

# Elastic form factors for resonances from LQCD

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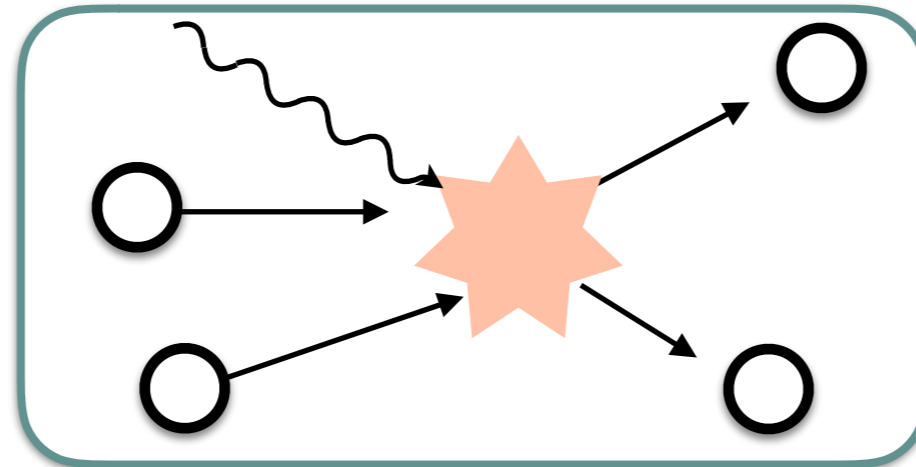
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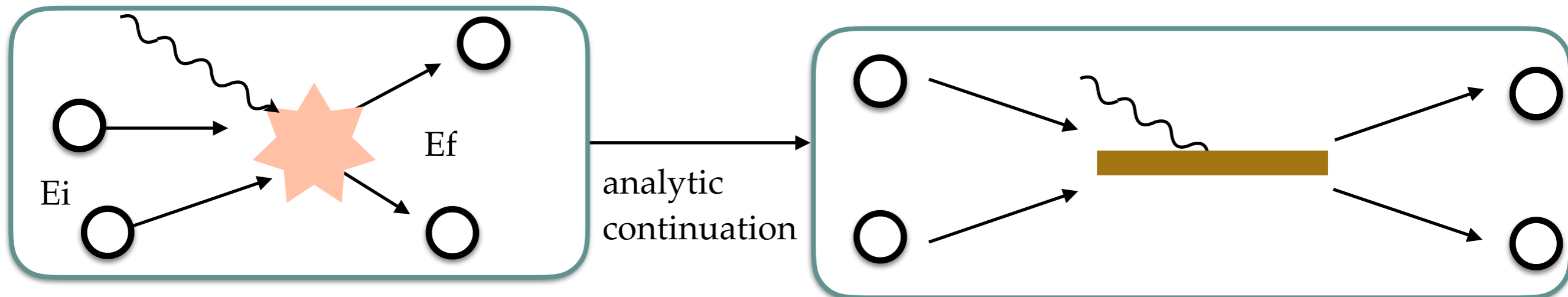
INT Workshop  
Multi-Hadron Systems from LQCD

# Form factor of a resonance

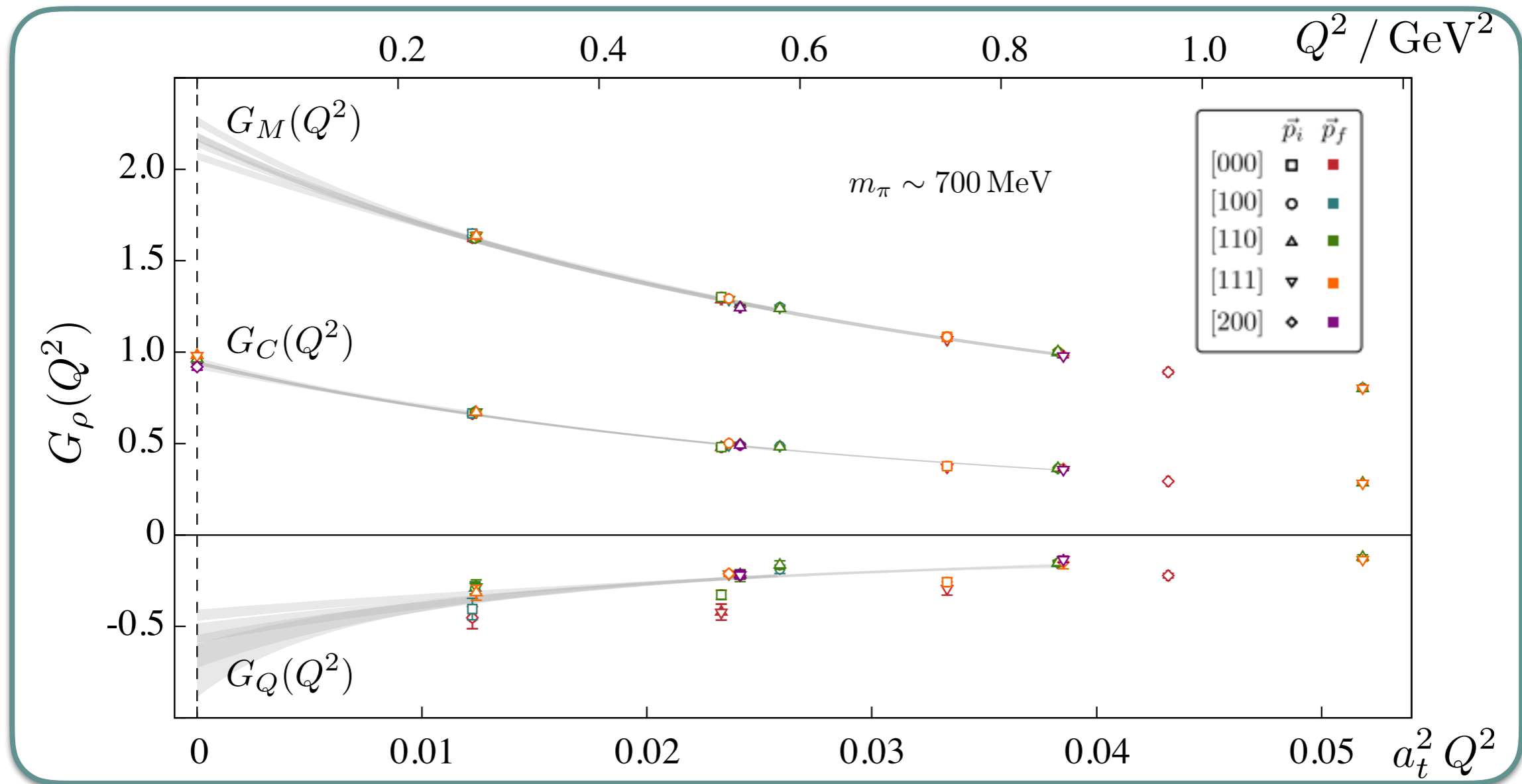
- Resonance not in the asymptotic states  $\rightarrow$  Form factor?



- We can get the form factor in this way



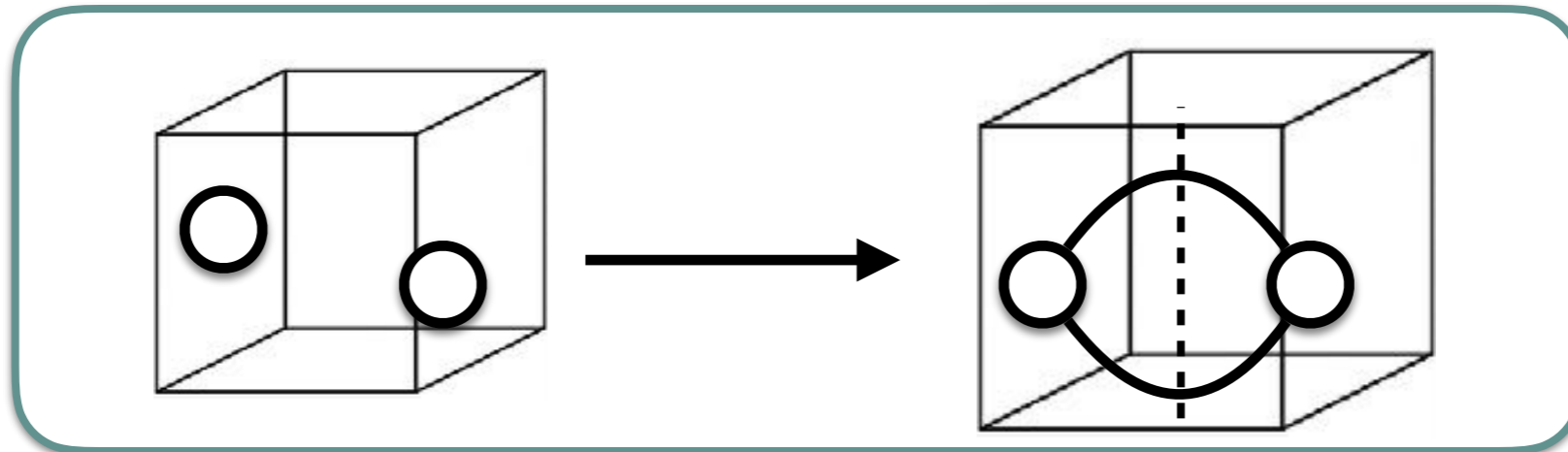
# Form factor of a stable “resonance”



● Shultz, Dudek, and Edwards PRD (2015)

# Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
  - On-shell intermediate states give singularities

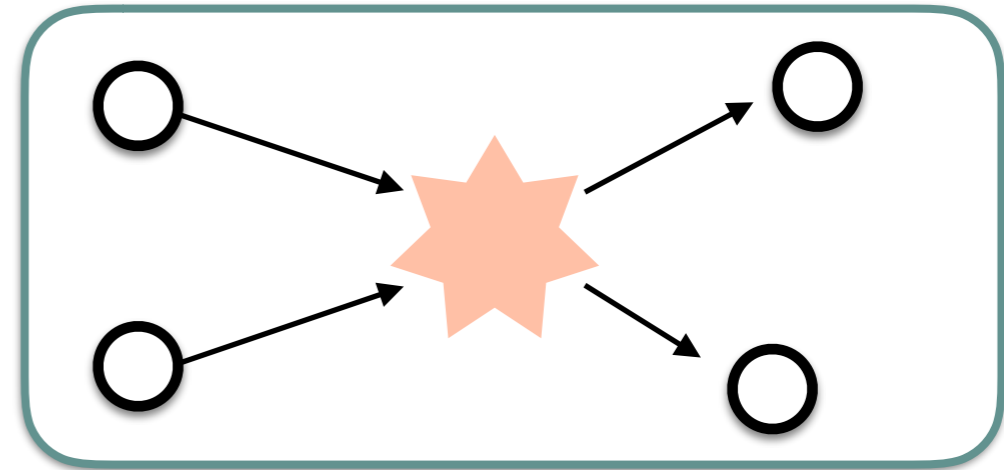
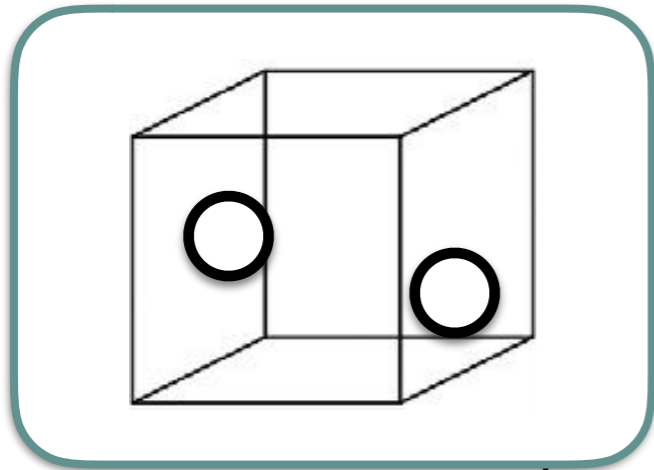


- Formalism needed to deal with these effects?

- Briceño and Hansen (2016) - general, inelastic, relativistic
- Rusetsky et al (2012) - EFT dependent, NR
- Briceño and Davoudi (2012) - EFT dependent, NR

# 2→2

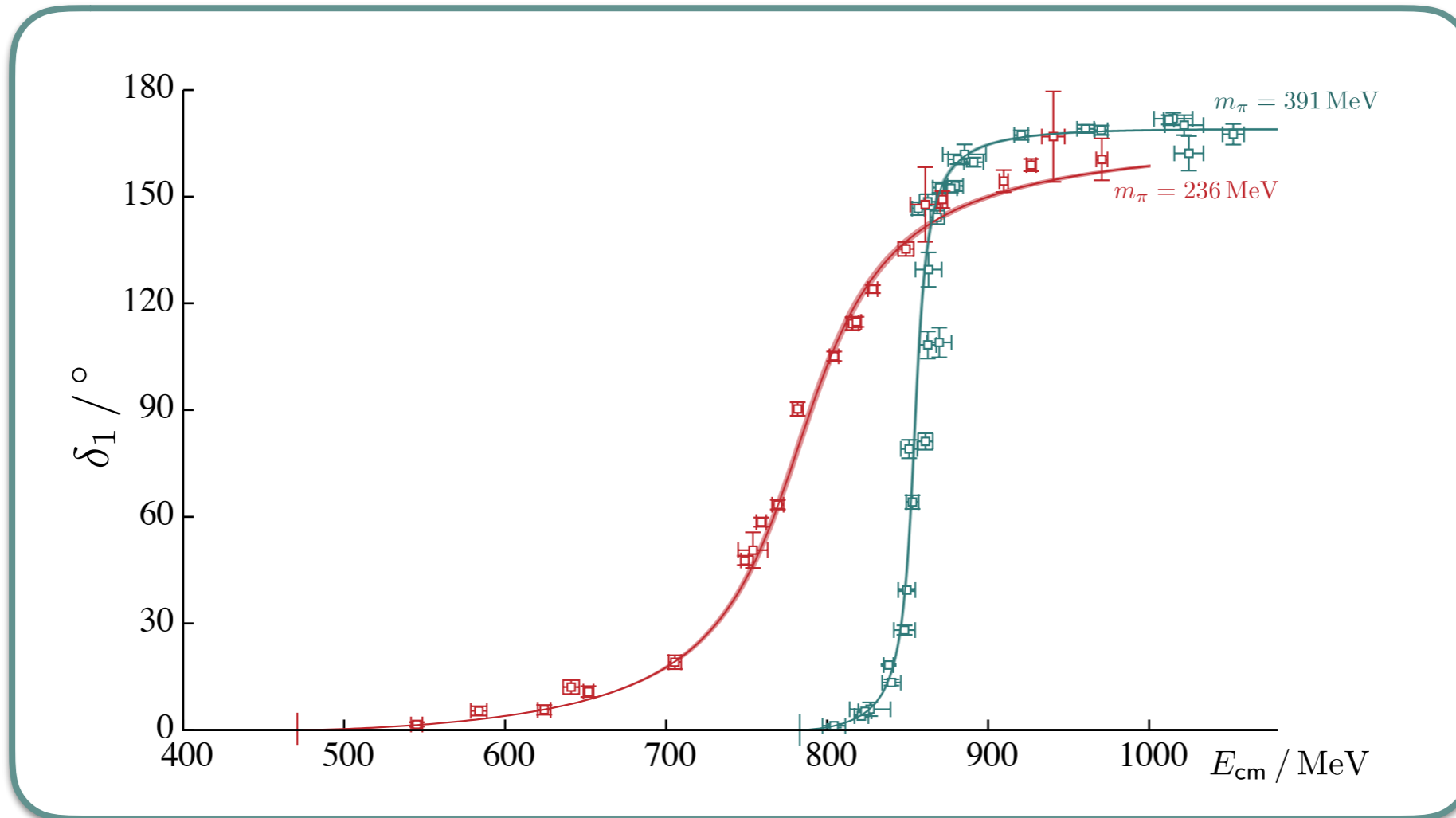
- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds  
→ no other assumptions!!



$$\det [F^{-1}(E_L, L) + M(E_L)] = 0$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]
- Briceño (2014) [general 2-body result]

# $2 \rightarrow 2$

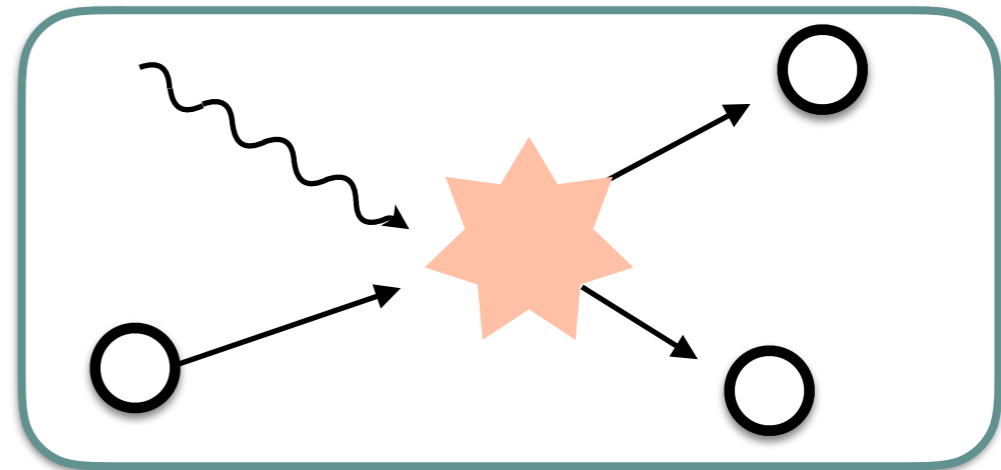
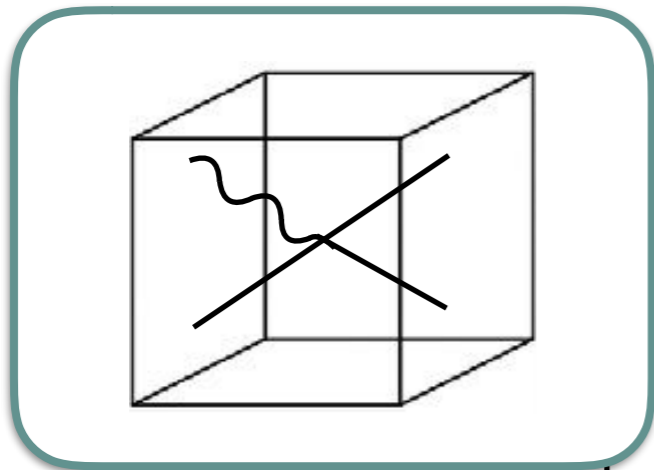


● Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

# $1 + \mathcal{J} \rightarrow 2$

- FV matrix elements to infinite volume electroweak amplitudes

$$\lim_{L \rightarrow \infty} \langle 2 | \mathcal{J} | 1 \rangle_L \neq \langle 2 | \mathcal{J} | 1 \rangle_\infty$$



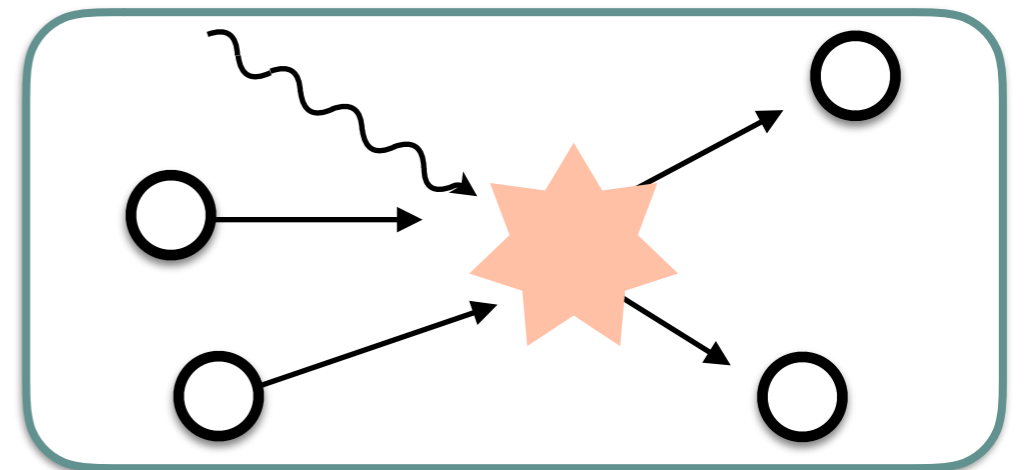
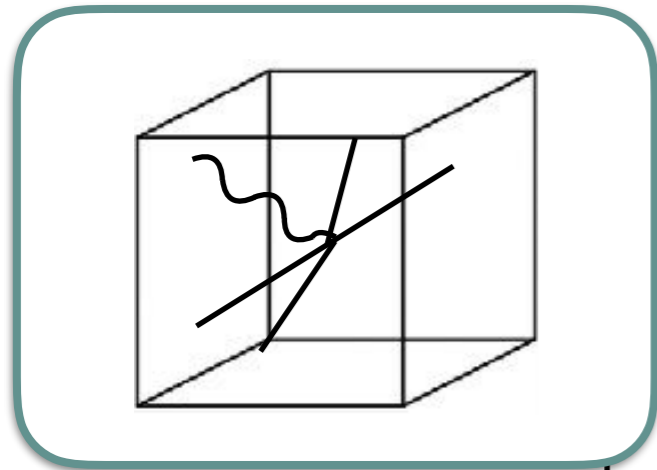
$$|\langle 2 | \mathcal{J} | 1 \rangle|_L = \frac{1}{L^3} \sqrt{\langle 1 | \mathcal{J} | 2 \rangle_\infty R(E_L, L) \langle 2 | \mathcal{J} | 1 \rangle_\infty}$$

- Lellouch & Lüscher (2000) [K-to- $\pi\pi$  at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to- $\pi\pi$ ]
- Hansen & Sharpe (2012) [D-to- $\pi\pi$  / KK]
- Briceño, Hansen Walker-Loud / Briceño & Hansen (2014-2015) [general 1-to-2]

# $2 + \mathcal{J} \rightarrow 2$

- FV matrix elements to infinite volume electroweak amplitudes

$$\lim_{L \rightarrow \infty} \langle 2 | \mathcal{J} | 2 \rangle_L \neq \langle 2 | \mathcal{J} | 2 \rangle_\infty$$

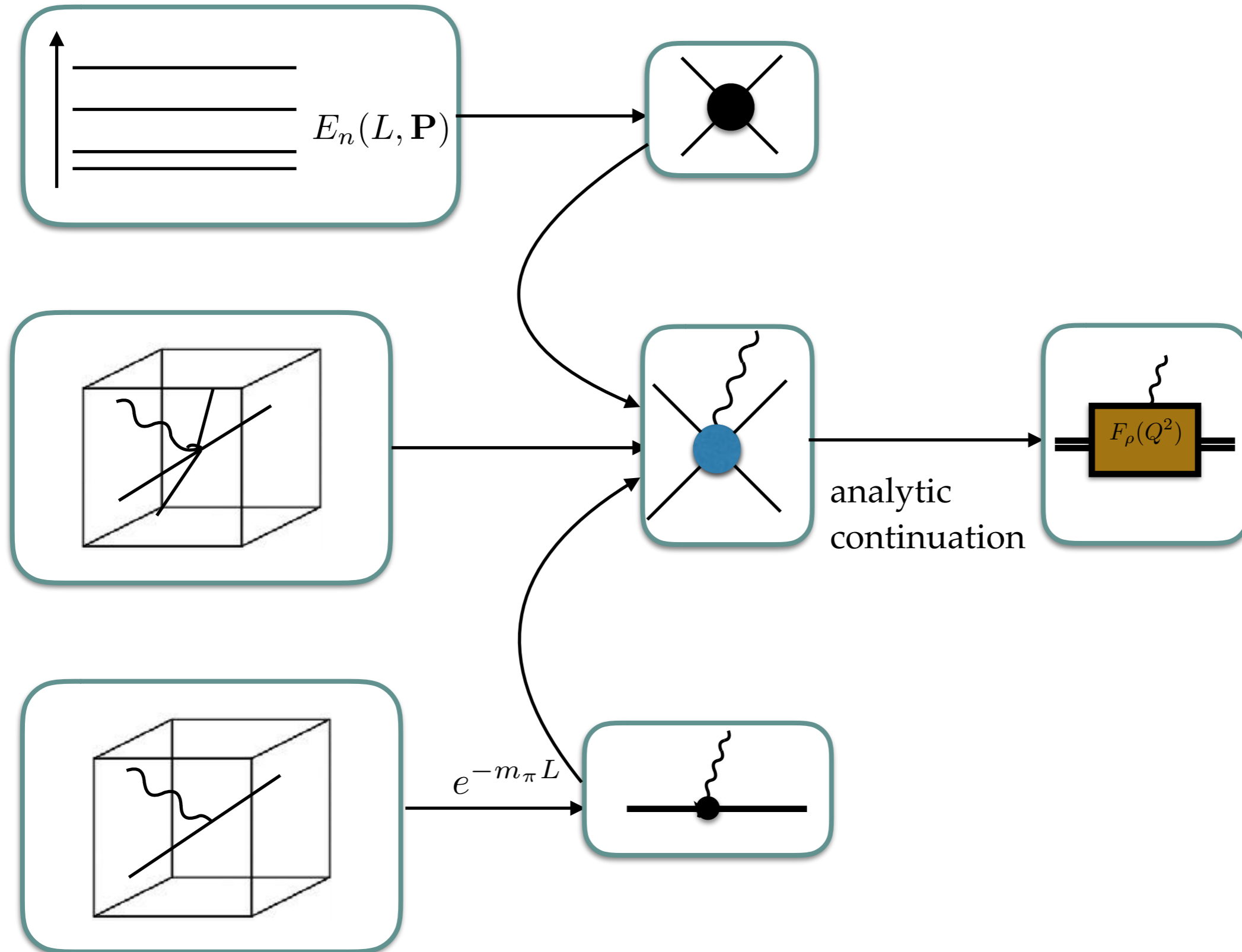


$$\langle 2 | \mathcal{J} | 2 \rangle_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L, \text{df}} R(E_L, L) W_{L, \text{df}}]$$

- Briceño & Hansen (2016)



# Workflow



# $2 + \mathcal{J} \rightarrow 2$

$$\langle 2 | \mathcal{J} | 2 \rangle_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + MG(L, w)M$$

$$W_{\text{df}} = \text{Diagram} = \text{Diagram} - \text{Diagram} - \text{Diagram} - \dots$$

The diagram shows the definition of  $W_{\text{df}}$ . On the left,  $W_{\text{df}}$  is represented by a blue circle with four external lines and a wavy line. This is equal to a series of diagrams: a black circle with four external lines and a wavy line, minus a black circle with four external lines and a wavy line attached to one of the external lines, minus another black circle with four external lines and a wavy line attached to another external line, and so on.

$$w = \text{Diagram}$$

The diagram shows  $w$  as a horizontal line with a wavy line attached to a vertex on the line.

$$G(L, w) = \text{Diagram} - \text{Diagram}$$

The diagram shows  $G(L, w)$  as the difference between two diagrams. The first is a loop with two vertices and a wavy line attached to one vertex, labeled  $V$ . The second is a similar loop labeled  $\infty$ .

$$M = \text{Diagram}$$

The diagram shows  $M$  as a black circle with four external lines.

# $2+\mathcal{J}\rightarrow 2$

$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L)W_{L,\text{df}}R(E_L, L)W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + MG(L, w)M$$

Naive Lellouch & Lüscher relation

$$W_{\text{df}} = \text{Diagram 1} = \text{Diagram 2}$$

Diagram 1: A central blue circle with four black lines extending outwards and a wavy line extending upwards from the top-right quadrant.

Diagram 2: A central black circle with four black lines extending outwards and a wavy line extending upwards from the top-right quadrant.

$$- \text{Diagram 3} - \text{Diagram 4} - \dots$$

Diagram 3: A central black circle with four black lines extending outwards and a wavy line extending upwards from the top-right quadrant, with a small black dot on the wavy line.

Diagram 4: A central black circle with four black lines extending outwards and a wavy line extending downwards from the bottom-right quadrant, with a small black dot on the wavy line.

$$w = \text{Diagram 5}$$

Diagram 5: A horizontal black line with a wavy line extending upwards from its center, with a small black dot on the wavy line.

$$G(L, w) = \text{Diagram 6} - \text{Diagram 7}$$

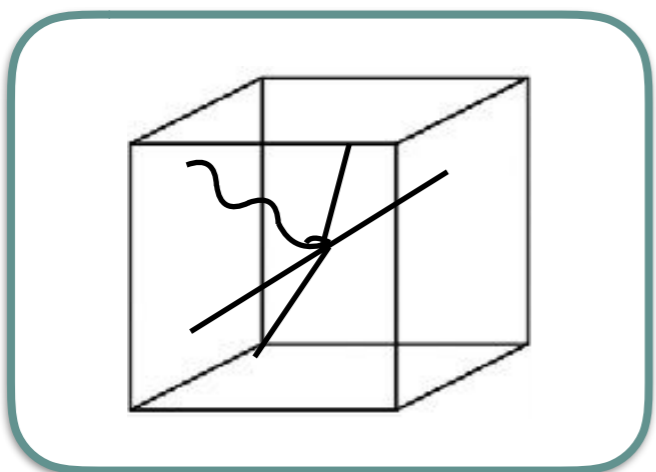
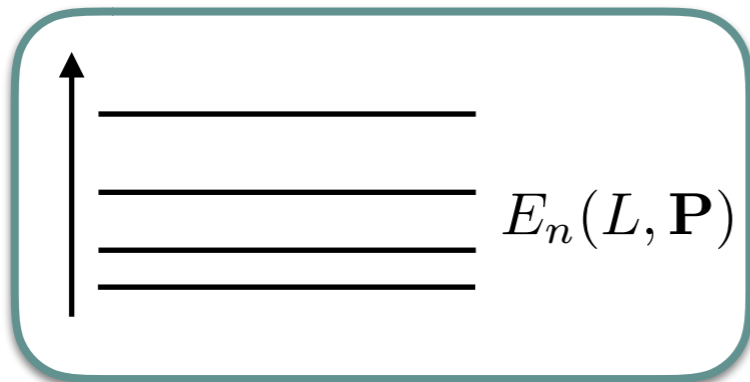
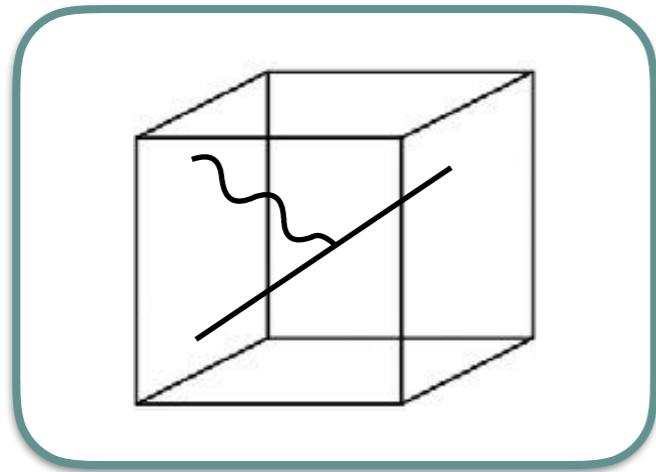
Diagram 6: A circle with two black dots on its horizontal diameter and a wavy line extending upwards from the top dot, with a small black dot on the wavy line. The letter 'V' is inside the circle.

Diagram 7: A circle with two black dots on its horizontal diameter and a wavy line extending upwards from the top dot, with a small black dot on the wavy line. The infinity symbol '∞' is inside the circle.

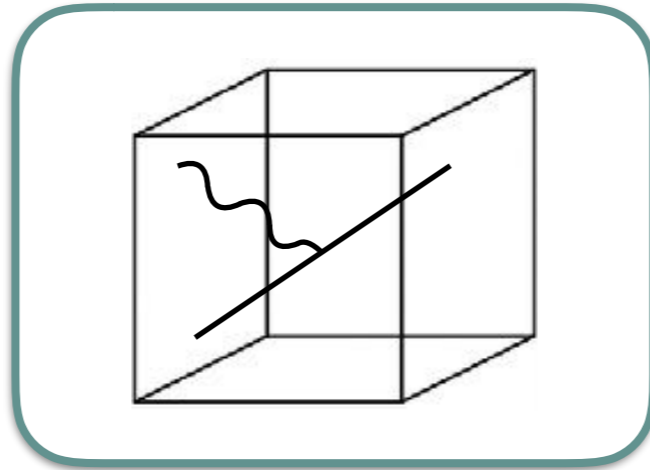
$$M = \text{Diagram 8}$$

Diagram 8: A central black circle with four black lines extending outwards.

# Ingredients of the calculation



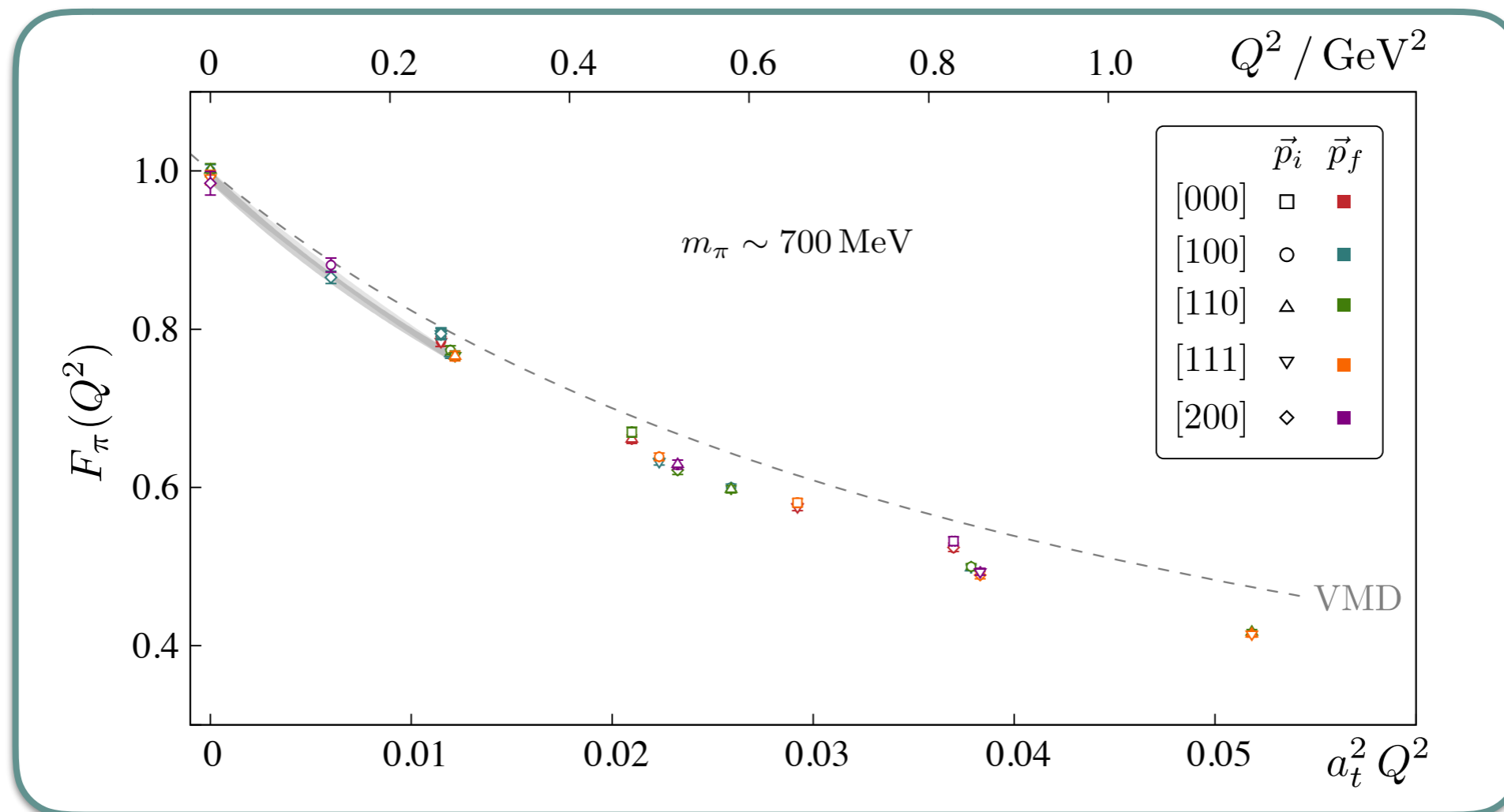
# Pion form factor



$$\begin{aligned}\langle \pi(p_f) | \mathcal{J}^\mu | \pi(p_i) \rangle &= \text{Lorentz, T-reversal and P-invariance} \\ &= \left[ (p_i + p_f)^\mu + (p_i - p_f)^\mu \frac{(m_{\pi'}^2 - m_\pi^2)}{Q^2} \right] F_{\pi\pi}(Q^2)\end{aligned}$$

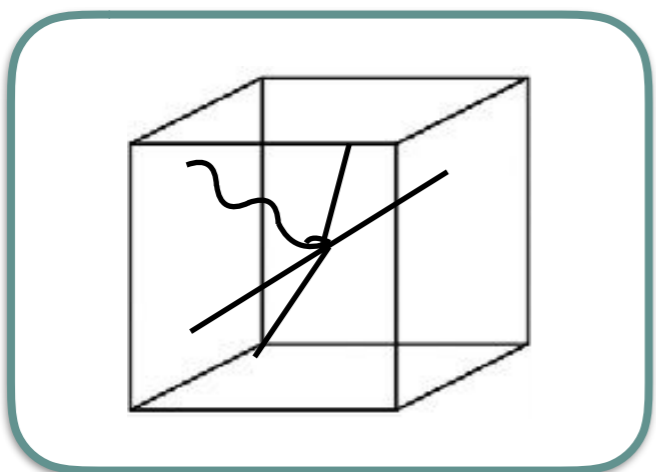
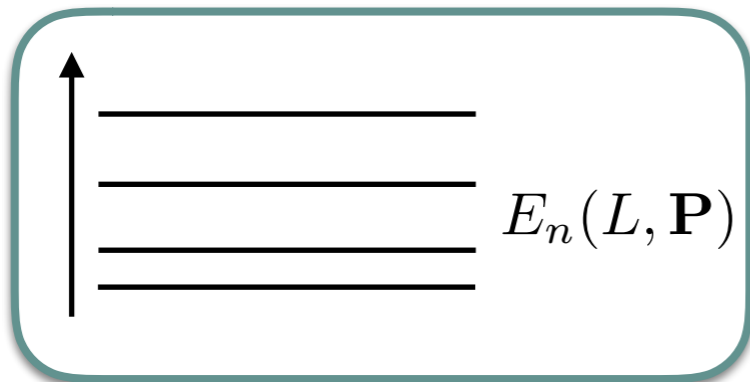
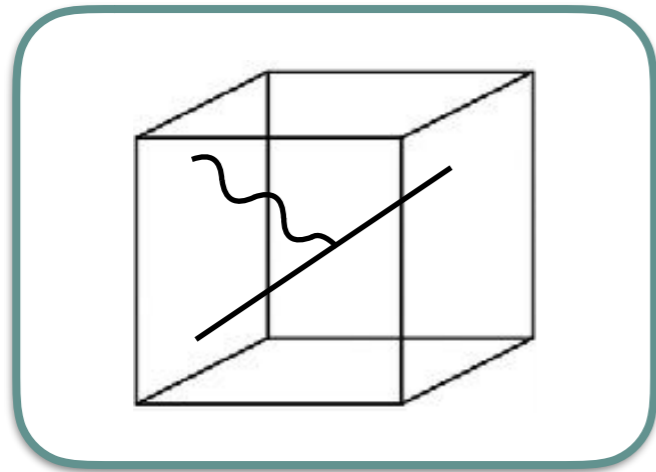
- Only one form factor
- Obtained from the lattice

# Pion form factor



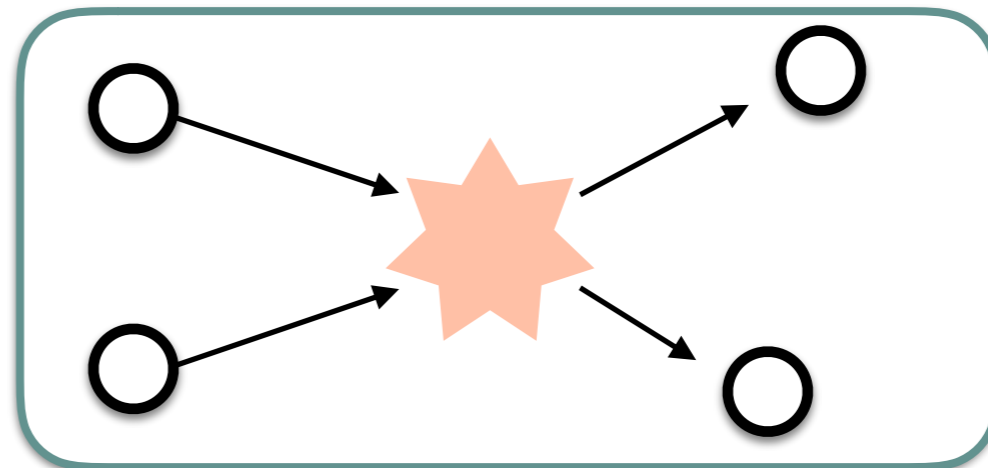
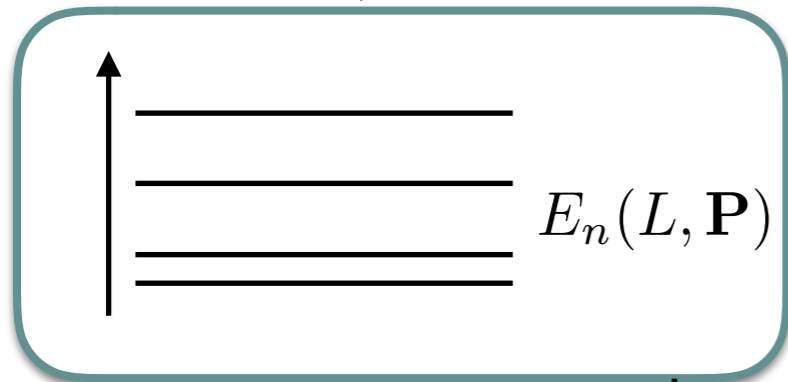
● Shultz, Dudek, and Edwards PRD (2015)

# Ingredients of the calculation



# 2→2

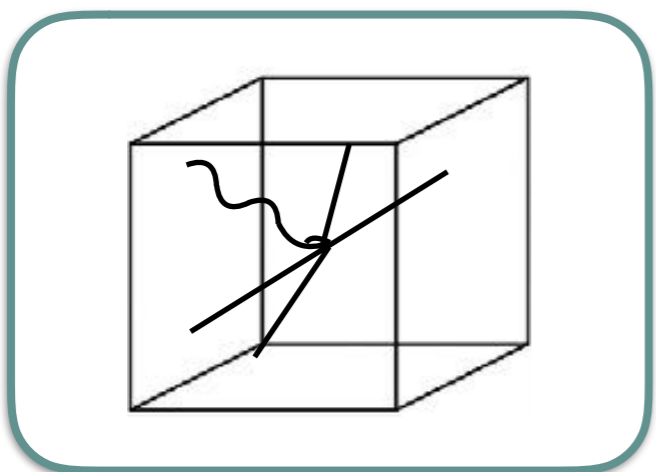
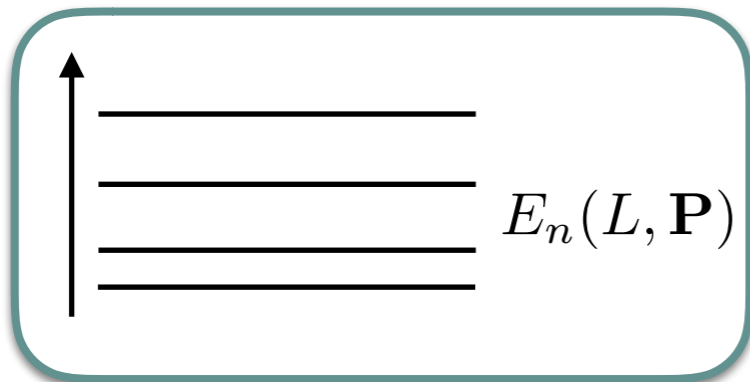
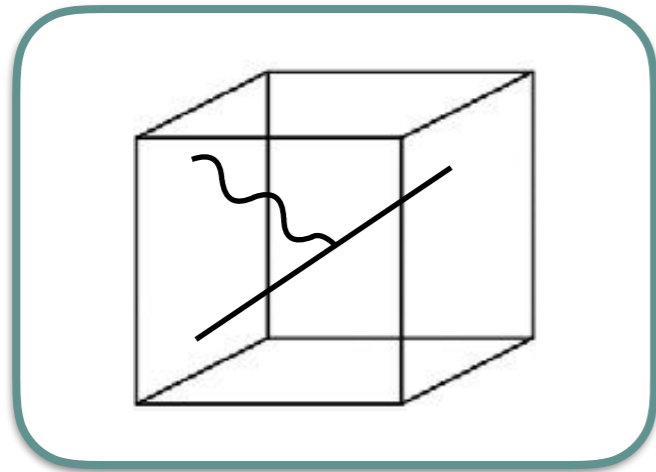
$$C_{2\rightarrow 2}^{2pt.} = \langle 0 | \mathcal{O}_f(\delta t) \mathcal{O}_i^\dagger(0) | 0 \rangle_L = \sum_n Z_{n,f} Z_{n,i}^* e^{-E_n \delta t}$$



● Lüscher formalism

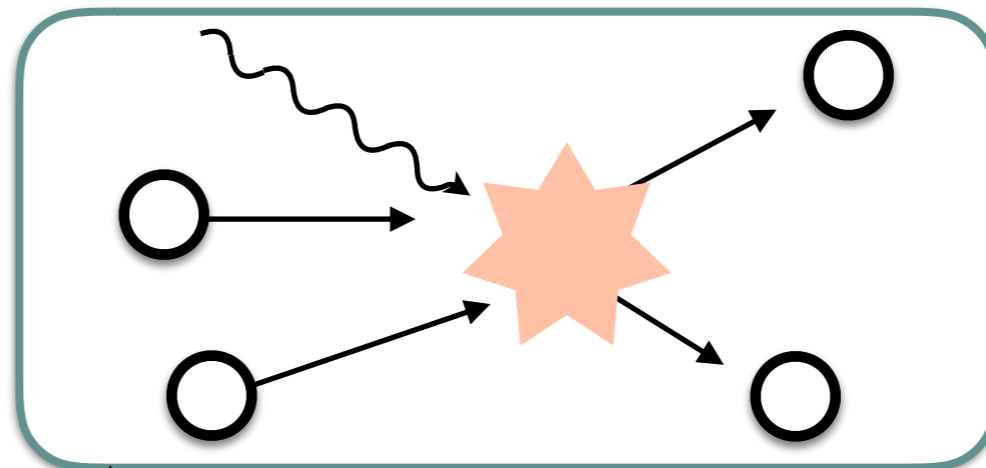
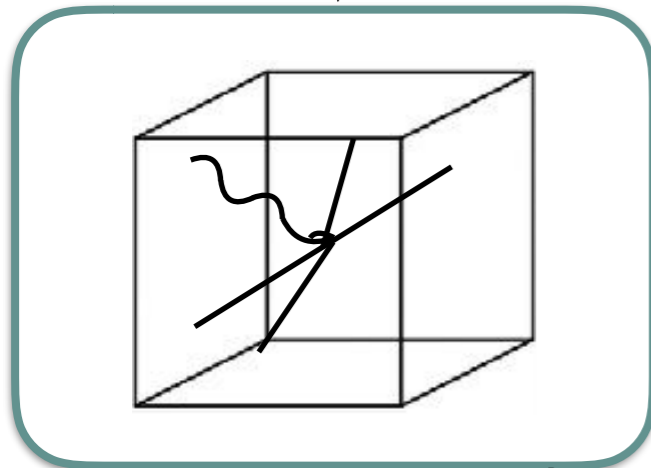


# Ingredients of the calculation



# $2+\mathcal{J}\rightarrow 2$

$$C_{2+\mathcal{J}\rightarrow 2}^{3pt.} = \langle 0 | \mathcal{O}_f(\delta t) \mathcal{J}(t) \mathcal{O}_i^\dagger(0) | 0 \rangle_L = \sum_{n,n'} Z_{n,f} Z_{n',i}^* e^{-(\delta t-t)E_n} e^{-tE'_n} \langle n | \mathcal{J} | n' \rangle_L$$

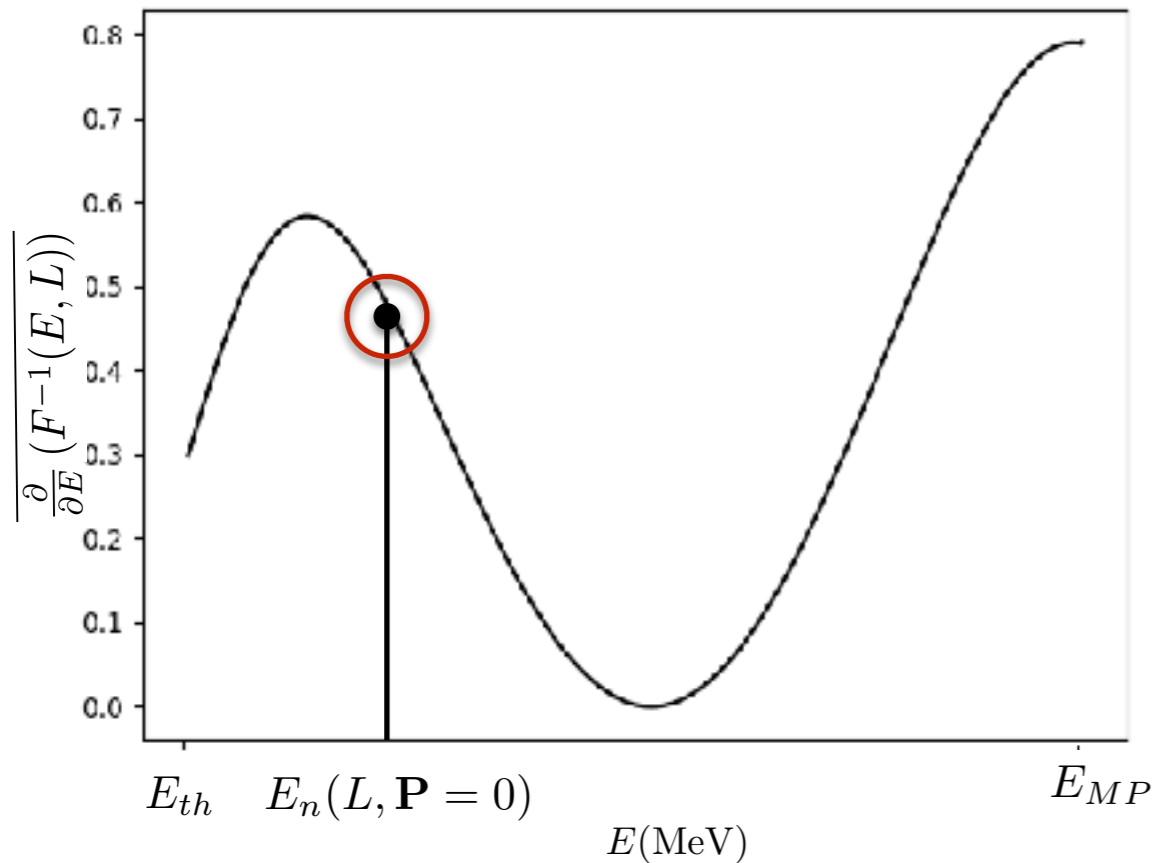


● Briceño & Hansen (2016)

# Kinematic functions

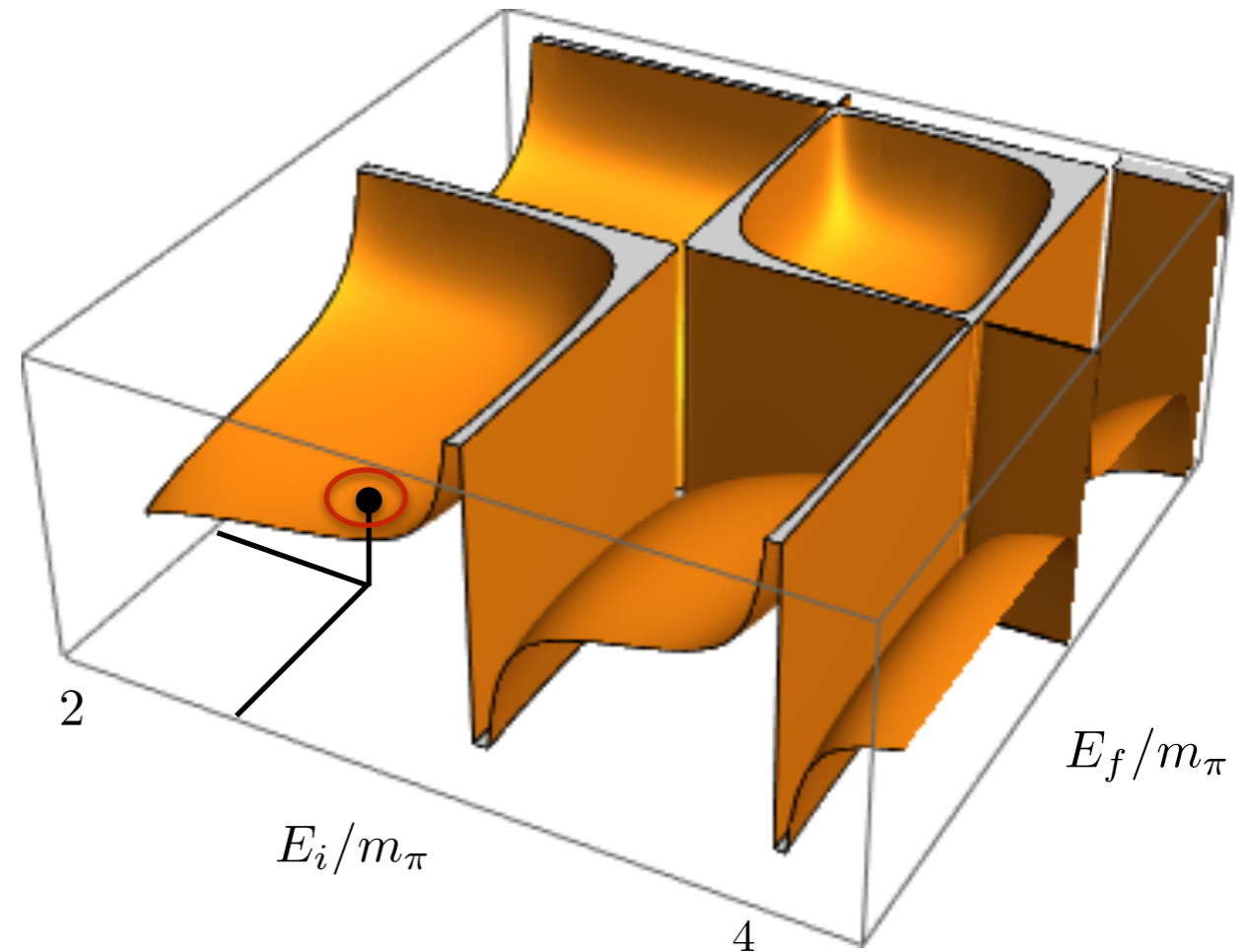
$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E} (F^{-1}(E, L) + M(E))} \Big|_{E=E_L}$$

$$G(E_i, E_f, L) = \left[ \frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots\dots)$$



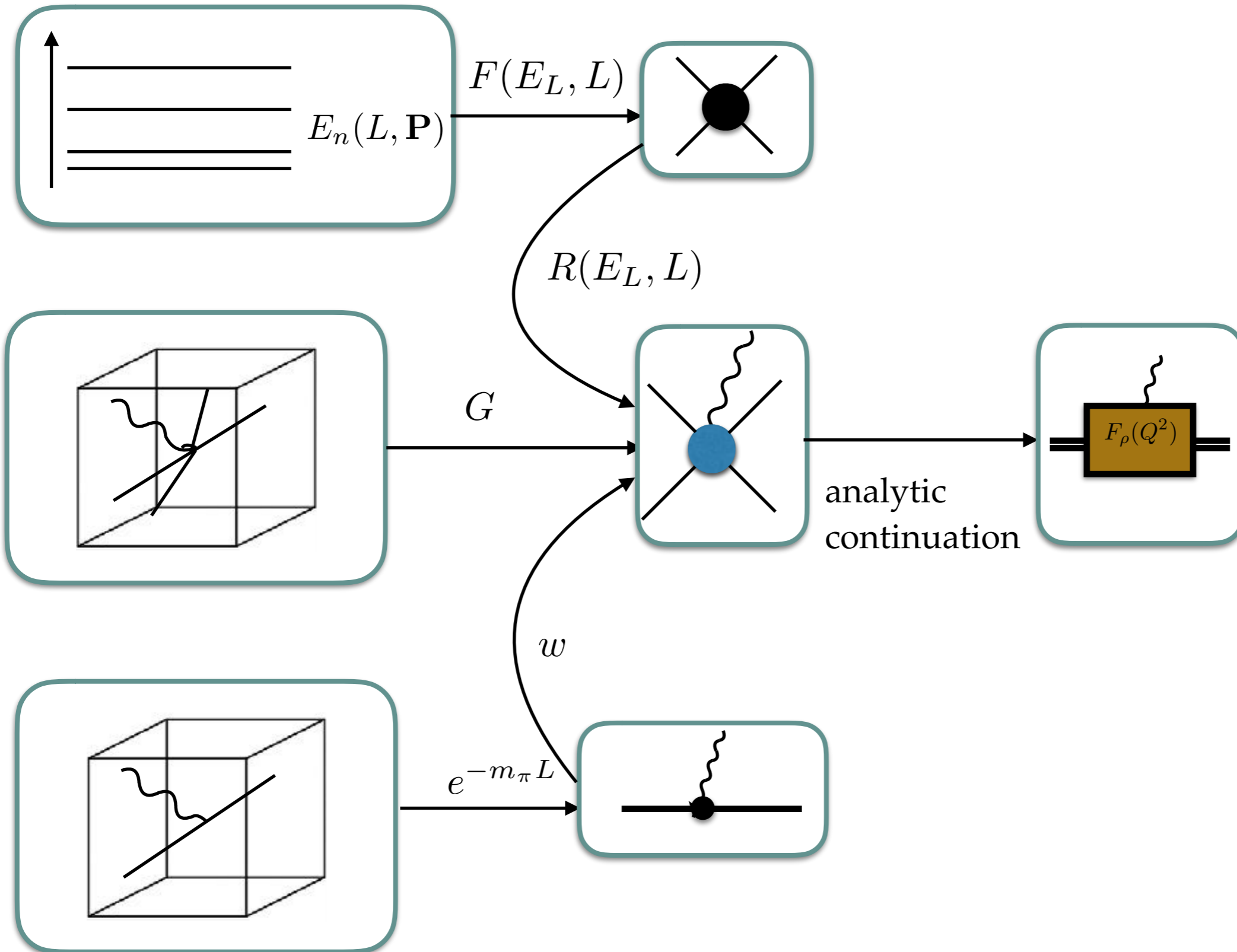
$E_{th}$  = threshold energy

$E_{MP}$  = multiparticle states energy

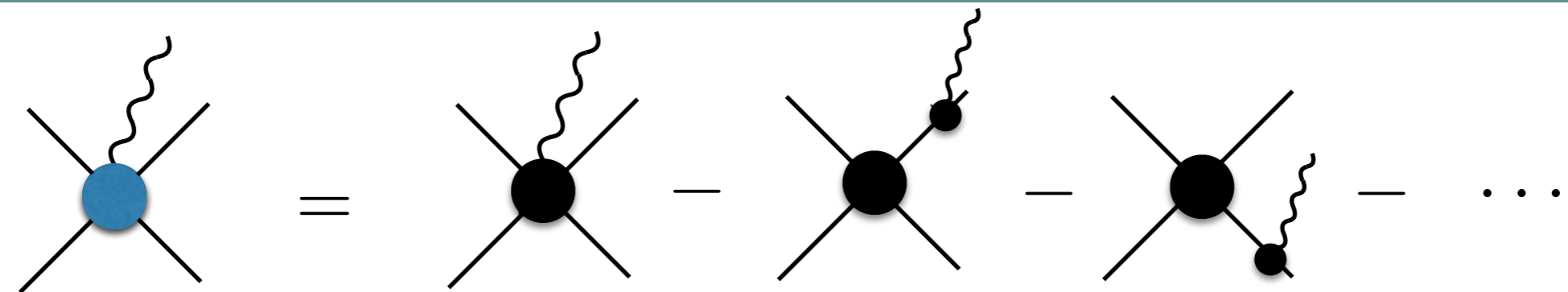


Singularities at free particle energies

# Workflow



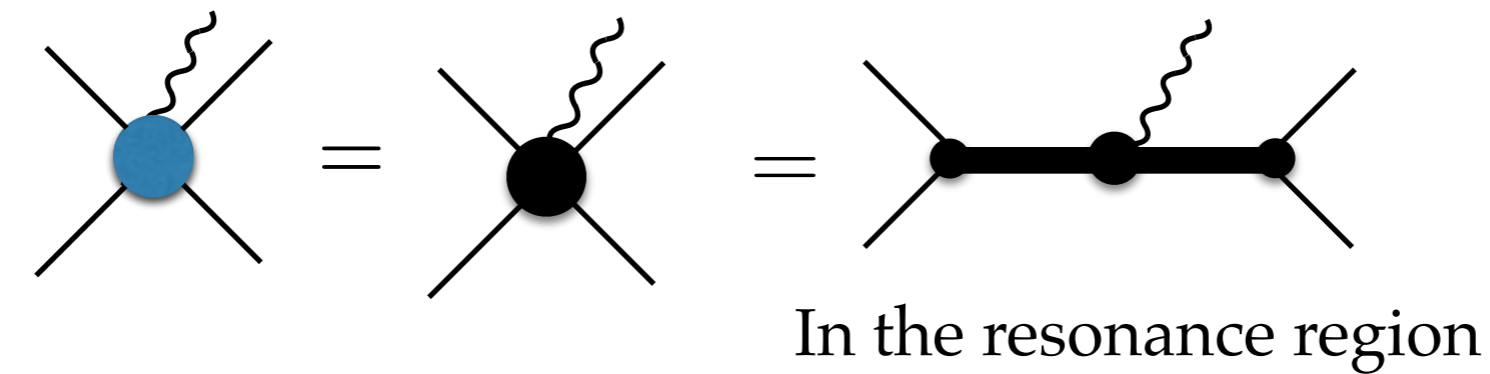
# A detail

$$W_{df} = \text{[Blue vertex]} = \text{[Black vertex]} - \text{[Black vertex with wavy line]} - \text{[Black vertex with wavy line]} - \dots$$
The diagram shows the expansion of a vertex  $W_{df}$ . On the left is a blue circle with four external lines and a wavy line. This is equal to a series of terms: a black circle with four external lines and a wavy line; minus a black circle with four external lines, a wavy line, and a small black dot on the top-right line; minus a black circle with four external lines, a wavy line, and a small black dot on the bottom-right line; and so on, indicated by an ellipsis.

Kinematic singularities not showing up in this limit

$$\lim_{E_i^{cm}, E_f^{cm} \rightarrow E_R} \text{[Blue vertex]} = \text{[Black vertex]} = \text{[Black line with three vertices and wavy line]}$$

In the resonance region

The diagram shows the limit of the vertex  $W_{df}$  as the center-of-mass energy  $E_i^{cm}, E_f^{cm}$  approaches the resonance energy  $E_R$ . The blue vertex is equal to the black vertex, which is then shown as a thick black horizontal line with three vertices and a wavy line attached to the middle vertex. This represents the resonance region.

# Some challenges

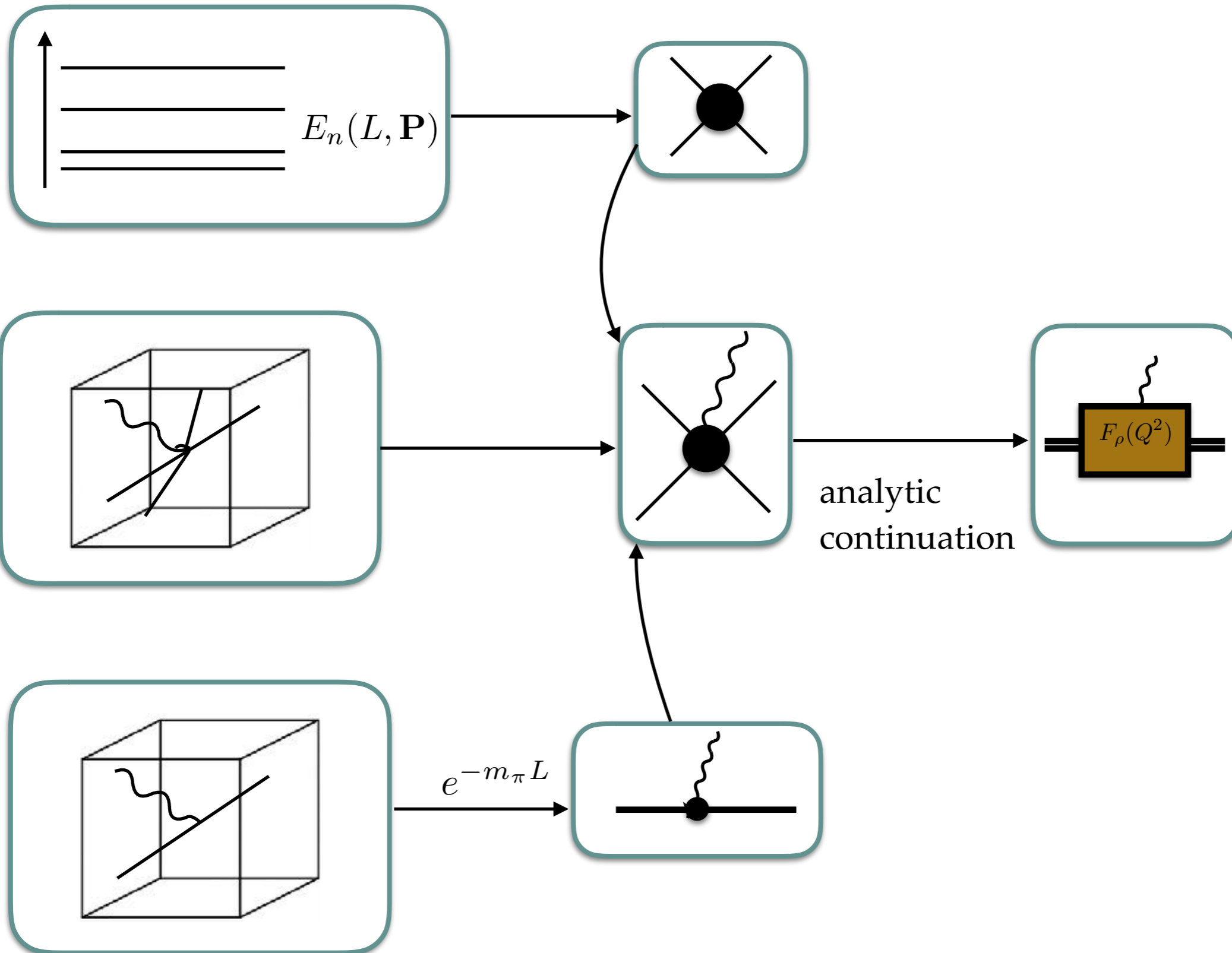
- A spin 1 particle between non-degenerate states has four form factors
- There is not a one-to-one mapping between matrix elements and amplitudes
  - Solved problem for spectrum analysis
- Analytical continuation of the amplitudes

Thank you!

Backup slides



# Workflow 101



# Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\text{FV}} \rightarrow \langle 2|\mathcal{J}|2\rangle_{\infty}$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves.....

- From  $\langle 2|\mathcal{J}|2\rangle_{\infty}$  how do we get the four form factors?
  - **Analytic continuation**