Evgeny Epelbaum, Ruhr-University Bochum

Multi-Hadron Systems from Lattice QCD,
INT Seattle, February 5-9, 2018

Nuclear forces at physical and unphysical pion masses

- **Part I: Chiral EFT for few-N systems**
  - physical M_π
  - unphysical M_π
- **Part II: Low-energy theorems for NN scattering**

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

From QCD to nuclei

QCD → **lattice**

symmetries (especially the chiral symmetry);
lost of information (LECs)

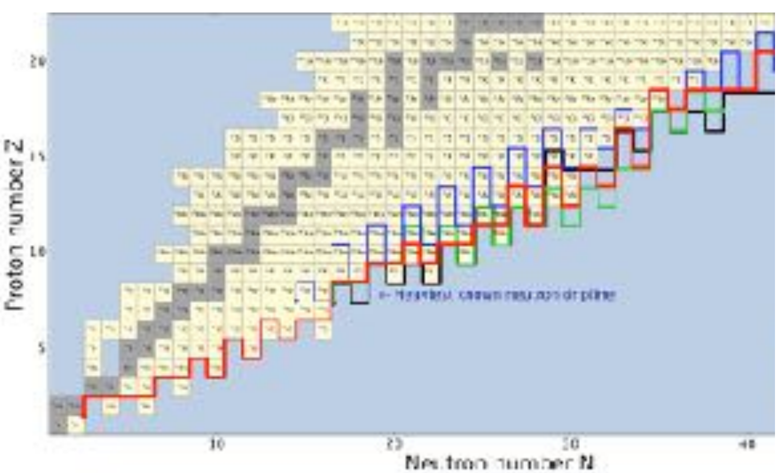
effective chiral Lagrangian $\mathcal{L}_{\text{eff}}(\pi, N)$

integrate out $|\vec{p}| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}| \sim M_\pi$):
Chiral Perturbation Theory

nuclear forces and currents

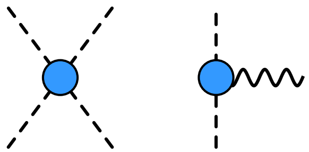
ab initio many-body methods:
lattice, FY, NCSM,…

nuclear structure and dynamics

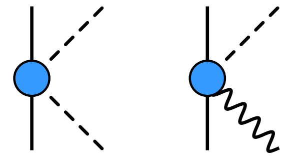


Modern approach to nuclear physics

GB dynamics
Weinberg, Gasser, Leutwyler, …



πN dynamics
Bernard-Kaiser-Meißner et al.



Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}$$

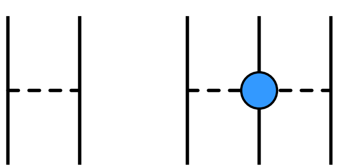
Effective Lagrangian:

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

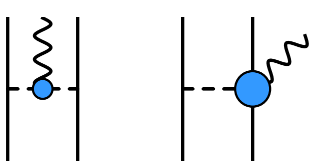
$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots ,$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

Nuclear forces
Weinberg, van Kolck, Kaiser, EGM, …

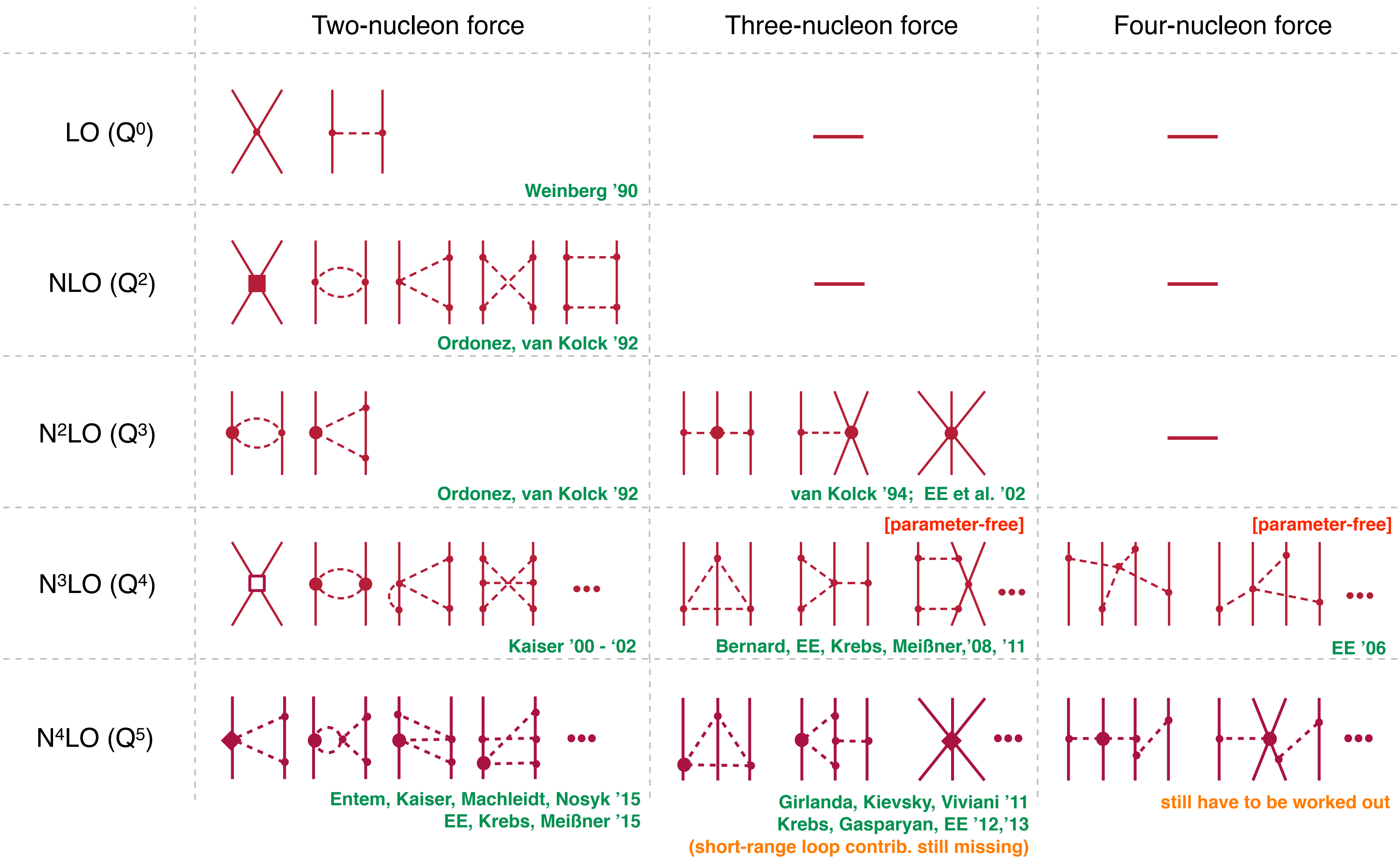


Nuclear currents
Park et al, Bochum-Bonn, JLab-Pisa



Auxilliary quantities (not observable),
must be consistent (unitary ambiguity)

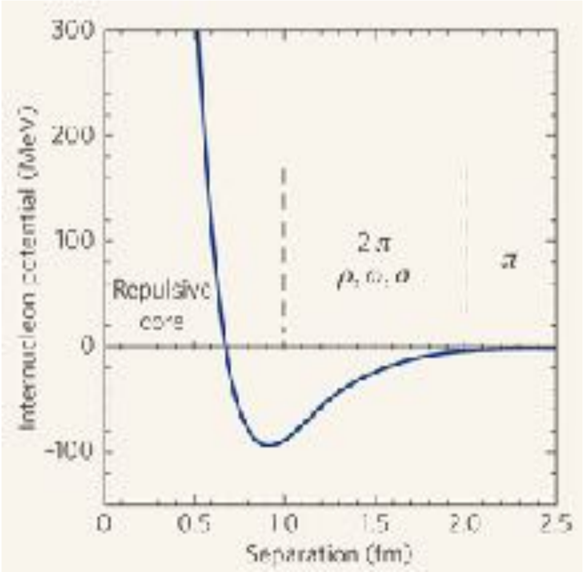
Chiral expansion of the nuclear forces



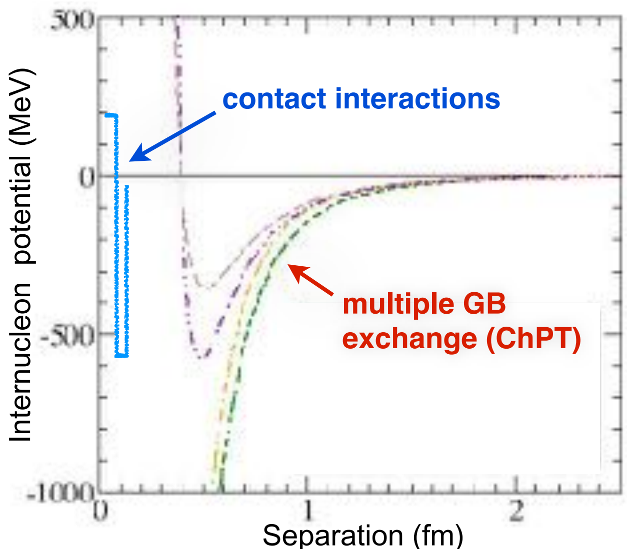
A similar program is being pursued for currents Kölling, EE, Krebs, Meißner '09,'11; Kölling, EE, Phillips '12; Krebs, EE, Meißner '17,...

The long and short of nuclear forces

conventional picture

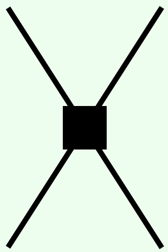


chiral EFT



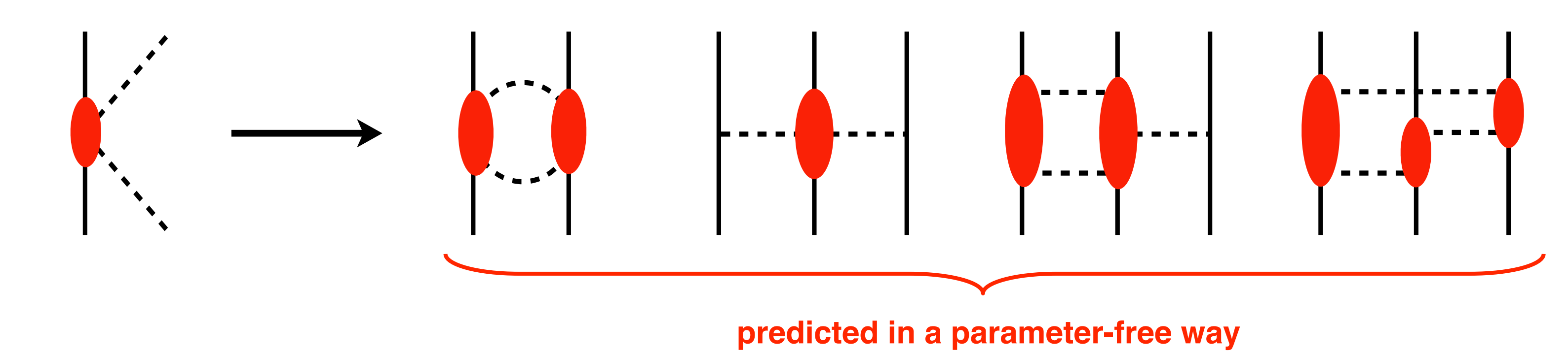
The long and short of nuclear forces

- Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:



LO [Q^0]:	2 operators (S-waves)
NLO [Q^2]:	+ 7 operators (S-, P-waves and ε_1)
N²LO [Q^3]:	no new terms
N³LO [Q^4]:	+ 12 operators (S-, P-, D-waves and ε_1, ε_2)
N⁴LO [Q^5]:	no new terms

- The long-range part of nuclear forces and currents is completely determined by the chiral symmetry of QCD + experimental information on πN scattering



Determination of πN LECs

Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

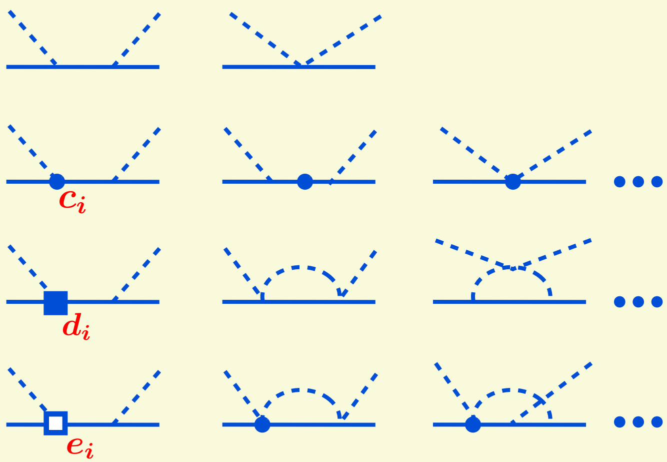
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

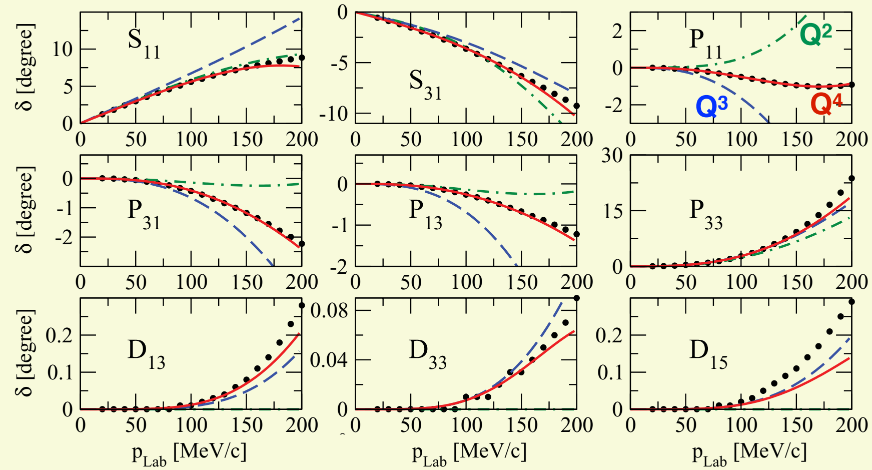
Order Q:

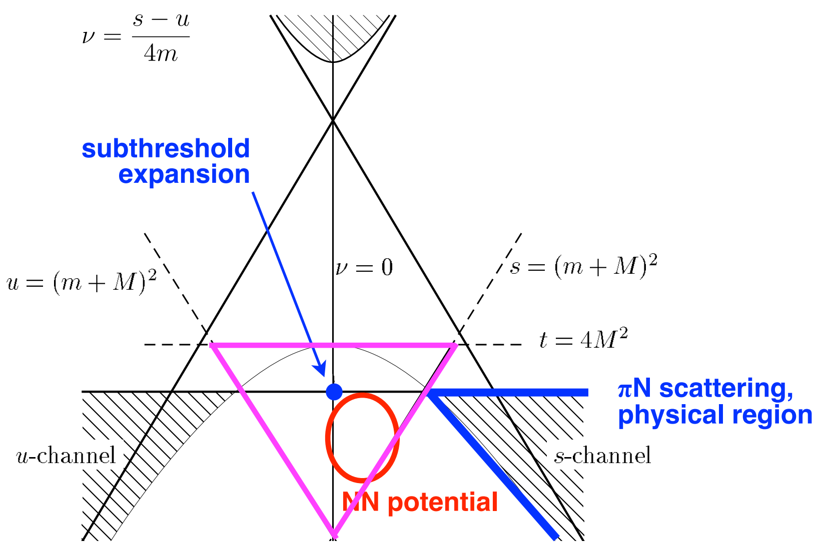
Order Q^2:

Order Q^3:

Order Q^4:







Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

$$\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm,\, B^\pm\}$$

- Closer to the kinematics relevant for nuclear forces…

NN data analysis

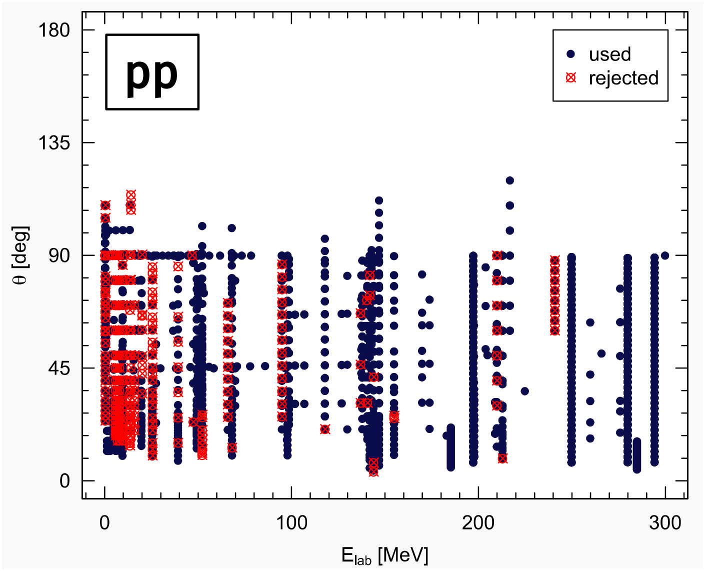
P. Reinert, H. Krebs, EE, arXiv:1711.08821[nucl-th]

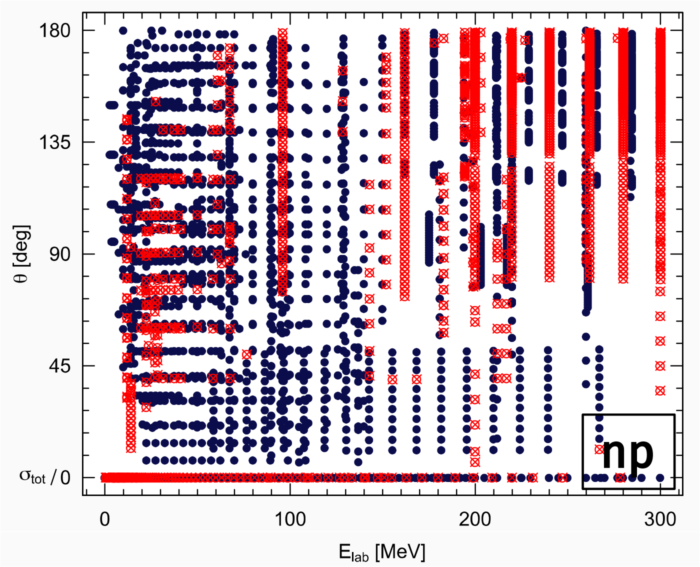
- To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm.

- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.

- However, certain data are mutually incompatible within errors and have to be rejected. 2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:

2158 proton-proton + 2697 neutron-proton data below $E_{lab} = 300$ MeV





NN data analysis

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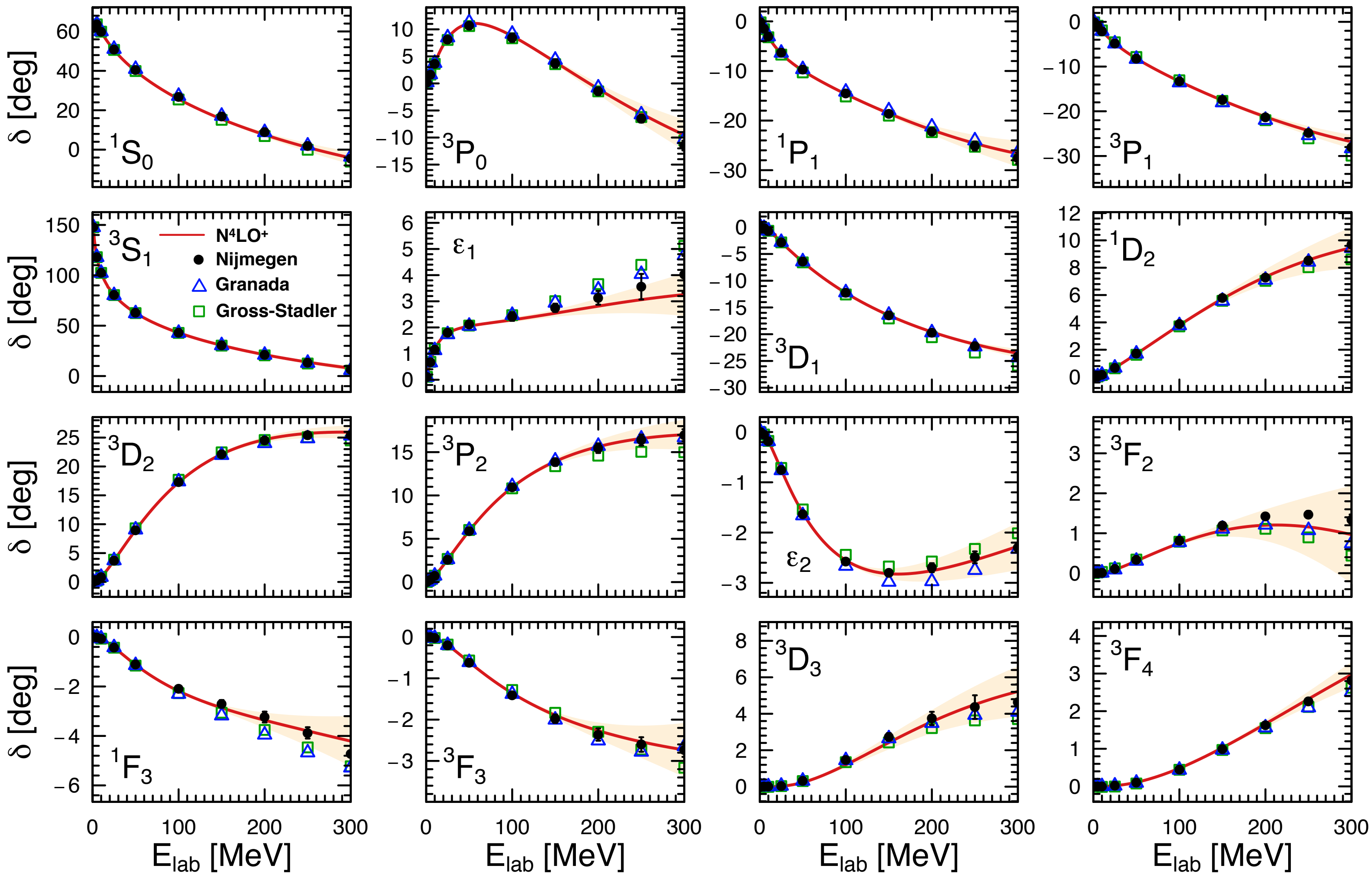


— N^4LO^+ yields currently the best description of np+pp data below E_{lab} = 300 MeV

— About 40% less parameters (LECs) than in high-precision potentials

— Clear evidence of the (parameter-free) chiral 2π exchange

Error analysis

Reinert, Krebs, EE, arXiv:1711.08821[nucl-th]

Careful error analysis: truncation error [EE, Krebs, Meißner EPJ A51 (15), PRL 115 (15)], statistical uncertainty (NN LECs), uncertainty due to πN LECs, choice of the energy in the fits.

Example: deuteron asymptotic normalizations (relevant for nuclear astrophysics)

Our determination:

statistical error, truncation error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \text{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \text{fm}^{-1/2}$, $\eta = 0.0256(4)$

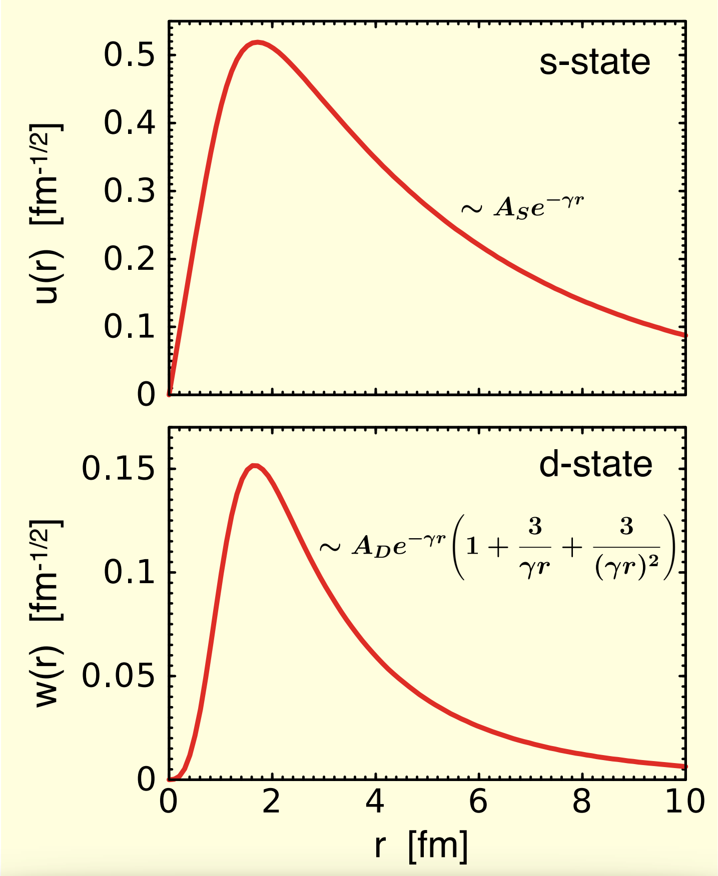
Borbely et al. '85 — Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses“] Stoks et al. '95

$$A_S = 0.8845(8)\ \text{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

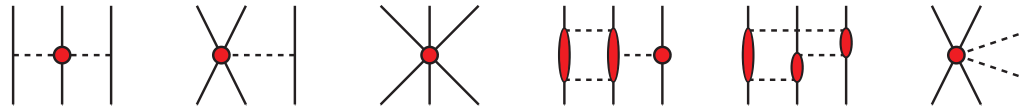
Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4)\ \text{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



Three-nucleon forces

N²LO: tree-level graphs, 2 new LECs
van Kolck '94; EE et al '02

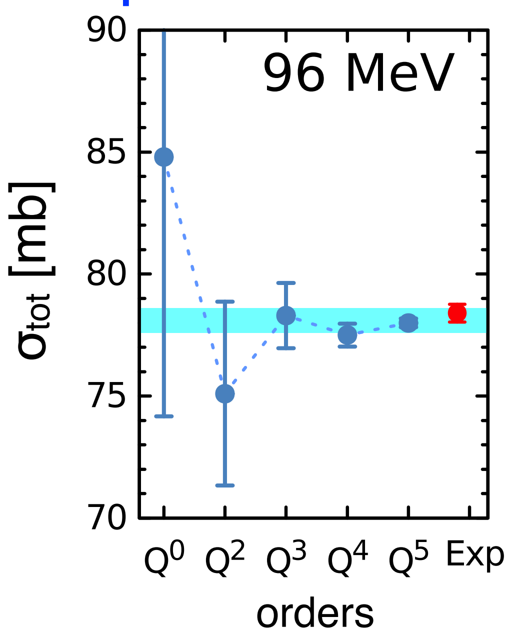


N³LO: leading 1 loop, parameter-free
Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08, '11

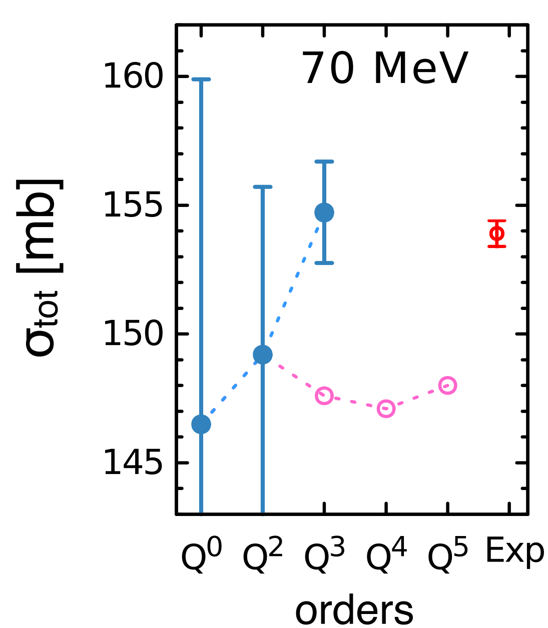
N⁴LO: full 1 loop, almost completely worked out, several new LECs
Girlanda, Kievski, Viviani '11; Krebs, Gasparyan, EE '12,'13; EE, Gasparyan, Krebs, Schat '14

Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration **(LENPIC)**, work in progress

np total cross section



nd total cross section



But the real challenge is to understand the spin structure of the 3NF…
Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (12) 016301

General structure of a 3NF

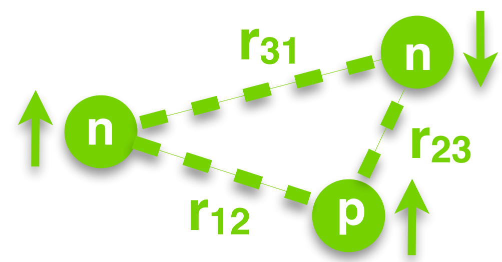
Why is it so difficult to model the 3NF as compared to NN potentials?

- More scarce Nd data base compared to np and pp data bases
- Solving the Faddeev equation for 3N more involved than solving the LS equation for NN
- General structure of the 3NF is much more involved

Most general structure of a local, isospin-symmetric 3NF

Krebs, Gasparyan, EE ’13; Phillips, Schat ’13; EE, Gasparyan, Krebs, Schat ’14

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$



Assuming hermiticity, time reversal & parity invariance, **20 structure functions** are needed:

$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i\, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i\, F_i(q_1, q_2, q_3) + \text{permutations}$$

Intermediate summary

Nuclear chiral EFT at physical M_π

— The 2N sector is in a very good shape

— Current frontiers: 3N forces, external probes (e.m., weak), ab-initio calculations of (heavier) nuclei and reactions, systematic underbinding and too small radii for heavier nuclei…

— Still under debate: power counting for short-range operators…

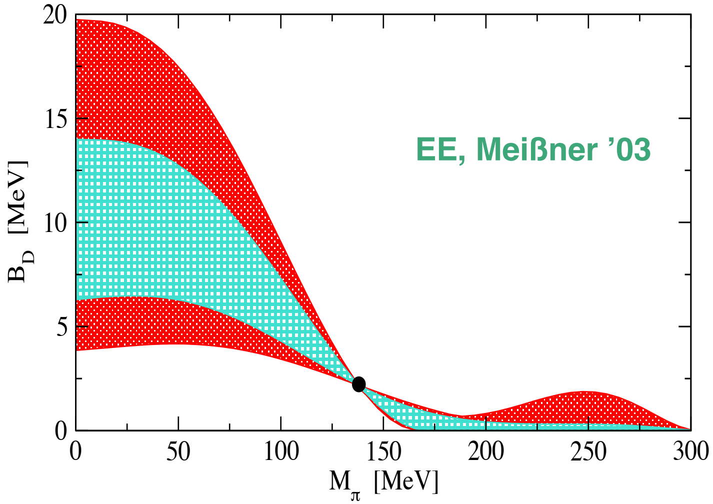
Input from lattice QCD hardly needed for:

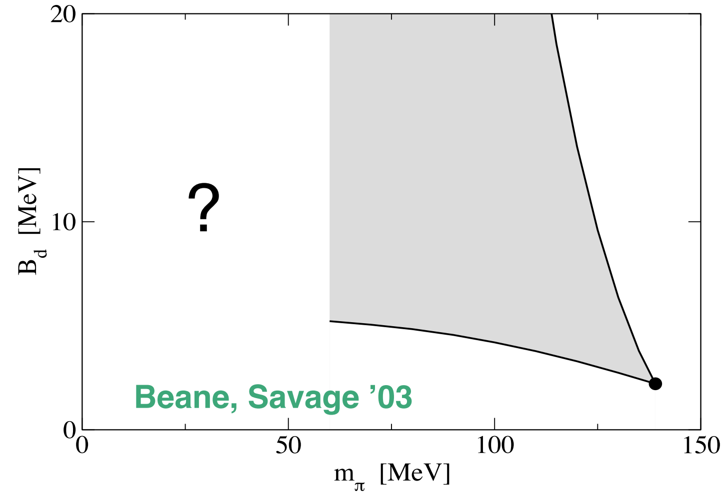
— Neutron-neutron scattering (especially the scattering length!)

— Hyperon-nucleon and hyperon-hyperon scattering

— On the other hand, no need for np, pp (and probably also for Nd) „data"…

Nuclear EFT at unphysical pion mass



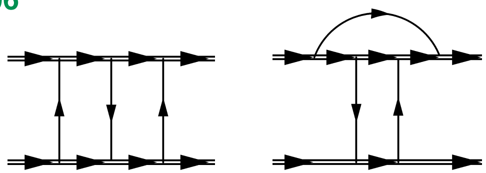


Chiral EFT @ unphysical M_π

Complications:

- Unknown M_π-dependent NN LECs: 2@NLO + 7@N^{3}LO + … [W. counting]
- Limited convergence range of the quark-mass expansion (e.g. $g_A(M_\pi)$…)
- Chiral expansion of short-range terms **Mondejar, Soto ’06**

— momenta $\sim \sqrt{M_\pi m_N}$ inside loops generate contributions to contact terms $\sim (M_\pi m_N)^{n/2}$, which likely need to be resummed…



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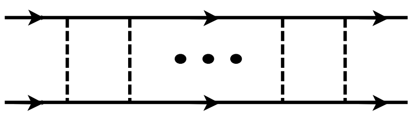
- Finite cutoff & implicit renormalization (resummation of pion-exchange)

$$T(\vec{p}\,', \vec{p}) = V_{2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



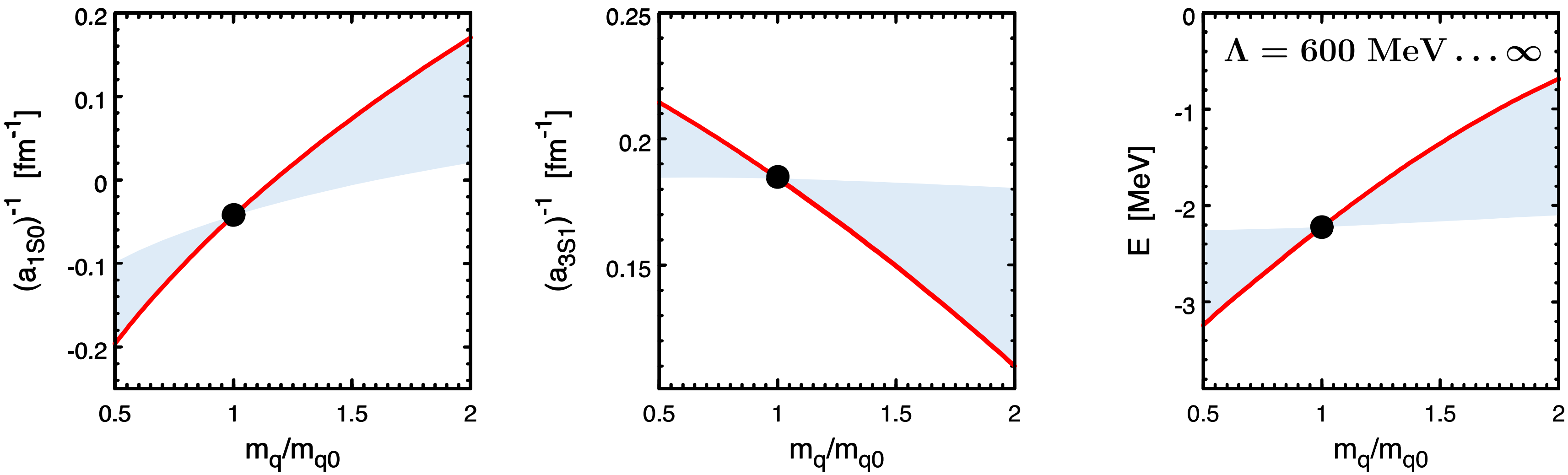
→ Finite-cutoff EFT **Lepage ’96**

Introduce a finite UV cutoff $\Lambda \sim \Lambda_b \sim 500$ MeV and tune **bare** LECs $C_i(\Lambda)$ to experimental data **(implicit renormalization)**.

Chiral EFT @ unphysical M_π

Alternatives:

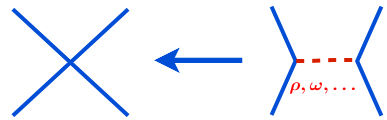
- Perturbative pions (KSW) Kaplan, Savage, Wise '96
  - — does not converge in certain S=1 channels for p ~ M_π (@ physical M_π) Cohen, Hansen '99, Fleming, Mehen, Stewart '99
  - — very slow convergence in the $^1S_0$ channel (if at all…) EE, Gasparyan, Gegelia, Krebs '15
- Dibaryon formalism (with perturbative pions) Soto, Tarrus, '08 -'12
  - — equivalent to KSW…
- Semi-relativistic approach with nonperturbative $V_{1\pi}$ EE, Gegelia et al., '12 -



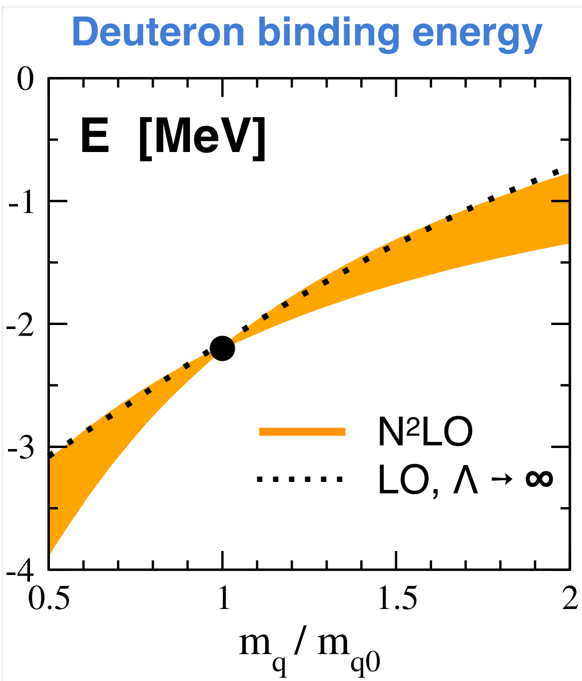
M_π dependence from resonance saturation

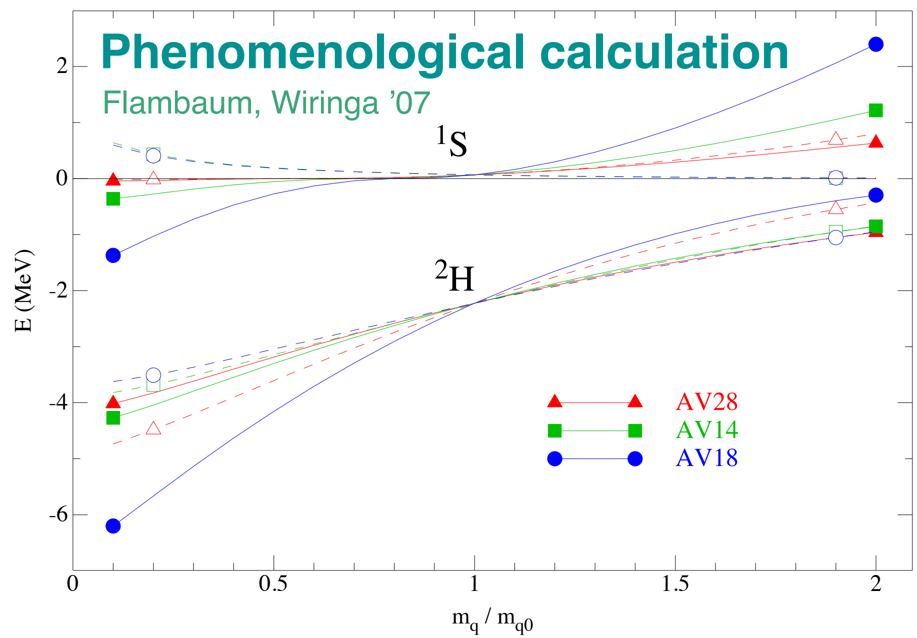
Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez ’13

- Use ChPT combined with lattice-QCD data to constrain the M_π-dependence of the nucleon mass and long-range part of the force
- M_π-dependence of contacts from:
  resonance saturation [EE et al. ’02]
  + unitarized ChPT
  + lattice QCD



LEC	N^{2}LO fits	$\sigma + \rho + \omega$
$\tilde{C}^{\rm res}_{1S0}$	$-(0.12\ldots0.16)$	-0.12
$C^{\rm res}_{1S0}$	$(1.16\ldots1.37)$	1.28
$\tilde{C}^{\rm res}_{3S1}$	$-(0.13\ldots0.16)$	-0.10
$C^{\rm res}_{3S1}$	$(0.42\ldots0.72)$	0.66
$C^{\rm res}_{\epsilon1}$	$-(0.36\ldots0.47)$	-0.41





Intermediate summary

Nuclear chiral EFT at unphysical M_π

- No conclusive predictions for M_π dependence of few-N observables…
- M_π dependence of the long-range interactions (π-exchanges) can be determined from lattice-QCD in combination with ChPT
- Strict chiral expansion of the short-range interactions is difficult to control; phenomenological parametrizations based on lattice-QCD data seem more feasible…

Matching with lattice QCD in finite volume

- Probably, most efficient at the level of bare LECs $C_i(\Lambda)$ which can be tuned to the spectrum in a finite volume…
- For lower M_π, pions need to be kept explicitly; pi-less EFT not enough!

Low-Energy Theorems
as a tool for extrapolation in energy

- **Heavy pions: pionless EFT**
  — extrapolation in the number of nucleons **Barnea, Kirscher, van Kolck, …**

- **Light pions: chiral EFT**
  — extrapolate in M_π (and in the number of nucleons)
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …

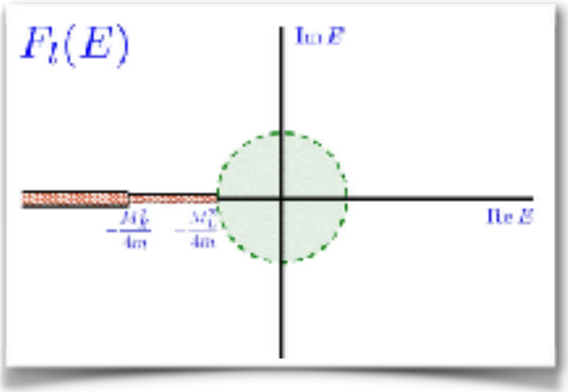
- **Light pions: Low-Energy Theorems (LETs)**
  — extrapolate the NN amplitude in energy at fixed M_π **Baru, EE, Filin, Gegelia**

ERE, MERE and LETs

Two-range potential $V(r) = V_L(r) + V_S(r), \; M_L^{-1} \gg M_H^{-1}$

$$S_l = e^{2i\delta_l(k)} = 1 - i\left(\frac{km}{8\pi^2}\right) T_l(k)\,, \quad T_l(k) = -\frac{16\pi^2}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}}$$

effective range function, $F_l \equiv k^{2l+1}\cot\delta_l$



$F_l(k^2)$ is a real meromorphic function of k^2 for $|k| < M_L/2$

$\longrightarrow$ ERE: $F_l = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$ Landau, Smorodinsky '44; Blatt, Jackson'49; Bethe'49

Generalization to the modified ERE by „subtracting“ effects due to the long-range force

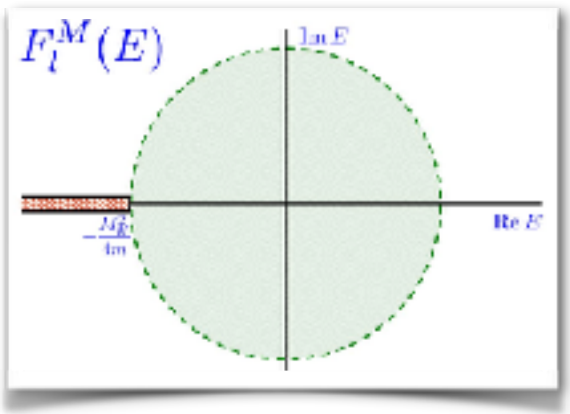
van Haeringen, Kok PRA 26 (1982) 1218

$$F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2}\cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

$$f_l^L(k) = \lim_{r\to 0}\left(\frac{l!}{(2l)!}(-2ikr)^l f_l^L(k,r)\right)$$

Jost function for $V_L(r)$ — Jost solution for $V_L(r)$

$$M_l^L(k) = Re\left[\frac{(-ik/2)^l}{l!}\lim_{r\to 0}\left(\frac{d^{2l+1}}{dr^{2l+1}}\frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$$



Per construction, F_l^M reduces to F_l for $V_L = 0$ and is a real meromorphic function for $|k| < M_H/2$

MERE and Low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)
Cohen, Hansen '99; Steele, Furnstahl '00

$$F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

meromorphic for $k^2 < (M_H/2)^2$

can be computed if the long-range force is known

– approximate $F_l^M(k^2)$ by first 1,2,3,… terms in the Taylor expansion in k^2/M_H^2, calculate all „light" quantities, reconstruct $\delta_l^L(k)$ and predict all coefficients in the ERE

Even in the $^3S_1$ channel, the accuracy is insufficient when using effective range as input:

$$r = 1.75 \text{ fm [input]} \longrightarrow a = 7.16 \text{ fm}, \quad B_d = 1.1 \text{ MeV [LET predictions]}$$

→ go to NLO LETs by including the (modified) effective range correction modeled via

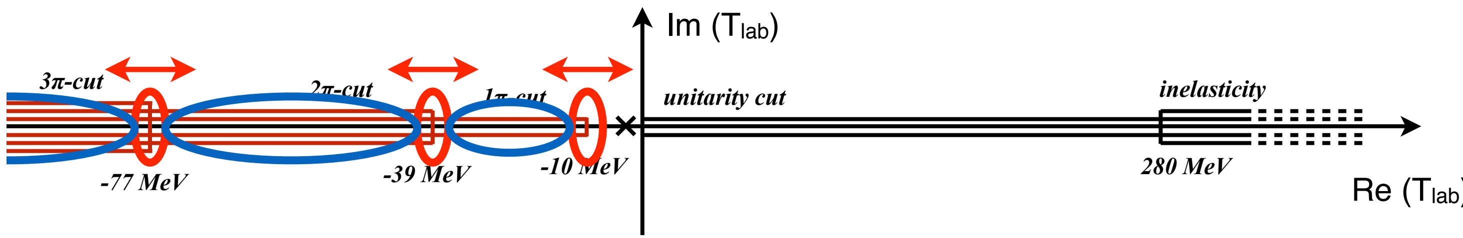
$$V^{\rm NLO} = V^{\rm LO} + \beta \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M^2}, \quad M = 700 \text{ MeV}$$

	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
LO LET	5.42*	1.60	−0.05	0.82	−5.0
NLO LET	5.42*	1.75*	0.06	0.70	−4.0
Nijmegen PWA	5.42	1.75	0.04	0.67	−4.0

*Fit parameter.

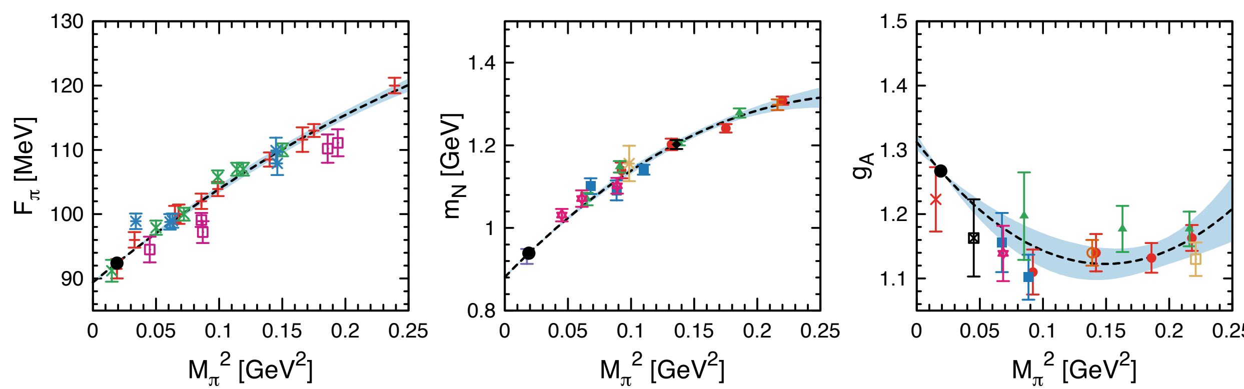
For the $^1S_0$ channel see: EE, Gasparyan, Gegelia, Krebs, EPJA 51 (2015) 71

LETs @ unphysical M_π



- When going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

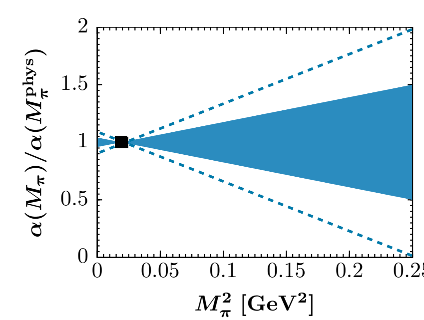
- We also include changes in discontinuity across the left-hand cuts (M_π-dependence of g_A, F_π) and M_π-dependence of the nucleon mass



- For NLO LETs, we need to know M_π-dependence of the subleading short-range term (a higher-order effect in EFT)

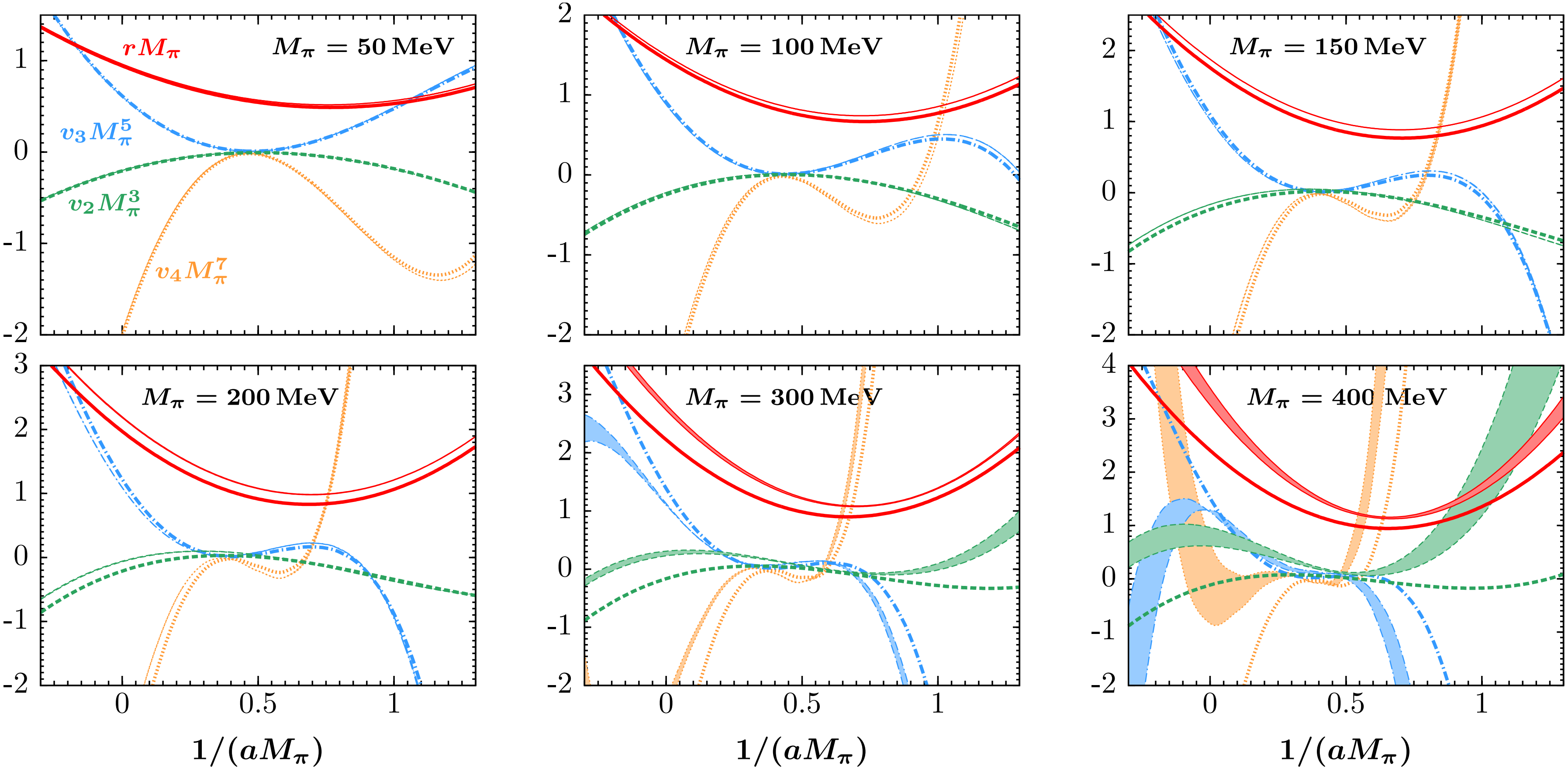
$$V_{\rm NLO} = \beta \, \frac{\vec{\sigma}_1 \cdot \vec{q}\;\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M^2}$$

We allow β to vary: $\delta\beta(M_\pi{=}500\text{ MeV}) = \pm 50\%$ ($\pm 100\%$)



LETs in the $^3S_1$ channel

LETs at nonphysical pion masses (at NLO, δβ = 0.5)

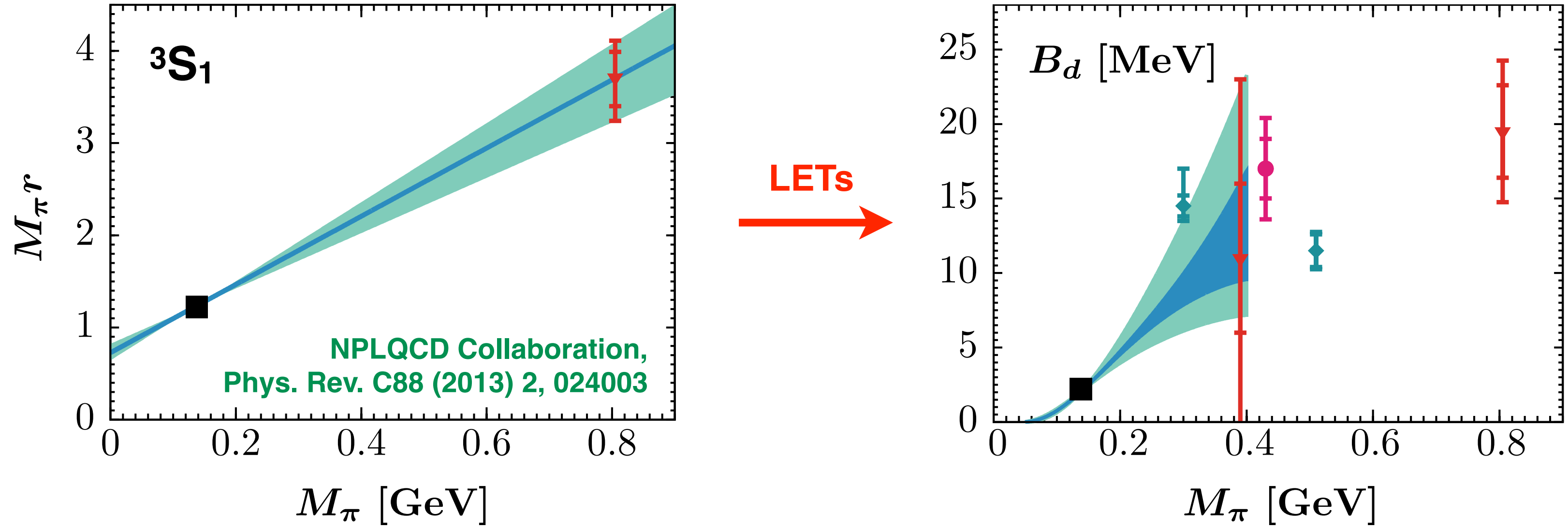


- good convergence and accuracy of the LETs for low values of M_π (below 200 MeV)
- sizable uncertainty at pion masses above 400 MeV (even at NLO)

Low-energy theorems for NN scattering

- Is the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ consistent with the trend in BEs?

Baru, EE, Filin, Gegelia '15

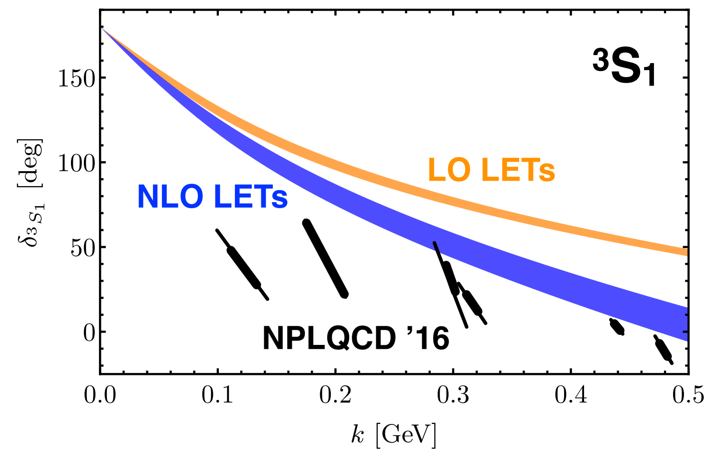


- Are the NPLQCD results for BE & phase shifts @ M_π=450 MeV consistent?

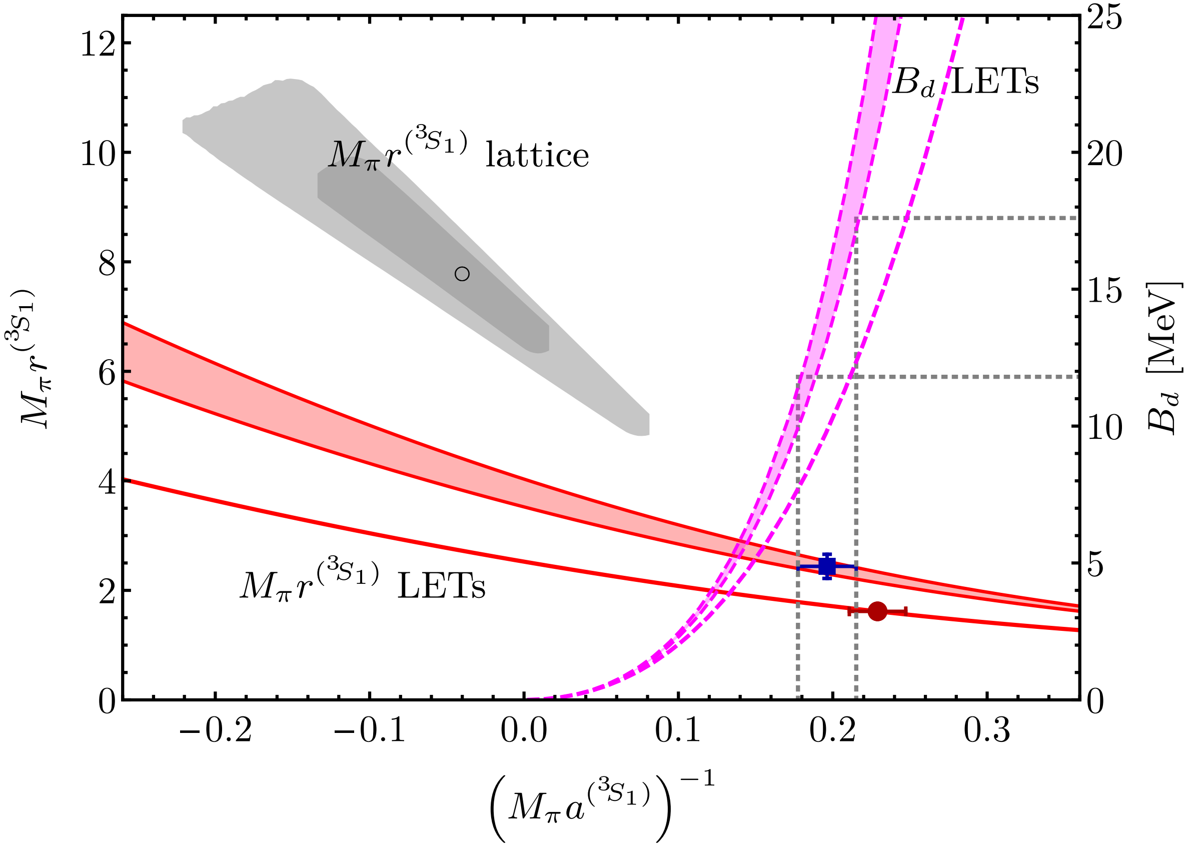
Baru, EE, Filin, to appear

Use $B_d = 14.4\left(^{+3.2}_{-2.6}\right)$ MeV **[Beane et al.'16]** as input to predict phase shifts via LETs

NPLQCD results for phase shifts at the two lowest energies are incompatible with their results for B_d: Underestimated systematics??



NPLQCD meets LETs: The $^3S_1$ channel



- Consequently, different results for the scattering length and effective range:

NPLQCD: $(M_\pi a^{(^3S_1)})^{-1} = -0.04(^{+0.07}_{-0.10})(^{+0.08}_{-0.17}),\quad M_\pi r^{(^3S_1)} = 7.8(^{+2.2}_{-1.5})(^{+3.5}_{-1.7})$

statistics systematics

NLO LETs: $(M_\pi a^{(^3S_1)})^{-1} = 0.196(^{+0.014}_{-0.013})(^{+0.018}_{-0.008}),\quad M_\pi r^{(^3S_1)} = 2.44(^{+0.08}_{-0.08})(^{+0.21}_{-0.47})$

error in B_d uncertainty of the LETs ($\delta\beta$=1)

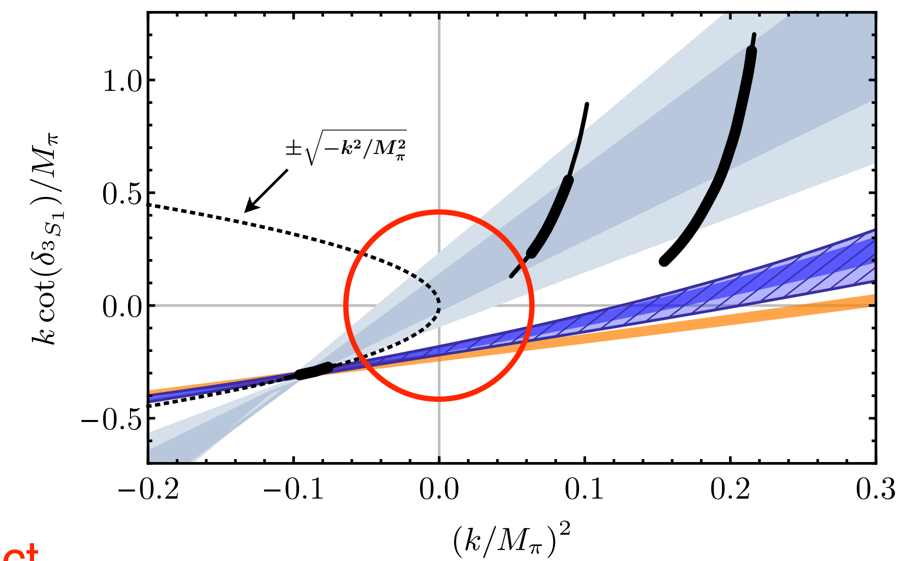
NPLQCD meets LETs: The ${}^3S_1$ channel

(If true), the very large effective range,

$$r^{({}^3S_1)} \sim 8 M_\pi^{-1}$$

would suggest:

— either the interaction range (much) longer than that of $V_{1\pi}$
— or the appearance of a pole in $k \cot \delta$ near threshold



In both cases, there is no reason to expect the approximation $k \cot \delta \simeq -\frac{1}{a} + \frac{1}{2} r k^2$ to be valid for $|k| \gtrsim 2/r \sim M_\pi/4$.

Moreover, the second, deeper bound state is (normally) to be viewed as an artifact of the effective range approximation:

$$-\frac{1}{a} + \frac{1}{2} r k^2 = ik \quad \longrightarrow \quad k_{1,2} = \frac{i}{r}\left(1 \pm \sqrt{1 - \frac{2r}{a}}\right) \quad \xrightarrow[|r/a| \ll 1]{} \quad \begin{cases} k_1 \simeq \frac{i}{a}\left(1 + \frac{r}{2a}\right) \\ k_2 \simeq i\left(\frac{2}{r} - \frac{1}{a}\right) \end{cases}$$

— physical pion mass: $k_1 \simeq 45i$ MeV (deuteron), $k_2 \simeq 200i$ MeV (artifact)

— NPLQCD solution: $k_1 \simeq -15i$ MeV (virtual state), $k_2 \simeq 135i$ MeV (deuteron)

Summary part II

- LETs allow to reconstruct the NN scattering amplitude at fixed M_π using a single observable (e.g. binding energy) as input

  → extrapolations of lattice-QCD results in energy, self-consistency checks

- The linear in M_π dependence of $M_\pi\, r^{(3S1)}$ conjectured by the NPLQCD collaboration based on their $M_\pi \sim 800$ MeV results is consistent with the common trend for B_d

- NPLQCD results at $M_\pi \sim 450$ MeV for the $^1S_0$ / $^3S_1$ phase shifts are incompatible with their B_{nn} / B_d energies (within errors). Underestimated systematics?

LETs: a useful addition to the lattice QCD toolbox!