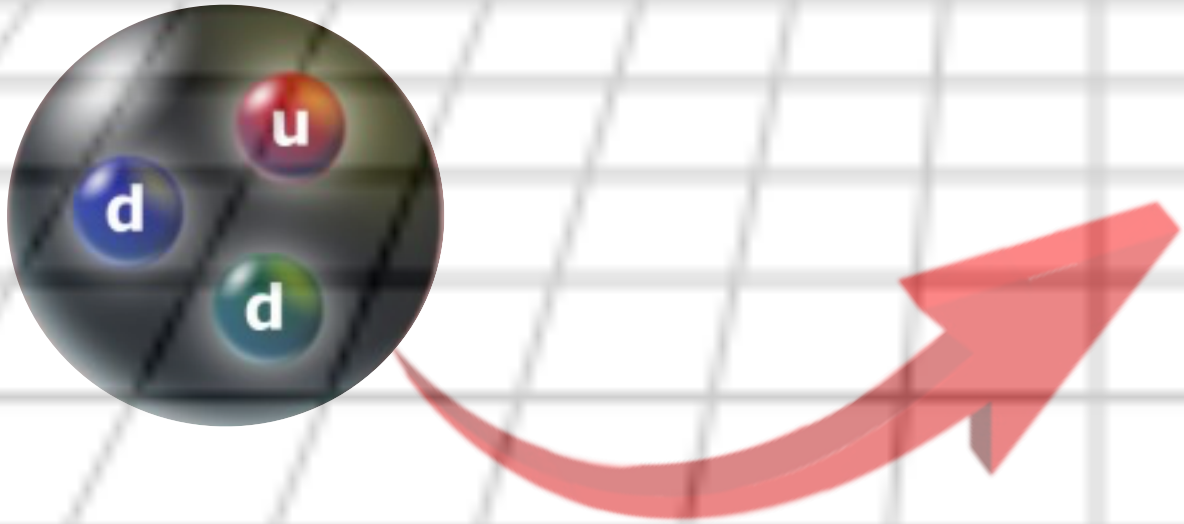
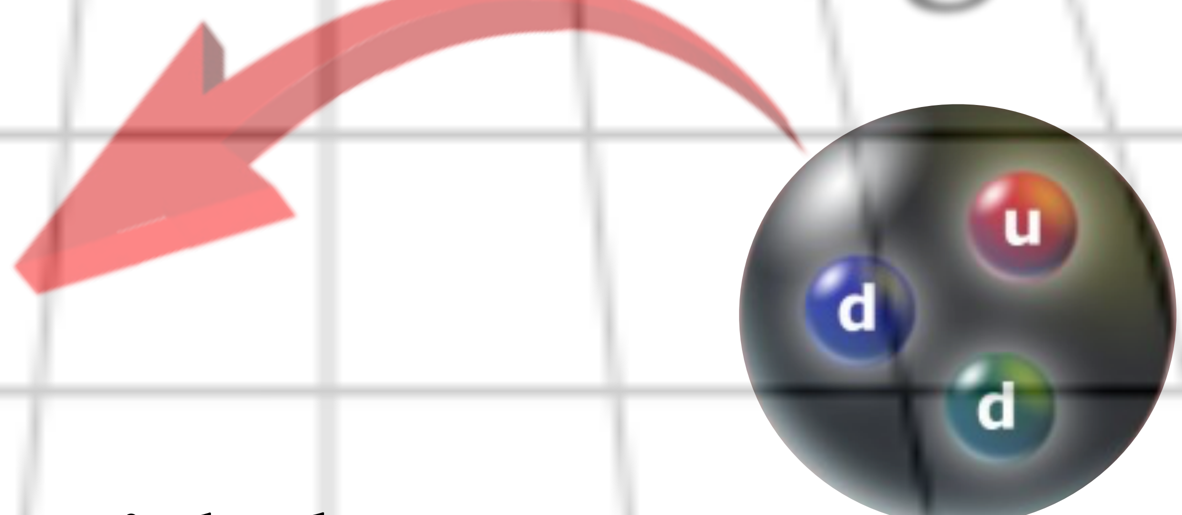


Advances in nucleon-nucleon scattering

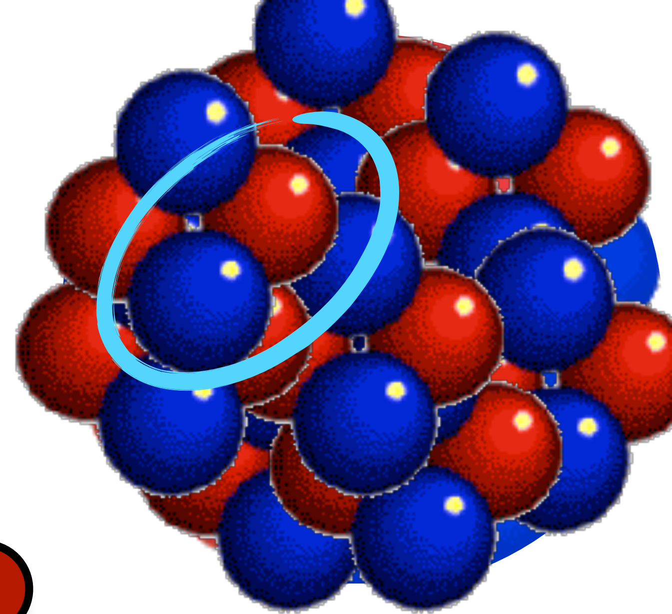


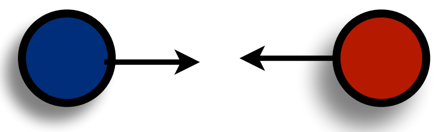
Amy Nicholson
UNC, Chapel Hill

Multi-Hadron Systems from Lattice QCD

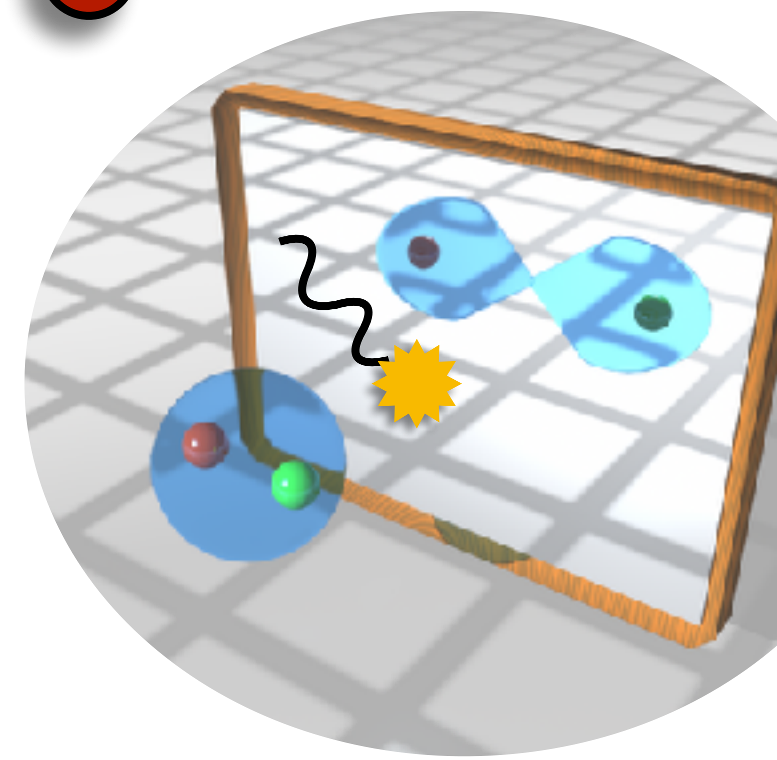
INT, Seattle, Feb. 7, 2018

NN systems

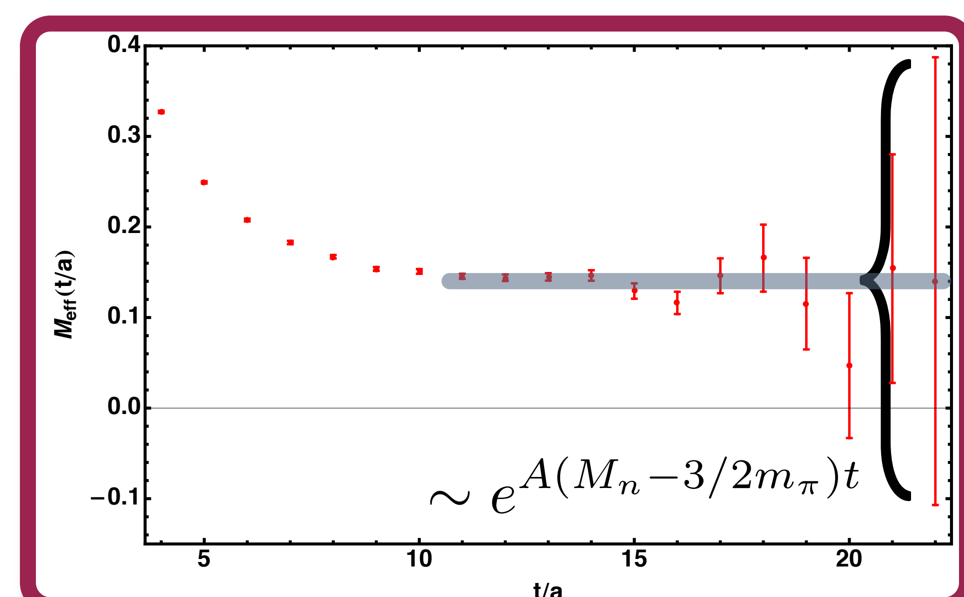




- **What do we need to do nuclear physics?**



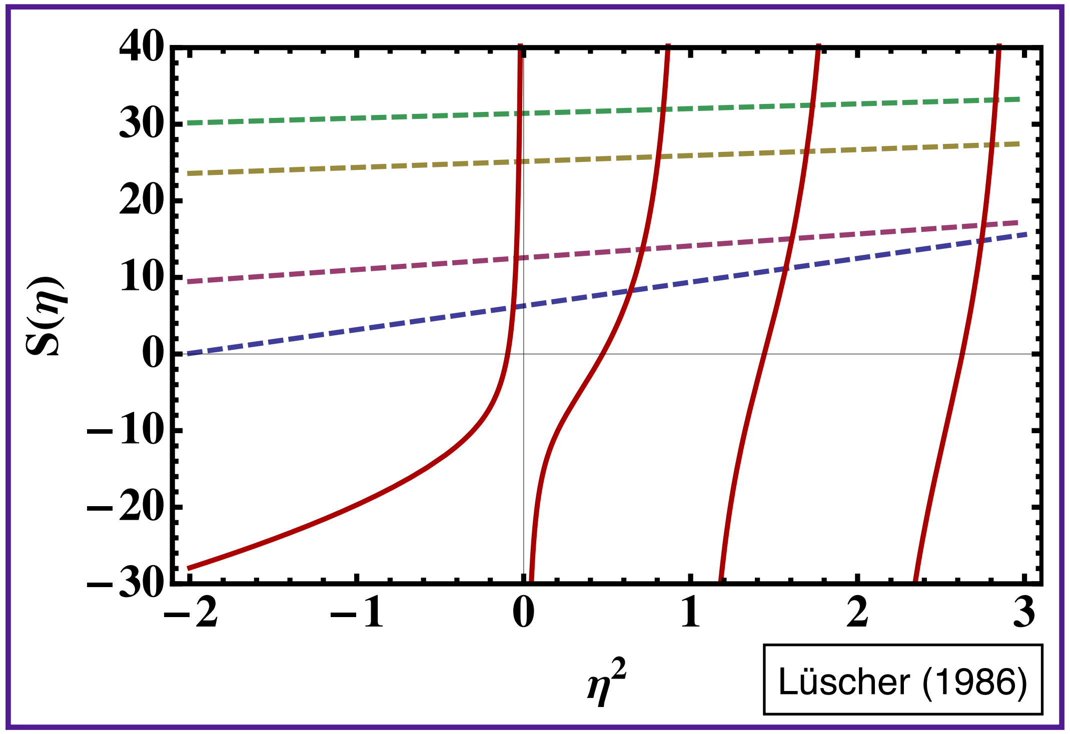
- **Must have full control over 2-body systems**
  - **How do we project onto desired states?**
  - **How do we disentangle signals from closely spaced energy levels?**
  - **How do we beat the noise?**

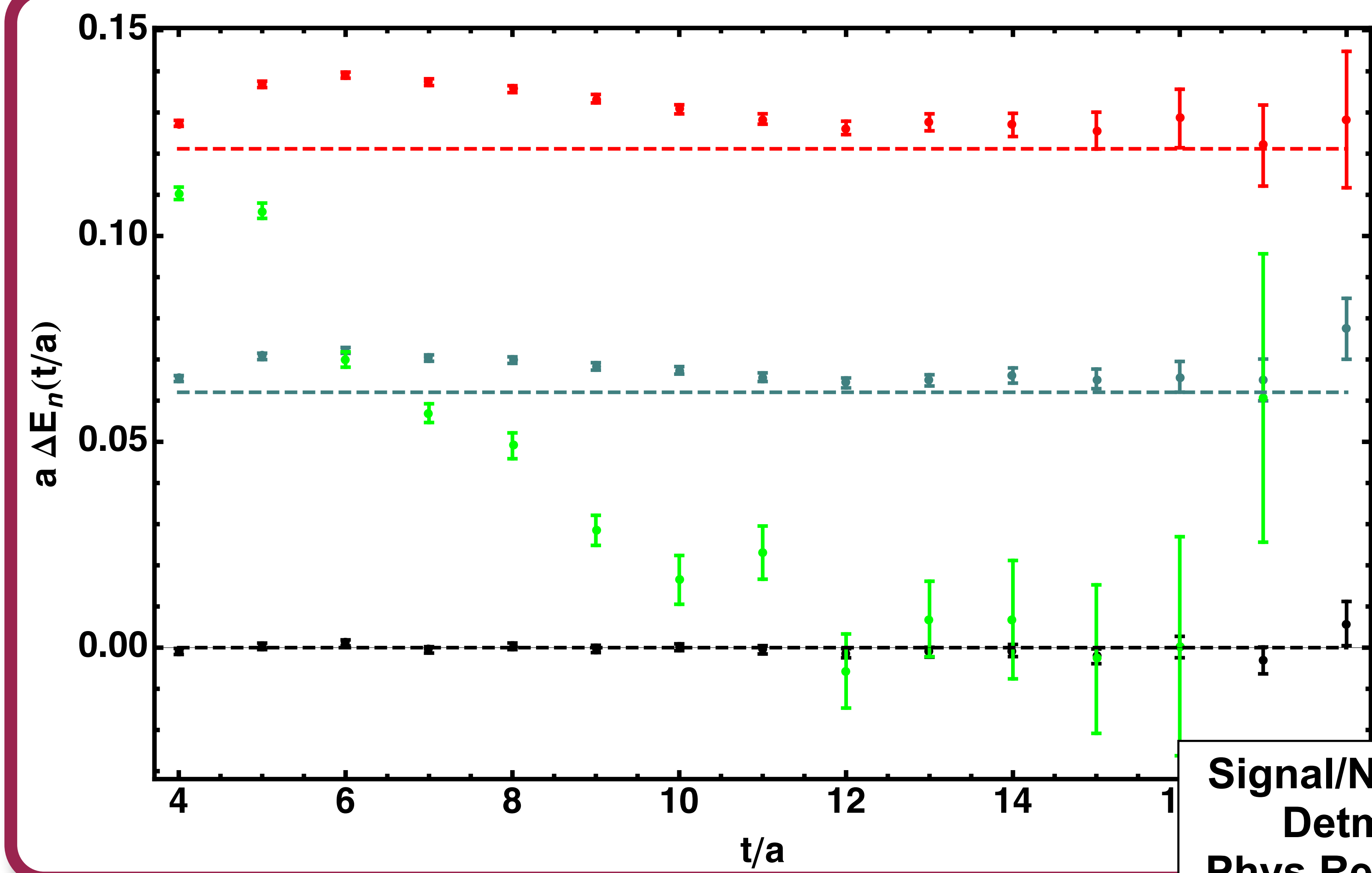


Good operators (and analysis)!

Trying to pull off tiny correction compared to large nucleon mass:

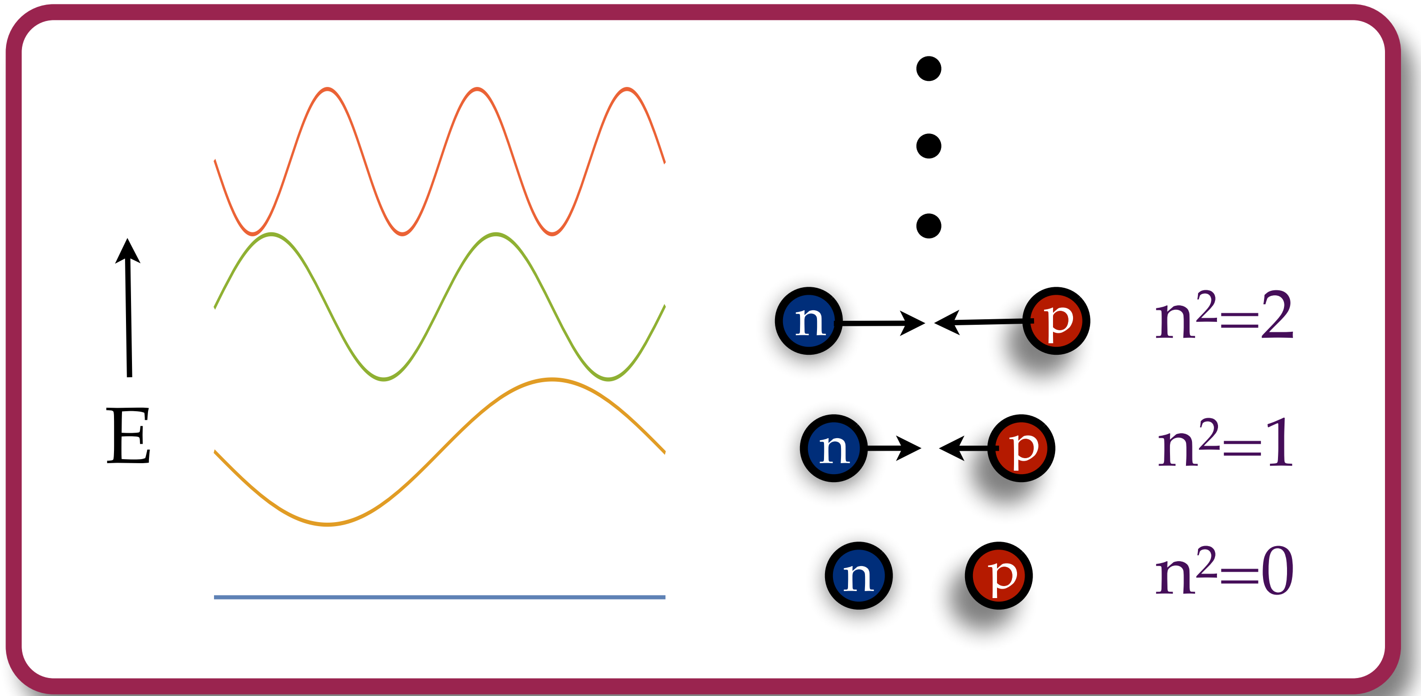
$\Delta E = E_{NN} - 2E_N$





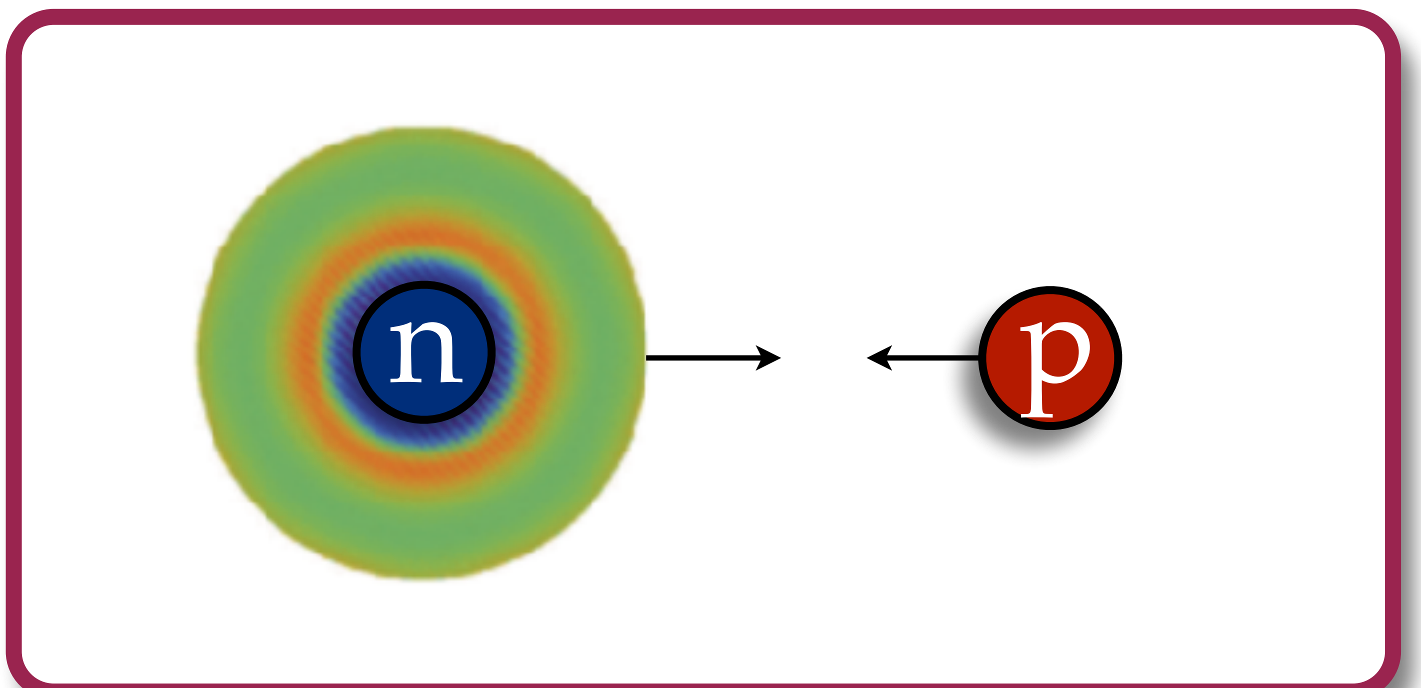
Signal/Noise optimization: Detmold & Endres, Phys.Rev. D90 (2014) no.3, 034503

Excited state contamination



Elastic scattering
(2-body)

$\Delta E \sim 50$ MeV

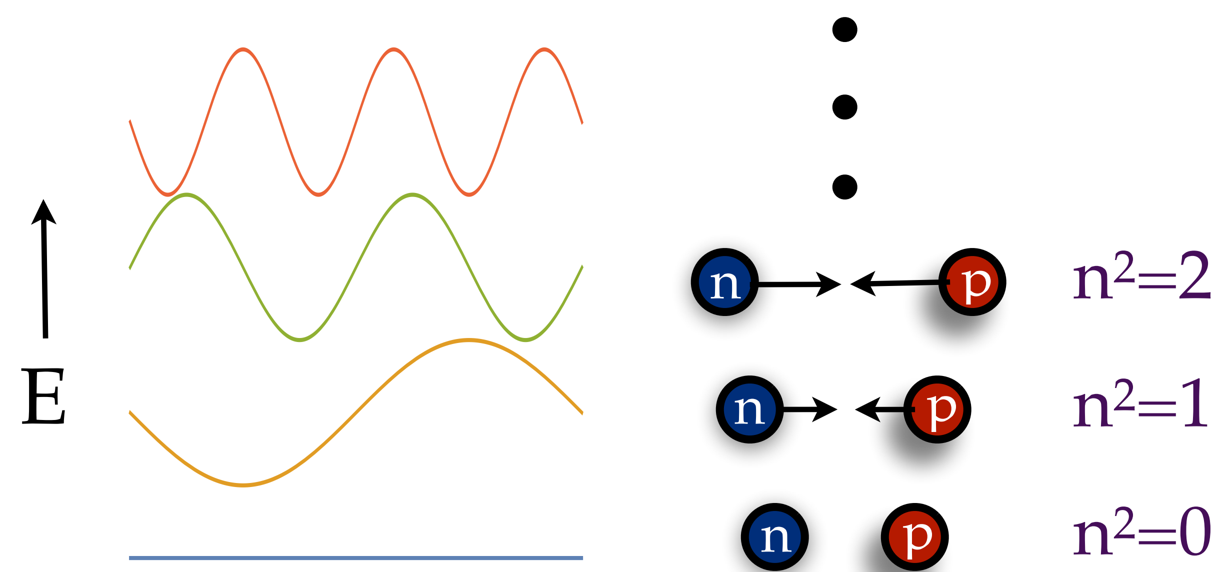


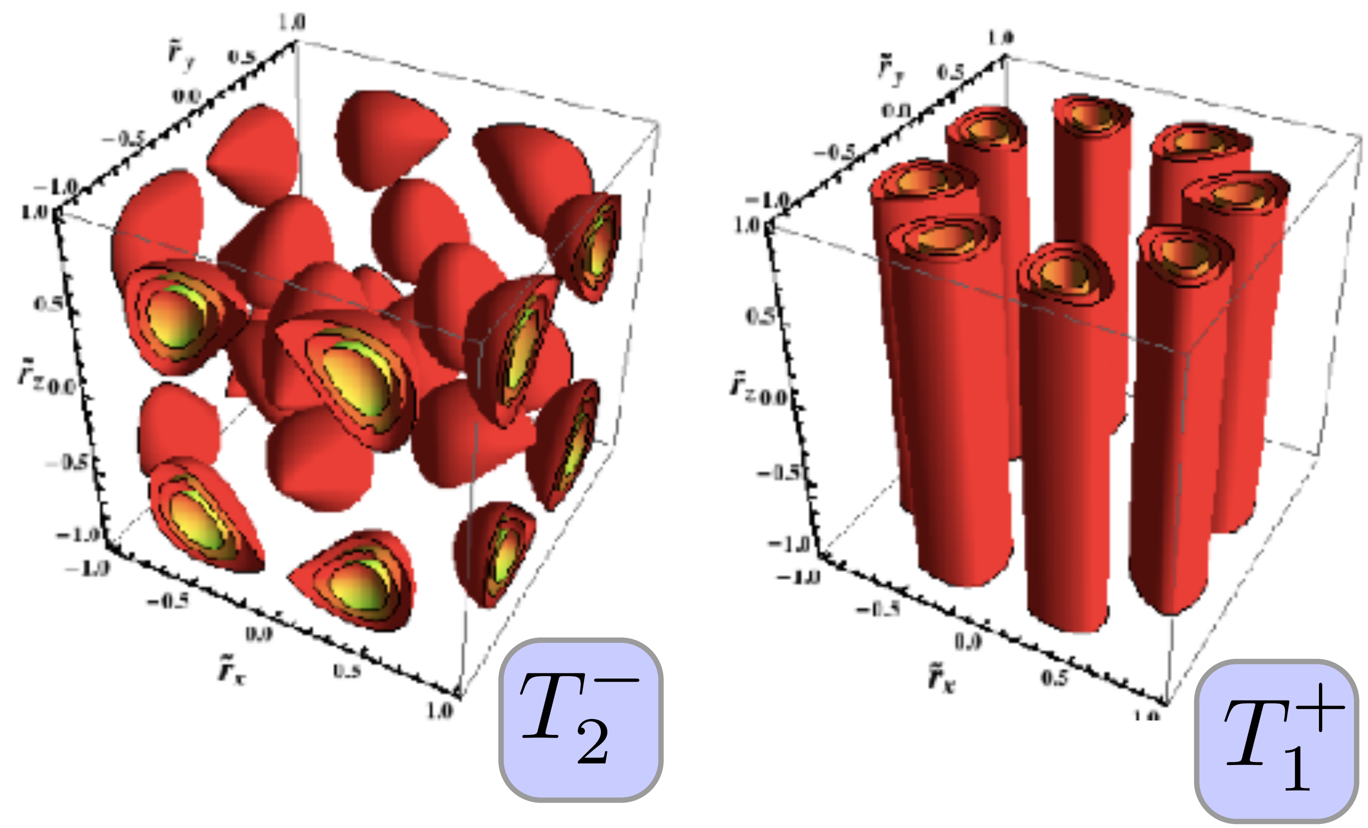
Inelastic single body

$\Delta E \sim m_\pi$

Reducing elastic 2-body excited states

- Project onto non-interacting eigenstates of the box
- Very costly to perform exact momentum/angular momentum projection at both source & sink (~V)
- Perform exact projection only at the sink





Figures from Luu & Savage (2011)

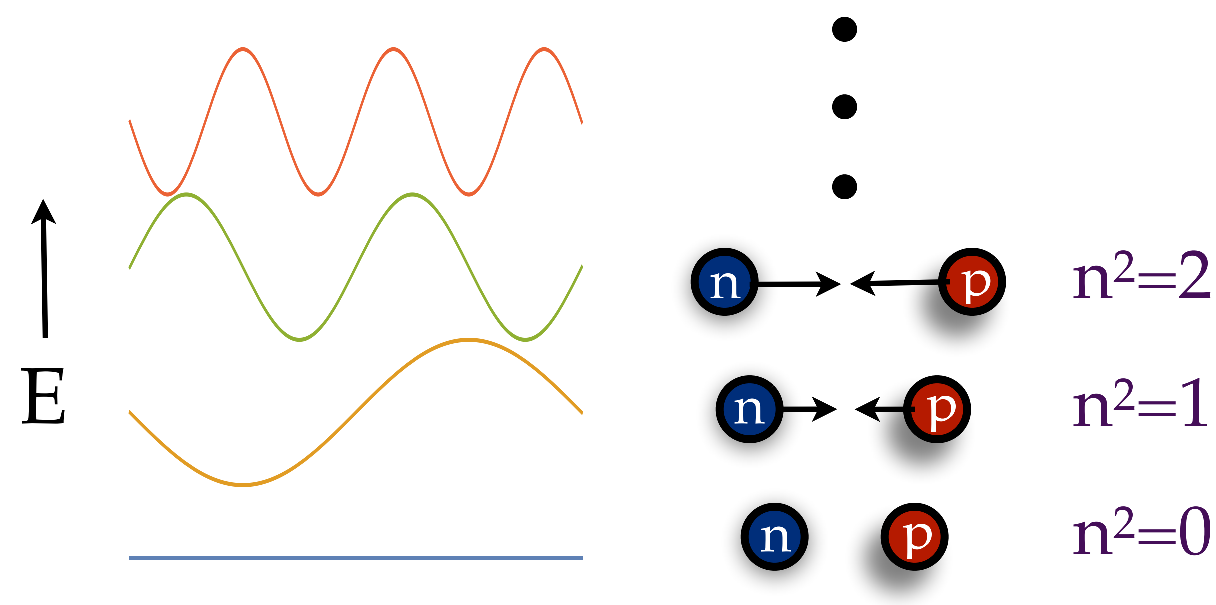
Reducing elastic 2-body excited states

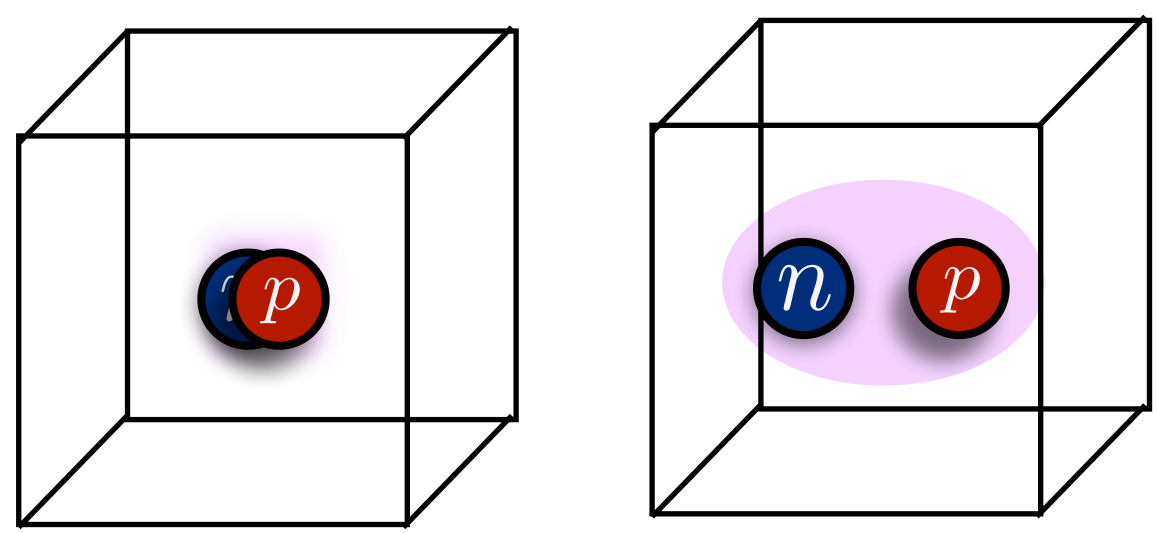
- Project onto non-interacting eigenstates of the box

- Very costly to perform exact momentum/angular momentum projection at both source & sink (~V)

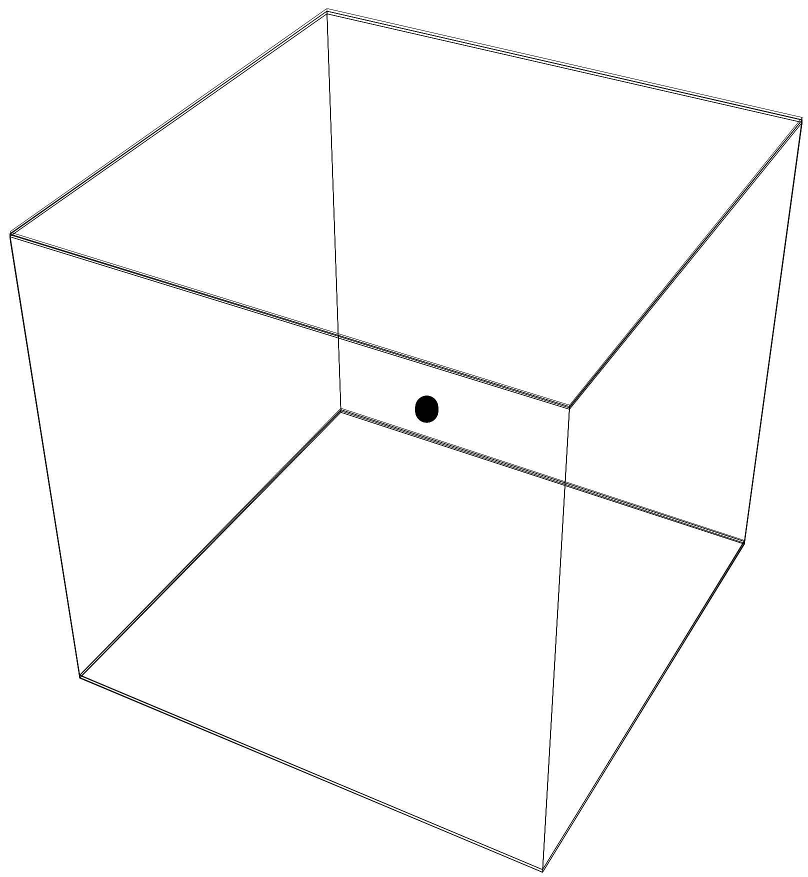
- Source: need spatially displaced source operators to have overlap with $\ell > 0$

- Even for s-wave, displaced sources are cleaner



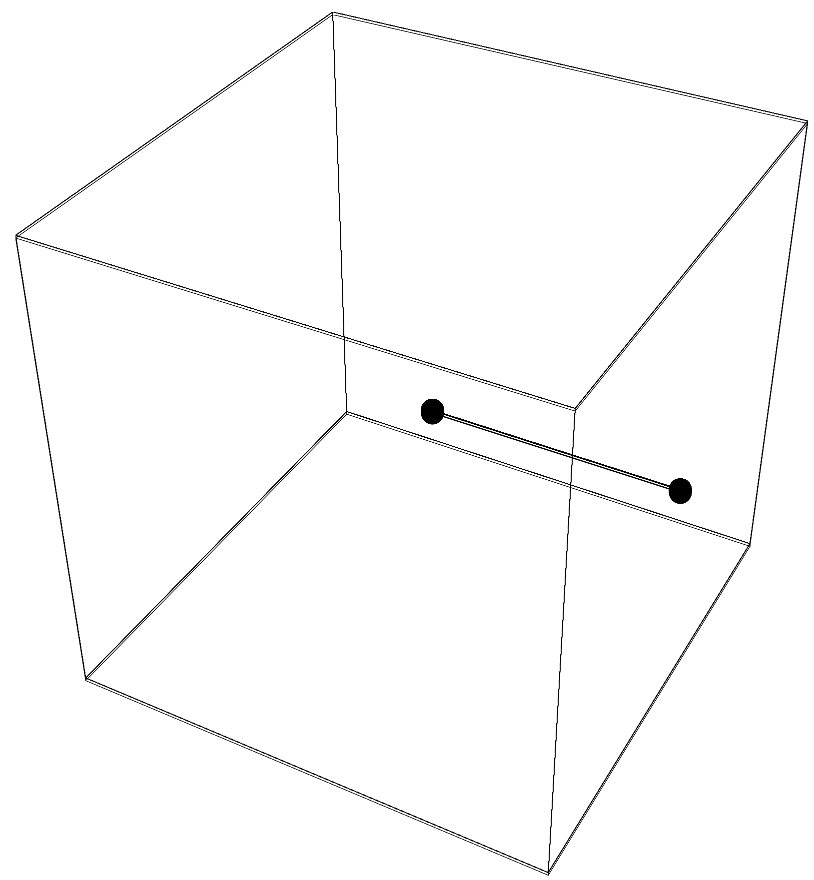


Source: position space



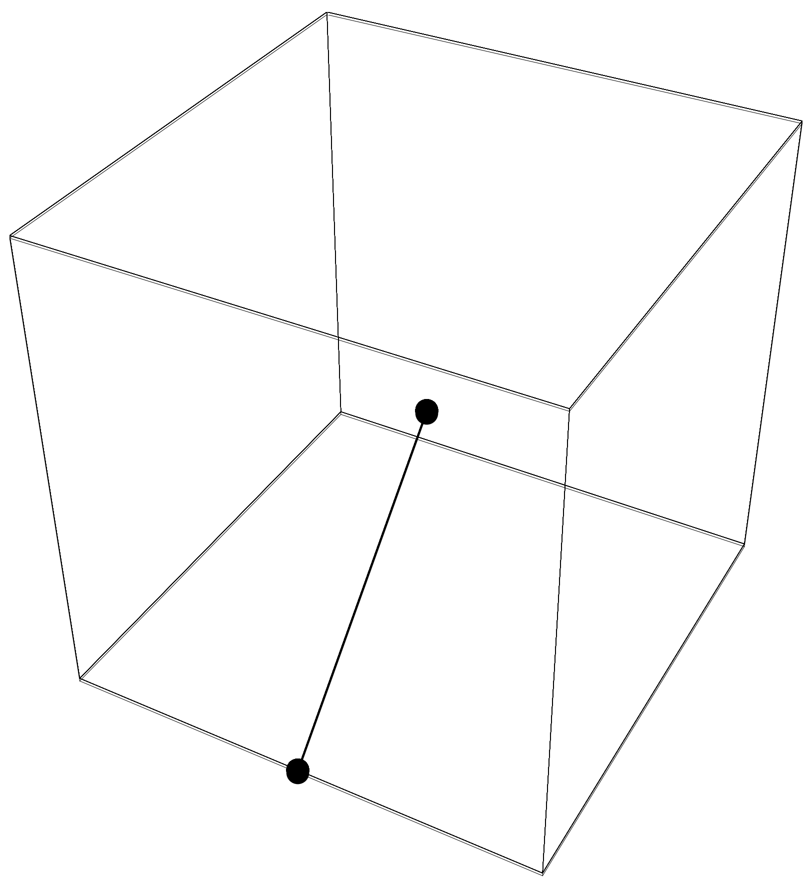
Starting with a good interpolating operator for a single nucleon at x_0....

Source: position space



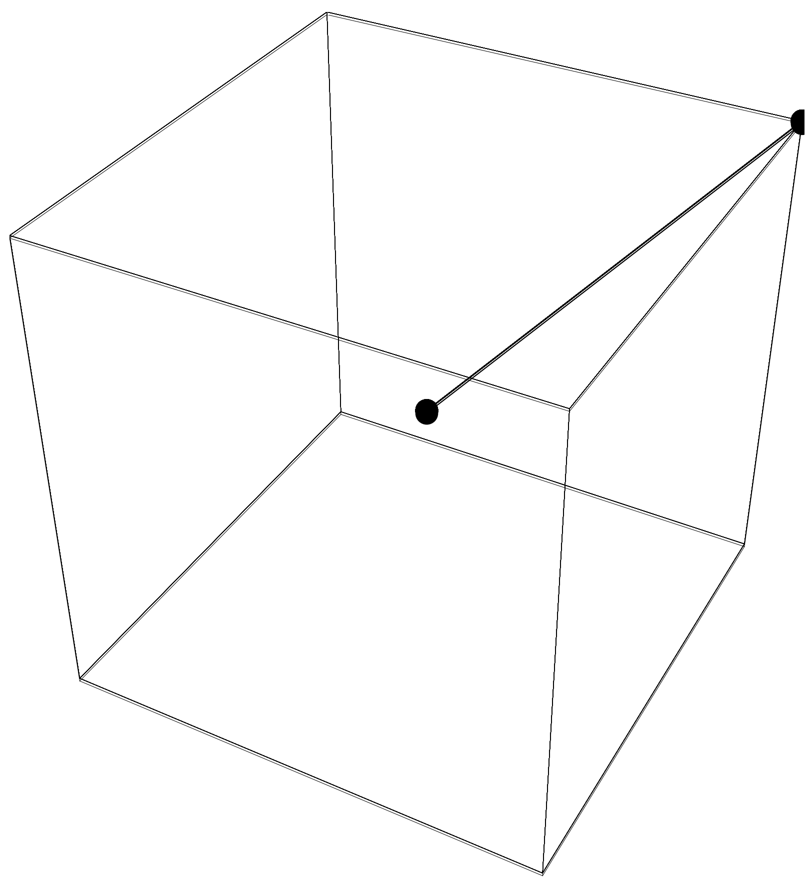
Add displaced nucleon:
"Face" (6)

Source: position space



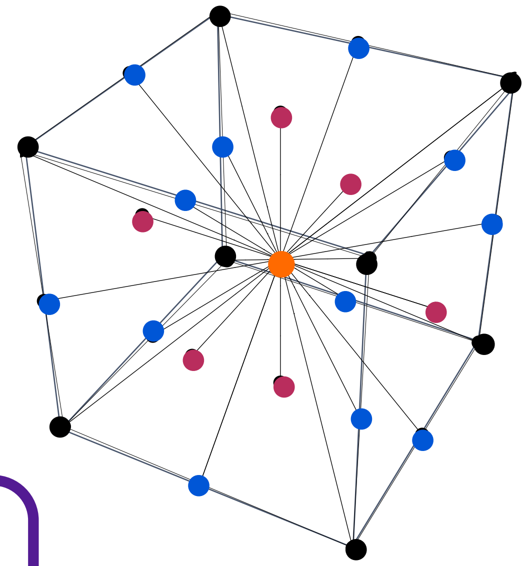
Add displaced nucleon:
"Edge" (12)

Source: position space

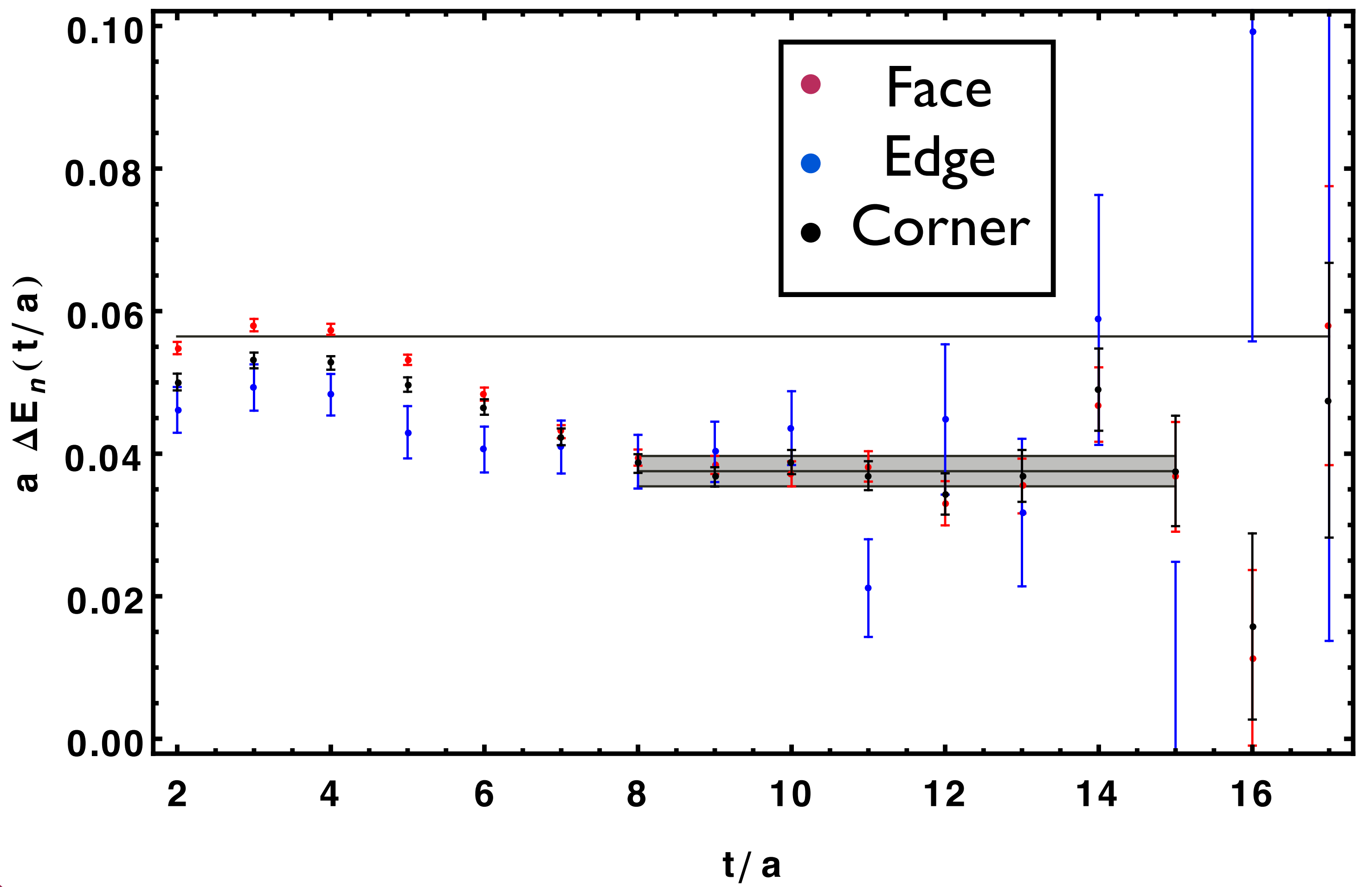


Add displaced nucleon:
"Corner" (8)

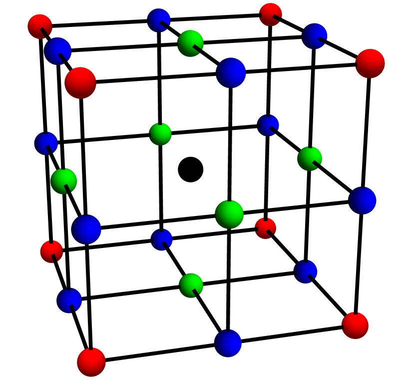
Source: position space



Different source types give us a handle for isolating the desired state

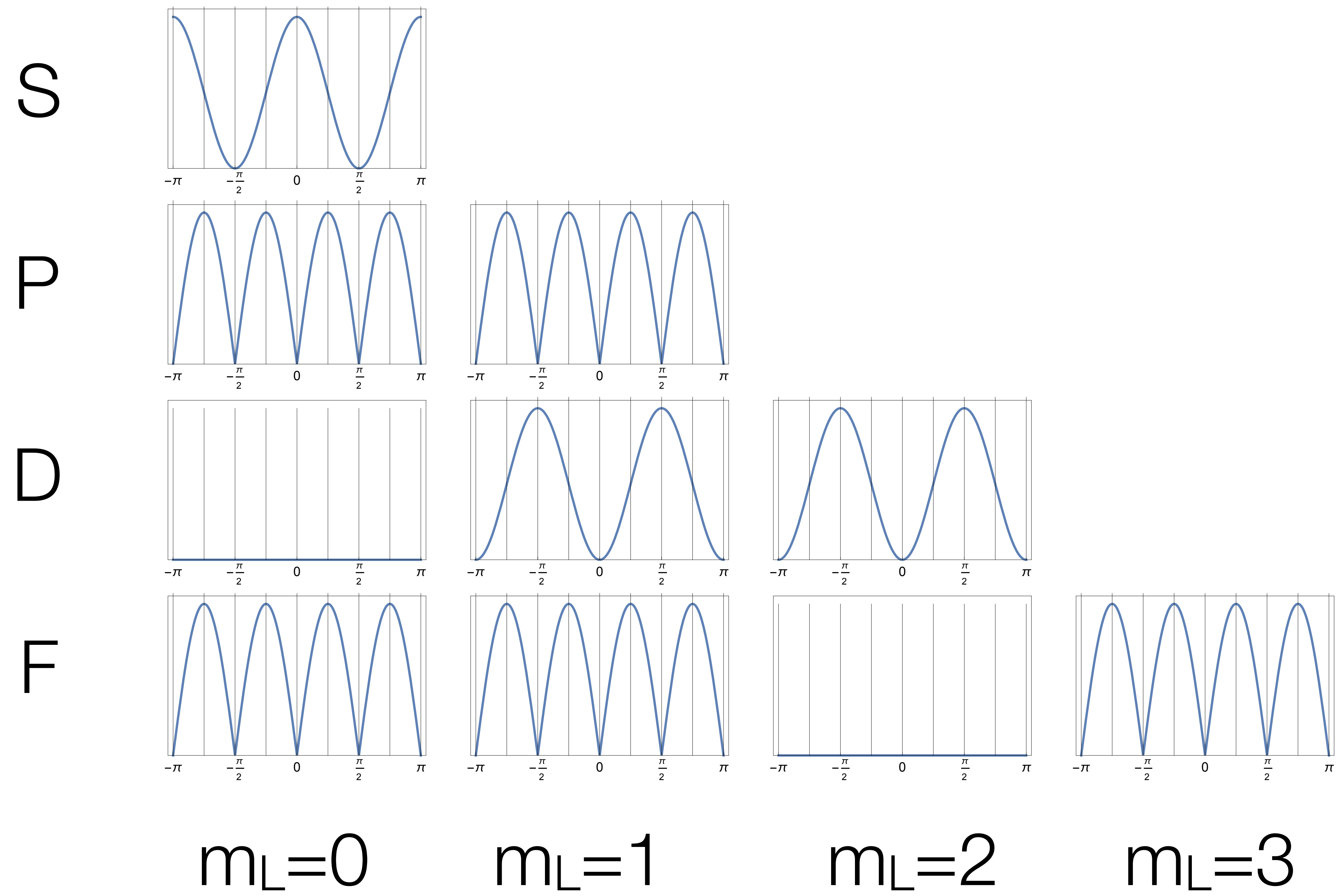


Slides courtesy E. Berkowitz

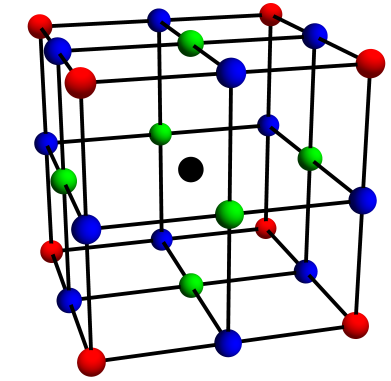


Source Overlap

Project Luu & Savage momentum sources to corner as a function of $\pi\Delta x/L$

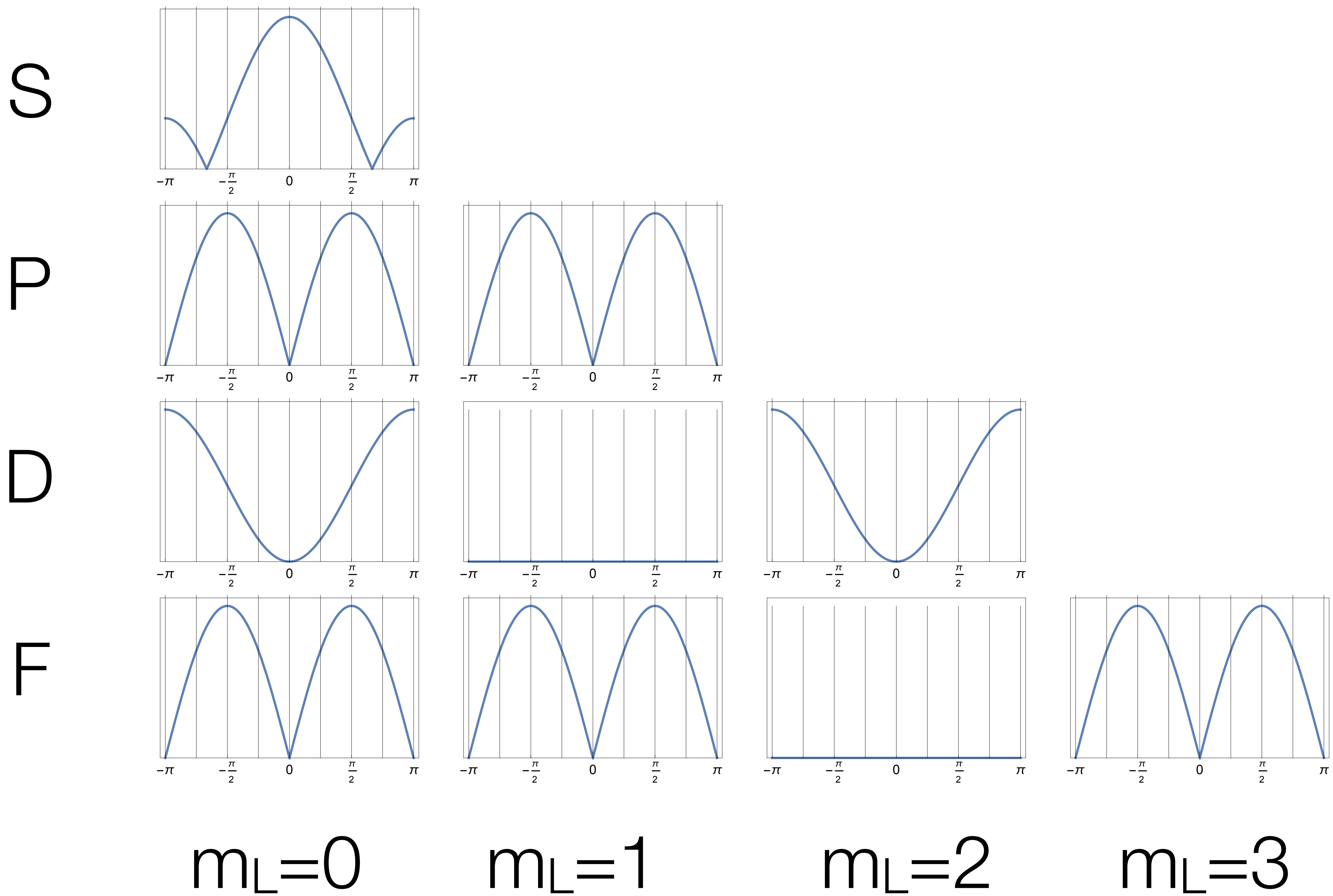


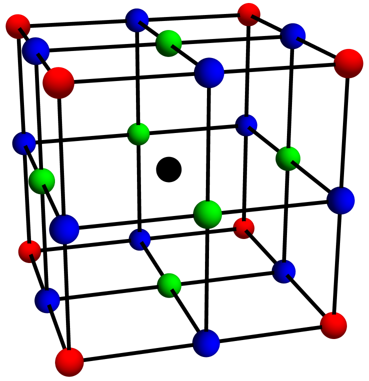
Slides courtesy E. Berkowitz



Source Overlap

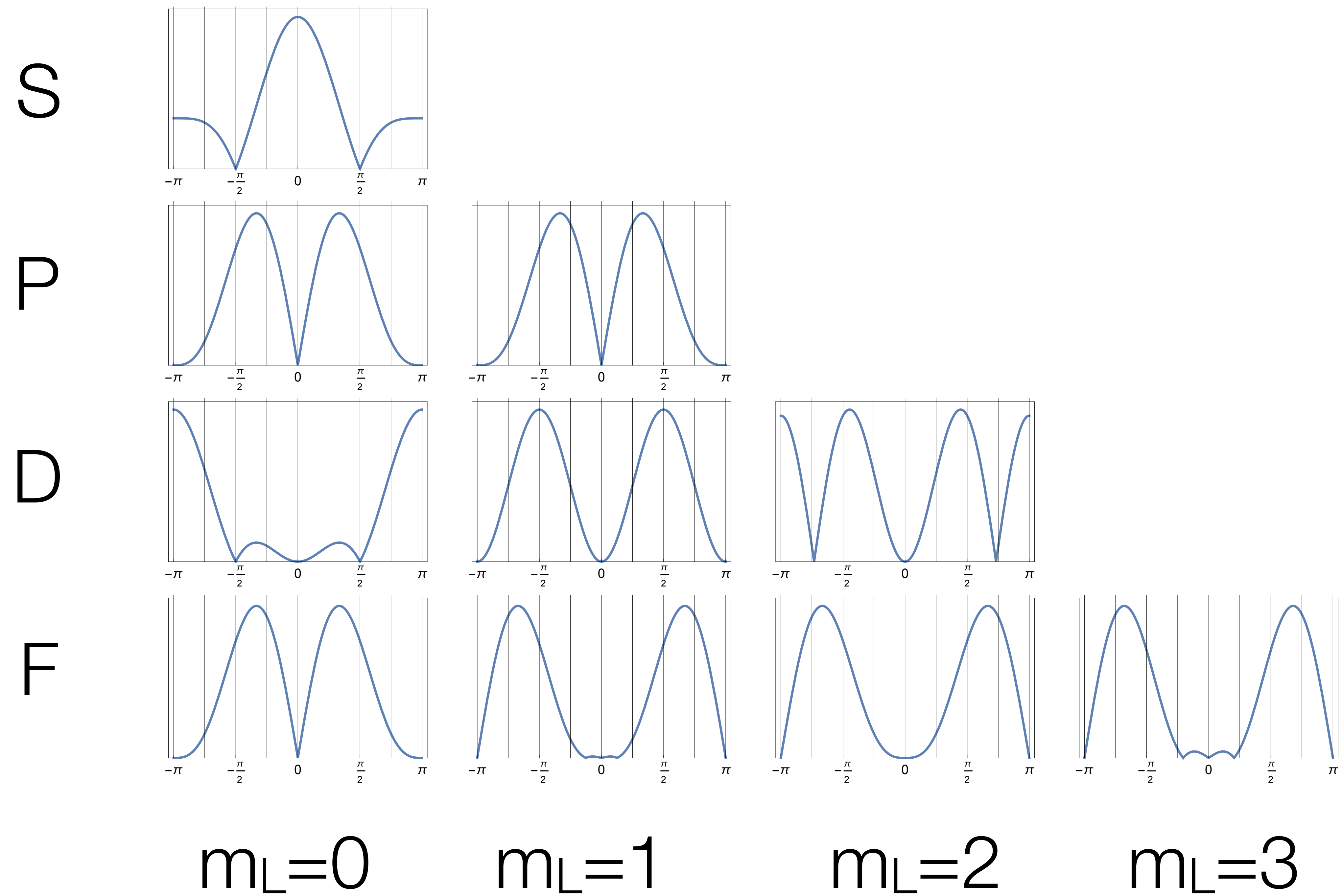
Project Luu & Savage momentum sources to faces as a function of $\pi\Delta x/L$





Source Overlap

Project Luu & Savage momentum sources to edges as a function of πΔx/L



Slides courtesy E. Berkowitz

Propagator Reuse

$0 = (0,0,0)$

$A = (L,L,L)/2$

$C = (\pm 1,\pm 1,\pm 1)\,\Delta x$

10 local sources

+1 maximally displaced

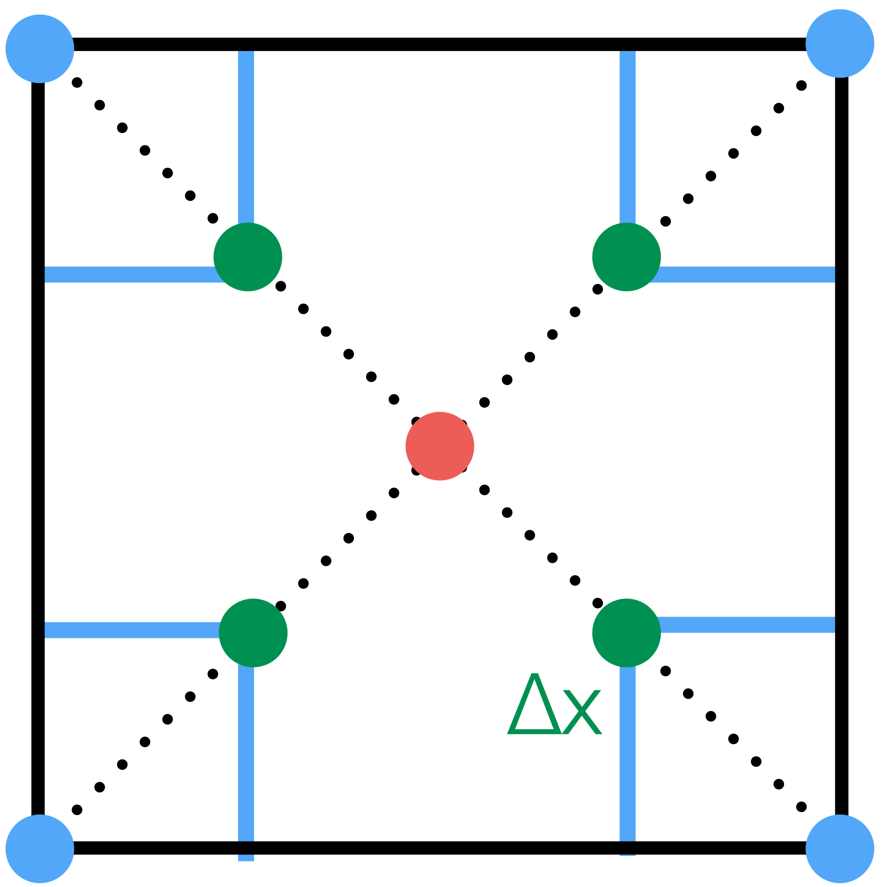
+1 corner(Δx) around 0

+1 corner($L/2-\Delta x$) around A

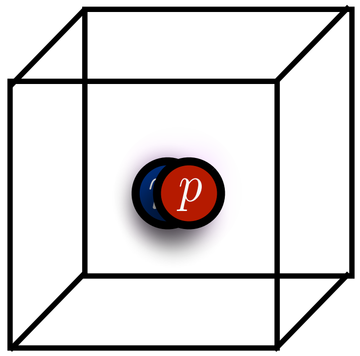
+1/2 corner($2\Delta x$) from C

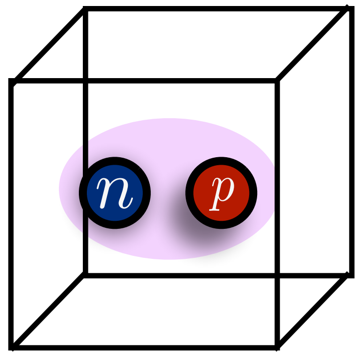
+2 faces($2\Delta x$) from C

+1 edges($2\Delta x$) from C

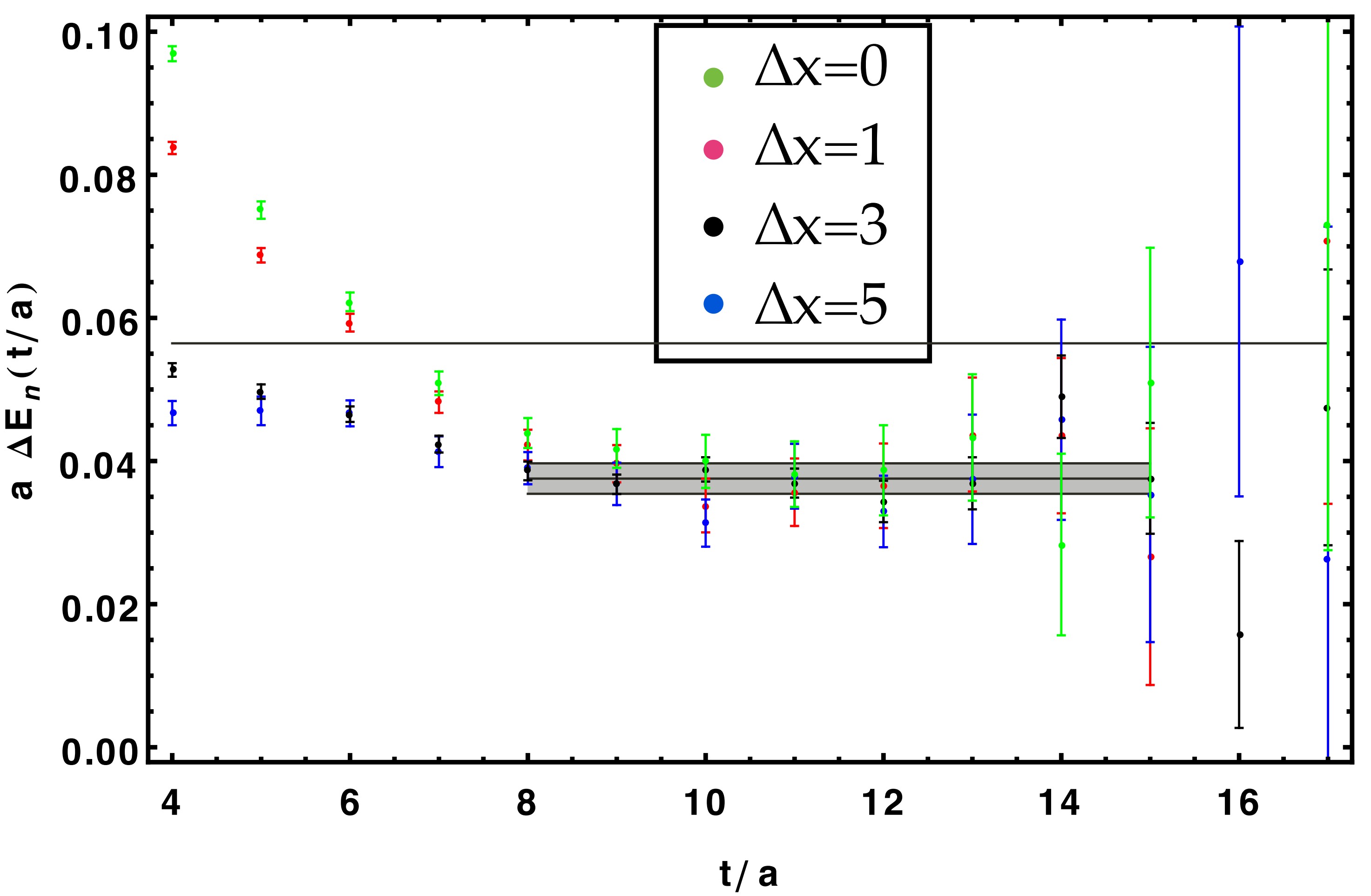


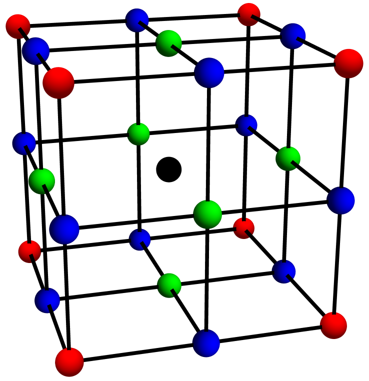
Source: position space





Large displacements are necessary for maximal overlap with low-energy states

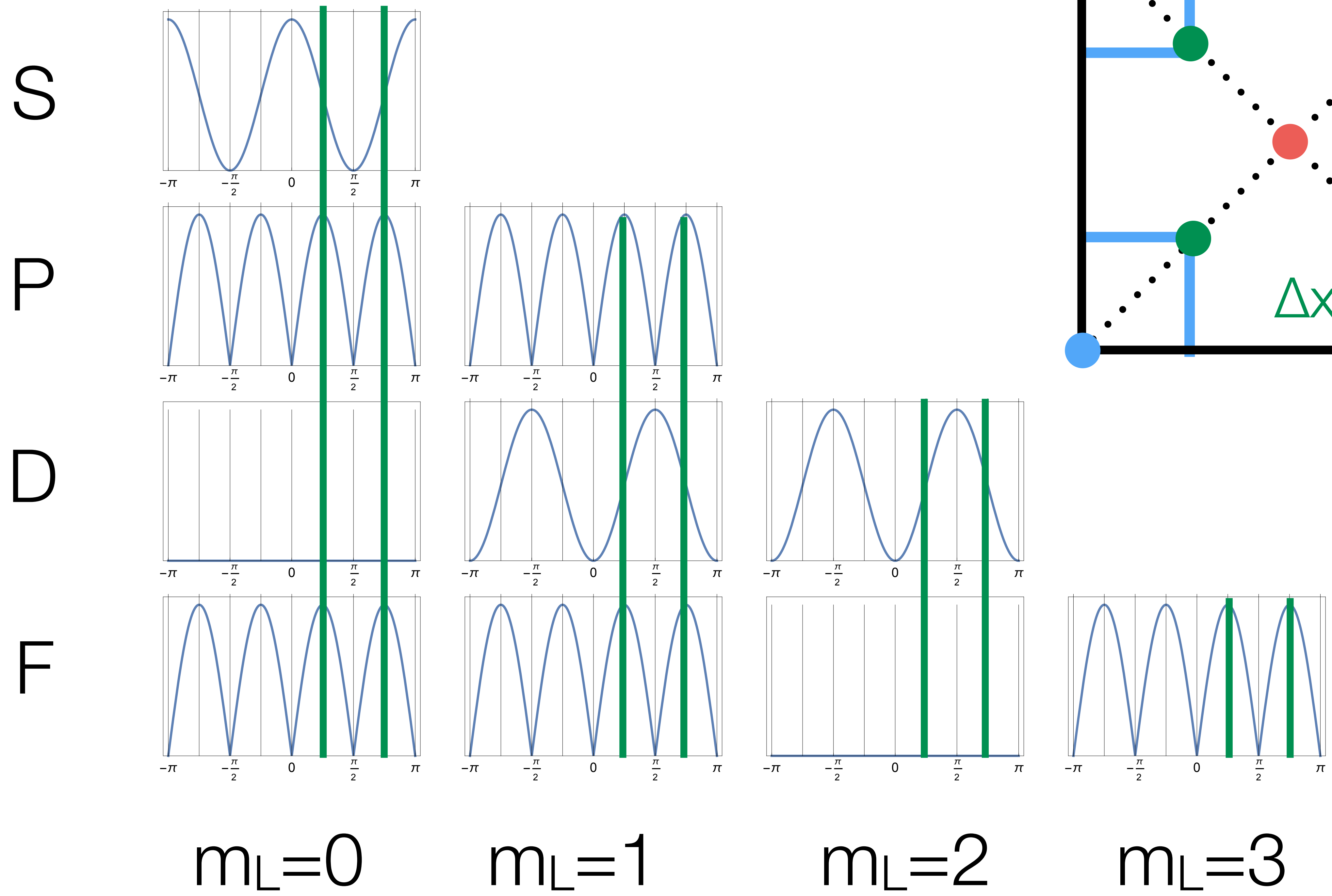


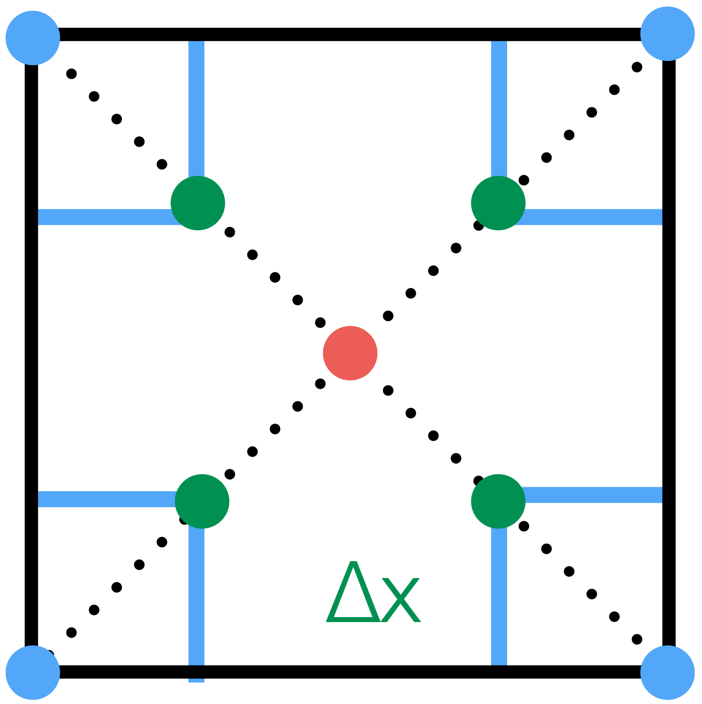


Slides courtesy E. Berkowitz

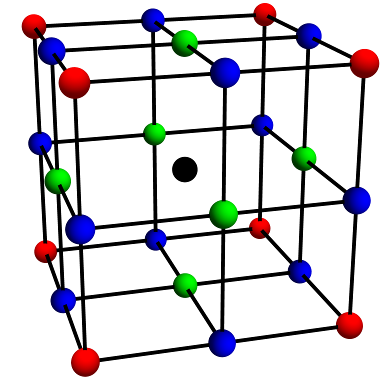
Magic Choice: Δx = L/8 (≡3L/8)

corner



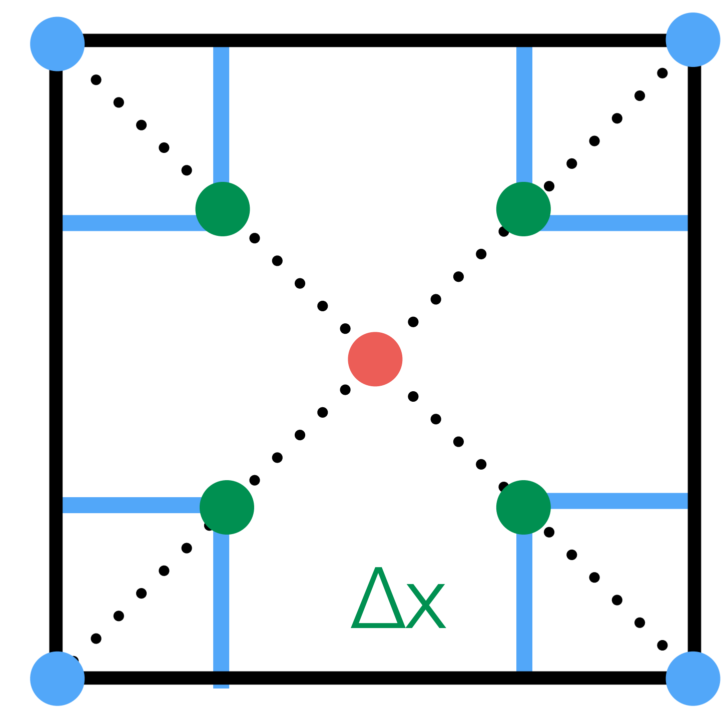


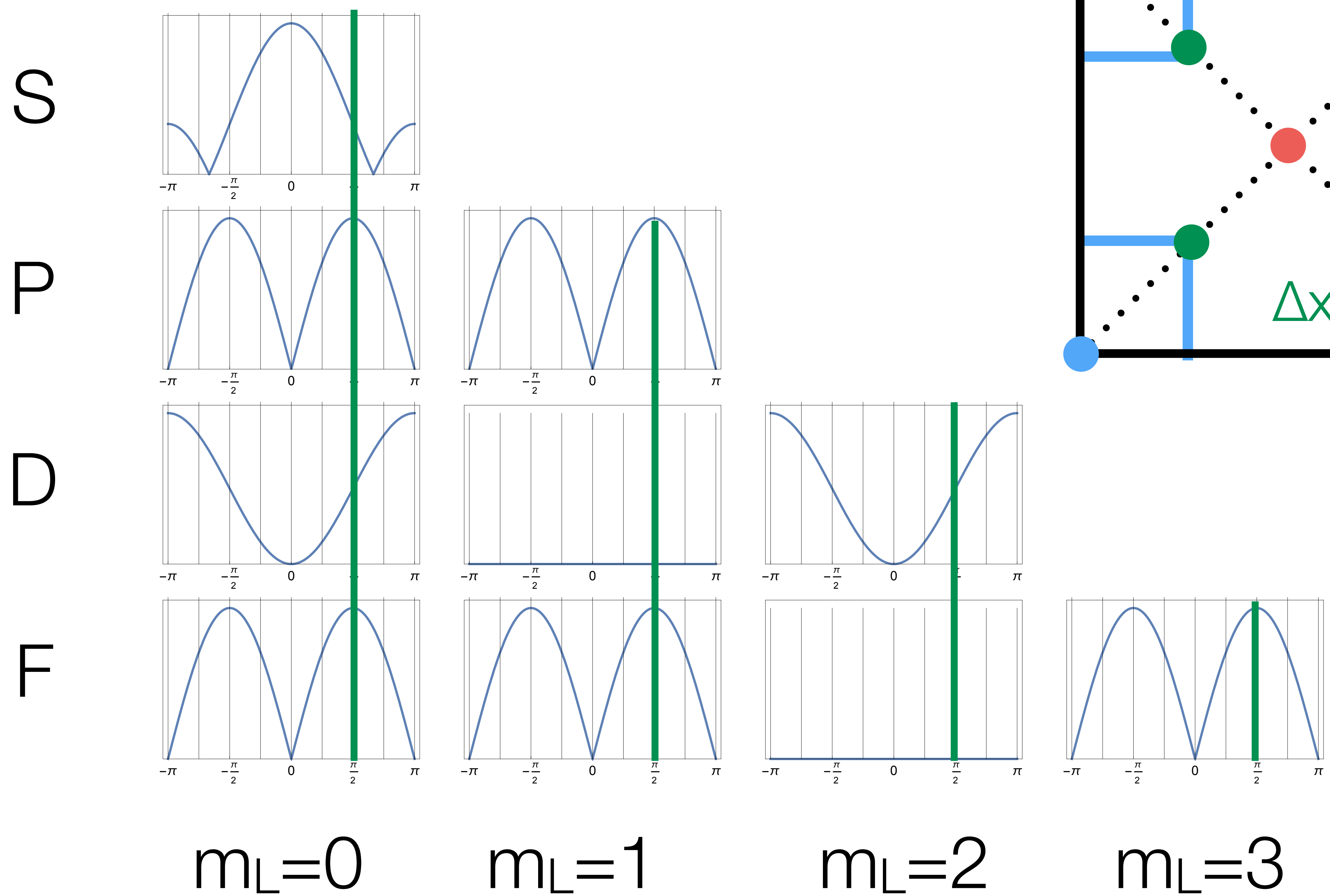
Slides courtesy E. Berkowitz



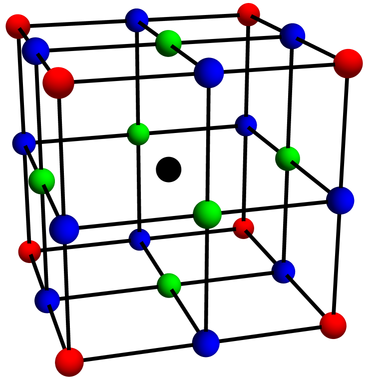
Magic Choice: $\Delta x = L/8\ (\equiv 3L/8)$

faces



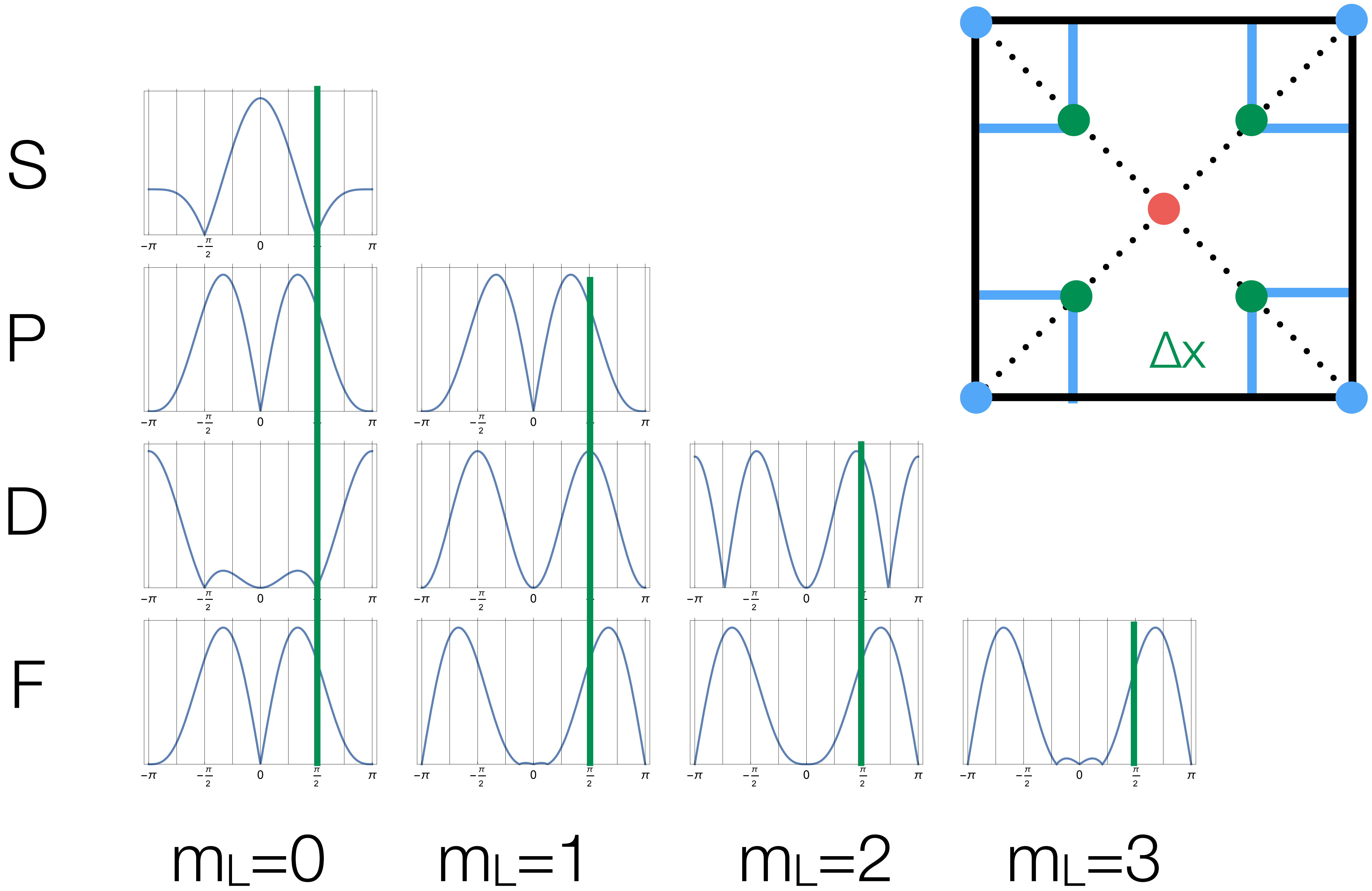


Slides courtesy E. Berkowitz



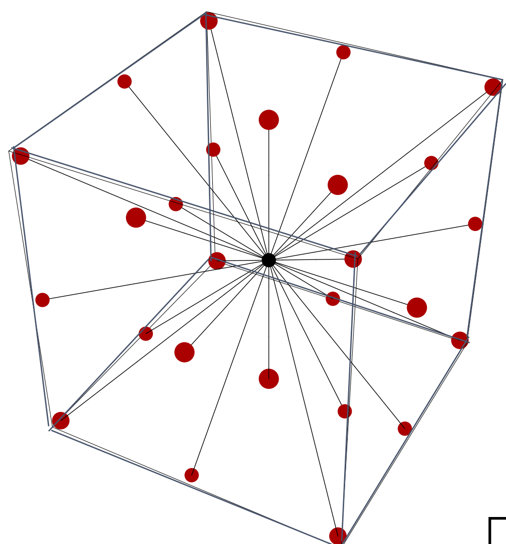
Magic Choice: $\Delta x = L/8$ ($\equiv 3L/8$)

edges

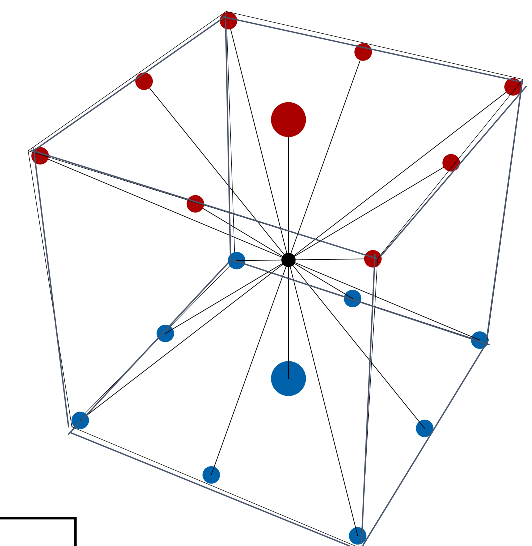


Source: position space

S

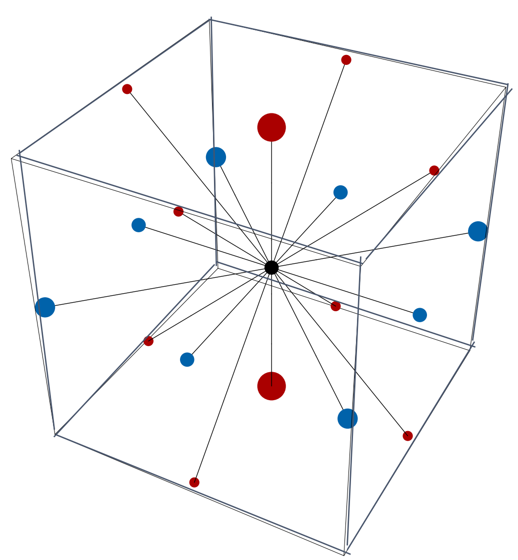


P

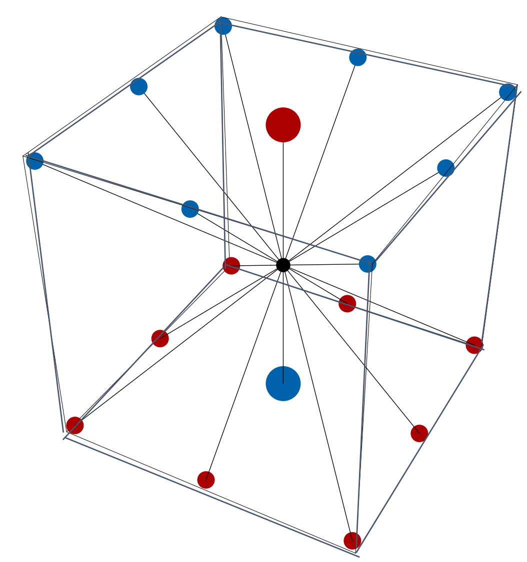


Spherical harmonics

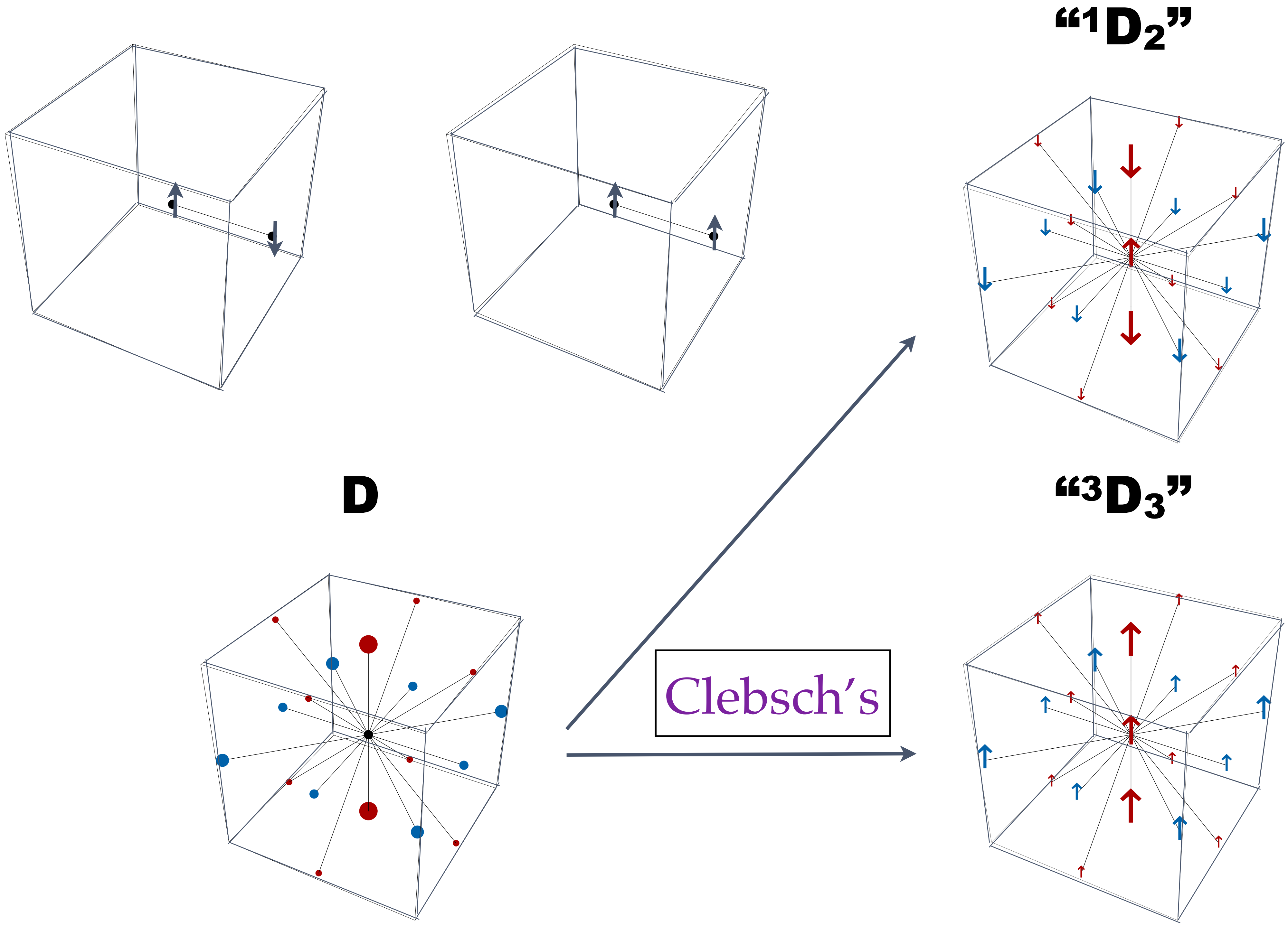
D



F

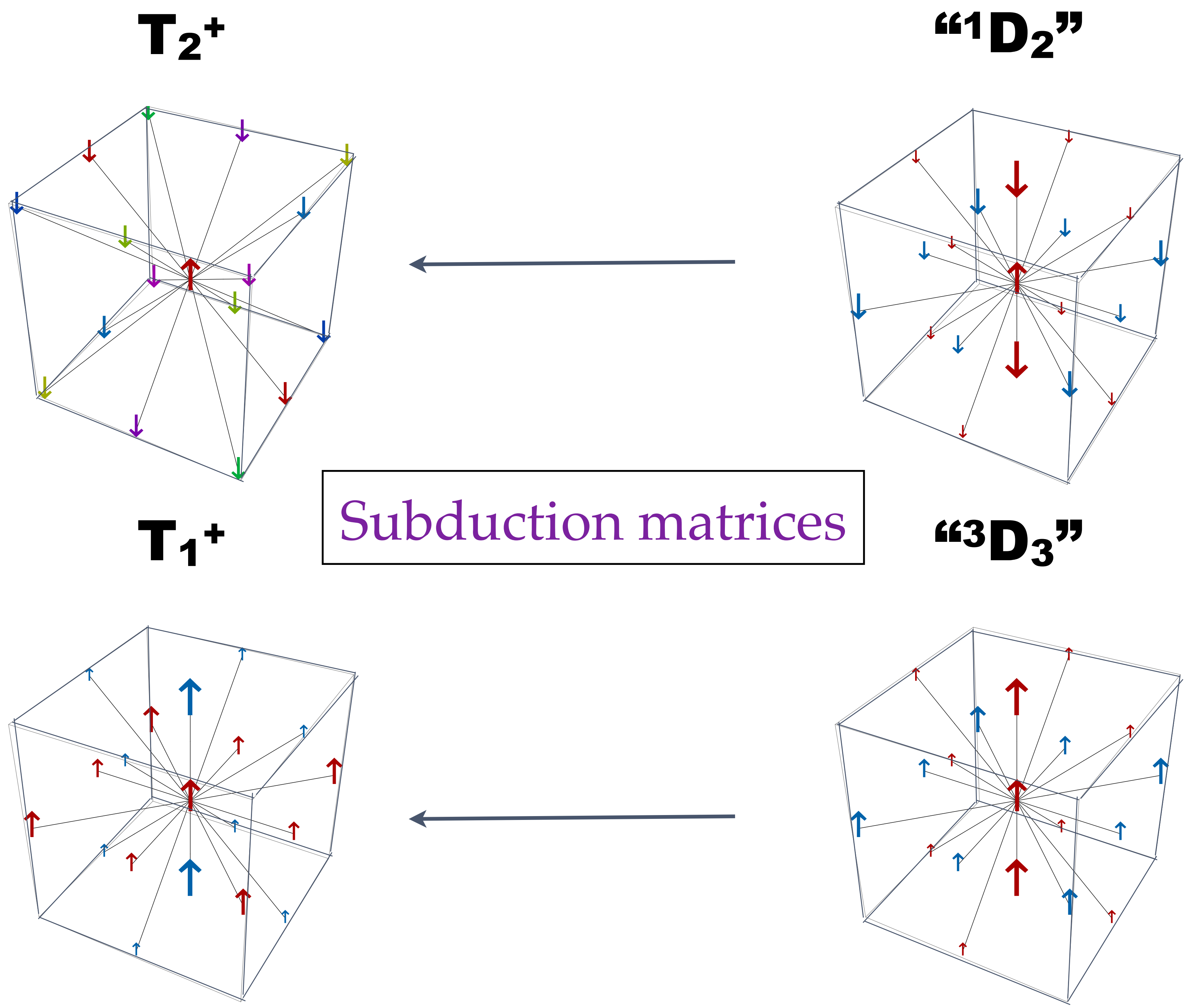


Source: position space

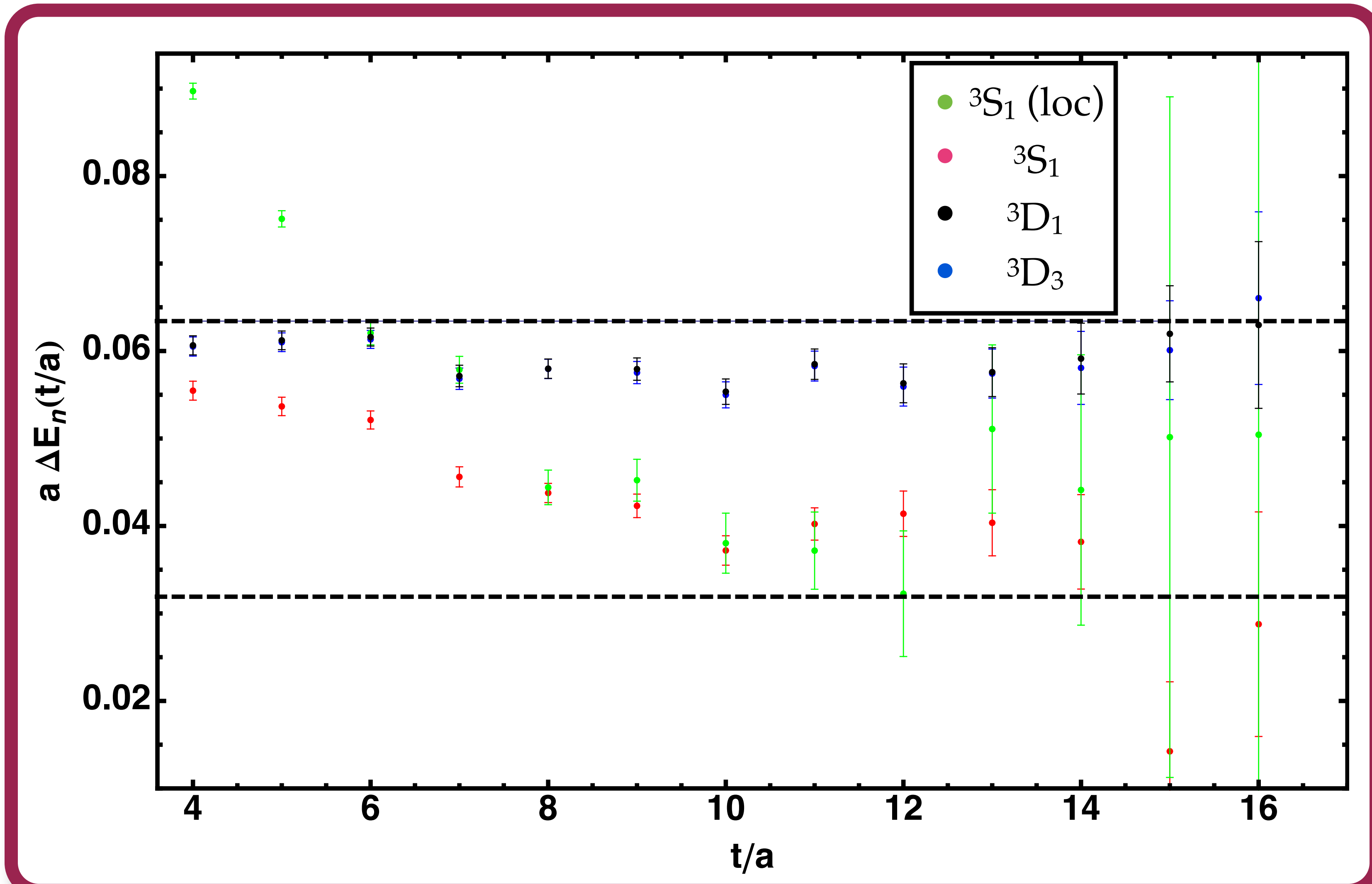


HadSpec 1004.4930

Source: position space



Source: position space



Set of multiple sources coupling to same cubic irrep

Isospin 0

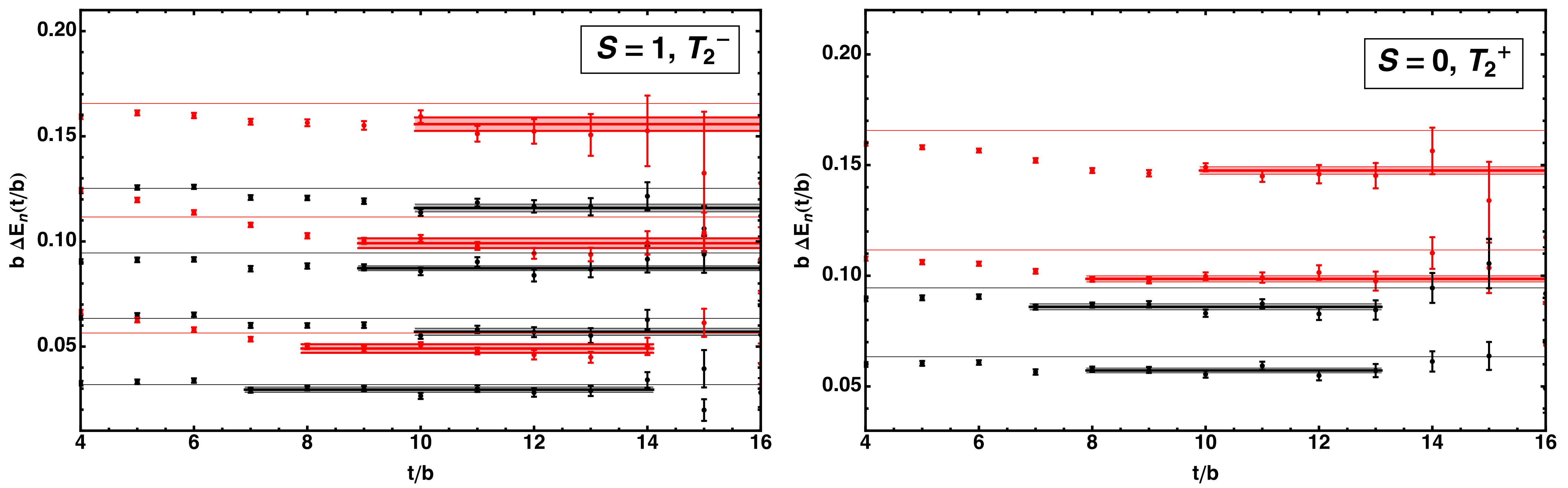
Partial wave	Irreps
$^1P_1$	T_1^-
$^3S_1, {}^3D_1$	T_1^+
$^3D_2$	$E^+ \oplus T_2^+$
$^3D_3$	$A_2^+ \oplus T_1^+ \oplus T_2^+$
$^1F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$

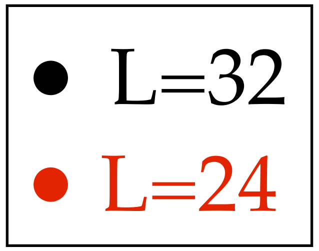
Isospin 1

Partial wave	Irreps
$^1S_0$	A_1^+
$^3P_0$	A_1^-
$^3P_1$	T_1^-
$^3P_2, {}^3F_2$	$E^- \oplus T_2^-$
$^1D_2$	$E^+ \oplus T_2^+$
$^3F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$
$^3F_4$	$A_1^- \oplus E^- \oplus T_1^- \oplus T_2^-$

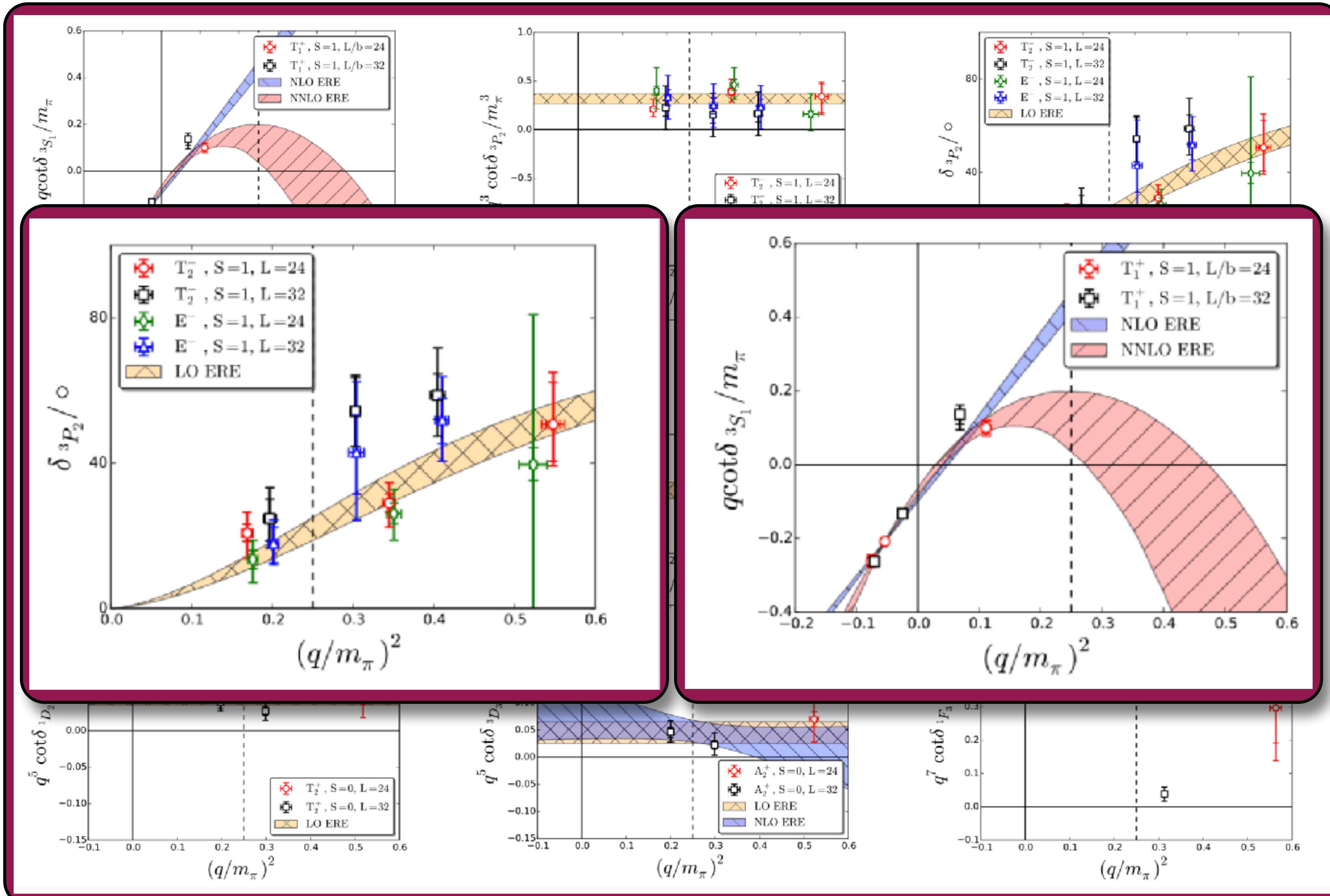
Signals

- m_π ~ 800 MeV
- a ~ 0.145 fm
- L ~ 2.5, 3.5 fm
- ~ 1M sources
- W&M/JLab configs



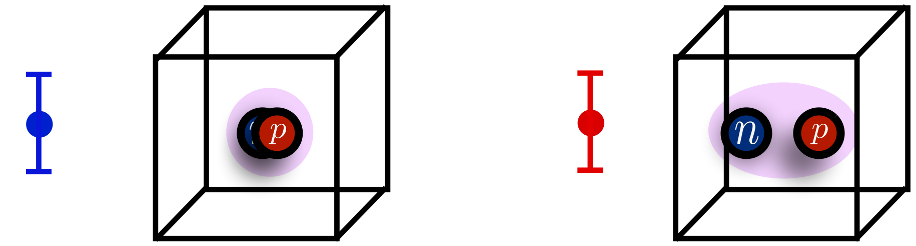


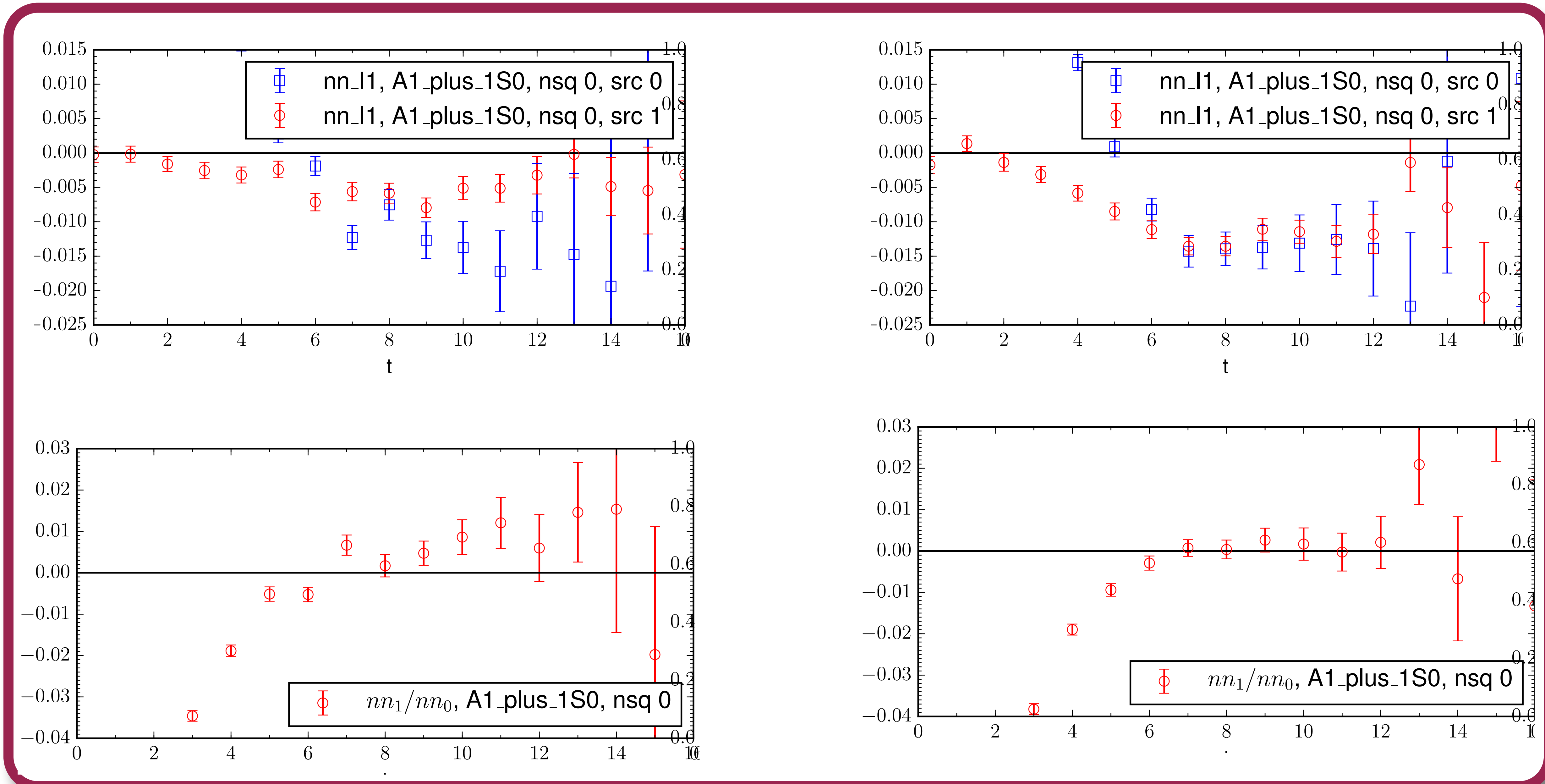
NN scattering at $m_\pi \sim 800$ MeV



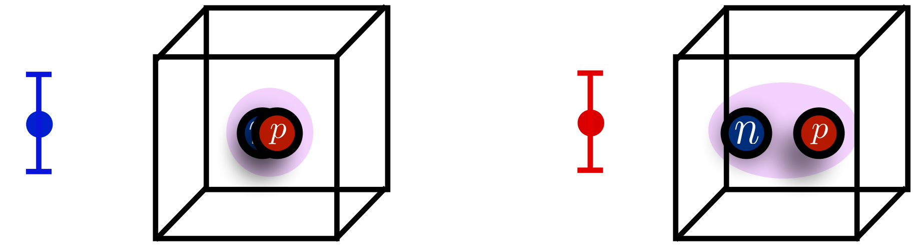
Callat (2015)

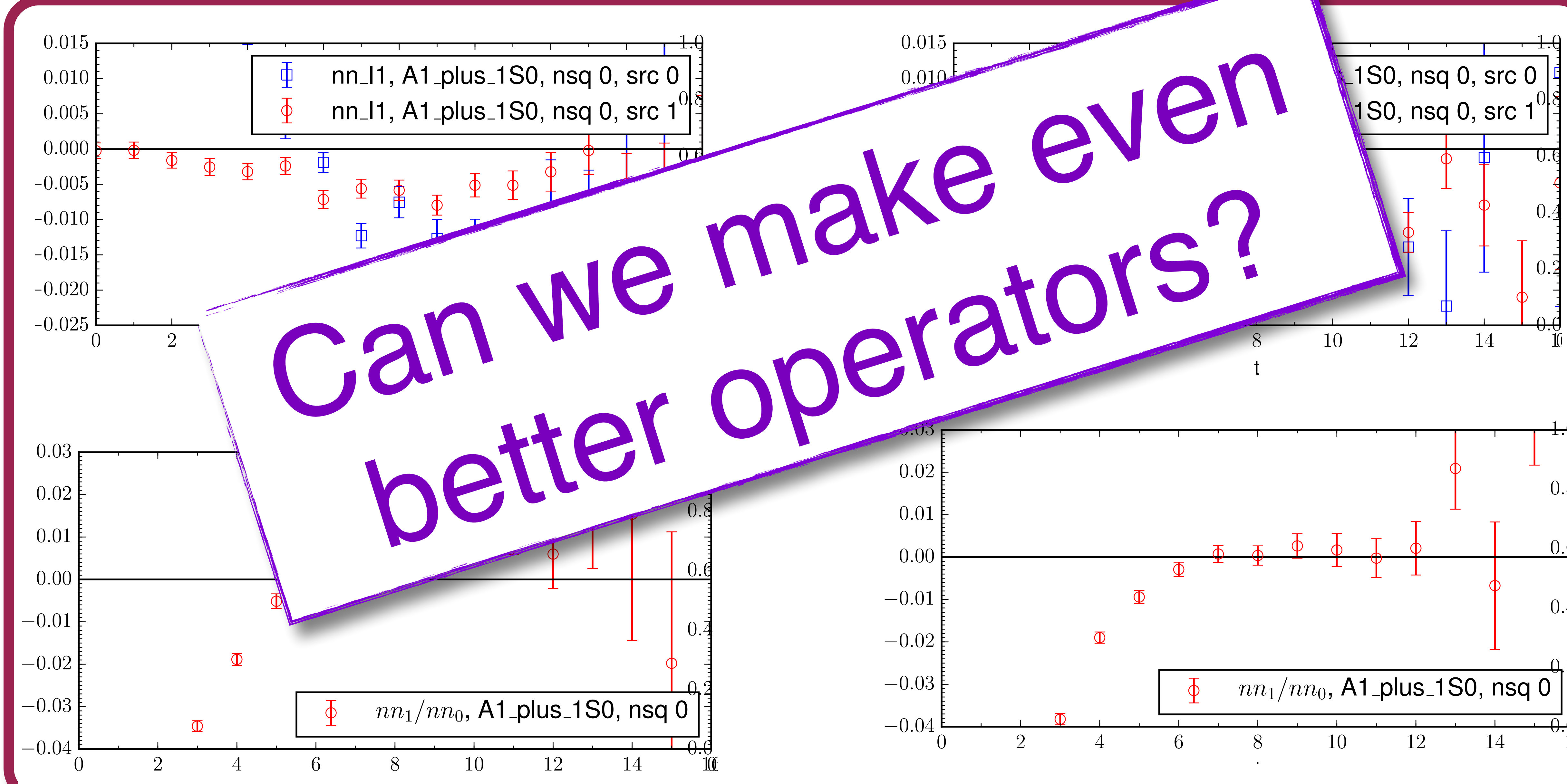
Local vs. displaced





HOBET* ?

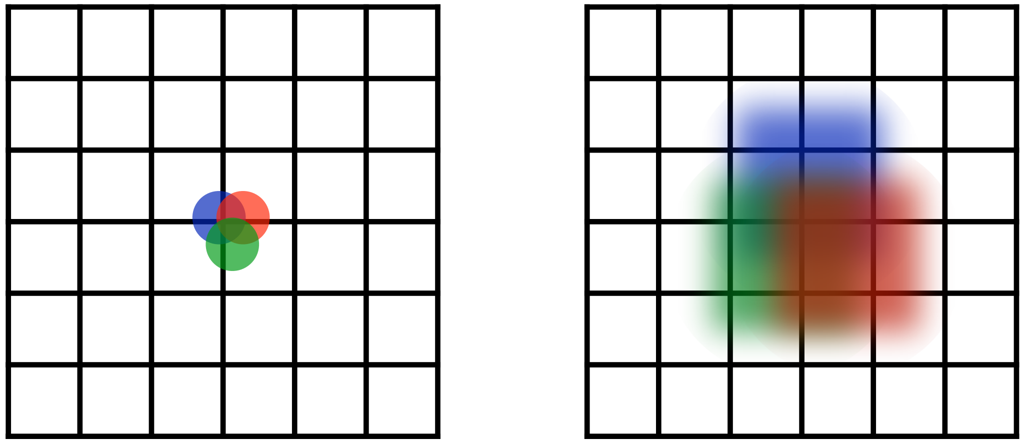


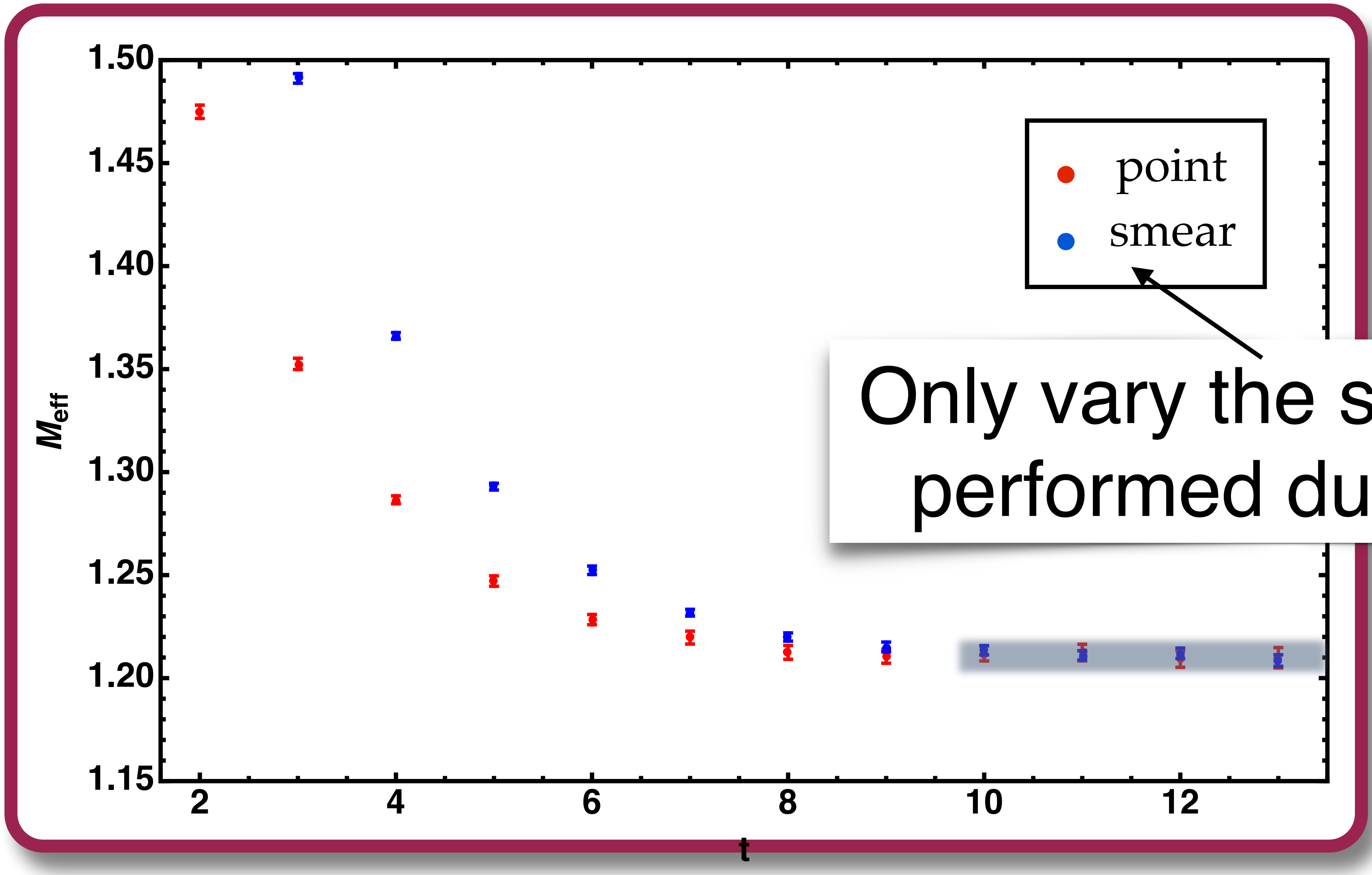


Can we make even better operators?

*not an official logo

Matrix Prony: poor man's GEVP





Only vary the sink: easily performed during FFT

Single nucleon correlator

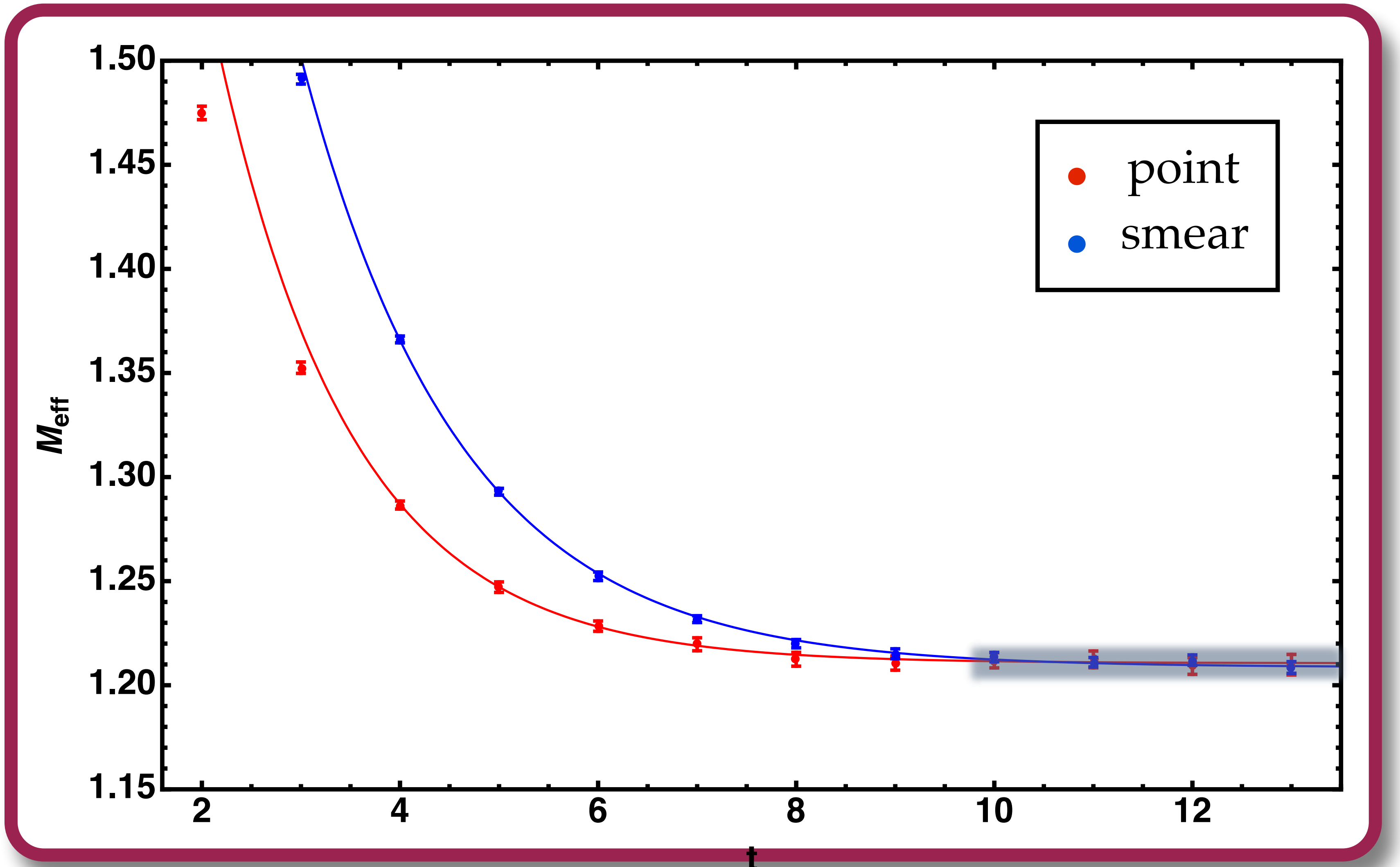
NPLQCD (2009)

Matrix Prony: poor man's GEVP

$$C_0(t+t_0) + \alpha C(t) = 0$$

$$\alpha = -e^{-E_0 t_0}$$

$$E_0 = -\frac{1}{t_0} \ln \frac{C(t+t_0)}{C(t)}$$



Single nucleon correlator

NPLQCD (2009)

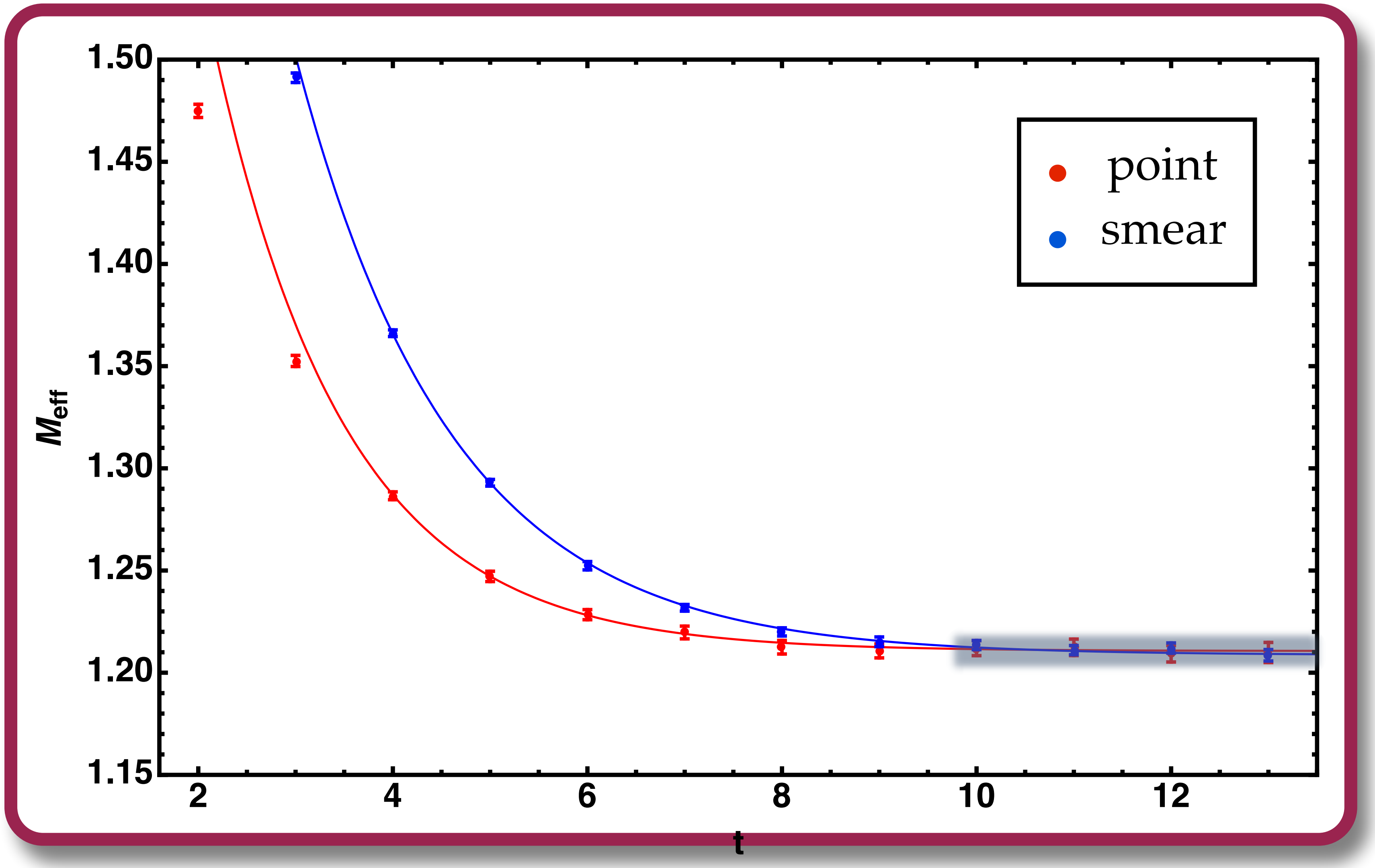
Matrix Prony: poor man's GEVP

$$MC(t+t_0) - VC(t) = 0$$

$$C(t) = \sum_{n=1}^{N} \alpha_n u_n \lambda_n^{-t} \qquad \lambda = e^{E_n}$$

$$Mu = \lambda^{t_0} V u$$

$$M = \left[\sum_{\tau=t}^{t+t_W} C(\tau+t_0)C(\tau)^T\right]^{-1} \qquad V = \left[\sum_{\tau=t}^{t+t_W} C(\tau)C(\tau)^T\right]^{-1}$$



Single nucleon correlator

NPLQCD (2009)

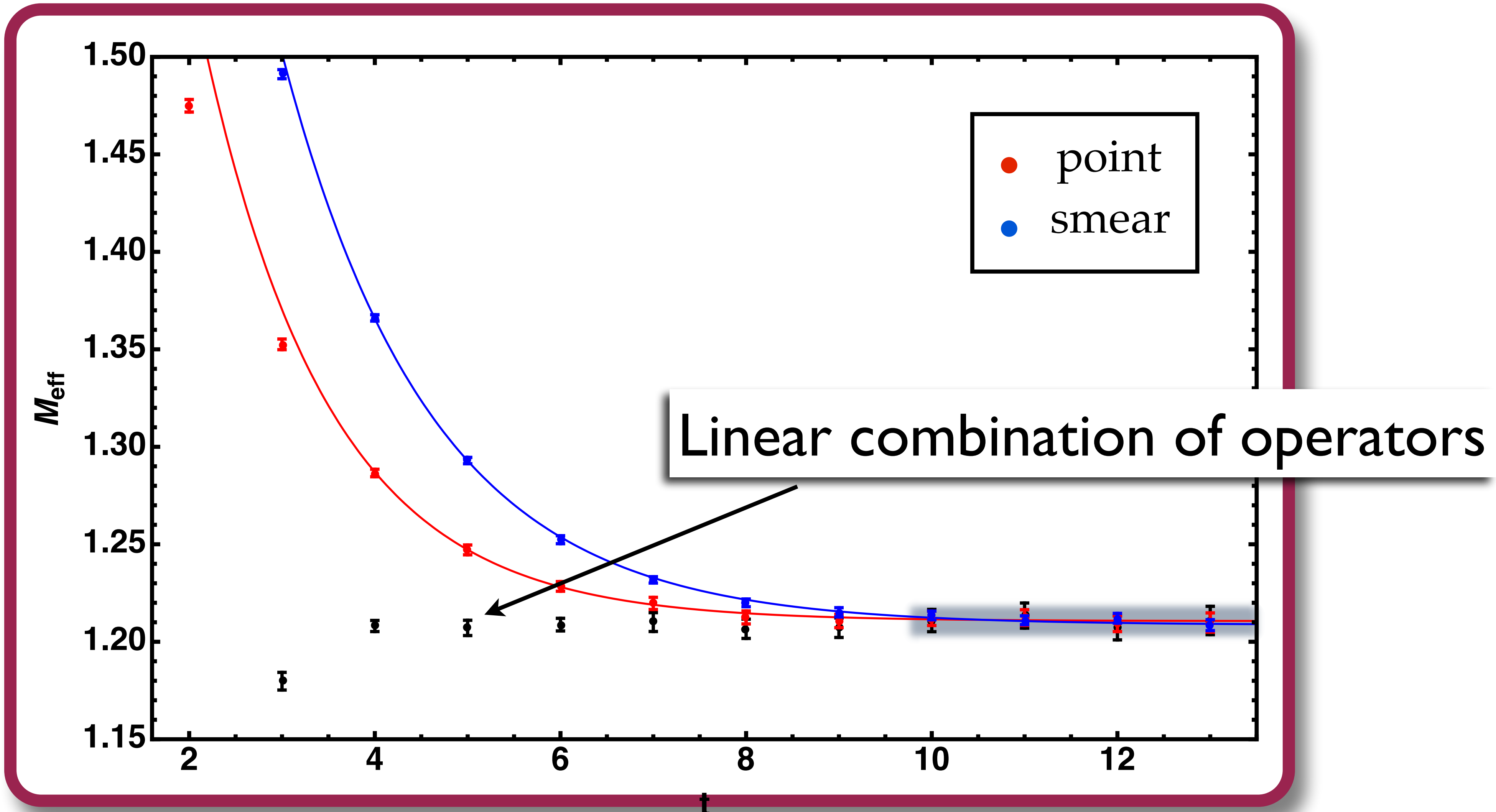
Matrix Prony: poor man's GEVP

$$MC(t+t_0) - VC(t) = 0$$

$$C(t) = \sum_{n=1}^{N} \alpha_n u_n \lambda_n^{-t} \qquad \lambda = e^{E_n}$$

$$Mu = \lambda^{t_0} V u$$

$$M = \left[\sum_{\tau=t}^{t+t_W} C(\tau+t_0)C(\tau)^T\right]^{-1} \qquad V = \left[\sum_{\tau=t}^{t+t_W} C(\tau)C(\tau)^T\right]^{-1}$$



Single nucleon correlator

NPLQCD (2009)

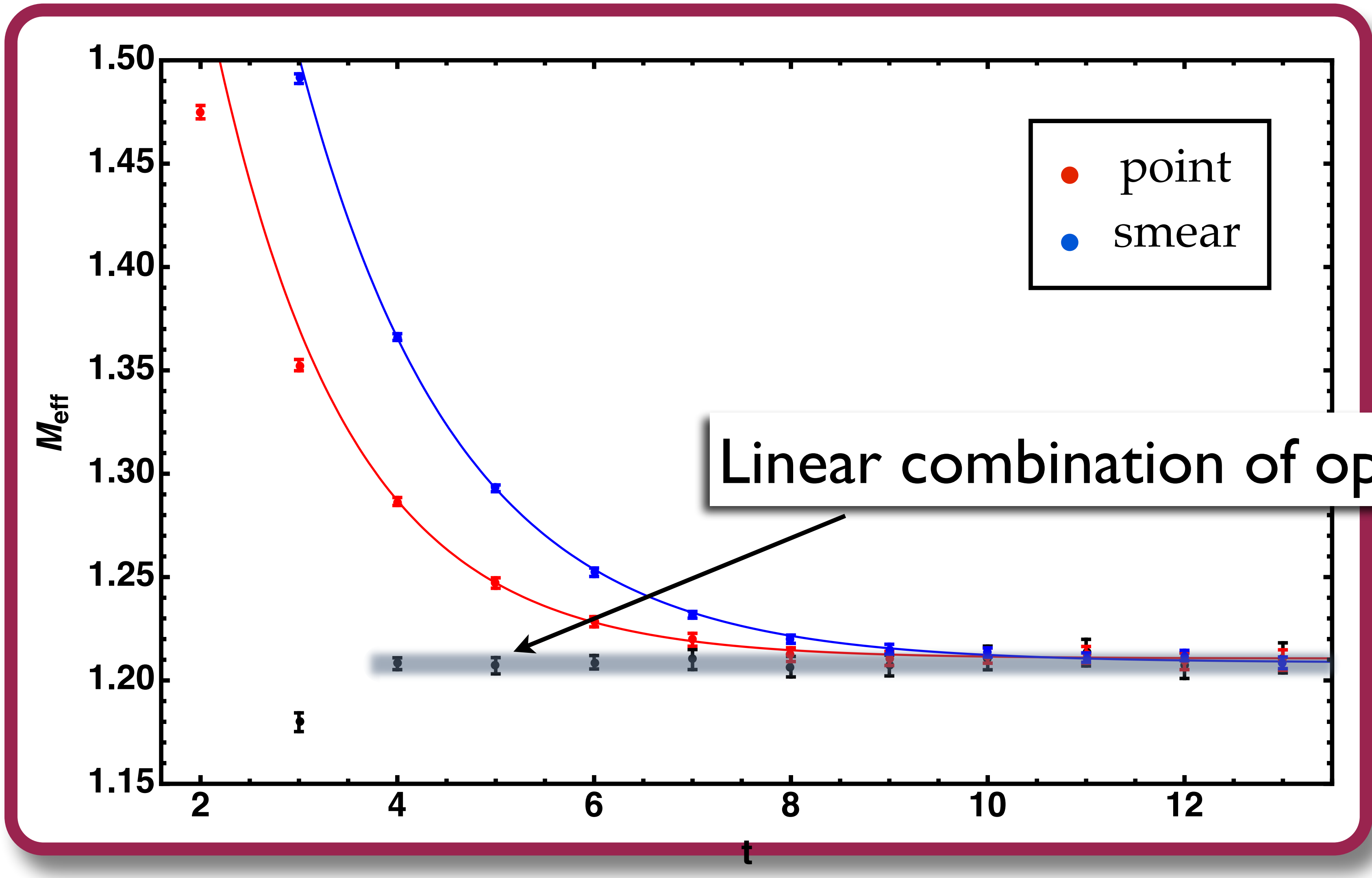
Matrix Prony: poor man's GEVP

$$MC(t+t_0) - VC(t) = 0$$

$$C(t) = \sum_{n=1}^{N} \alpha_n u_n \lambda_n^{-t} \qquad \lambda = e^{E_n}$$

$$Mu = \lambda^{t_0} V u$$

$$M = \left[\sum_{\tau=t}^{t+t_W} C(\tau+t_0) C(\tau)^T\right]^{-1} \quad V = \left[\sum_{\tau=t}^{t+t_W} C(\tau) C(\tau)^T\right]^{-1}$$

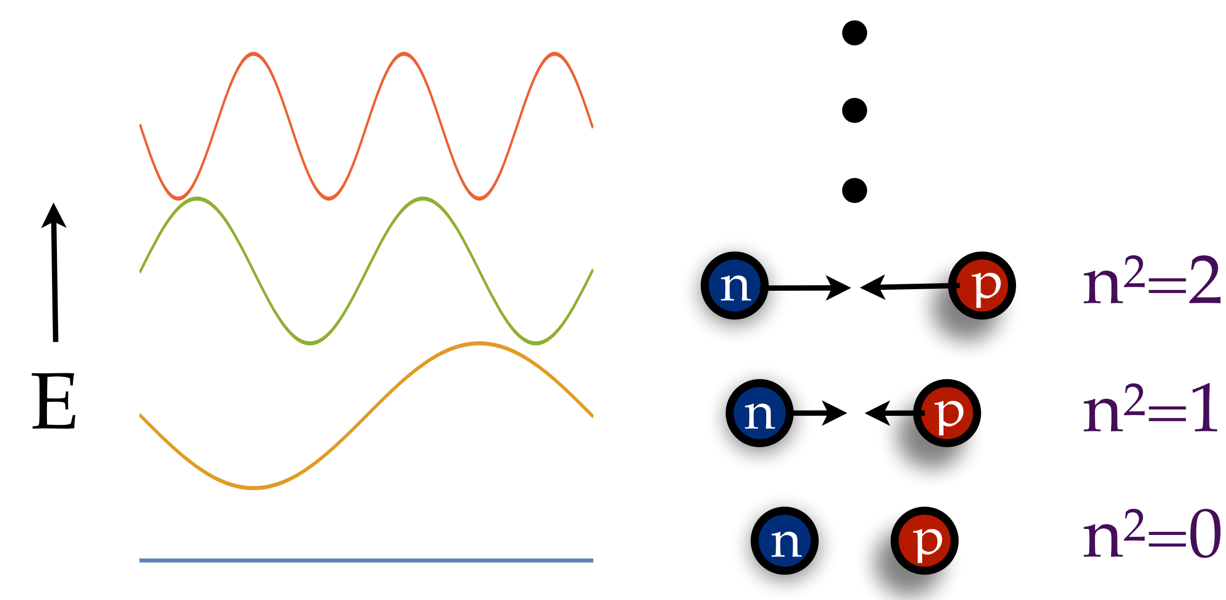


Linear combination of operators

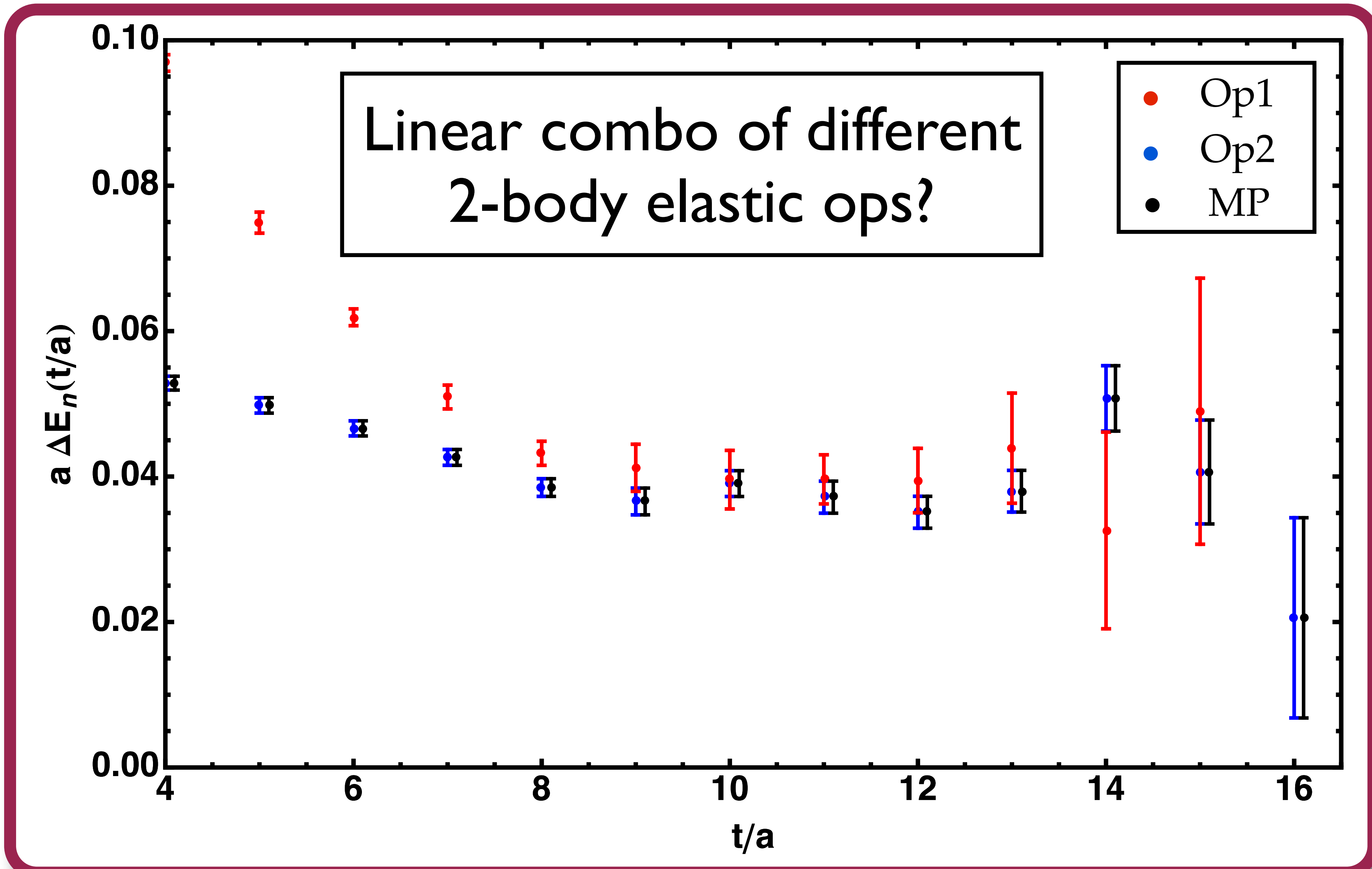
Single nucleon correlator

NPLQCD (2009)

MP for NN

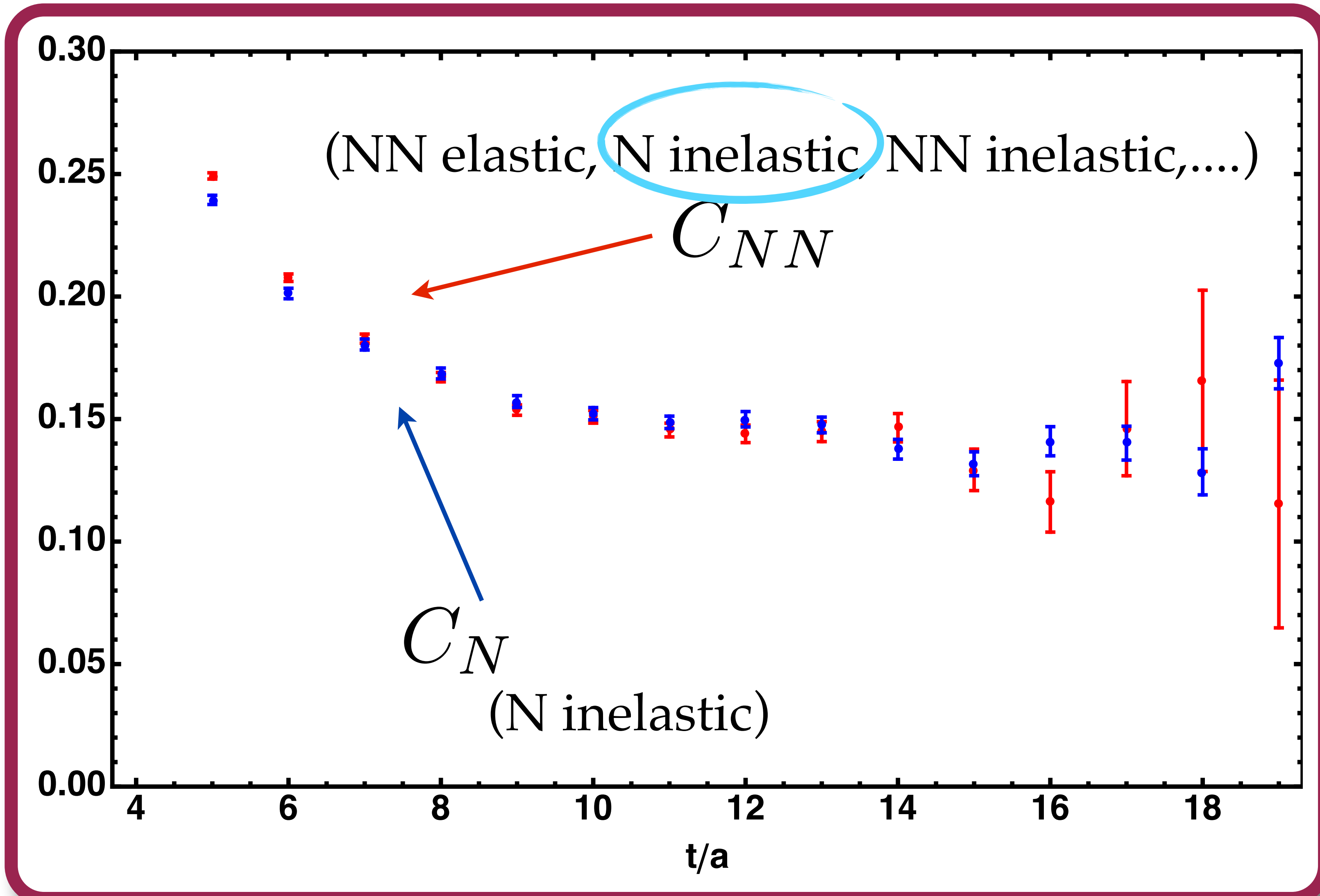


E
n
p
n²=2
n²=1
n²=0



Linear combo of different 2-body elastic ops?
Op1
Op2
MP
a ΔE_n(t/a)
t/a
0.00
0.02
0.04
0.06
0.08
0.10
4
6
8
10
12
14
16

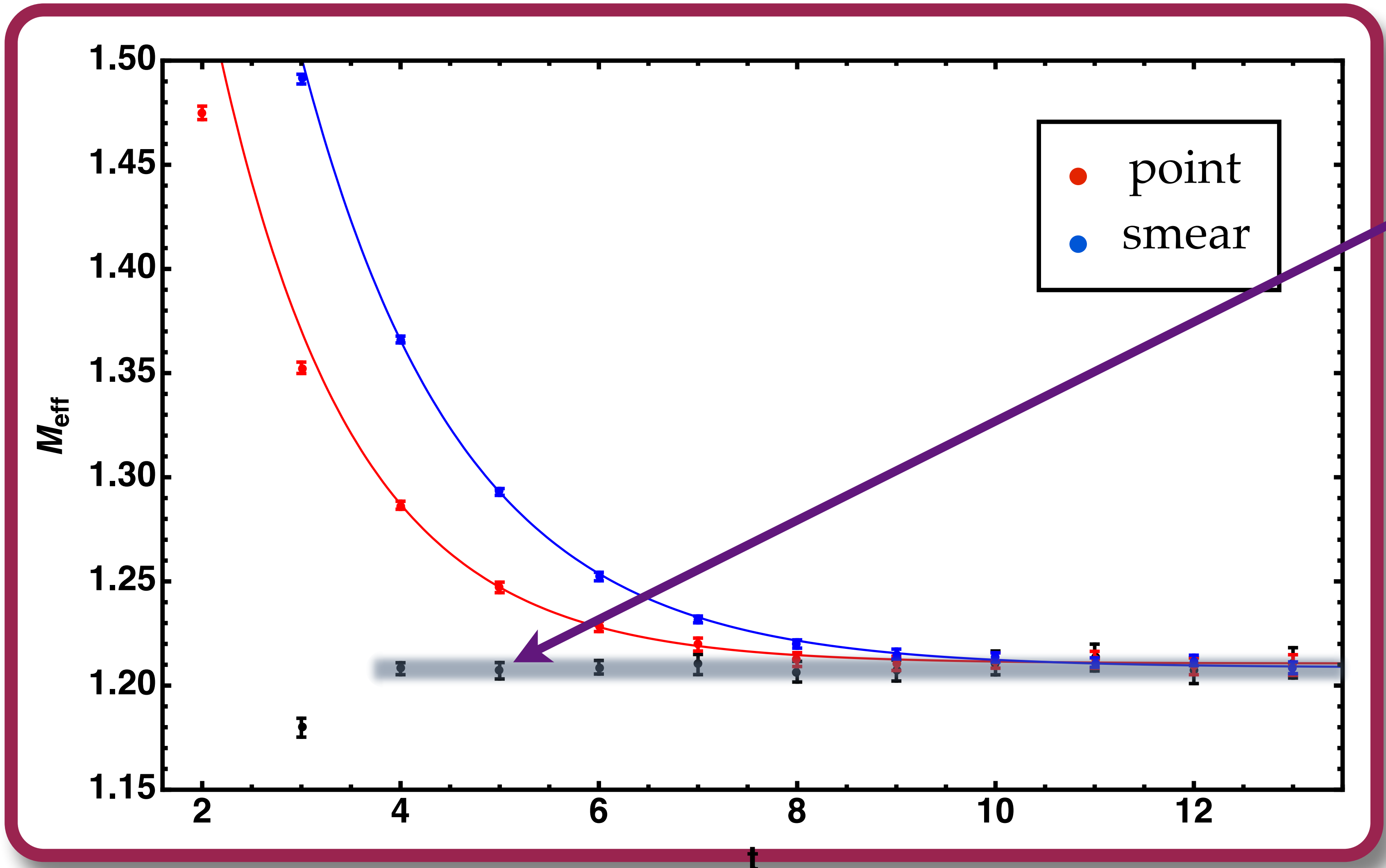
MP for NN



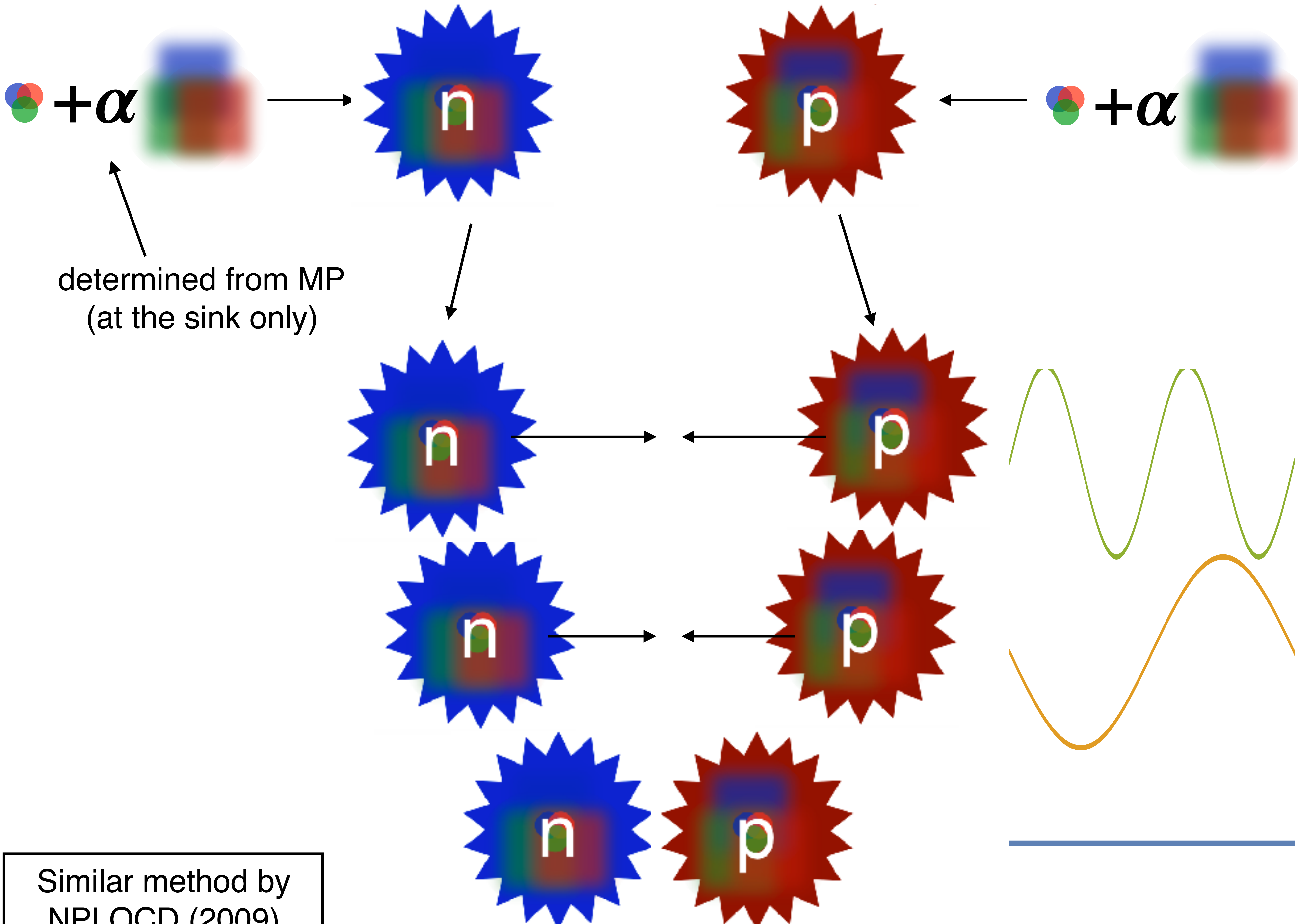
Long time behavior of NN correlator dominated by inelastic single nucleon excited state

→ Need to improve single nucleon interpolating operator for earlier plateaus

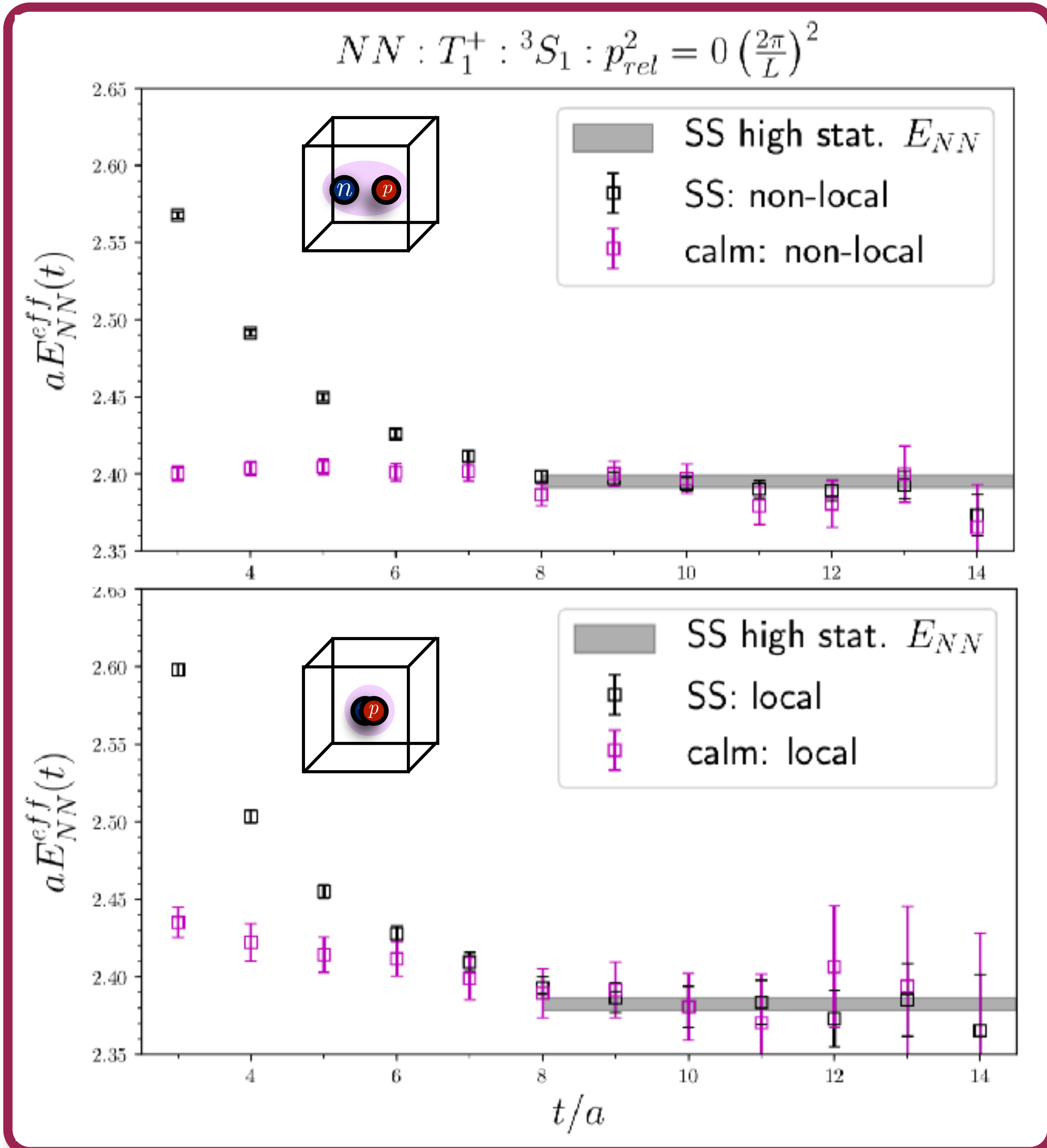
MP for NN



Single nucleon interpolating field: we should be using this sink operator!

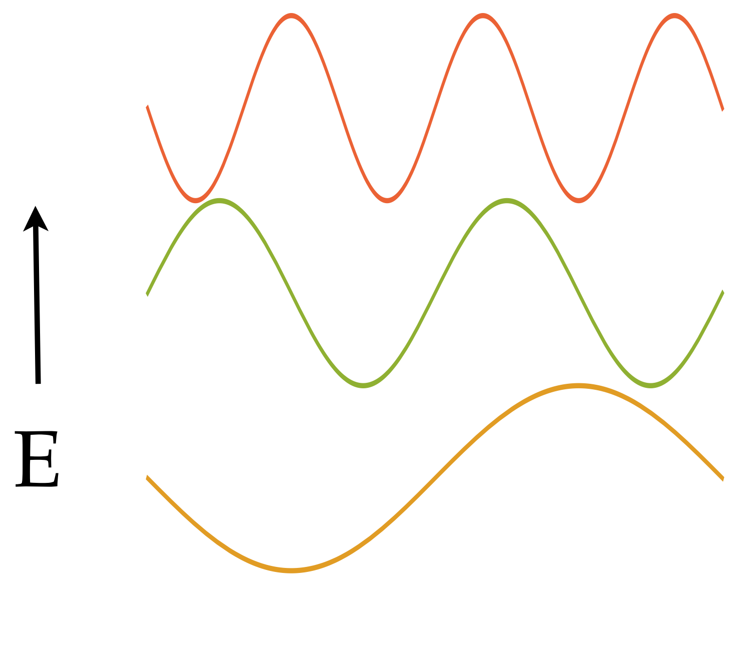


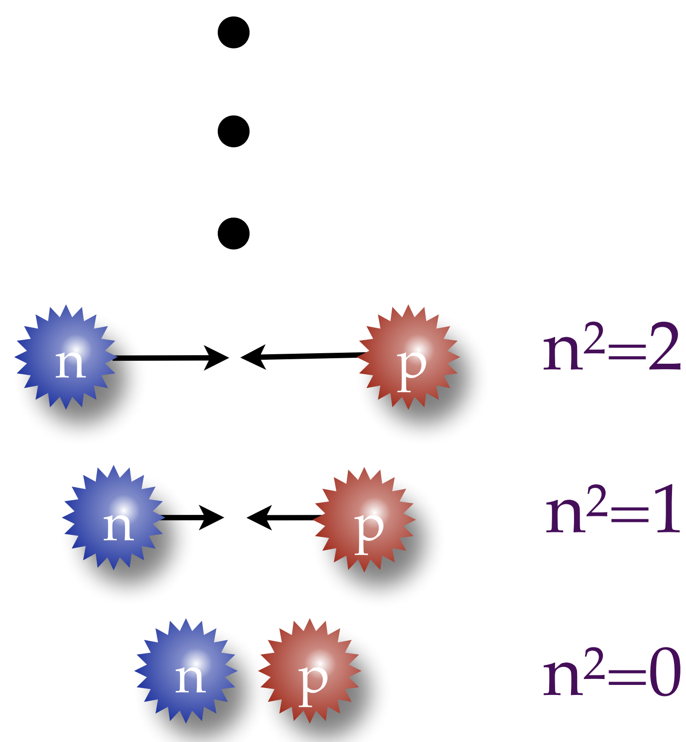
Similar method by
NPLQCD (2009)

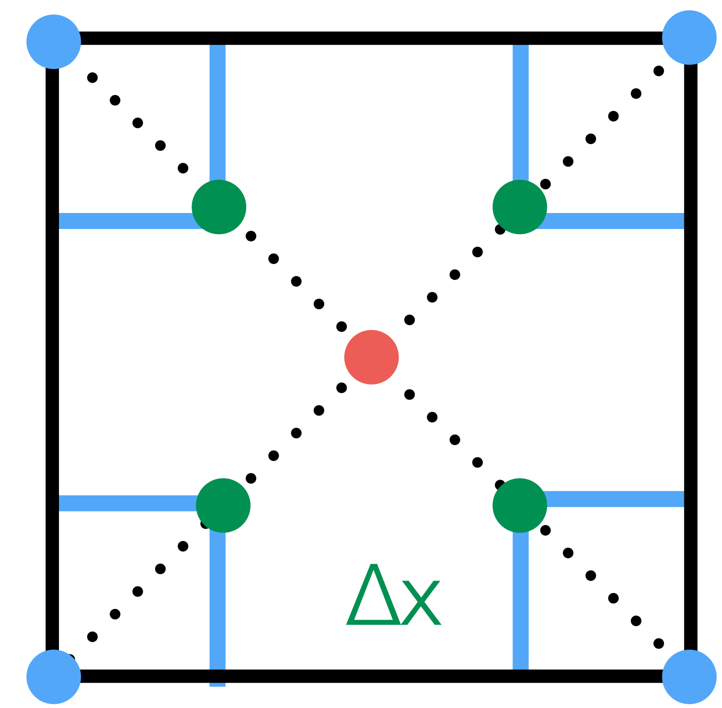

$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$
SS high stat. E_{NN}
SS: non-local
calm: non-local
SS high stat. E_{NN}
SS: local
calm: local
$aE_{NN}^{eff}(t)$
t/a
n
p

CalLat (2017)

MP method for NN

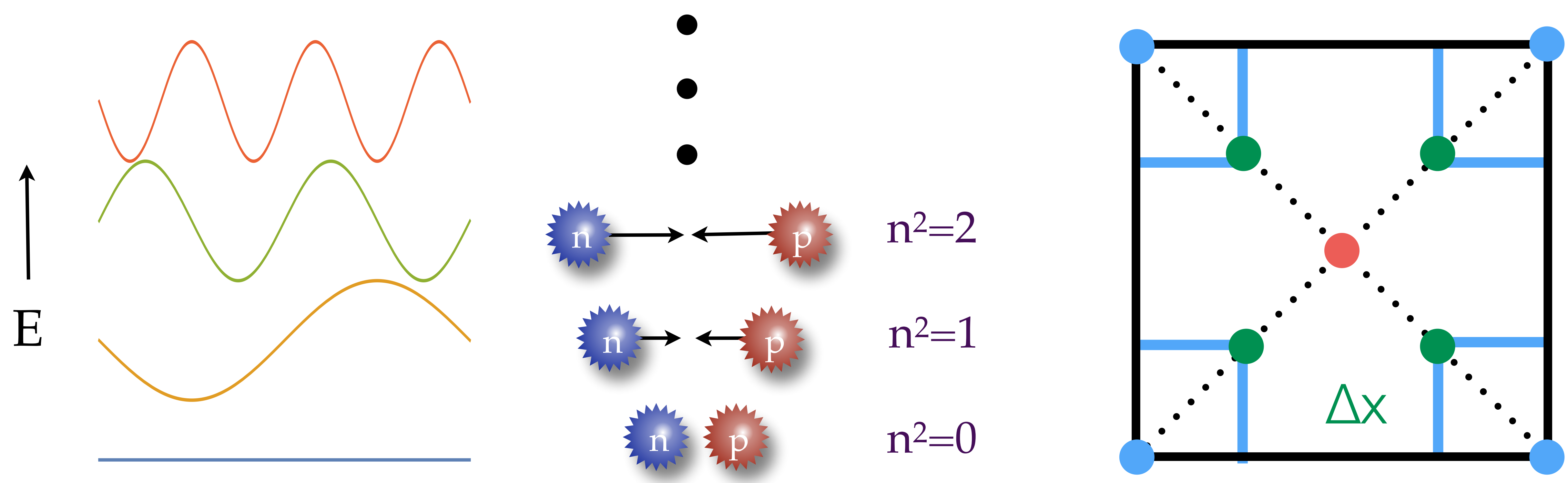






- Works best when you've eliminated leading elastic states
- Prony often doesn't work well for more than 2 ops:
  - should be able to do two stages of Prony to further reduce elastic excited states
  - or, do simultaneous fits of different elastic ops

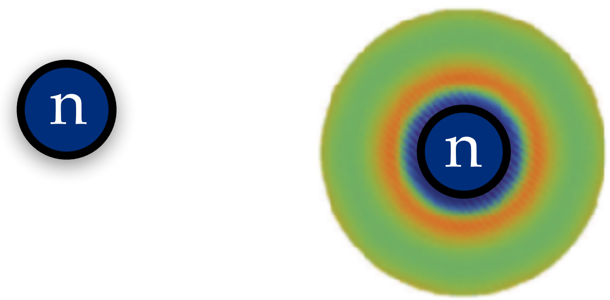
MP method for NN



- Contributions from inelastic excited states are important
- Forming a ratio with the single N correlator can lead to delicate cancellations between numerator and denominator
  - Improved single N operator lets us more confidently use the ratio

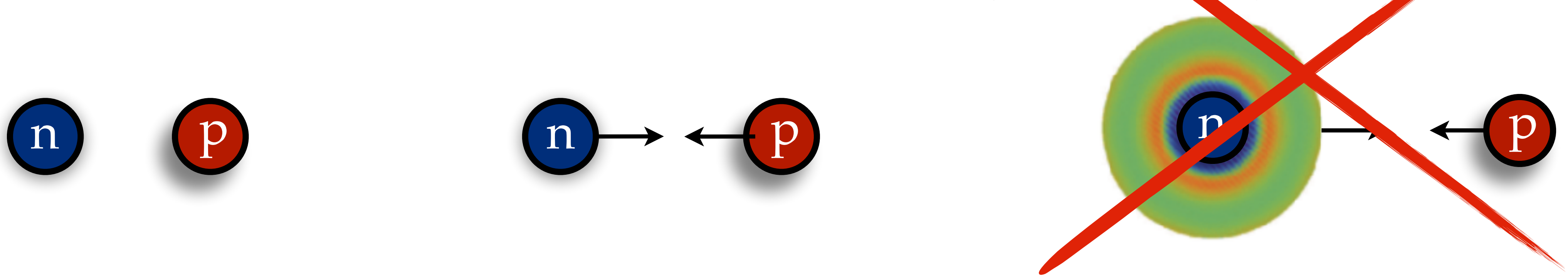
MP method for NN

- Single N: $C_N(t) = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \cdots$



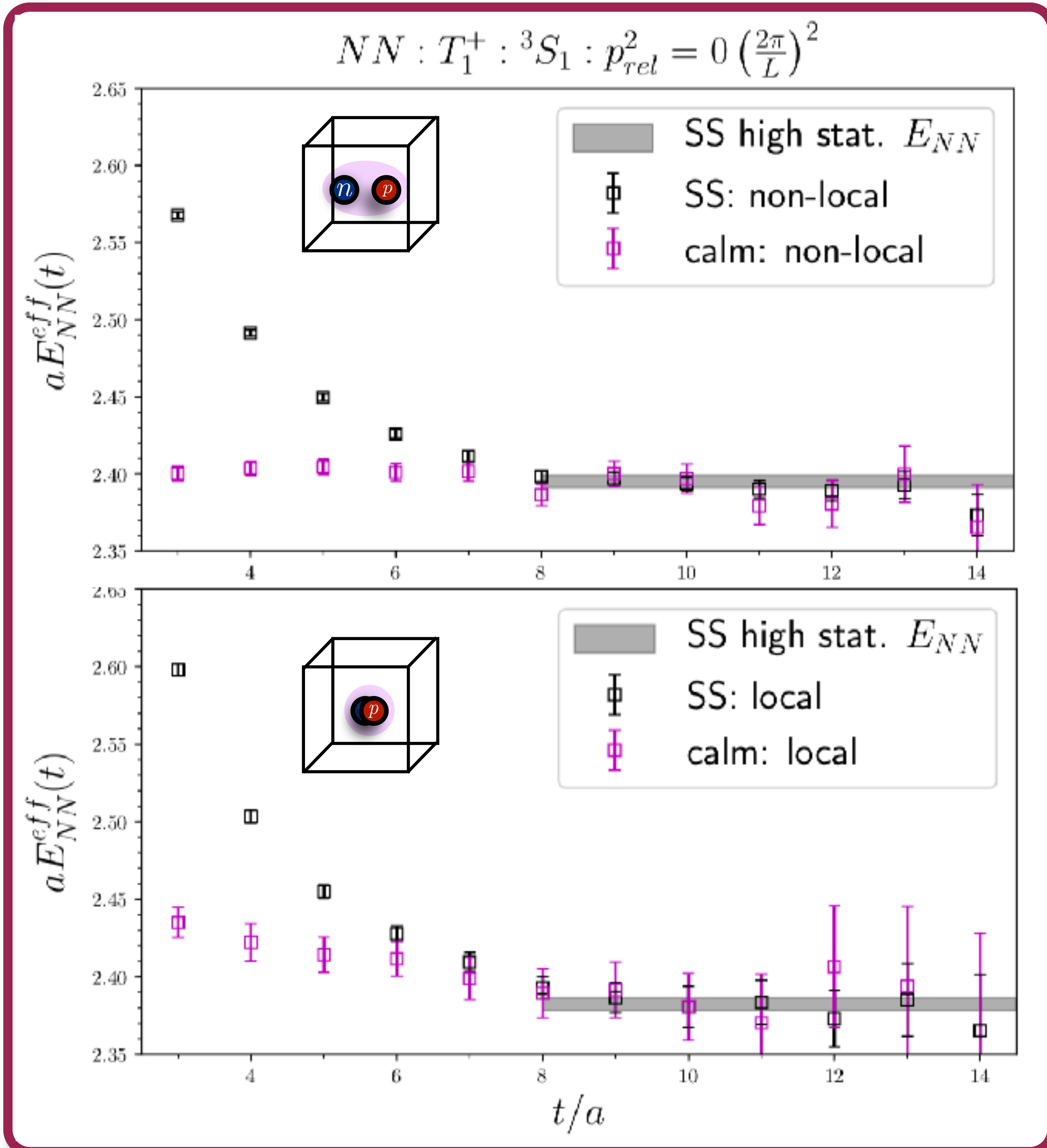
- NN:

$$C_{NN}(t) = \left(A_0^2 + \delta_0\right) e^{-(2E_0+\epsilon_0)t} + \left(A_0^2 + \delta_1\right) e^{-(2E_0+\epsilon_1)t} + \left(2A_0 A_1 + \delta_1^{'}\right) e^{-(E_0+E_1+\epsilon_1^{'})t} + \cdots$$



- "Fake plateaus" resulting from cancellation in non-positive correlation functions require at least three terms of roughly the same order over a given time range
  - Eliminating contamination from single nucleon excitations reduces this possibility
- Can we use the ratio?

$$\log\left[\frac{C_{NN}(t)}{C_N^2(t)}\right] \sim \log\left[\sum_n \left(1 + \frac{\delta_n}{A_0^2}\right) e^{-(\epsilon_n)t}\right] + \frac{2A_1}{A_0} e^{-\Delta_{01} t} - B\left(\frac{2A_1}{A_0} + \delta_1^{''}\right) e^{-(\Delta_{01}+\epsilon_1^{'})t}$$

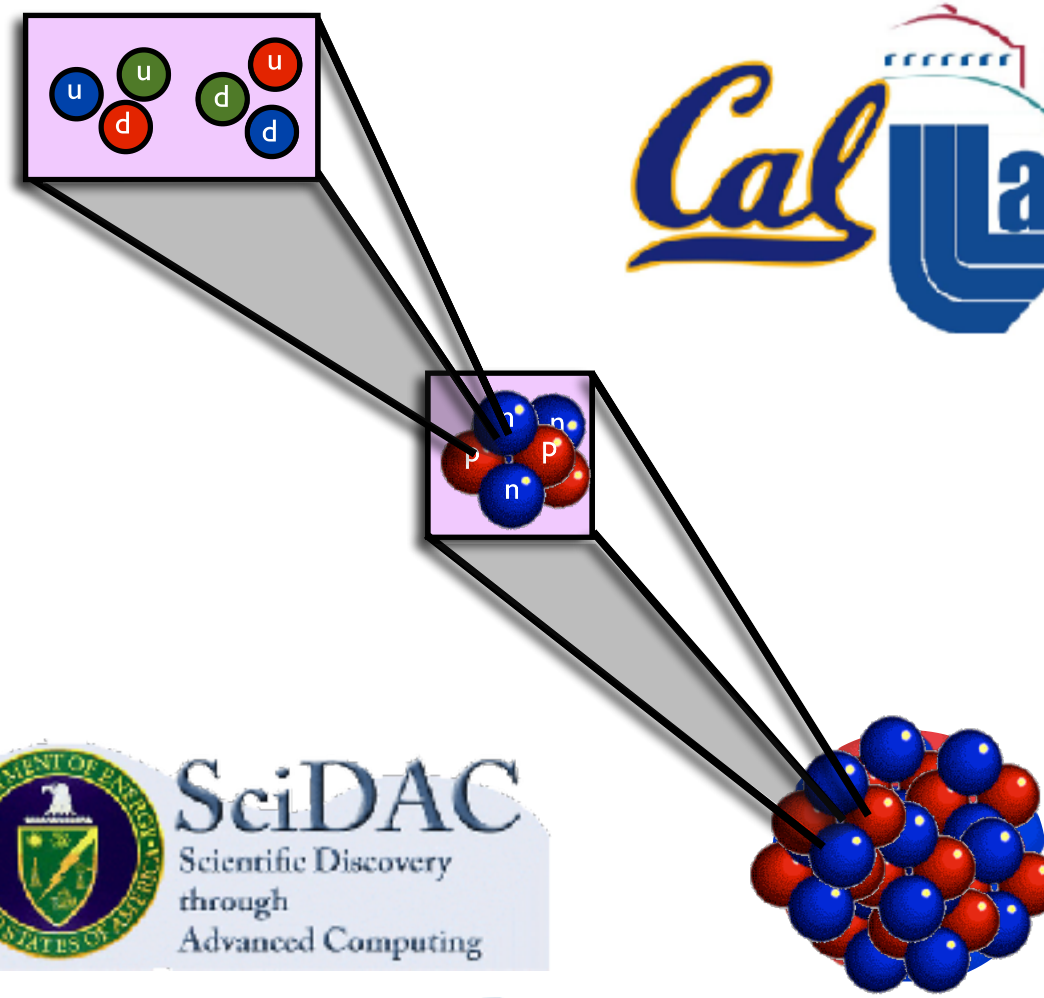


CallLat (2017)

Summary

- Lüscher method requires excellent resolution of energy levels ⟷ excellent operators
  - Displaced operators help reduce elastic excited state contamination
  - Need to improve single nucleon!
- Still need to sort out composite states at $m_\pi \sim 800$ MeV
- Variational?

Jülich	Evan Berkowitz
LBL/UCB	Chia Cheng Chang
	Thorsten Kurth
	Mark Strother
	André Walker-Loud
NVIDIA	Kate Clark
UNC	Amy Nicholson
JLab	Balint Jóo
RIKEN/BNL	Enrico Rinaldi
LLNL	Arjun Gambhir
	Pavlos Vranas



CalLat

SciDAC
Scientific Discovery through Advanced Computing

NERSC

U.S. DEPARTMENT OF ENERGY
INCITE
LEADERSHIP COMPUTING

OLCF
OAK RIDGE LEADERSHIP COMPUTING FACILITY

red = postdoc
blue = grad student