

# Few-body resonances from finite-volume calculations

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in collaboration with P. Klos, J. Lynn, H.-W. Hammer, and A. Schwenk

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work in progress



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## terra incognita at the doorstep...

$^3\text{Li}$	$^4\text{Li}$	$^5\text{Li}$	$^6\text{Li}$	$^7\text{Li}$
$^3\text{He}$	$^4\text{He}$	$^5\text{He}$	$^6\text{He}$	
$^1\text{H}$	$^2\text{H}$	$^3\text{H}$	$^4\text{H}$	$^5\text{H}$
n	?	?	?	?

- bound dineutron state not excluded by pionless EFT

Hammer + SK, PLB 736 208 (2014)

- recent indications for a three-neutron resonance state...

Gandolfi *et al.*, PRL 118 232501 (2017)

- ... although excluded by previous theoretical work

Offermann + Glöckle, NPA 318, 138 (1979); Lazauskas + Carbonell, PRC 71 044004 (2005)

- possible evidence for tetraneutron resonance

Kisamori *et al.*, PRL 116 052501 (2016)

# Finite-volume resonance signatures

Lüscher formalism: phase shift  $\leftrightarrow$  box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left( \frac{Lp}{2\pi} \right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B 354 531 (1991); ...

resonance contribution  $\rightsquigarrow$  **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) 9, 609 (1989); ...

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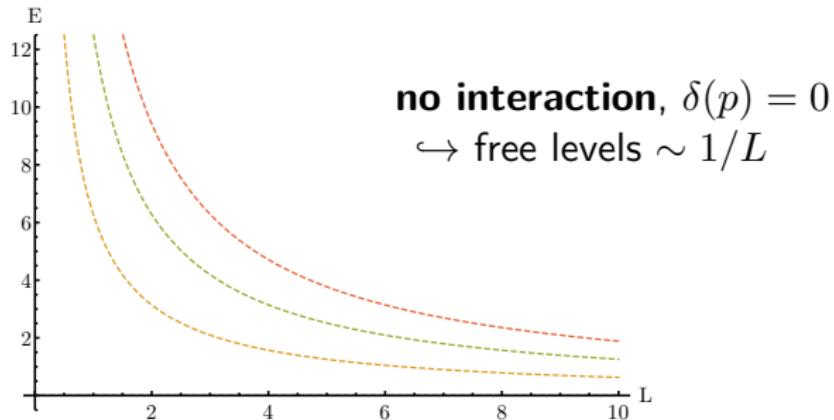
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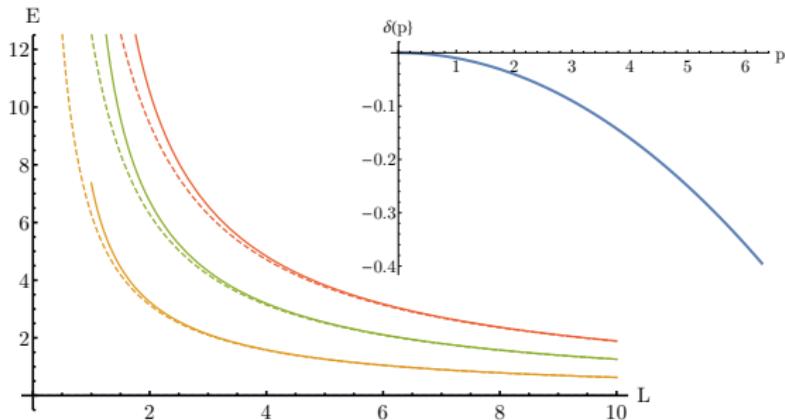
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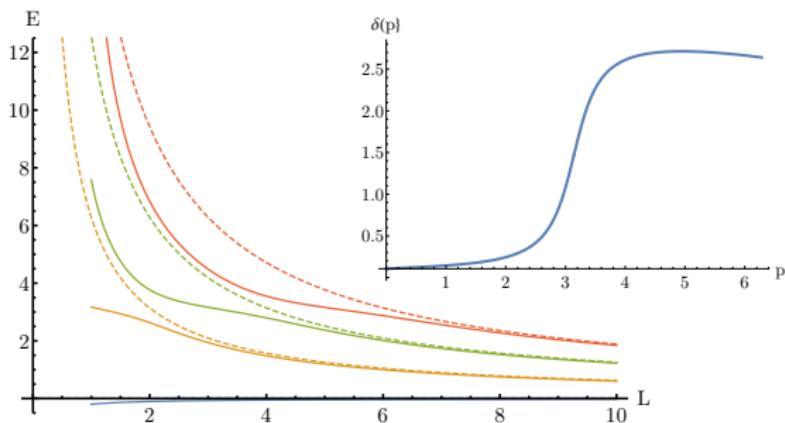
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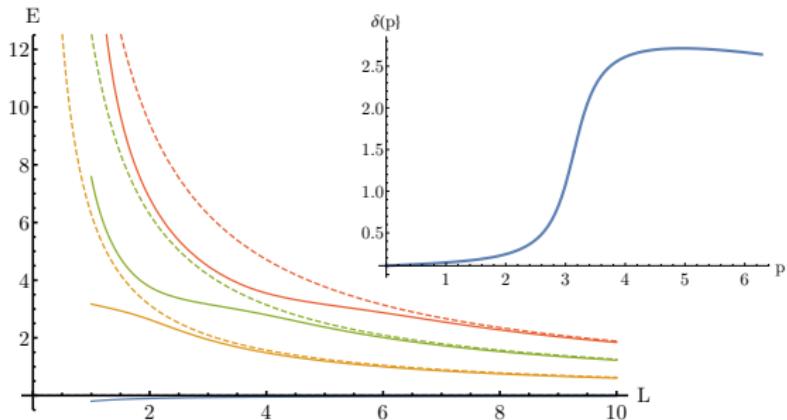
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Effect can be very subtle in practice...

Bernard et al., JHEP 0808 024 (2008); Döring et al., EPJA 47 139 (2011); ...

Few-body resonances from finite-volume calculations — p. 3

# Discrete variable representation

**Needed: calculation of several few-body energy levels**

- difficult to achieve with GFMC/AFDMC methods [Klos et al., PRC 94 054005 \(2016\)](#)
- simple direct discretization possible, but not very efficient

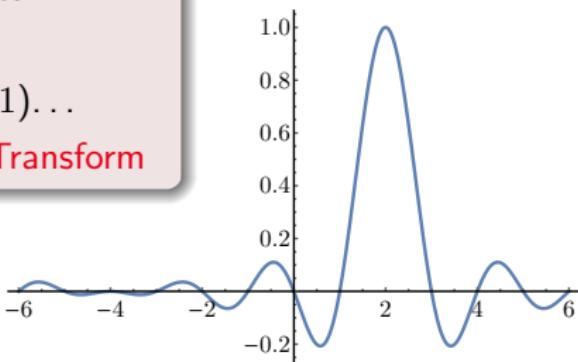
→ use a **Discrete Variable Representation (DVR)**

well established in quantum chemistry, suggested for nuclear physics by Bulgac+Forbes, [PRC 87 87, 051301 \(2013\)](#)

## Main features

- basis functions localized at grid points
- potential energy matrix diagonal
- kinetic energy matrix sparse (in  $d > 1$ )...
- ...or implemented via Fast Fourier Transform

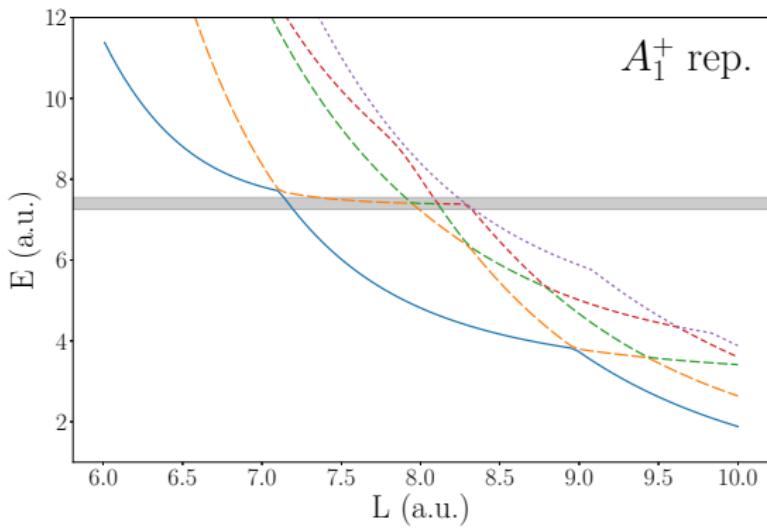
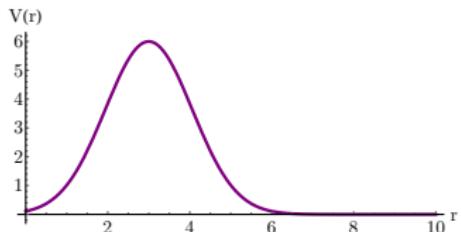
**periodic boundary conditions**  
↔ plane waves as starting point



# Three-body resonance example

## three-boson system

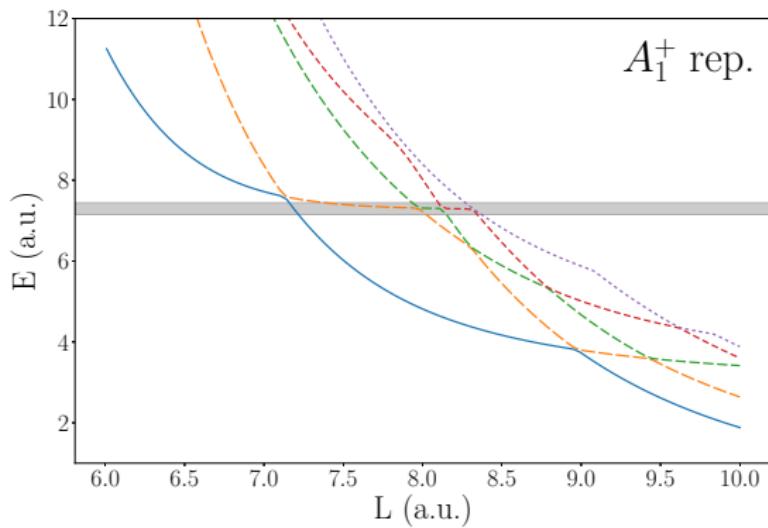
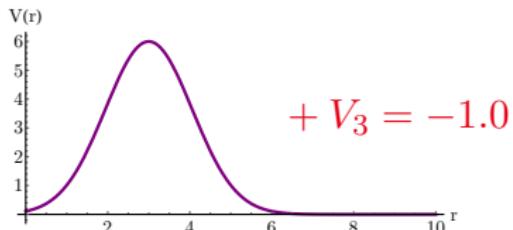
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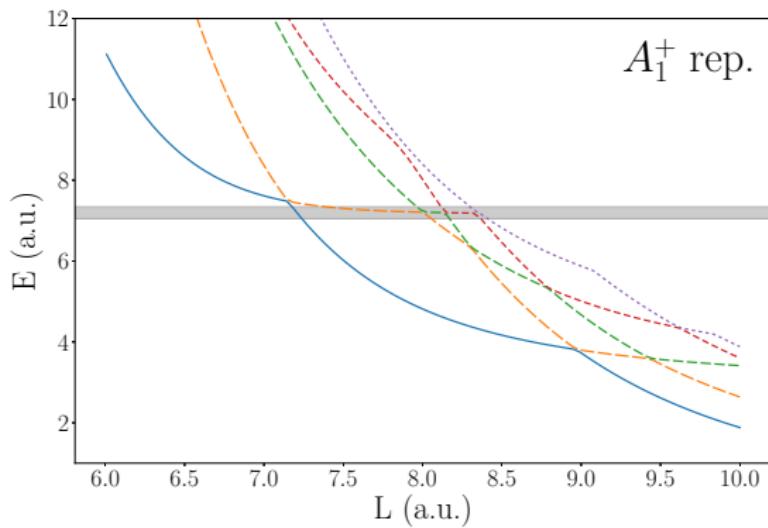
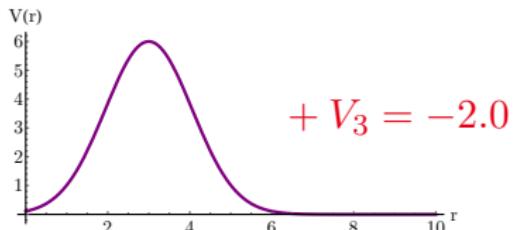
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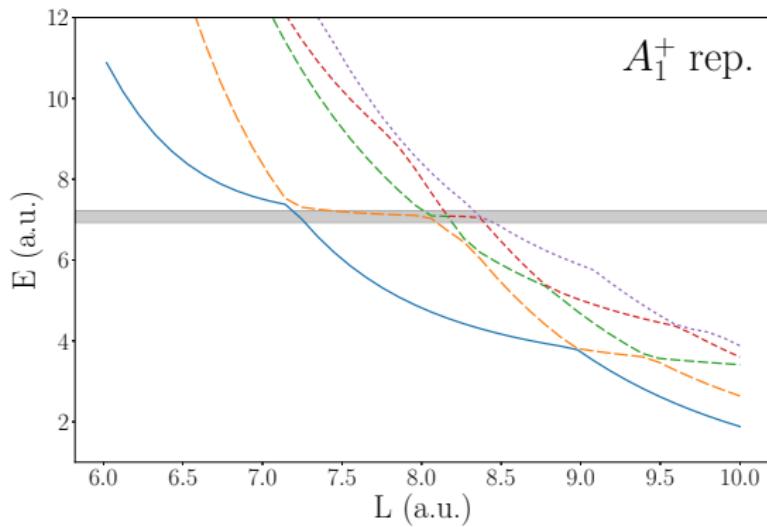
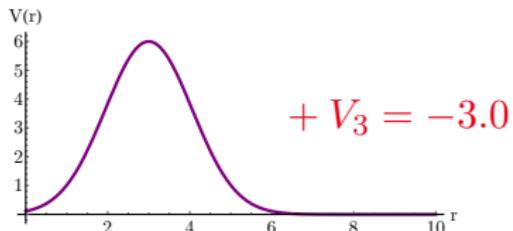
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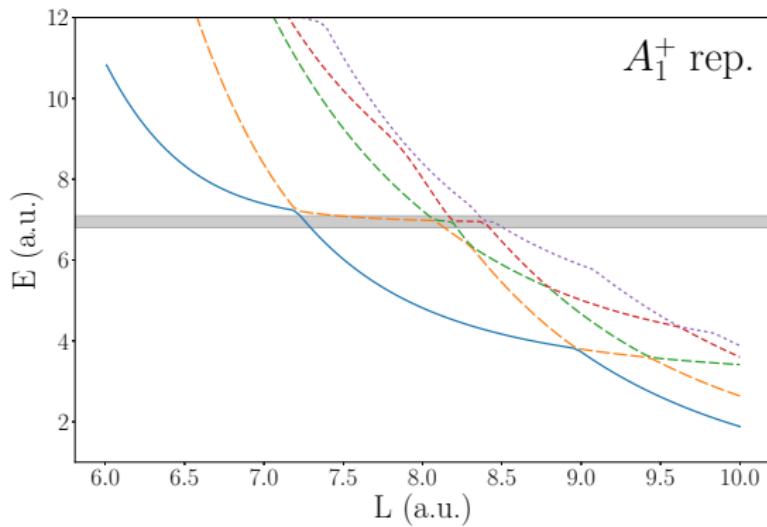
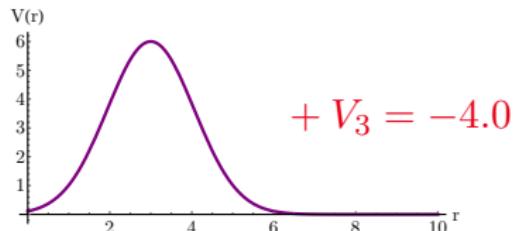
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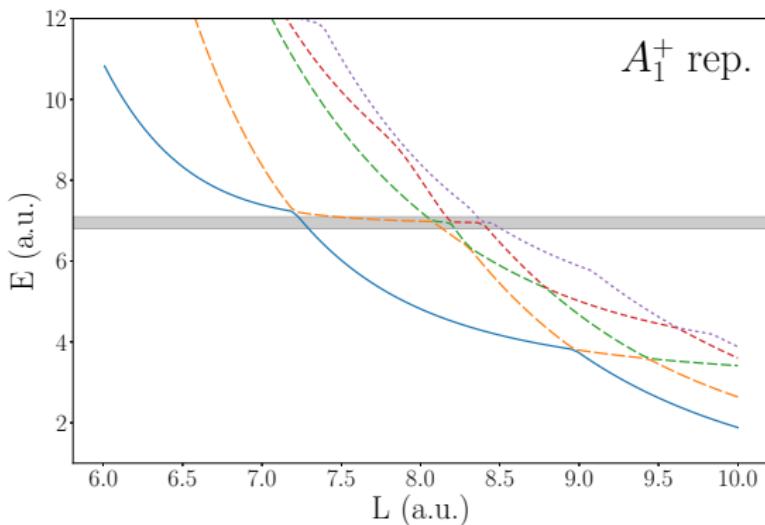
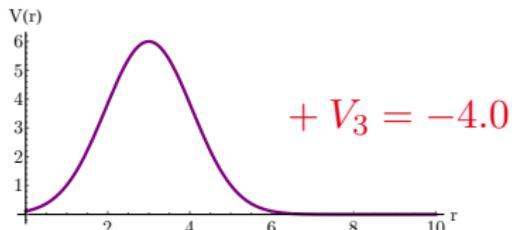
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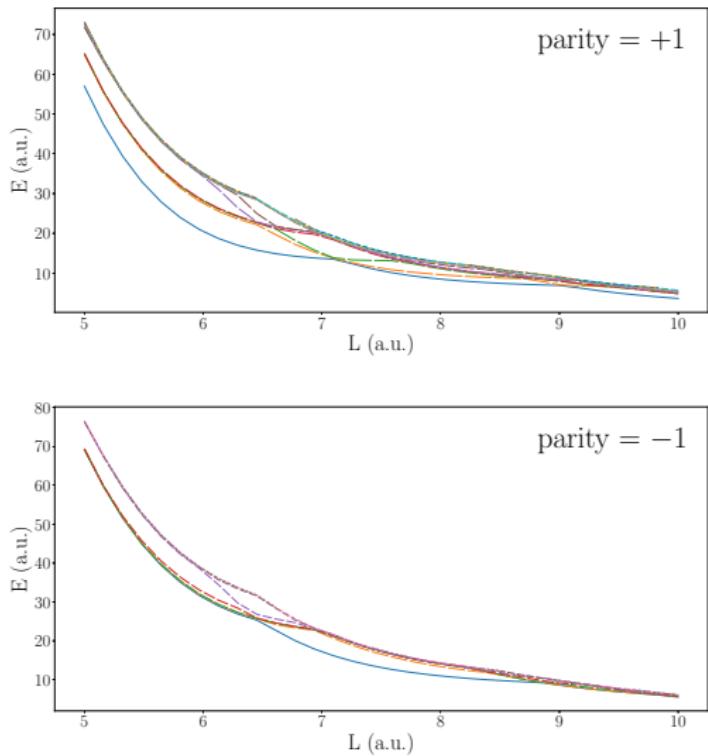
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- shifted Gaussian 2-body potential
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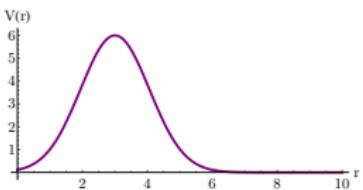


↪ possible to move three-body resonance

# Four-body spectra (very preliminary)

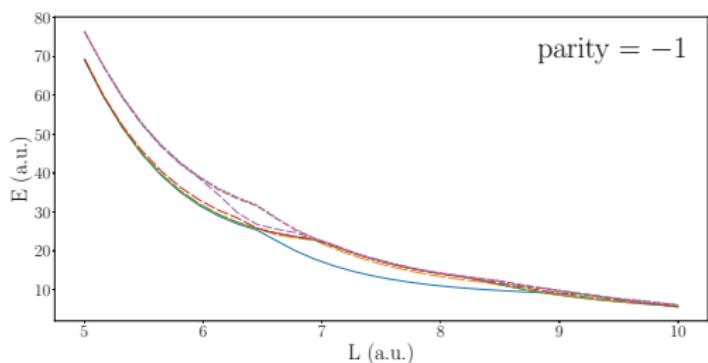
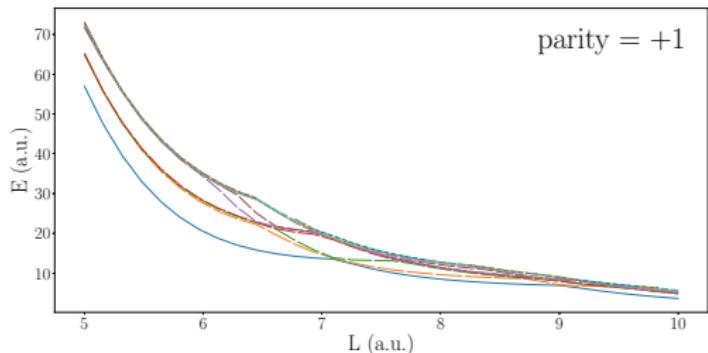


**four bosons**

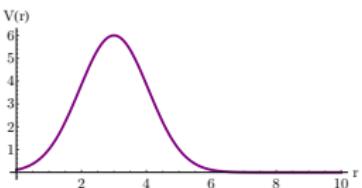


**crossings need not  
be avoided!**

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**four bosons**



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— Thank you —