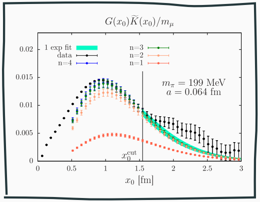
Some multi-hadron results from CLS ensembles

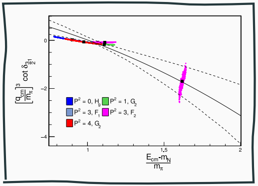
Ben Hörz (Johannes Gutenberg-Universität Mainz)

INT Workshop *Multi-Hadron Systems from Lattice QCD*

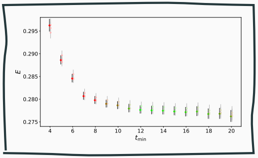
February 5, 2018



PRECISION



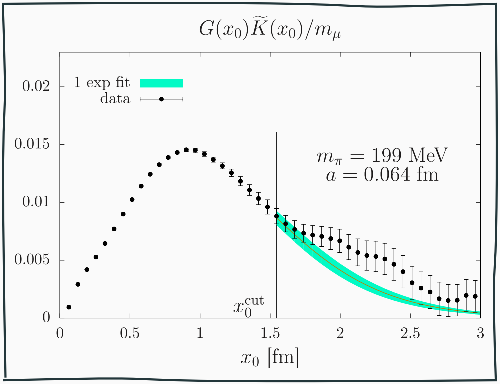
EXPLORATION



SPECTROSCOPY TECHNIQUES

Precise results for $g-2$

$$G(x_0) = \sum_{\mathbf{x}} \langle 0|J(x)J^\dagger(0)|0\rangle \approx \frac{1}{L^3}\sum_i^n \left|\langle 0|J_1(\mathbf{P}=0)|T_{1u}^+, i\rangle\right|^2 \mathrm{e}^{-E_i x_0}$$



[plot by A. Gérardin, cf. 1710.10072]

multihadron observables

- precision at large x_0

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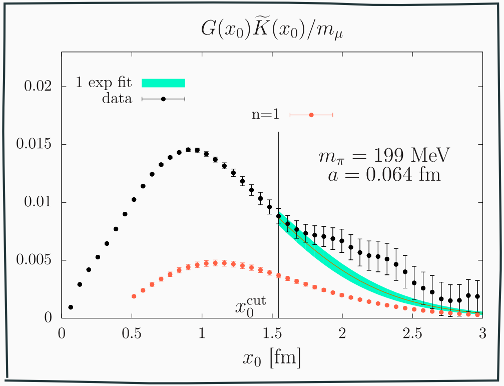

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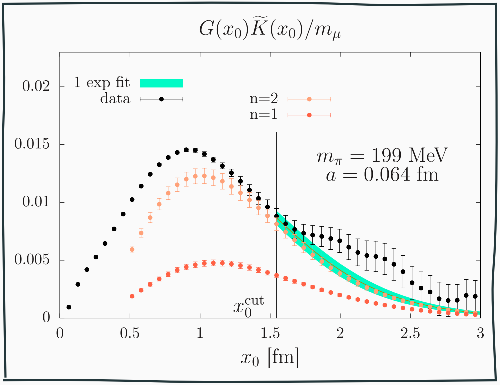

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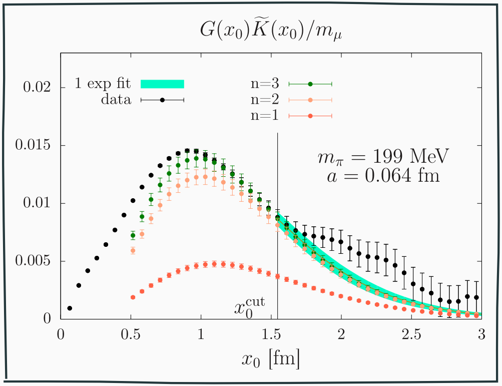

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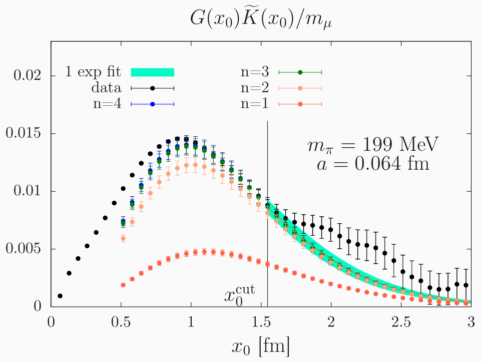


[plot by A. Gérardin, cf. 1710.10072]

multihadron observables

- precision at large x_0
- control FV effects

[Meyer, 1105.1892]

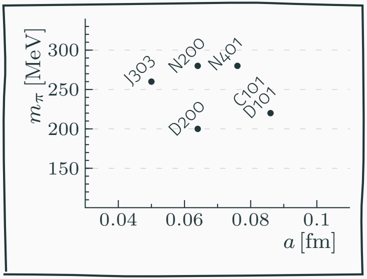
- 0-to-2 matrix element

We need good control over our own systematics!

Investigating systematic effects

goal: survey systematics for the ρ resonance

(with C. Andersen, J. Bulava, C. Morningstar)



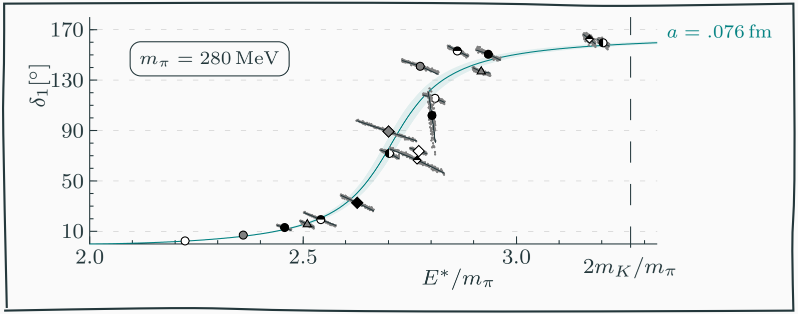
[full CLS landscape: Mohler et al, 1712.04884]

- lattice spacing a
- pion mass m_π
- higher partial waves
- residual (exponential) FV effects

Coordinated Lattice Simulations [Bruno et al, 1411.3982, 1608.08900]

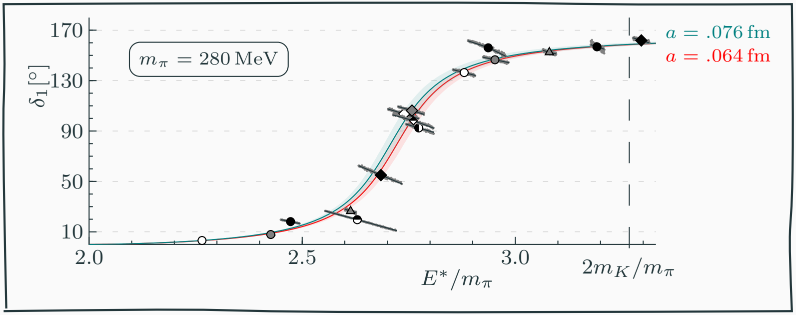
- $N_\mathrm{f} = 2 + 1$ nonpert. $\mathcal{O}(a)$-improved Wilson
- chiral trajectory along $\mathrm{Tr}\, M = \mathrm{const}$

Cutoff effects



Breit-Wigner resonance shape

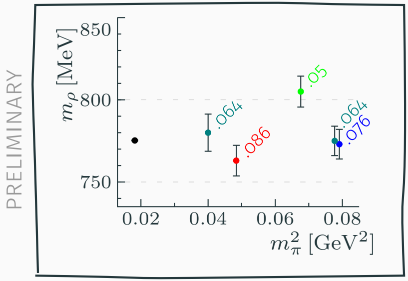
	m_ρ/m_π	$g_{\rho\pi\pi}$	χ^2/dof
$a = .076\,\mathrm{fm}$	2.712(15)	5.92(14)	1.22

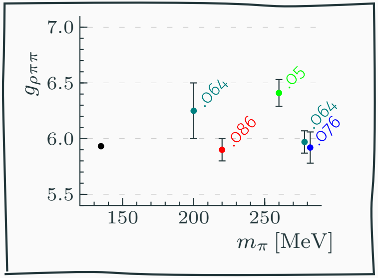


Breit-Wigner resonance shape

	m_ρ/m_π	$g_{\rho\pi\pi}$	χ^2/dof
$a = .076$ fm	2.712(15)	5.92(14)	1.22
$a = .064$ fm	2.741(16)	5.97(10)	0.75

Breit-Wigner parameters at various m_π





[scale setting: Bruno et al, 1608.08900]

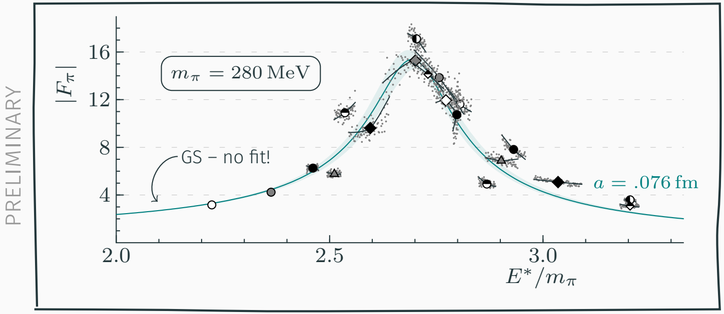
- remarkably little chiral dependence of m_ρ

  (on $\mathrm{Tr}\, M = \mathrm{const}$ trajectory)

- relevance of cutoff effects remains to be seen

  (low statistics at $a = 0.05\,\mathrm{fm}$ – autocorrelation increasingly important!)

- final analysis currently under way



- example of 0-to-2 matrix element

  (parametrization of $\delta_1(E^*)$ required as input)

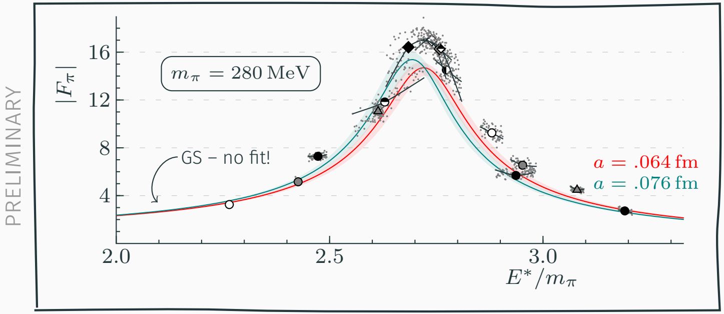
- so far: pert. $\mathcal{O}(a)$ improvement, nonpert. renormalization

  (improvement coefficient could be as large as $c_V \approx -0.25$)

  [Gérardin, Harris, Meyer, priv. comm.]

- delicate analysis (currently ongoing)

Preview: Timelike pion form factor



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Extraction of resonant meson-baryon scattering amplitudes

- at a more exploratory stage
- complicated by kinematics (inelastic thresholds!) ...

$$m_\pi \sqrt{1 + \left(\frac{2\pi}{m_\pi L}\right)^2}$$

at $m_\pi L \approx 3.6$,
one unit of momentum is worth an extra pion

- ...and rotational symmetry breaking in a box

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Two case studies

- $\Delta(1232)$ (with C. Andersen, J. Bulava, C. Morningstar)
- $\Lambda(1405)$ (with JB, M. Hansen, D. Mohler, CM, H. Wittig)

Rotational symmetry in a box

$\Delta(1232)$: $I = 3/2$ p-wave N-π scattering

J^P	[000]	$[00n]$	$[0nn]$	$[nnn]$	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	$\Delta(1750)$ (?)
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Delta(1620)$
$\frac{3}{2}^+$	H_g	$G_1 \oplus G_2$	$2G$	$F_1 \oplus F_2 \oplus G$	$\Delta(1232)$, $\Delta(1600)$
$\frac{3}{2}^-$	H_u	$G_1 \oplus G_2$	$2G$	$F_1 \oplus F_2 \oplus G$	$\Delta(1700)$

- ☐ disentangle various $N - \pi$ amplitudes [Morningstar et al, 1707.05817]
- ☐ avoid mixing with lower partial wave [Goeckeler et al, 1206.4141]

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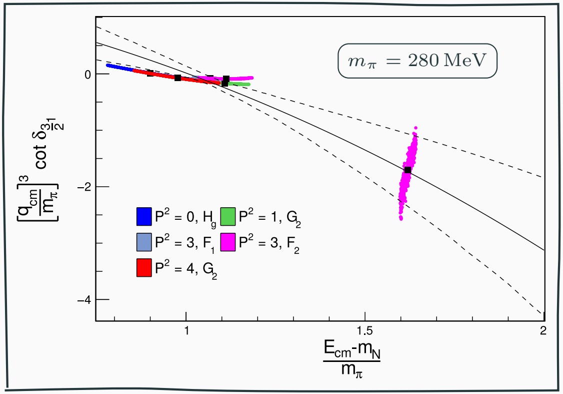
$I = 3/2$ N-π SCATTERING AMPLITUDE



[Andersen et al, 1710.01557]

- Breit-Wigner parametrization

$$m_\Delta = 1344(20)\,\mathrm{MeV} \qquad g^{\mathrm{BW}}_{\Delta N\pi} = 19.0(4.7) \qquad \chi^2/\mathrm{dof} = 1.11$$

- $\Delta(1232)$ – the 'ρ of baryons'

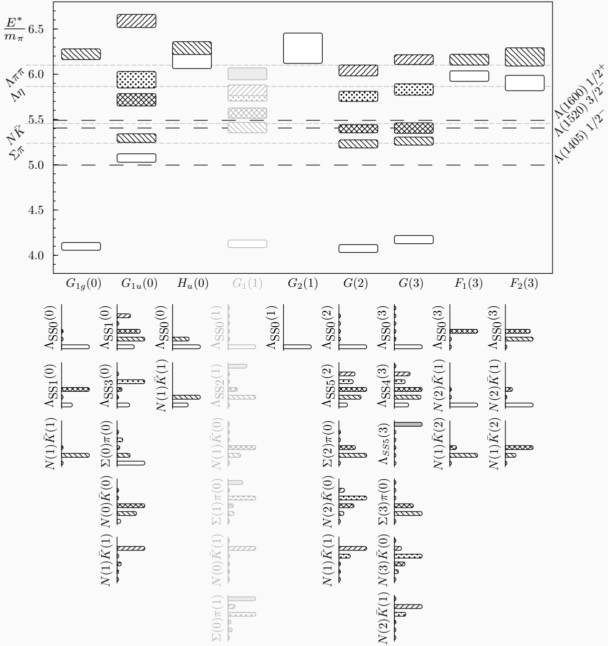
$\Lambda(1405)$ SETUP

J^P	[000]	[00n]	[0nn]	[nnn]	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	Λ, $\Lambda(1600)$
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Lambda(1405)$, $\Lambda(1670)$
$\frac{3}{2}^+$	H_g	$G_1 \oplus G_2$	$2G$	$F_1 \oplus F_2 \oplus G$	$\Lambda(1890)$
$\frac{3}{2}^-$	H_u	$G_1 \oplus G_2$	$2G$	$F_1 \oplus F_2 \oplus G$	$\Lambda(1520)$, $\Lambda(1690)$

$I = 0$, $S = -1$ Σ-π / N-$\bar{K}$ (/ Λ-η) scattering

- small kinematic window below three-particle threshold
- coupled-channel system
- complicated partial wave mixing pattern

The whole two-particle machinery will be required!



$\frac{E^*}{m_\pi}$
6.5
6.0
5.5
5.0
4.5
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$\Lambda\pi\pi$
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$\Lambda(1600)\ 1/2^+$
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$G_{1g}(0)$ $G_{1u}(0)$ $H_u(0)$ $G_1(1)$ $G_2(1)$ $G(2)$ $G(3)$ $F_1(3)$ $F_2(3)$
ΛSS0(0) ΛSS1(0) $N(1)\bar{K}(1)$
ΛSS1(0) ΛSS3(0) $\Sigma(0)\pi(0)$ $N(0)\bar{K}(0)$ $N(1)\bar{K}(1)$
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ΛSS0(1)
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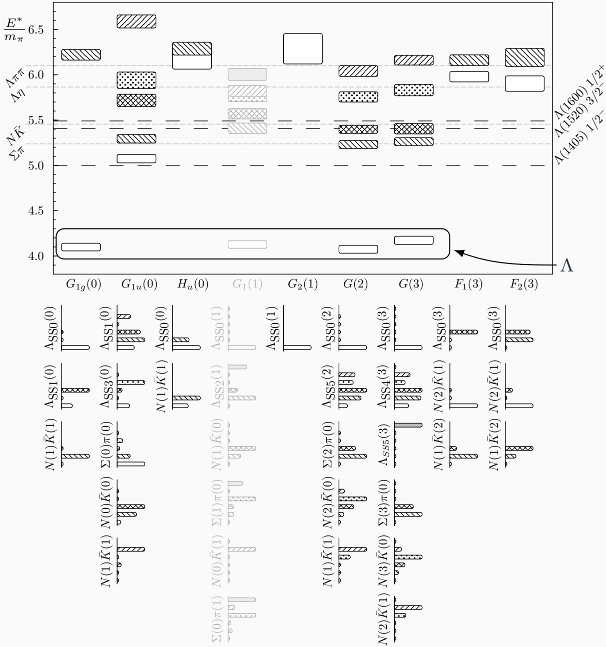

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Λ
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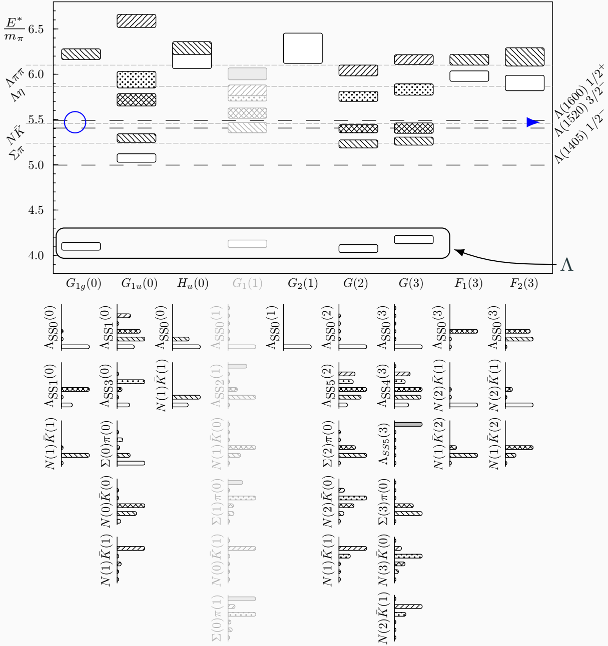

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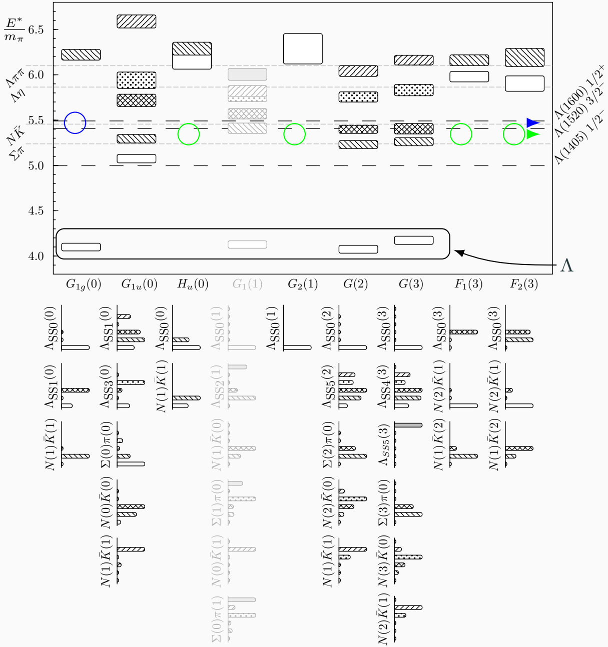

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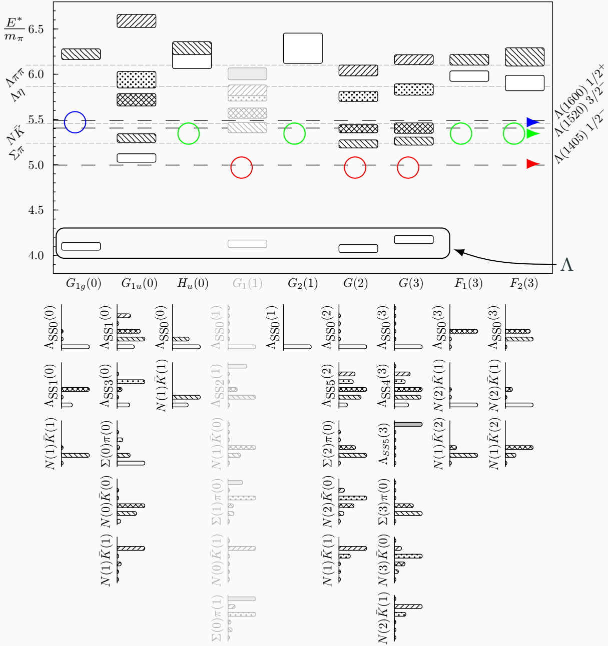

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Tentative $\Lambda(1405)$ lessons

- no FV states associated with $\Lambda(1520)\,3/2^-$ & $\Lambda(1600)\,1/2^+$
  - same pattern in previous studies

    [Edwards et al, 1212.5236; Engel et al, 1301.4318]
  - $\mathrm{Tr}\,M = \mathrm{const}$ here

    [ChPT predictions: Lutz et al, 1801.06417]

- if corroborated, absence of FV states constrains models

  (check of interpolator basis required)

- high precision required for Lüscher-style analysis

  (narrow kinematic range: $5.2 \leq E^*/m_\pi \leq 5.8$)

A few words on lattice spectroscopy

- systematic analysis uncertainties
- correlation function construction in large volumes

- diagonalization procedure
  - GEVP with fixed diagonalization times
  - *window method*
  - *fixed* t_0 *method*
  - Prony methods
  - ...
- fit procedure
  - single-exponential
  - two-exponential
  - multi-state fits
  - ratio fits
  - ...
- ...

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- ...

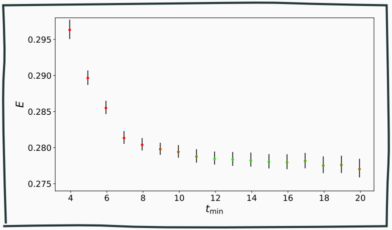
Controlling excited-state contamination

GEVP efficacy

$I = 1$ '$\pi(0)\pi(1)$ correlator' $\mathbf{d} = [001]$ A_1^+

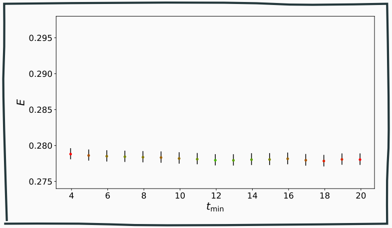
$n_{\rm op} = 1$

singleExp



ratio

$R(t) = C(t)/(C_{\pi(0)}(t)C_{\pi(1)}(t))$

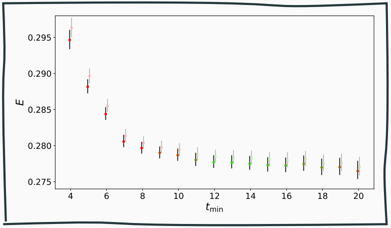


Gevp efficacy

$I = 1$ '$\pi(0)\pi(1)$ correlator' $\mathbf{d} = [001]$ A_1^+

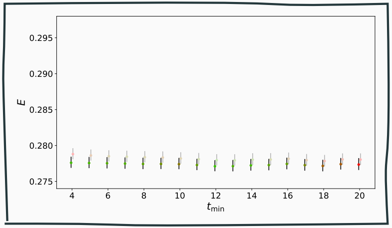
$n_{\text{op}} = 2$

singleExp



ratio

$$R(t) = C(t)/(C_{\pi(0)}(t)C_{\pi(1)}(t))$$



Gevp efficacy

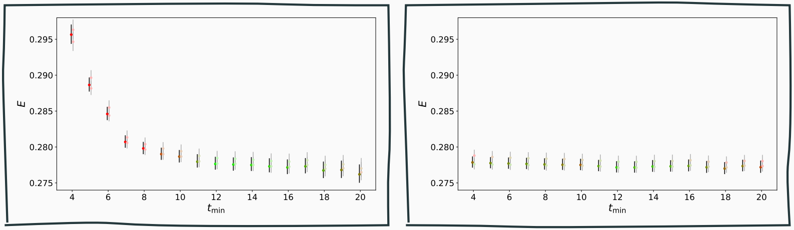
$I = 1$ 'π(0)π(1) correlator' $\mathbf{d} = [001]$ A_1^+

$n_{\text{op}} = 3$

singleExp

ratio

$R(t) = C(t)/(C_{\pi(0)}(t)C_{\pi(1)}(t))$

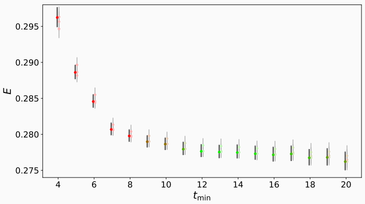


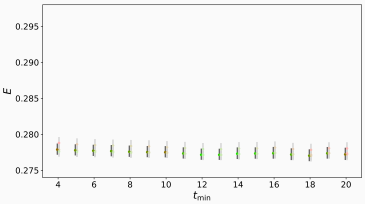
GEVP EFFICACY

$I = 1$ ‘$\pi(0)\pi(1)$ correlator’ $\mathbf{d} = [001]$ A_1^+

singleExp $n_{\mathrm{op}} = 4$ ratio

$R(t) = C(t)/(C_{\pi(0)}(t)C_{\pi(1)}(t))$



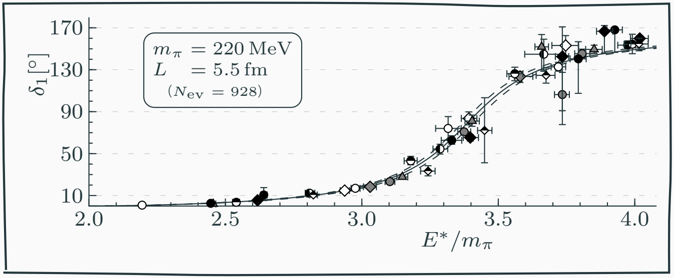


All excited-state contamination is not equal

- GEVP can only help with ‘multi-hadron excited states’
- should use *improved* single-hadron operators to build MH

[Shultz et al, 1501.07457; Woss, Thomas, 1612.05437; Berkowitz et al, 1710.05642]

- ...but how (for light pions)?



[Bulava et al, 1710.04545]

- large volumes crucial for small pion masses
  (exponential effects, energy resolution)
- challenge for treatment of quark propagation
  (efficient techniques also relevant for 3-particle spectrum)
- ideally: precision & generality of distillation
  @ cost of stochastic LapH

Closing remarks

- 2-particle results range from robust to exploratory
- control over 3-particle thresholds desirable
- spectroscopy practitioners:
  let us not forget about furthering our craft