# Two-baryon spectroscopy and distillation

Jeremy Green

NIC, DESY, Zeuthen

Multi-Hadron Systems from Lattice QCD
Institute for Nuclear Theory
February 5–9, 2018

# Outline

1. Previous study using smeared point sources
2. Distillation and two-baryon correlators
3. Previous study revisited with distillation
4. Preliminary results with $N_f = 3$
5. Challenges
6. Outlook

# Mainz project

Goal: study the conjectured $H$ dibaryon

- $0^+$ SU(3) singlet, quark content $uuddss$
- Initially using $N_f = 2$ ensembles from CLS and a quenched strange.

People:

- Hartmut Wittig
- current Mainz postdoc: Andrew Hanlon
- former Mainz postdocs: Anthony Francis, JG, Parikshit Junnarkar, Chuan Miao, Thomas Rae

Preliminary work presented at conferences:

C. Miao *et al.*, PoS **LATTICE2013** 440 [1311.3933]
JG *et al.*, PoS **LATTICE2014** 107 [1411.1643]
P. Junnarkar *et al.*, PoS **LATTICE2015** 082, PoS **CD15** 079 [1511.01849]
P. Junnarkar *et al.*, talk at Confinement XII (2016)

Final results from point sources are in preparation.

# Interpolating operators

Use smeared quark fields. Consider two kinds of operators:

1. Hexaquark,

$$\mathcal{O}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}(qqqqqq)(t,\vec{x}).$$

   - Looks like bag-model picture of $H$ dibaryon.
   - Under broken SU(3), two operators couple: singlet $H^1$ and 27-plet $H^{27}$.

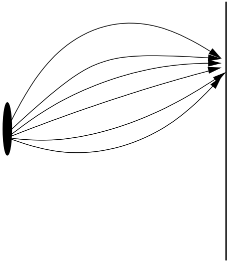
2. Two-baryon,

$$\mathcal{O}(t,\vec{p}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_1\cdot\vec{x}} e^{-i\vec{p}_2\cdot\vec{y}}(qqq)(t,\vec{x})(qqq)(t,\vec{y}), \quad \vec{p}_1 + \vec{p}_2 = \vec{p}.$$

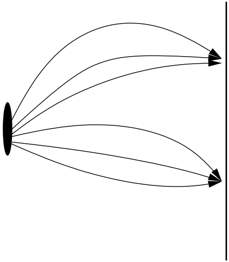
   - Looks more like noninteracting state.
   - Three relevant flavour combinations $\Lambda\Lambda$, $\Sigma\Sigma$, and $N\Xi$. Can rotate to SU(3) basis to get $BB^1$, $BB^8$, $BB^{27}$.
   - Many combinations of $(\vec{p}_1, \vec{p}_2)$ possible.
   - Can't evaluate at a point source.

# Point-source correlators

Using a (smeared) point-source propagator, we can compute:



$\langle H(t) H^\dagger(0) \rangle$



$\langle BB(t) H^\dagger(0) \rangle$

Can't compute $\langle BB(t) BB^\dagger(0) \rangle$ since it is not local on the source timeslice.

# Variational method

Given $N$ sink operators $\tilde{O}_i$ and source operators $O_j$, compute the correlator matrix

$$C_{ij}(t) = \left\langle \tilde{O}_i(t) O_j^\dagger(0) \right\rangle.$$

It has the spectral decomposition

$$C_{ij}(t) = \sum_n \tilde{Z}_i^{(n)} Z_j^{(n)*} e^{-E_n t}.$$

Solve the GEVP,

$$C(t+\Delta) v^{(n)}(t) = \lambda_n(t) C(t) v^{(n)}(t),$$
$$\tilde{v}^{(n)\dagger}(t) C(t+\Delta) = \lambda_n(t) \tilde{v}^{(n)\dagger}(t) C(t)$$

- If only $N$ states contribute, then $v^{(n)}$ and $\tilde{v}^{(n)}$ are the dual vectors to $Z^{(n)}$ and $\tilde{Z}^{(n)}$, and $\lambda_n = e^{-E_n \Delta}$.
- If $\tilde{O}_i = O_i$, then C is Hermitian, $v = \tilde{v}$, and

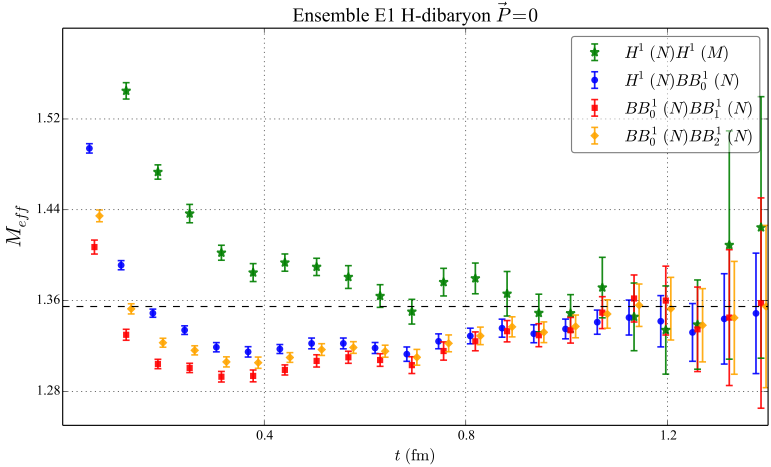
$$E_n^{\text{eff}}(t) \equiv \frac{-1}{\Delta} \log \lambda_n = E_n + O(e^{-(E_{N+1}-E_n)t}).$$

# Results with point sources

- Old CLS ensembles: $N_f = 2$, $O(a)$-improved Wilson fermions. Mainly show two of them with $a = 0.0658$ fm, $L = 2.1$ fm.
  - E1: $m_\pi = 960$ MeV, quenched $m_s = m_{ud}$.
  - E5: $m_\pi = 440$ MeV, quenched $m_s \approx m_s^{\text{phys}}$.
- Increase number of source operators using two different smearing widths.
- Apply GEVP to $2 \times 2$ or $4 \times 4$ square subsets of the $\mathrm{C}_{ij}$ at a fixed time to find $v$, $\tilde{v}$. Then compute the effective energy of the projected correlators $\Lambda_n(t) \equiv \tilde{v}^{(n)\dagger}\mathrm{C}(t)v^{(n)}$:

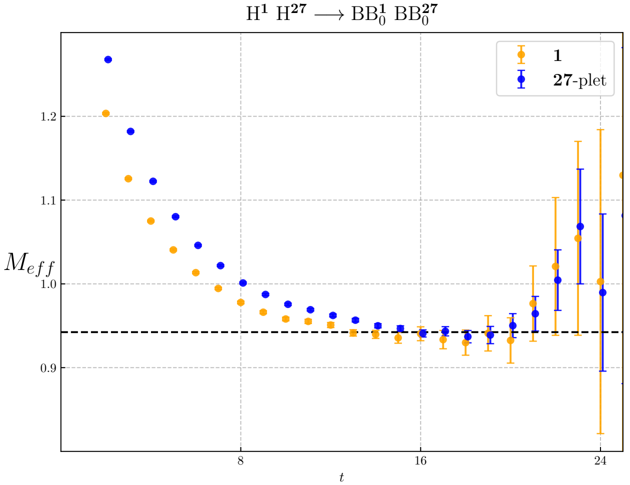
$$E_n^{\text{eff}}(t) \equiv \frac{1}{\Delta} \log \frac{\Lambda_n(t)}{\Lambda_n(t+\Delta)}.$$

# Effective energy, SU(3) singlet




Ground state from $2 \times 2$ GEVP. Fixed $\{O_i\}$, varying $\{\tilde{O}_i\}$.

# Effective energies, broken SU(3)




For broken SU(3), find two low-lying levels. One couples mostly to singlet operators, the other mostly to 27-plet.

# Outlook from point-source study

- Two-baryon operators seem more relevant than hexaquark operators.
- It is messier to analyze non-Hermitian correlator matrices.

Therefore we want a Hermitian matrix of correlators with two-baryon operators at source and sink.

One method for efficiently computing this: distillation

# Distillation

We construct interpolating operators using smeared quark fields $\tilde{q}(\vec{x}) = S(\vec{x},\vec{y})q(\vec{y})$. Common Gaussian-like smearing:

$$S = S_{\text{Wuppertal}} \approx e^{\sigma\Delta}.$$

Suppresses high modes of spatial Laplacian $-\Delta$.
Laplacian-Heaviside (LapH) smearing: project onto lowest modes,

$$S_{\text{LapH}} = \sum_{n=1}^{N_{\text{LapH}}} u_n u_n^\dagger, \quad -\Delta u_n = \lambda_n u_n, \quad 0 < \lambda_1 < \lambda_2 < \cdots$$

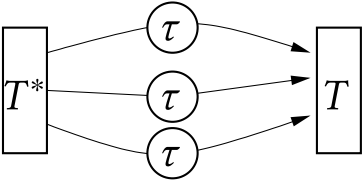
For constant smearing width, need $N_{\text{LapH}} \propto L^3$.
It is feasible to compute the timeslice-to-all or all-to-all propagator for LapH-smeared quarks. This requires the perambulator,

$$\tau_{n',n}(t',t) \equiv \sum_{\vec{x},\vec{x}'} u_{n'}^{(t')\dagger}(\vec{x}')D^{-1}(\vec{x}',t';\vec{x},t)u_n^{(t)}(\vec{x}).$$

Using perambulators to compute correlators forms the basis of *distillation*.

The simplest baryon correlators can be constructed from perambulators together with mode triplets,

$$T_{lmn}(t,\vec{p}) \equiv \sum_{\vec{x},a,b,c} e^{-i\vec{p}\cdot\vec{x}} \epsilon_{abc} u^{(t)}_{la}(\vec{x}) u^{(t)}_{mb}(\vec{x}) u^{(t)}_{nc}(\vec{x}).$$

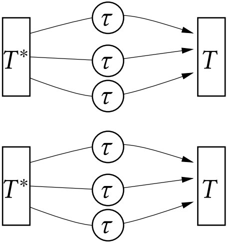



$\langle B(t)B^\dagger(0)\rangle$

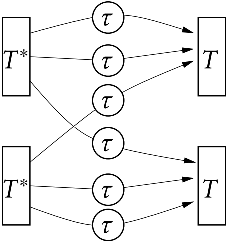
Contraction cost $\propto N_{\text{LapH}}^4$.

# Distillation and two baryons

For a correlator $\langle BB(t) BB^\dagger(0) \rangle$, Wick contractions fall into two classes:

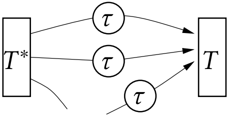



straight



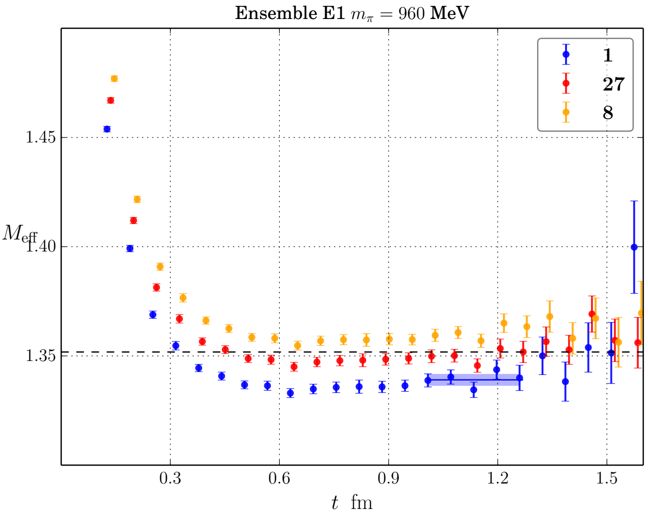

exchange

To compute these, first compute the partially-contracted source-sink blocks,



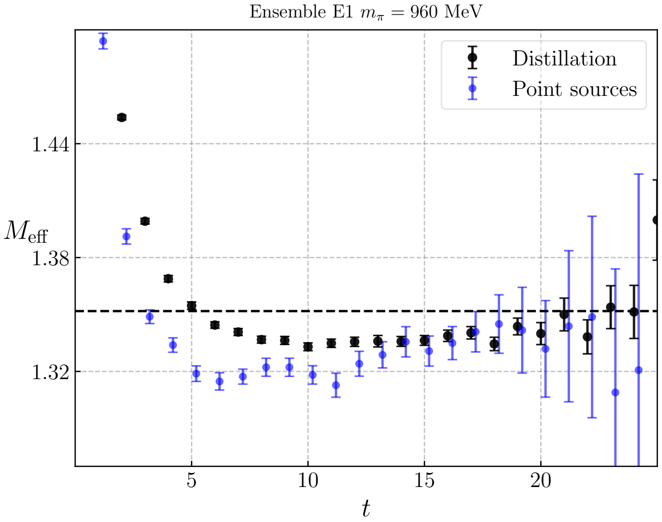

at a cost $\propto N_{\text{LapH}}^4$. Combining them to form correlators is inexpensive.

# Effective energy, distillation

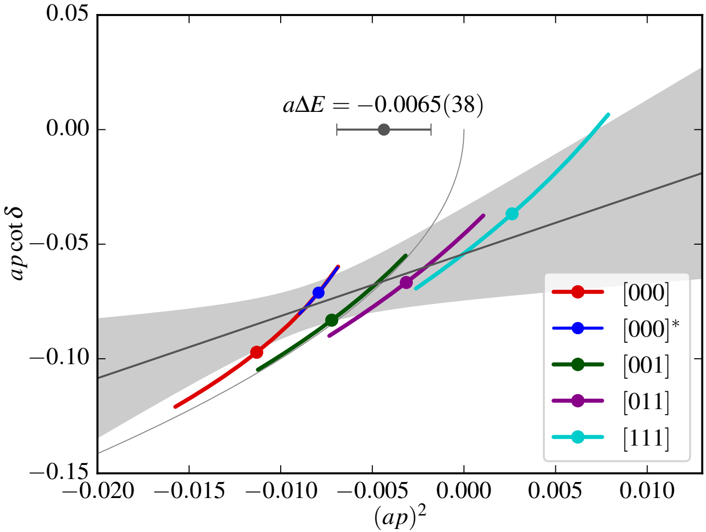



Exact SU(3): $E_{\rm eff}$ from a single operator in each flavour channel.

# Comparison: point-source and distillation

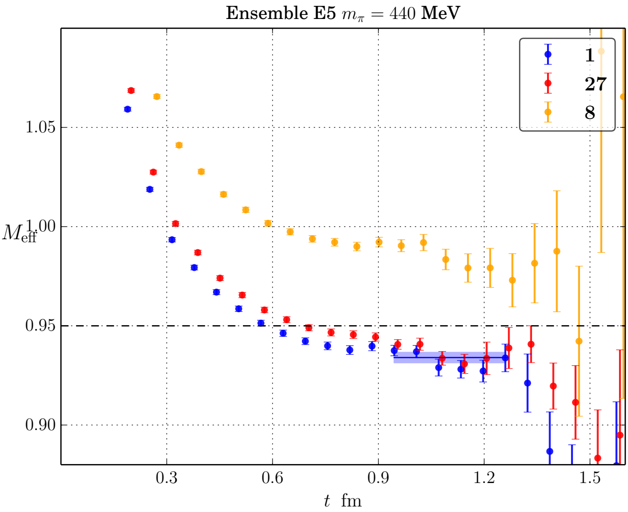


# E1: singlet $^1S_0$ phase shift




Distillation point [000]* provides strongest constraint. $\Delta E = 19(11)$ MeV.

# Effective energy, distillation (variational)




Broken SU(3): $E_{\text{eff}}$ from projected correlators using GEVP solution.

# Ongoing project with distillation

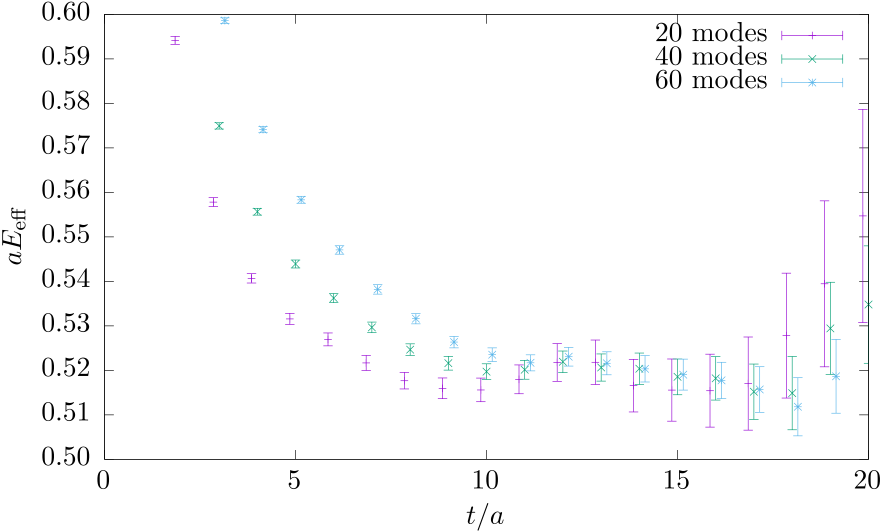
New CLS ensembles: $N_f = 2 + 1$ $O(a)$-improved Wilson fermions.

- $m_s$ varied simultaneously with $m_{ud}$ to keep $m_u + m_d + m_s$ constant.
- Multiple volumes available at same lattice parameters.
- Initial study on two ensembles at SU(3)-symmetric point, $m_\pi = m_K = m_\eta \approx 420$ MeV, $a = 0.086$ fm:
  - U103: $L = 2.06$ fm
  - H101: $L = 2.75$ fm
- Keep contraction costs down by using $P_+ \equiv \frac{1+\gamma_4}{2}$-projected quark fields: need only 2-component spinors.

Challenge: $N_{\text{LapH}}^4$ cost scaling for contractions.
(Recall $N_{\text{LapH}} \propto L^3$ to obtain constant smearing width.)

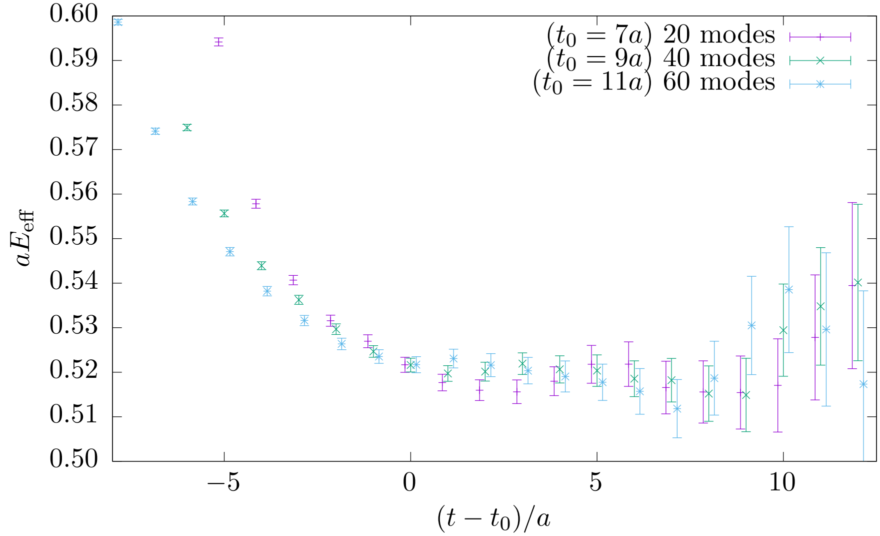
Results shown are VERY PRELIMINARY.

# U103: octet baryon effective mass




How small can we make $N_{\text{LapH}}$?

# U103: effective mass, shifted to plateau start




How small can we make $N_{\mathrm{LapH}}$? We will use 20 on U103 and 48 on H101.

# Two-baryon interpolating operators

First, form single-baryon operators, e.g.

$$\mathcal{O}_p(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \epsilon_{abc} \left[u_a(u_b^T C\gamma_5 P_+ d_c)\right](t,\vec{x}),$$

then construct spin-zero and spin-one two-baryon operators:

$$\mathcal{O}_{B_1B_2,0}(t,\vec{P}) = \sum_{\vec{p}} f(\vec{p})\mathcal{O}^T_{B_1}(t,\vec{p}) C\gamma_5 P_+ \mathcal{O}_{B_2}(t,\vec{P}-\vec{p})$$

$$\mathcal{O}_{B_1B_2,1}(t,\vec{P}) = \sum_{\vec{p}} g_i(\vec{p})\mathcal{O}^T_{B_1}(t,\vec{p}) C\gamma_i P_+ \mathcal{O}_{B_2}(t,\vec{P}-\vec{p})$$

Simplest rest-frame operators:

$$f^{(n)}(\vec{p}) = \begin{cases} 1 & p^2 = n\left(\frac{2\pi}{L}\right)^2 \\ 0 & \text{otherwise} \end{cases}$$

# Two-baryon interpolating operators: moving frames

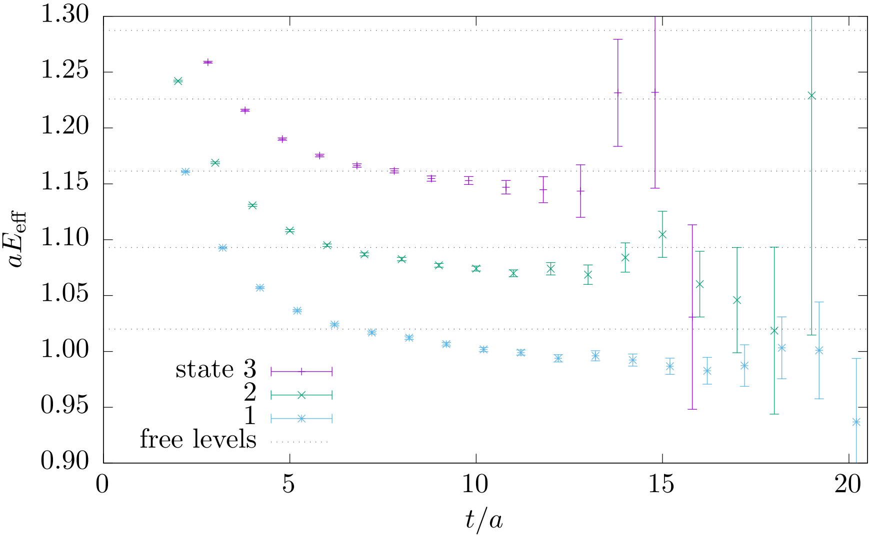
In moving frames, can often construct spin-zero and spin-one operators with the same lattice quantum numbers.

Consider flavor-symmetric operators in frame $\vec{P} = \frac{2\pi}{L}(1,1,0)$, $A_1$ irrep.

Let $\vec{p}_1 = \frac{2\pi}{L}\hat{x}$, $\vec{p}_2 = \frac{2\pi}{L}\hat{y}$. Operators corresponding to lowest free levels:

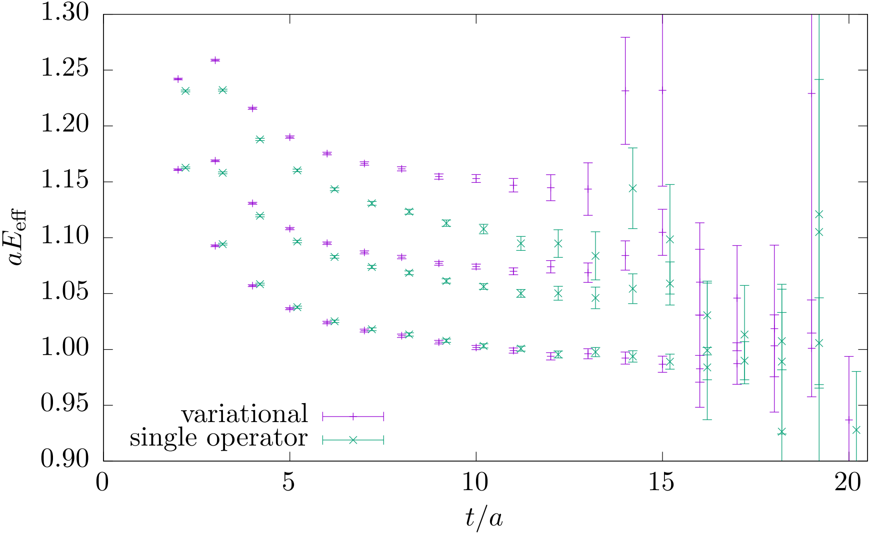
1. Orbital angular momentum in $A_1$: $\mathcal{O}_B^T(t,\vec{P})C\gamma_5 P_+ \mathcal{O}_B(t,0)$
2. Orbital angular momentum in $A_1$: $\mathcal{O}_B^T(t,\vec{p}_1)C\gamma_5 P_+ \mathcal{O}_B(t,\vec{p}_2)$
3. Orbital angular momentum in $B_1$: $\mathcal{O}_B^T(t,\vec{p}_1)C\gamma_3 P_+ \mathcal{O}_B(t,\vec{p}_2)$

# H101: SU(3) singlet, rest frame, $A_1^+$ irrep



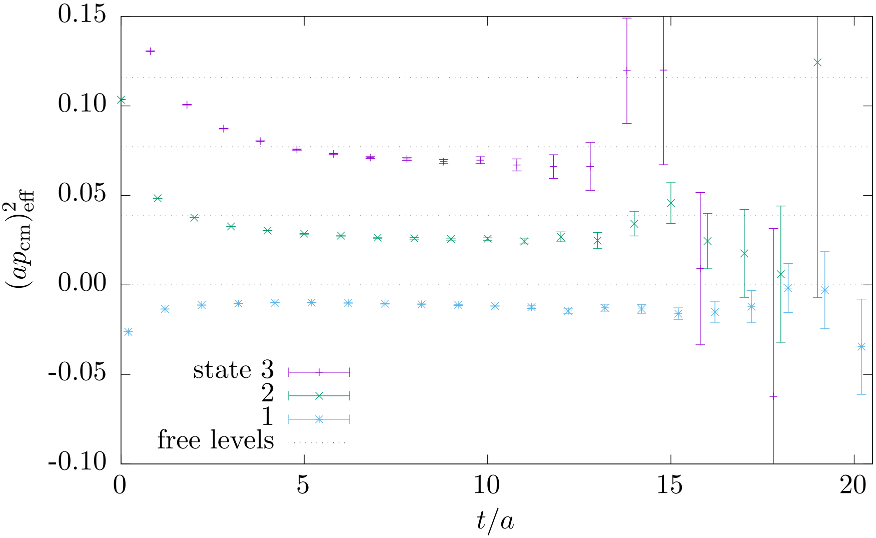

From a $3 \times 3$ Hermitian correlator matrix.

# H101: SU(3) singlet, rest frame, $A_1^+$ irrep



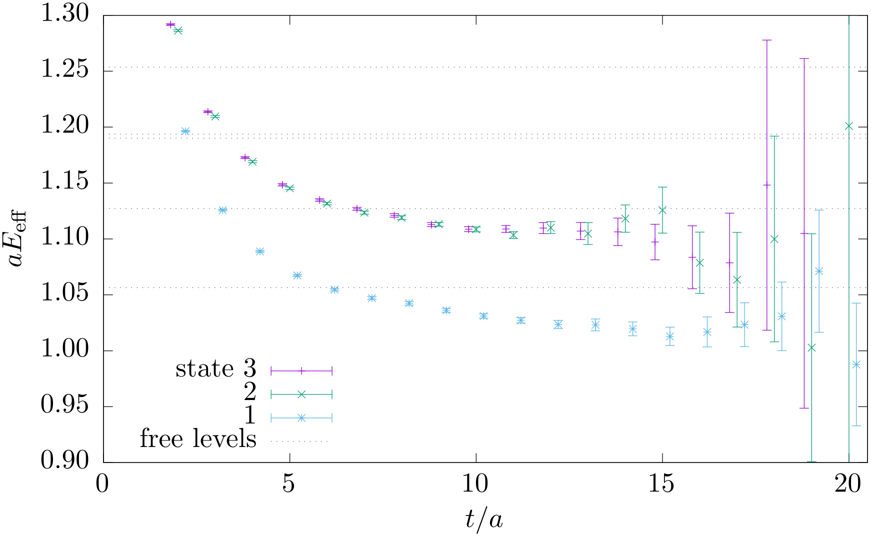

From a $3 \times 3$ Hermitian correlator matrix.

# H101: SU(3) singlet, rest frame, $A_1^+$ irrep



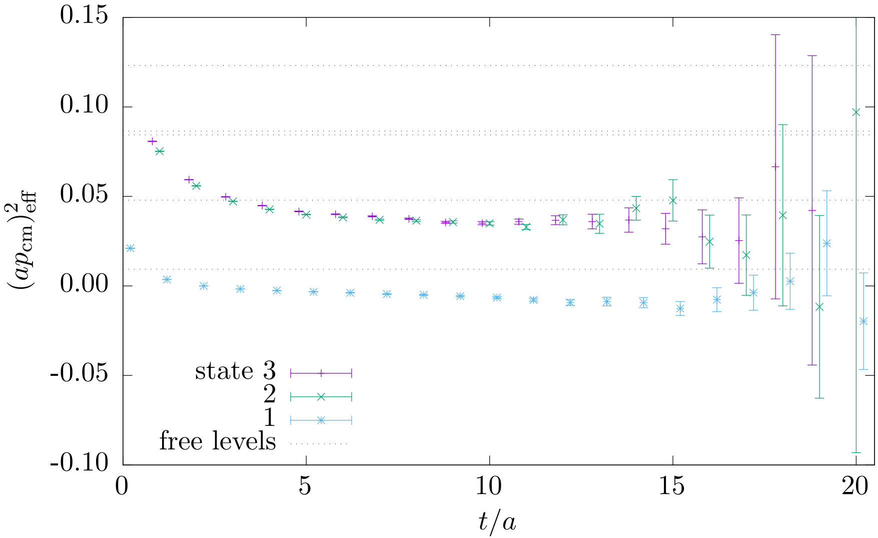

From a $3 \times 3$ Hermitian correlator matrix.

# H101: SU(3) singlet, frame $\vec{P} = \frac{2\pi}{L}(1, 0, 0)$, $A_1$ irrep



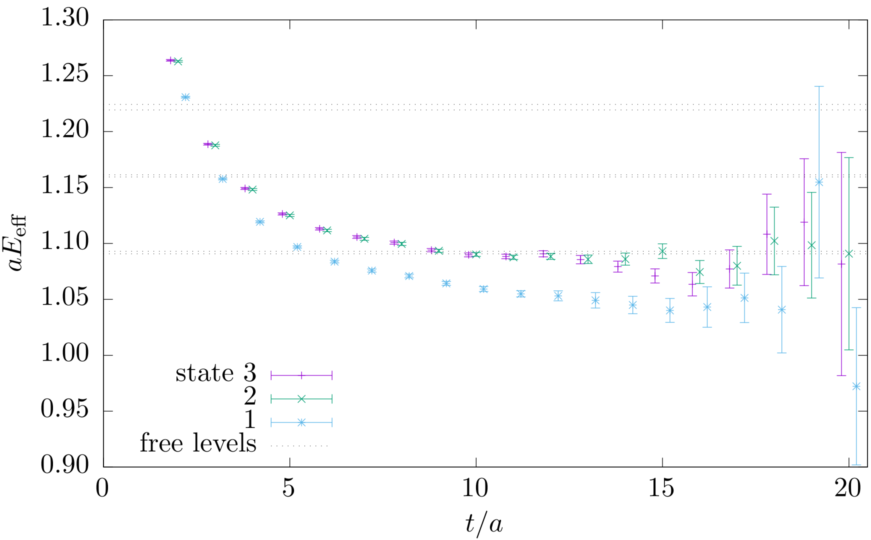

First two states couple mostly to $^1S_0$ operators; third state to $^3P_1$.

# H101: SU(3) singlet, frame $\vec{P} = \frac{2\pi}{L}(1, 0, 0)$, $A_1$ irrep



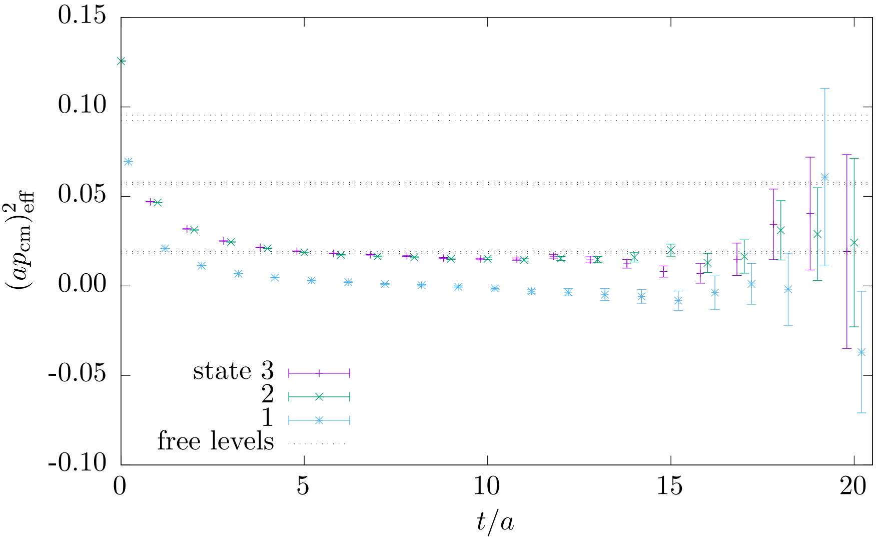

First two states couple mostly to $^1S_0$ operators; third state to $^3P_1$.

# H101: SU(3) singlet, frame $\vec{P} = \frac{2\pi}{L}(1,1,0)$, $A_1$ irrep




First two states couple mostly to $^1S_0$ operators; third state to $^3P_1$.

# H101: SU(3) singlet, frame $\vec{P} = \frac{2\pi}{L}(1, 1, 0)$, $A_1$ irrep



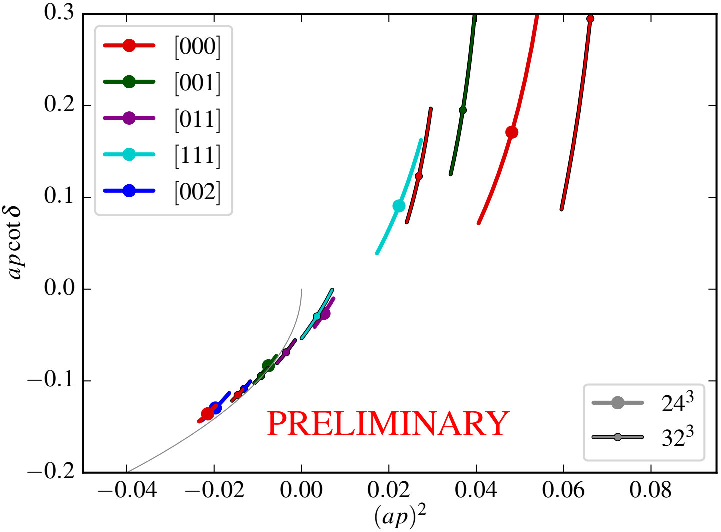

First two states couple mostly to $^1S_0$ operators; third state to $^3P_1$.

# Strategy: phase shift

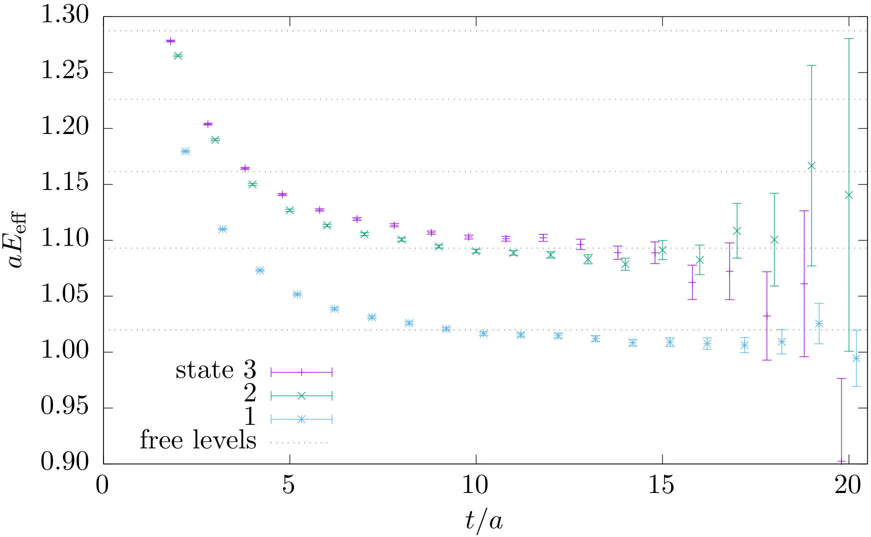
- For two-octet-baryon SU(3) singlet sector, spin-zero and spin-one scattering channels do not couple.
- Finite-volume quantization is diagonal in spin.

→ Expect that states can be separated by spin, with phase shifts analyzed independently.

# SU(3) singlet $^1S_0$ phase shift

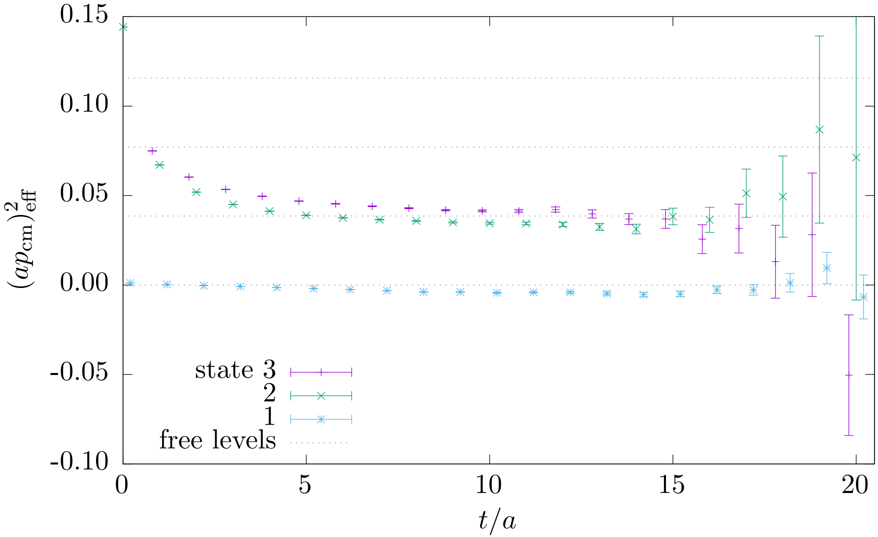


# H101, $\overline{10}$, rest frame, $T_1^+$ irrep




Deuteron channel. Using two ${}^3S_1$ operators and one ${}^3D_1$ operator.

# H101, $\overline{10}$, rest frame, $T_1^+$ irrep




Deuteron channel. Using two $^3S_1$ operators and one $^3D_1$ operator.

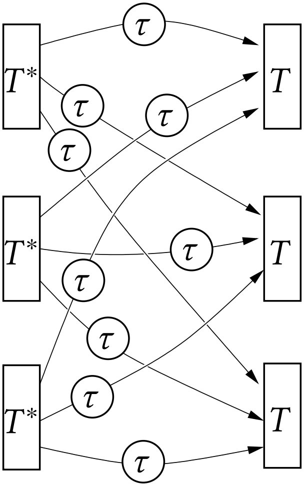
# Challenge: large volume

Biggest challenge for distillation: large volume, $N_{\text{LapH}} \propto L^3$.

- Current approach: cost $\propto N_{\text{LapH}}^4$.
  Might reach $N_{\text{LapH}} \approx 100$, or $L \approx 3.5$ fm with manageable contraction cost (several million core-hours). For $m_\pi L = 4$, this is $m_\pi \approx 225$ MeV.
- Stochastic distillation could have better cost scaling, if precise results can be obtained when keeping $N_{\text{dil}}$ independent of volume.
  Baryon-specific costs:
  1. Computing baryon sources and baryon sinks rather than mode triplets.
     Cost $\propto N_{\text{dil}}^3 L^3$.
  2. Contracting baryon sources and sinks. Cost in two-baryon sector $\propto N_{\text{dil}}^4$.

  The worst cost scaling could be the parts $\propto N_{\text{LapH}} L^3$ that also occur for meson correlators.

# Challenge: three baryons




Same ingredients: mode triplets and perambulators.

Some contractions could have worse cost scaling, like $N_{\text{LapH}}^6$.

Stochastic approach would need at least one noise source per quark line.

## Summary and outlook

- Hexaquark interpolating operators don't seem very important for spectroscopy in two-baryon sector.
- Distillation allows us to compute a Hermitian matrix of correlators using two-baryon interpolators.
- Multiple energy levels can be isolated for each combination of flavour, moving frame, and little-group irrep.

Looking forward, on both $N_f = 2$ and $N_f = 2 + 1$ ensembles, plan to study the following:

- Simplest uncoupled channels: ${}^1S_0$, singlet (at SU(3) point) and $NN$ $I = 1$.
- Deuteron channel, ${}^3S_1$–${}^3D_1$ coupled.
- $H$ dibaryon with broken SU(3).