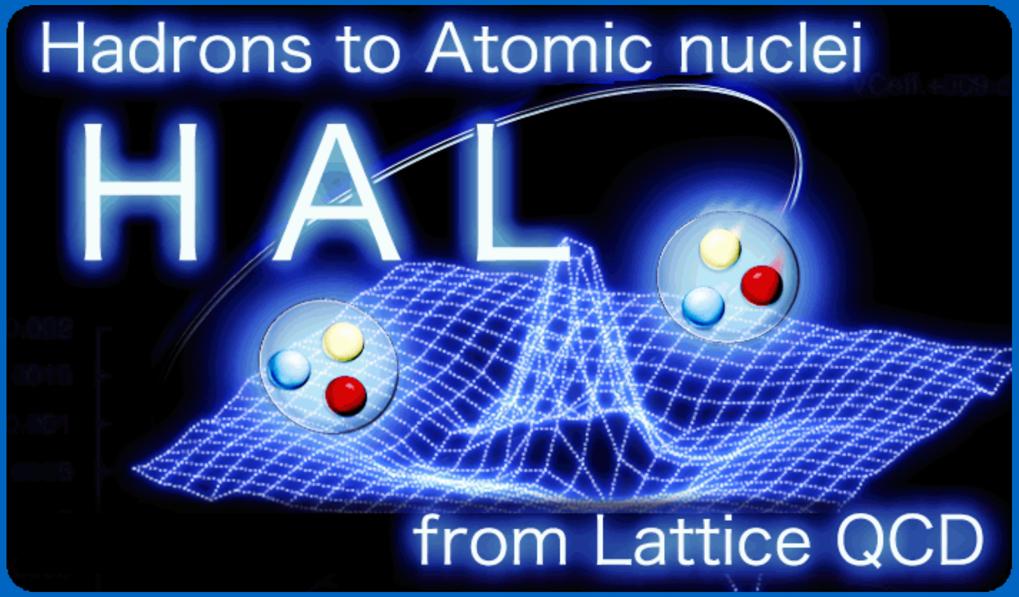
Systematics in the HAL QCD method and diagnoses of the direct method

Takumi Iritani (RIKEN)

Feb. 5-9 2018 @ INT-18-70W "Multi-Hadron Systems from Lattice QCD"



HAL QCD Collaboration
Sinya Aoki (Wed.), K. Sasaki, T. Aoyama,
Daisuke Kawai (Fri.), T. Miyamoto (YITP), T. Hatsuda,
Takumi Doi (Thu.), **TI**, S. Gongyo, T. M. Doi (RIKEN),
T. Inoue (Nihon Univ.), N. Ishii, Y. Ikeda,
H. Nemura, K. Murano (RCNP), F. Etminan (Univ. of Birjand)

Contents

Challenges in Multi-Baryon Systems

measurement of **ground state** from **temporal correlation**

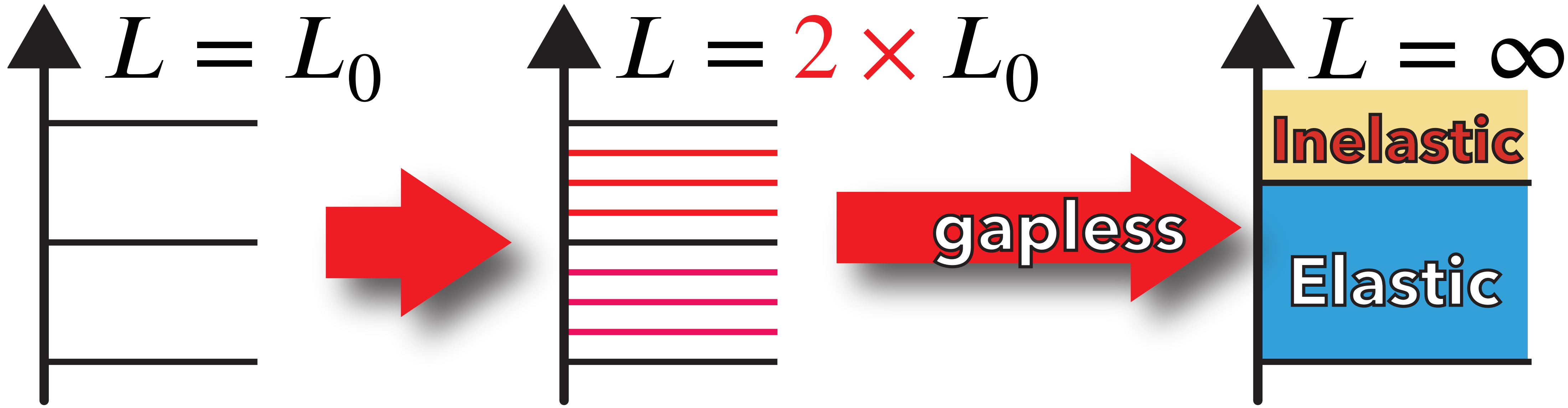
$$C_{NN}(t) = c_0 \exp(-E_0^{NN} t) + c_1 \exp(-E_1^{NN} t) + \cdots \simeq c_0 \exp(-E_0^{NN} t)$$

- **S/N problem** (**A**: mass number) Parisi '84, Lepage '89

$$S/N \sim \exp[-A \times (m_N - (3/2)m_\pi) \times t]$$

Scattering state contamination

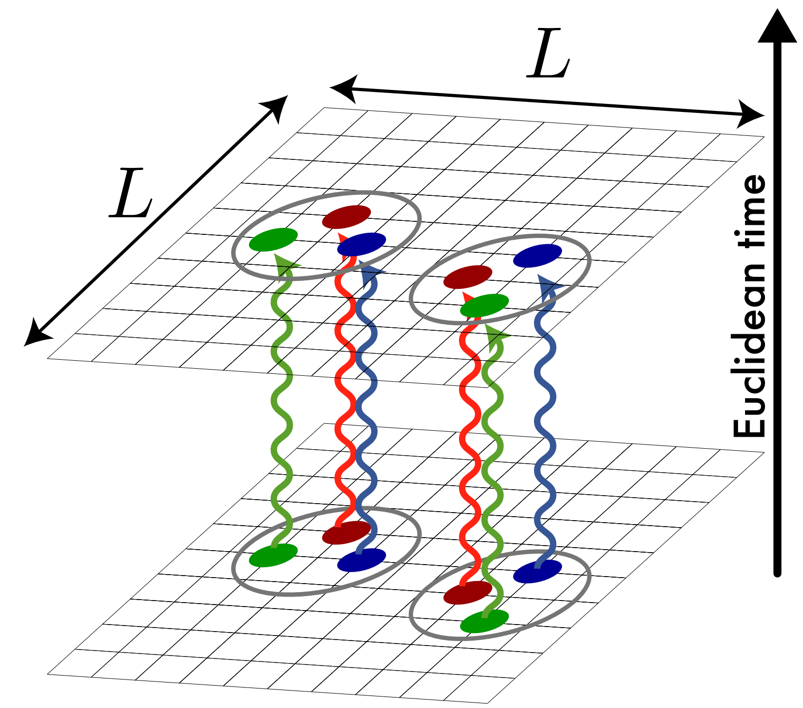
$$\Delta E \sim p^2/m_N \sim \mathcal{O}(1/L^2) \ll \mathcal{O}(\Lambda_{\rm QCD})$$



2 Hadrons in Lattice QCD (1) Direct Method

- **temporal** correlation of the two-hadron

$$R(t) \equiv C_{BB}(t)/\{C_B(t)\}^2$$



- energy shift from **plateau** of

$$\Delta E_{\rm eff}(t) = \log \frac{R(t)}{R(t+1)} \longrightarrow E_{BB}^L - 2m_B$$

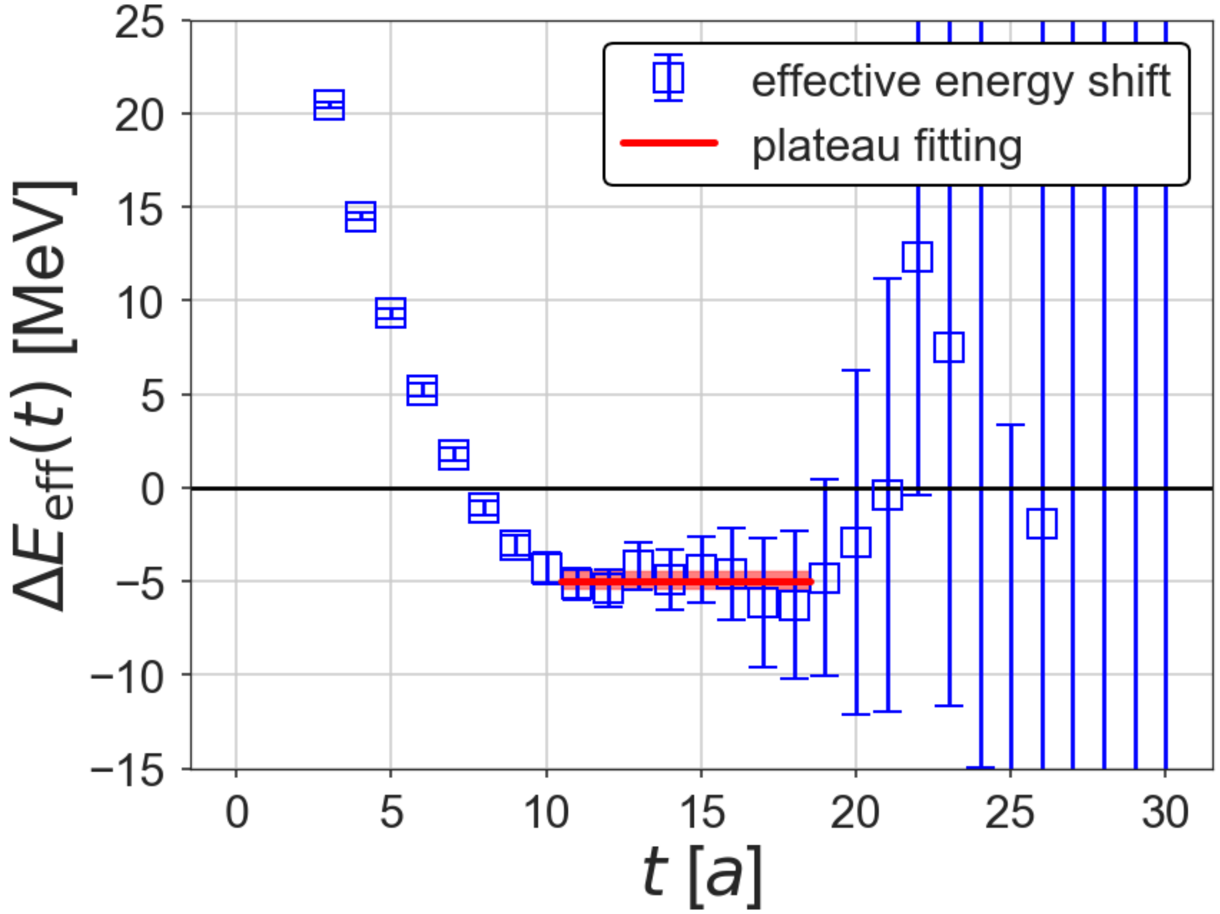
g.s. saturation is mandatory!

ex. **L = 8 fm** @ phys. mass

$$\delta E_{\rm el.} \simeq 25 \text{ MeV} \longrightarrow t > 10 \text{ fm}$$

$$S/N \sim \exp[-2(m_N - (3/2)m_\pi)t]$$

$$\longrightarrow 10^{-32}$$ **extremely difficult!**

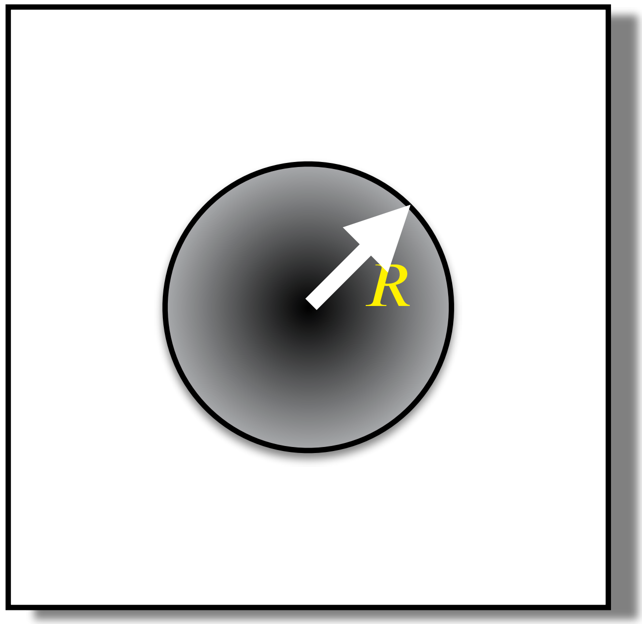


2 Hadrons in Lattice QCD (2) HAL QCD Method

Hadrons to Atomic nuclei from Lattice QCD

Ishii-Aoki-Hatsuda '06

- **spatial** correlation of the two-hadron



Nambu-Bethe-Salpeter wave func.

$$\psi_k(\vec{r}) = \langle 0 | B(\vec{r}) B(\vec{0}) | B(\vec{k}) B(-\vec{k}); \mathrm{in} \rangle$$

- **asymptotic region** $r \gg R$

$$\psi_k(\vec{r}) \simeq C \sin(kr - l\pi/2 + \delta(k))/(kr)$$

- **interacting region** $r < R$

$$(\nabla^2 + k^2)\psi_k(\vec{r}) = m \int d\vec{r'}\, U(\vec{r}, \vec{r'}) \psi_k(\vec{r'})$$

energy-indep. & non-local potential & faithful to δ

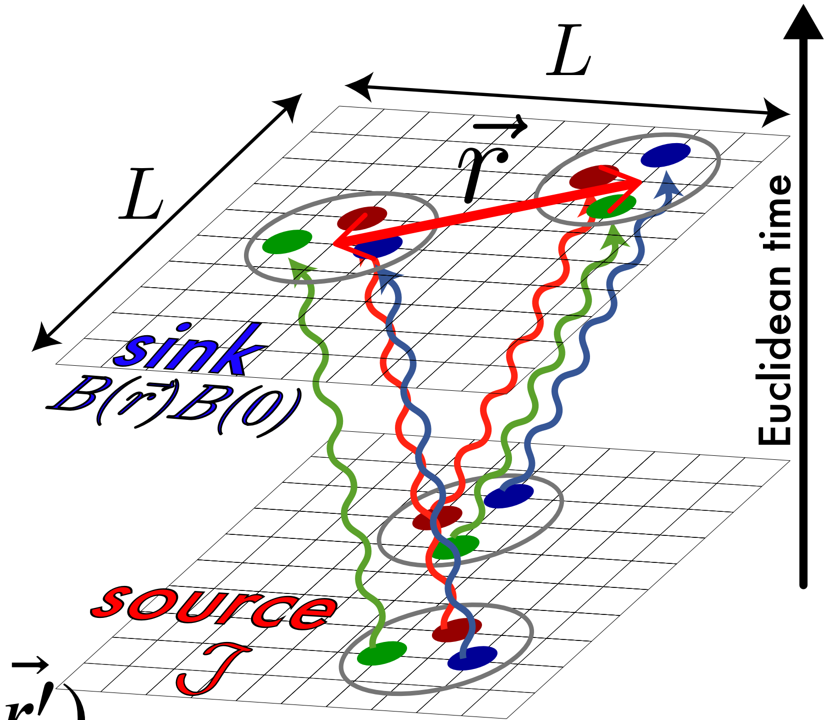
$$U(\vec{r}, \vec{r'}) = \sum_{|\vec{p}| \le p_{\mathrm{th}}} [E_p - H_0] \psi_p(\vec{r}) \psi_p^*(\vec{r'})$$

HAL QCD method: NBS wave func. → U(r,r') → observables

"Time-dependent" HAL QCD Method (1)

Ishii for HAL QCD '12

$$R(\vec{r},t) \equiv \langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle/\{C_B(t)\}^2$$
$$= \sum_n a_n \psi_{W_n}(\vec{r}) e^{-(W_n-2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th} t})$$



g.s. & scattering state NBS funcs. satisfy

they are signals

$$[E_{W_0} - H_0]\psi_{W_0}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_{W_0}(\vec{r'})$$

$$[E_{W_1} - H_0]\psi_{W_1}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_{W_1}(\vec{r'})$$

$$\vdots$$

$$\mathcal{O}(e^{-\delta E_{\rm inel} t}) \ll \mathcal{O}(e^{-\delta E_{\rm el} t})$$

with **elastic saturation (exponentially easier** than **g.s. saturation!)**

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

"Time-dependent" HAL QCD Method (2)

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

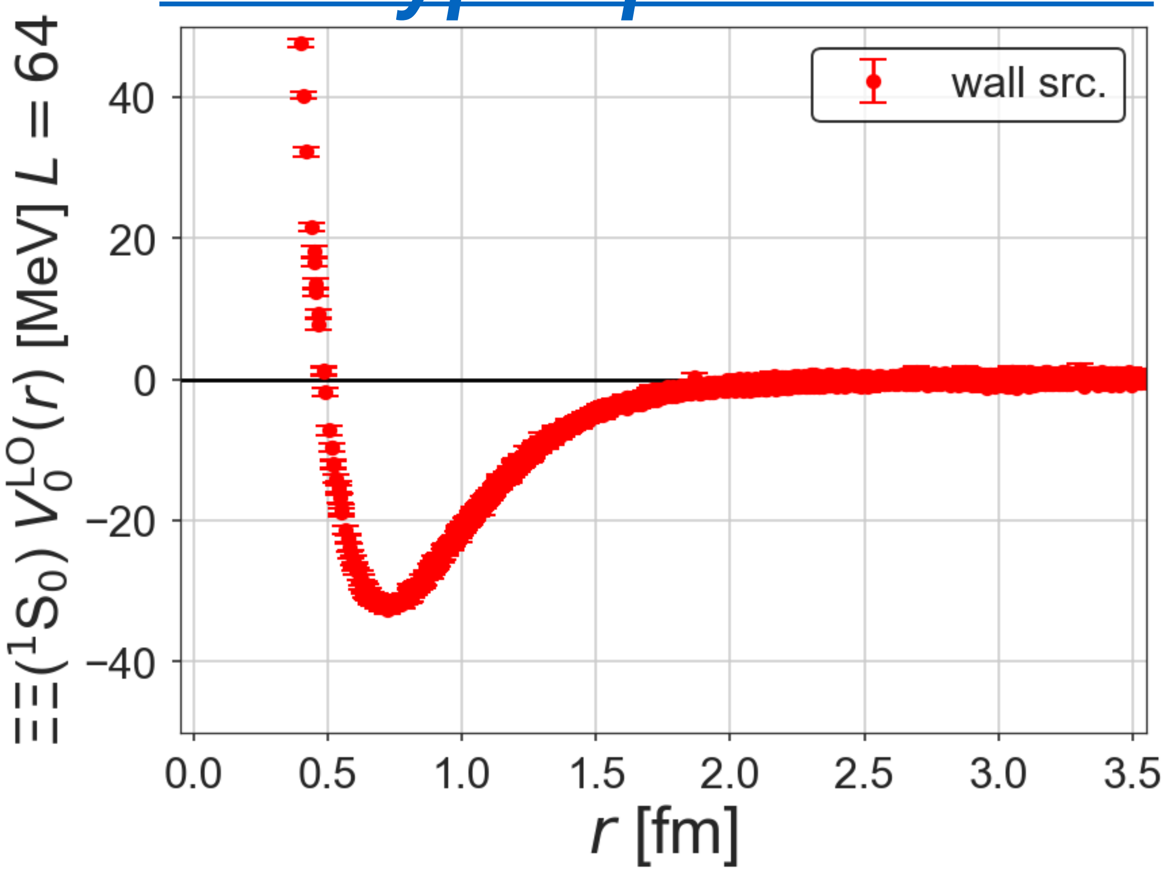
local pot. by ***velocity expansion*** of **E-indep.** & **non-local** pot. **U(r,r')**

$$U(\vec{r},\vec{r'}) = \left[V_0(\vec{r}) + V_1(\vec{r})\mathbf{L}\cdot\mathbf{S} + V_2(\vec{r})\nabla^2 + \cdots\right]\delta(\vec{r}-\vec{r'})$$

previous results by **HAL QCD** mainly employ

"**leading order approximation**" of ***wall-type quark source***

$$U(\vec{r},\vec{r'}) \simeq V_0^{\rm LO}(\vec{r})\delta(\vec{r}-\vec{r'})$$

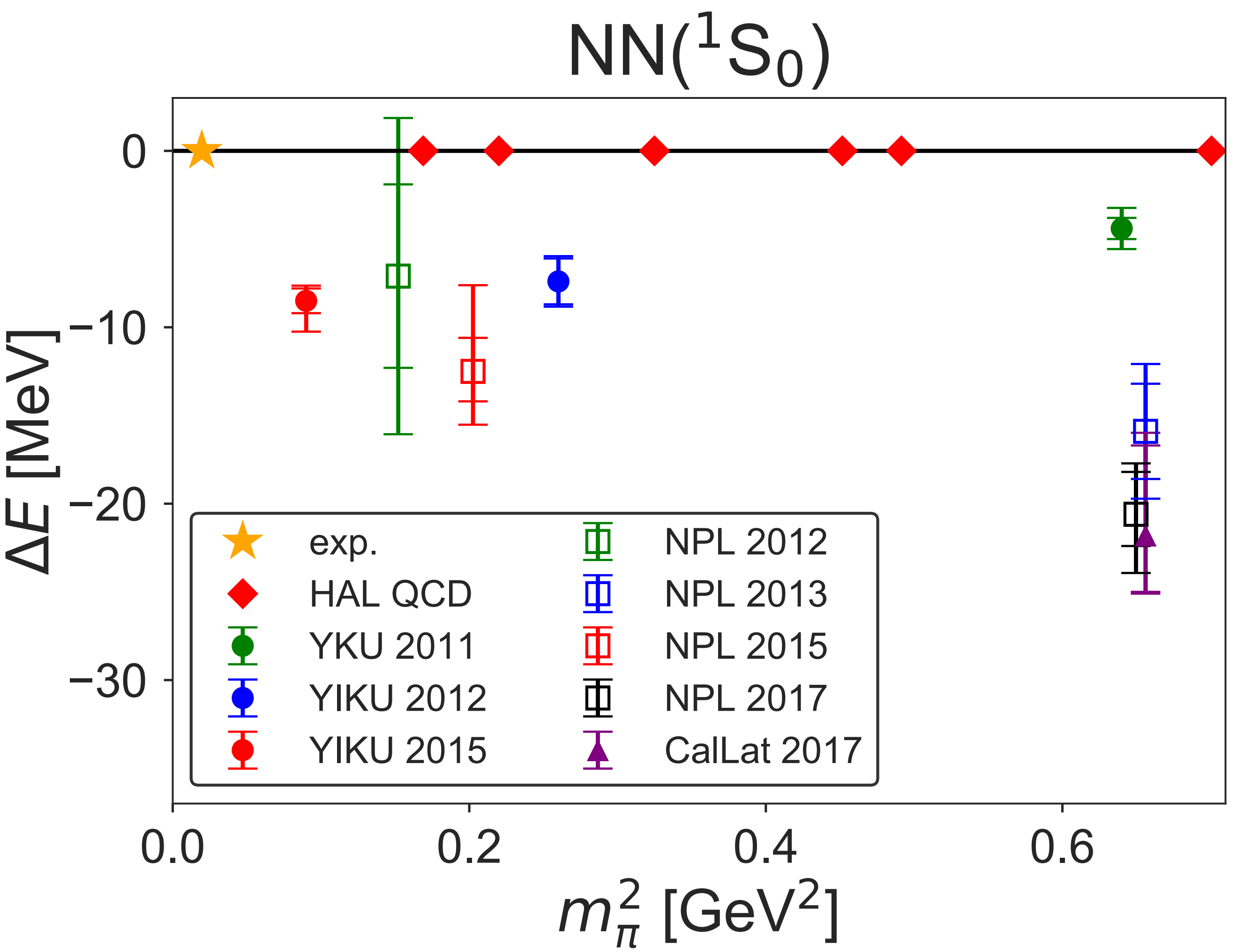


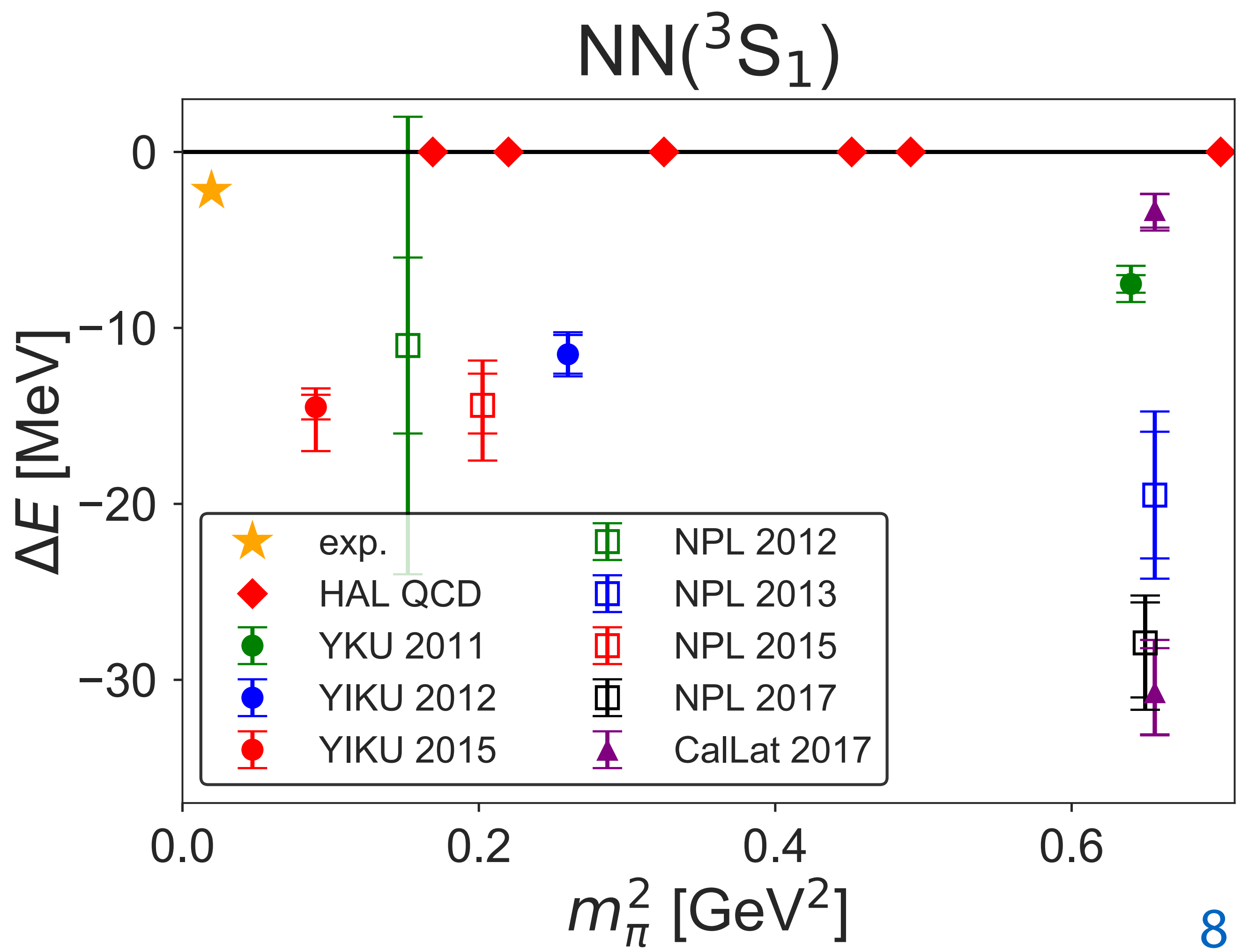
Q: Is this expansion reasonable & converged?

NN Systems from Lattice QCD

- *Inconsistency between these methods*
- *It was a long-standing problem in lattice QCD.*

	"Direct"	HAL QCD	phys. point
Dineutron ($^1S_0$)	bound	unbound	unbound
Deuteron ($^3S_1$)	bound	unbound	bound

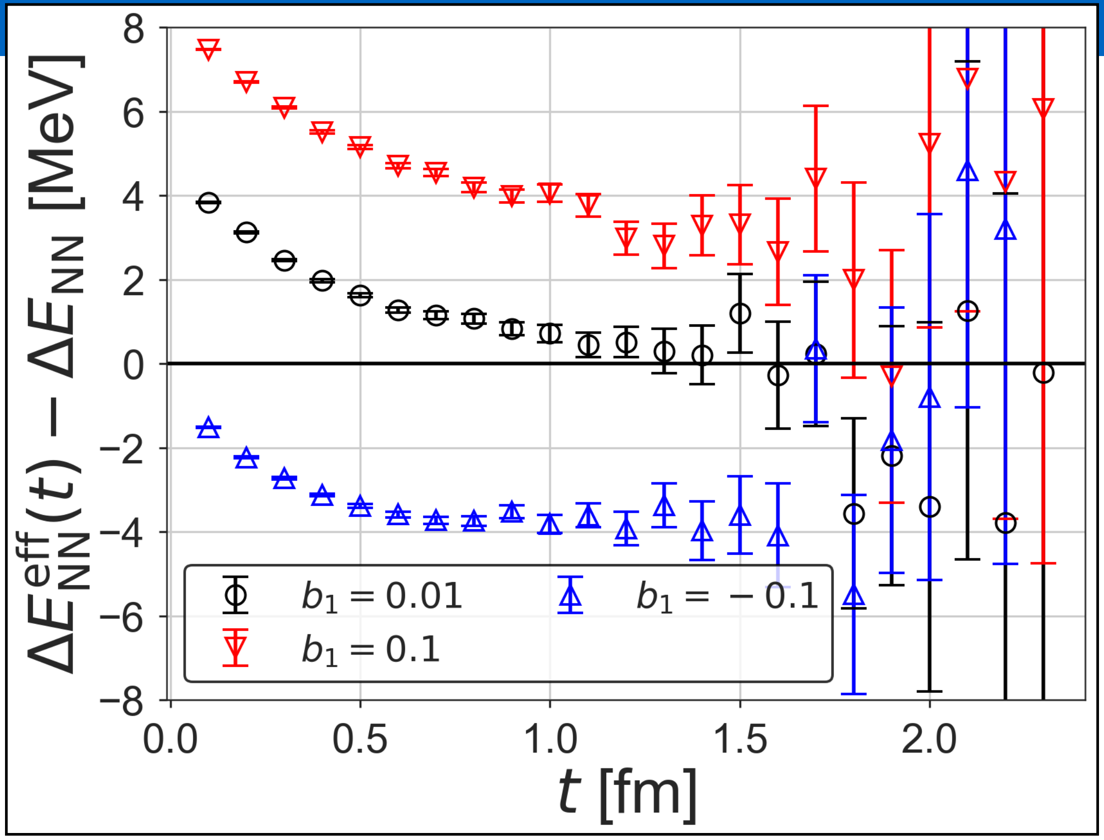




Fundamental Problem in Direct Method: Fake Plateau

Simple plateau fitting is challenging.

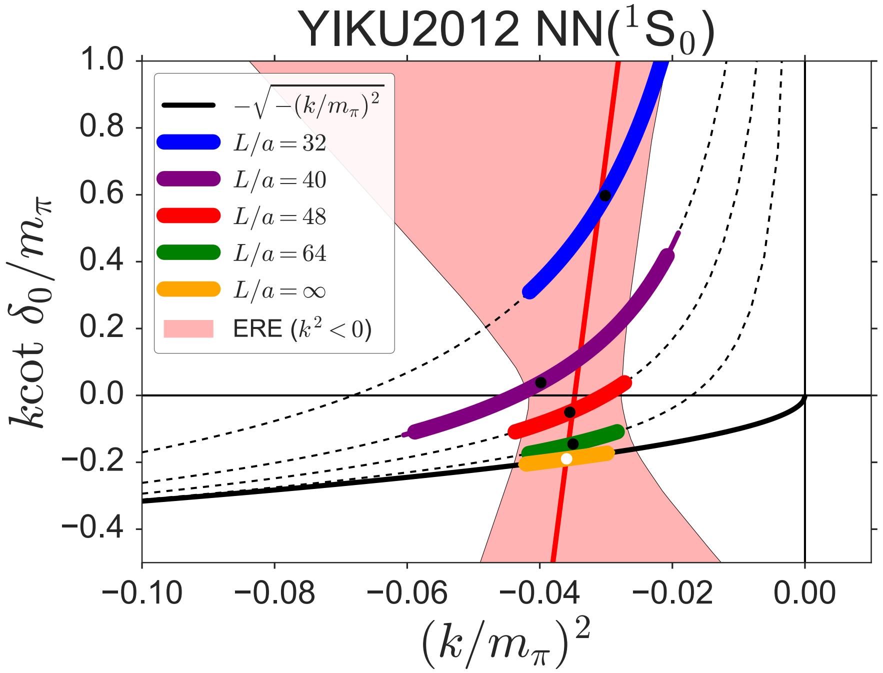
ground state saturation requires
huge temporal correlation



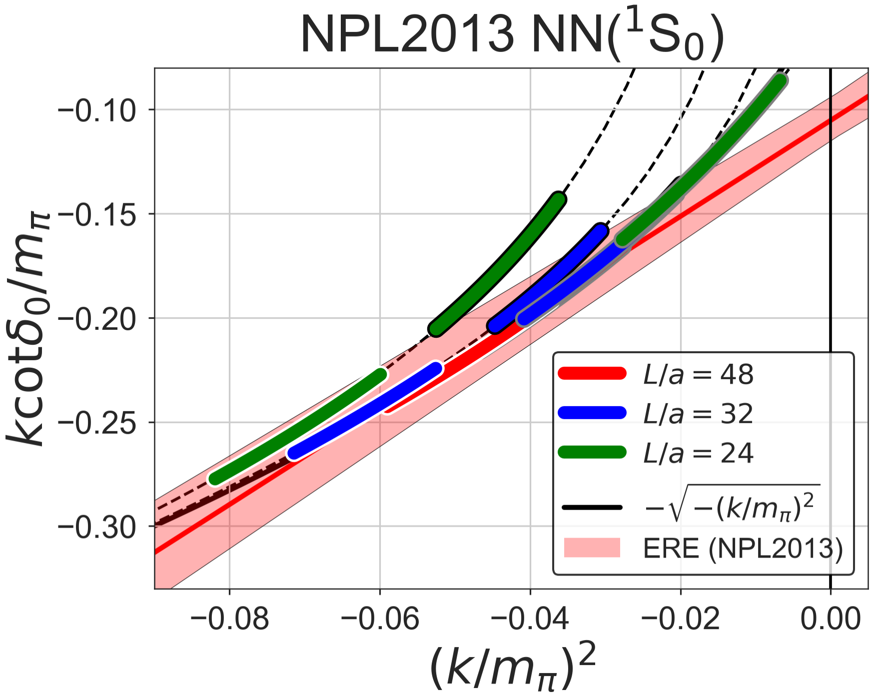
Normality Check Aoki's Talk (Wed)

previous studies show "anomalous behavior."

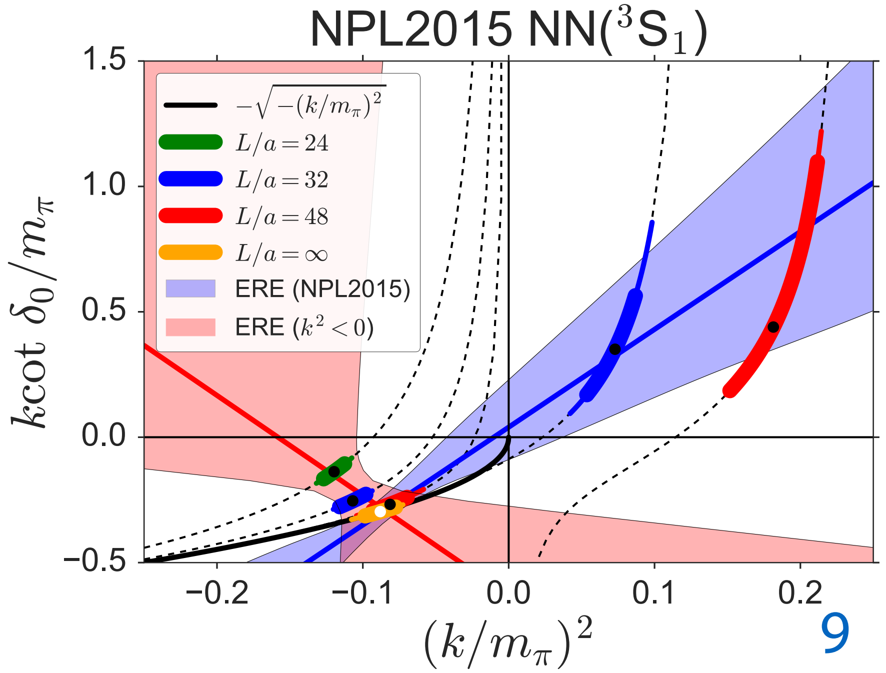
singular ERE



unphysical pole



unphys. & mismatch



Next: Reliability of the HAL QCD Method

Part I: Systematics in the HAL QCD Method

Ref.

TI for HAL QCD Coll., PoS(Lattice 2016) 109, arXiv:1610.09763.

& in prep

1. convergence of velocity expansion

2. inelastic state contamination

Setup & Channel: $\Xi\Xi(^1\mathrm{S}_0)$ at $m_\pi = 0.51$ GeV

better signal & the same SU(3) rep. as $\mathrm{NN}(^1\mathrm{S}_0)$

time-dep. HAL QCD method

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r}, t) = \int d\vec{r'} U(\vec{r}, \vec{r'}) R(\vec{r'}, t)$$

local pot. by ***velocity expansion*** of **E-indep.** & **non-local** pot. **U(r,r')**

LO approximation (**standard in previous. HAL QCD works**)

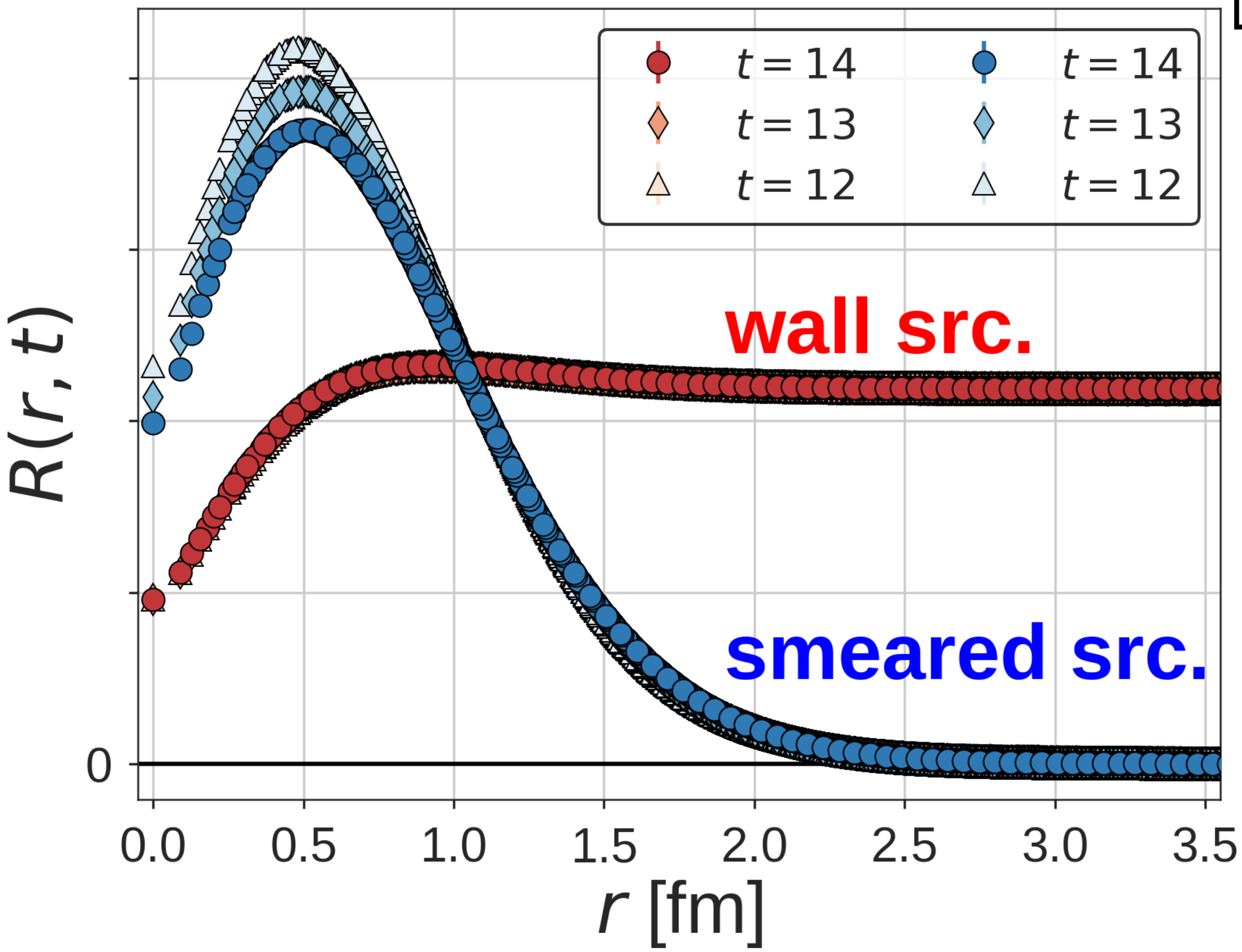
$$U(\vec{r}, \vec{r'}) \simeq V_0^{\mathrm{LO}}(\vec{r})\delta(\vec{r} - \vec{r'})$$

N²LO approximation & **$V_2(r)$**

$$U(\vec{r}, \vec{r'}) \simeq [V_0^{\mathrm{N^2LO}}(\vec{r}) + V_2^{\mathrm{N^2LO}}(\vec{r})\nabla^2]\delta(\vec{r} - \vec{r'})$$

→ **Check: higher order correction of the HAL QCD pot.**
by using **wall-type** & **smeared-type** quark sources

Time Dependence of the Spatial Correlation



wall src. – **delocalized** & **weak** t-dep.
smeared src. – **localized** & **strong** t-dep.

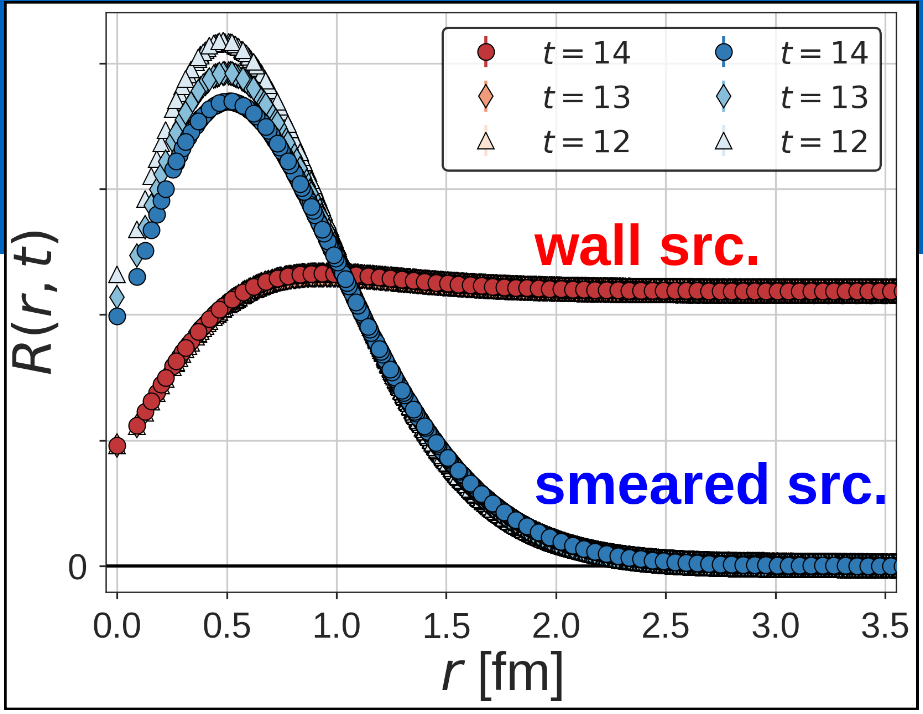
$$R(\vec{r}, t) \equiv C_{\mathrm{BB}}(\vec{r}, t) / \{C_{\mathrm{B}}(t)\}^2$$
$$= \sum_n a_n \Psi_n(\vec{r}) e^{-\Delta E_n t}$$

src. & **time dependence**

These correlators are mixtures of excited states.

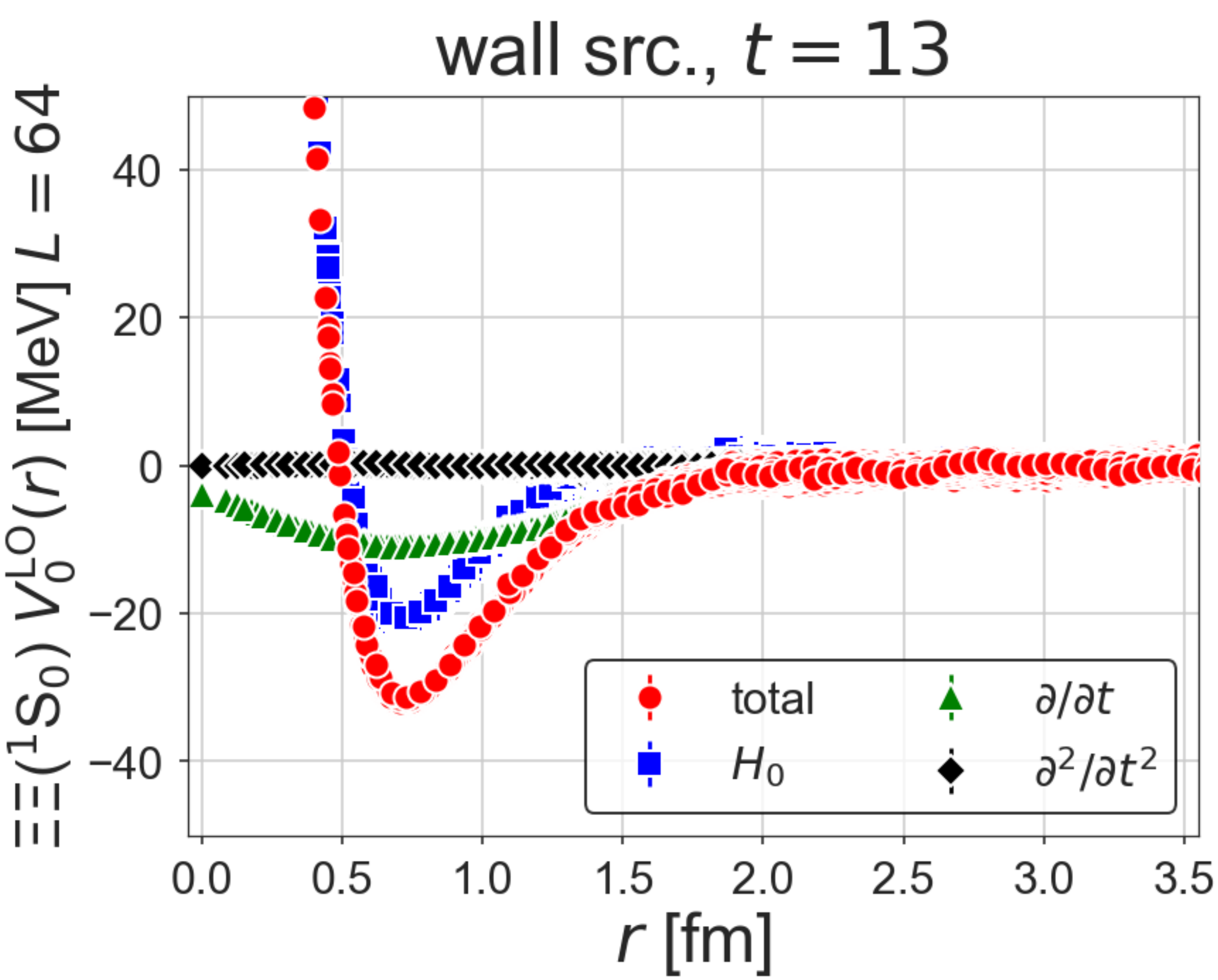
time-dep. HAL QCD method is essential!

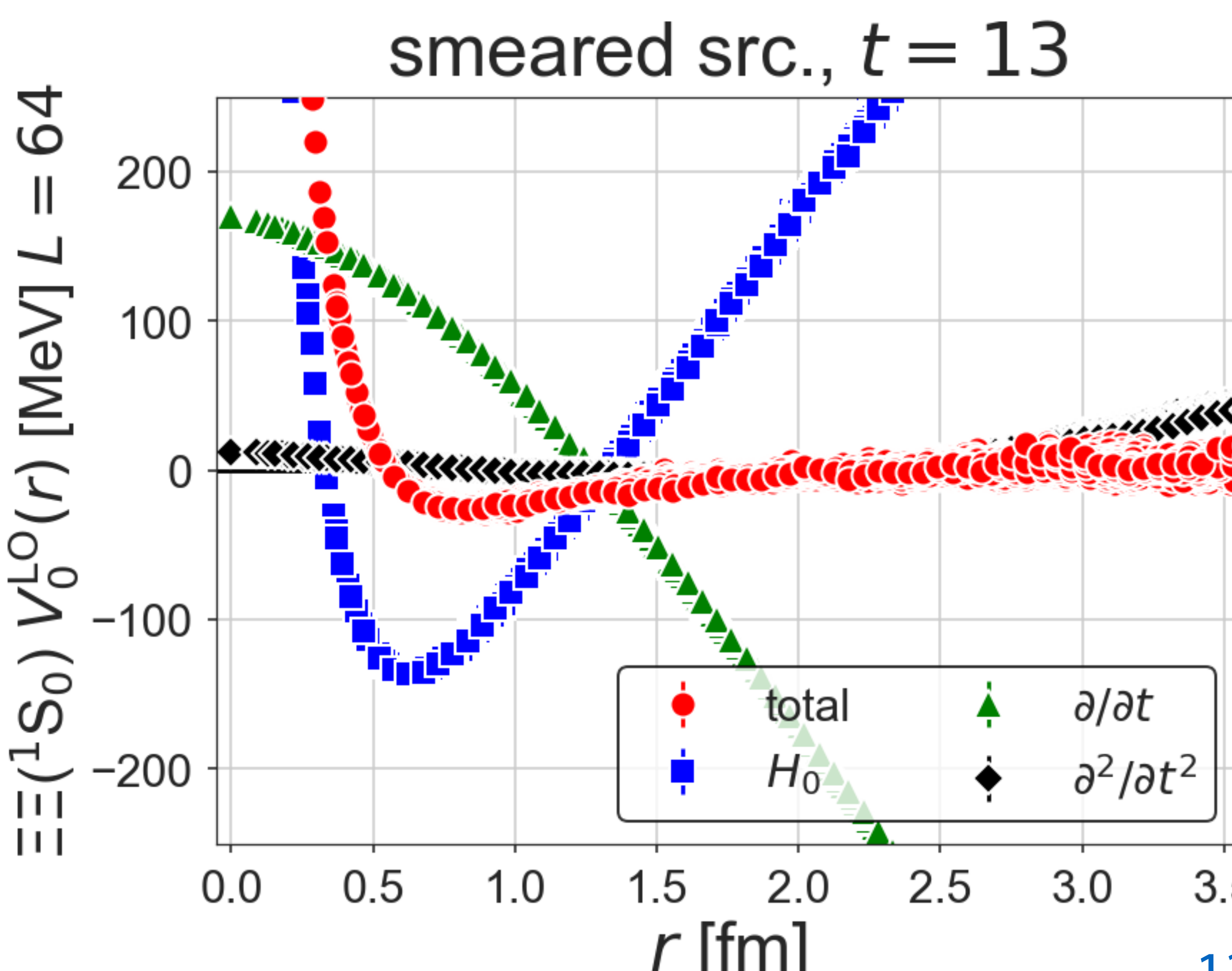
Potential (Leading Order) (1)



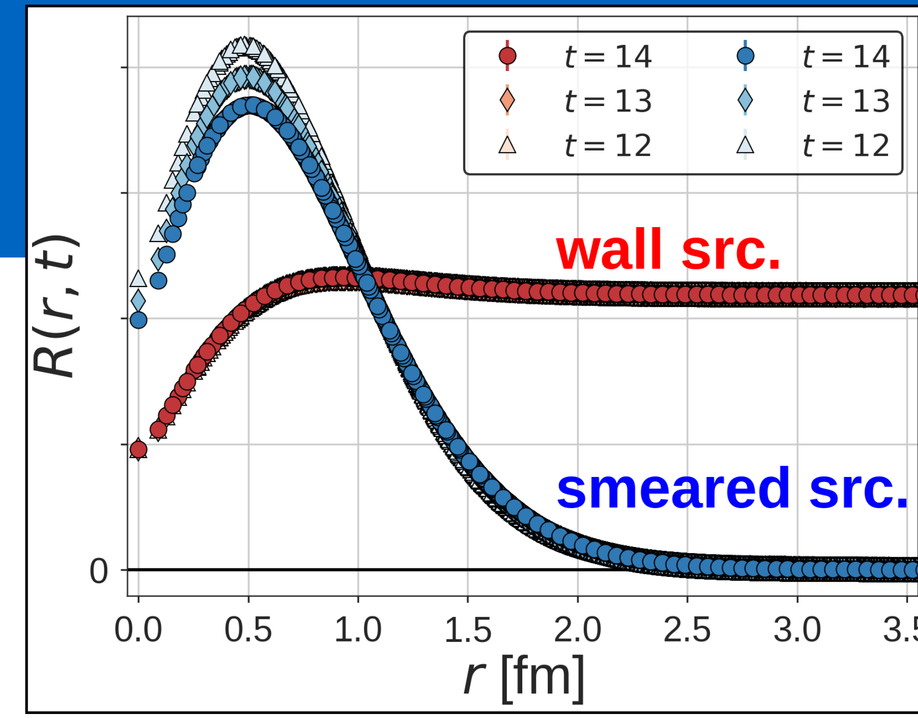
$$V_0^{\mathrm{LO}}(r) = \frac{1}{4m_B}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R}$$

R-corr. differs, but *qualitatively* the same behavior





Potential (Leading Order) (2)

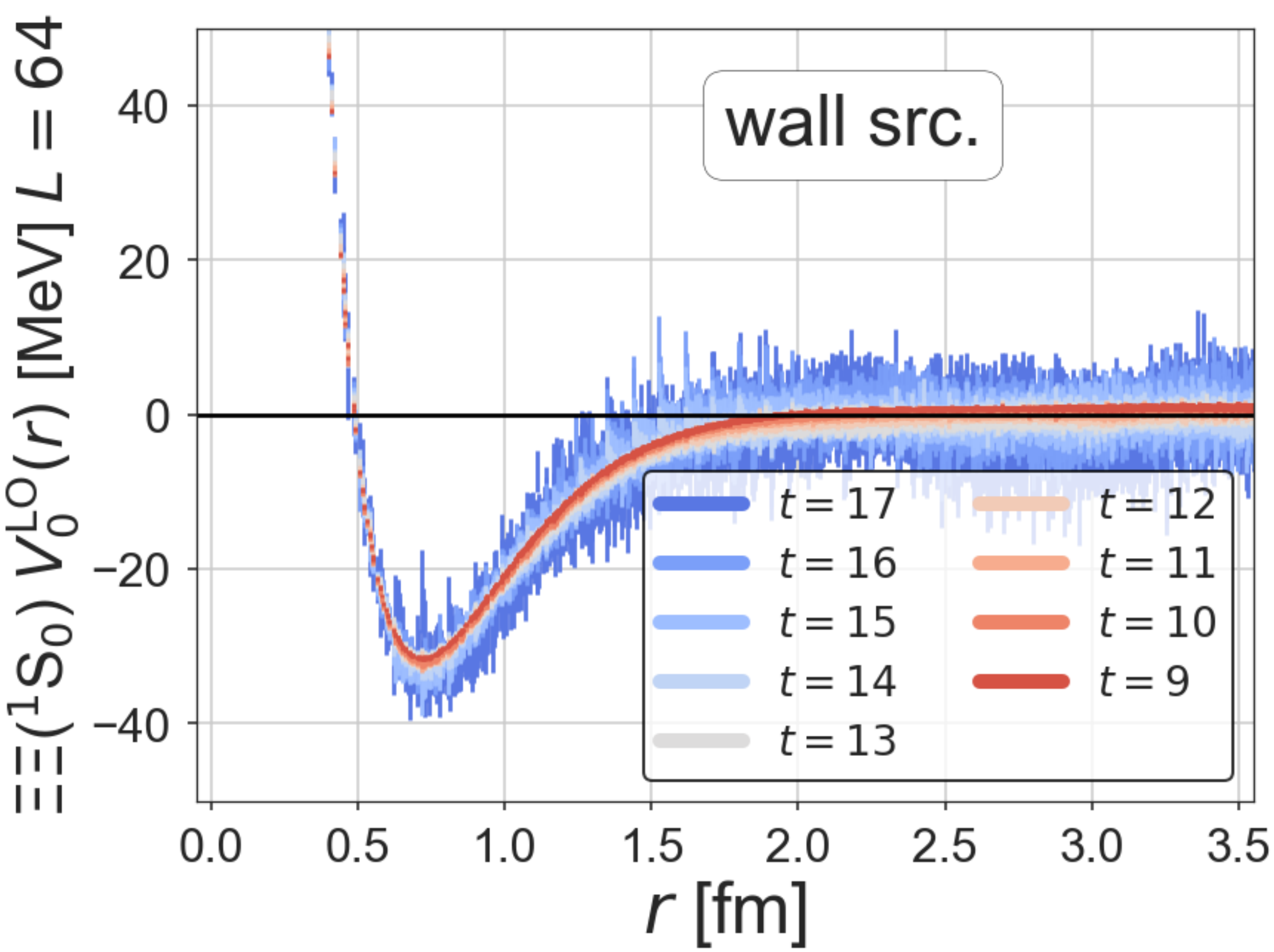


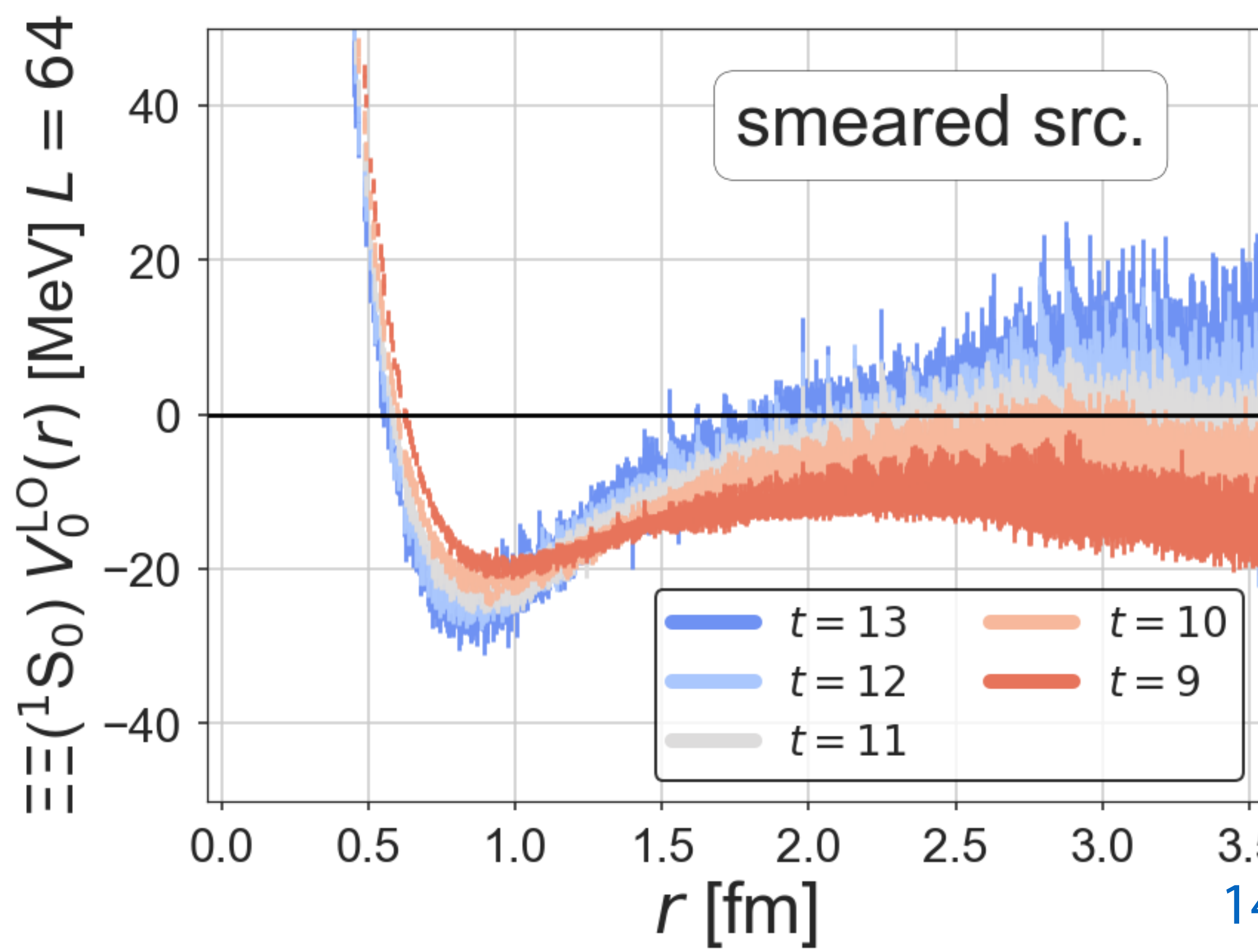
$$V_0^{\mathrm{LO}}(r) = \frac{1}{4m_B}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R}$$

R-corr. differs, but *qualitatively* the same behavior

wall src. – t-independent

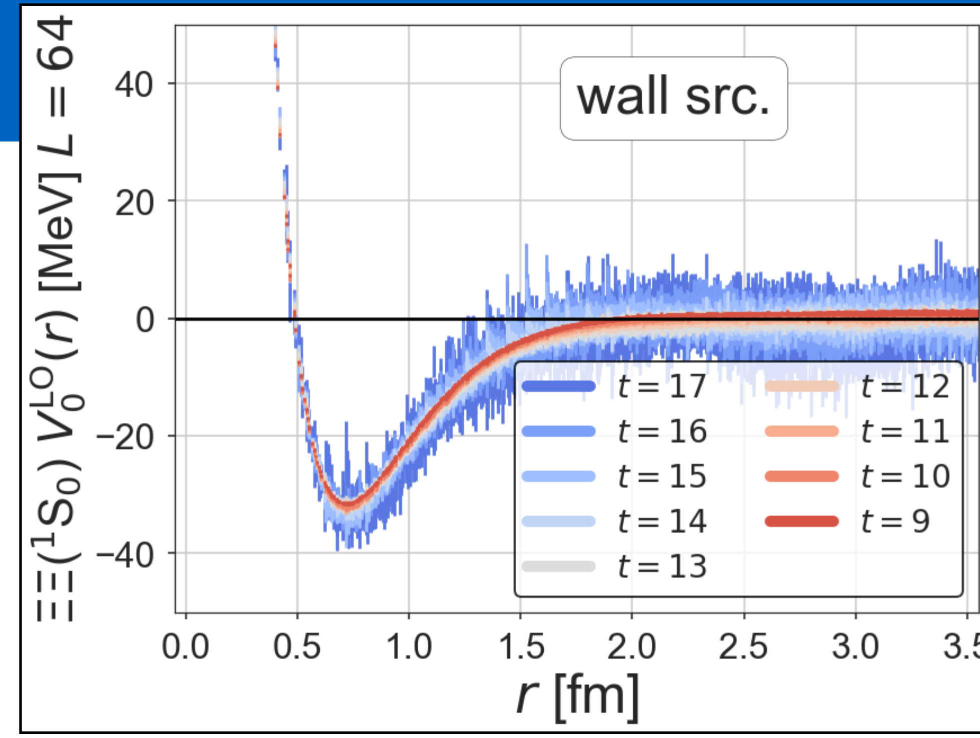
smeared src. – t-dependent



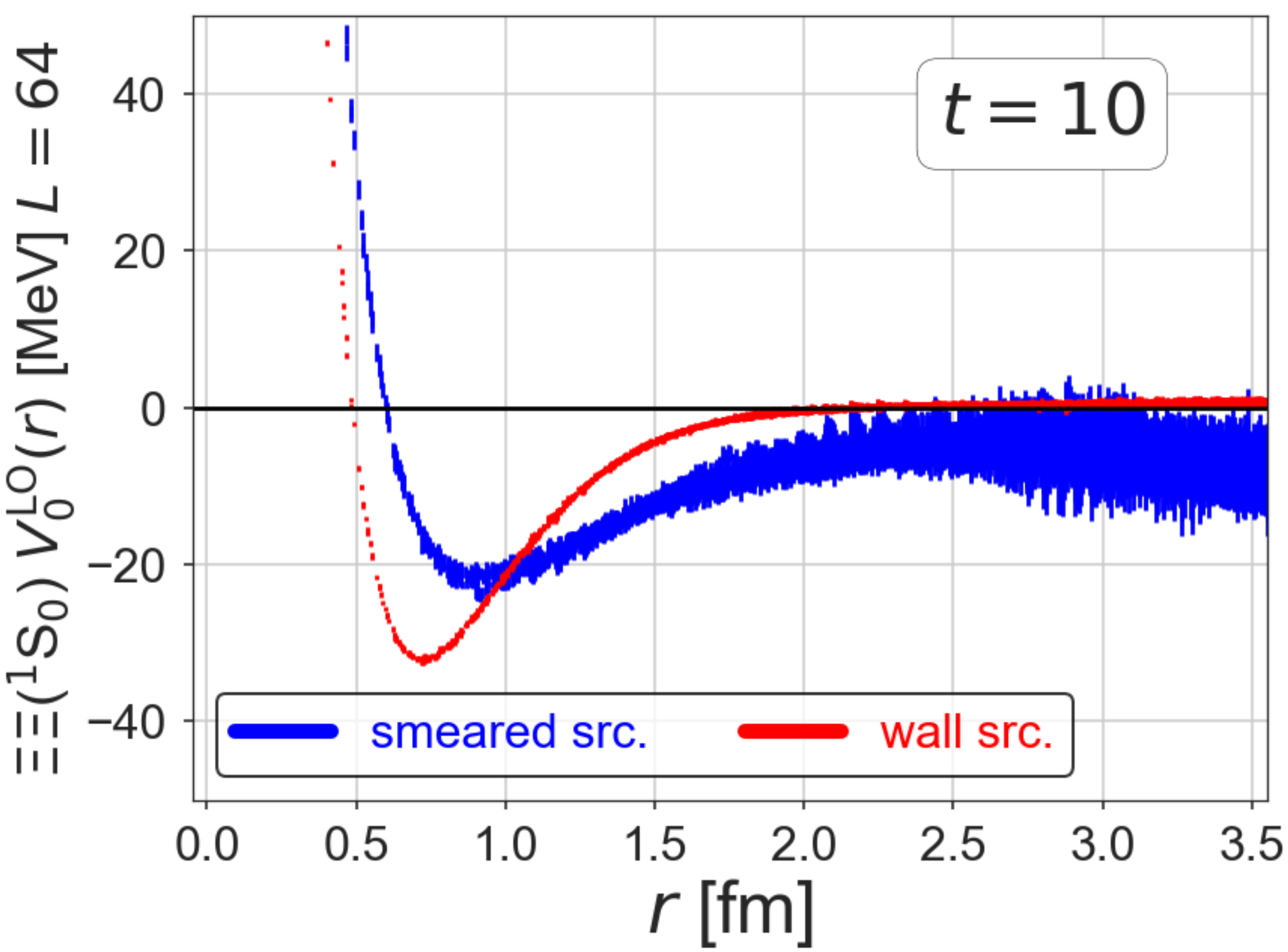


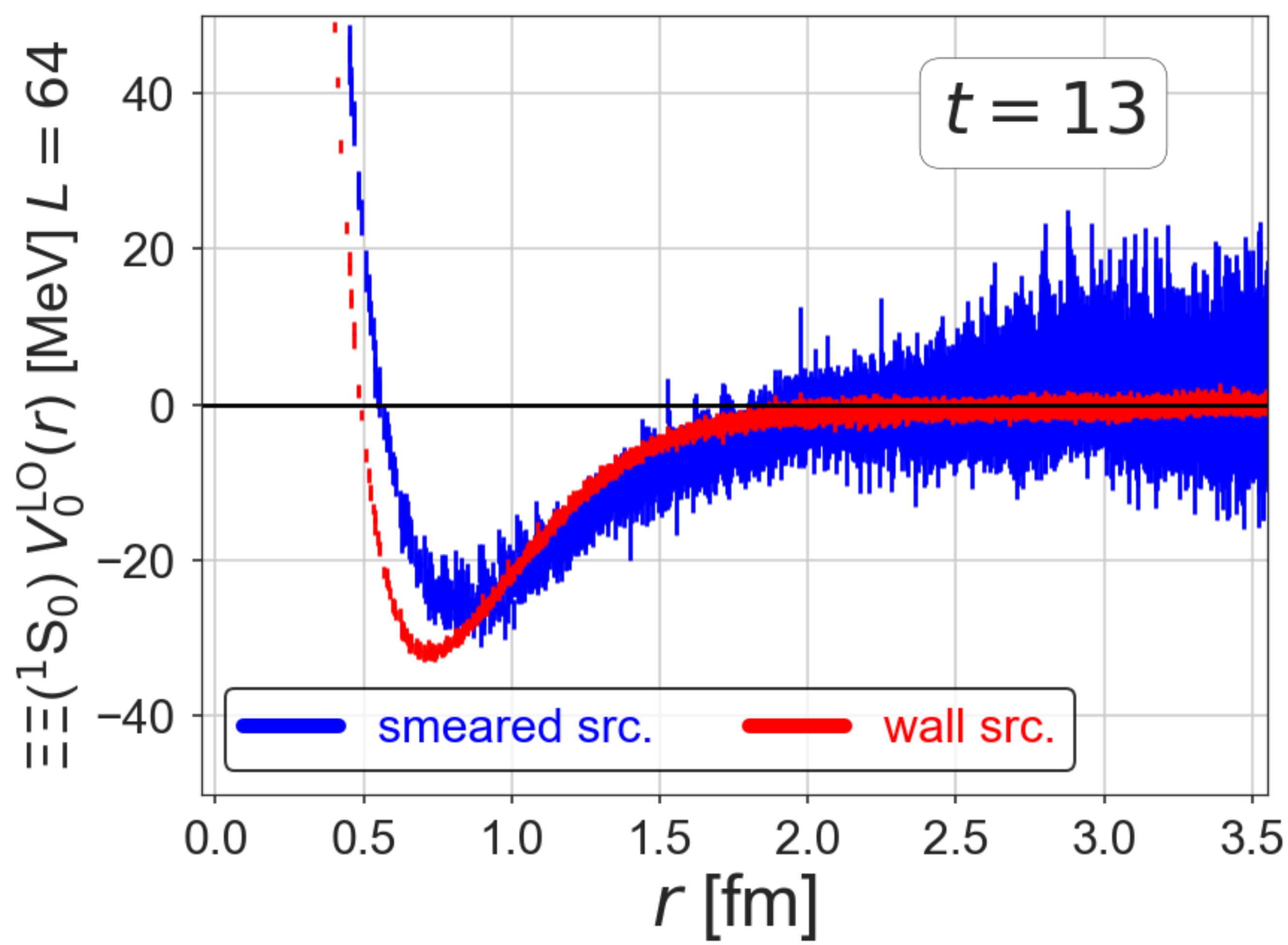
Systematic in LO Potential

- **Wall** src. = time-indep.
- **Smeared** src. → **Wall** src.



t-dep. implies systematics from truncation

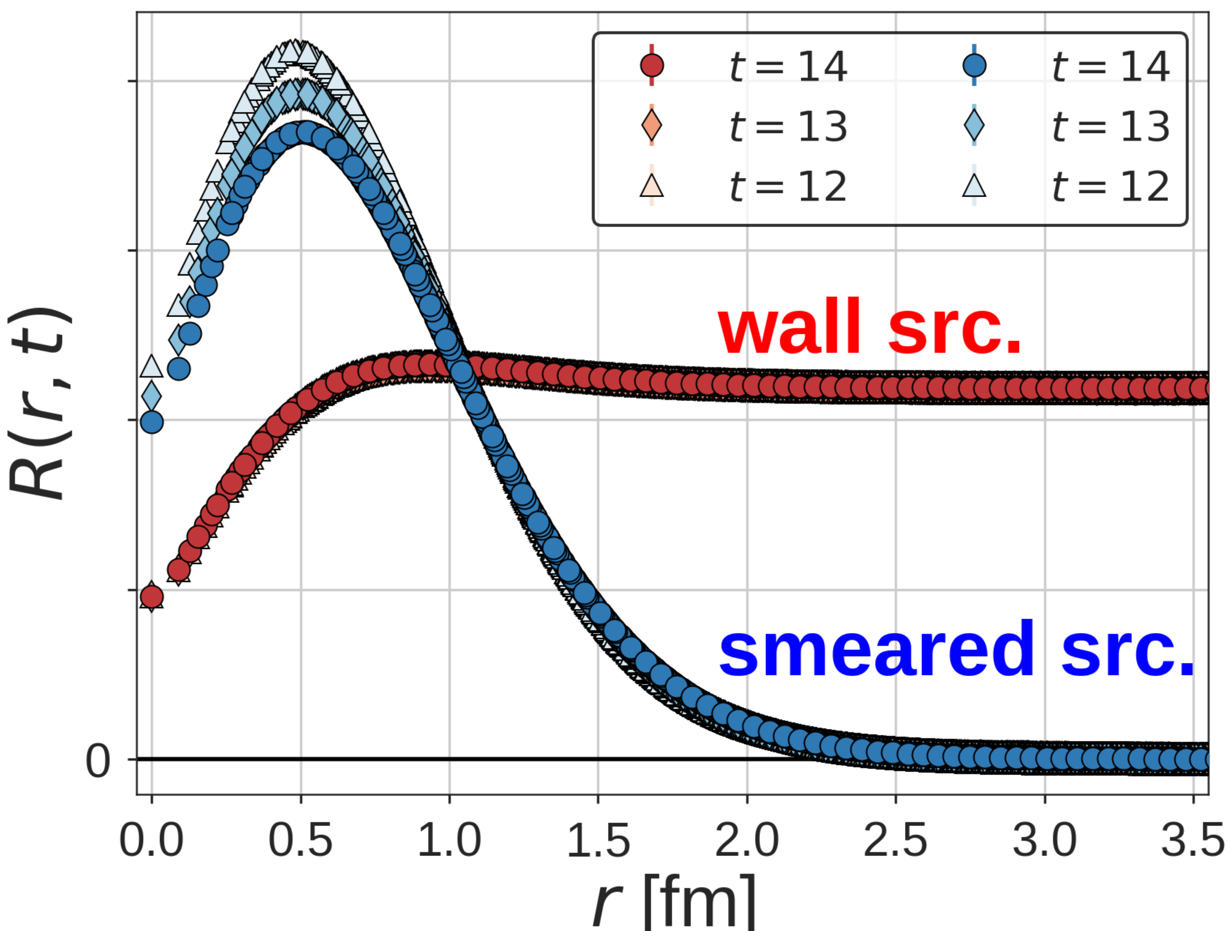




Higher Order Approximation (N²LO) (1)

$$U(r,r') \simeq \left[V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\nabla^2\right]\delta(r-r')$$

$$\begin{cases} \dfrac{1}{4m_B}\dfrac{(\partial^2/\partial t^2)R^{\mathrm{wall}}}{R^{\mathrm{wall}}} - \dfrac{(\partial/\partial t)R^{\mathrm{wall}}}{R^{\mathrm{wall}}} - \dfrac{H_0 R^{\mathrm{wall}}}{R^{\mathrm{wall}}} = V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\dfrac{\nabla^2 R^{\mathrm{wall}}}{R^{\mathrm{wall}}} \\ \dfrac{1}{4m_B}\dfrac{(\partial^2/\partial t^2)R^{\mathrm{smear}}}{R^{\mathrm{smear}}} - \dfrac{(\partial/\partial t)R^{\mathrm{smear}}}{R^{\mathrm{smear}}} - \dfrac{H_0 R^{\mathrm{smear}}}{R^{\mathrm{smear}}} = V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\dfrac{\nabla^2 R^{\mathrm{smear}}}{R^{\mathrm{smear}}} \end{cases}$$



$V_0^{\mathrm{N^2LO}}(r)$ and $V_2^{\mathrm{N^2LO}}(r)$

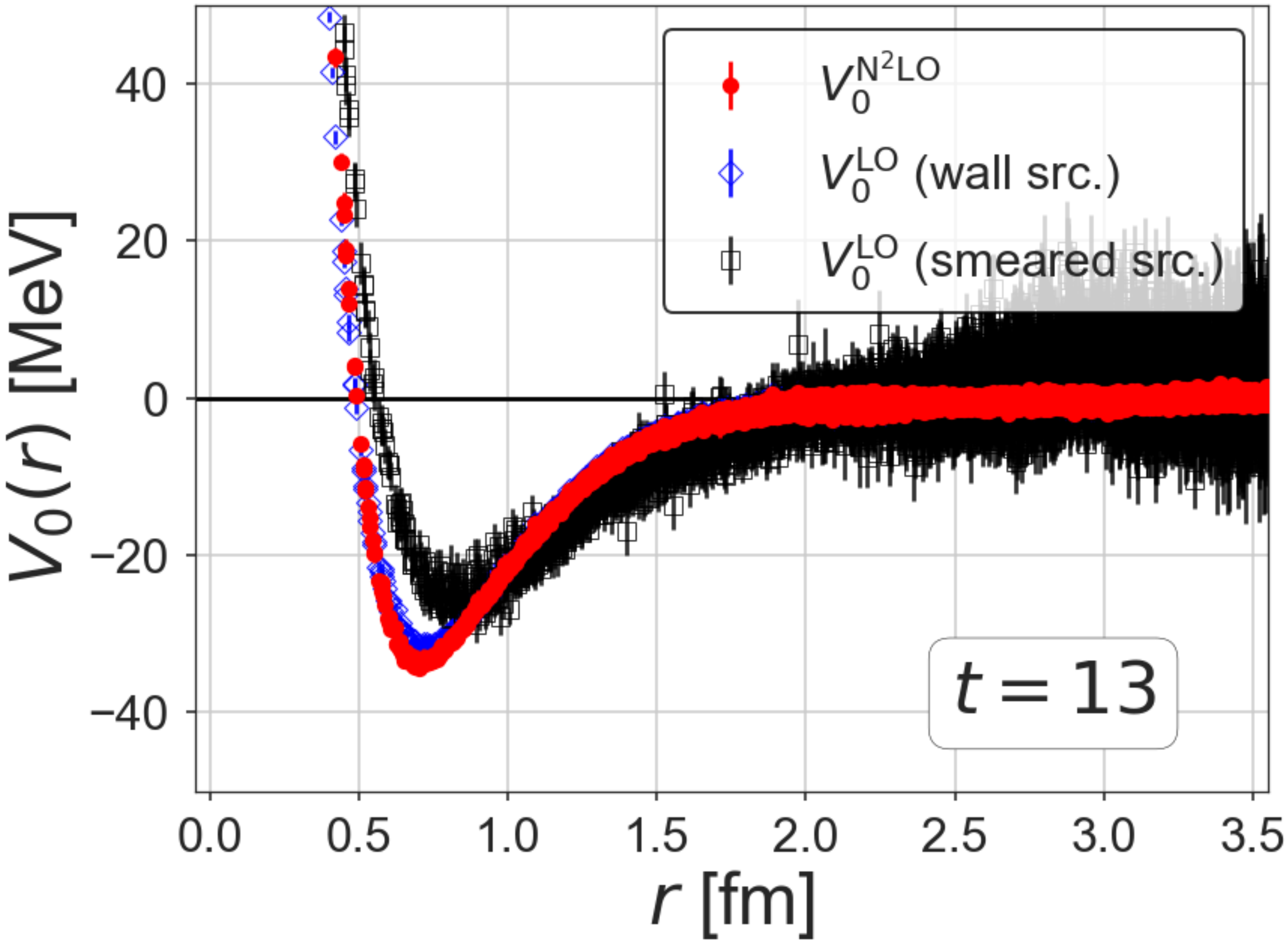
Higher Order Approximation (N^2LO) (2)

$$U(r,r') \simeq \left[V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\nabla^2\right]\delta(r-r')$$

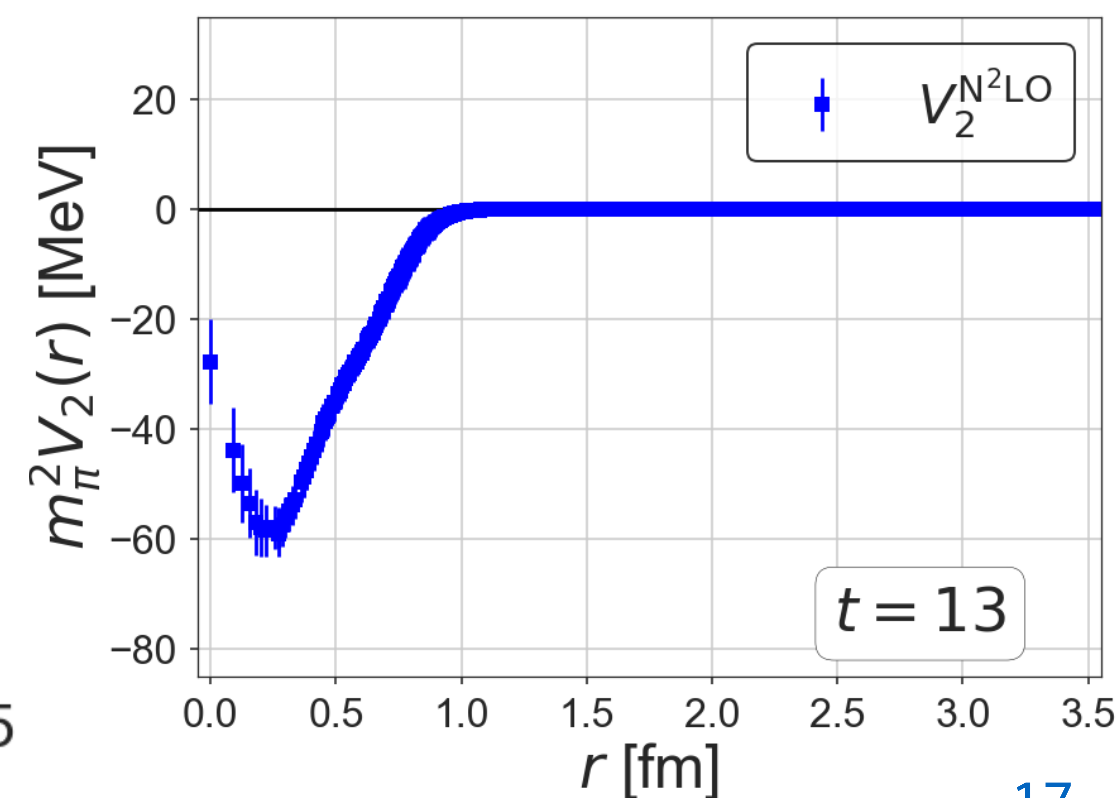
wall src. –> small $\mathbf{V_2\nabla^2}$ **correction**
smeared src. –> large $\mathbf{V_2\nabla^2}$ **correction**

$$V_2(r)\underline{\nabla^2 R^{\mathrm{wall/smear}}(r)}$$

dep. on shape of **R**

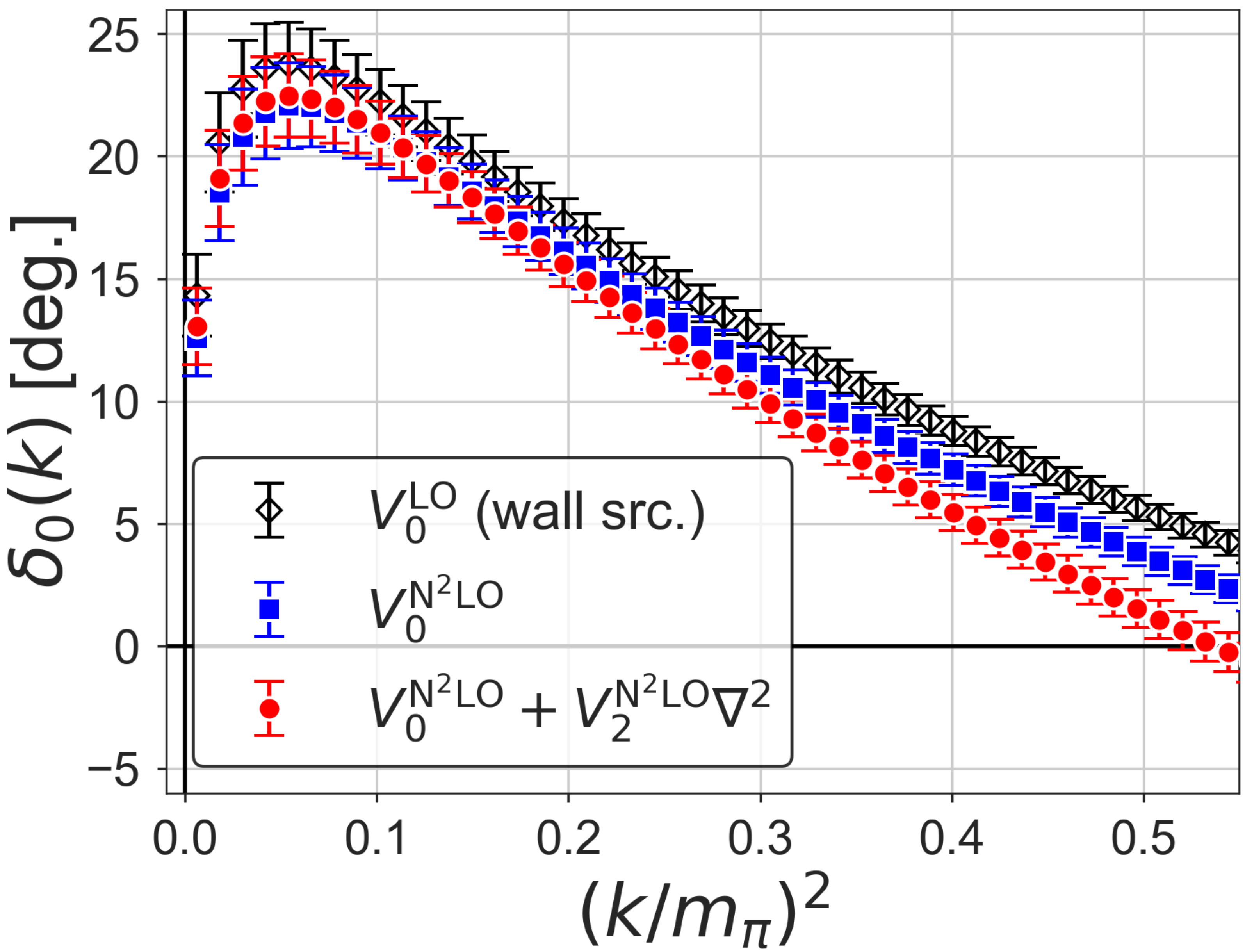


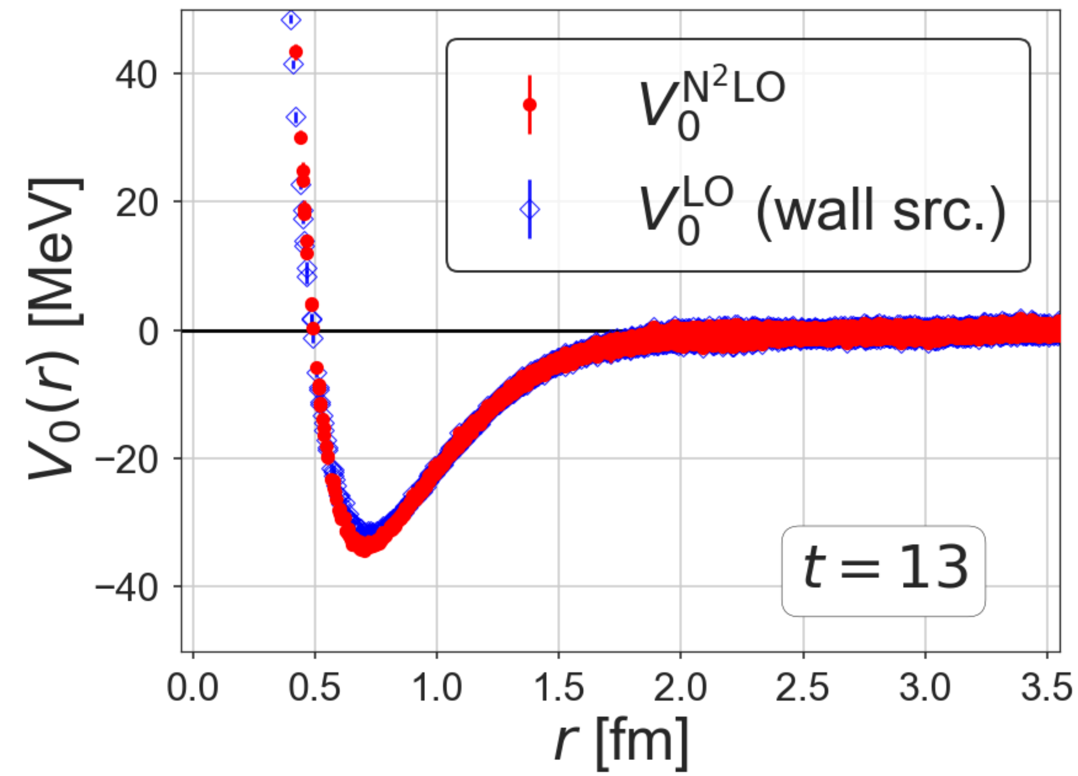
V_2-pot.

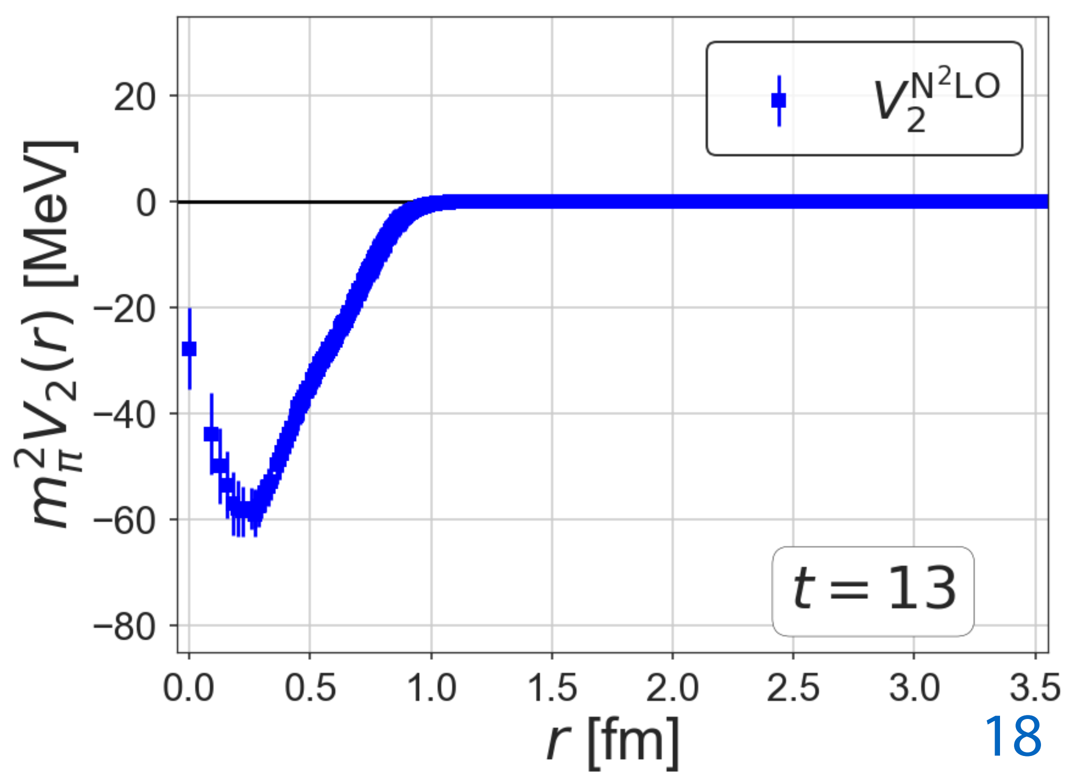


Phase Shift and Uncertainties in Velocity Expansion

- **Wall src. LO approx.** (standard of HAL QCD studies) **works well at low energy.**
- **V_2 correction** at high energy





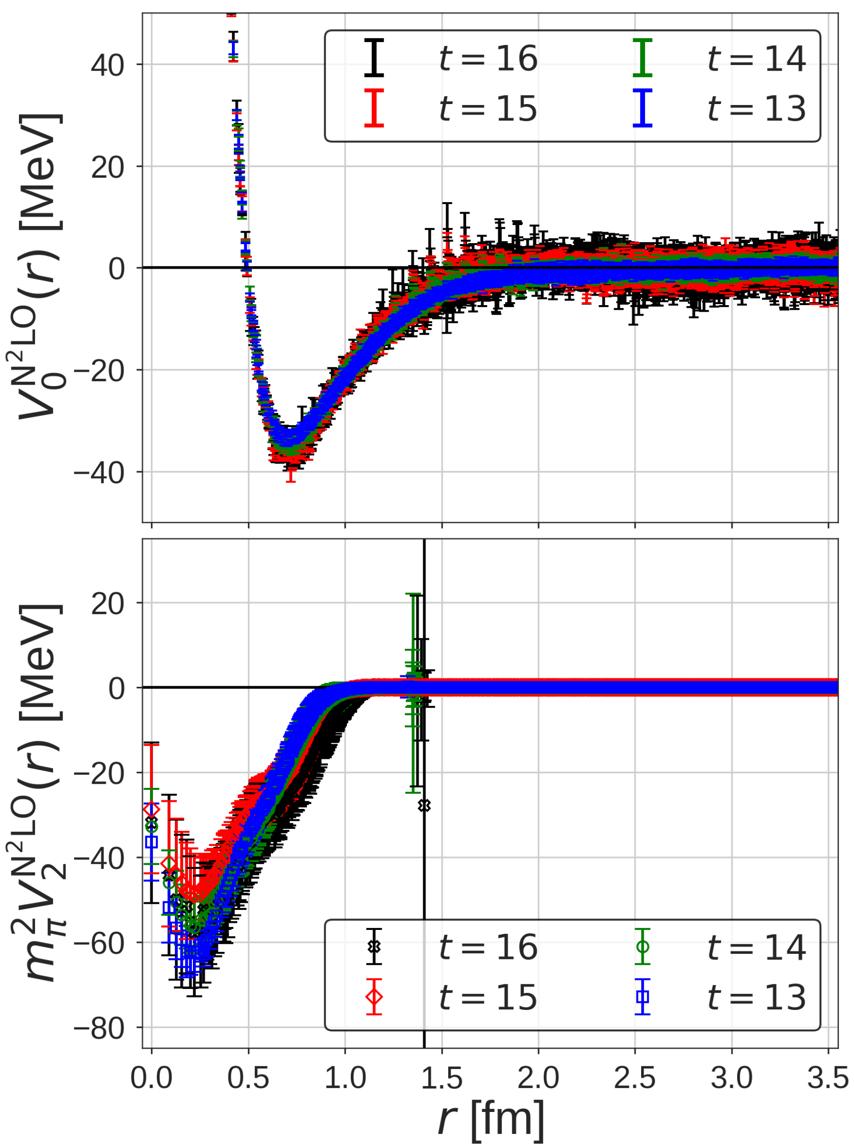


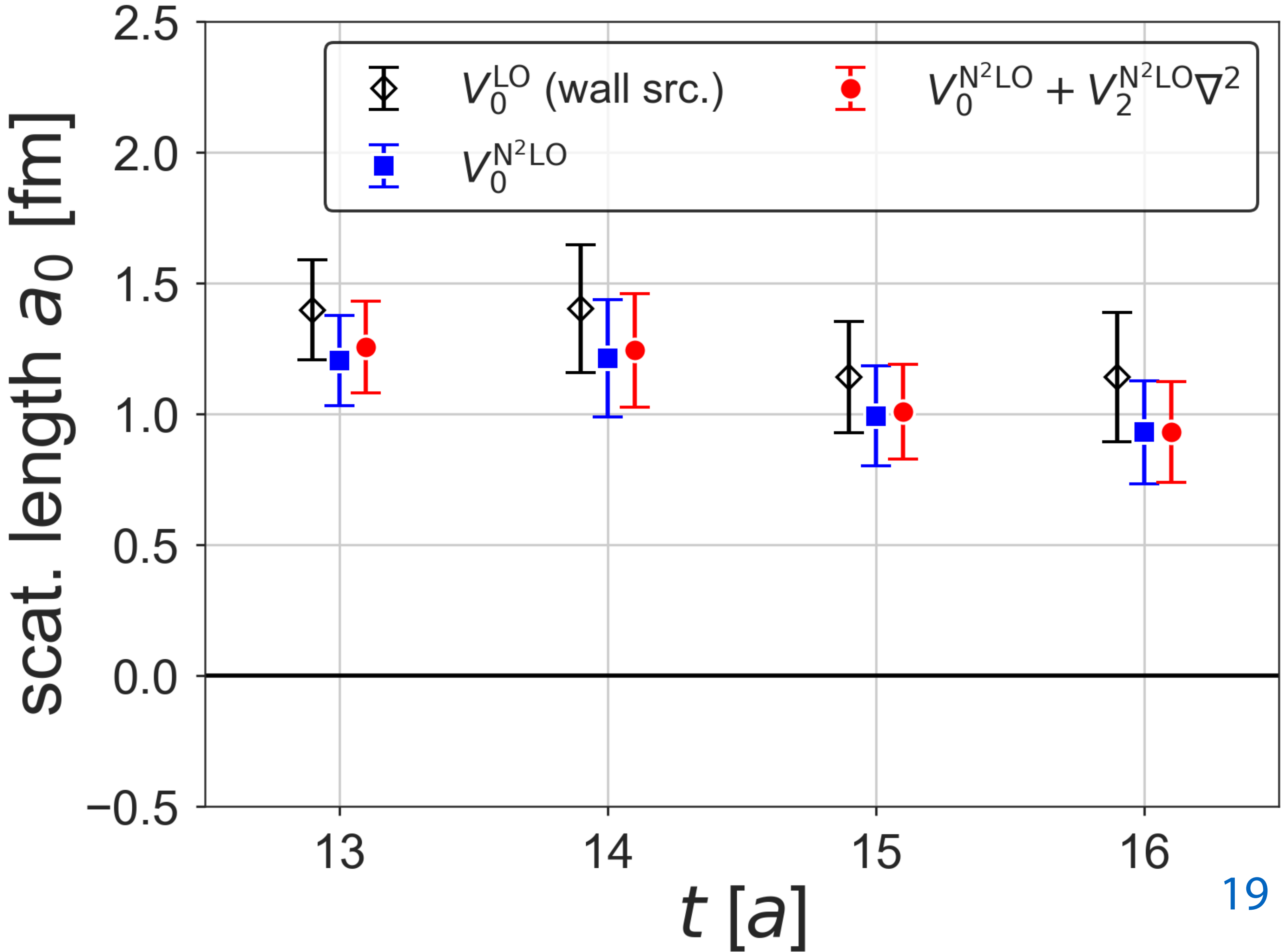
Time-dependence of Potential (N^2LO)

- **N^2LO** velocity expansion $U(r,r') \simeq \left[V_0^{N^2LO}(r) + V_2^{N^2LO}(r)\nabla^2\right]\delta(r-r')$

  t-indep. => convergence of the velocity expansion up to **N^2LO**

Systematics in velocity expansion are under control.





Part I: Systematics in the HAL QCD Method

Ref.

TI for HAL QCD Coll., PoS(Lattice 2016) 109, arXiv:1610.09763.

& in prep

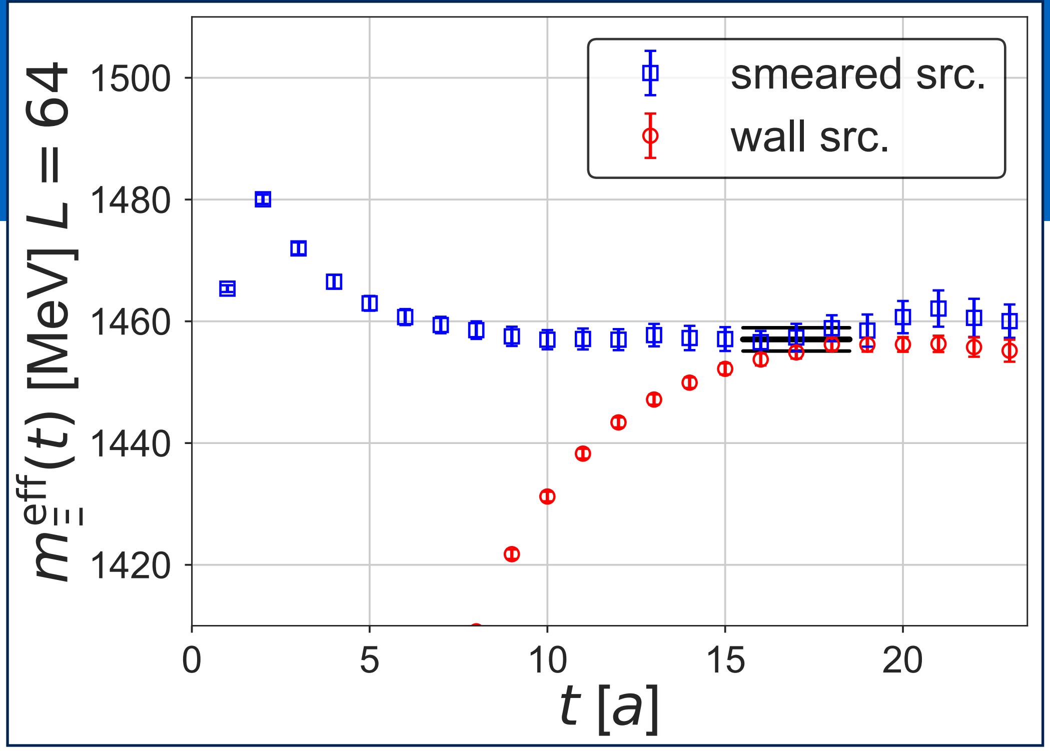
1. convergence of velocity expansion

2. inelastic state contamination

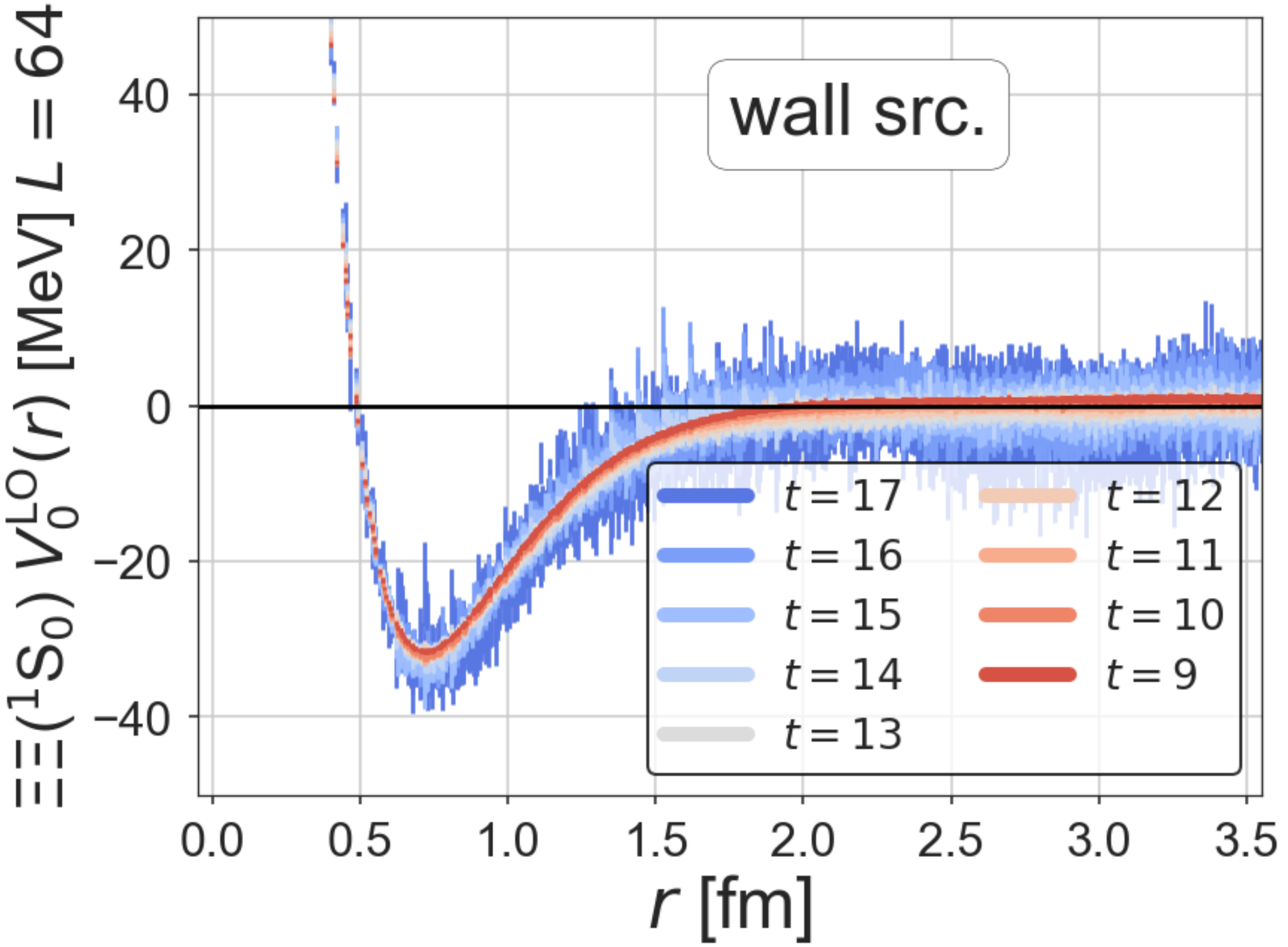
t-dep. of the Wall src.

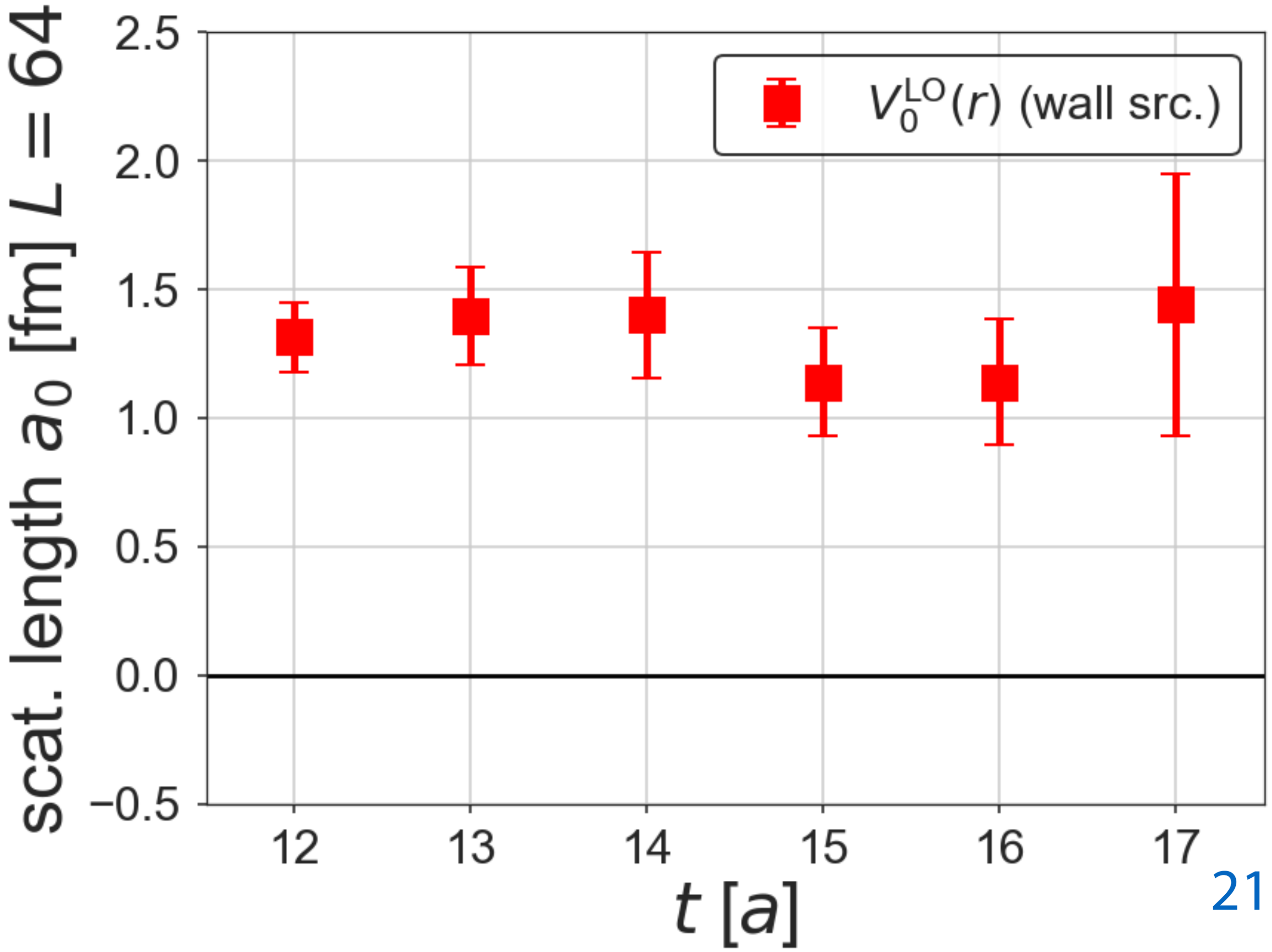
single saturation is later

than smeared src.



potential & observable are stable even at early time





Intermission:
“Correct” finite volume method analysis for two-baryon in lattice QCD

Lüscher's Method from HAL QCD: G.S. Energy

- **eigenvalue** at **finite box L³** with **HAL QCD pot.**

$$[H_0 + V_0(r) + V_2(r)\nabla^2]\Psi = \Delta E \Psi$$

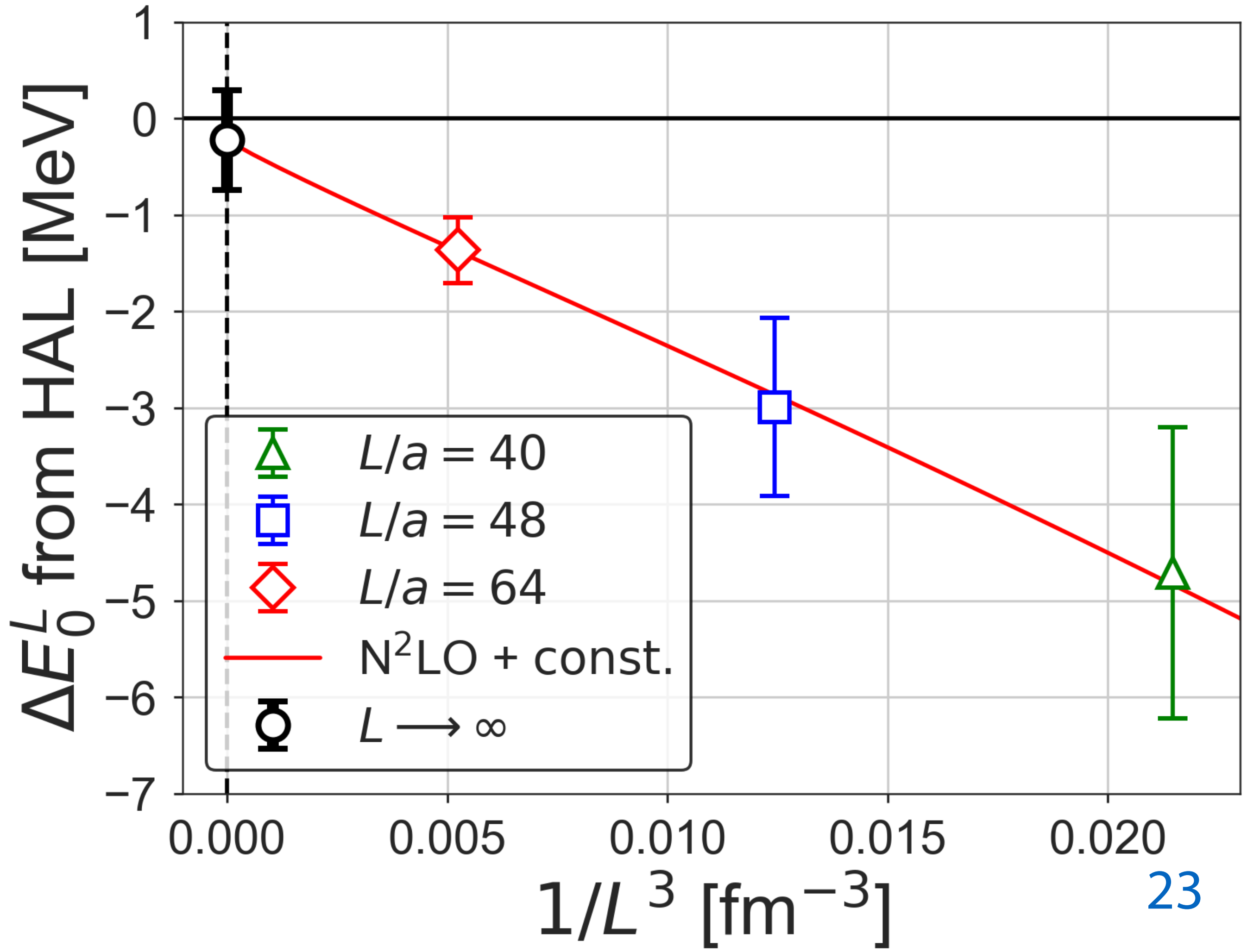
$$\Delta E_0^L \simeq -\frac{2\pi a_0}{\mu L^3}\left[1 + c_1 \frac{a_0}{L} + c_2 \left(\frac{a_0}{L}\right)^2\right]$$

→ **attractive** but **unbound**

scattering length

$$a_0 \equiv \lim_{k\to 0} \frac{\tan\delta(k)}{k}$$

$$= 0.93(29)\ \mathrm{fm}$$

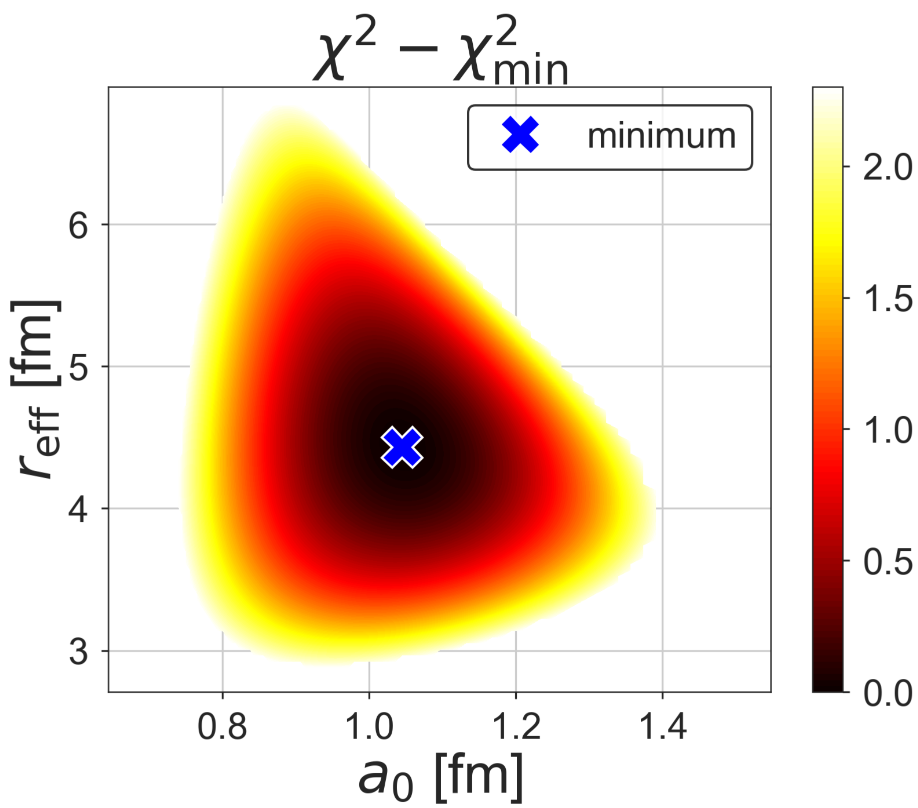


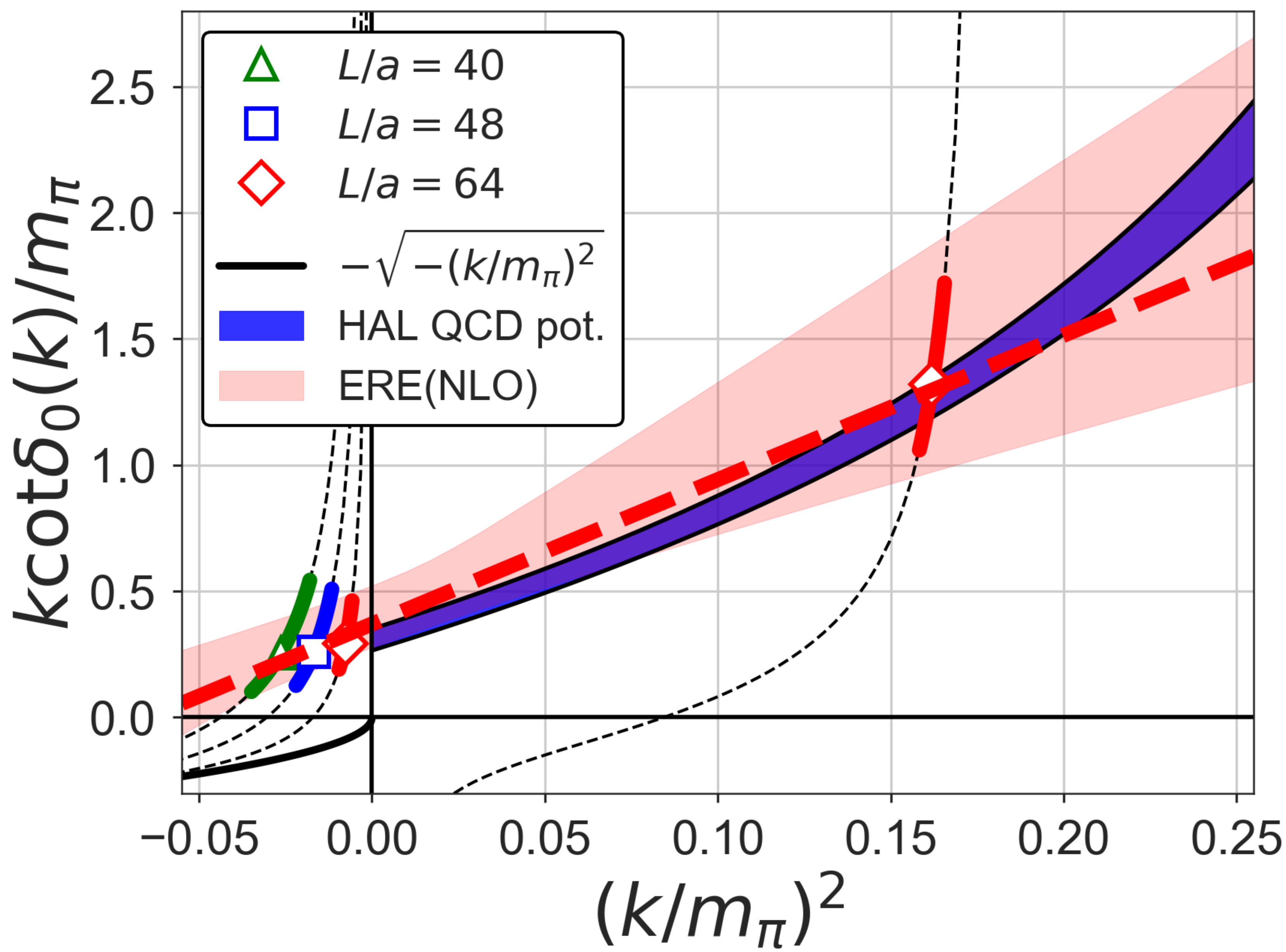
Lüscher's Method from HAL QCD: ERE Analysis

finite vol. energy shift $\Delta E_L \longrightarrow k \cot \delta(k)$ by Lüscher's formula

ERE(NLO) works well

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2$$





Part II: Diagnosis of the Fake Plateaux in the Direct Method

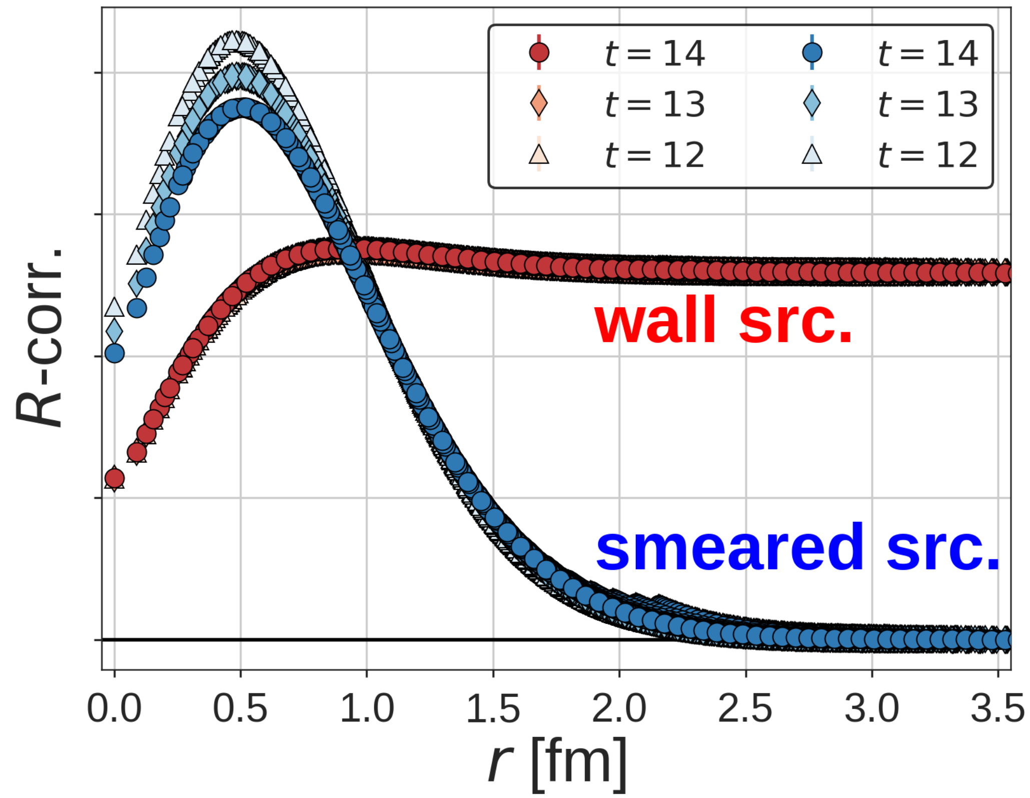
Ref.

TI for HAL QCD Coll., PoS(Lattice 2016) 109, arXiv:1610.09763.

& in prep.

Contamination and Fake Plateaux

Spatial(&Temporal) Correlation

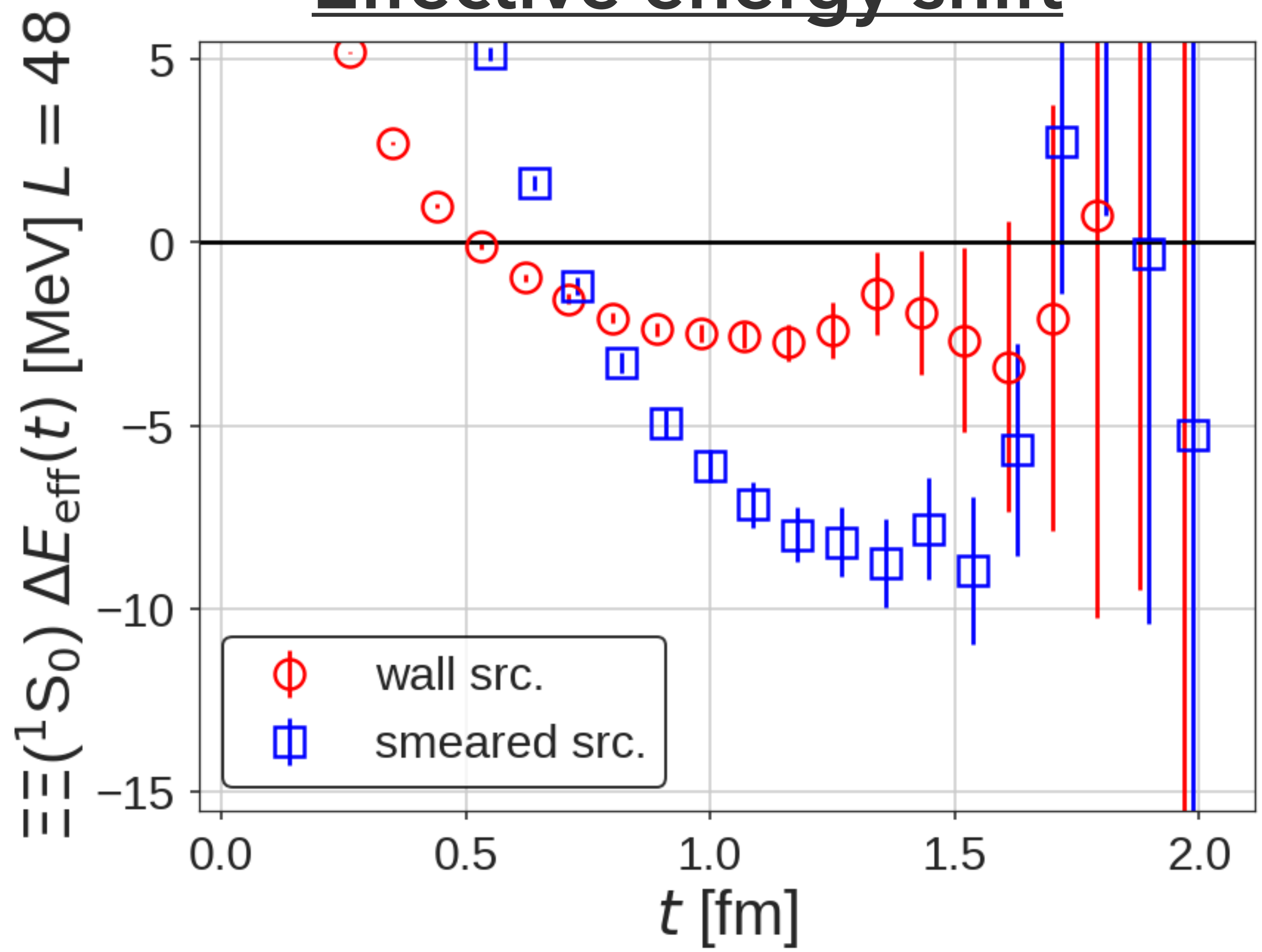


$$R(\vec{r},t) = \frac{C_{\mathrm{BB}}(\vec{r},t)}{\{C_{\mathrm{B}}(t)\}^2}$$

$$= \sum_n a_n \Psi_n(\vec{r}) e^{-\Delta E_n t}$$

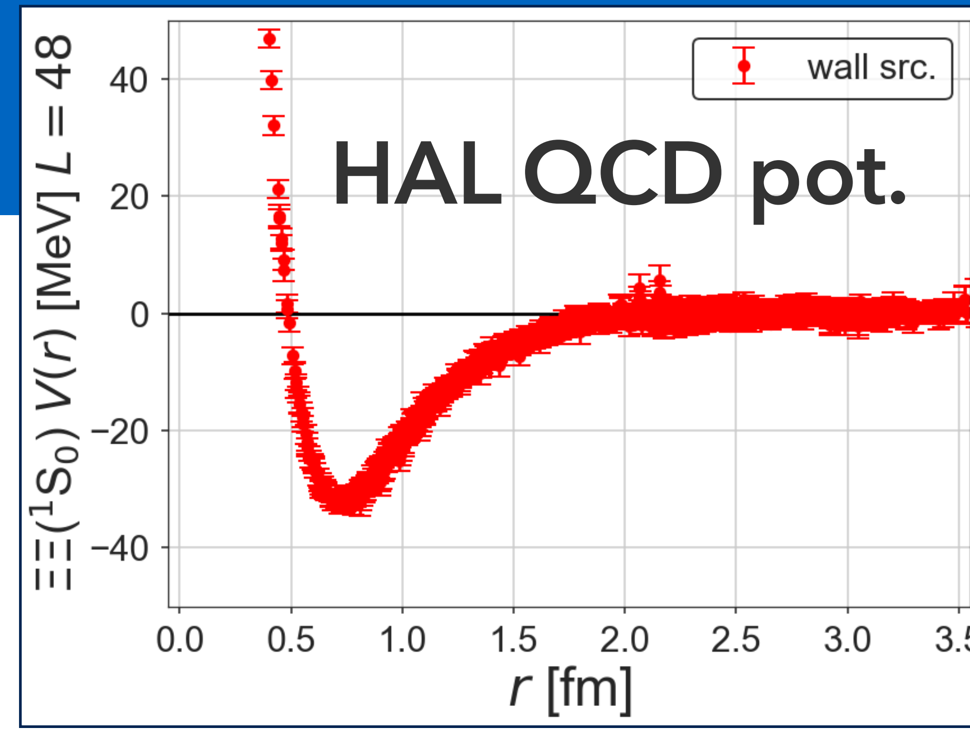
$$\Delta E_{\mathrm{eff}}(t) = \log \frac{\sum_{\vec{r}} R(\vec{r},t)}{\sum_{\vec{r}} R(\vec{r},t+1)}$$

Effective energy shift



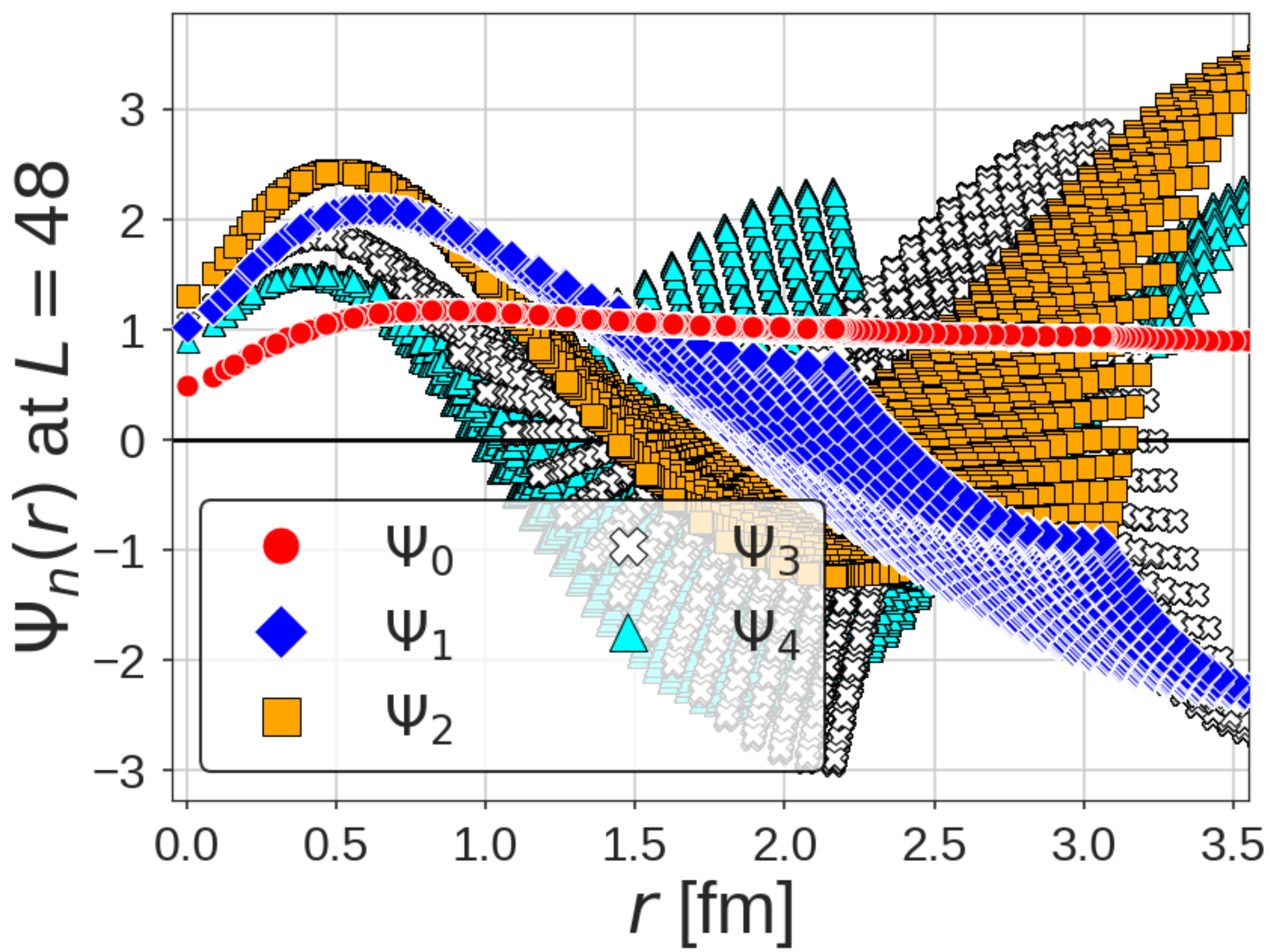
Final Check: magnitudes of the **contamination** & origin of the fake plateaux

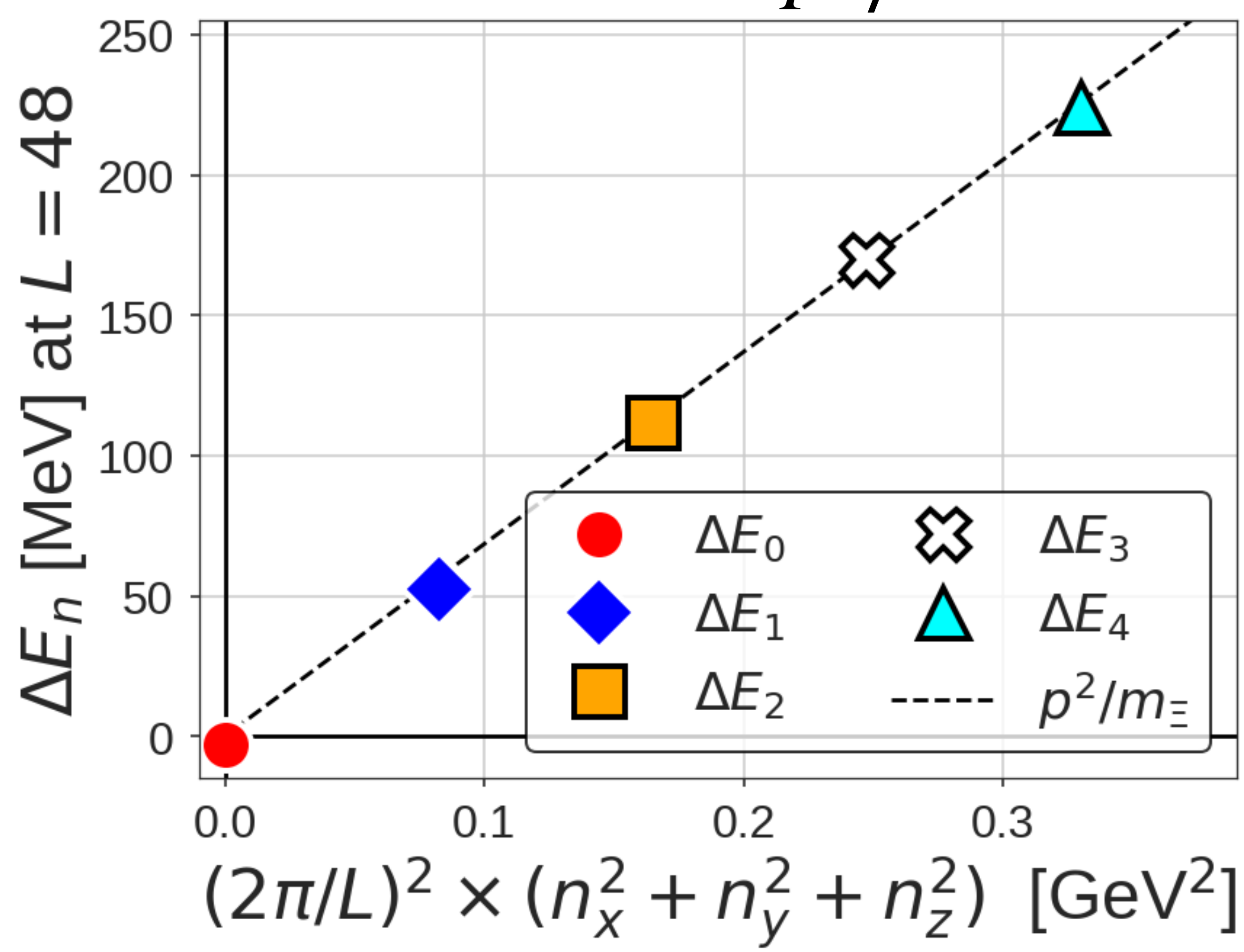
Eigenfunctions & Eigenvalues



Solving $H_0 + V(r)$ in the finite box

Low-lying eingenfunctions & eigenvalues $\sim p^2/m$

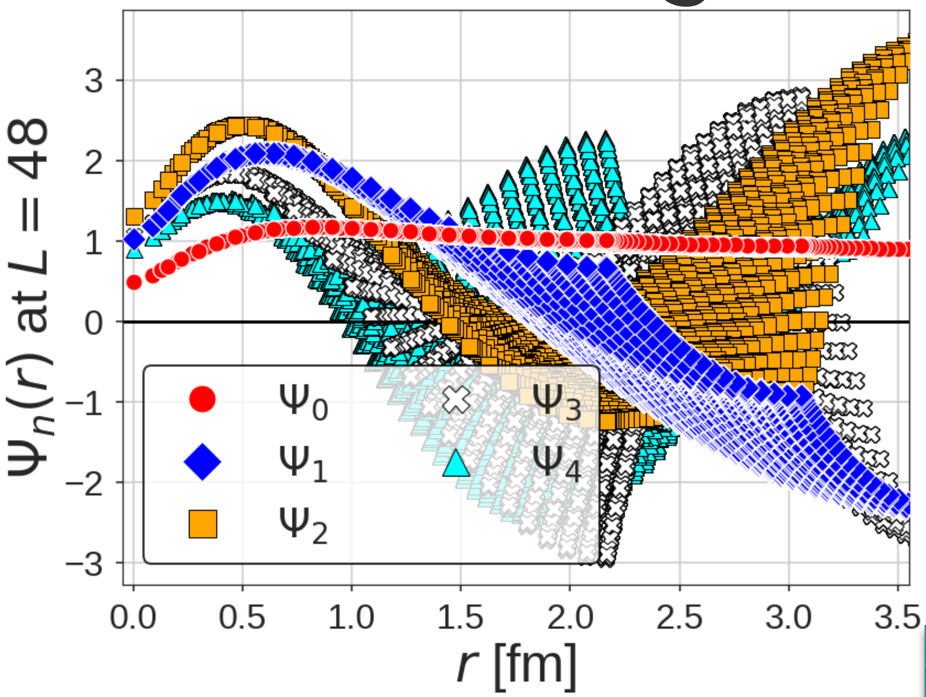


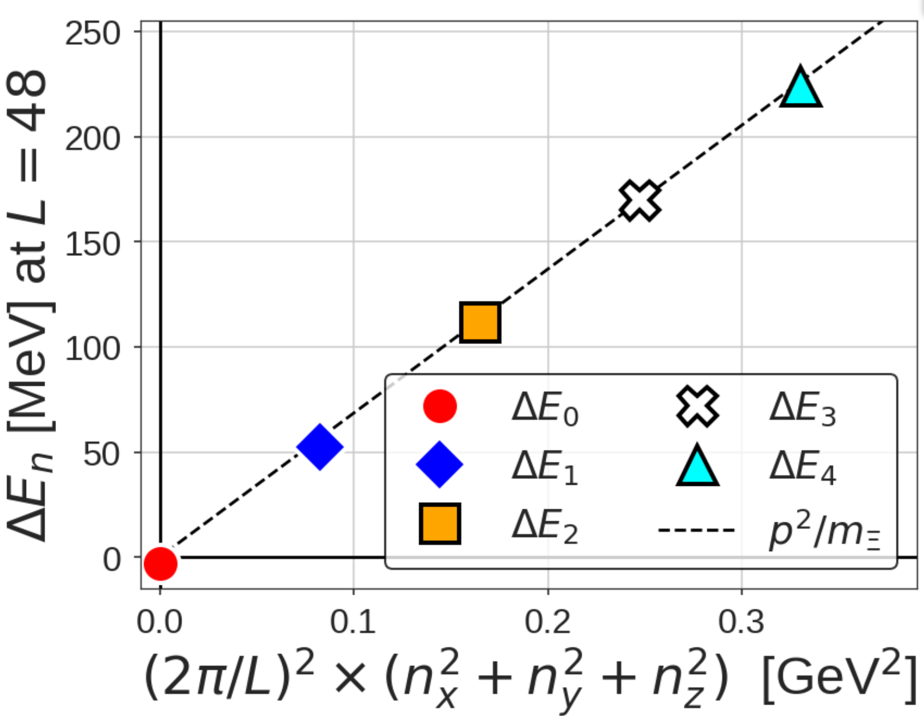


normalization $\sum |\Psi_n(r)|^2 = 1$

Eigenmode Decomposition of the Spatial Correlator

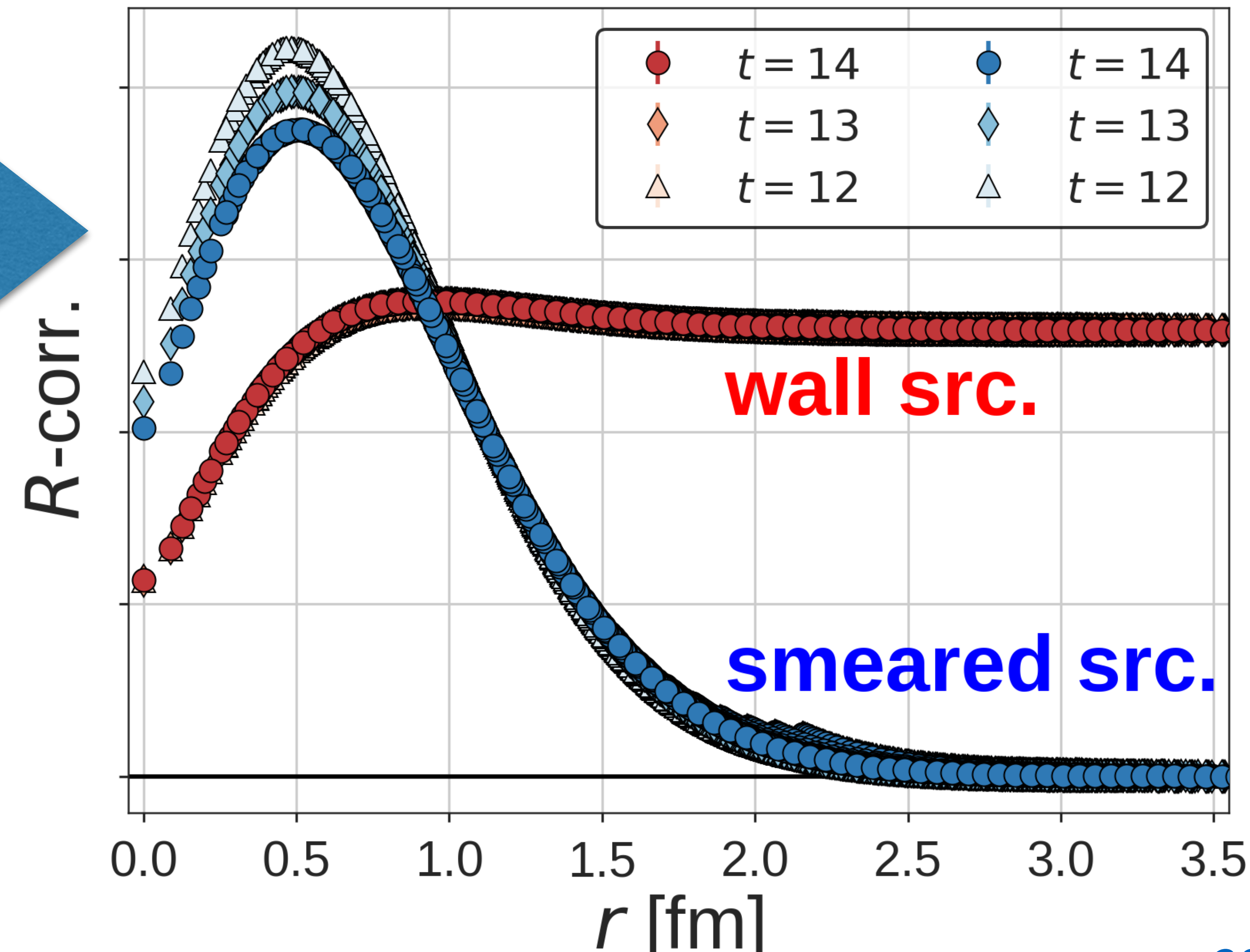
finite volume eigenstates





determine overlap coeff.

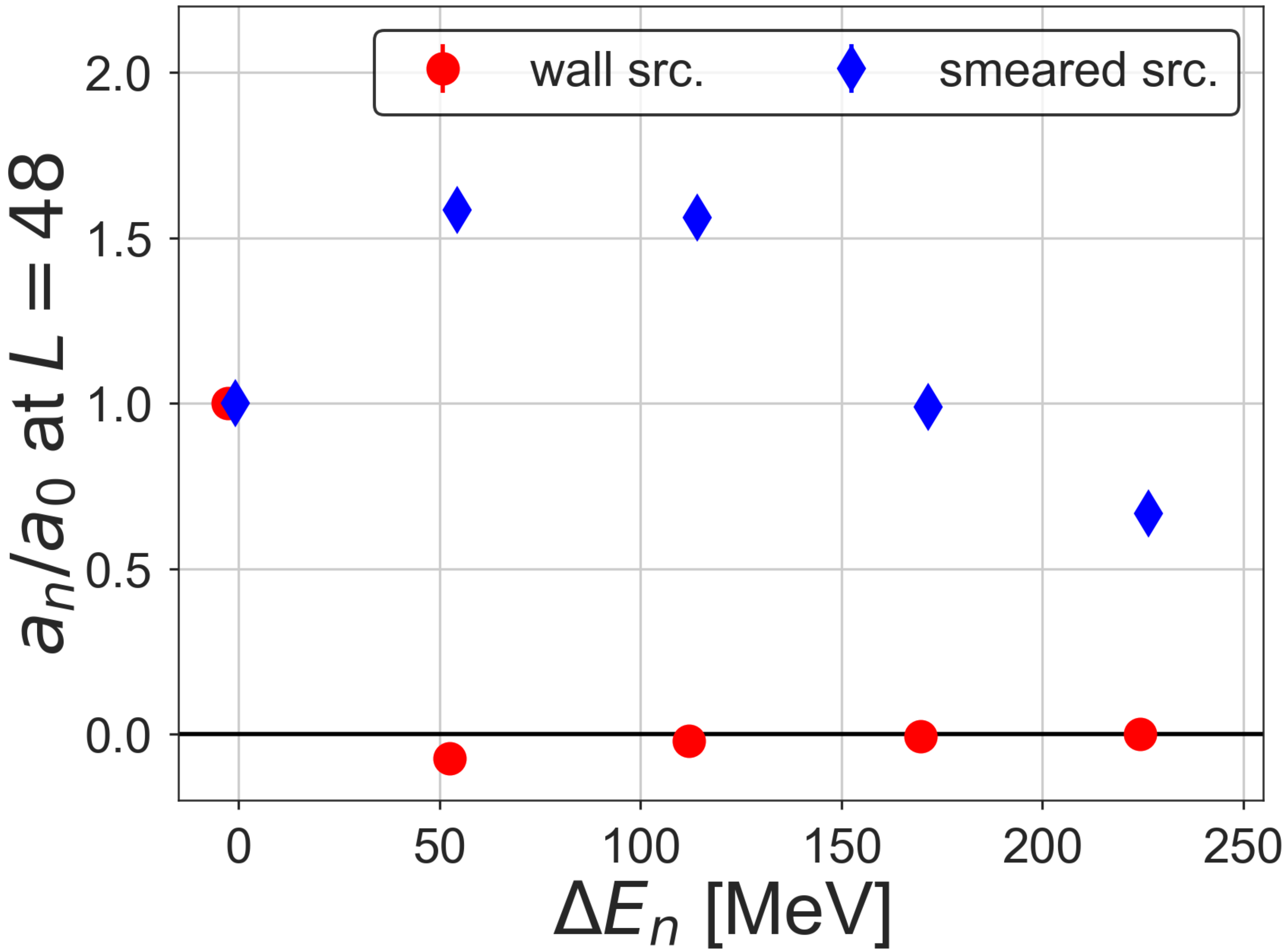
$$R^{\text{wall/smear}}(\vec{r},t) = \sum_n a_n^{\text{wall/smear}} \Psi_n(\vec{r}) e^{-\Delta E_n t}$$



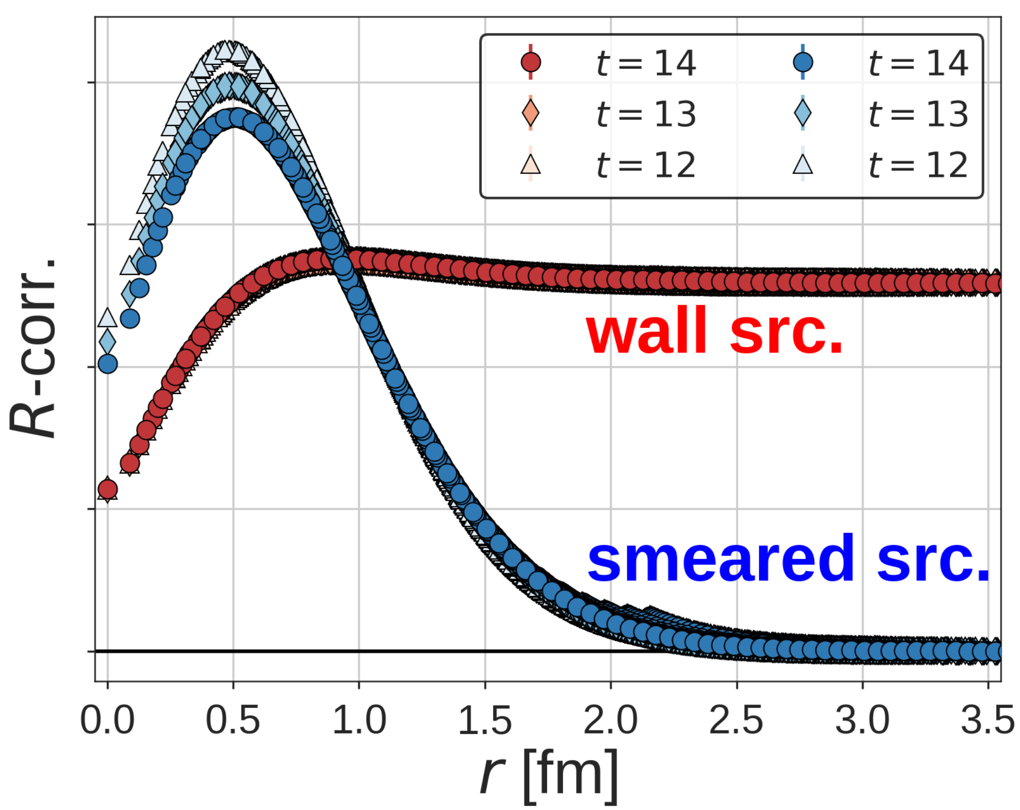
Overlap of Excited States

$$R^{\mathrm{wall/smear}}(\vec{r},t) = \sum_n a_n^{\mathrm{wall/smear}} \Psi_n(\vec{r}) e^{-\Delta E_n t}$$

"**smeared src.**" has large overlaps with **scat. states**.



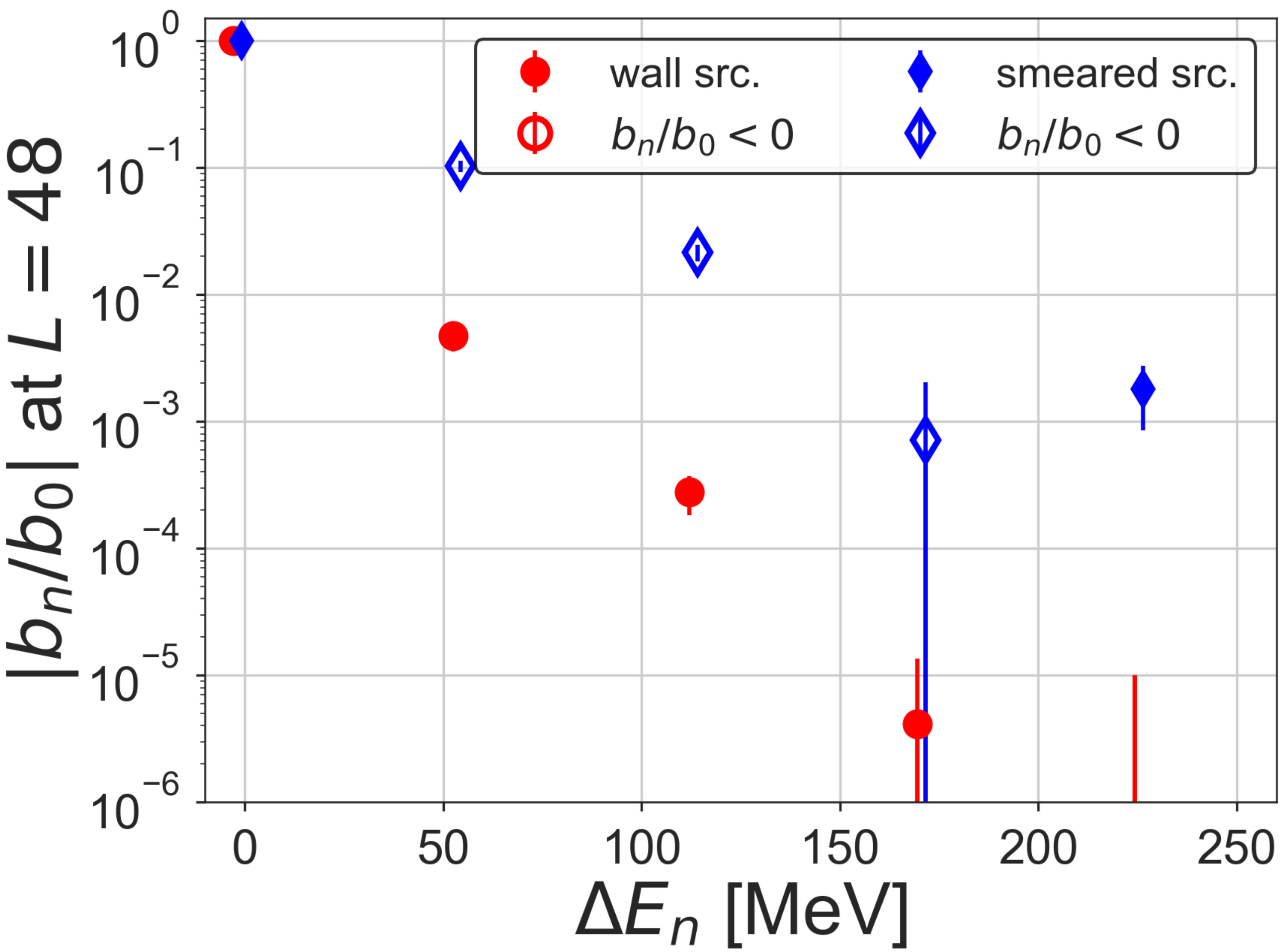
$$\mathcal{O}(a_0^{\mathrm{smear}}) \sim \mathcal{O}(a_n^{\mathrm{smear}})$$



Contaminations of Excited States

$$R(t) = \sum_n b_n^{\mathrm{wall/smear}} e^{-\Delta E_n t} \quad \text{with} \quad b_n^{\mathrm{wall/smear}} = a_n^{\mathrm{wall/smear}} \sum_{\vec{r}} \Psi_n(\vec{r})$$

b_n: magnitude of contamination in $\Delta E_{\mathrm{eff}}(t)$



ex.

- **wall source**

  $b_1/b_0 \ll 1\ \%$

- **smeared source**

  $|b_1/b_0| \simeq 10\ \%$

Origin of Fake Plateaux and Saturation

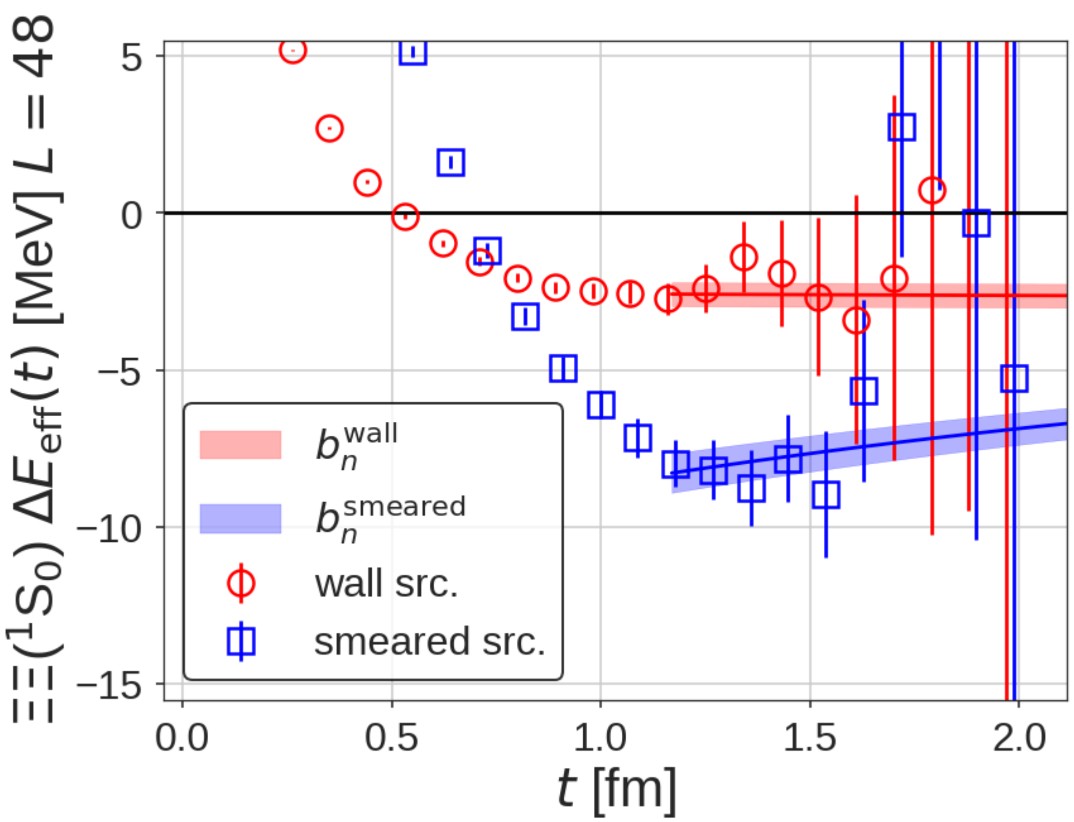
$$\Delta E_{\mathrm{eff}}^{\mathrm{wall/smear}}(t) \equiv \log \frac{R(t)}{R(t+1)} = \log \frac{\sum_n b_n^{\mathrm{wall/smear}} \exp(-\Delta E_n t)}{\sum_n b_n^{\mathrm{wall/smear}} \exp(-\Delta E_n (t+1))}$$

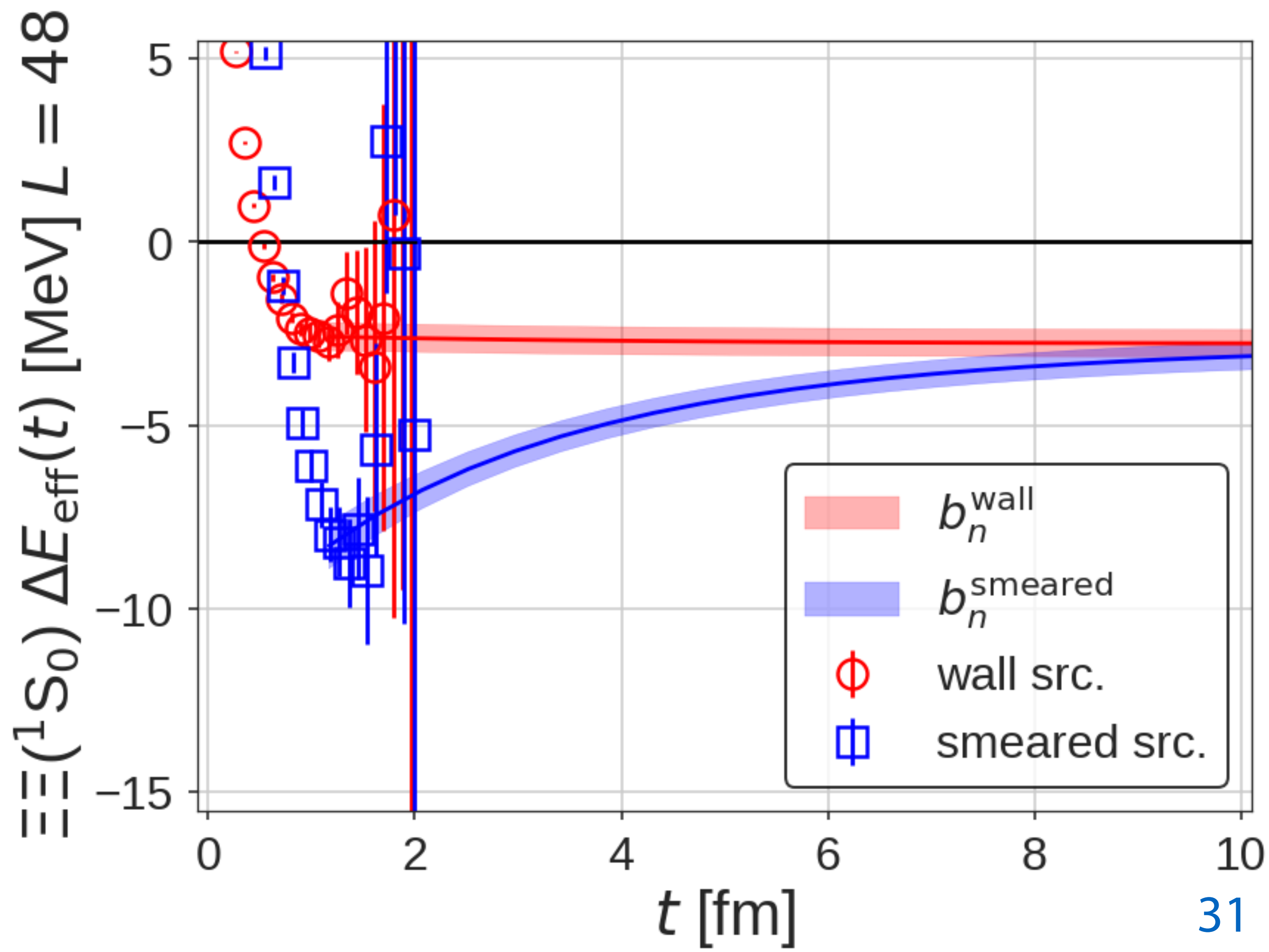
- "**direct measurement**" is reproduced by **low-modes**

g.s. saturation for **smeared src.** ~ **10 fm !!!**

naïve expectation

$$1/\delta E_{\mathrm{el.}} \sim 10\ \mathrm{fm}$$



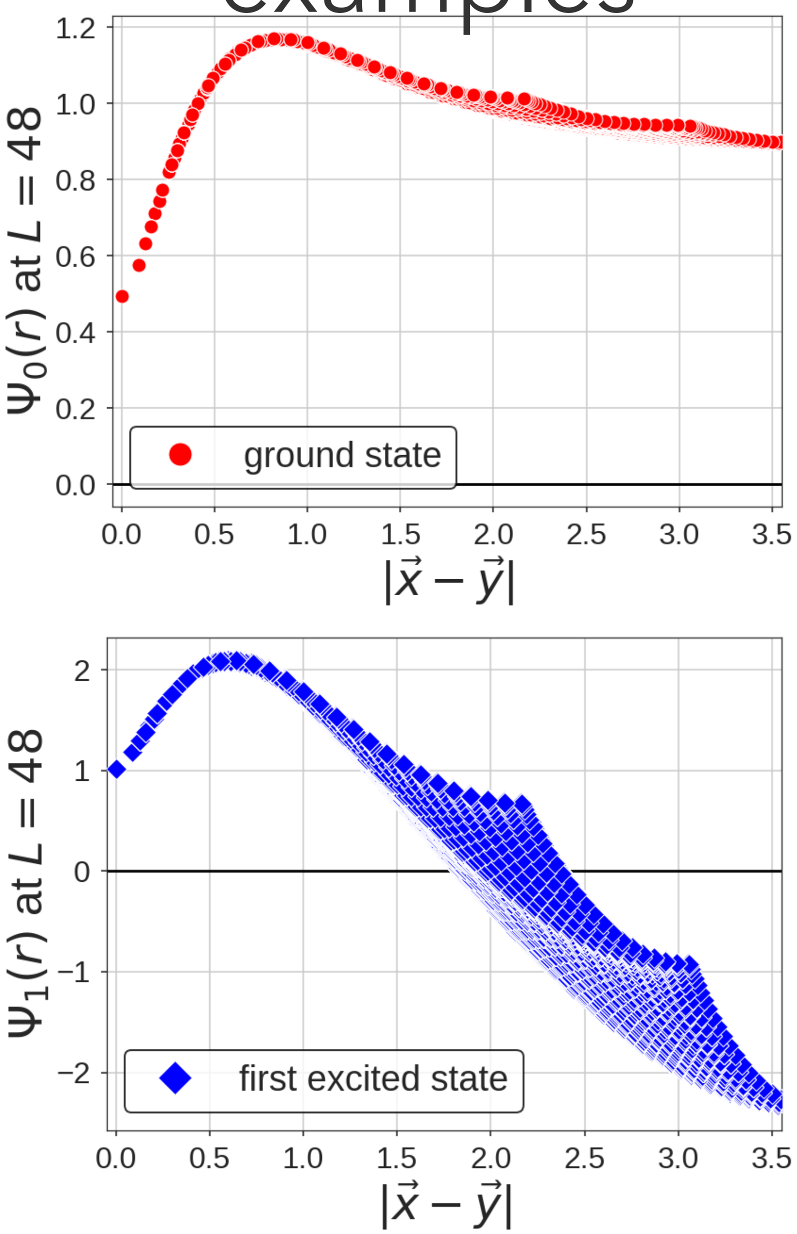


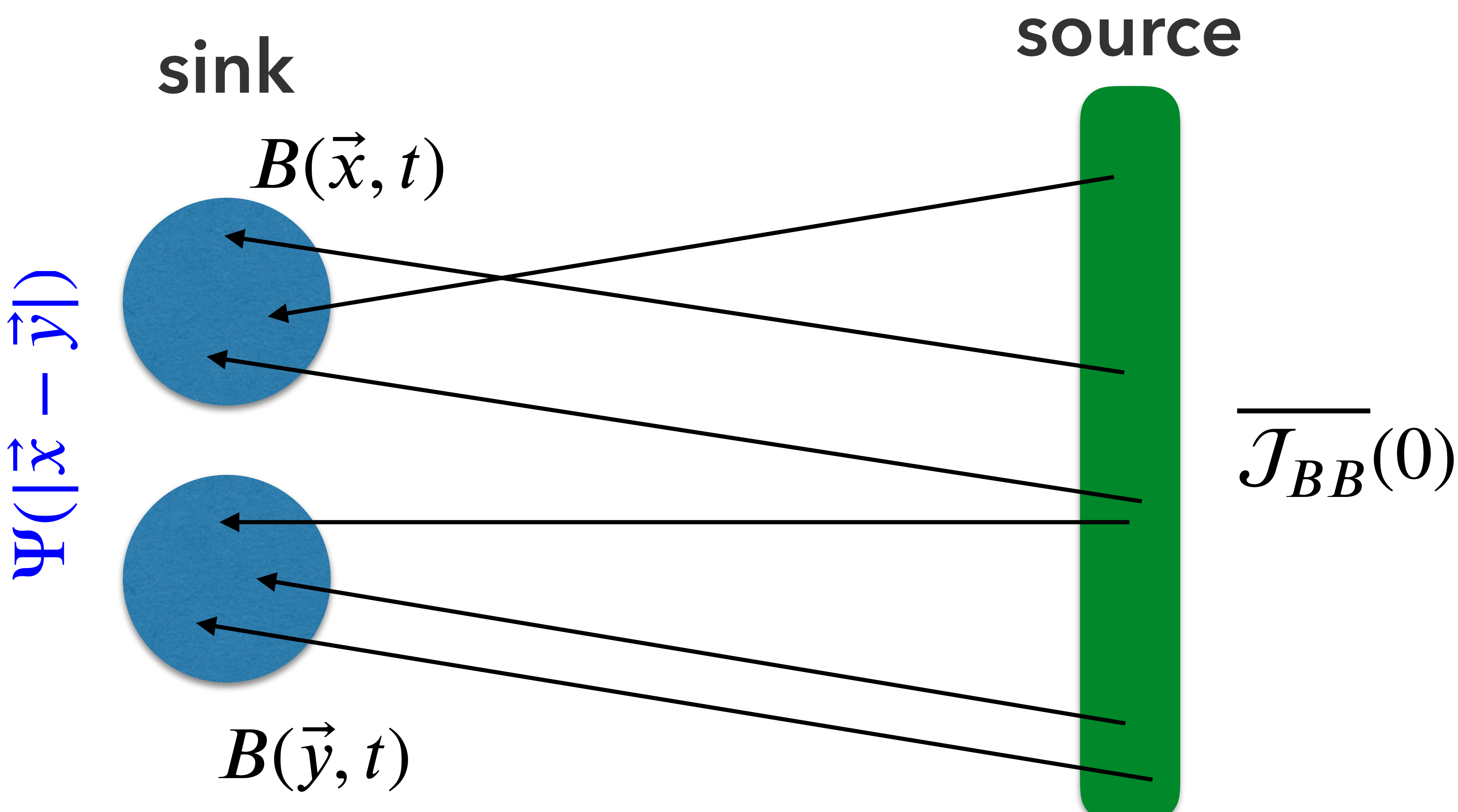
Sink Op. Projection with Proper Eigenmode (1)

- "correct sink operator"

$$R^{(n)}(t) = \sum_{\vec{x},\vec{y}} \Psi_n(|\vec{x}-\vec{y}|)\langle B(\vec{x},t)B(\vec{y},t)\overline{\mathcal{J}_{BB}}(0)\rangle / \{C_B(t)\}^2$$

examples

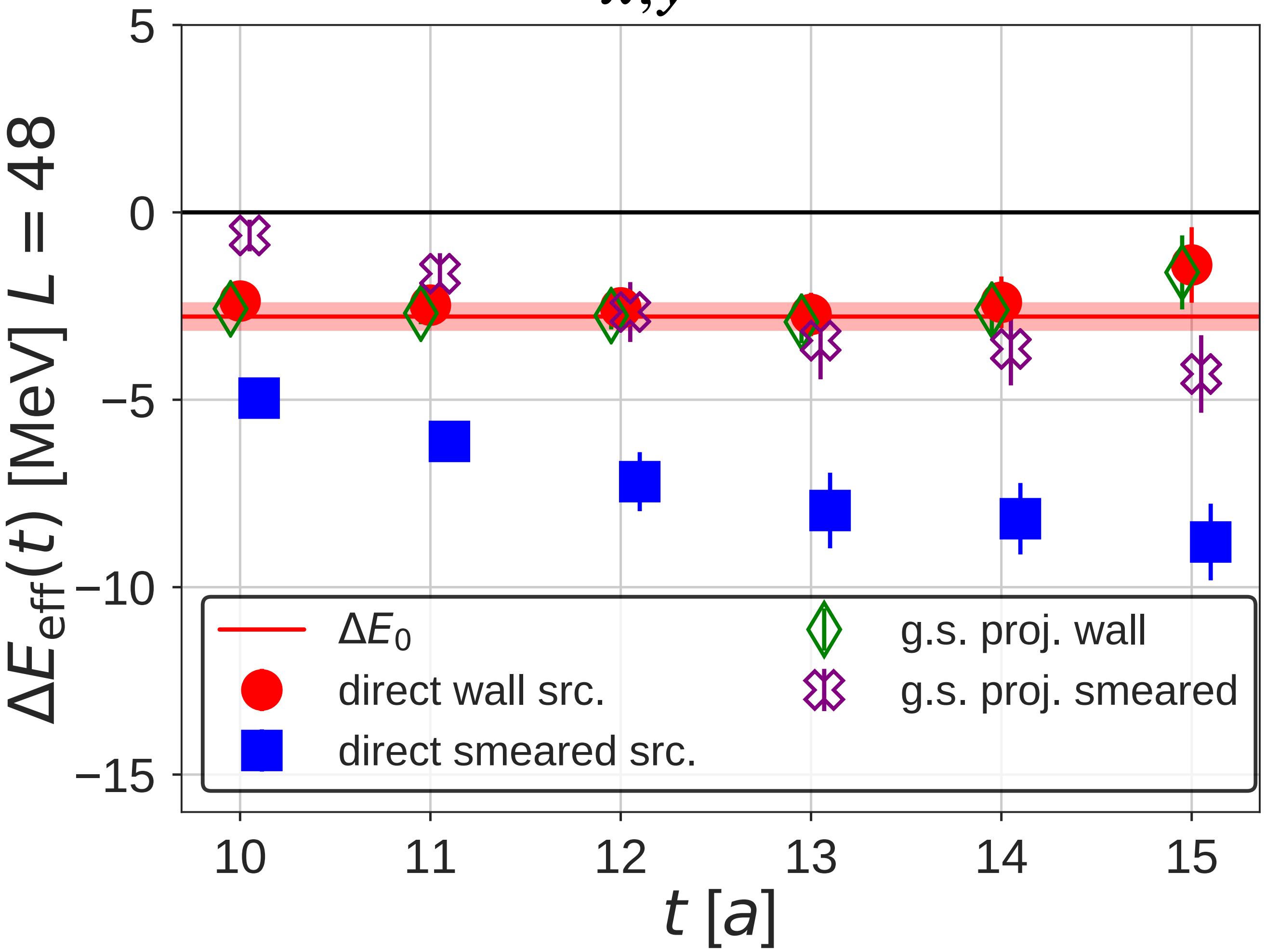


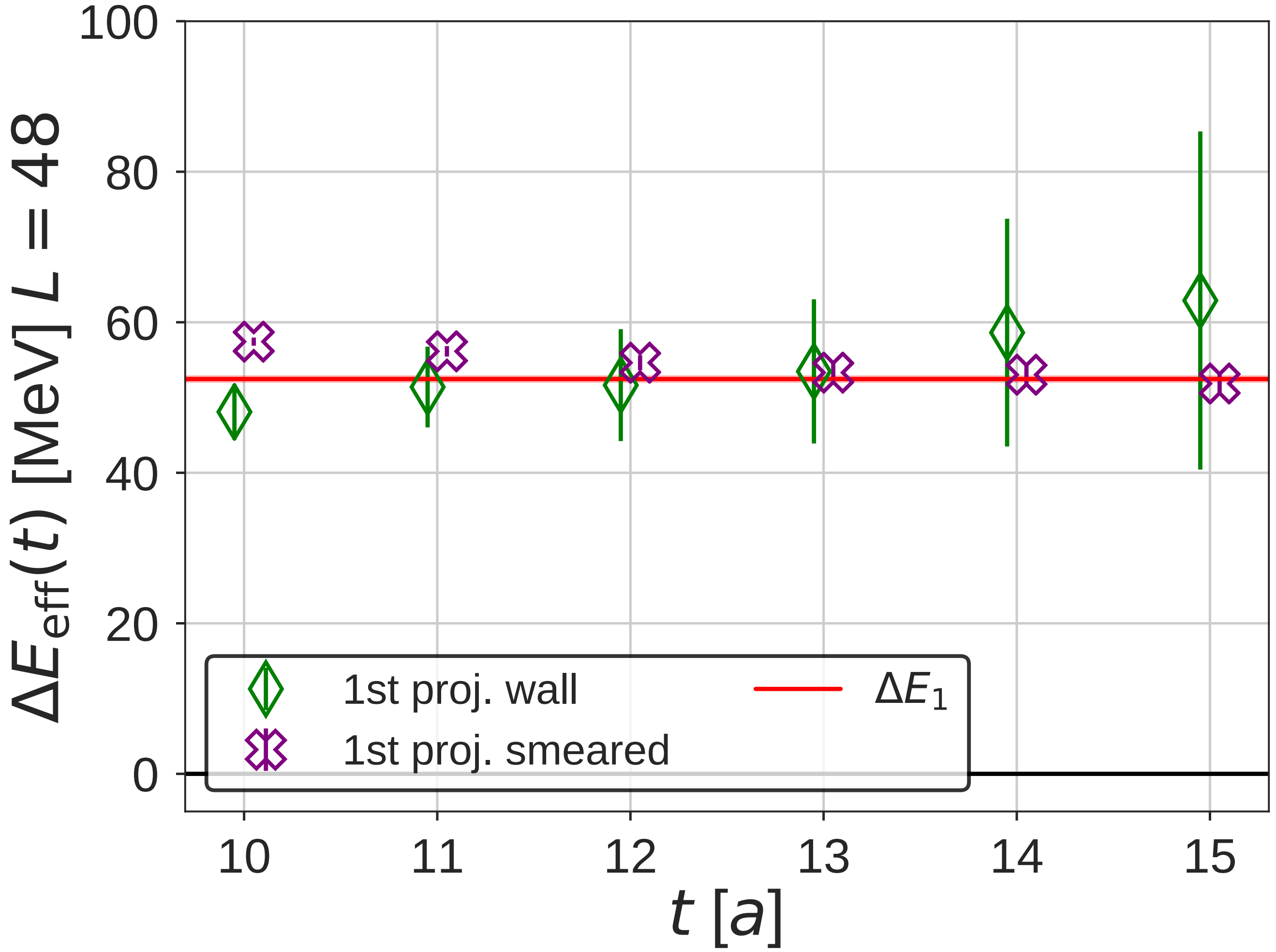


Sink Op. Projection with Proper Eigenmode (2)

- **“correct sink operator”** –> “**true plateau**” appears

$$R^{(n)}(t) = \sum_{\vec{x},\vec{y}} \Psi_n(|\vec{x}-\vec{y}|)\langle B(\vec{x},t)B(\vec{y},t)\overline{\mathcal{J}_{BB}}(0)\rangle / \{C_B(t)\}^2$$





HAL QCD = Lüscher’s method with ***proper projection***

≠ naive plateau fitting

Summary

Direct Method vs. HAL QCD Method

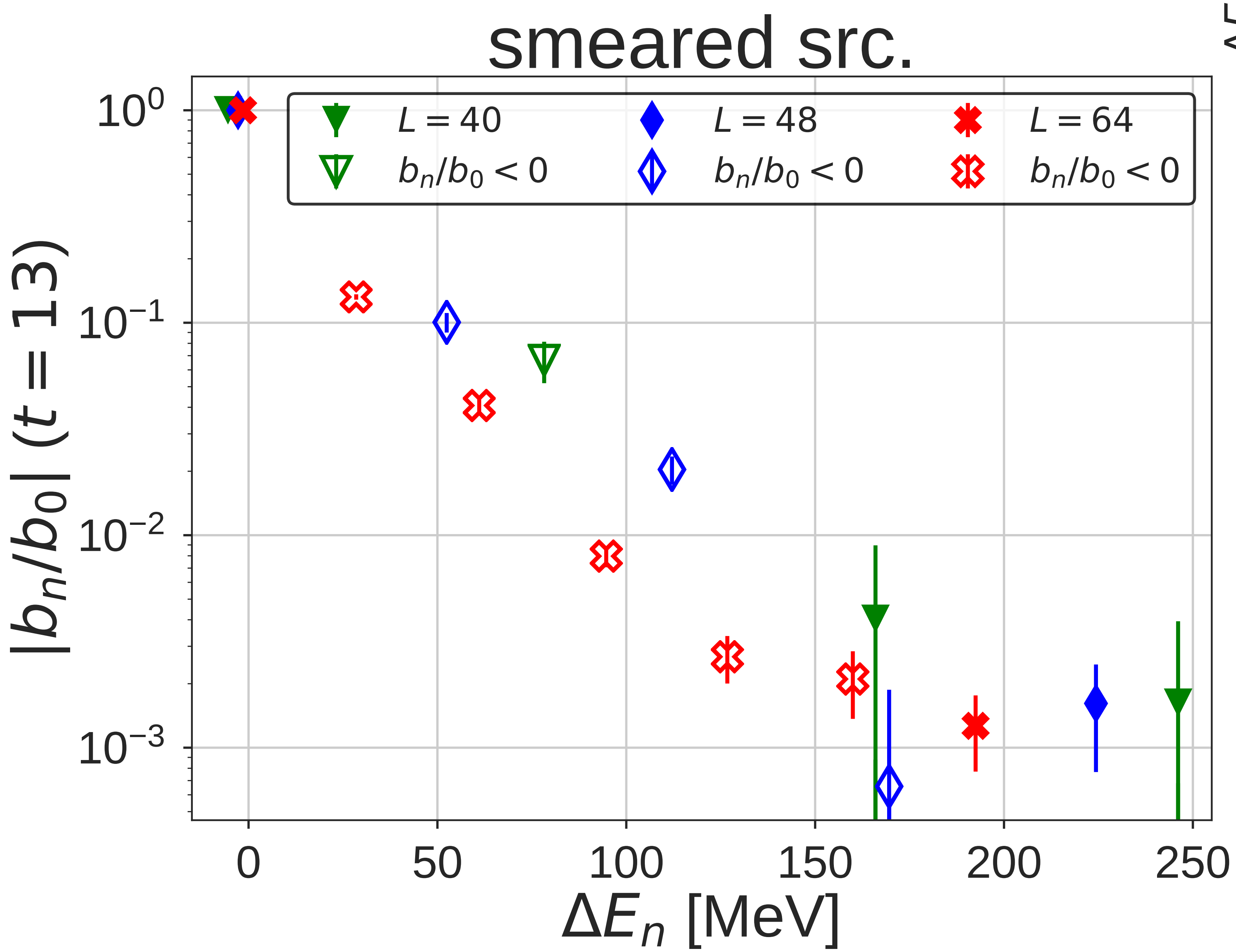
- HAL QCD method can control **scattering states.**
- We checked
  - **The velocity expansion of non-local pot.** has **good convergence at low energies.**
  - **Leading order expansion of the wall src.** (standard of HAL QCD studies) **works well.**
  - **The inelastic state contamination** is **under control.**
- We show the origin of the fake plateau in the direct method comes from the scattering state contamination.

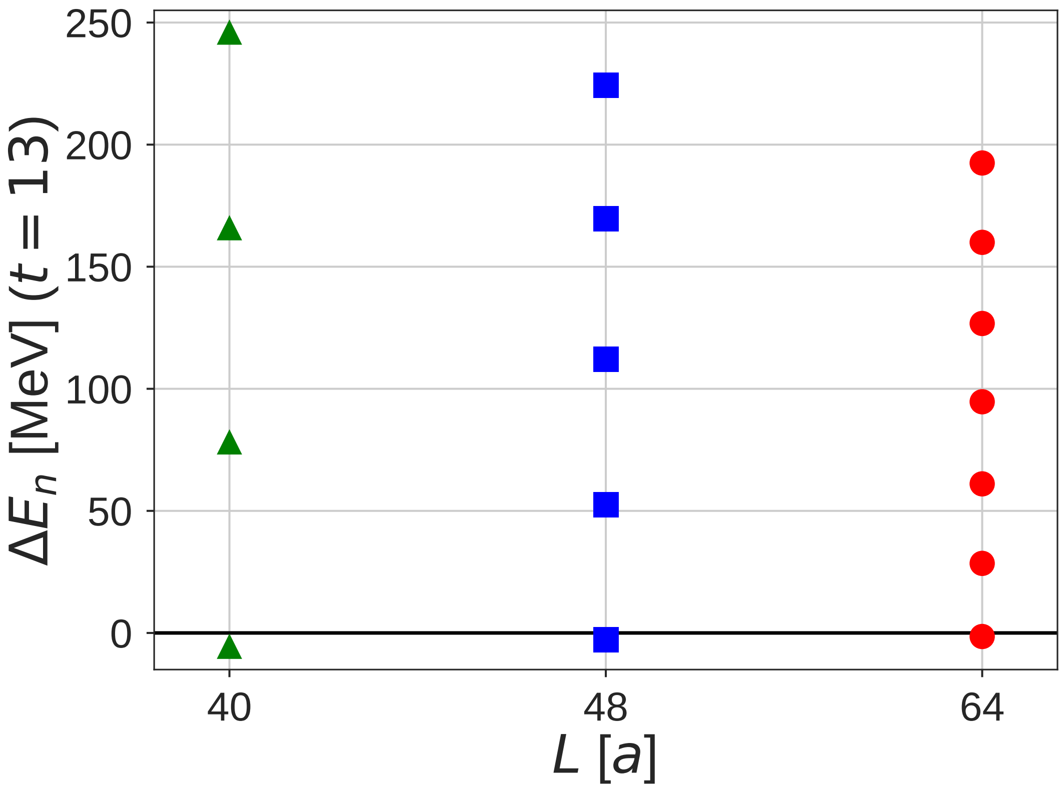
- **HAL QCD** results at (almost) **physical point**
  Next Talk by Takumi Doi

BACKUPS

Volume Dep. of the Fake Plateaux (1)

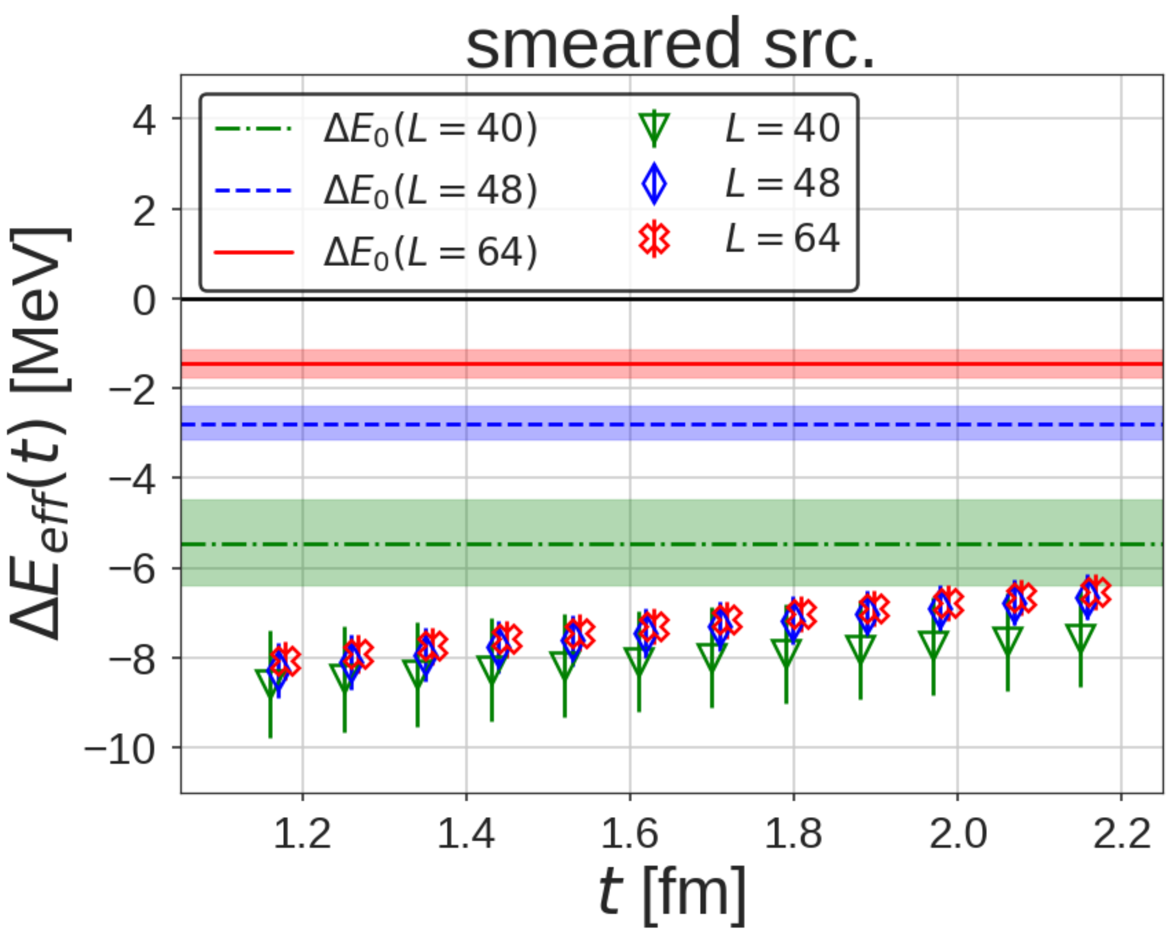
- **Scattering state gap** – $1/L^2$
- scat. states becomes severe for a larger volume





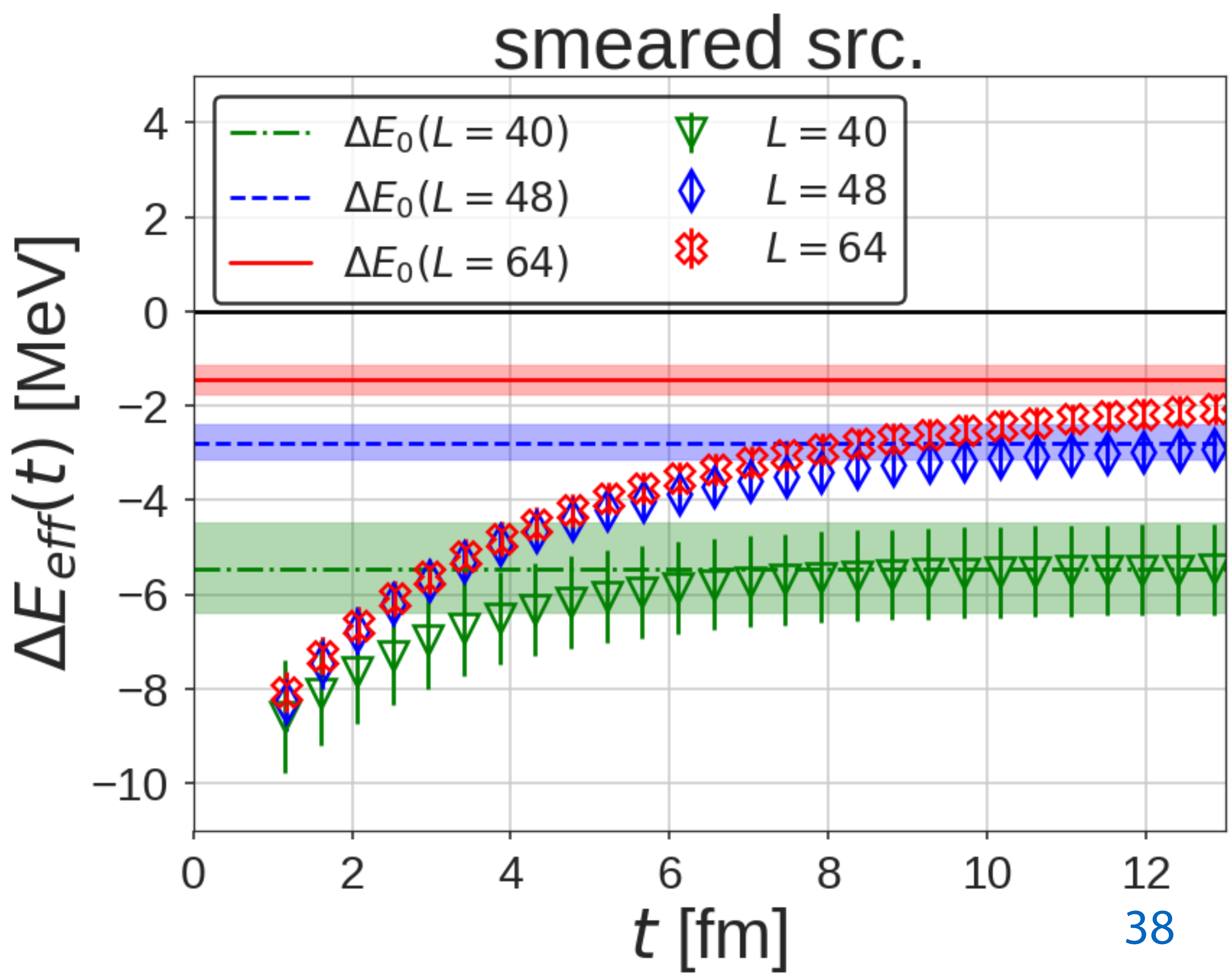
Volume Dep. of the Fake Plateaux (2)

- **Volume indep.** does not mean **ground state saturation**.
- **Volume dep.** appears at later time.



volume indep.

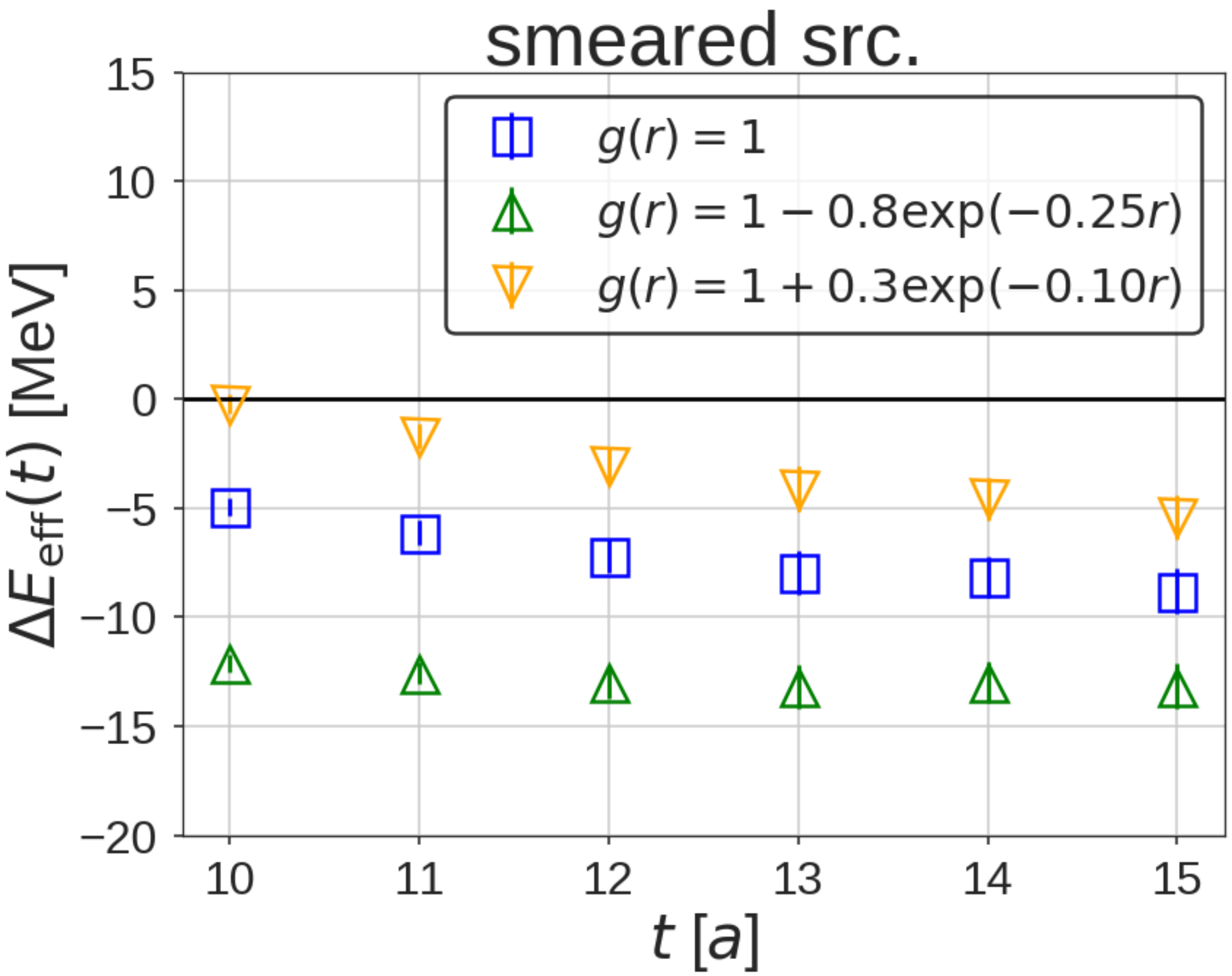
plateaux around t ~ 1.5 fm

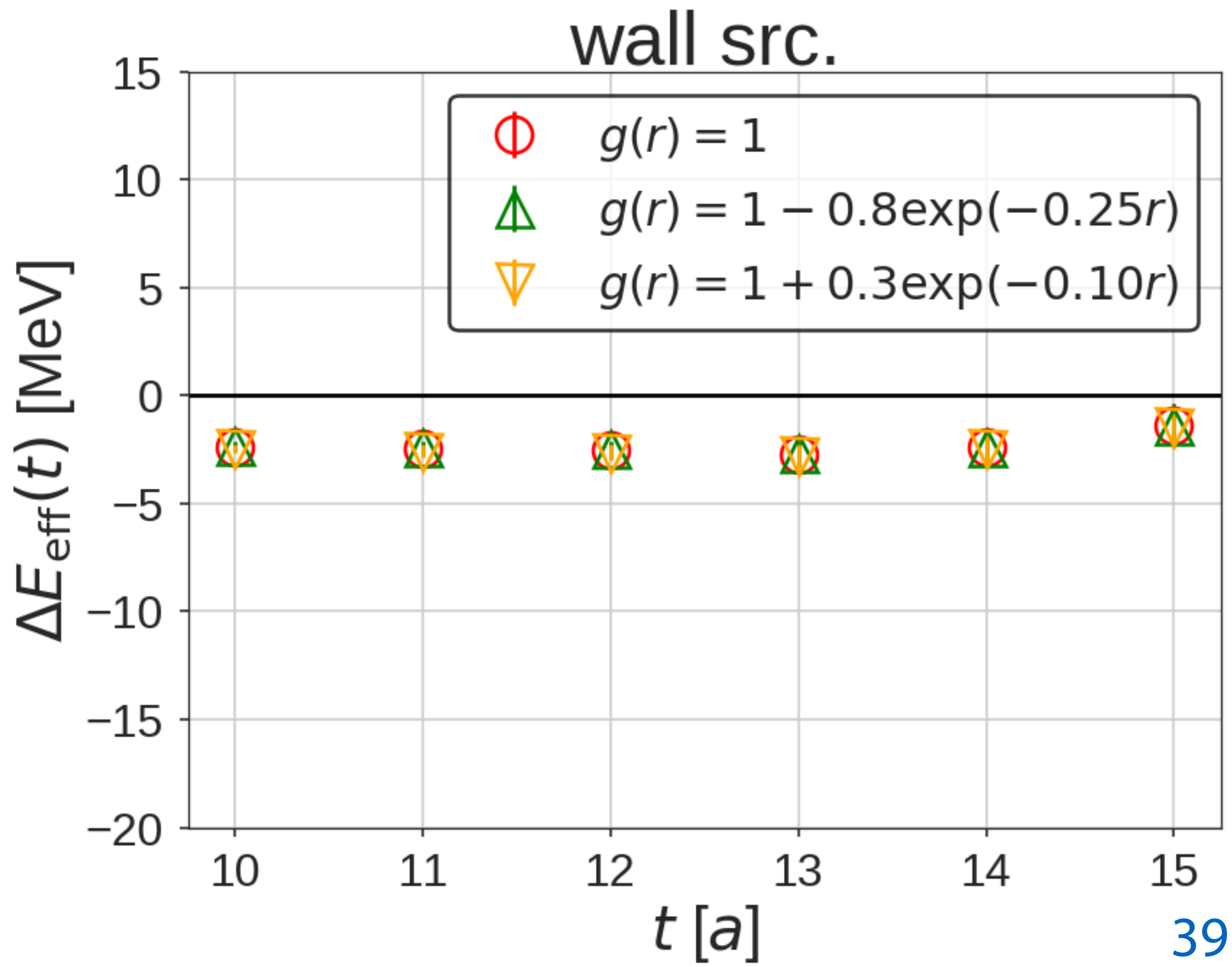


Sink Operator Dependence of the Plateaux

- **smeared src.** depend on "**sink**"
- **wall src.** is stable

$$C_{\mathrm{BB}}(t) = \sum_{\vec{x},\vec{y}} g(|\vec{x} - \vec{y}|)\langle B(\vec{x}, t)B(\vec{y}, t)\overline{\mathcal{J}_{BB}}(0)\rangle$$





Lüscher's Finite Volume Method & Bound State

1. Lüscher's formula
$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\mathbf{n} \in \mathbf{Z}^3} \frac{1}{|\mathbf{n}|^2 - (kL/2\pi)^2}$$

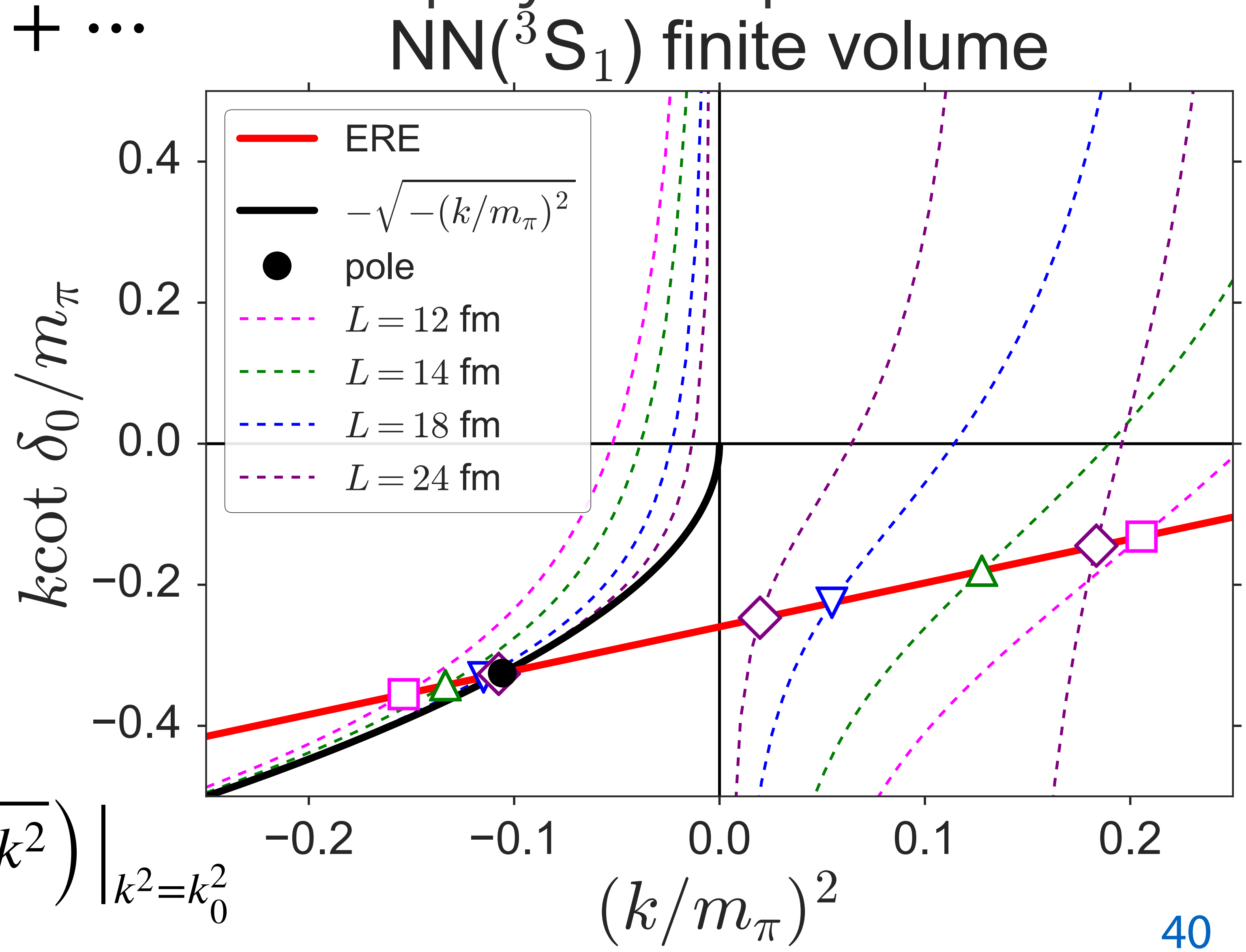
2. Effective Range Expansion (**ERE**)
$$k \cot \delta_0(k) \simeq \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2 + \cdots$$

3. Search **bound state**

@ S-matrix's pole
$$k_0 \cot \delta_0(k_0) = -\sqrt{-k_0^2}$$

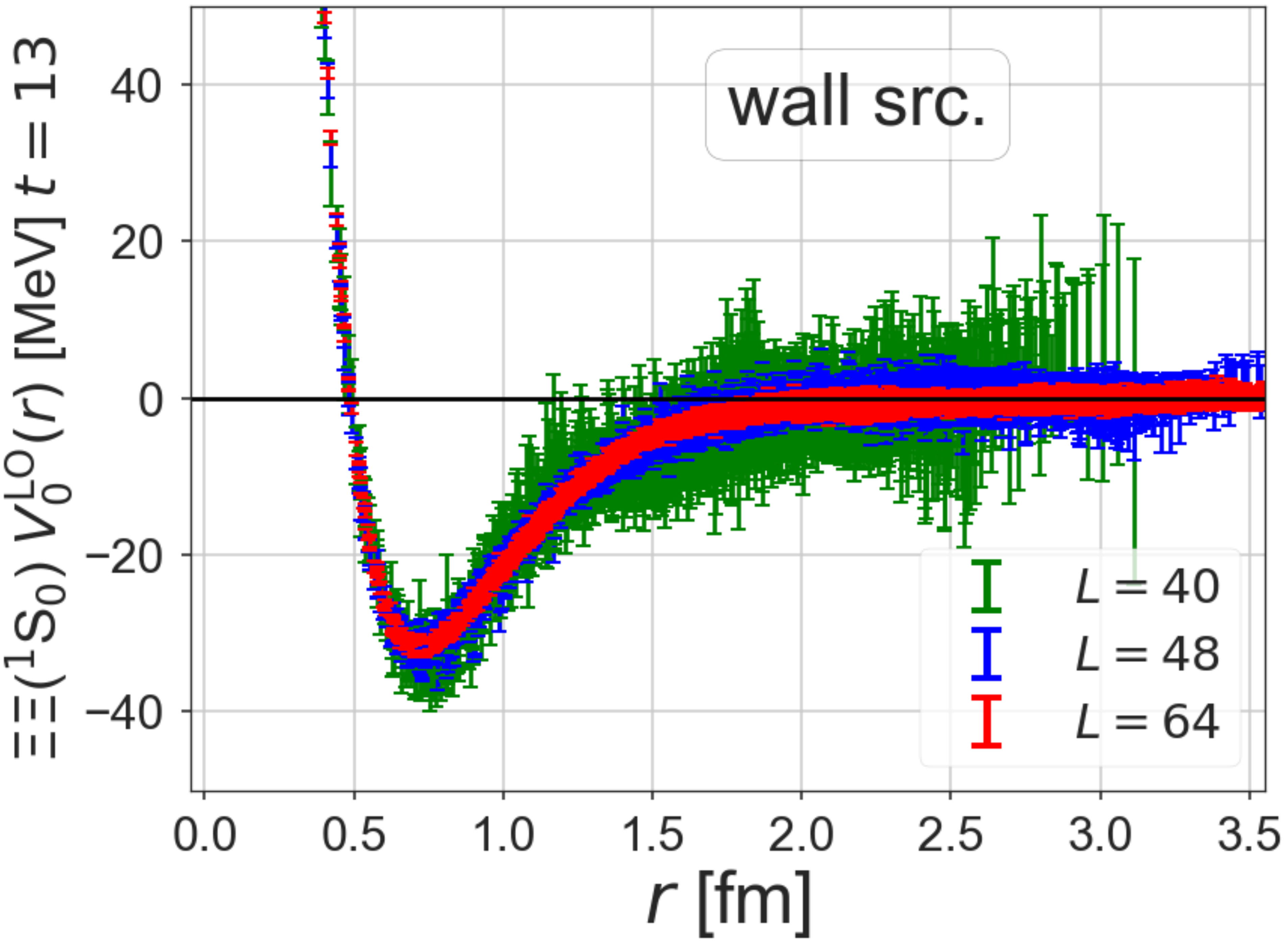
& physical pole cond.
$$\frac{d}{dk^2}(k \cot \delta_0(k))\Big|_{k^2=k_0^2} < \frac{d}{dk^2}\left(-\sqrt{-k^2}\right)\Big|_{k^2=k_0^2}$$

ex. physical quark mass



Volume Independence of the Potential

- The potential from L = 40, 48 and 64 are consistent.

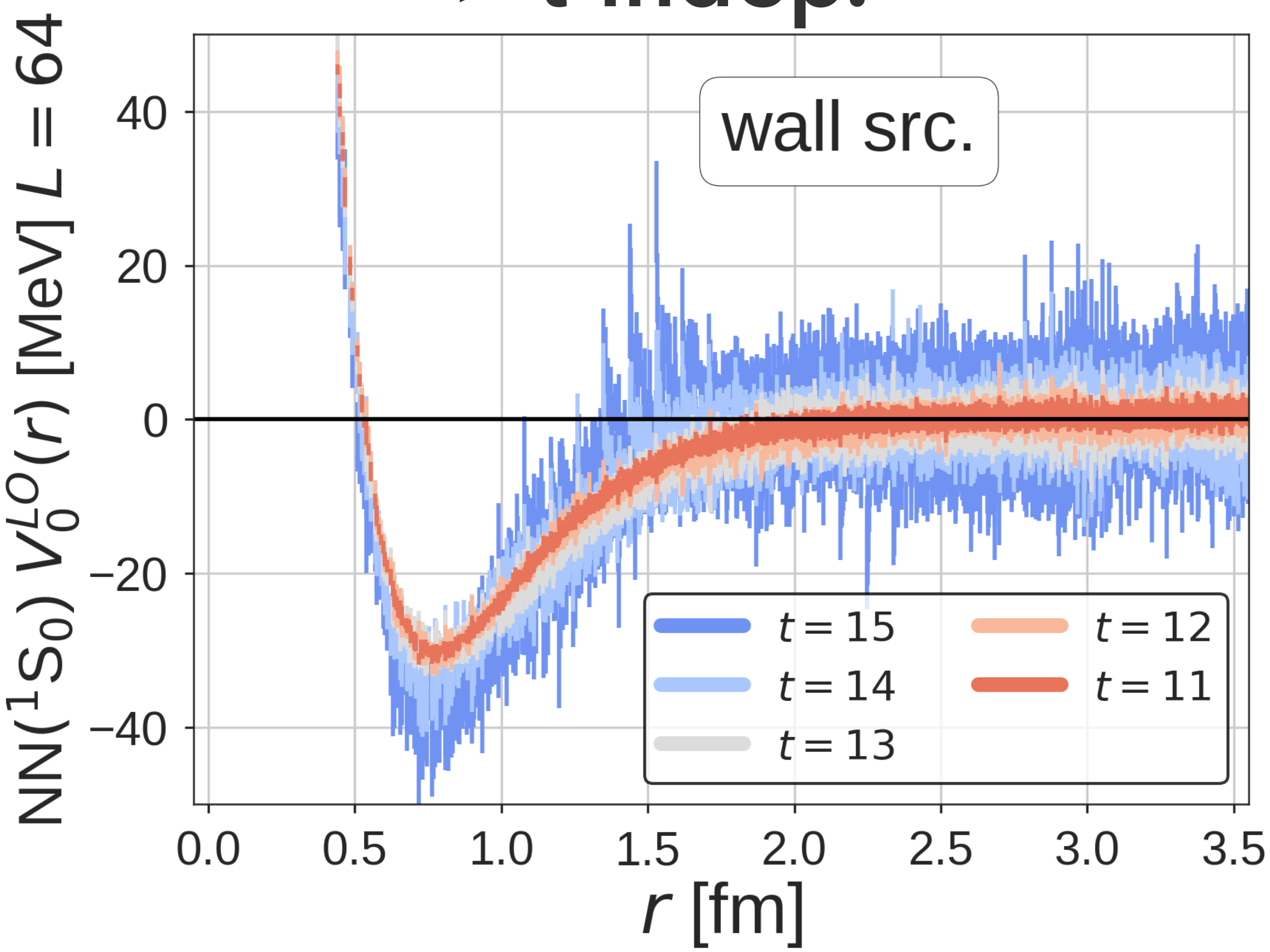


NN($^1S_0$) System at M_{PI} = 510 MeV (1) LO Approx.

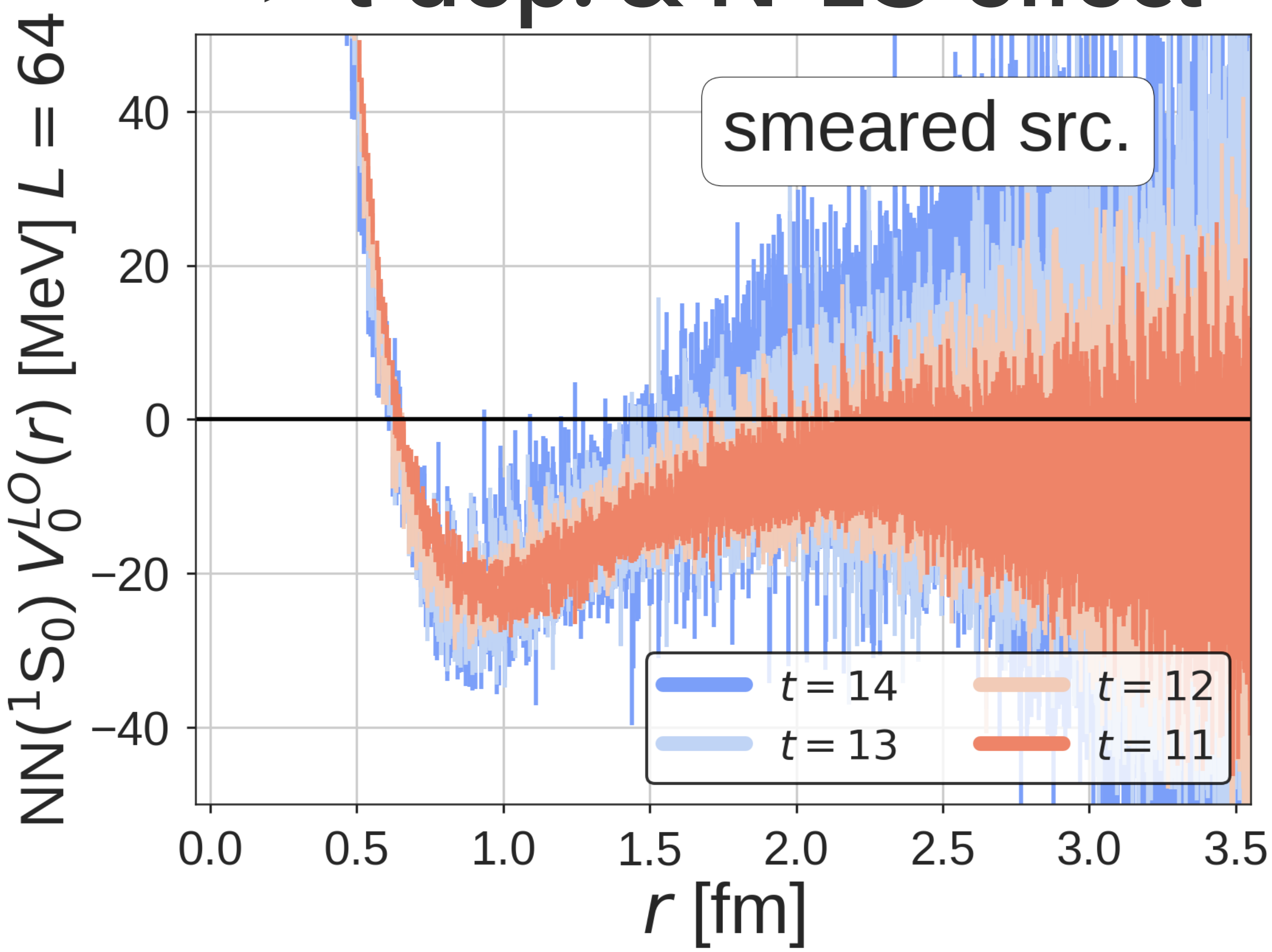
$$U(r,r') \simeq V_0^{\mathrm{LO}}(r)\delta(r-r')$$

- Quantitatively, the same as $\Xi\Xi$ ($^1S_0$)

wall src.
–> t-indep.



smeared src.
–> t-dep. & N^{2}LO effect

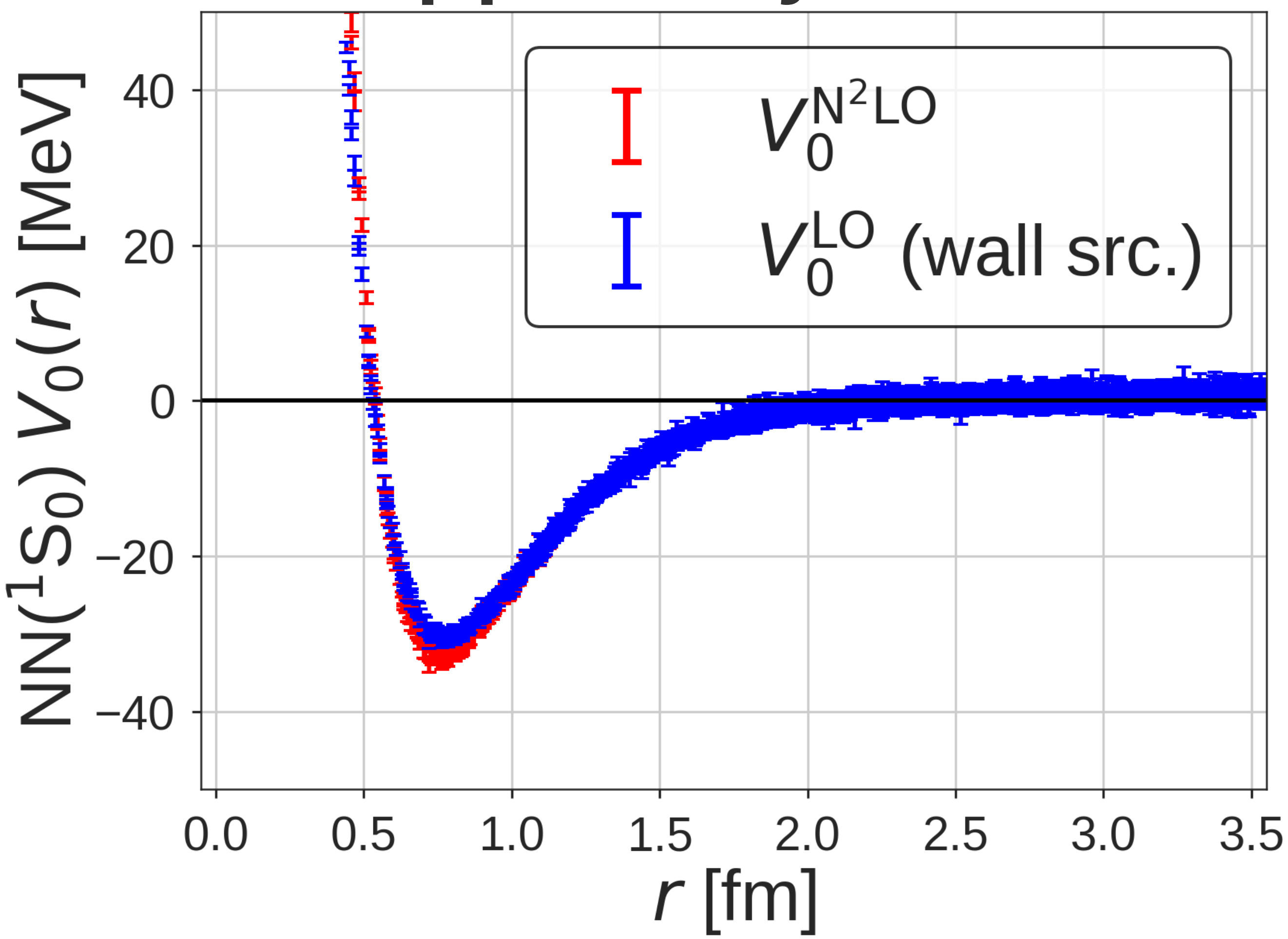


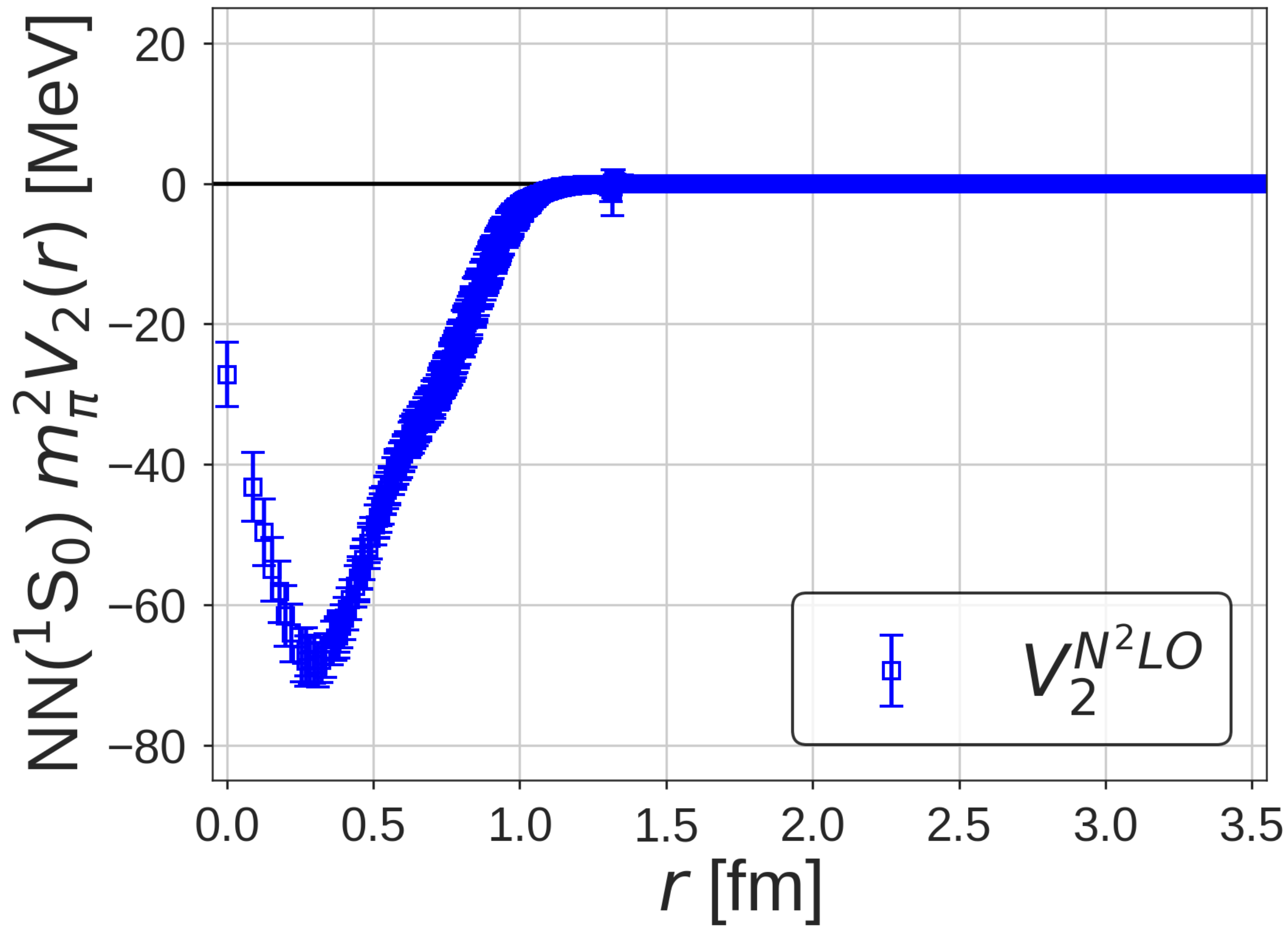
NN($^1S_0$) System at M_{PI} = 510 MeV (2) N^2LO Approx.

$$U(r,r') \simeq \left[V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\nabla^2 \right] \delta(r-r')$$

- Quantitatively, the same as $\Xi\Xi$ ($^1S_0$)

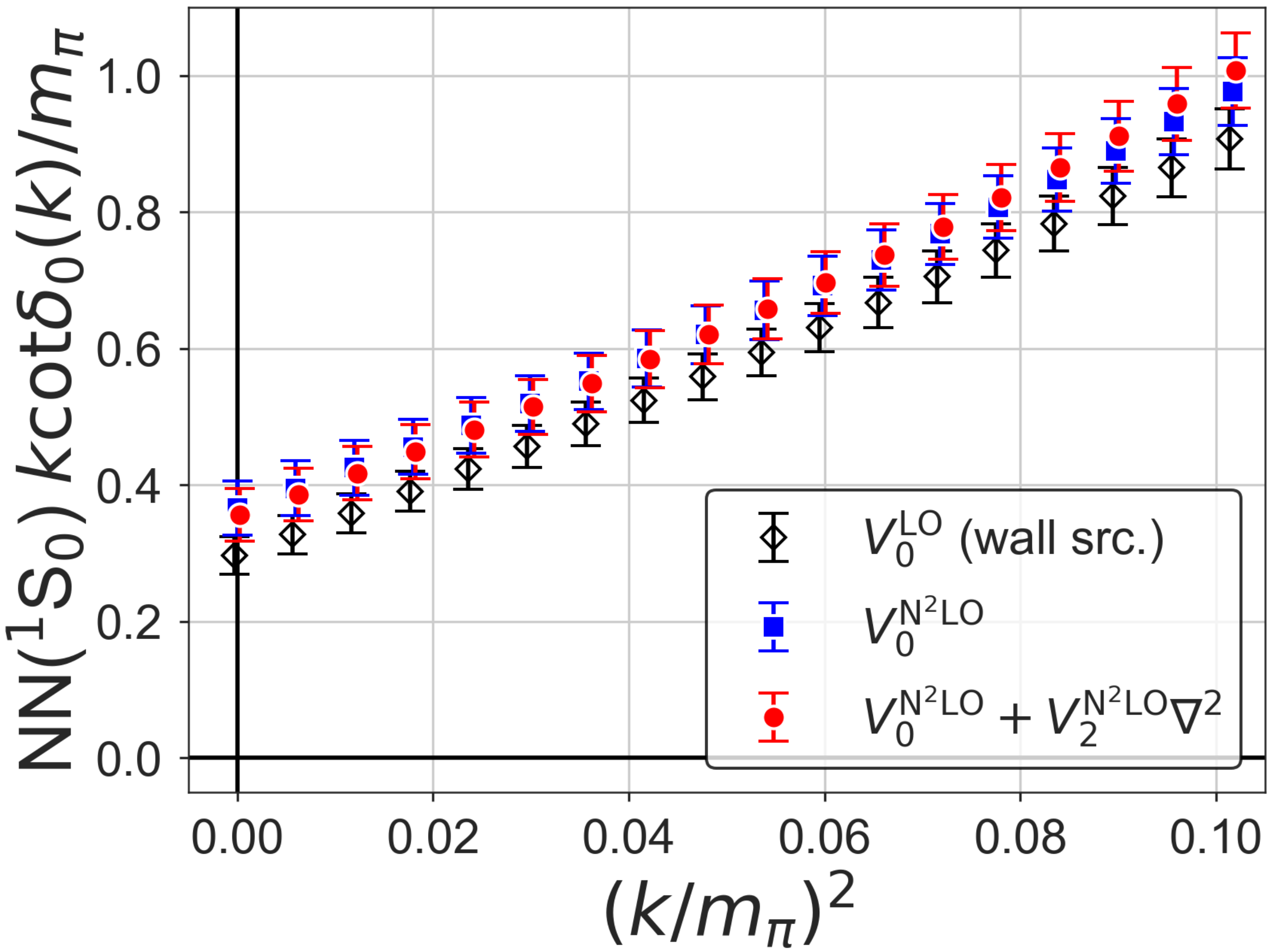
good convergence of LO approx. by wall src.

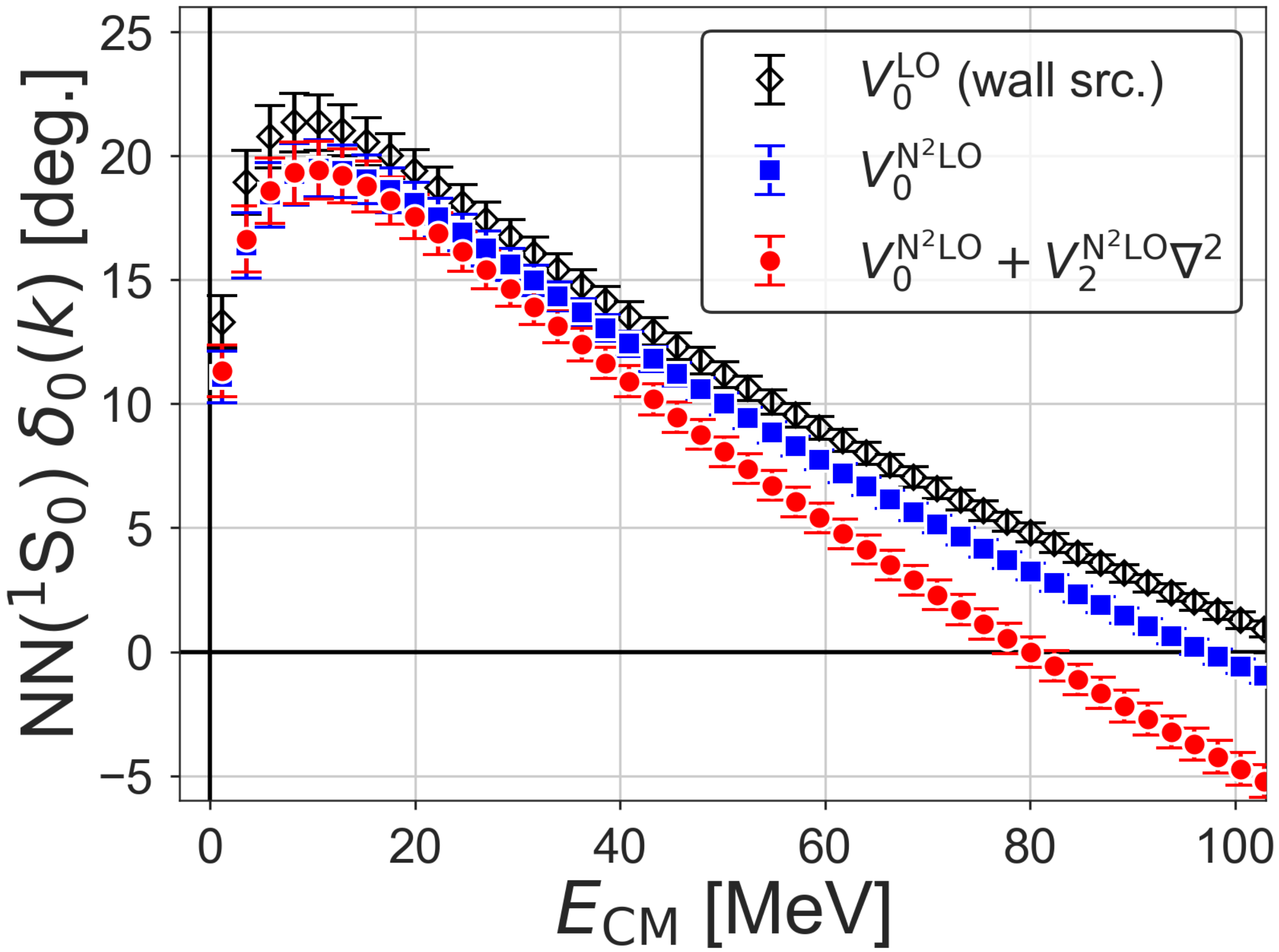




NN($^1S_0$) System at M_{PI} = 510 MeV (3) Phase Shift

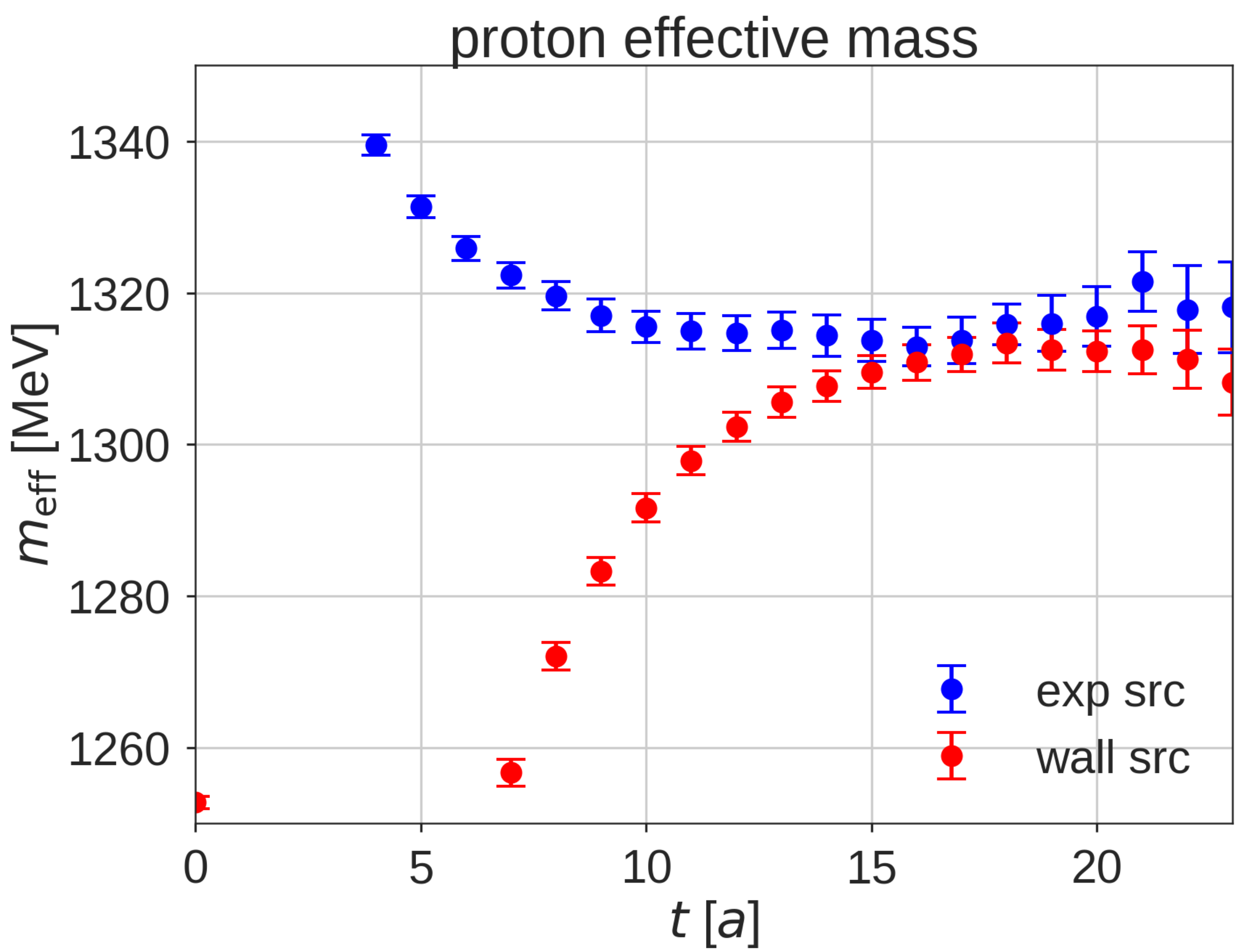
- Quantitatively, the same as ΞΞ ($^1S_0$)
- NN($^1S_0$) is **unbound** at this pion mass.
- **Wall src. LO approx.** is consistent at low energy

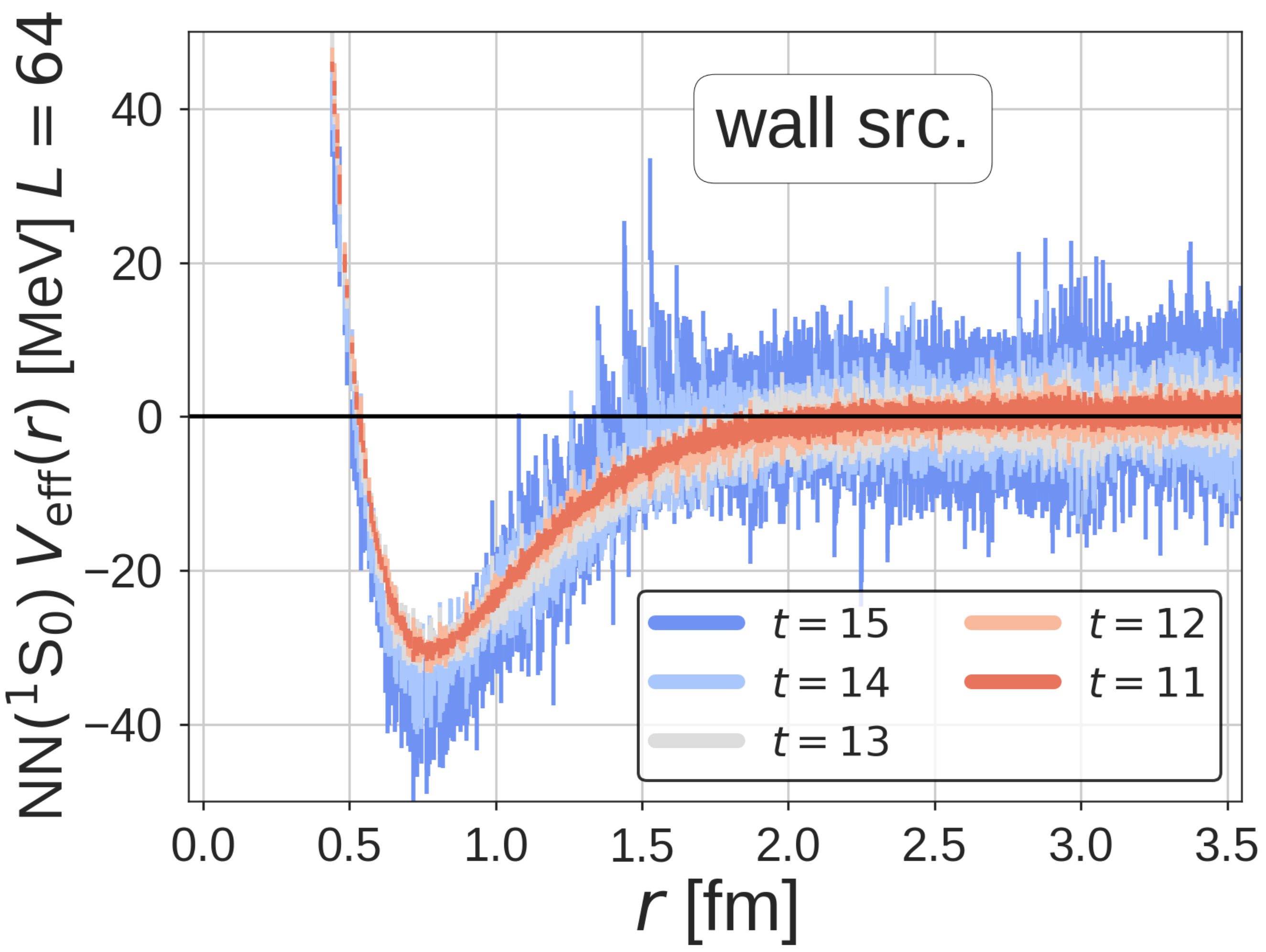




NN($^1S_0$) System at M_{PI} = 510 MeV (4) Proton Mass

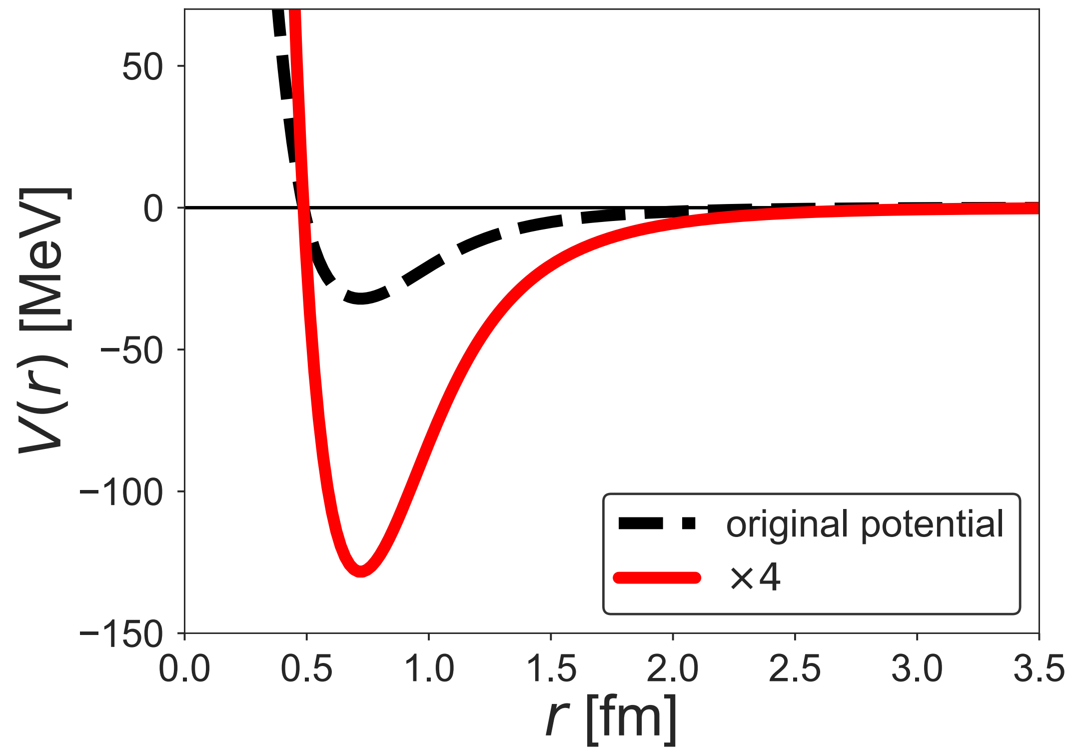
- LO approx. wall src. potential is stable against time.
- Inelastic contamination is under control.



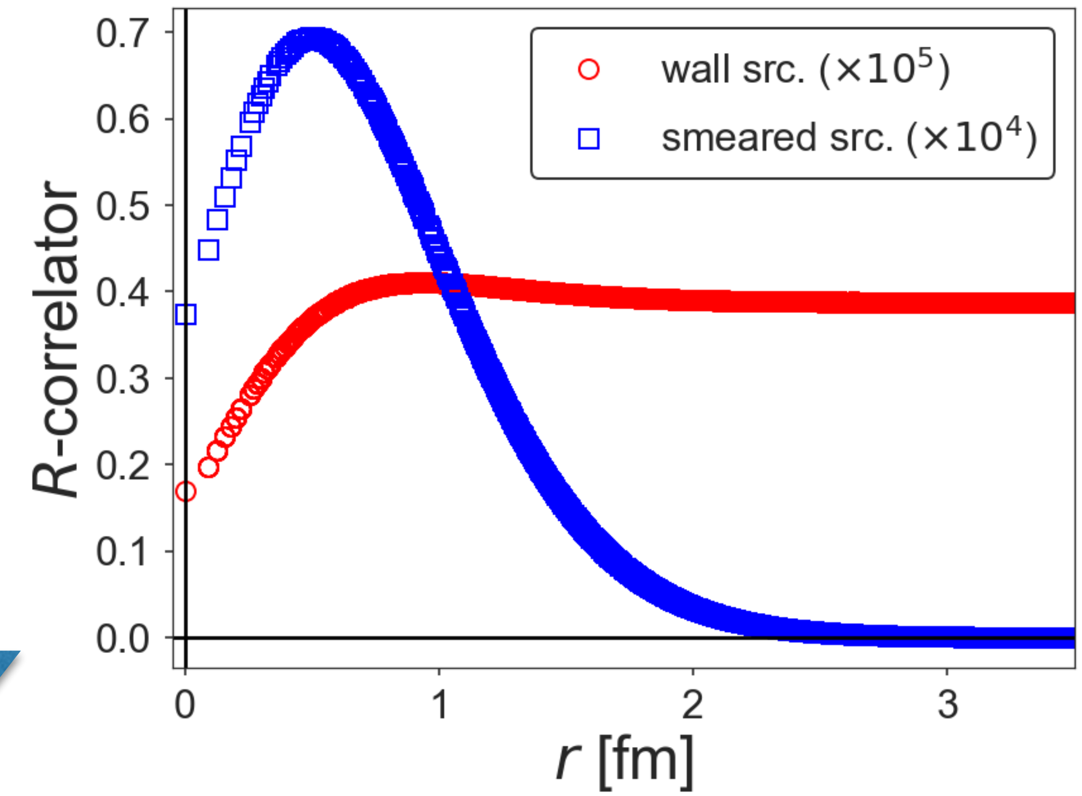


Fake Plateaux for Binding System

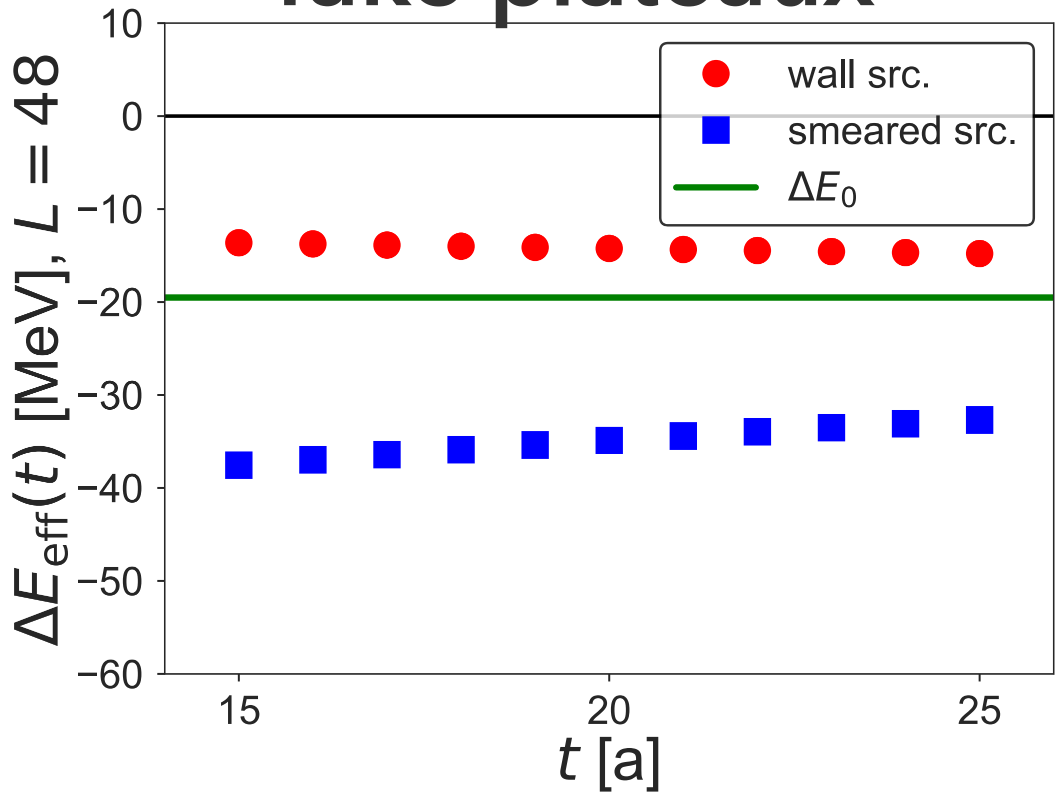
mock potential w/ bound state



mock wave func.



fake plateaux



g.s. convergence