Baryon spectroscopy in the DSE / BSE approach

Gernot Eichmann
IST Lisboa, Portugal

INT Workshop "Multi-Hadron Systems from Lattice QCD"
Seattle, WA, USA

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Outline

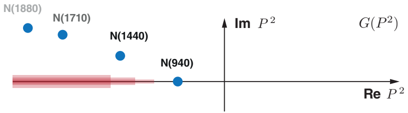
- **Lattice** vs. **DSE / BSE**
- **Bethe-Salpeter equations,**
  applications to mesons
- **Baryons:**
  Faddeev equation, form factors, light and strange baryons
- **Future challenges:**
  Resonances, multiquarks, scattering amplitudes

Lattice vs. DSE / BSE

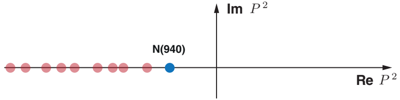
Extract baryon poles from (gauge-invariant) two-point correlators:

$$G(x-y) = \langle 0 \,|\, T \underbrace{[\Gamma_{\alpha\beta\gamma}\, \psi_\alpha\, \psi_\beta\, \psi_\gamma]\,(x)}_{B(x)} \underbrace{[\bar\Gamma_{\rho\sigma\tau}\, \bar\psi_\rho\, \bar\psi_\sigma\, \bar\psi_\tau]\,(y)}_{\bar{B}(y)} \,|\, 0 \rangle \;=\; \int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S}\, B(x)\, \overline{B}(y)$$

$$G(\tau) \sim e^{-m\tau} \quad \Leftrightarrow \quad G(P^2) \sim \frac{1}{P^2+m^2}$$



- **Infinite volume:**
  Bound states, resonances, branch cuts



- **Finite volume:**
  bound states & scattering states

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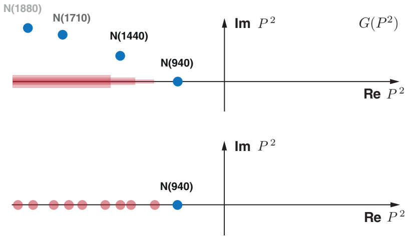
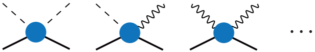


Spectral representation →
same singularity structure in



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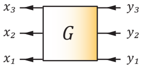
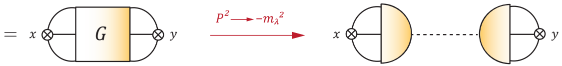
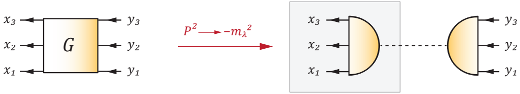
$$= \lim_{\substack{x_i \to x \\ y_i \to y}} \Gamma_{\alpha\beta\gamma}\,\bar\Gamma_{\rho\sigma\tau}\; \langle 0 \,|\, T\,\psi_\alpha(x_1)\,\psi_\beta(x_2)\,\psi_\gamma(x_3)\,\bar\psi_\rho(y_1)\,\bar\psi_\sigma(y_2)\,\bar\psi_\tau(y_3) \,|\, 0 \rangle$$





Alternative: extract **gauge-invariant** baryon poles from **gauge-fixed** quark 6-point function:



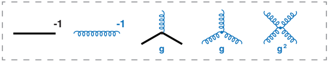
Bethe-Salpeter wave function:
residue at pole, contains all information about baryon

$$\langle 0 \,|\, T\,\psi_\alpha(x_1)\,\psi_\beta(x_2)\,\psi_\gamma(x_3) \,|\, \lambda \rangle$$

QCD's n-point functions

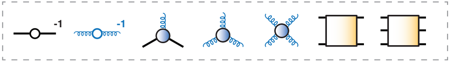
QCD's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + ig A\!\!\!/ + m \right) \psi + \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

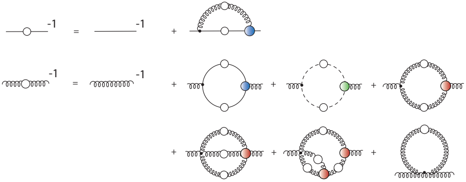
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$



DSEs = quantum equations of motion:
derived from path integral, relate n-point functions

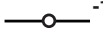


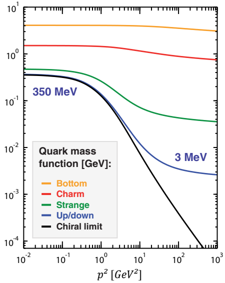
- infinitely many coupled equations
- reproduce perturbation theory, but **nonperturbative**
- systematic truncations: neglect higher n-point functions to obtain **closed system**

Reviews:
Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
Alkofer, von Smekal, Phys. Rept. 353 (2001)
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
Prog. Part. Nucl. Phys. 91 (2016), 1606.09602 [hep-ph]

QCD's n-point functions

- **Quark propagator**

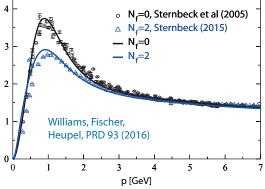
$A(p^2)\,(i\not{p} + M(p^2))$  $^{-1}$



Dynamical chiral symmetry breaking generates 'constituent-quark masses'

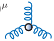
- **Gluon propagator**

$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$ 



- **Three-gluon vertex**

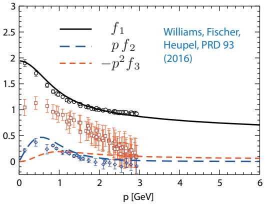
$$F_1\,[\,\delta^{\mu\nu}(p_1-p_2)^\rho + \delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu] + \dots$$

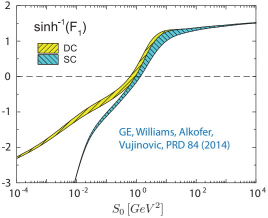


Agreement between lattice, DSE & FRG within reach

- **Quark-gluon vertex**

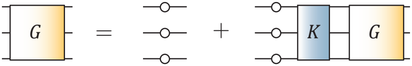
$f_1\,\gamma^\mu + f_2\,ip^\mu + f_3\,p^\mu\not{p} + \dots$ 





DSEs $\longrightarrow$ Hadrons?

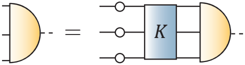
Bethe-Salpeter approach:
use scattering equation $G = G_0 + G_0\, K\, G$



- still exact - to begin with, kernel is black box
- but can be derived together with QCD's n-point functions.
  Important to preserve symmetries!

$P^2 \longrightarrow -m^2$

Homogeneous BSE for **BS wave function:**



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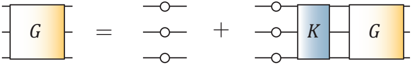

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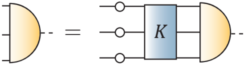
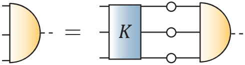
Homogeneous BSE for **BS wave function** . . . or **BS amplitude:**



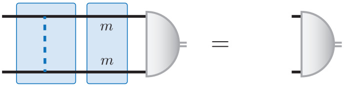


Bethe-Salpeter equations

Simplest: **Wick-Cutkosky model**

Wick 1954, Cutkosky 1954, Nakanishi 1969, . . .

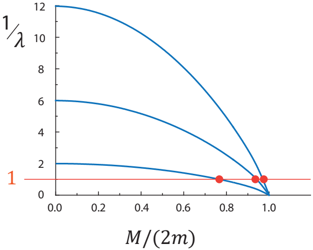
- scalar tree-level propagators, scalar exchange particle
- bound states for $M < 2m$



$$K(M)\ G(M)\ \phi_i(M) = \lambda_i(M)\ \phi_i(M)$$

But:

- no confinement: threshold $2m$
- not a consistent QFT: would need to solve DSEs for propagators, vertices etc.



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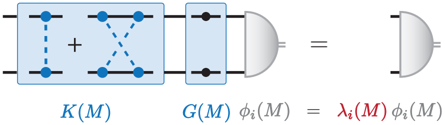

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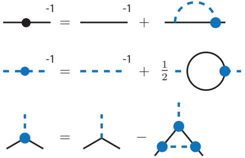

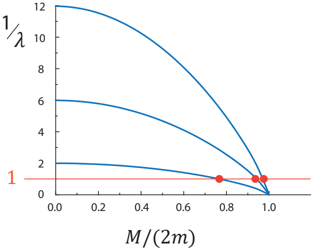

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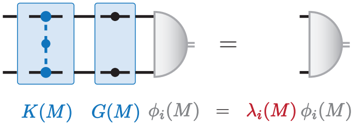

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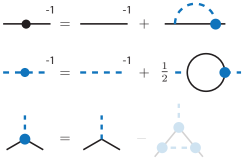
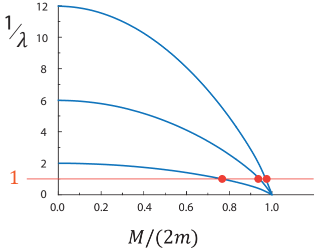
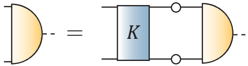




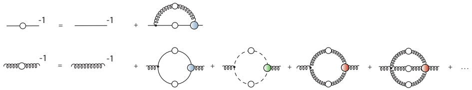
Ahlig, Alkofer, Ann. Phys. 275 (1999)

Mesons

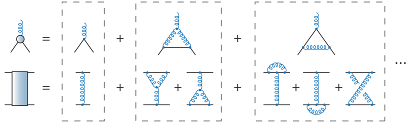
- Meson **Bethe-Salpeter equation** in QCD:



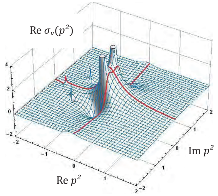
- Depends on QCD's n-point functions, satisfy **DSEs:**



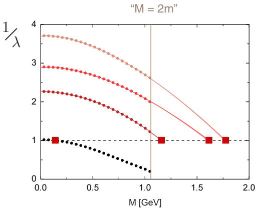
- Kernel derived in accordance with **chiral symmetry:**



Quark propagator has **complex singularities:** no physical threshold



Eigenvalues in **pion** channel:



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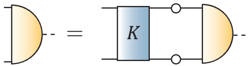

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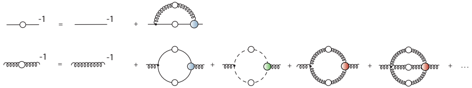

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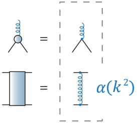

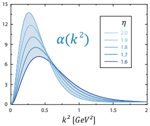

Rainbow-ladder:
effective gluon exchange

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(\frac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

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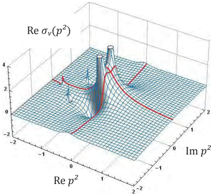

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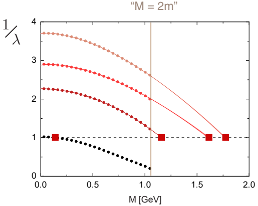

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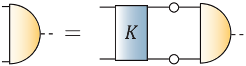

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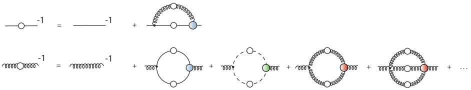

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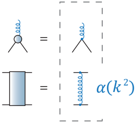

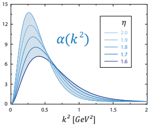

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Maris, Tandy, PRC 60 (1999),
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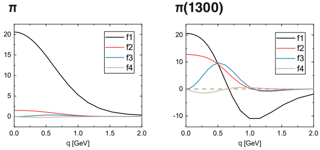
Eigenvectors = BS amplitudes



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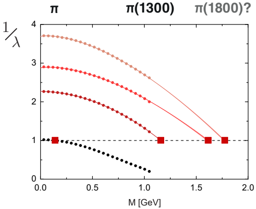
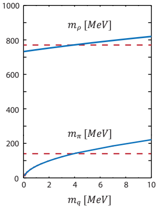
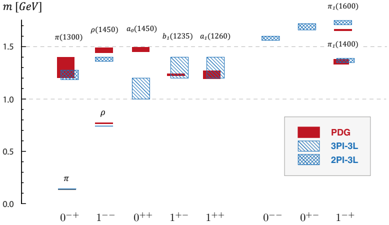


Mesons

- Pion is **Goldstone boson:** $m_\pi^{\,2} \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

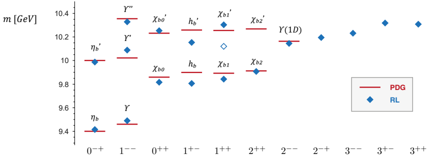


Williams, Fischer, Heupel, PRD 93 (2016)

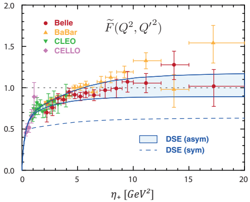
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Charmonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



- **Pion transition form factor**

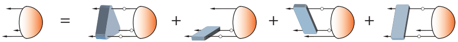


GE, Fischer, Weil, Williams, PLB 774 (2017)

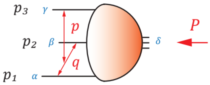
Baryons

Covariant Faddeev equation for baryons:

GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)



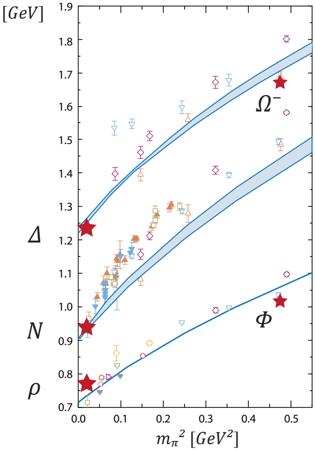
- 3-gluon diagram vanishes ⇒ **3-body effects small?**
- 2-body kernels same as for mesons, no further approximations:



$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\;\tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

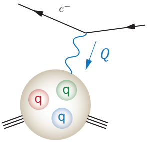
Lorentz-invariant dressing functions

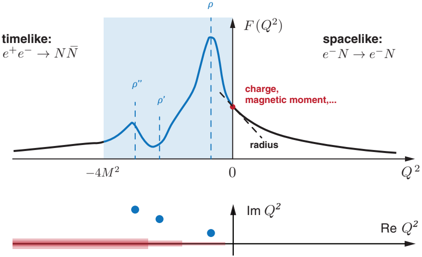
Dirac-Lorentz tensors carry OAM: s, p, d,...



Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

Form factors

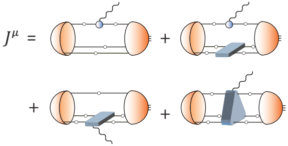




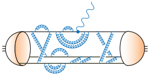
$$J^\mu = e\,\bar{u}(p_f)\left(F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i}{4m}\,[\gamma^\mu, \not{Q}]\right) u(p_i)$$

Consistent derivation of **current matrix elements & scattering amplitudes**

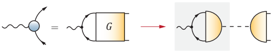
Kvinikhidze, Blankleider, PRC 60 (1999),
GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



- **rainbow-ladder** topologies (1st line):

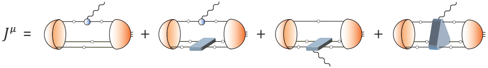


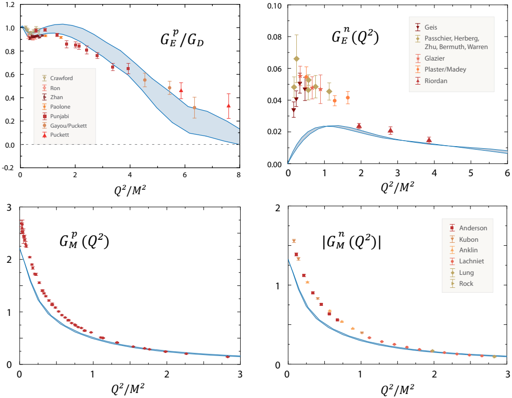
- **quark-photon vertex** preserves em. gauge invariance, dynamically generates **VM poles:**



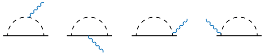
Form factors

Nucleon em. form factors
from three-quark equation
GE, PRD 84 (2011)





- **"Quark core without pion cloud"**



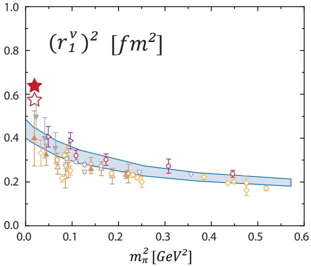
- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

Form factors

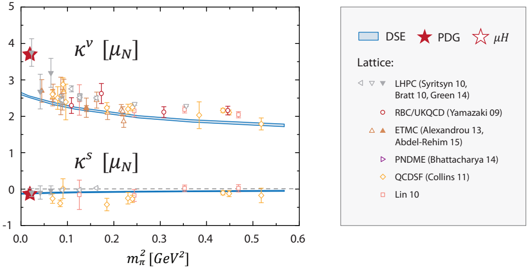
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

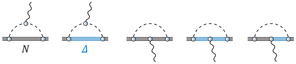


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.

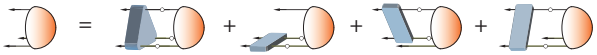


- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

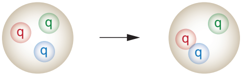
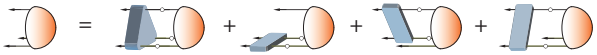
  Exp: $\kappa^s = -0.12$
  Calc: $\kappa^s = -0.12(1)$ **!!**

  GE, PRD 84 (2011)

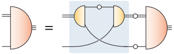
DSE / Faddeev landscape $N \to N^*\gamma$

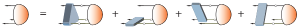
	Three-quark		
			
	RL	bRL	bRL + 3q
N, Δ masses	√	...	...
N, Δ em. FFs	√		
$N \to \Delta\gamma$	√		
Roper	√	...	
$N \to N^*\gamma$	...		
$N^*(1535), \ldots$	√	...	
$N \to N^*\gamma$	...		

DSE / Faddeev landscape $N \to N^*\gamma$

	Quark-diquark	Three-quark		
				
		RL	bRL	bRL + 3q
N, Δ masses		√	. . .	. . .
N, Δ em. FFs		√		
$N \to \Delta\gamma$		√		
Roper		√	. . .	
$N \to N^*\gamma$		. . .		
$N^*(1535), \ldots$		√	. . .	
$N \to N^*\gamma$		. . .		

DSE / Faddeev landscape $N \to N^*\gamma$

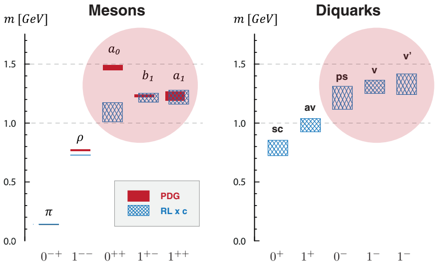


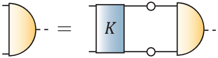


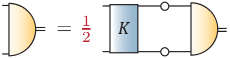
	Quark-diquark			Three-quark		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	...	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	√		
Roper	√	√	√	√	...	
$N \to N^*\gamma$	√	√	...	...		
$N^*(1535), \ldots$	...	...	√	√	...	
$N \to N^*\gamma$	...	...	...	...		
	Roberts, Bashir, Segovia, Chen, Wilson, Lu, ...	Oettel, Alkofer, Roberts, Cloet, Segovia, ...	GE, Alkofer, Nicmorus, ...	GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, ...		

The role of diquarks

Mesons and 'diquarks' closely related:
after taking traces, only factor 1/2 remains
⇒ **diquarks 'less bound' than mesons**





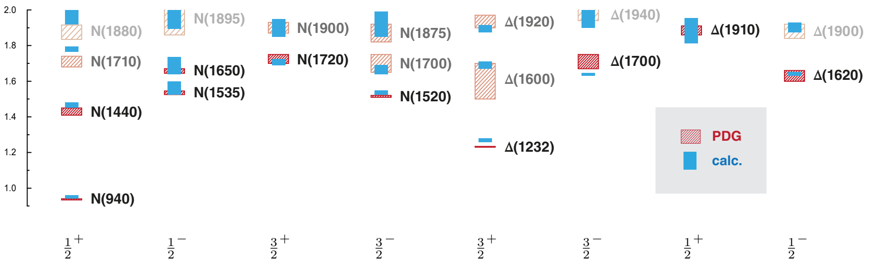


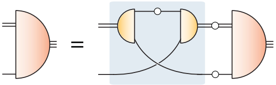
Pseudoscalar & vector mesons already good in rainbow-ladder ⇔ **Scalar & axialvector diquarks** sufficient for nucleon and Δ

Scalar & axialvector mesons too light, repulsion beyond RL ⇔ **Pseudoscalar & vector diquarks** important for remaining channels

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

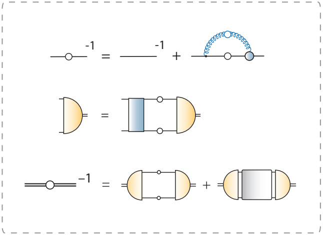


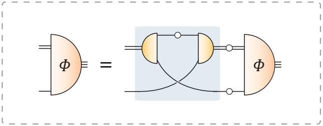


- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

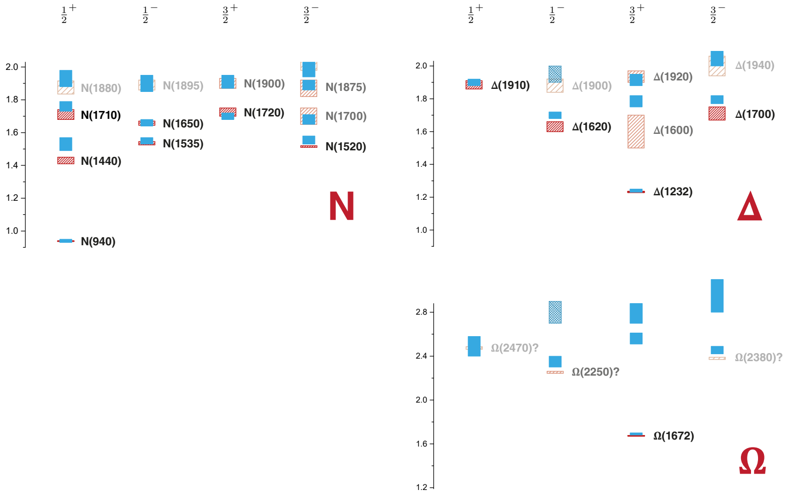
Strange baryons

	[nn]	{nn}	[ns]	{ns}	{ss}
N	●	●			
Δ		●			
Λ	●		●	●	
Σ		●	●	●	
Ξ			●	●	●
Ω					●

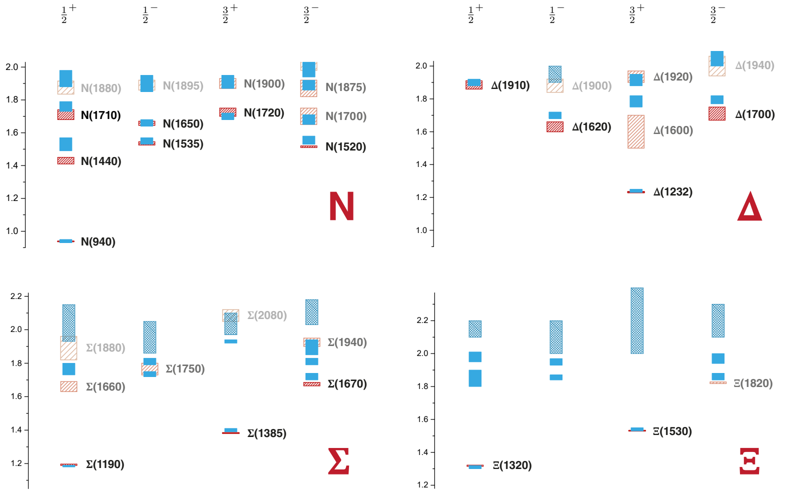




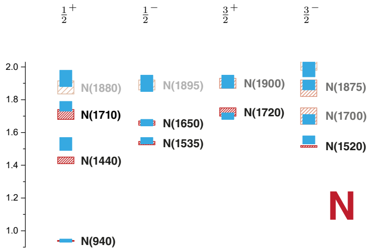
Strange baryons

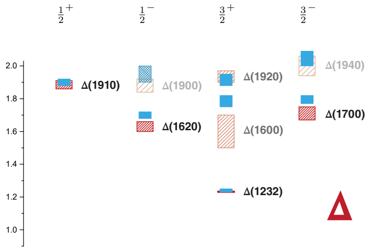


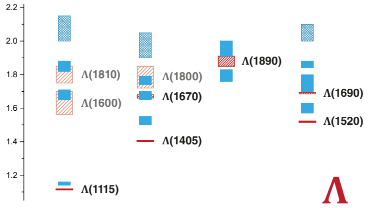
Strange baryons



Strange baryons







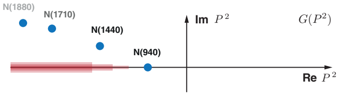
- Strange baryons similar to **light baryons:**

$$\begin{array}{l} \mathbf{\Omega} \rightarrow \mathbf{\Delta} \\ \mathbf{\Sigma}, \mathbf{\Xi} \rightarrow \mathbf{N} + \mathbf{\Delta} \\ \mathbf{\Lambda} \rightarrow \mathbf{N} + \text{singlets} \end{array} \qquad \rightarrow \text{rich spectrum!}$$

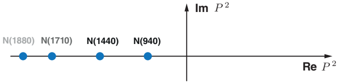
- Roper, $\mathbf{\Delta}$(1600), $\mathbf{\Lambda}$(1405), $\mathbf{\Lambda}$(1520): additional dynamics?

Resonances?

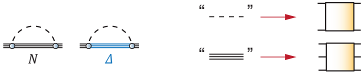
- Branch cuts & widths generated by **meson-baryon interactions:** Roper $\rightarrow N\pi$, etc.



- So far: bound states

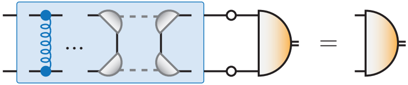


- **Resonance dynamics** difficult to implement at quark-gluon level:

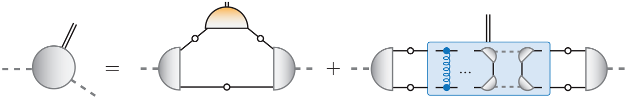


Resonances?

$\rho \to \pi\pi$: **resonance dynamics**
only beyond rainbow-ladder,
would shift ρ pole into complex plane
(above $\pi\pi$ threshold)

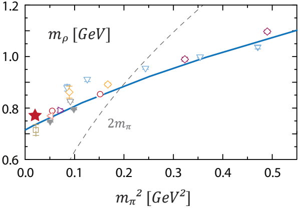


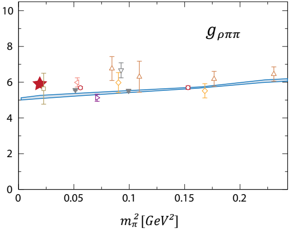
But ρ **decay width**
already calculable
in rainbow-ladder



Rainbow-ladder vs. lattice:

References: GE et al., PPNP 91 (2016) 1606.09602





actual resonance dynamics
subleading effect?

ρ may just be a special case,
but baryon spectrum?

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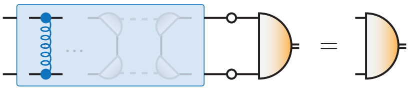

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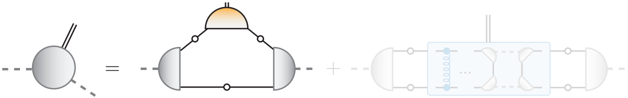

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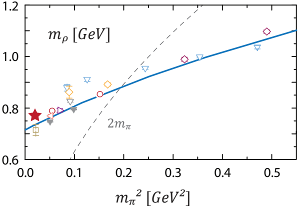

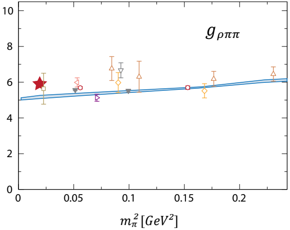

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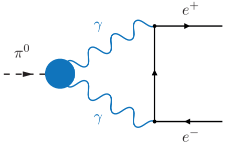
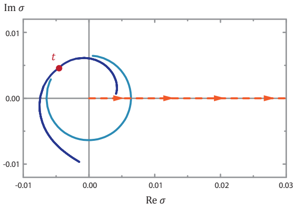
Developing numerical tools

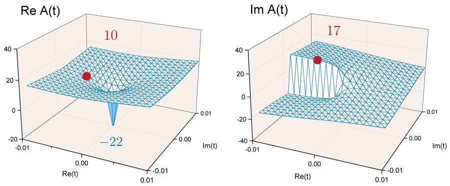
Rare pion decay $\pi^0 \to e^+e^-$:



$$A(t) = \int d\sigma \int dz \; \cdots \; \frac{1}{k^2+m^2} \, \frac{1}{Q^2} \, \frac{1}{Q'^2}$$

Photon and lepton poles produce branch cuts in complex plane:
deform integration contour!





- Result agrees with dispersion relations
- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known

Weil, GE, Fischer, Williams, PRD 96 (2017)

Developing numerical tools

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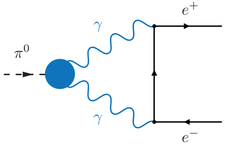

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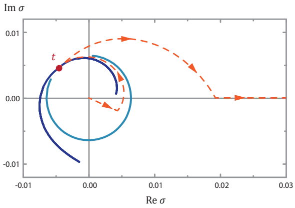

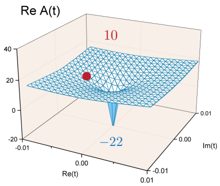

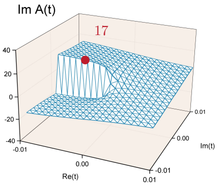

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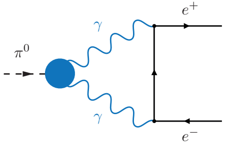

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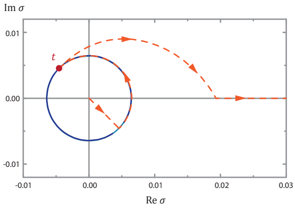

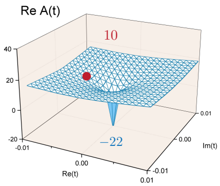

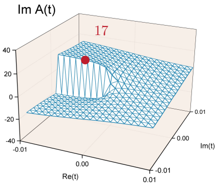

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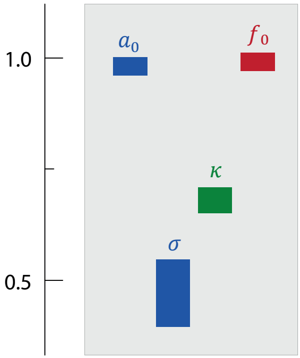
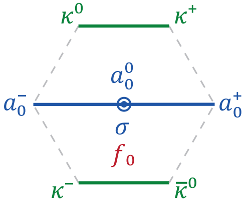
Weil, GE, Fischer, Williams, PRD 96 (2017)

Tetraquarks?

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





f_0 (980 MeV) $s\bar{s}$

κ (680 MeV) $u\bar{s}, d\bar{s}$

a_0 (980 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$

σ (500 MeV) }

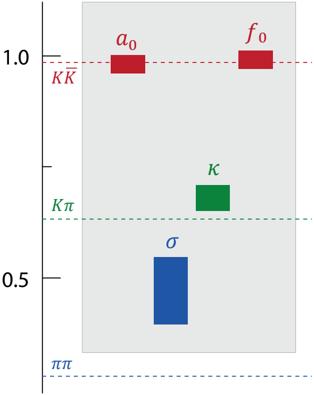
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

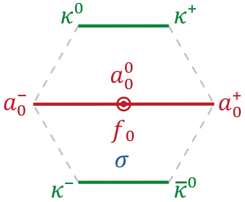
  $\Gamma(\sigma, \kappa) \approx 550$ MeV

  $\Gamma(a_0, f_0) \approx 50\text{–}100$ MeV
- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have
  masses similar to axialvector & tensor mesons ~ 1.3 GeV

Tetraquarks?

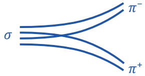
What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004





f_0 (980 MeV) } $us\bar{u}\bar{s}$, ...
a_0 (980 MeV) }
κ (800 MeV) $us\bar{u}\bar{d}$, ...
σ (500 MeV) $ud\bar{u}\bar{d}$

- Explains **mass ordering** & **decay widths**:
  f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ

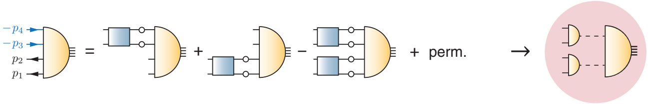


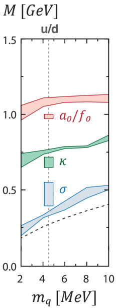
- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-q$\bar{\text{q}}$ nature** of σ supported by dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models
  Pelaez, Phys. Rept. 658 (2016)

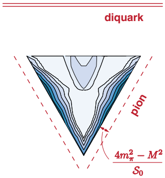
Tetraquarks

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)



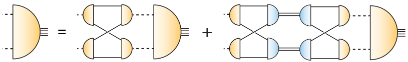


BSE dynamically generates **meson poles** in wave function:



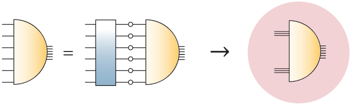
Four quarks rearrange to "**meson molecule**"

- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)



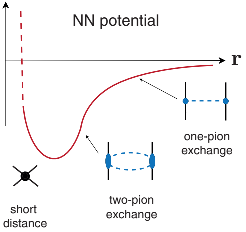
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

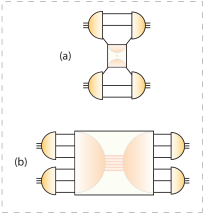


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



Weise, Nucl. Phys. A805 (2008)



- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Scattering amplitudes

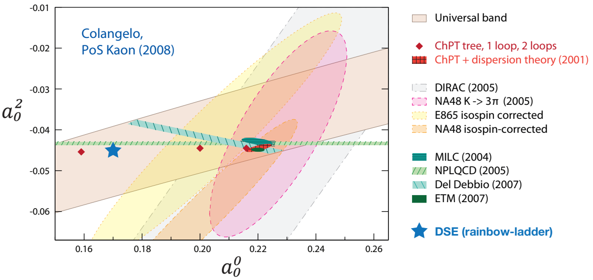
Scattering amplitudes from quark level:

- **$\pi\pi$ scattering**

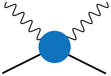
  DSE: Bicudo, Cotanch, Llanes-Estrada, Maris, Ribeiro, Szczepaniak, PRD 65 (2002),

  Cotanch, Maris, PRD 66 (2002)

  CST: Biernat, Pena, Ribeiro, Stadler, Gross, PRD 90 (2014)



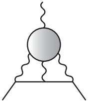
- **Nucleon Compton scattering**



  GE, Fischer, PRD 85 (2012) & PRD 87 (2013), GE, FBS 57 (2016)

- **Hadronic light-by-light scattering**

  Goecke, Fischer, Williams, PLB 704 (2011), GE, Fischer, Heupel, PRD 92 (2015)

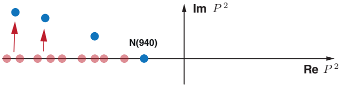


Lattice vs. DSE / BSE

Lattice

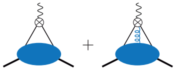
Full dynamics contained in path integral

Proper treatment of **resonances** essential



Simpler access to **position-space** and **gluonic operators**

$\langle N|\bar{\psi}\, \not{D}\, \psi|N\rangle \sim$

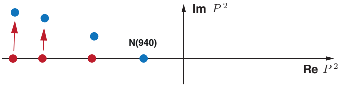


Precision!

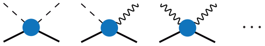
DSE / BSE

Dynamics constructed from underlying **n-point functions**

Resonance dynamics “on top of” **quark-gluon dynamics**



Simpler access to multi-scale problems and higher n-point functions

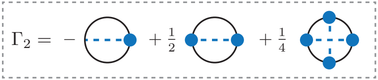


Can tell us about underlying dynamics!

Backup slides

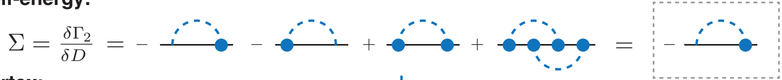
nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.

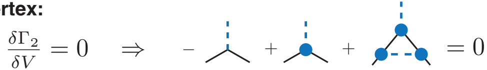


see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

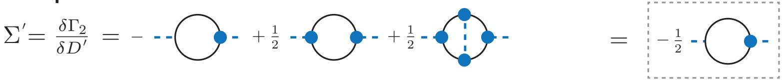
Self-energy:



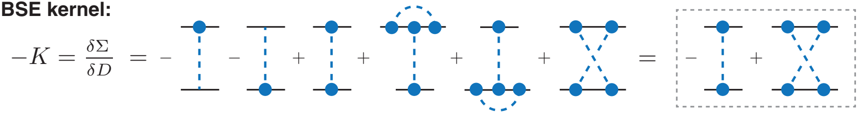
Vertex:



Vacuum polarization:



BSE kernel:



nPI effective action

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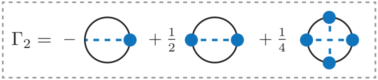
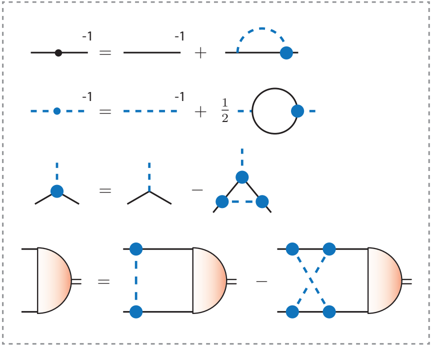


see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

So we arrive at a closed system of equations:



- Crossed ladder cannot be added by hand, requires **vertex correction!**

nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
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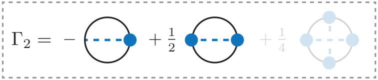

see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

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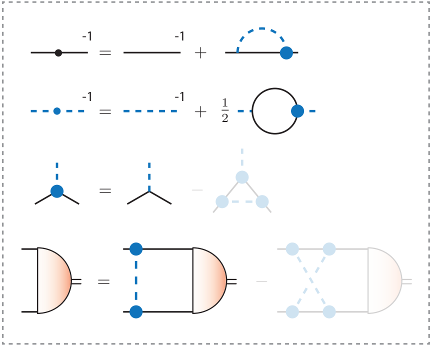
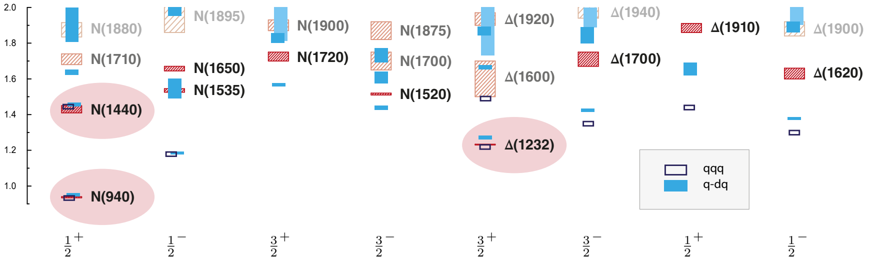


- Crossed ladder cannot be added by hand, requires **vertex correction!**
- without 3-loop term: **rainbow-ladder** with tree-level vertex ⇒ 2PI
- but still requires **DSE solutions** for propagators!
- Similar in QCD. nPI truncation guarantees chiral symmetry, massless pion in chiral limit, etc.

Baryon spectrum I

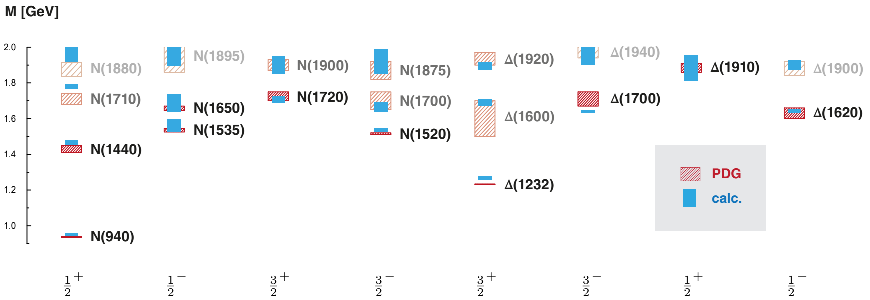
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

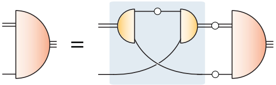


- **qqq** and **q-dq** agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)

Baryon spectrum

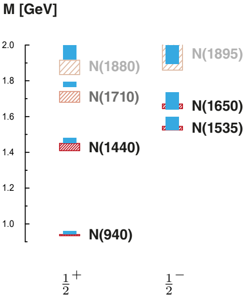
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)





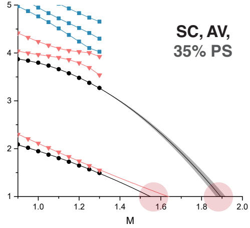
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

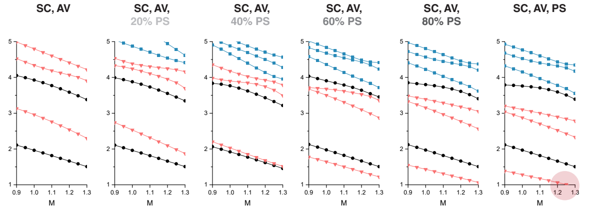
Baryon spectrum



Level ordering between
Roper and N(1535):

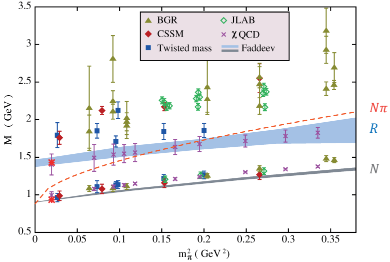
dynamics of ps diquark produces
2 nearby states: **N(1535), N(1650)**



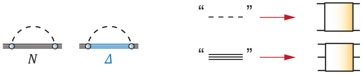


Resonances

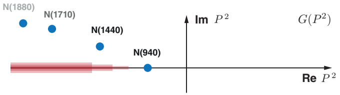
- **Current-mass evolution** of Roper:
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



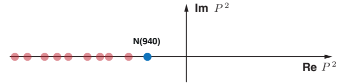
- **'Pion cloud'** effects difficult to implement at **quark-gluon level:**

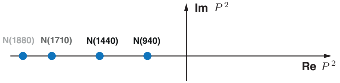


- Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



- **Lattice:** finite volume, **DSE** (so far): bound states





Resonance dynamics
shifts poles into complex plane,
but effects on real parts small?

QED

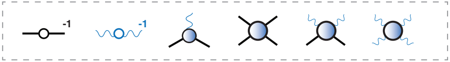
QED's classical action:

$$S = \int d^4x \left[\bar\psi \, (\not\partial + ig\not A + m) \, \psi + \tfrac{1}{4} \, F_{\mu\nu} \, F^{\mu\nu} \right]$$

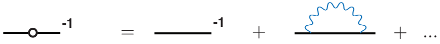
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A] \, e^{-S} = e^{-\Gamma}$$

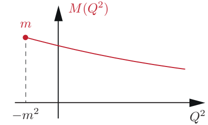


Perturbation theory: expand Green functions in powers of the coupling



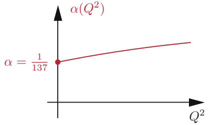
$A(p^2) \, (i\not p + M(p^2))$ $\qquad$ $i\not p + m$

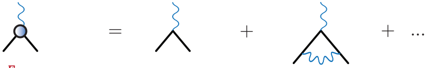
mass function



$D^{-1}(p^2) \, (p^2 \, \delta^{\mu\nu} - p^\mu p^\nu)$ $\qquad$ $p^2 \, \delta^{\mu\nu} - p^\mu p^\nu$

running coupling





$F_1 \gamma^\mu - \frac{F_2}{2m} \, \sigma^{\mu\nu} Q^\nu + \ldots$ $\qquad$ γ^μ

anomalous magnetic moment

$F_2(0) = \frac{\alpha}{2\pi}$

QED

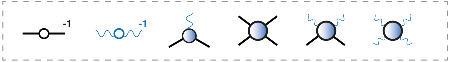
QED's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + igA\!\!\!/ + m\right)\psi + \tfrac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu}\right]$$

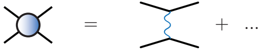
$=$ 

Quantum "effective action":

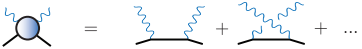
$$\int \mathcal{D}[\psi,\bar\psi,A]\, e^{-S} = e^{-\Gamma}$$



Perturbation theory: expand Green functions in powers of the coupling



Moller scattering



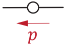
Compton scattering

Light-by-light scattering

$\Rightarrow$ **extremely precise theory predictions!**

Dynamical quark mass

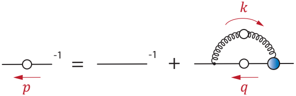
- General form of dressed **quark propagator:**



$$S(p) = \frac{1}{A(p^2)} \, \frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

$$S^{-1}(p) = A(p^2)\,(i\not{p} + M(p^2))$$

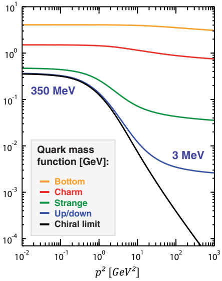
- **Quark DSE:** determines quark propagator, input → gluon propagator, quark-gluon vertex



$$S^{-1}(p) = S_0^{-1}(p) + \int_q \gamma^\mu \, S(q)\, D^{\mu\nu}(k)\, \Gamma^\nu(p,q)$$

- Reproduces **perturbation theory:**

$$\boldsymbol{S}^{-1} = S_0^{-1} - \boldsymbol{\Sigma} \quad \Rightarrow \quad \boldsymbol{S} = S_0 + S_0 \, \boldsymbol{\Sigma} \, \boldsymbol{S}$$
$$= S_0 + S_0 \, \boldsymbol{\Sigma} \, S_0 + S_0 \, \boldsymbol{\Sigma} \, S_0 \, \boldsymbol{\Sigma} \, \boldsymbol{S}$$
$$= \dots$$



- If strength large enough ($\alpha > \alpha_{\text{crit}}$), **chiral symmetry is dynamically broken**

- Generates $M(p^2) \neq 0$ even in chiral limit. Cannot happen in perturbation theory!

- Mass function ~ **chiral condensate:**

$$-\langle \bar{q} q \rangle = N_C \int \frac{d^4 p}{(2\pi)^4} \, \text{Tr}\, S(p)$$

Dynamical quark mass

Simplest example: **Munczek-Nemirovsky model**
Gluon propagator = δ-function, analytically solvable
Munczek, Nemirovsky, PRD 28 (1983)

$$S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

$$S^{-1}(p) = A(p^2)\,(i\not{p}+M(p^2))$$

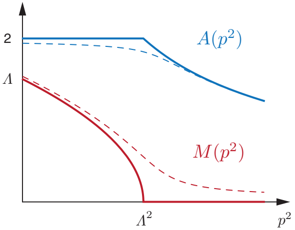
$$D^{\mu\nu}(k)\,\Gamma^\nu(p,q) \;\longrightarrow\sim \Lambda^2\,\delta^4(k)\,\gamma^\mu$$

Quark DSE becomes

$$S^{-1}(p) - S_0^{-1}(p) = \Lambda^2\,\gamma^\mu S(p)\gamma^\mu = \Lambda^2\,\frac{2i\not{p}+4M}{(p^2+M^2)\,A}\,,$$

leads to self-consistent equations for A, M:

$$A = 1 + \frac{2\Lambda^2}{(p^2+M^2)\,A}\,, \qquad AM = m_0 + 2M\,\frac{2\Lambda^2}{(p^2+M^2)\,A}$$



Two solutions in chiral limit: IR + UV

$$M(p^2) = \sqrt{\Lambda^2-p^2} \qquad M(p^2) = 0$$
$$A(p^2) = 2 \qquad A(p^2) = \tfrac{1}{2}\left(1+\sqrt{1+8\,\Lambda^2/p^2}\right)$$

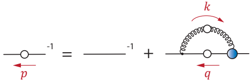
Quark condensate:

$$-\langle\bar{q}q\rangle = N_C \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,S(p) \;=\; \frac{2}{15}\,\frac{N_C}{(2\pi)^2}\,\Lambda^3$$

Another extreme case: **NJL model,**
gluon propagator = const,
$M(p^2)$ = const, but critical behavior
Nambu, Jona-Lasinio, 1961

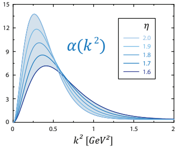
Dynamical quark mass

- Simplest realistic example: **rainbow-ladder**



Tree-level quark-gluon vertex + **effective interaction:**

$$D^{\mu\nu}(k)\,\Gamma^{\nu}(p,q) \longrightarrow \sim \frac{\alpha(k^2)}{k^2}\left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\gamma^\nu$$



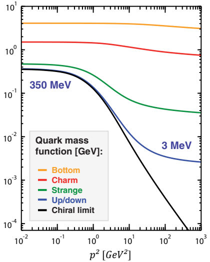
$$\alpha(k^2) = \alpha_{\text{IR}}\left(\frac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\text{UV}}(k^2)$$

adjust scale Λ to observable, keep width η as parameter

Maris, Tandy, PRC 60 (1999)

- If strength is large enough ($\alpha > \alpha_{\text{crit}}$): **DCSB**

- All dimensionful quantities ~ Λ in chiral limit
  ⇒ **mass generation for hadrons!**



Classical PCAC relation for $SU(N_f)_A$:

$$\partial_\mu\ \bar{\psi}\,\gamma^\mu\gamma_5\,\mathsf{t}_a\,\psi \;=\; i\bar{\psi}\,\{\mathsf{M},\mathsf{t}_a\}\,\gamma_5\,\psi$$

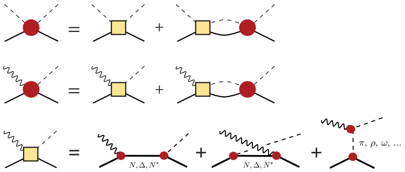
At quantum level:

$$f_\pi\, m_\pi^2 = 2m\, r_\pi$$

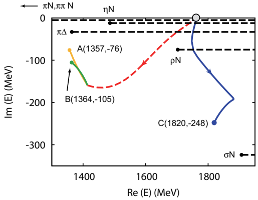
Also $f_\pi \sim \Lambda$ ⇒ $m_\pi = 0$ in chiral limit!
⇒ **massless Goldstone bosons!**

Extracting resonances

Hadronic coupled-channel equations:



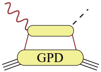
Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI, JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina, JPAC, . . .



Suzuki et al., PRL 104 (2010)

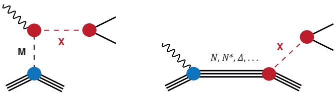
Microscopic effects?

What is an “offshell hadron”?



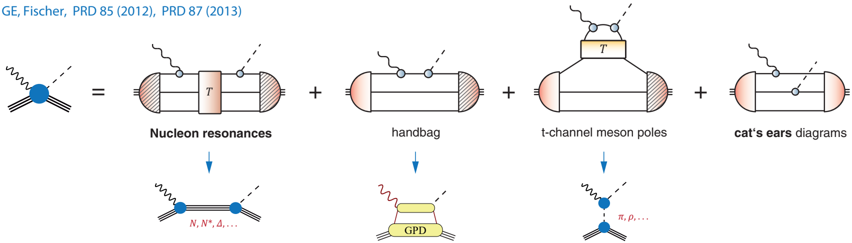
Extracting resonances

Photoproduction of **exotic mesons** at JLab/GlueX:



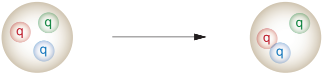
What if exotic mesons are **relativistic $q\bar{q}$ states?**
⇒ study with DSE/BSE!

Scattering amplitudes at quark-gluon level:
GE, Fischer, PRD 85 (2012), PRD 87 (2013)



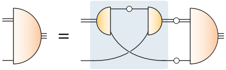
Diquarks?

- Suggested to resolve **'missing resonances'** in quark model: fewer degrees of freedom ⇒ fewer excitations



Anselmino et al., Rev. Mod. Phys. 65 (1993),
Klempt, Richard, Rev. Mod. Phys. 82 (2010)

- QCD version: assume qq scattering matrix as sum of diquark correlations ⇒ three-body equation simplifies to **quark-diquark BSE**



Oettel, Alkofer, Hellstern Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klähn, Roberts, FBS 46 (2009)

Quark exchange binds nucleon, gluons absorbed in building blocks.
Scalar diquark ~ 800 MeV, axialvector diquark ~ 1 GeV

Maris, FBS 32 (2002), GE, Krassnigg, Schwinzerl, Alkofer, Ann. Phys. 323 (2008), GE, FBS 57 (2016)

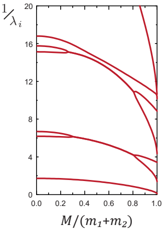
- N and Δ properties similar in quark-diquark and three-quark approach: **quark-diquark approximation is good!**

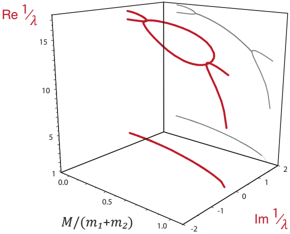
Complex eigenvalues?

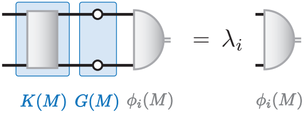
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

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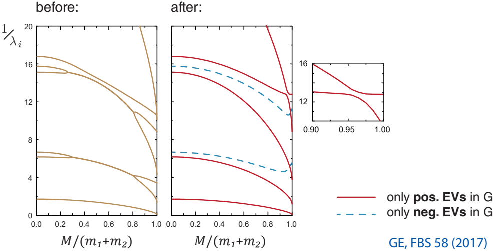
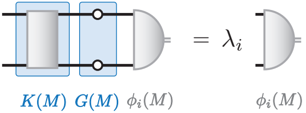


GE, FBS 58 (2017)



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⇒ Hermitian problem with same EVs!

⇒ all EVs strictly **real**
⇒ level repulsion
⇒ “anomalous states” removed?

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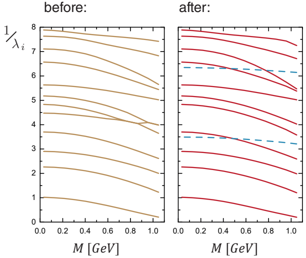
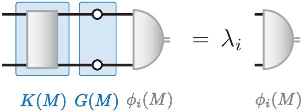
Ahlig, Alkofer, Ann. Phys. 275 (1999)



Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)

only **pos. EVs** in G
only **neg. EVs** in G



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⇒ all EVs strictly **real**
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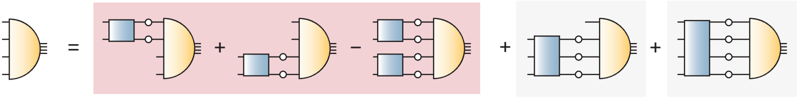
⇒ "anomalous states" removed?

Tetraquarks: four-body equation

Four-quark bound-state equation:



Two-body interactions

3-body
(+ permutations)

4-body

- plus permutations:

  $(qq)(\bar{q}\bar{q}),\ (q\bar{q})(q\bar{q}),\ (q\bar{q})(q\bar{q})$
  (12)(34) (23)(14) (13)(24)

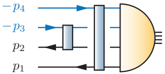
- $K \otimes I + I \otimes K - K \otimes K$ prevents overcounting in T-matrix $T = K + K\,G_0\,T$, avoids Van-der-Waals forces
  Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Keep **two-body interactions** with **rainbow-ladder kernel:**
well motivated by meson & baryon studies

Structure of the amplitude

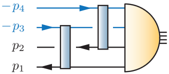
4-quark Bethe-Salpeter amplitude $\Gamma(p,q,k,P)$:
one total momentum & three relative momenta:

$$P = p_1 + p_2 + p_3 + p_4$$



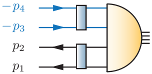
$$p = \tfrac{1}{2}\,(p_2 + p_3 - p_1 - p_4)$$

's channel'



$$q = \tfrac{1}{2}\,(p_3 + p_1 - p_2 - p_4)$$

'u channel'



$$k = \tfrac{1}{2}\,(p_1 + p_2 - p_3 - p_4)$$

't channel'

General structure**:**

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right)\, \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2$$

$$\begin{aligned} \omega_1 &= q\cdot k & \eta_1 &= p\cdot P \\ \omega_2 &= p\cdot k & \eta_2 &= q\cdot P \\ \omega_3 &= p\cdot q & \eta_3 &= k\cdot P \end{aligned}$$

$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3}, \quad 6\otimes\bar{6}$ or
$1\otimes 1, \quad 8\otimes 8$
(Fierz-equivalent)

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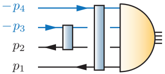

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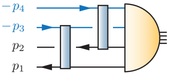

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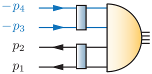

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$\otimes$ Color $\otimes$ Flavor

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$3 \otimes \overline{3}, \quad 6 \otimes \overline{6}$ or
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Kaeding, nucl-th/9502037

Structure of the amplitude

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256 Dirac-Lorentz tensors

Keep **s waves** only:
16 Dirac-Lorentz tensors,
Fierz-complete

e.g. $\left\{ \begin{array}{c} C^T\gamma_5 \otimes \gamma_5 C \\ C^T\gamma^\mu \otimes \gamma^\mu C \\ \dots \end{array} \right\}$ in (12)(34)

automatically includes also
$\gamma_5 \otimes \gamma_5$ in (23)(14), (31)(24)

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automatically includes also
$\gamma_5 \otimes \gamma_5$ in (23)(14), (31)(24)

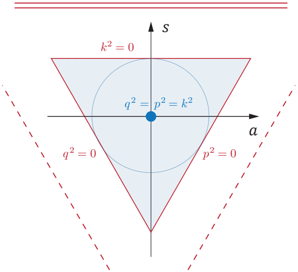
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{1}{4}(p^2+q^2+k^2)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} \sqrt{3}\,(q^2-p^2) \\ p^2+q^2-2k^2 \end{bmatrix}$

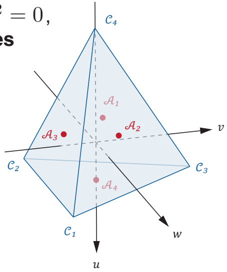
Mandelstam triangle,
outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\omega_1+\omega_2+\omega_3) \\ \sqrt{2}\,(\omega_1+\omega_2-2\omega_3) \\ \sqrt{6}\,(\omega_2-\omega_1) \end{bmatrix}$

tetrahedron bounded by $p_i^2=0$,
outside: **quark singularities**



- Second **triplet:**
3dim. sphere

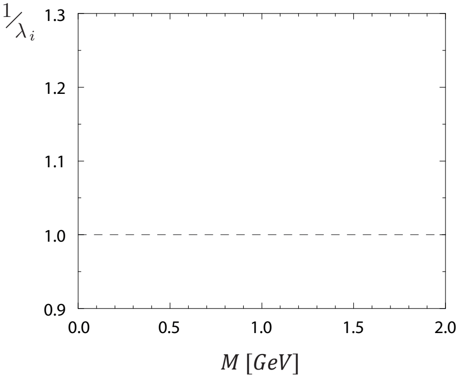
$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\eta_1+\eta_2+\eta_3) \\ \sqrt{2}\,(\eta_1+\eta_2-2\eta_3) \\ \sqrt{6}\,(\eta_2-\eta_1) \end{bmatrix}$$

Tetraquark mass

Idea: use symmetries to figure out **relevant** momentum dependence

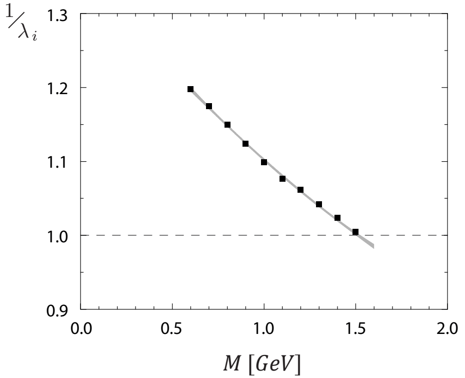
similar:

- **Three-gluon vertex**
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)
- HLbL scattering for **muon g-2**
  GE, Fischer, Heupel, PRD 92 (2015)



Tetraquark mass

$f_i(\mathcal{S}_0, \bigtriangledown, \triangle, \bigcirc)$



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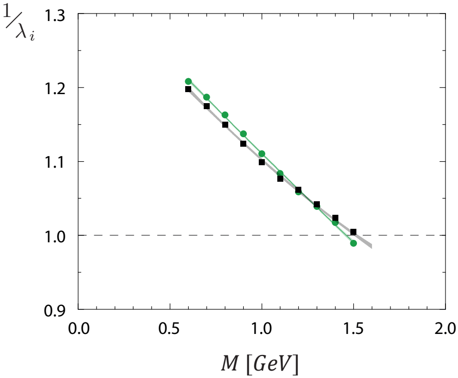
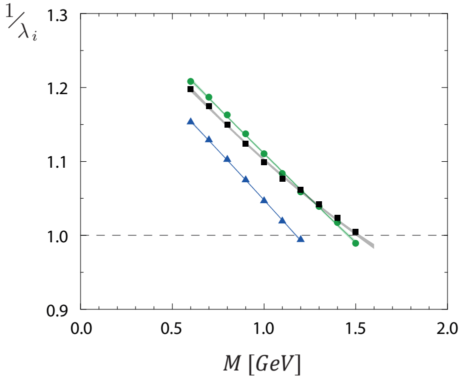
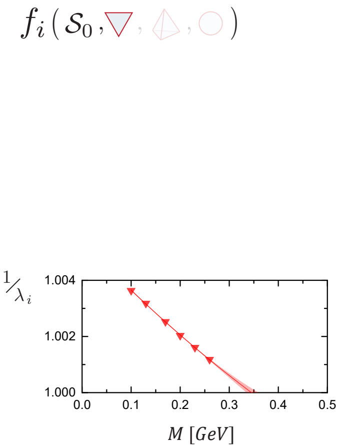


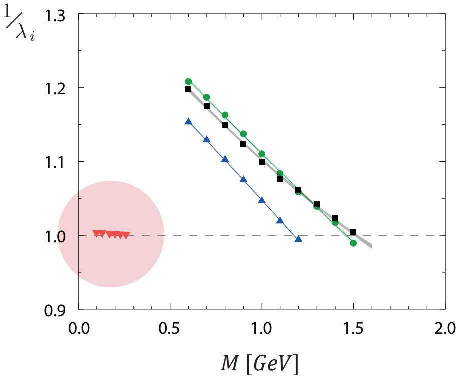
Tetraquark mass

$f_i\,(\,\mathcal{S}_0\,,\ \triangledown\,,\ \triangle\,,\ \bigcirc\,)$



Tetraquark mass





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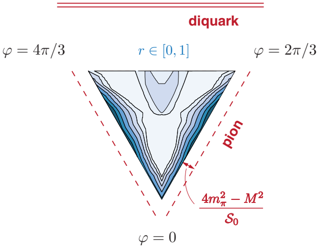
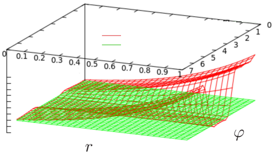


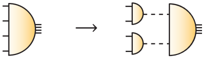
Gap in Mandelstam triangle due to **pion poles!**



$f_i(\mathcal{S}_0, \nabla, \triangle, \bigcirc)$

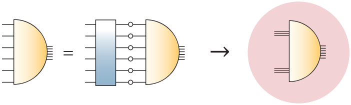
- Four-quark BSE dynamically generates pion poles in BS amplitude, although **equation knows nothing about pions!**
- drive σ mass from 1.5 GeV to ~350 MeV
  ⇒ light tetraquarks are indirect consequence of SχSB
- **Poles enter integration domain** above threshold $M > 2m_\pi$: the tetraquark becomes a **resonance**

- Four quarks rearrange to **“meson molecule”**



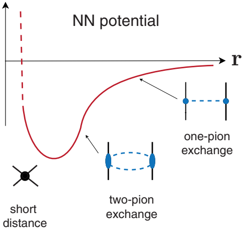
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

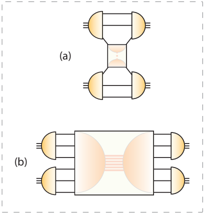


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



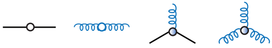
Weise, Nucl. Phys. A805 (2008)

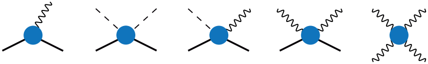


- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Hadron physics with functional methods

Understand properties of **elementary n-point functions** ↔ Calculate hadronic **observables**: mass spectra, form factors, scattering amplitudes, . . .



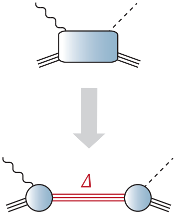


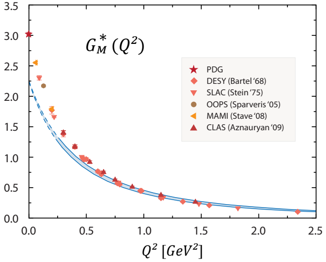
- **QCD**
- **symmetries** intact (Poincare invariance & chiral symmetry important)
- access to all momentum scales & all quark masses
- compute mesons, baryons, tetraquarks, . . . **from same dynamics**

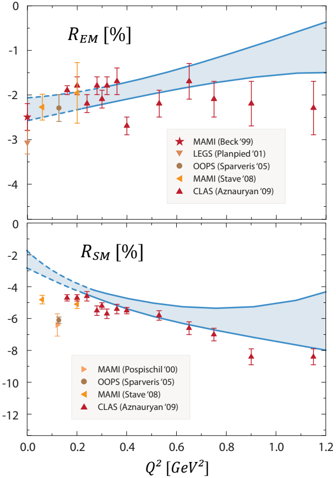
- **systematic** construction of truncations
- technical challenges: coupled integral equations, complex analysis, structure of 3-, 4-, ... point functions, **need lots of computational power!**

access to underlying nonperturbative dynamics!

Nucleon-Δ-γ transition





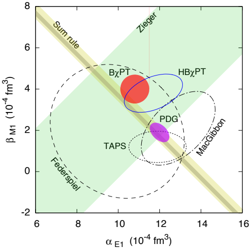


- **Magnetic dipole transition (G_M^*)** dominant:
  quark spin flip (s wave). "Core + 25% pion cloud"

- **Electric & Coulomb quadrupole ratios**
  small & negative, encode deformation.
  Reproduced without pion cloud: **OAM** from **p waves!**
  GE, Nicmorus, PRD 85 (2012)

Compton scattering

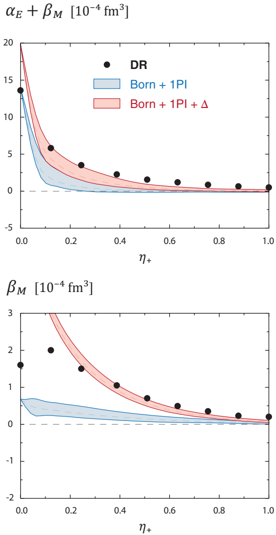
Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)



First DSE results:
GE, FBS 57 (2016)

- Quark Compton vertex (Born + 1PI) calculated, added Δ exchange
- compared to DRs
  Pasquini et al., EPJ A11 (2001), Downie & Fonvieille, EPJ ST 198 (2011)
- α_E dominated by handbag, β_M by Δ contribution

⇒ **large "QCD background"!**

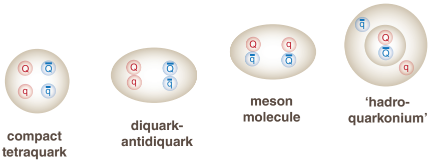


In total: polarizabilities ≈

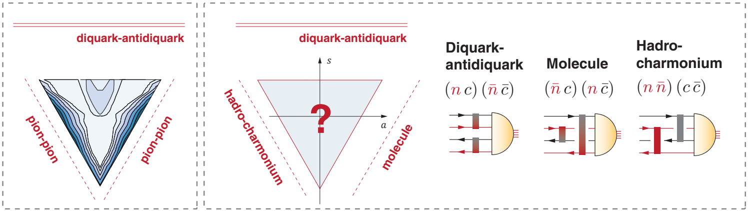
Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly Δ)
+ pion cloud (at low η_+)?

Tetraquarks in charm region?

- Can we **distinguish** different tetraquark configurations?

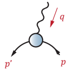


- **Four quarks** dynamically rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium; strengths determined by four-body BSE:



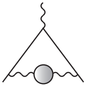
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

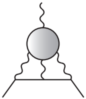


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



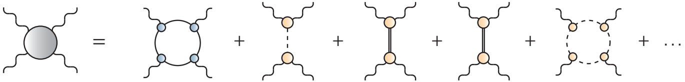
Hadronic light-by-light scattering

a_μ $[10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

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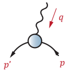
  total SM prediction deviates from exp. by ~3σ



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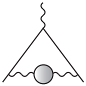

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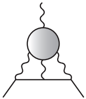

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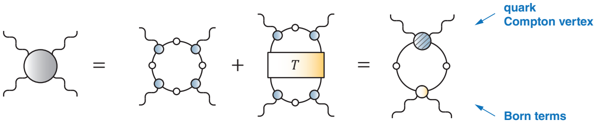
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)