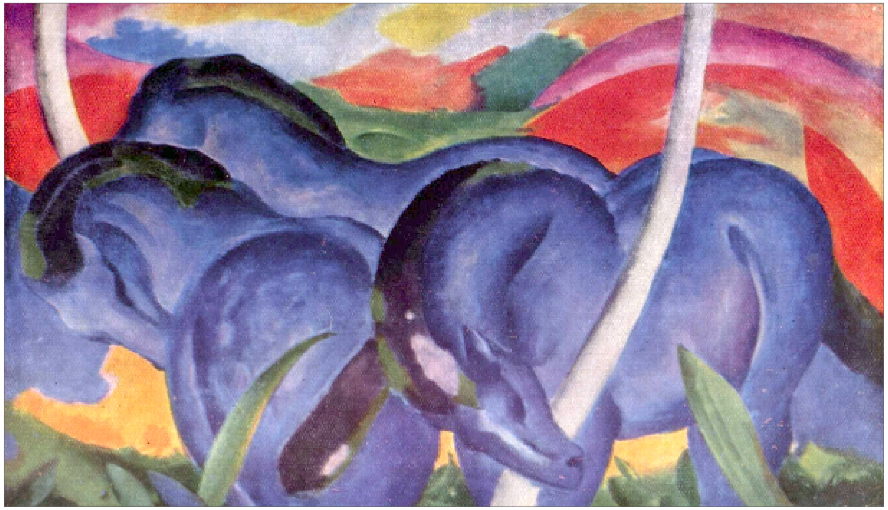
3→3 SCATTERING AMPLITUDE IN ISOBAR FORMULATION



Maxim Mai
The George Washington University

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Deutsche Forschungsgemeinschaft
DFG

INTRODUCTION

INTRODUCTION

- Non-perturbative dynamics of QCD → **rich spectrum of excited states**

Q1: how many are there?

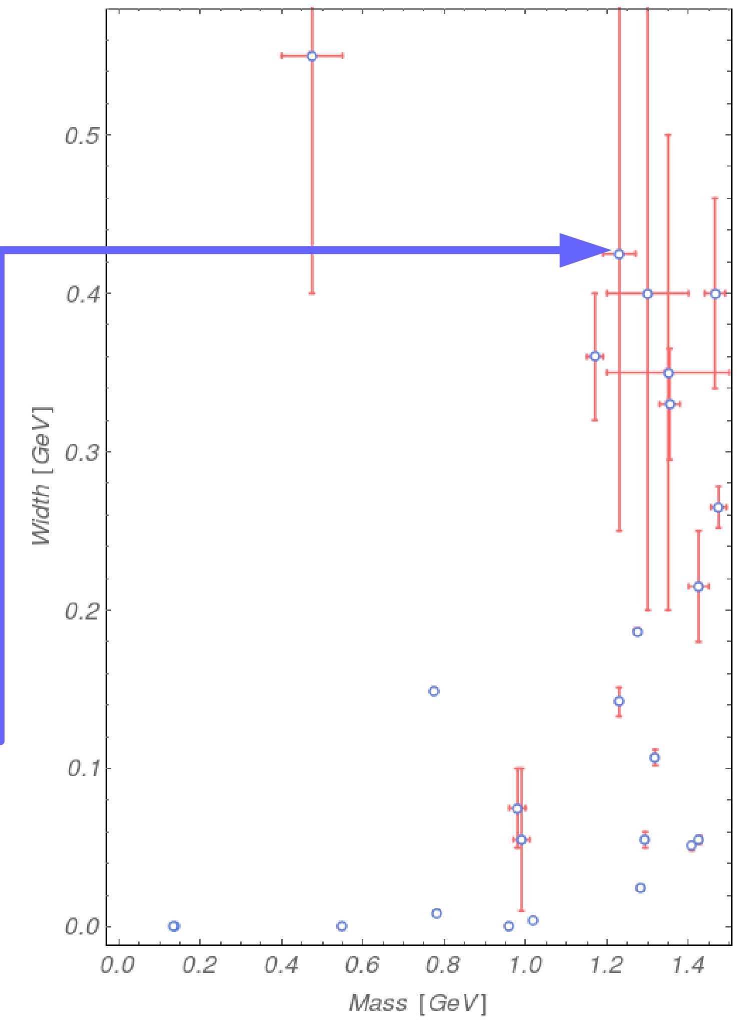
missing resonances

Q2: what are they?

quark-antiquark, gluons, meson-baryon dynamics

- **Many states couple strongly to 3-body channels**

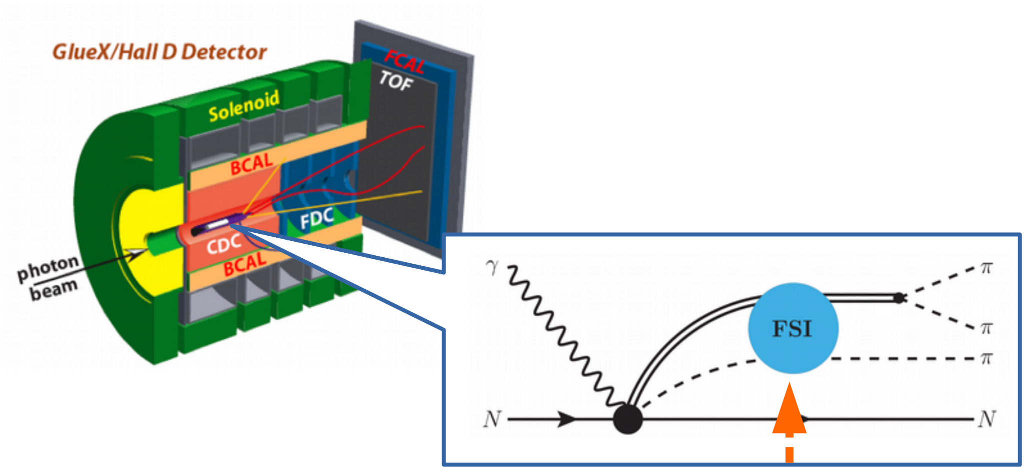
→ e.g. N*(1440), a1(1260)



Experiment

Search for QCD exotics GlueX @ Jlab

* a1(1260) in FSI



Further applications:

* Roper puzzle (ππN)

* X(3872)

→ talk by A. Pilloni

Lattice QCD

Ab-initio numerical calculations

– Euclidean ST & finite lattice spacing

– finite volume: discrete spectrum $\{E^*\}$

* 2-body quantization condition

$$p(E^*)cot\delta(E^*) = Z_{00}(E^*)$$

[Lüscher (1986)]

* 3-body QC not **yet** established

see also talks by

M. Hansen & A. Rusetsky

THIS TALK: 3-body scattering amplitude in isobar-formulation

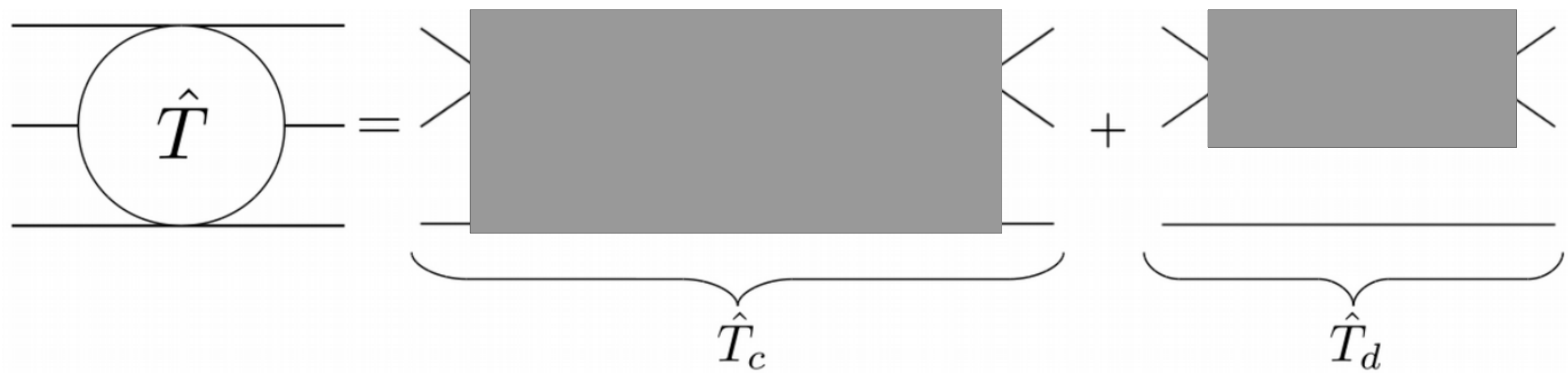
$3 \to 3$ SCATTERING AMPLITUDE:

INFINITE VOLUME

[MM, Hu, Döring, Pilloni, Szczepaniak EPJ A53 (2017)]

T-MATRIX

- 3 asymptotic states (scalar particles of equal mass (***m***))
- ***Connectedness structure*** of matrix elements: (all permutations considered)



- **isobar-parametrization of two-body amplitude** **[Bedaque, Griesshammer (1999)]**

  → "isobars"~ $S(M_{inv})$ for definite QN & correct r.h.-singularities

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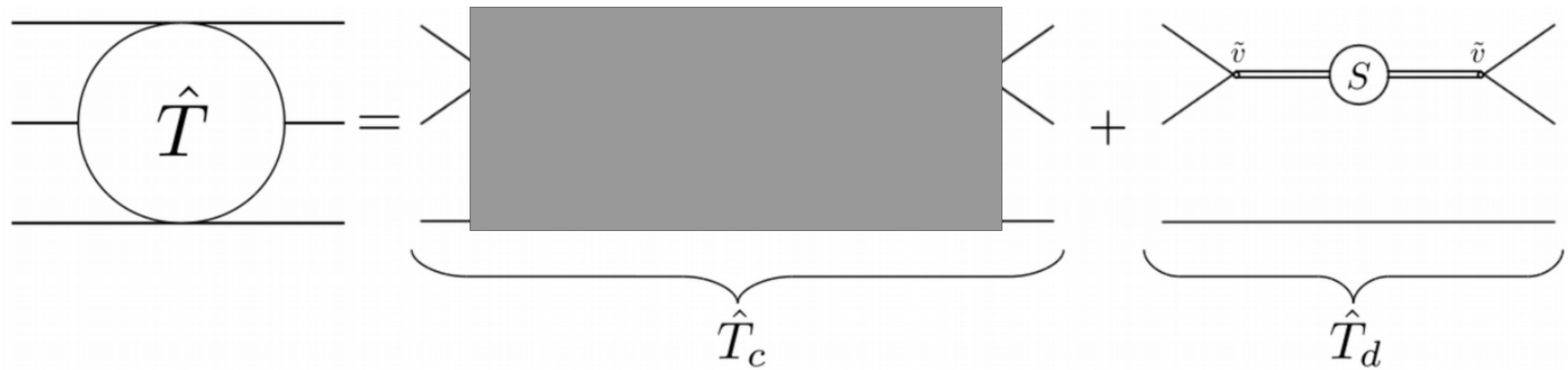

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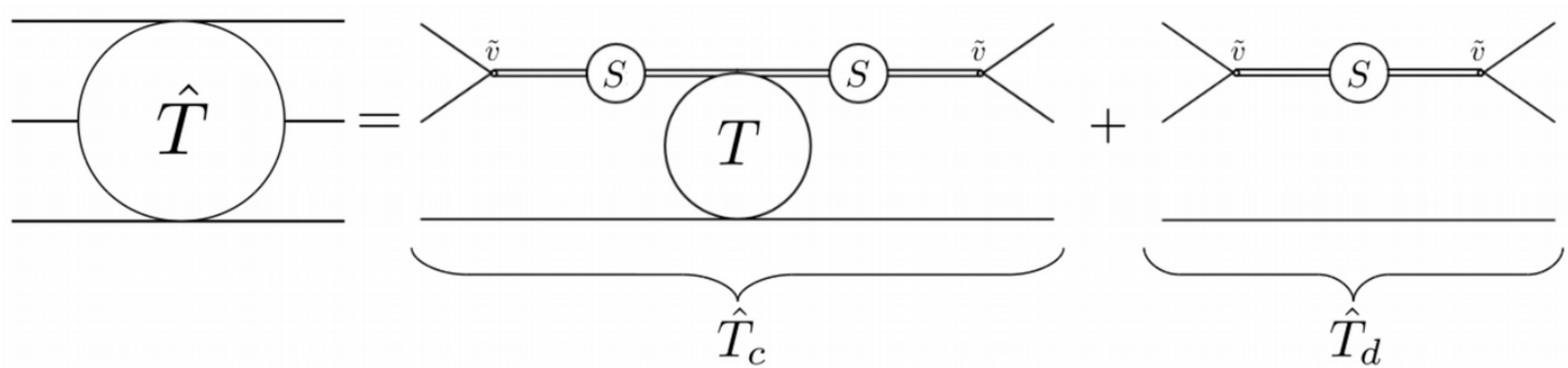


- isobar-parametrization of two-body amplitude by **[Bedaque, Griesshammer (1999)]**

  → "isobars"~ $S(M_{inv})$ for definite QN & correct r.h.-singularities

  → coupling to asymptotic states: cut-free-function $v(q,p)$

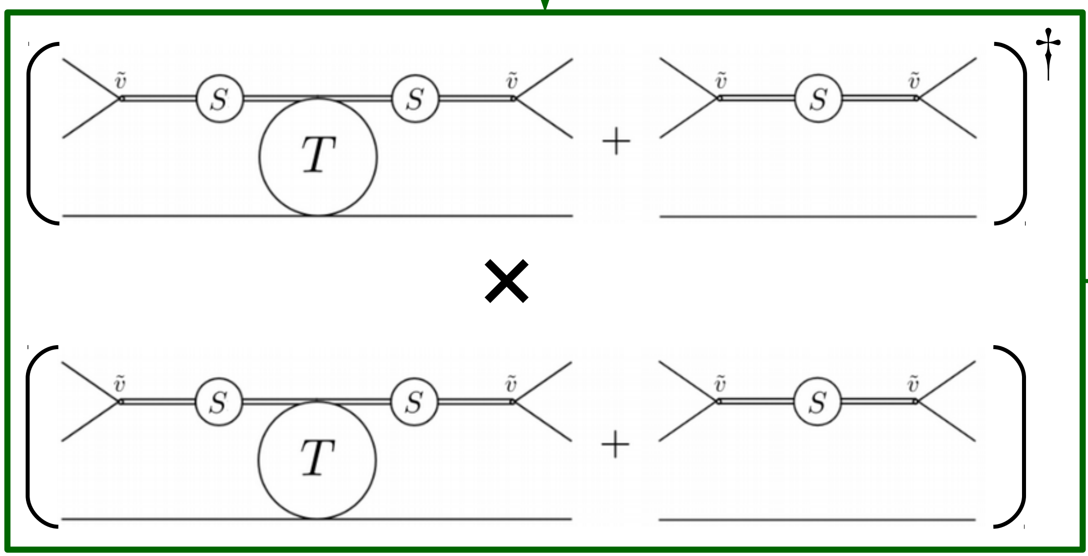
- Connected part: due to isobar-spectator interaction → $T(q_{in}, q_{out}; s)$

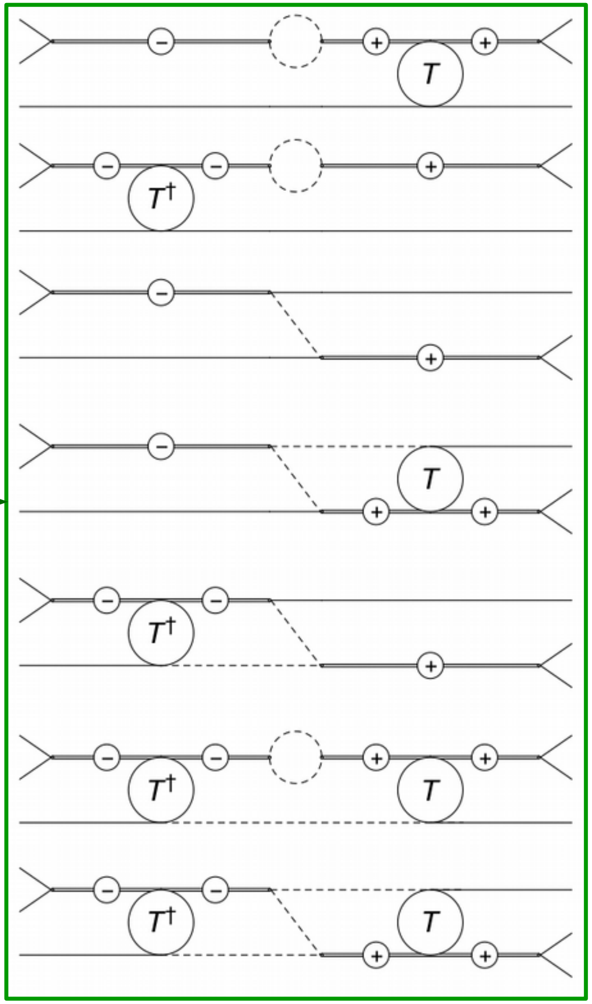
→ 3 unknown functions

→ 8 kinematic variables (talk by A. Jackura)

3-body Unitarity (normalization condition ↔ phase space integral)

$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \quad = \quad i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$





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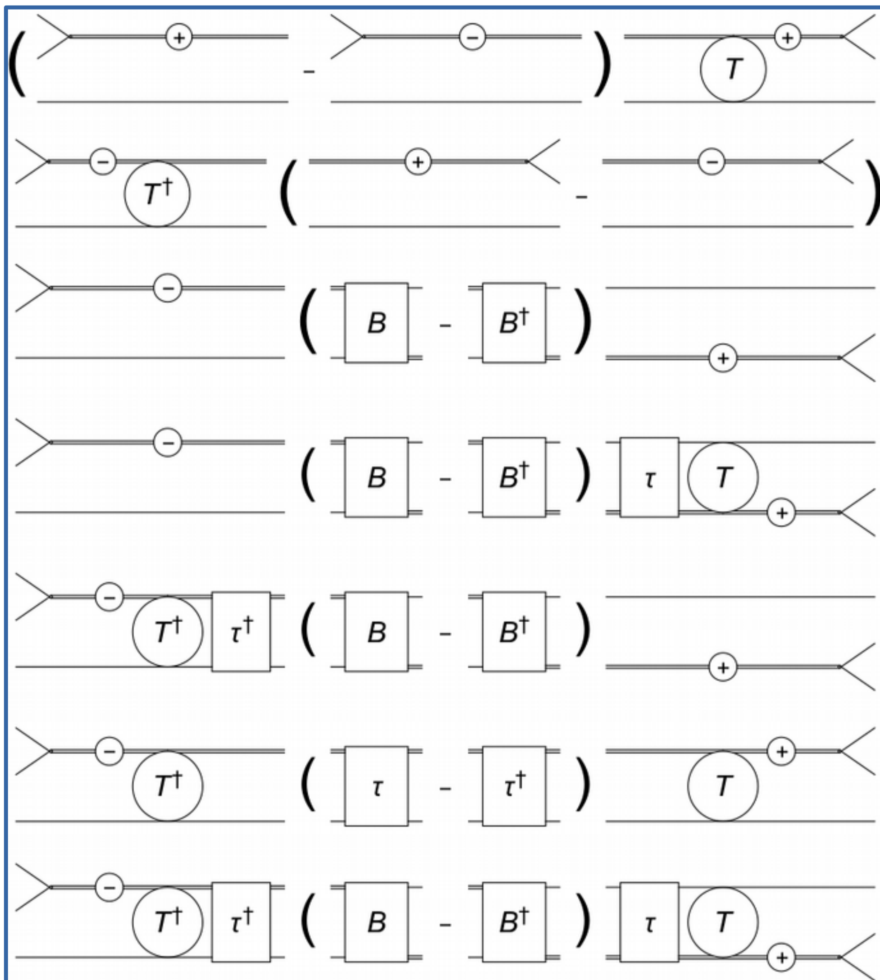
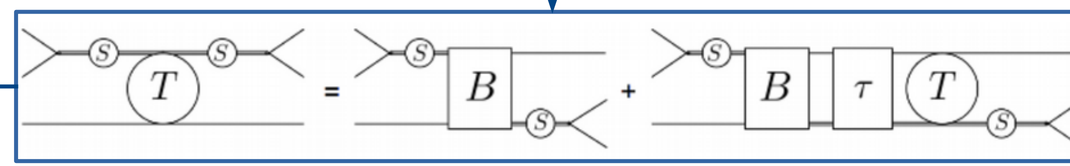


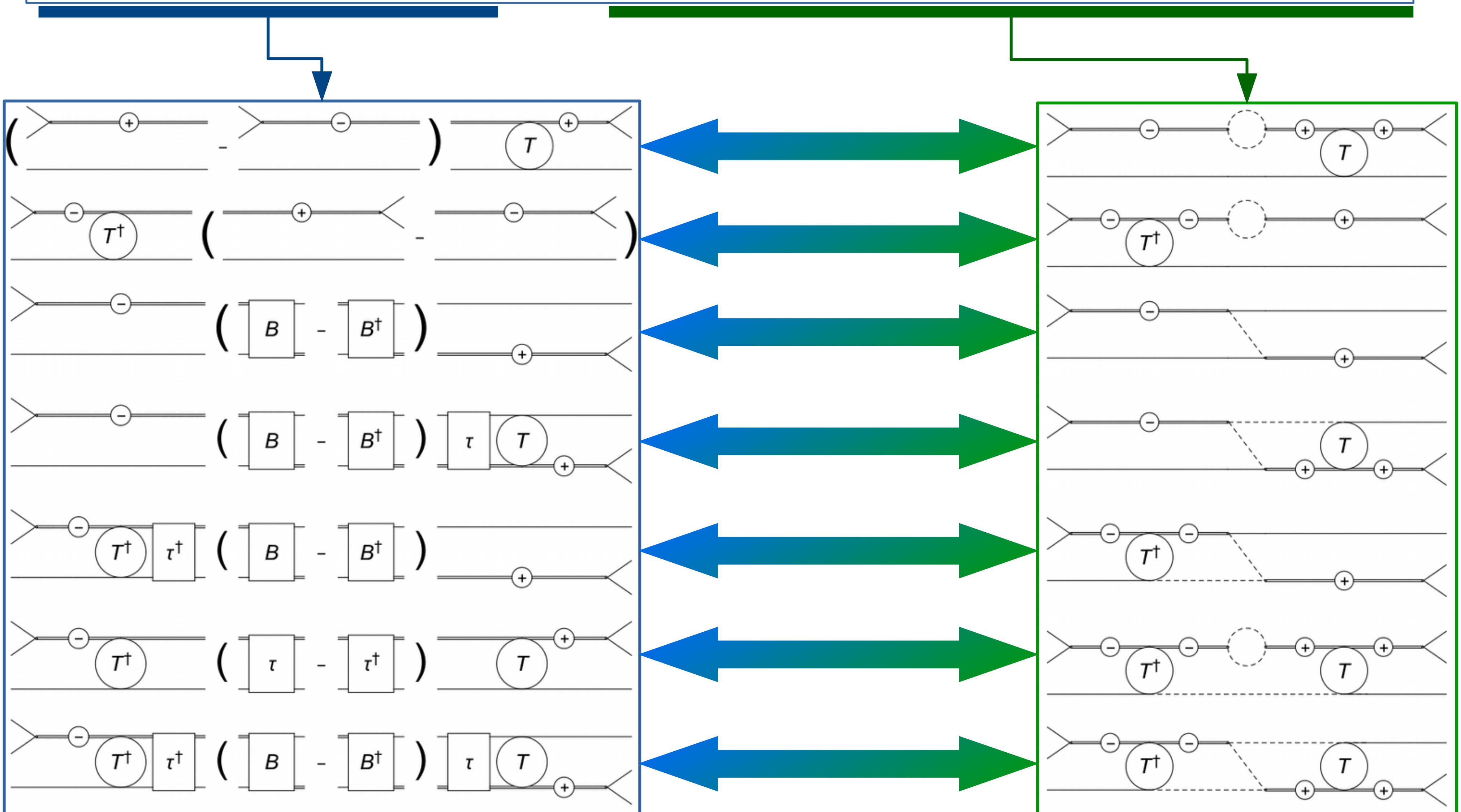


General Ansatz for the isobar-spectator interaction

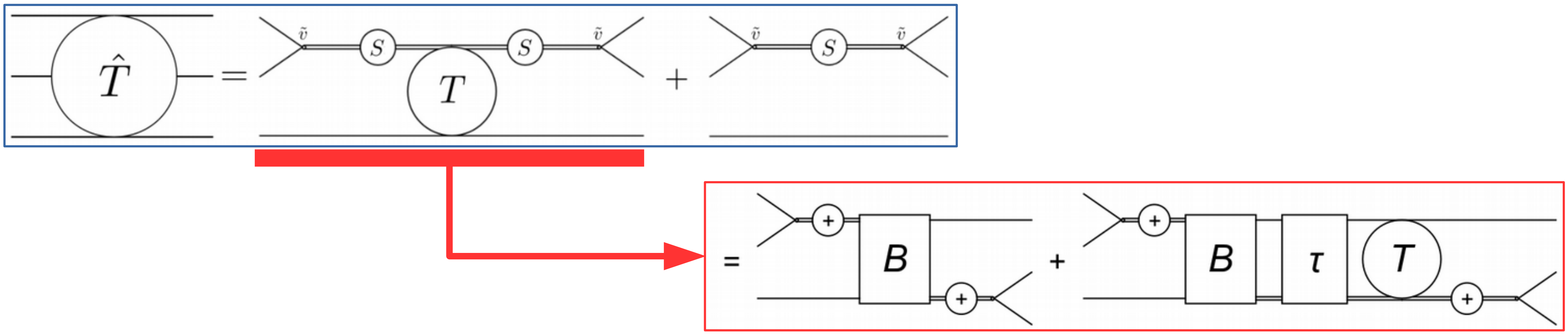
→ B & τ are unknown!!!

3-body Unitarity (normalization condition ↔ phase space integral)

$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \;=\; i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$



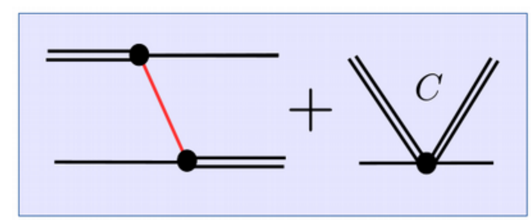
$3 \to 3$ scattering amplitude as a 3-dimensional integral equation



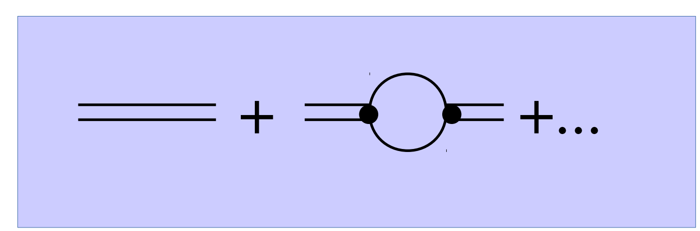
Unitarity/matching

Dispersion relation

$$\text{Disc}\, B(u) = 2\pi i\, \frac{\delta\left(E_Q - \sqrt{m^2 + \mathbf{Q}^2}\right)}{2\sqrt{m^2 + \mathbf{Q}^2}} v^2$$



$$\text{Disc}\, \frac{1}{S(\sigma(k))} = \frac{-i}{64\pi^2 K_{\text{cm}}} \int d^3\bar{\mathbf{K}}\, \frac{\delta\left(|\bar{\mathbf{K}}| - K_{\text{cm}}\right)}{\sqrt{(\bar{\mathbf{K}})^2 + m^2}} v^2$$



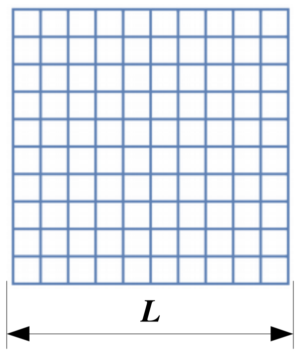
THE ONLY IMAGINARY PARTS REQUIRED BY 3b UNITARITY

3→3 SCATTERING AMPLITUDE: ~~IN~~FINITE VOLUME

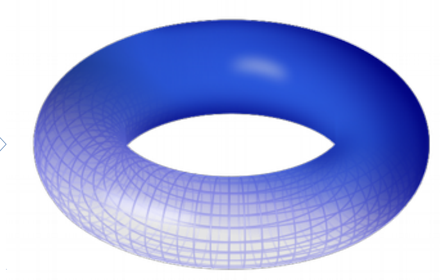
[MM & Döring EPJ A53 (2017)]

LATTICE QCD SETUP

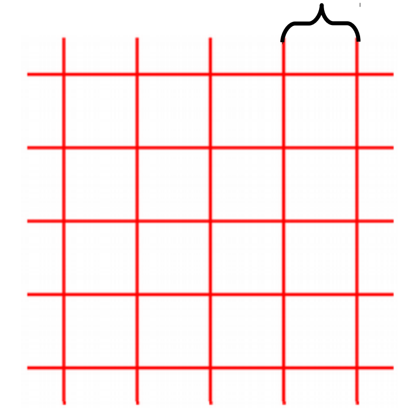
Lattice QCD calculations are performed in finite volume



boundary conditions



discretized momenta

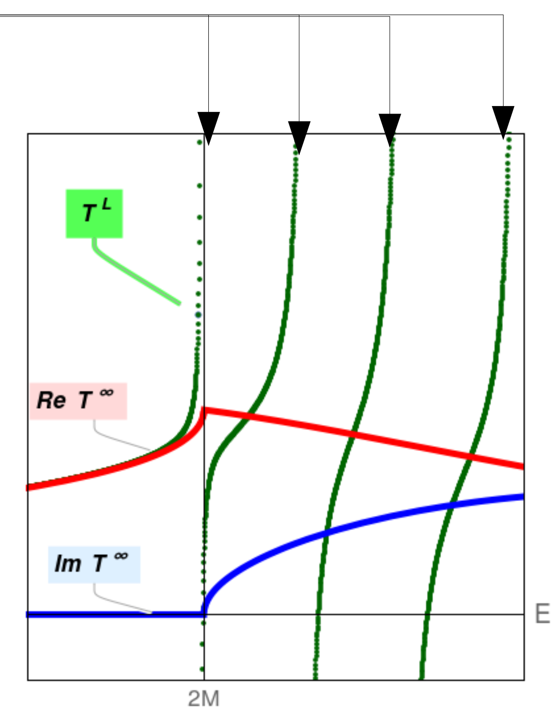


momenta & spectra are discretized

– replace $\int \frac{d^3\mathbf{q}}{(2\pi)^3} \to \frac{1}{L^3}\sum_{\vec{n}\in Z^3}$

– LSZ formalism relates Greens fct. & S-matrix

→ ***1/T(E*) = 0*** ↔ ***E**** **∊** energy-eigenvalues



– **well established in 2-body**

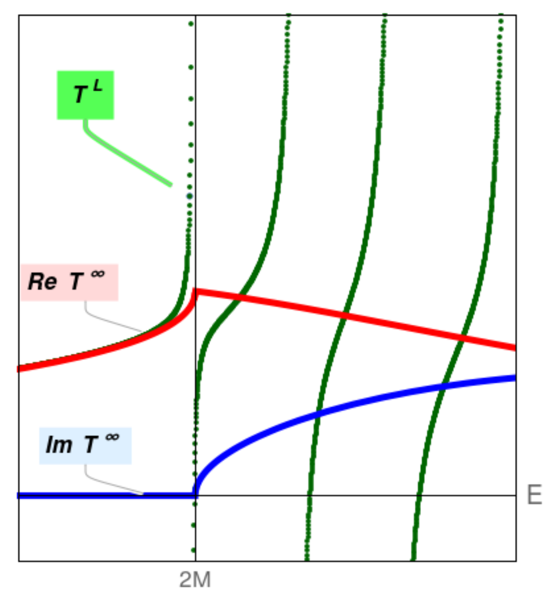
→ R. Molina's talk on rho/sigma(Thursday)

LATTICE QCD SETUP

Unitarity

Imaginary parts in inf. vol

Power-law fin. vol. effects



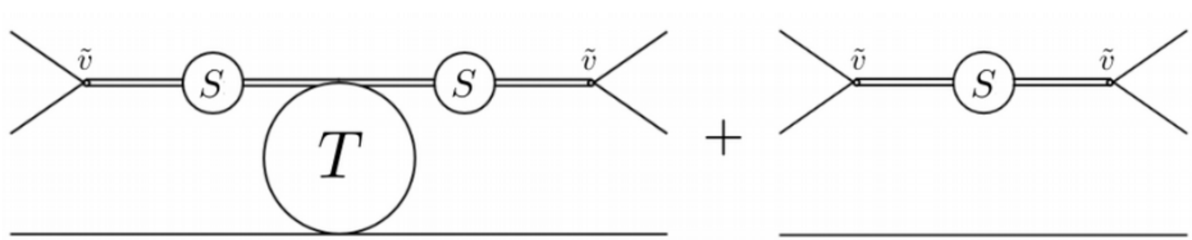
LATTICE QCD SETUP

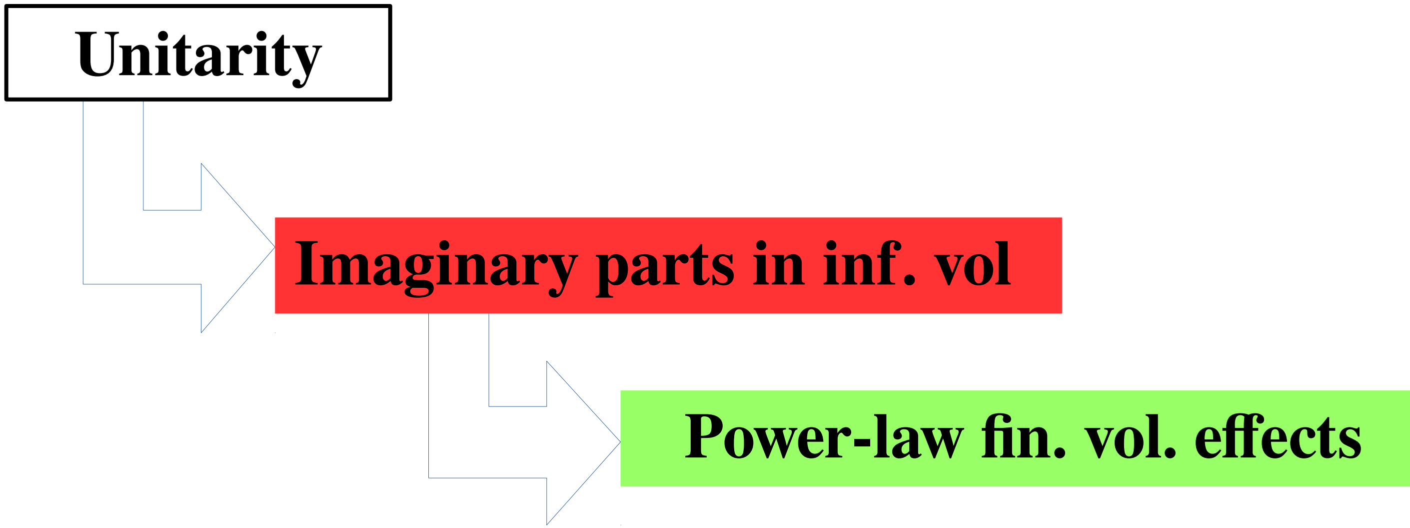
Unitarity

→ **Imaginary parts in inf. vol**

→ **Power-law fin. vol. effects**

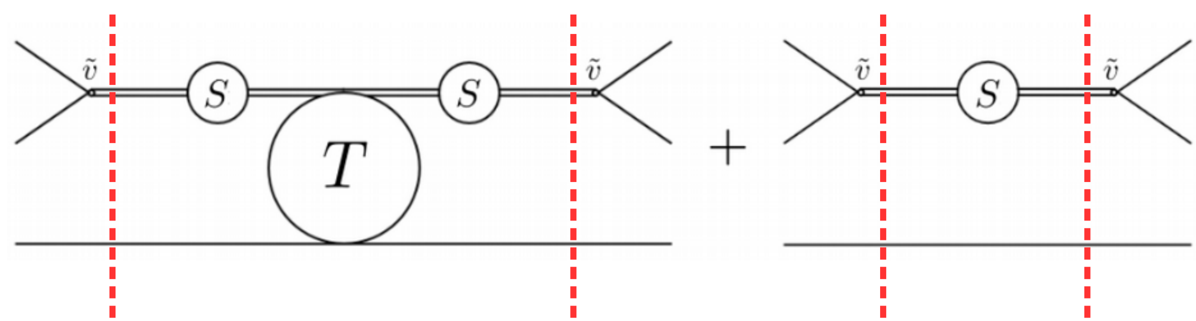
Discretize 3b-scattering amplitude → 3b Quantization Condition





Discretize 3b-scattering amplitude → 3b Quantization Condition

→ v is cut-free



Unitarity

Imaginary parts in inf. vol

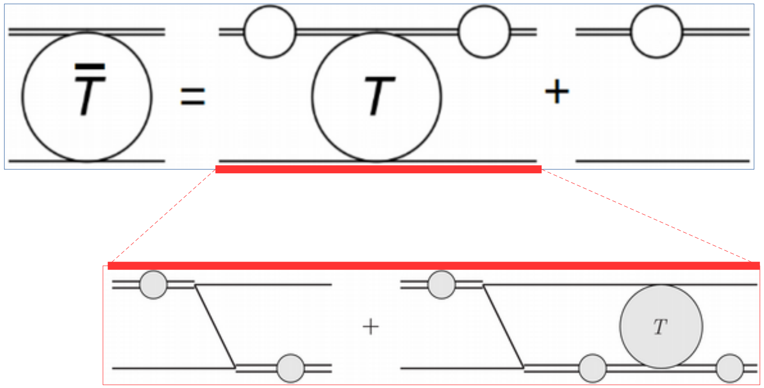
Power-law fin. vol. effects

Discretize 3b-scattering amplitude → Quantization Condition

→ v is cut-free

→ matrix-equation in q_{in} q_{out}

$$\bar{T}(s) = \infty \quad \Leftrightarrow \quad \text{Det}\left[\tau(s)^{-1} + \frac{B(s)}{2E(s)}\right]_{q_{in},q_{out}} = 0$$



QUANTIZATION CONDITION

$$\mathrm{Det}\left[\tau(s)^{-1} + \frac{B(s)}{2E(s)}\right]_{q_{in},q_{out}} = 0$$

- high-dimensional problem → **separate variable / use symmetry**
  - $SO(3) \rightarrow O_h$ (octahedral group)
  - $Y_{lm}(\theta,\varphi)$ not a basis of functional space (~~**PWA**~~)
- actual Lattice QCD calculations determine energy spectrum for irreps separately
  - irreps of cubic group A_1, A_2, E, T_1, T_2

→ **projection to irreps is vital!**

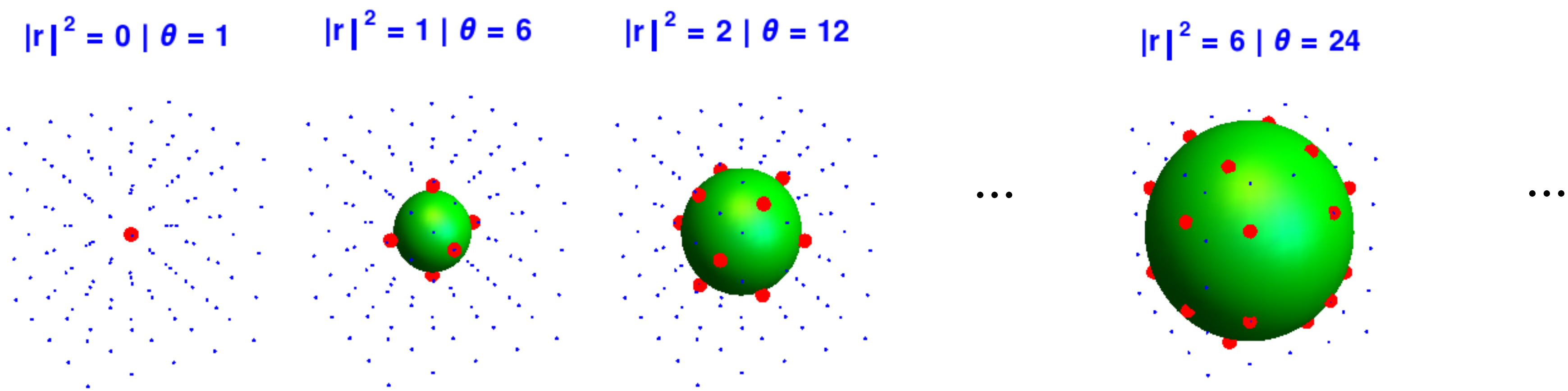
PROJECTION TO IRREPS

work with Pang, Hammer,Rusetsky, Doring,Wu, MM in preparation

→ see also talk by A. Rusetsky

I. ON-basis of functions

1) Separation of variables: shells (sets of points related by $\boldsymbol{O_h}$)



2) Multiple spheric harmonics contribute to a given irrep

→ $\boldsymbol{l}$ is not a good quantum number in finite volume

→ cubic harmonics: $X_\ell^{\Gamma\nu\alpha}(\hat{\mathbf{p}}) = \sum_m c_{\ell m}^{\Gamma\nu\alpha} Y_{\ell m}(\hat{\mathbf{p}})$

PROJECTION TO IRREPS

I. ON-basis of functions ...

3) Construct a maximal set of linear indep. cubic harmonics

→ depends on the shell type

→ for each shell type:	*000*	*00b*	*0bb*	*bbb*	*0bc*	*bbc*	*bcd*
→ number of elements:	*1*	*6*	*12*	*24*	*8*	*24*	*48*

4) Ortho-normalize (3) w.r.t

→ depends on the shell index (s)

$$\langle f,g\rangle_s = \frac{4\pi}{\vartheta(s)}\sum_j^{\vartheta(s)} f(\hat{\mathbf{p}}_j)^* g(\hat{\mathbf{p}}_j)$$

$$\rightarrow ONB_s = \{\chi_u^{\Gamma\alpha s}(\hat{\mathbf{p}})\}$$

II. Projection of 3-body-Quantization-Condition = FINAL RESULT

$$\det\left(B_{uu'}^{\Gamma ss'}(W^2) + \frac{2E_s\,L^3}{\vartheta(s)}\tau_s(W^2)^{-1}\delta_{ss'}\delta_{uu'}\right) = 0$$

NUMERICAL EXAMPLE

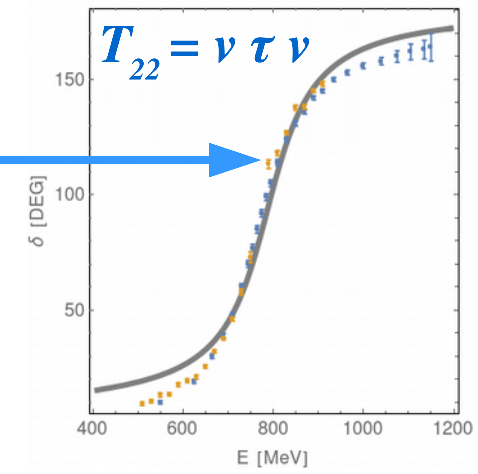
[MM & Döring EPJ A53 (2017)]

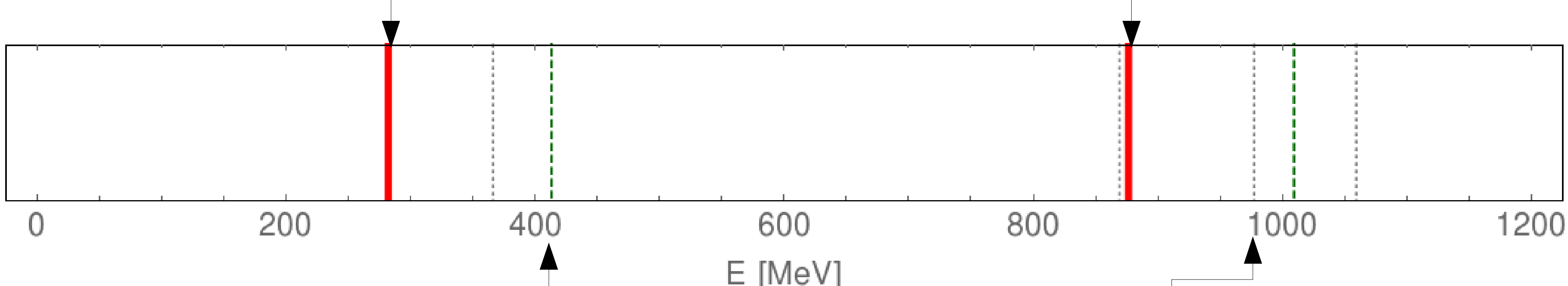
EXAMPLE

$$\det\left(B^{\Gamma ss'}_{uu'}(W^2)+\frac{2E_s\,L^3}{\vartheta(s)}\tau_s(W^2)^{-1}\delta_{ss'}\delta_{uu'}\right)=0$$

- 3 particles in finite volume: ***m=138 MeV, L=3 fm***
- one S-wave isobar → two unknowns:
  - – vertex(Isobar→2 stable particles)
  - – subtraction constant (~mass)
- **Project to $\Gamma = A^{1+}$**

  → prediction of 3body energy-eigenlevels





unphysical lvls cancel out (exact proof available)

SUMMARY/OUTLOOK

3-body scattering amplitude derived from 2&3 body Unitarity

– interaction kernel = one-particle-exchange

– flexible parametrization: real contributions can be added to the kernel

TBD: analysis of physical systems

3-body Quantization Condition in fin. vol. derived

– cancellations of unphysical poles revealed

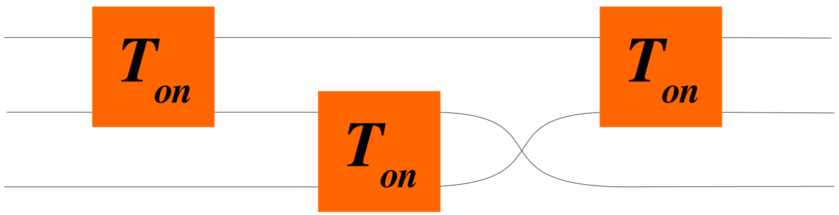
– projection to irreps done

– technical feasibility on a numerical example

– the only approximation = number of isobars

TBD: multiple channels

TBD: inclusion of isospin & angular momentum



“power of Unitarity”

THANK YOU!

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