I = 1 $\pi\pi$ scattering in HAL QCD method with LapH smearing

Daisuke Kawai (Kyoto U.)

Studies on the ππ scattering in I < 2 channel with HAL QCD method

All-to-all propagator is necessary for 4-pt correlation function. ➡ **Large computational cost**

We considered the combination of **HAL QCD method** and **LapH smearing (distillation)**

[M. Peardon et al. (Hadron Spectrum Collaboration) (2009)]

R-correlator

$$R(\mathbf{r}, t-t_0) = e^{2m_\pi(t-t_0)} \sum_{\mathbf{x}} < 0|\underbrace{\pi(\mathbf{x},t)\pi(\mathbf{x}+\mathbf{r},t)}_{\text{LapH smeared sink}}\underbrace{\pi(\mathbf{P},t_0)\pi(-\mathbf{P},t_0)}_{\text{LapH smeared src.}}|0 >$$

➡ potential

$$\left(\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right) R(\mathbf{r},t) = \int d^3r' U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t)$$

First, we checked the sink operator (scheme) independence of the potential method.

[Ref : DK et al, (HAL QCD Collab.)1711.01883]

Point sink ➡ LapH-smeared sink

$U(\mathbf{r},\mathbf{r}')$: non-local potential ➡ *derivative expansion* ➡ $U(\mathbf{r},\mathbf{r}') \simeq \{\underbrace{V_0(\mathbf{r})}_{\text{LO}} + \underbrace{V_1(\mathbf{r})\nabla^2}_{\text{NLO}} + \mathcal{O}(\nabla^4)\}\,\delta(\mathbf{r}-\mathbf{r}')$

⬇ **faithful** [S. Aoki et al. (2012)] ⬇ **truncation**

$\delta_0(k)$: phase shift ⟷ How accurate ? ⟷ $\delta_0'(k)$: phase shift ***from truncated potential***

scheme independent **Accuracy depend on source/sink operator**

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→ potential

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Second, we applied this method to ππ scattering in I = 1 channel

Preliminary

- Resonant behavior in the phase shift

  → First result of conventional resonance with HAL QCD method

- Direct search of S-matrix pole is performed

  → Pole is found in the second Riemann sheet.

$k\cot\delta_0(k)$ plot of phase shift in I = 2 ππ scattering

N_f = 2+1 gauge config., a = 0.12 fm, $16^3 \times 32$, m_π = 870 MeV

[CP-PACS/JLQCD Collaboration : T.Ishikawa, et al, (2008)]

Point sink (Conventionally used in HAL QCD)

⇨ NLO term is negligible at this energy region.

($\because$ source momentum dependence is negligible)

- The phase shift in low energy region

The deviation of phase shift in smeared-sink scheme

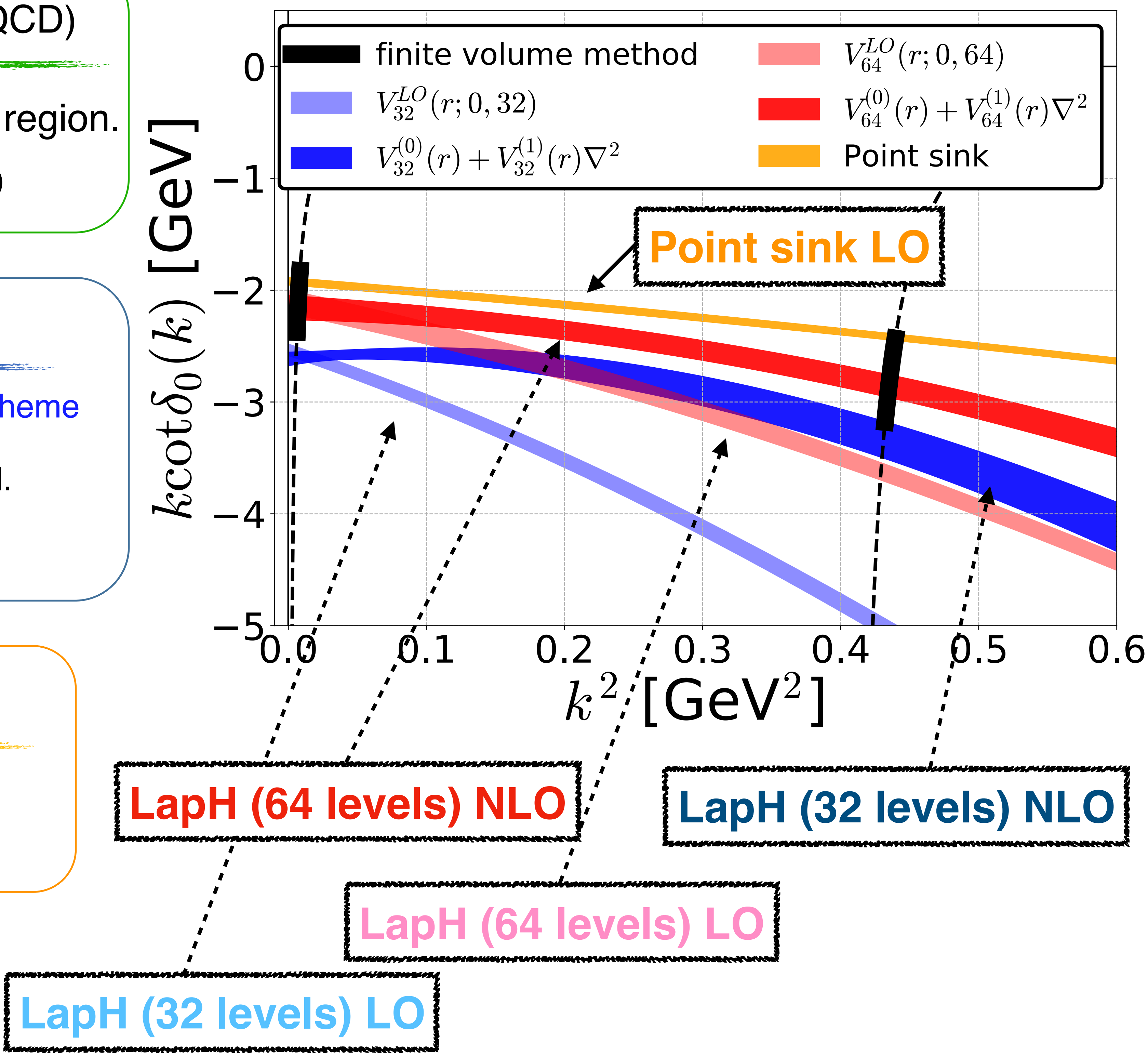
from $V^{LO}_{n_{\max}}(r;0,16)$ and the finite volume method.

⇨ controlled by smearing level

- The phase shift in high energy region

The deviation is dominated by

the contribution from the NLO analysis.



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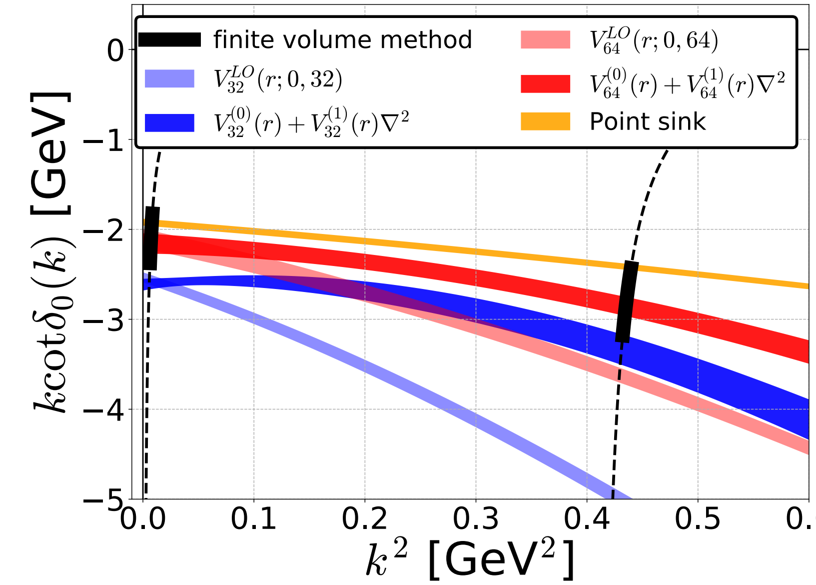

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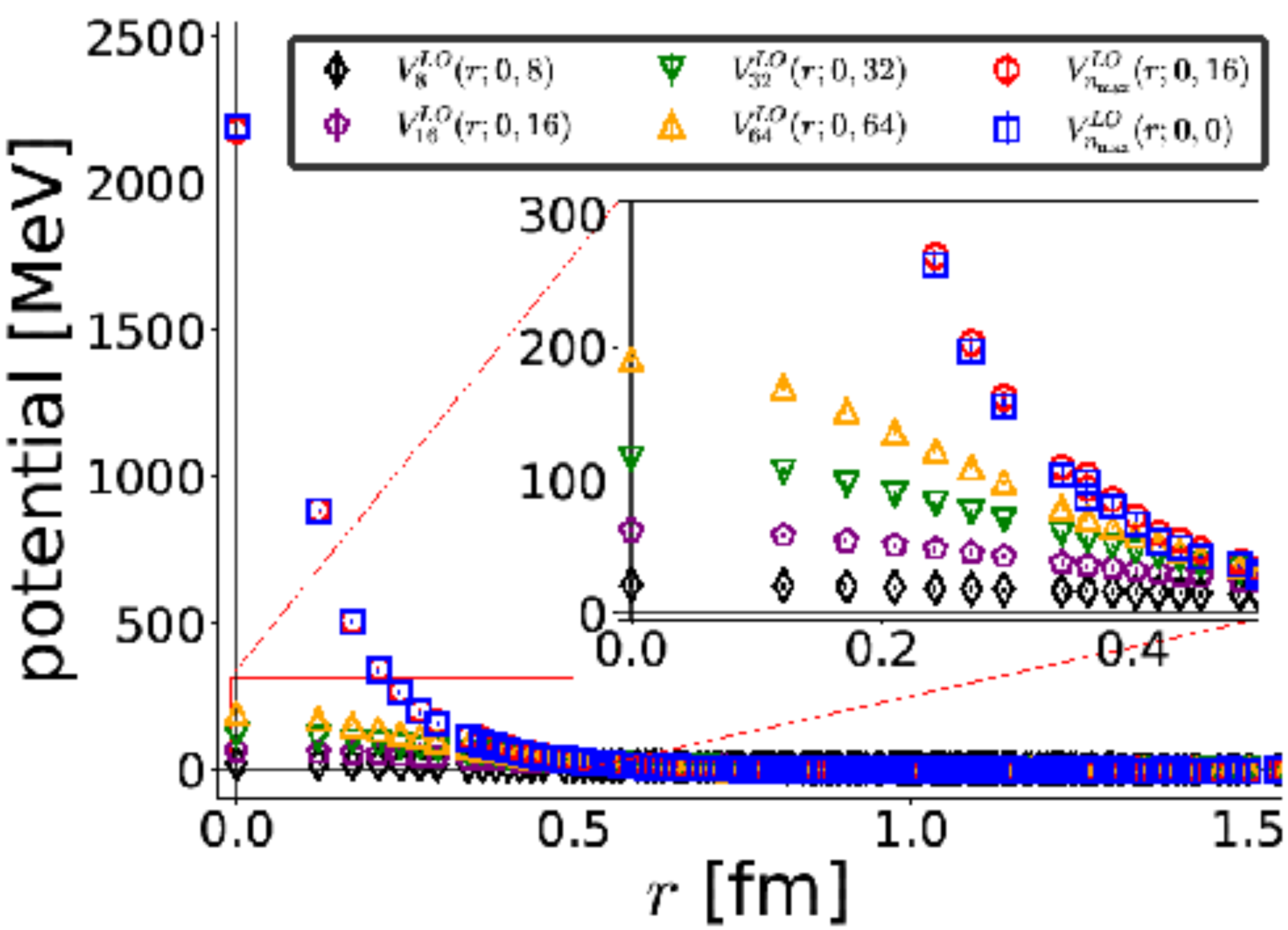
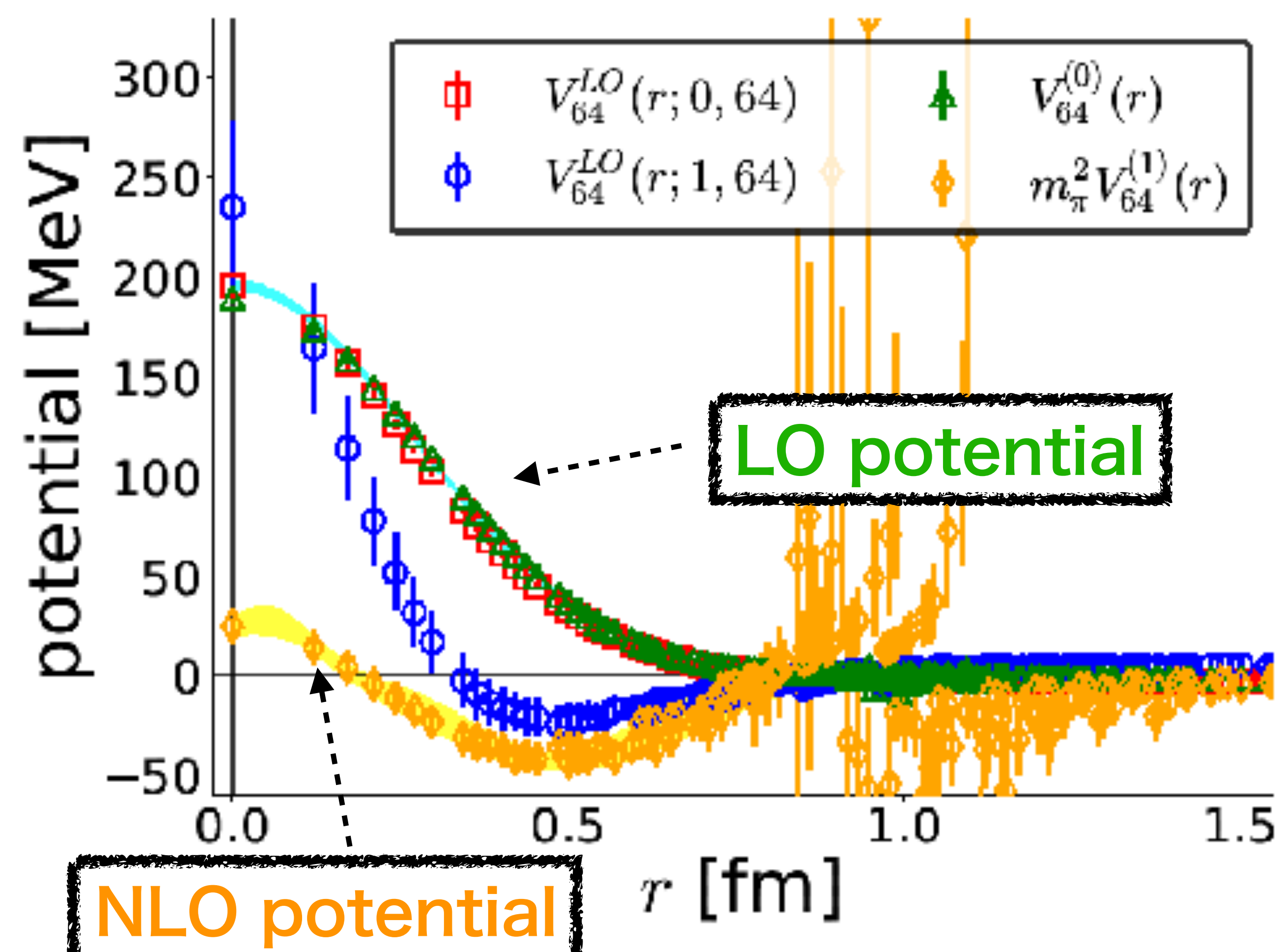




Phase shift I = 1 ππ scattering from HAL QCD method with LapH smearing

The phase shift in the LO analysis for LapH smeared-sink

(# of conf. = 10 for Level = 256, so it is preliminary) N_f = 2+1 gauge config., a = 0.09 fm, $32^3 \times 64$, m_π = 410 MeV

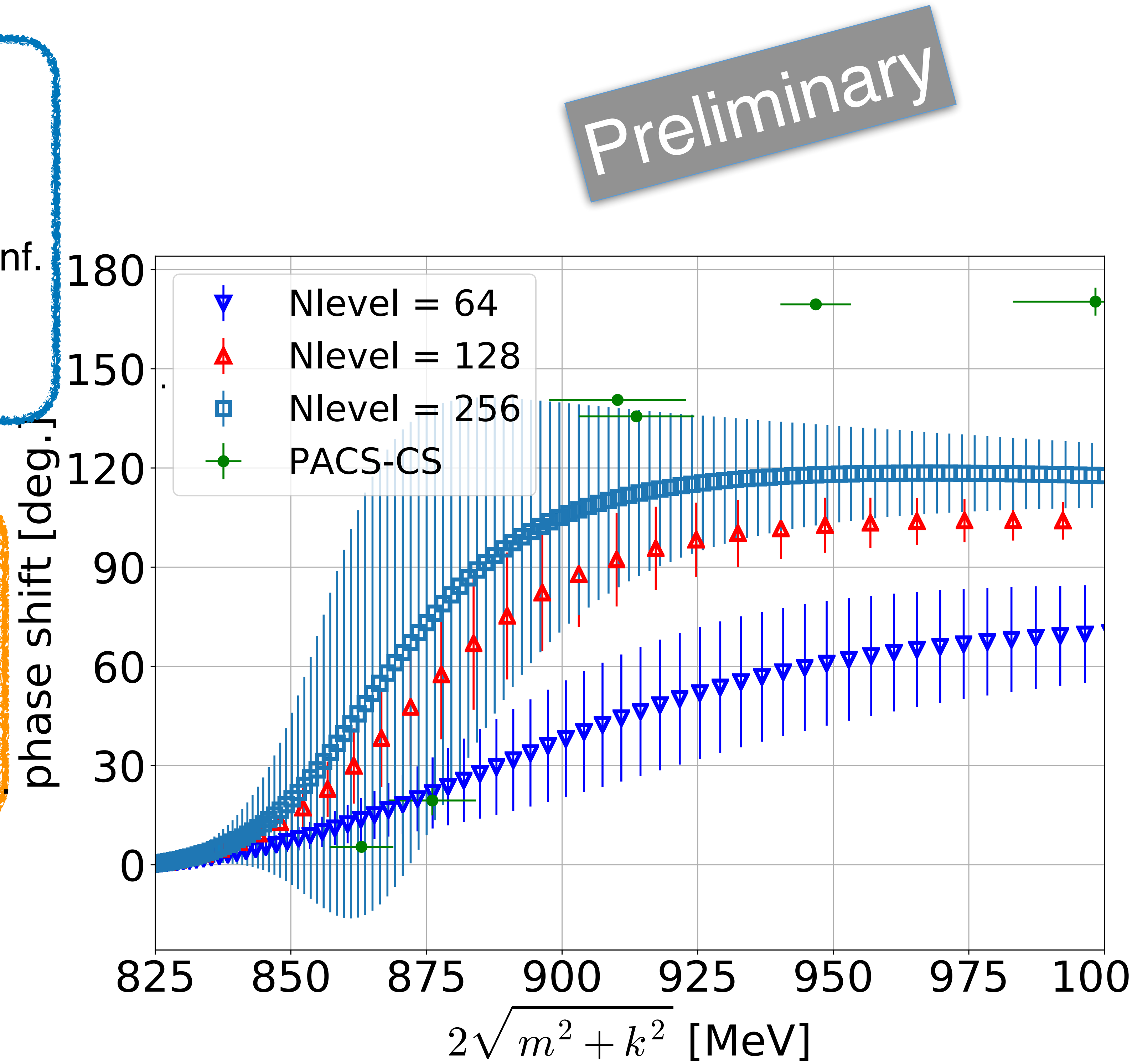
[PACS-CS Collaboration: S. Aoki et. al., (2009)]

Results

- Phase shift given by the potential crosses 90 degrees at $\sqrt{s} \sim 900\,\mathrm{MeV}$

⇨ Consistent with known value for this conf.

$$(\because\ m_\rho \simeq 890\,\mathrm{MeV})$$



Open problem

- It largely deviates from PACS-CS result for $\sqrt{s} > 900\,\mathrm{MeV}$

  [S. Aoki et al., (PACS-CS Collaboration), (2011)]

- increase of phase shift stops around 120°.

⇩

NLO analysis and larger number of n_s is necessary as I = 2 case.

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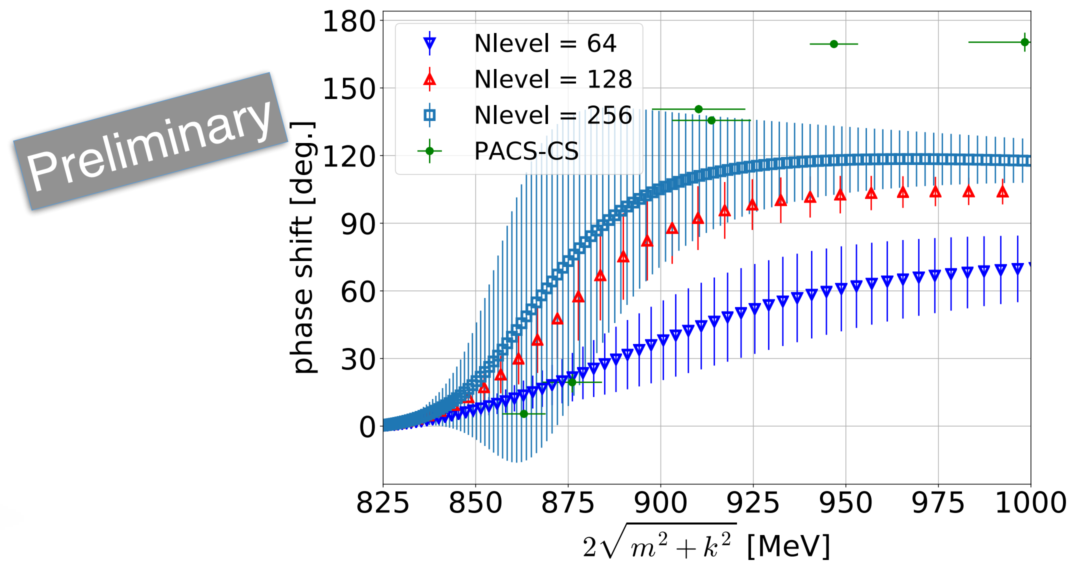

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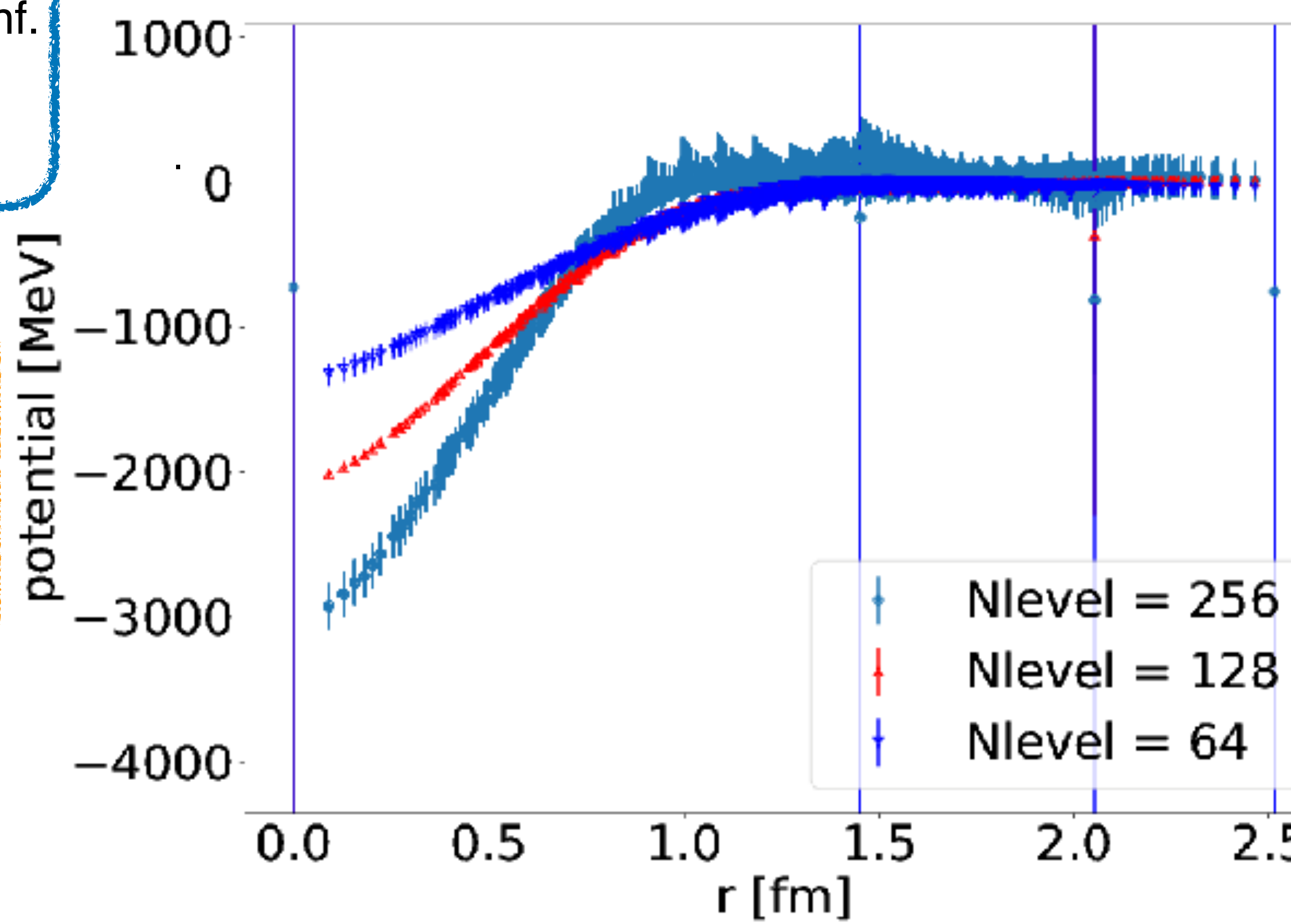
Potential



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⬇

NLO analysis and larger number of n_s is necessary as I = 2 case.

Direct search of S-matrix pole

Pole is located in the second Riemann sheet.

Invariant mass

$$\sqrt{s} = 886.4(4.4) - \frac{i}{2}82.1(10.2)\,\mathrm{MeV}$$

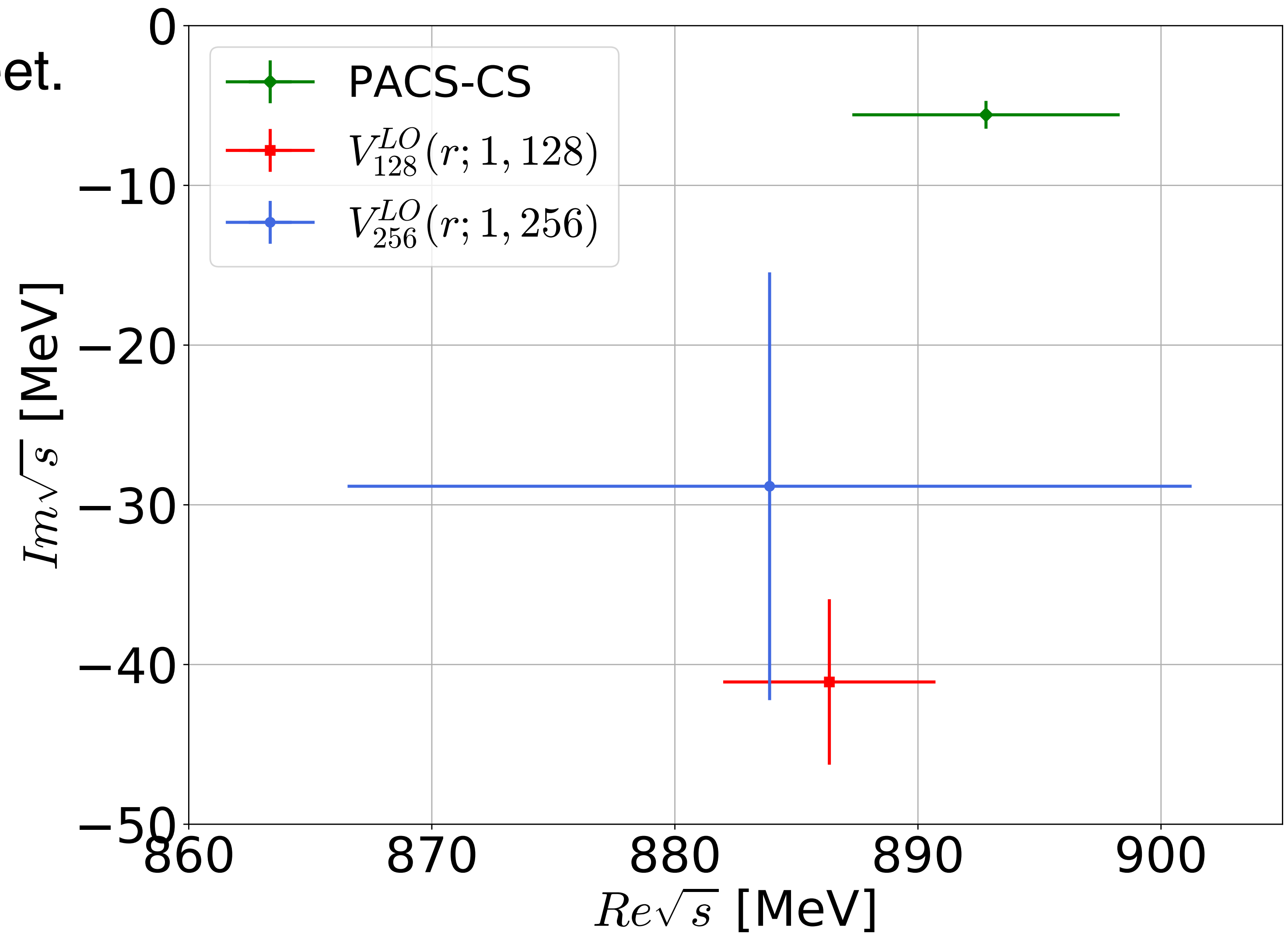
(NLevel = 128)

$$\sqrt{s} = 883.9(17.4) - \frac{i}{2}57.6(26.8)\,\mathrm{MeV}$$

(NLevel = 256, preliminary)

c.f.) PACS-CS result

$$\sqrt{s} = 892.8(5.5) - \frac{i}{2}11.2(1.7)\,\mathrm{MeV}$$



Preliminary

- Real part (= mass) is consistent with the result with Lüscher's method by PACS-CS collaboration.
- The deviation in imaginary part (= decay width) will be reduced by increasing n_s and employing NLO analysis.

This method can be applied for exotic hadrons and resonance with large width such as σ.

Summary

Sink operator independence (I=2 channel)

- The truncated potential with LapH smeared sink has large sink operator dependence.

  ⇒ The height of repulsive core drastically changes.

- Even with the dependence, phase shift can be correctly obtained by considering higher order terms.

  ⇒ We show that phase shifts in high energy are improved by considering the NLO term.

I=1 ππ scattering

- Resonant behavior in the phase shift

  ⇒ Peak point is consistent with configuration data.

  However, improvement will be necessary to get correct behavior in higher energy.

- Direct search of S-matrix pole without any fitting such as Breit-Wigner form is possible.

  ⇒ Resonant pole in the second Riemann sheet

Back up

Spatial distribution of smearing operator

N_f = 2+1 gauge config., a = 0.12 fm, $16^3 \times 32$, m_π = 870 MeV

[CP-PACS/JLQCD Collaboration : T.Ishikawa, et al, (2008)]

How is the dependence of smearing operator on the number of eigenvalue included ?

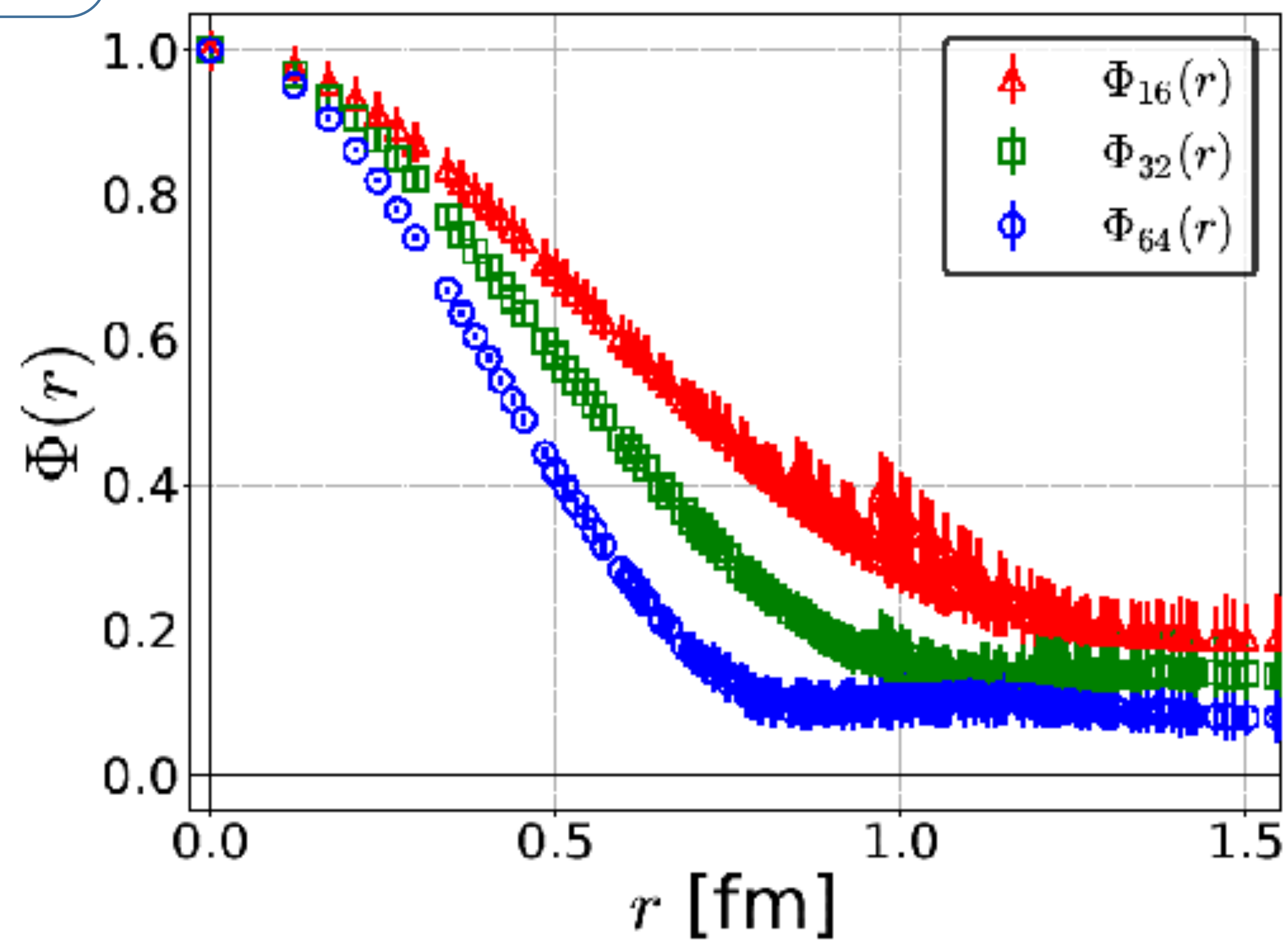
A gauge invariant measure

[M. Peardon et al. (Hadron Spectrum Collaboration), (2009)]

$$\Phi_{n_s}(\mathbf{r}) = \sum_{\mathbf{x},t} \sqrt{\mathrm{Tr}\left\{S_{n_s}(\mathbf{x},\mathbf{x}+\mathbf{r},t)S_{n_s}(\mathbf{x}+\mathbf{r},\mathbf{x},t)\right\}}$$

(normalized at $r = 0$)

→ The smeared quark is more localized as n_s increases.



Note : The long tail structure might be the origin of systematic change in physical observables.

(Investigation is on going)

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS & JLQCD
  [CP-PACS/JLQCD Collaboration : T.Ishikawa, et al., PRD 78 (2008) 011502(R)]
- Wilson clover fermion and Iwasaki gauge action
- a = 0.1214 fm , $16^3 \times 32$ lattice
- $m_\pi \simeq$ 870 MeV
- 60conf × 32 time slices
- Calculated on Cray XC40 in YITP
- No gauge fixing is used

Cray XC40 in YITP

$$C^4_M(\mathbf{r}, t; t_0) = \sum_{\mathbf{x}} \sum_{\mathbf{y_1}, \mathbf{y_2}} \langle 0 | \pi^+(\mathbf{x}, t) \pi^+(\mathbf{x}+\mathbf{r}, t) \pi^{-s}(\mathbf{y_1}, t_0) \pi^{-s}(\mathbf{y_2}, t_0) | 0 \rangle$$

Remark : the sum over source space improves statistics.

Details of HAL QCD potentials with LapH smearing

4-pt correlator : $C^{4,A_1^+,1}_{n_a,n_b}(\mathbf{r},t;\mathbf{P},t_0) = \sum_{\mathbf{x}} < 0|\pi^-_{n_a}(\mathbf{x},t)\pi^-_{n_a}(\mathbf{x}+\mathbf{r},t)(\pi_{n_b}\pi_{n_b})^{A_1^+,1}_{2,2}(|\mathbf{P}|,t_0)|0 >$

smearing level for sink

smearing level for source

relative momentum in the unit of $2\pi/L$

R-correlator : $R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0) \equiv C^{4,A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)/\left\{C^2_{n_a,n_b}(t,t_0)\right\}^2$

(effective) leading order potential : $V^{\mathrm{LO}}_{n_a}(r;|\mathbf{P}|,n_b) = \dfrac{\left(\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right) R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)}{R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)}$

Notation

"point-sink scheme" : $n_a = N_c N_x N_y N_z \equiv n_{\max}$ = Conventionally used sink

"smeared-sink scheme" : $n_a < n_{\max}$

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS collaboration

[PACS-CS Collaboration: S. Aoki et. al., (2009)]

- Wilson clover fermion and Iwasaki gauge action
- a = 0.0907 fm , $32^3 \times 64$ lattice
- m_π = 410MeV, m_ρ = 890 MeV

  ➡ ρ meson will appear as a resonant state

K-computer

- 64 time slices are fully used for the average value.
- Periodic boundary condition is used for all direction.
- No gauge fixing is used.

Big Warterfall @RIKEN