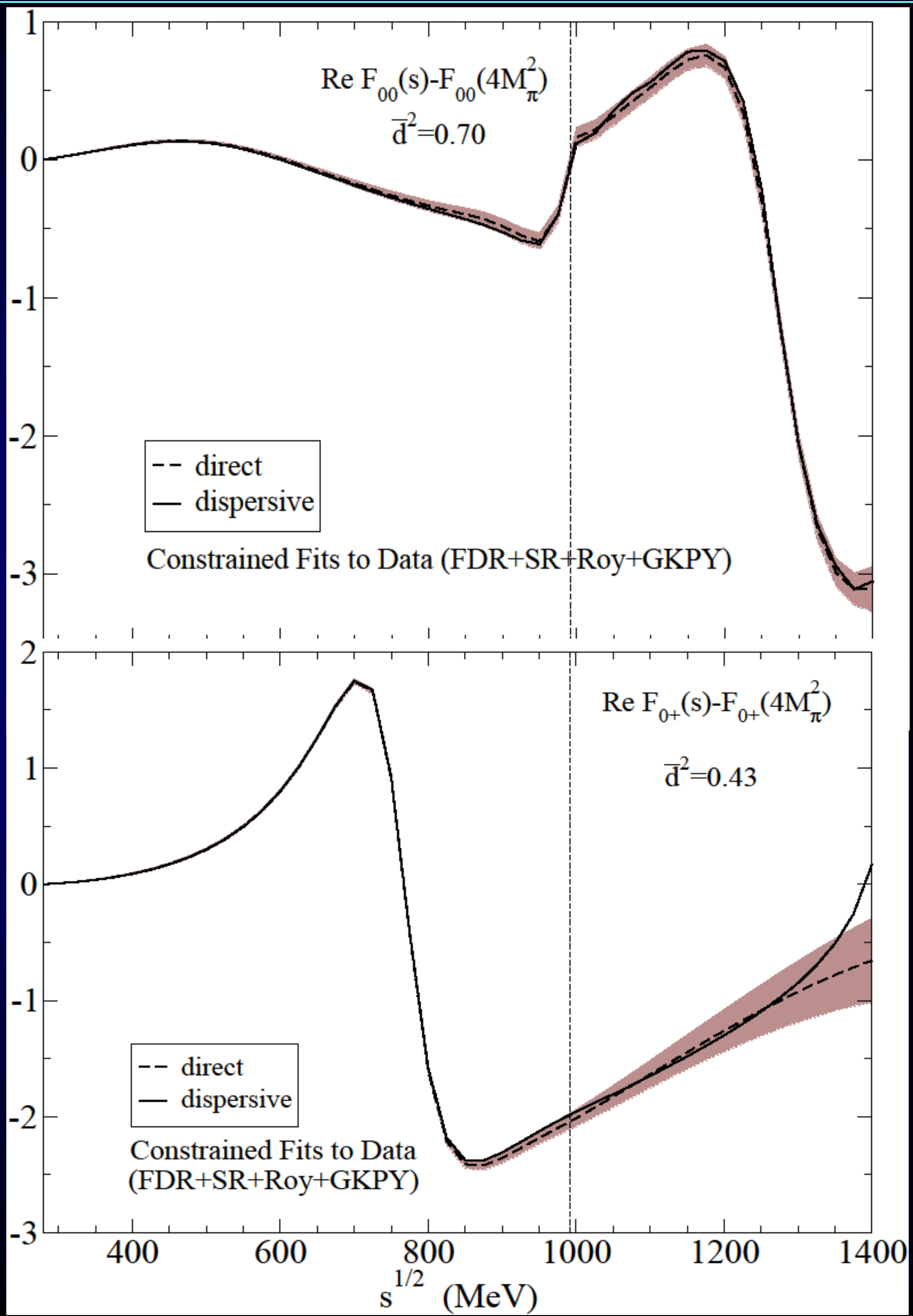


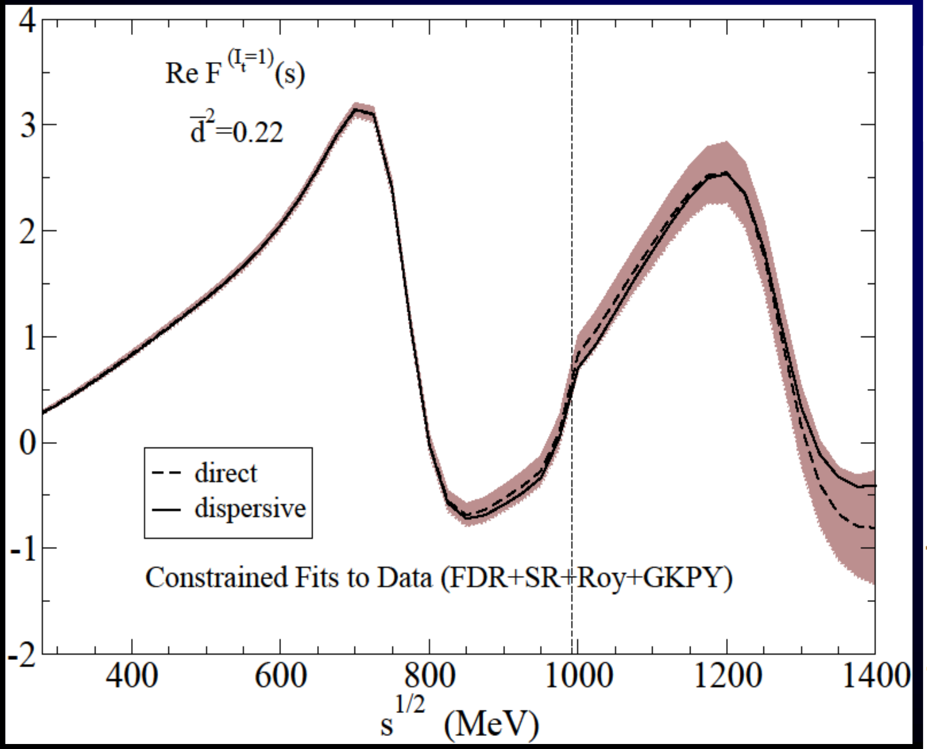
Forward Dispersion Relations for CONSTRAINED fits

FDRs averaged d^2

	<932MeV	<1400MeV
$\pi^0\pi^0$	0.32	0.51
$\pi^0\pi^+$	0.33	0.43
I_t=1	0.06	0.25

VERY GOOD!!!



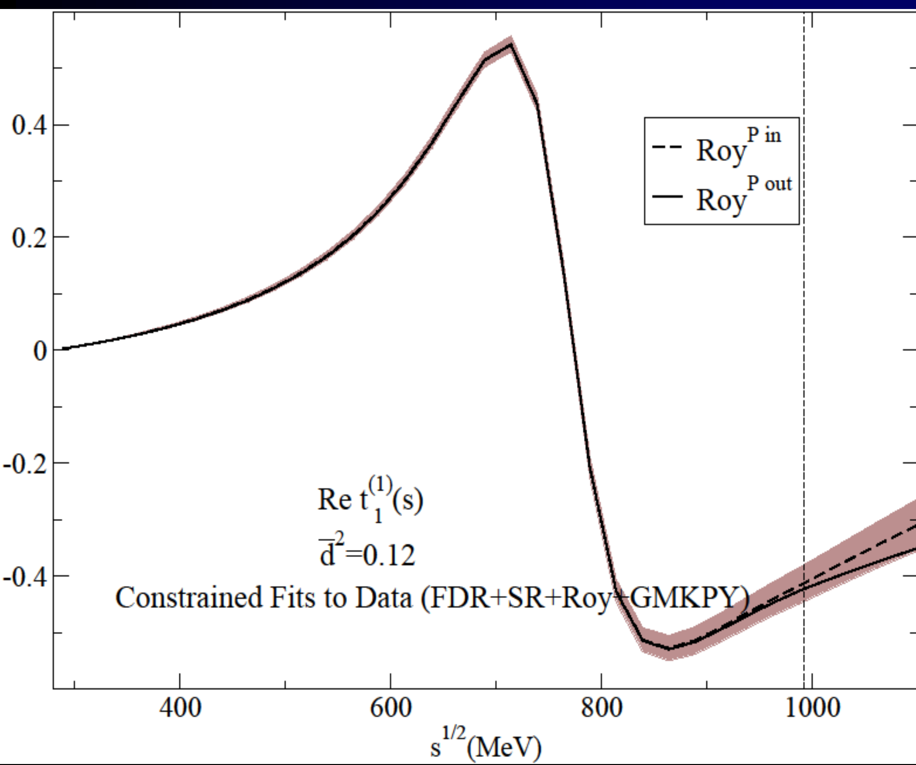


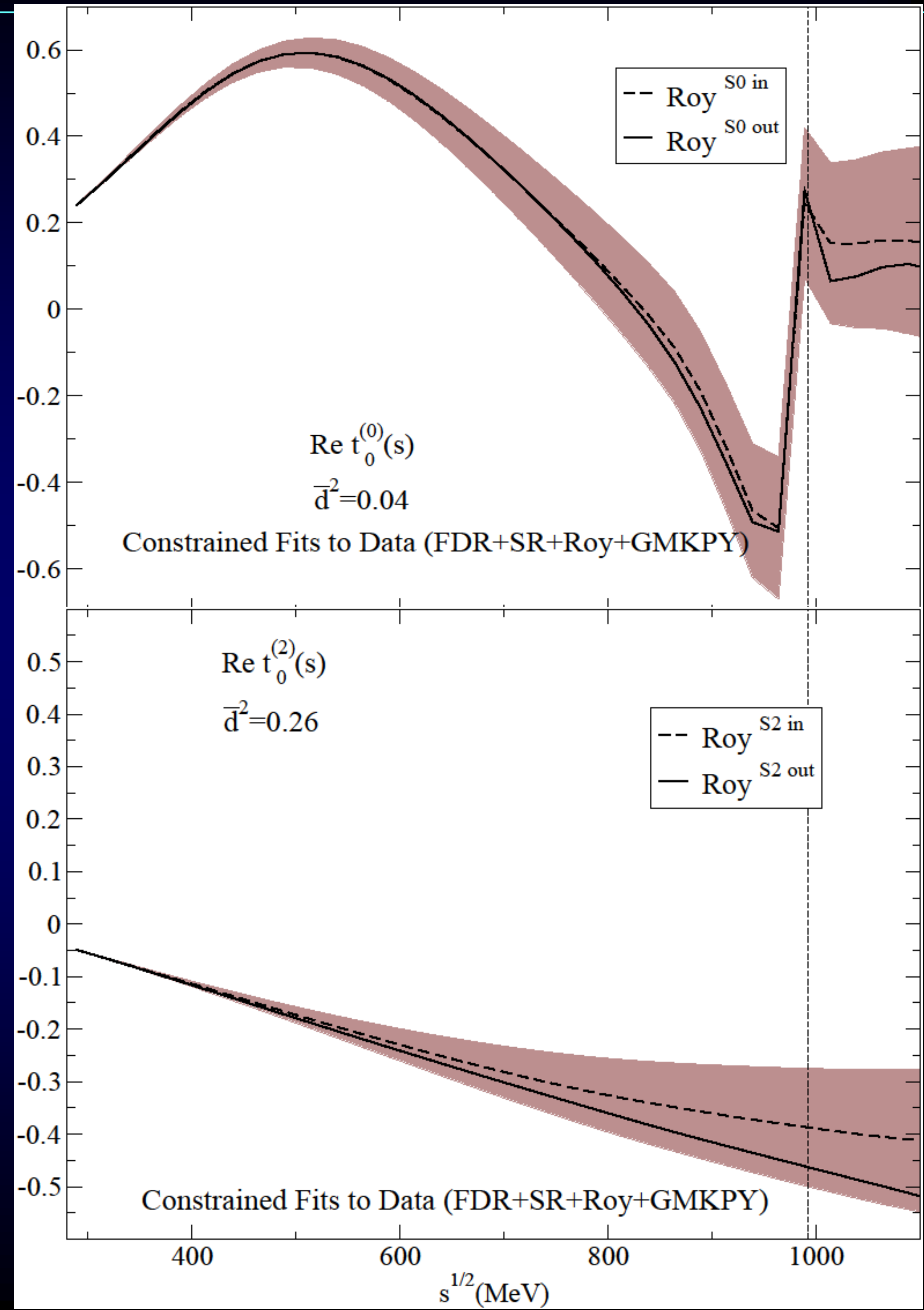
Roy Eqs. for CONSTRAINED fits

Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.02	0.04
P wave	0.04	0.12
S2 wave	0.21	0.26

VERY GOOD!!!



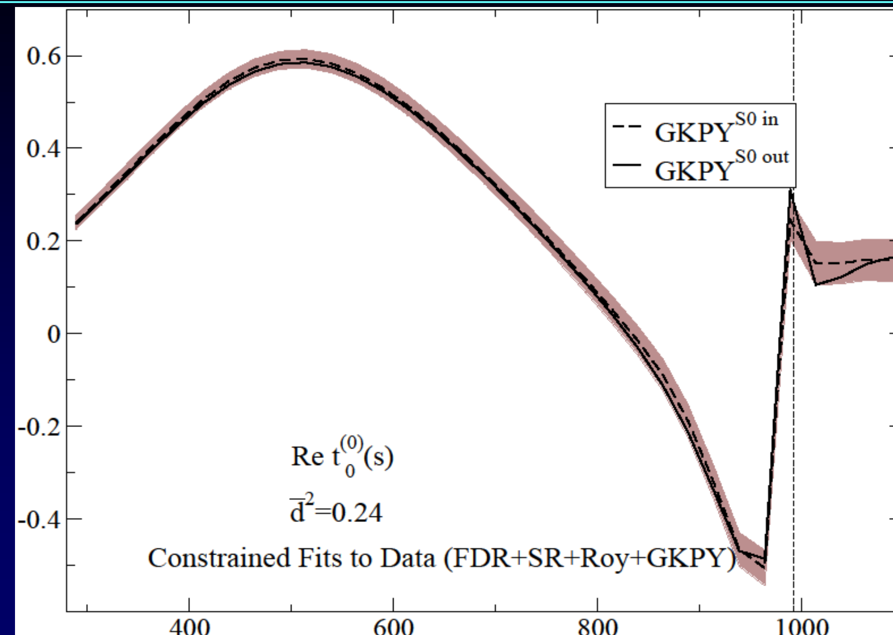


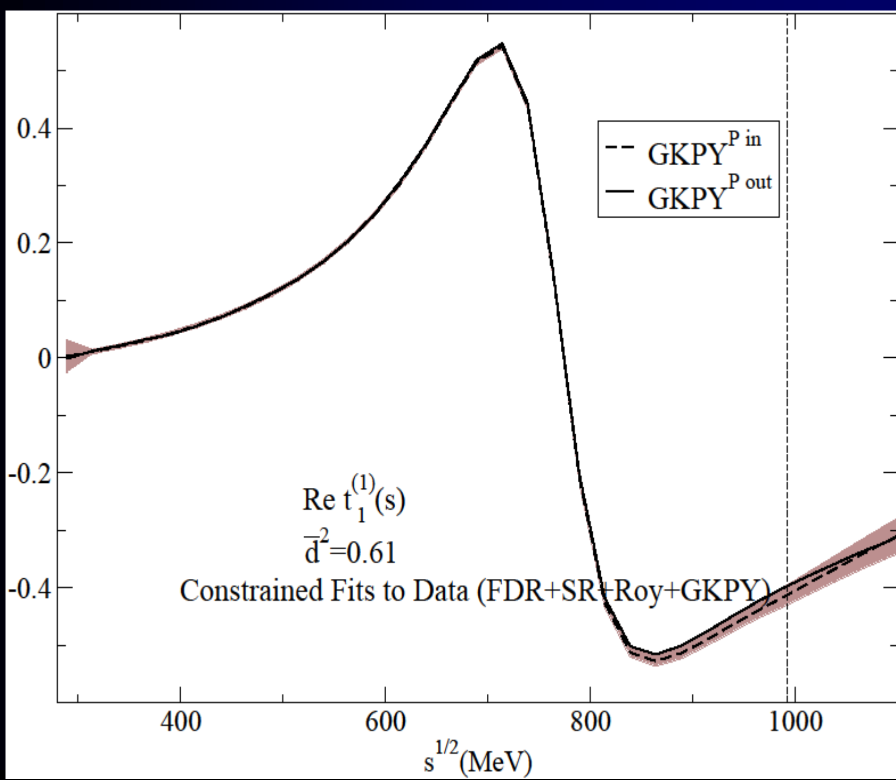
GKPY Eqs. for CONSTRAINED fits

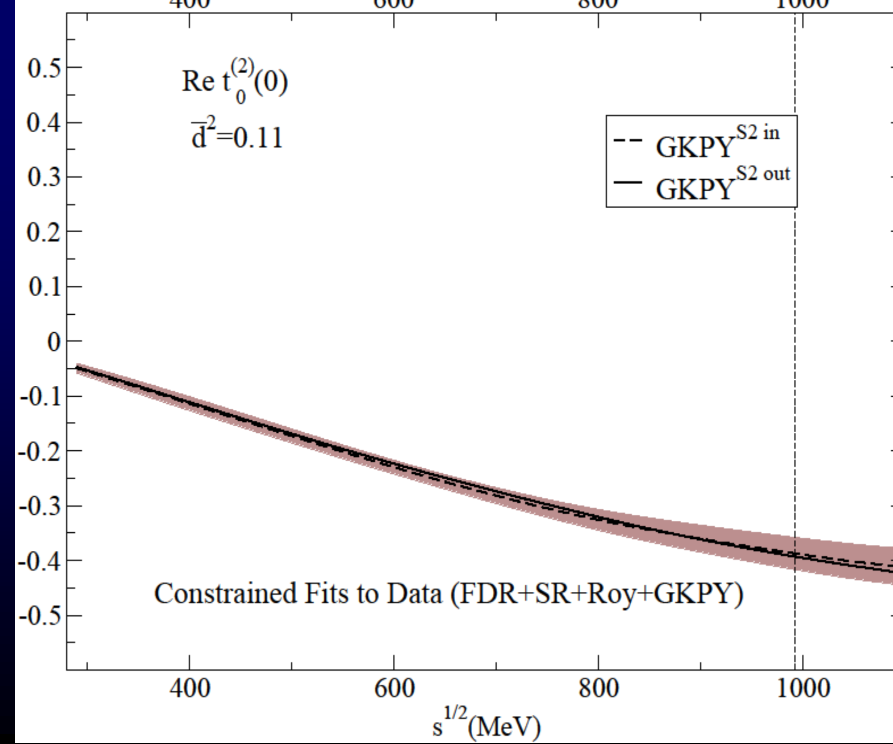
Roy Eqs. averaged d^2

	<932MeV	<1100MeV
S0wave	0.23	0.24
P wave	0.68	0.60
S2 wave	0.12	0.11

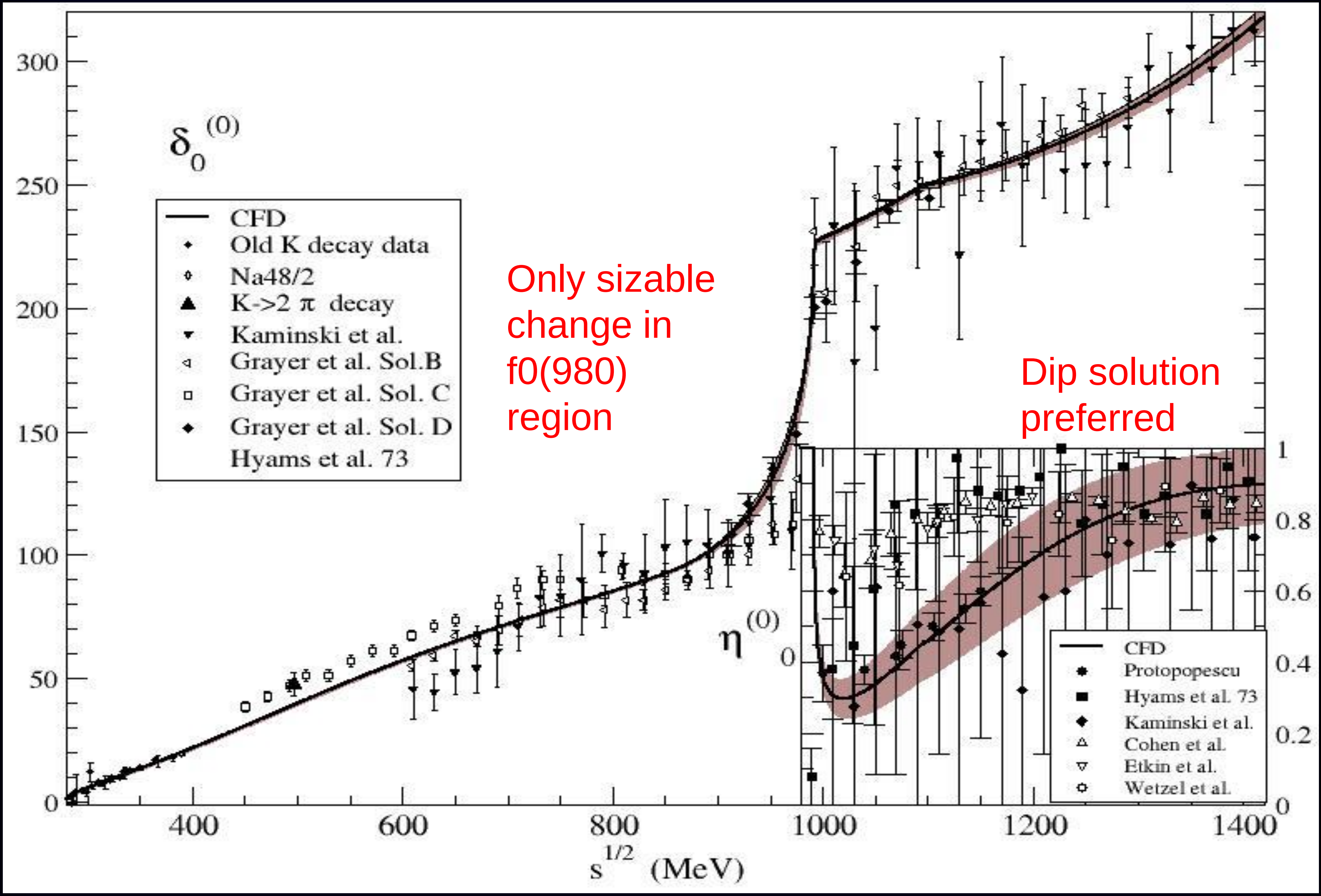
VERY GOOD!!!







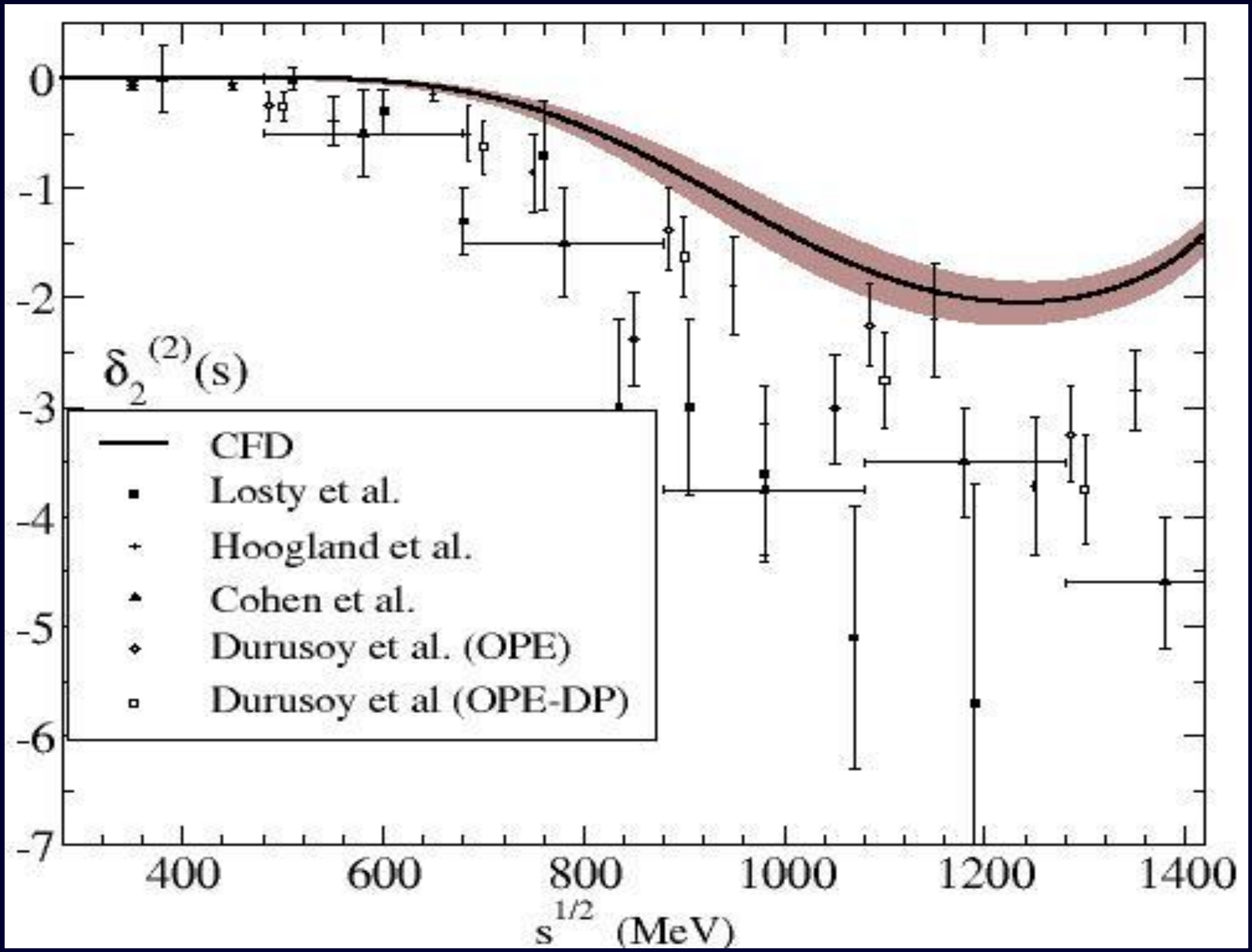
S0 wave: from UFD to CFD



Only sizable change in f0(980) region

Dip solution preferred

As expected, the wave suffering the largest change is the D2

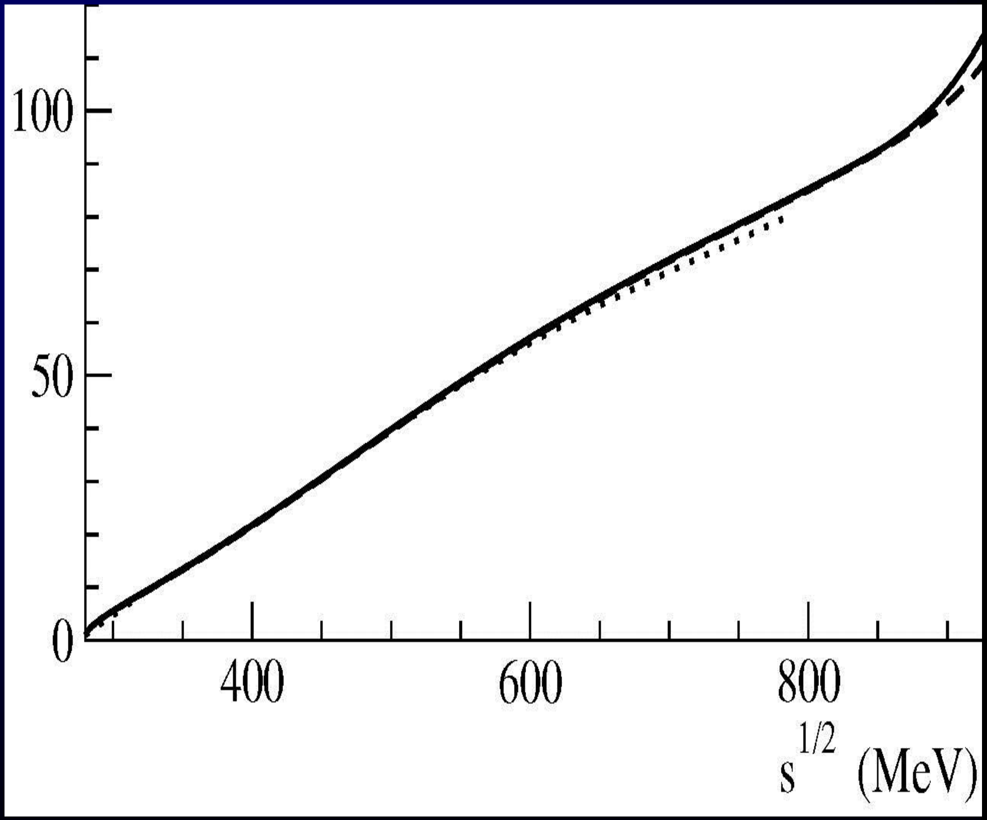


Apart from S0 and D2, changes in other waves from UFD to CFD is imperceptible

Other groups (Ananthanarayan, Gasser, Laetwyler, Caprini, Colangelo, Maussallam)
have used Roy Eqs. alone to obtain
SOLUTIONS for the S and P waves below 800 or 1000 MeV,
using the rest as input.

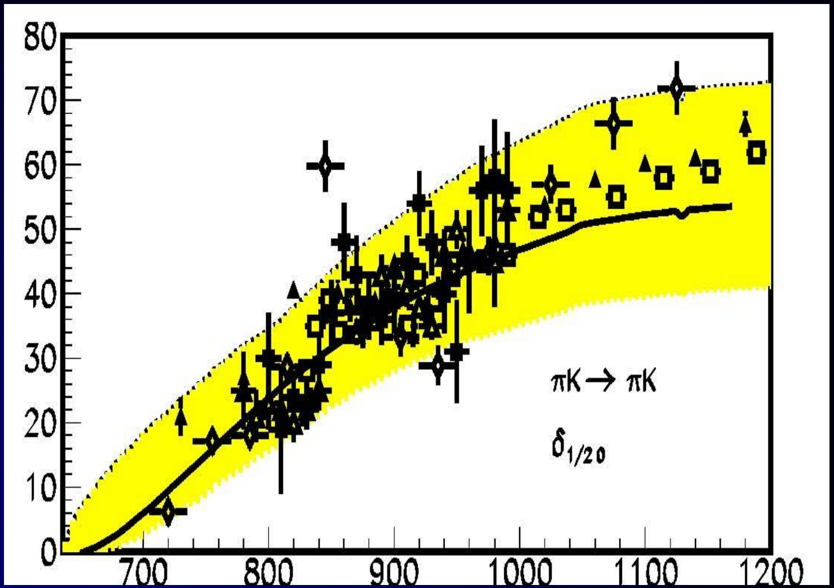
For their most precise results, they use Chiral Perturbation Theory as INPUT
(or universal band)

The results shown so far are
quite consistent with theirs



πK Scattering

A similar approach can be followed for πK.



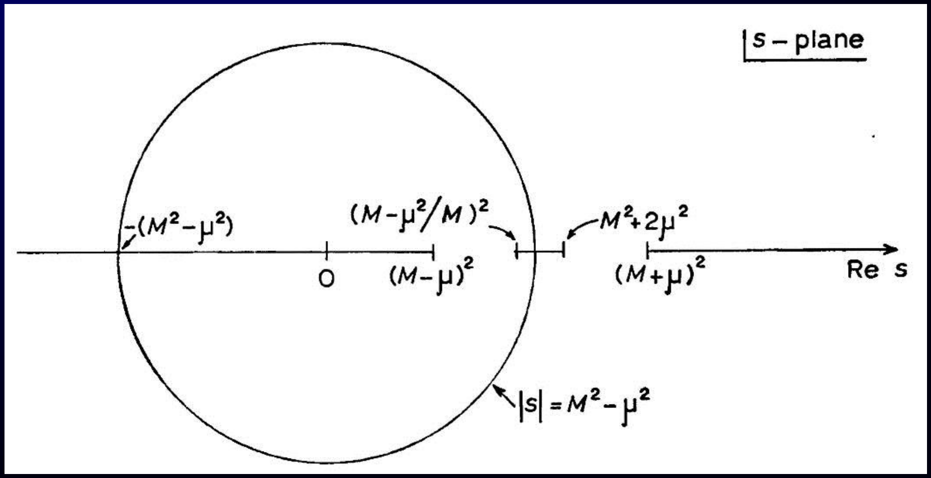
The scalar I=1/2 wave is again a mess

There is also a SOLUTION of Roy-Steiner equations in the elastic region. Uses ChPT as input

(Descotes-Genon, Moussallam)

Roy-Steiner equations are more complicated because using crossing to rewrite the left cut one also needs ππ→KK.

In addition, the different masses give rise to new analytic structures (Circular cut)

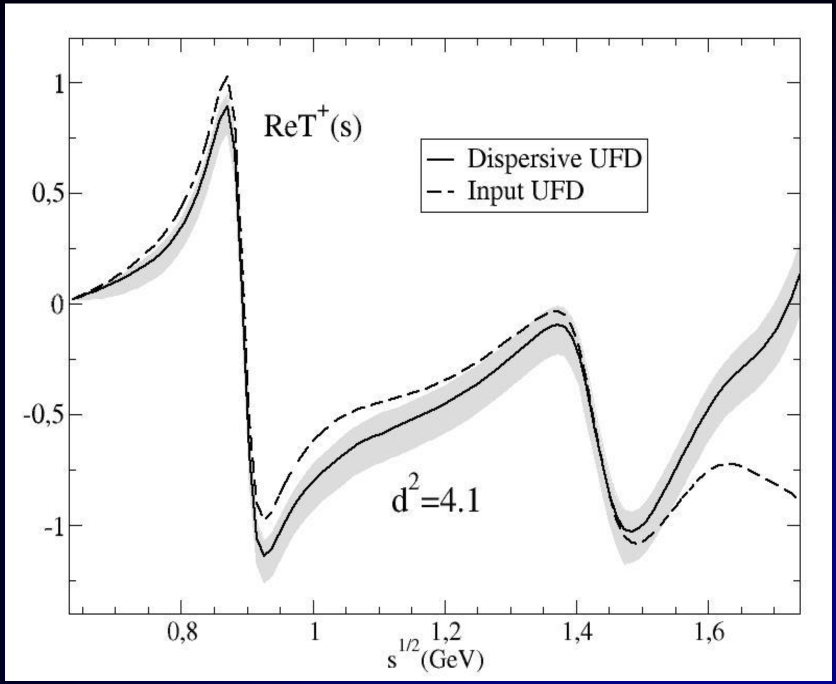


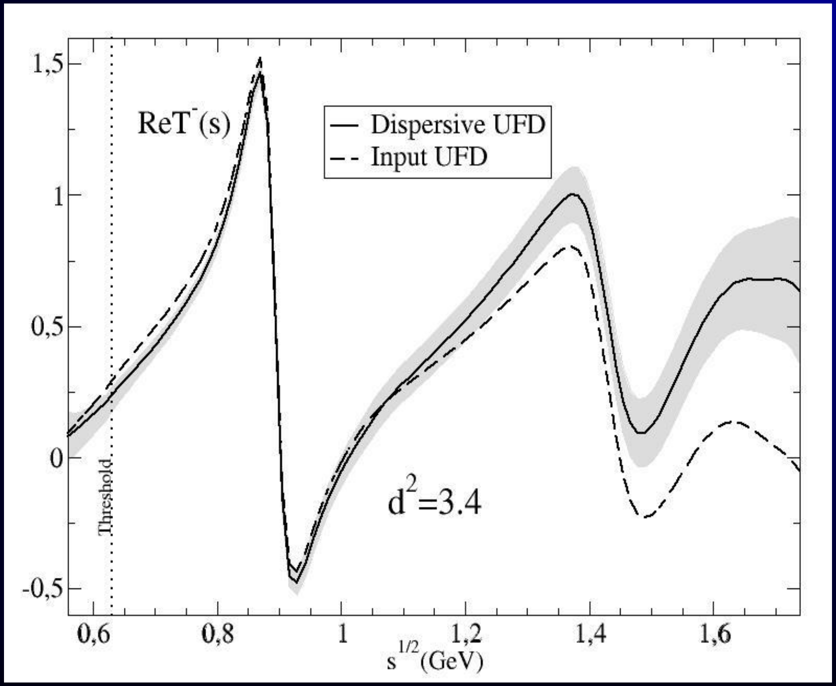
But FDRs are equally simple…

Dispersive analysis of πK scattering DATA

(**<u>not a solution</u>** of dispersión relations, but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)





First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

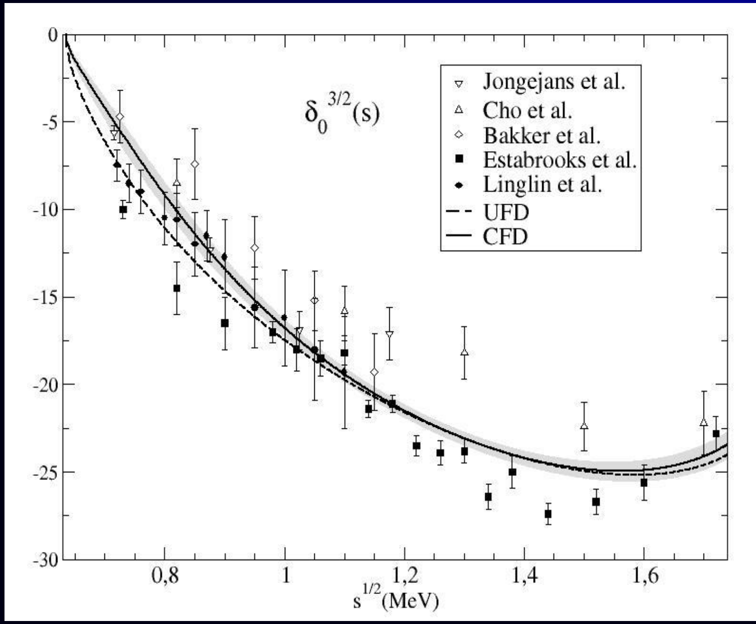
So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

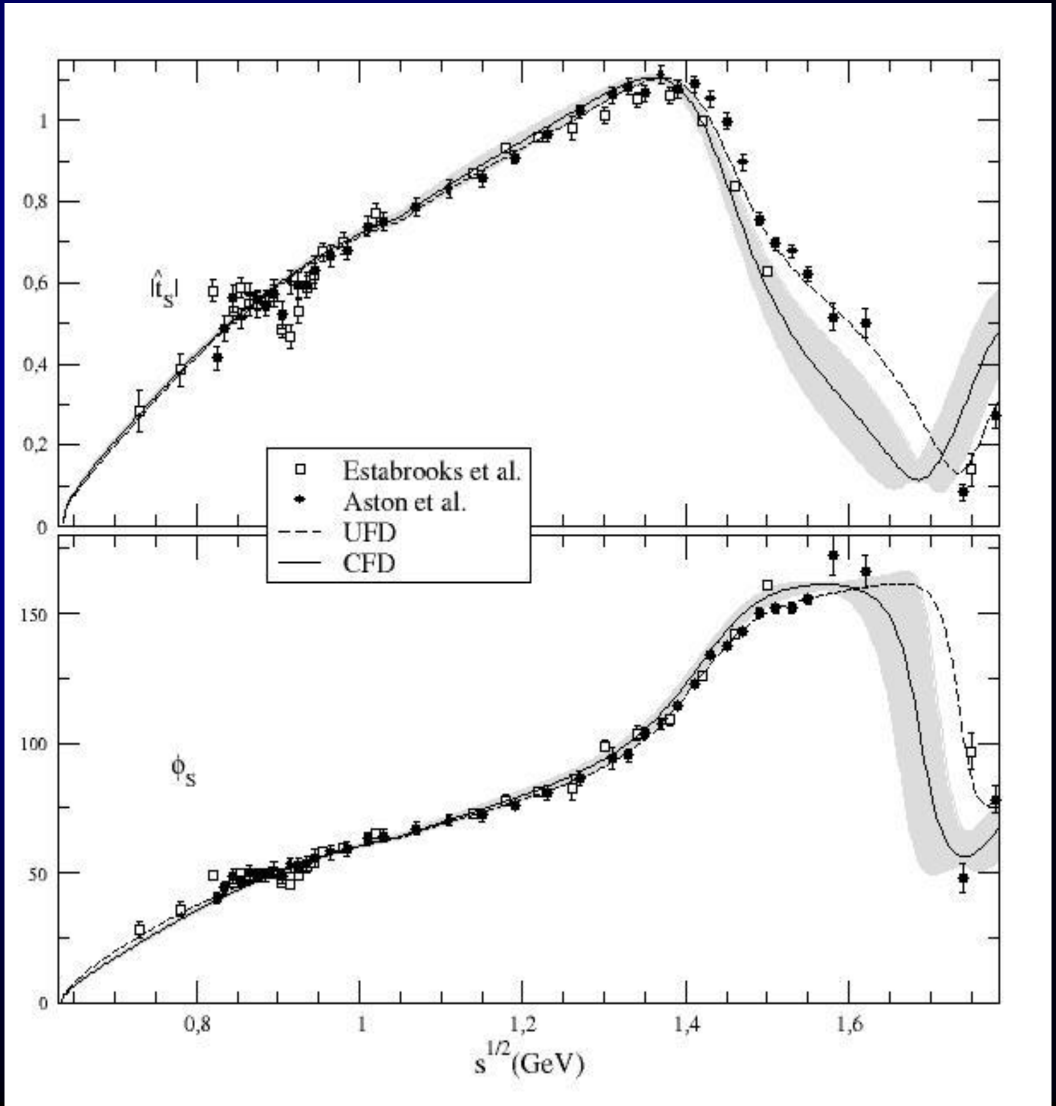
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

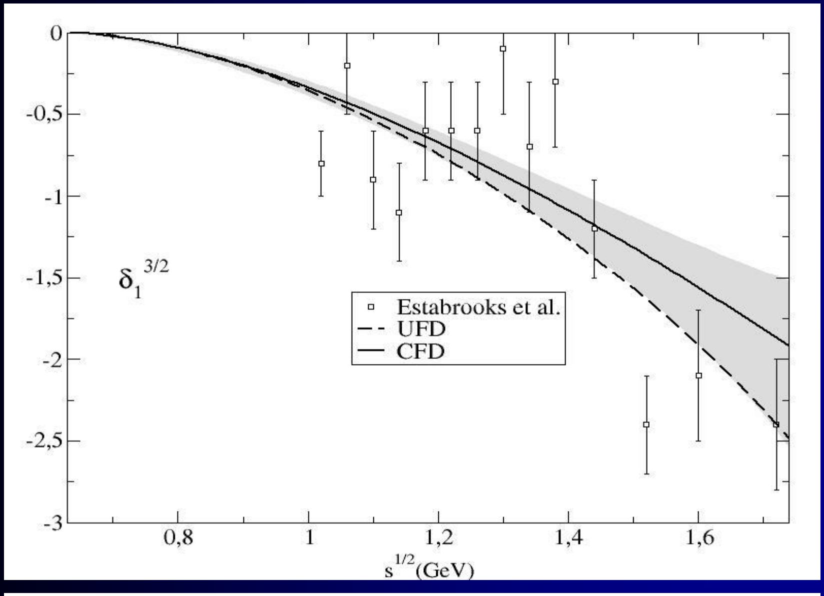
at higher energies

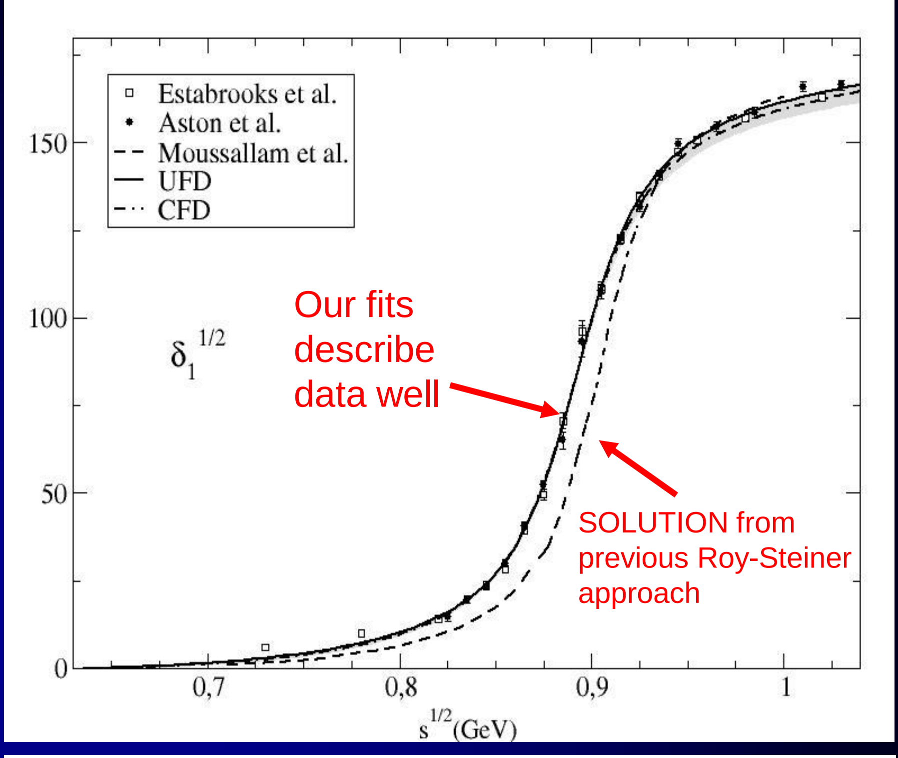


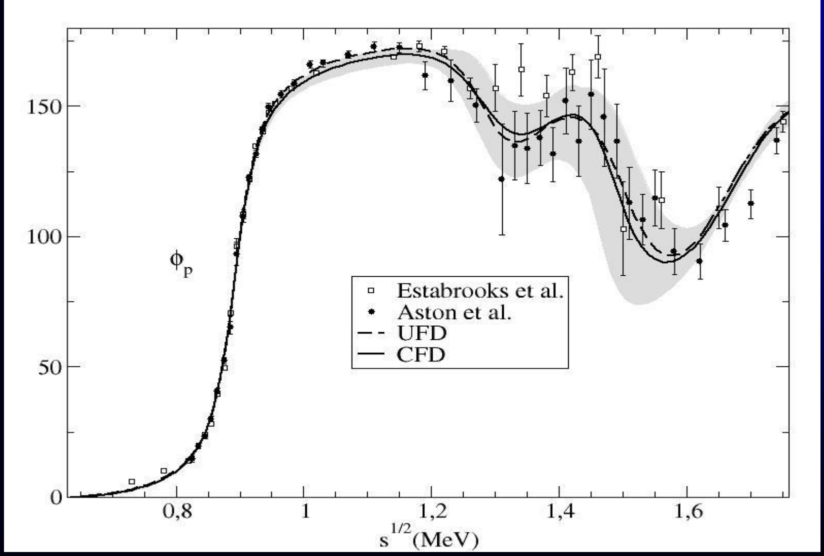


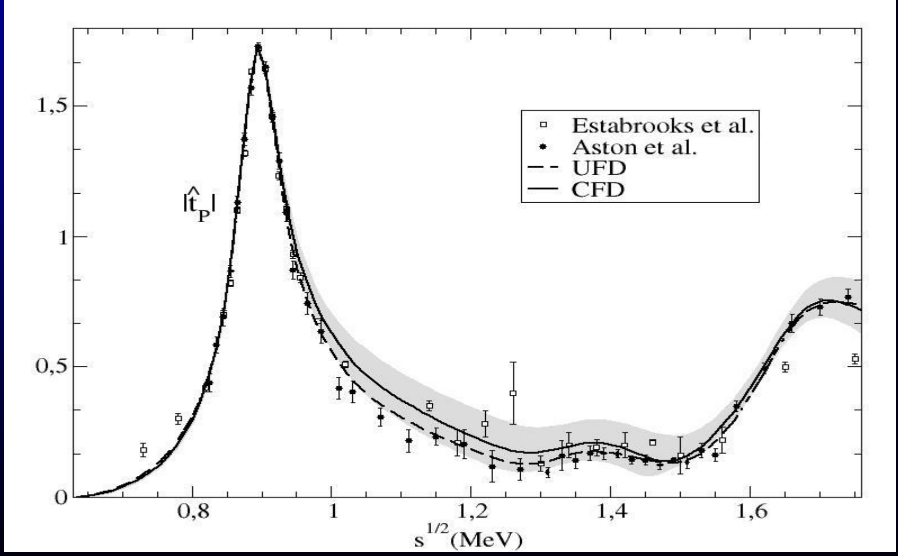
From Unconstrained (UFD) to Constrained Fits to data (CFD)

P-waves: Small changes



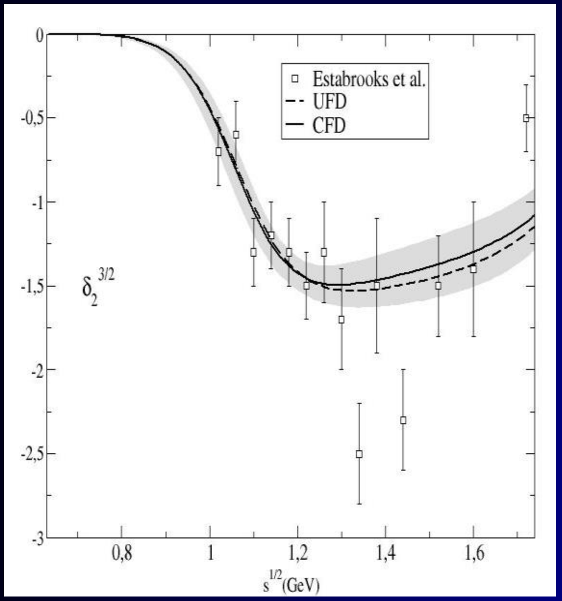


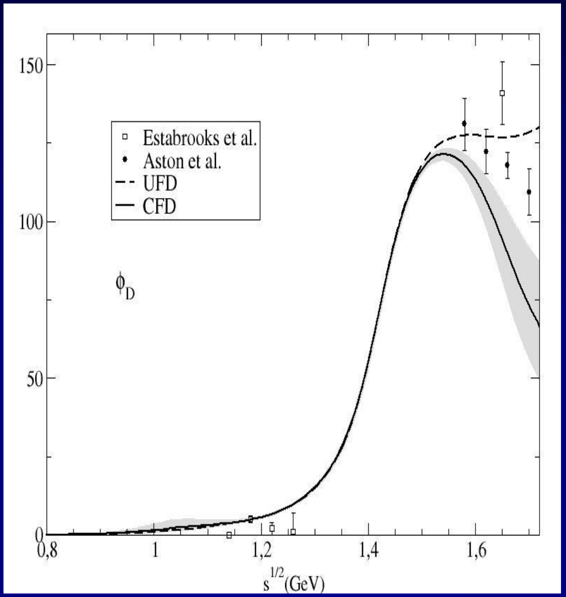


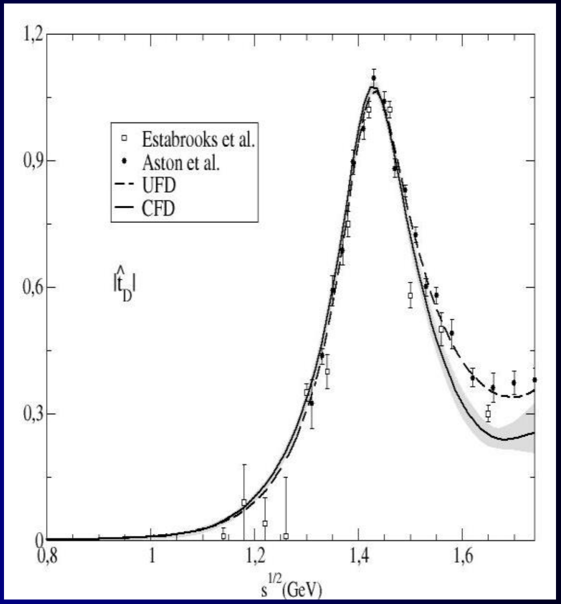


From Unconstrained (UFD) to Constrained Fits to data (CFD)

D-waves: Largest changes of all, but at very high energies

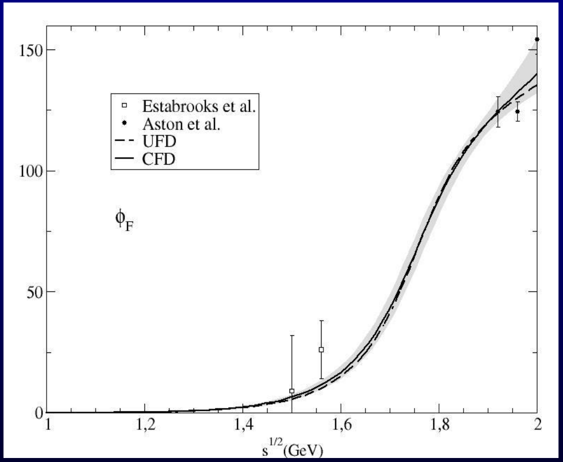


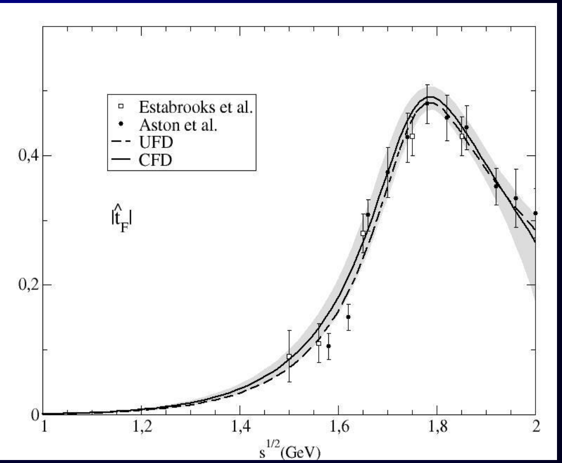




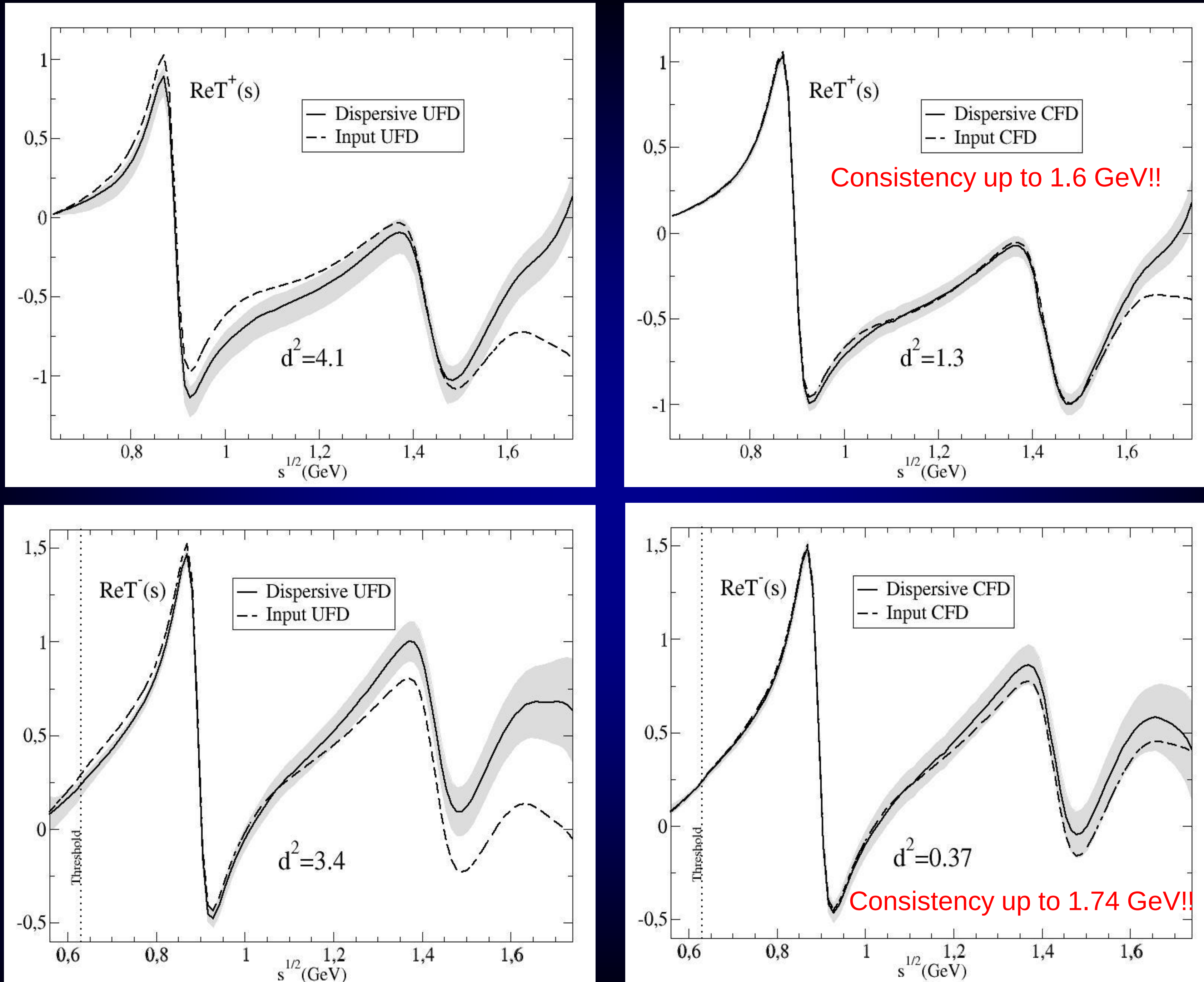
F-waves:

Imperceptible changes





Regge parameterizations allowed to vary: Only πK-ρ residue changes by 1.4 deviations


ReT+(s)
Dispersive UFD
Input UFD
d2=4.1
s1/2(GeV)
ReT+(s)
Dispersive CFD
Input CFD
Consistency up to 1.6 GeV!!
d2=1.3
s1/2(GeV)
ReT-(s)
Dispersive UFD
Input UFD
Threshold
d2=3.4
s1/2(GeV)
ReT-(s)
Dispersive CFD
Input CFD
Threshold
d2=0.37
Consistency up to 1.74 GeV!!
s1/2(GeV)

Our series of works: 2005-2011

R. Kaminski, JRP, F.J. Ynduráin Eur.Phys.J.A31:479-484,2007. PRD74:014001,2006
JRP ,F.J. Ynduráin. PRD71, 074016 (2005) , PRD69,114001 (2004),
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira, F.J. Ynduráin, Phys.Rev. D83 (2011) 074004,
R. García Martín, R. Kaminski, JRP, J. Ruiz de Elvira ,Phys.Rev.Lett. 107 (2011) 072001

Independent and **simple** fits
to data in different channels.
"**Unconstrained Data Fits=UDF**"

↓

Check Dispersion Relations

↓

Impose FDRs, Roy & GKPY Eqs
on data fits
"**Constrained Data Fits CDF**"
Describe data and are consistent with Dispersion relations

For resonance poles: Continuation to complex plane
USING THE DISPERSIVE INTEGRALS

Some relevant Roy-like POLE Determinations
which the PDG took into account in their 2012 σ revision

- **Roy Eqs.** I. Caprini, G. Colangelo, H. Leutwyler PRL97 011601 (2006)

  An S0 Wave solution up to 800 MeV, uses ChPT input

$$(441^{+16}_{-8})\text{-i}(272^{+9}_{-12.5})\ \text{MeV}$$

- **GKPY equations = Roy like with one subtraction**

  R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).

  Includes latest NA48/2 **constrained data fit** .One subtraction allows use of data only

  NO ChPT input but good agreement with previous Roy Eqs.+ChPT results.

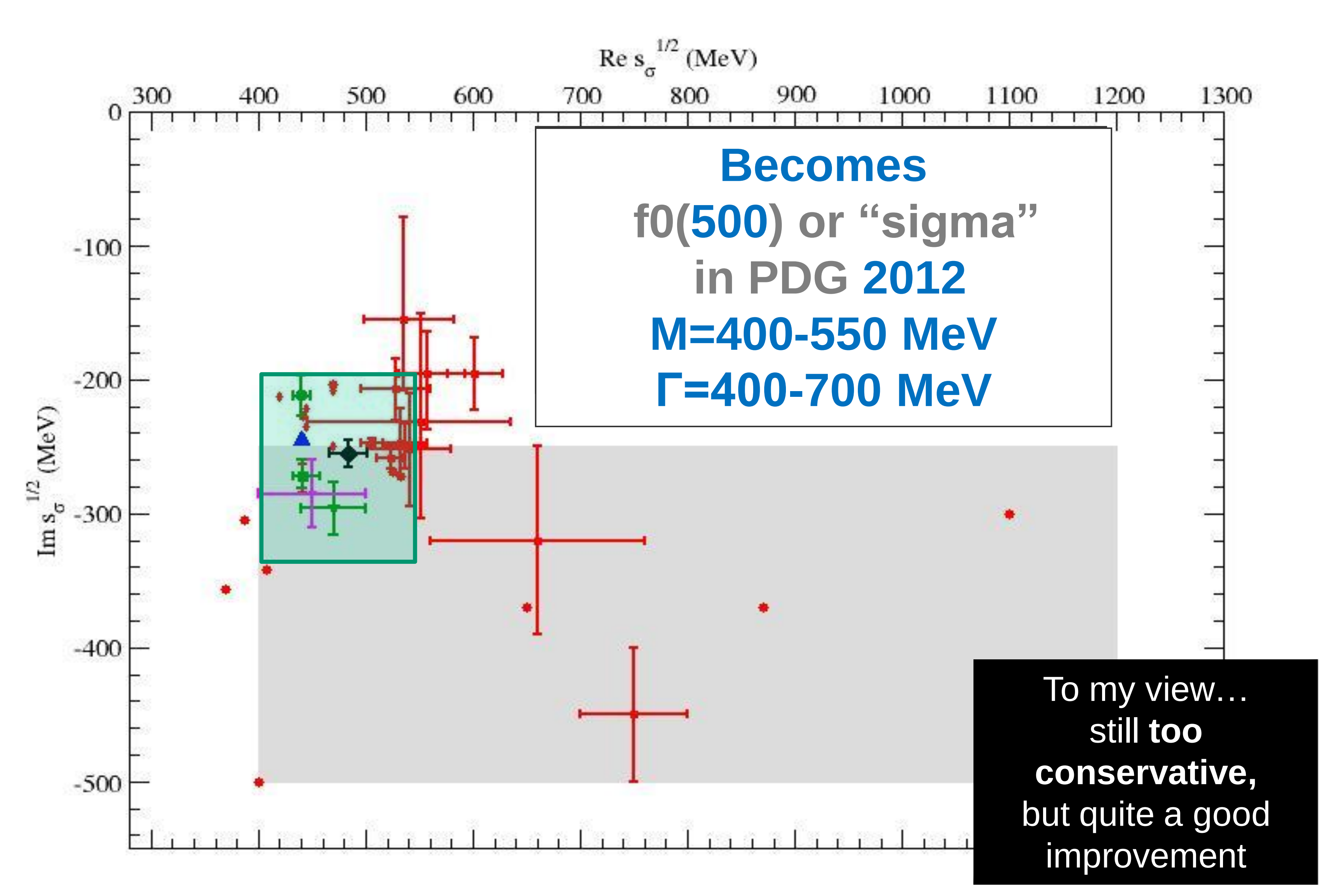
$$(457^{+14}_{-15})-i(279^{+11}_{-7})\text{MeV}$$

- **Roy equations** B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

  An S0 Wave solution up to KK threshold with input from previous Roy Eq. works

$$(442^{+5}_{-8})-i(274^{+6}_{-5})\text{MeV}$$

These and other experimental results triggered a LONG AWAITED CHANGE ON “sigma” RESONANCE @ PDG!!

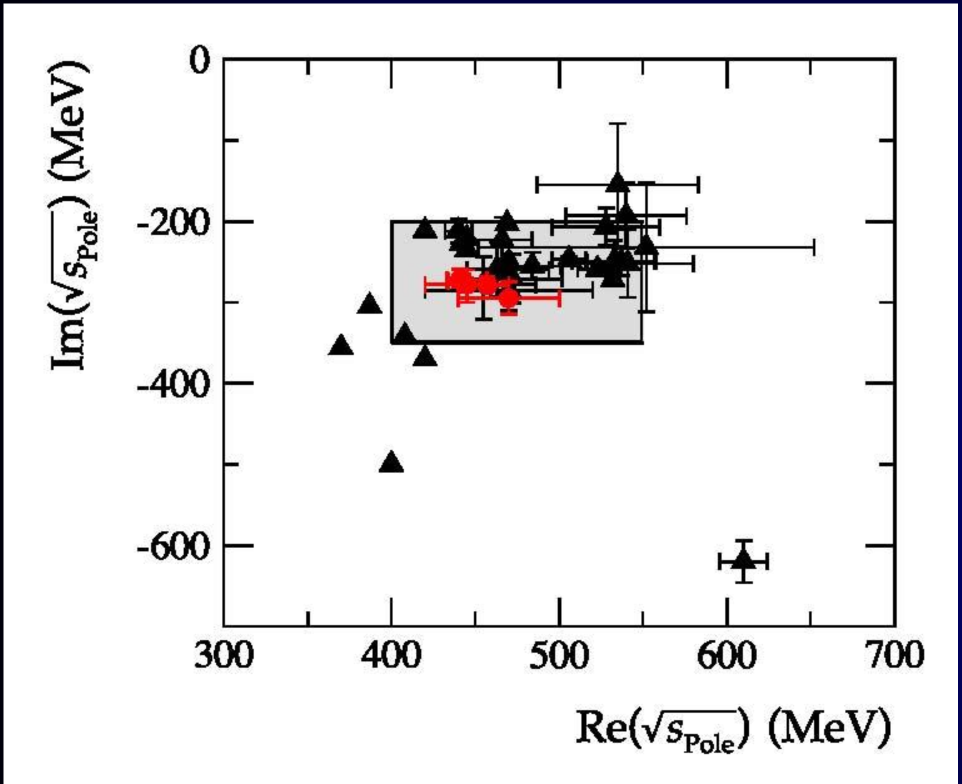


Becomes
f0(500) or “sigma”
in PDG 2012
M=400-550 MeV
Γ=400-700 MeV

To my view…
still **too conservative,**
but quite a good improvement

Actually, in the PDG 2017: "Note on scalars"

"One might just consider the most advanced dispersive analyses, Refs. [9–13]. They agree on a pole position close to (450−i 280) MeV."



9. G. Colangelo, J. Gasser, and H. Leutwyler, NPB603, 125 (2001).
10. I. Caprini, G. Colangelo, and H. Leutwyler, PRL 96, 132001 (2006).
11. R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(
12. B. Moussallam, Eur. Phys. J. C71, 1814 (2011)
13. P. Masjuan, J. Ruiz de Elvira, J.J. Sanz-Cillero, PRD90, 097901 (2014).
.

Combining conservatively statistical and systematic uncertainties I estimate:

$$(449^{+22}_{-16})\text{-}i(275\pm12)\ \text{MeV}$$

JRP, Physics Reports 658-2016-1

This was a long awaited improvement !!!!

Unfortunately, to keep the confusion
the PDG still quotes a “Breit-Wigner mass” and width…

I have no words…

Comments on the minor additions to the $K_0^*(800)$ @PDG12

- Still "omittted from the summary table" since, "needs confirmation"

But, all sensible implementations of unitarity, chiral symmetry, describing the data

find a pole between 650 and 770 MeV with a 550 MeV width or larger.

Since 2009 two EXPERiMENTAL results are quoted from D decays @ BES2

Surprisingly BES2 gives a pole position of $(764 \pm 63^{+71}_{-54}) - i(306 \pm 149^{+143}_{-85})\,\text{MeV}$

Fortunately, the most sounded determination comes from a Roy-Steiner dispersive formalism, consistent with UChPT

Decotes Genon et al 2006

$$(682 \pm 29) - i(273 \pm 22)\,\text{MeV}$$

But AGAIN!! PDG goes on giving Breit-Wigner parameters!! More confusion!!

Recently found as virtual state on the lattice (J.Dudek et al., 2014) consistently with UChPT

Kappa pole from CFD

We have amplitudes that describe data and satisfy dispersion relations up to 1.6 GeV

THERE IS ALSO A KAPPA POLE in our CFD parameterizations

Extracted from conformal parameterization (STILL MODEL DEPENDENT)
A.Rodas & JRP, PRD93,074025 (2016)

$$M-i\,\Gamma/2=(680\pm15)-i(334\pm i7.5)\ \text{MeV}$$

Using Padé Sequences… A.Rodas & JRP & J. Ruiz de ElviraEur. Phys. J. C (2017) 77:91

$$M-i\,\Gamma/2=(680\pm13)-i(325\pm i\ 7)\ \text{MeV}$$

Compare to PDG: $M-i\,\Gamma/2=(682\pm29)-i(273\pm i12)\ \text{MeV}$

Summary

- Dispersion relations have been useful for establishing the existence of resonances and for rigorous determinations of their parameters

- For light scalars, they have settled the longstanding σ-meson controversy and are on the way to settle that of the κ-meson

Still in progress:

A second dispersive determinationwith Roy-Steiner and FDRs will finally settle the $\kappa/K_0^*(800)$ issue at the PDG. Our group has been asked to do it.

We are about to finish the $\pi\pi\rightarrow KK$ analysis needed asinput for $\pi K\rightarrow\pi K$