Elastic form factors for resonances from LQCD

Alessandro Baroni, University of South Carolina

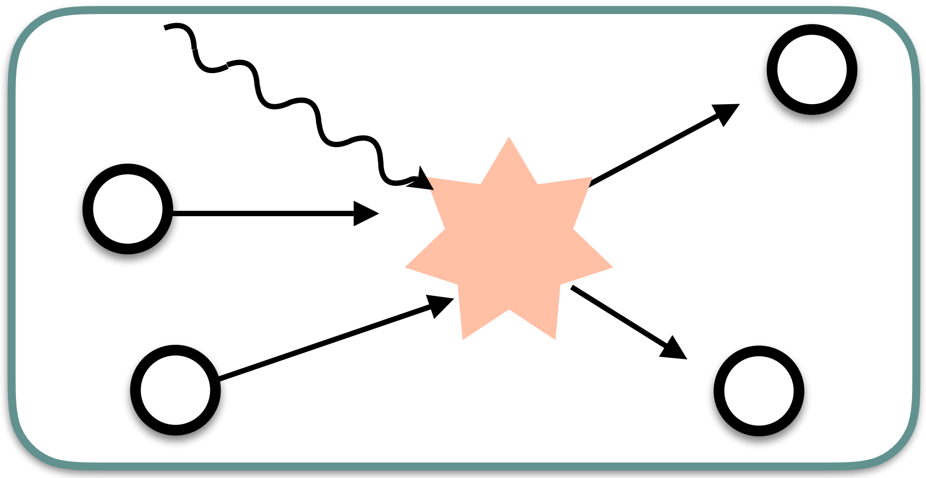
In collaboration with:

R. A. Briceño Jefferson Lab/Old Dominion University, USA
M.T. Hansen Cern, Switzerland
F. G. Ortega Perimeter Institute, Canada
D. J. Wilson Trinity College, Ireland

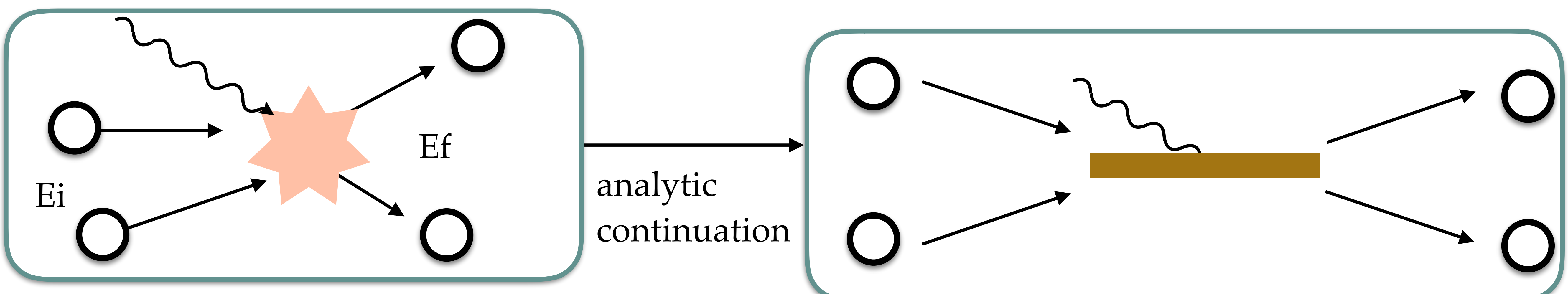
INT Workshop
Multi-Hadron Systems from LQCD

Form factor of a resonance

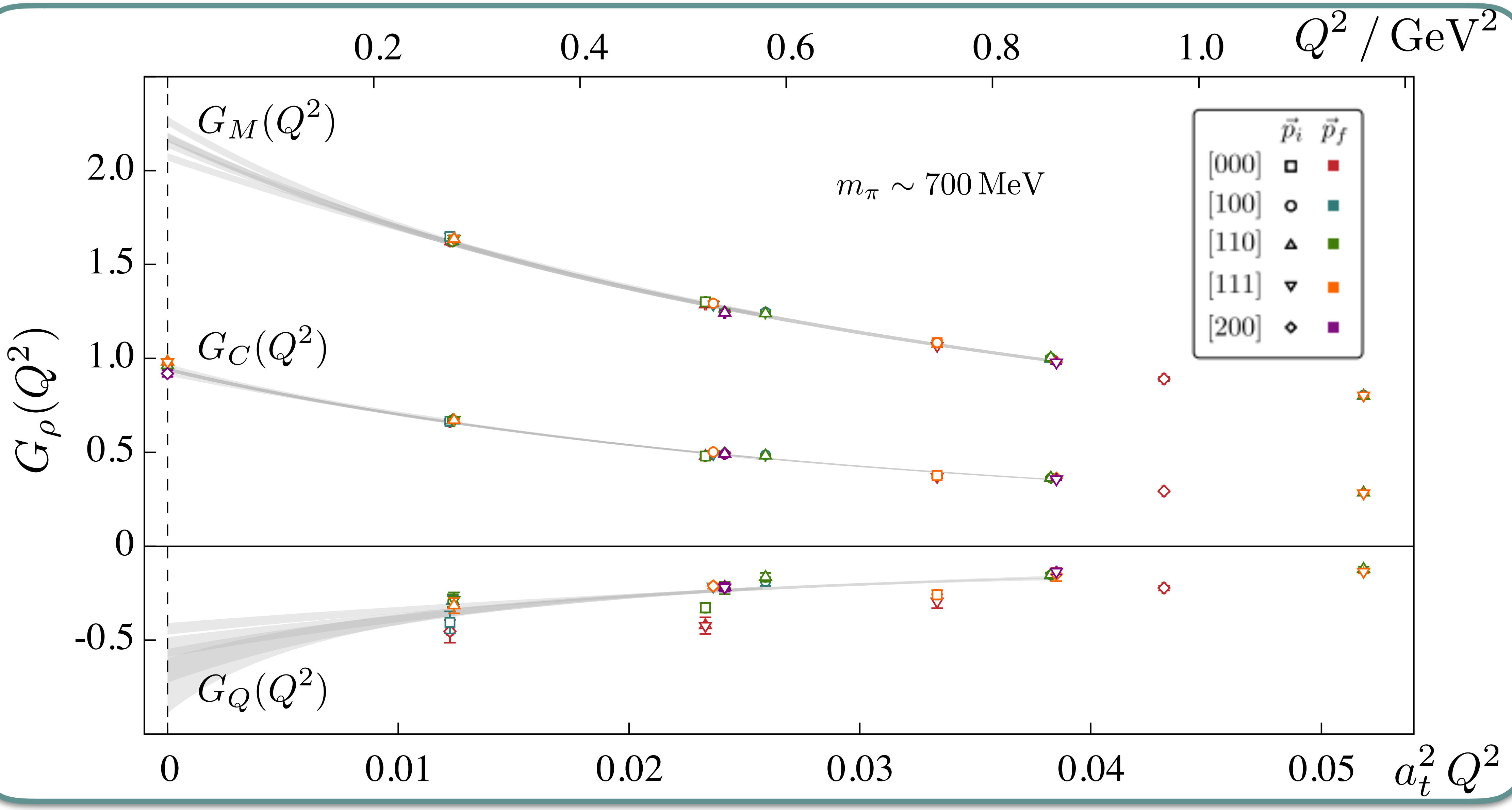
- Resonance not in the asymptotic states → Form factor?



- We can get the form factor in this way



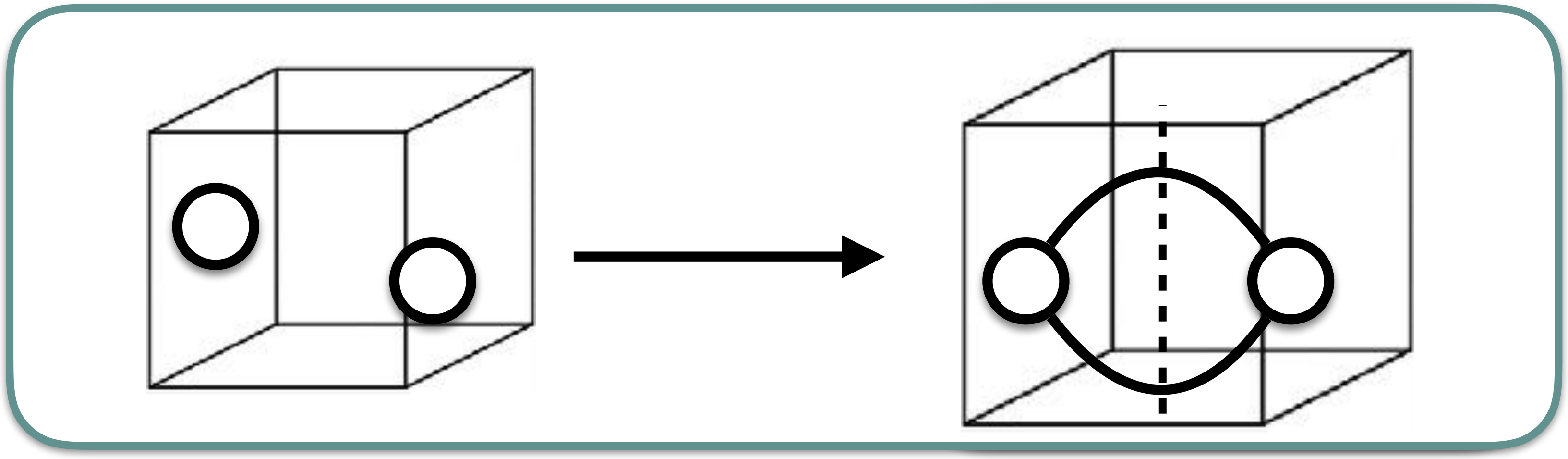
Form factor of a stable "resonance"



- Shultz, Dudek, and Edwards PRD (2015)

Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
  - On-shell intermediate states give singularities

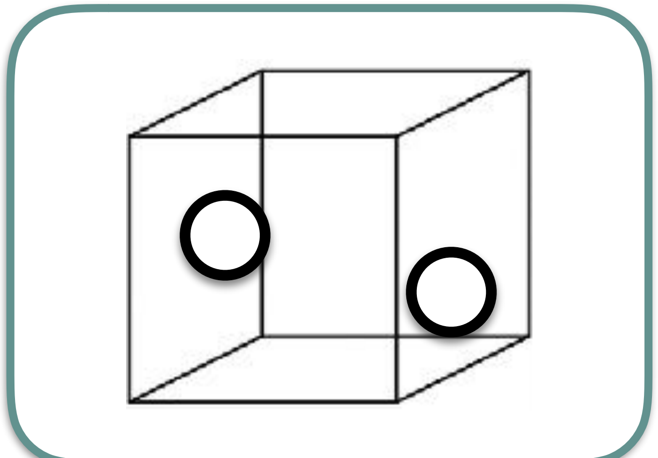


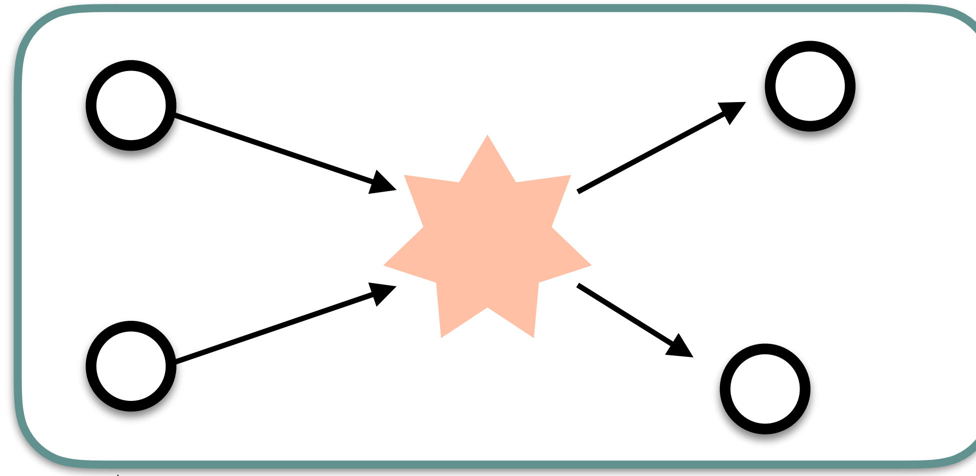
- Formalism needed to deal with these effects?

  - Briceño and Hansen (2016) - general, inelastic, relativistic
  - Rusetsky et al (2012) - EFT dependent, NR
  - Briceño and Davoudi (2012) - EFT dependent, NR

2→2

- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds
  → no other assumptions!!

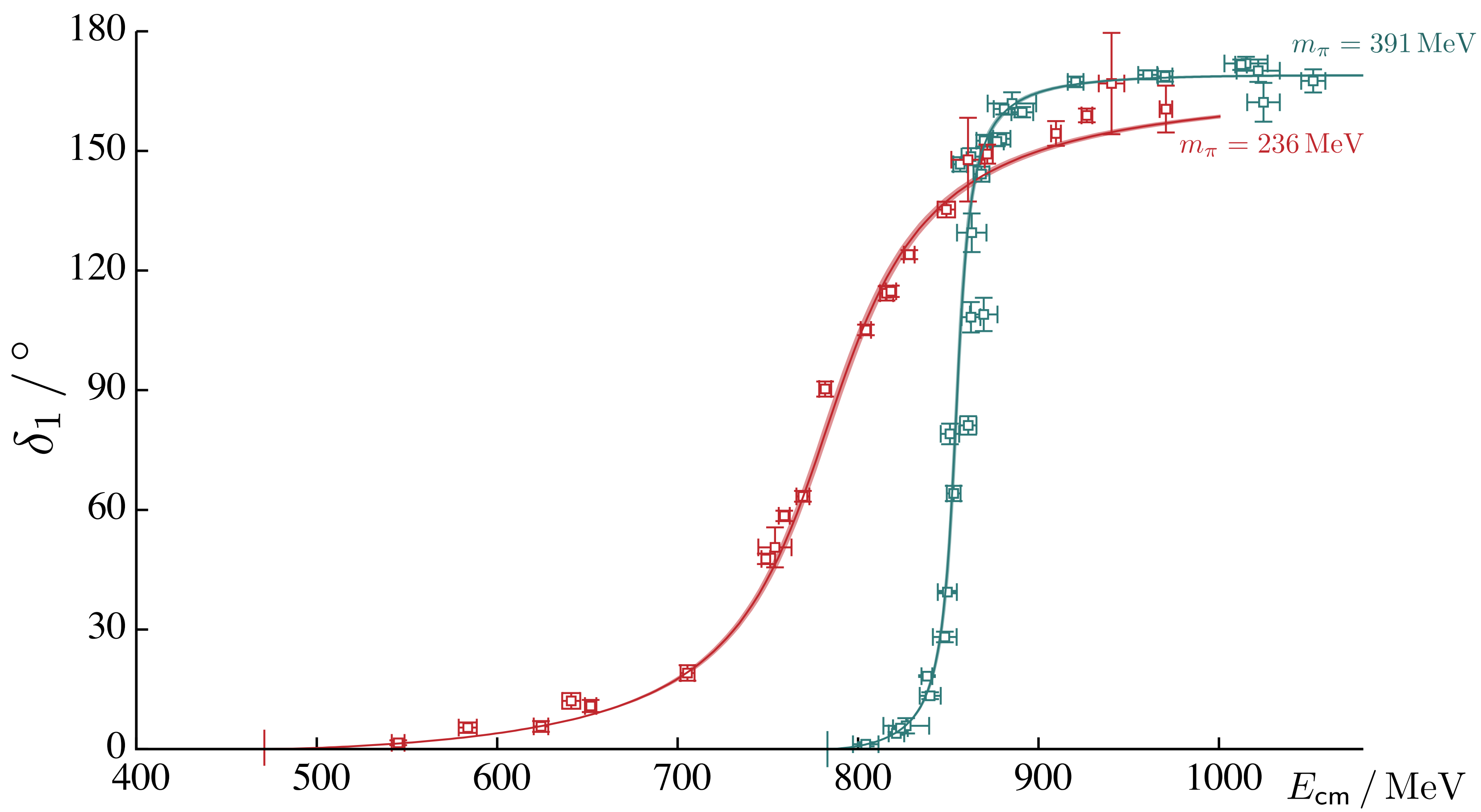




$$\det\left[F^{-1}(E_L, L) + M(E_L)\right] = 0$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]
- Briceño (2014) [general 2-body result]

2→2

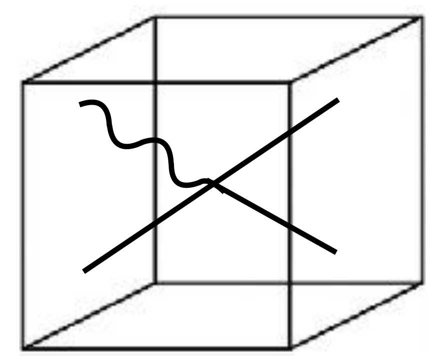


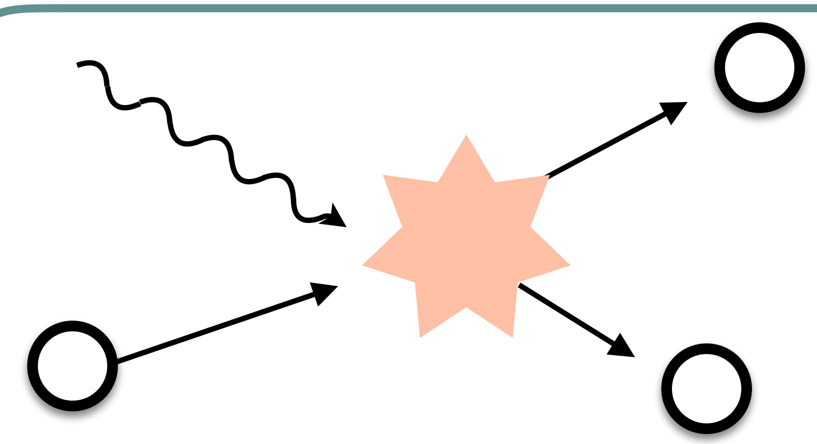
- Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$1+\mathcal{J}\to 2$

- FV matrix elements to infinite volume electroweak amplitudes

$$\lim_{L\to\infty}\langle 2|\mathcal{J}|1\rangle_L \neq \langle 2|\mathcal{J}|1\rangle_\infty$$





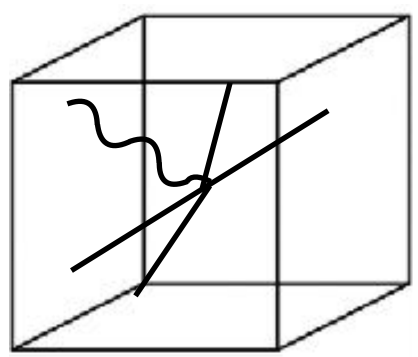
$$|\langle 2|\mathcal{J}|1\rangle|_L = \frac{1}{L^3}\sqrt{\langle 1|\mathcal{J}|2\rangle_\infty R(E_L, L)\langle 2|\mathcal{J}|1\rangle_\infty}$$

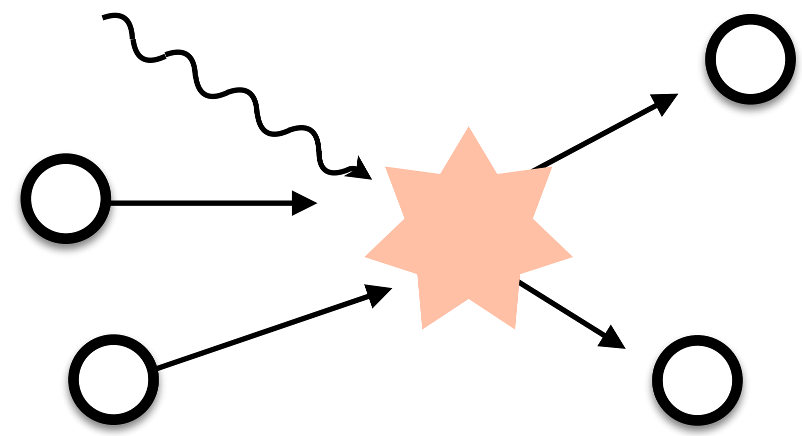
- Lellouch & Lüscher (2000) [K-to-ππ at rest]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [moving K-to-ππ]
- Hansen & Sharpe (2012) [D-to-ππ/KK]
- Briceño, Hansen Walker-Loud /Briceño & Hansen(2014-2015)[general 1-to-2]

$2+\mathcal{J}\rightarrow 2$

- FV matrix elements to infinite volume electroweak amplitudes

$$\lim_{L\to\infty}\langle 2|\mathcal{J}|2\rangle_L \neq \langle 2|\mathcal{J}|2\rangle_\infty$$

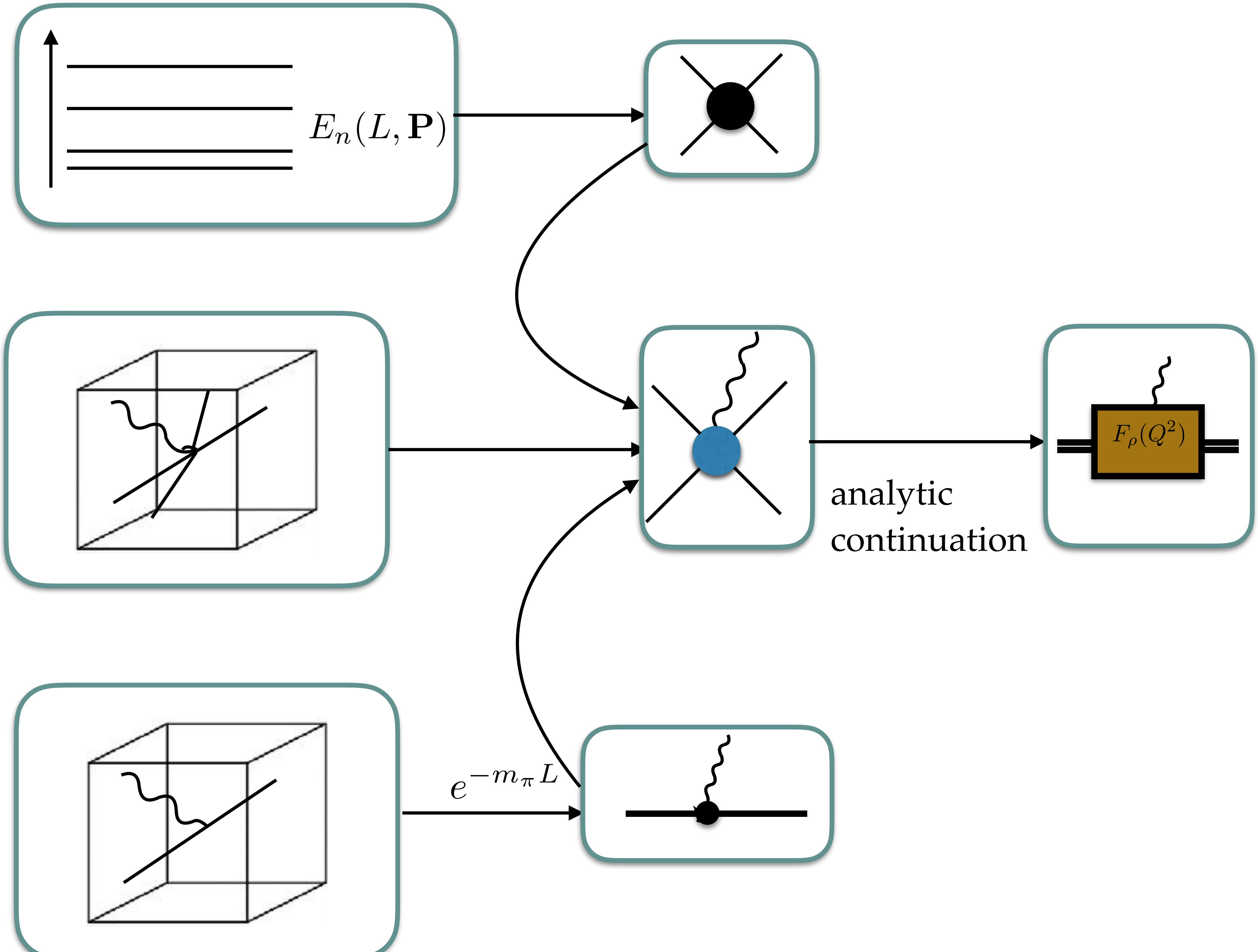




$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

- Briceño & Hansen (2016)

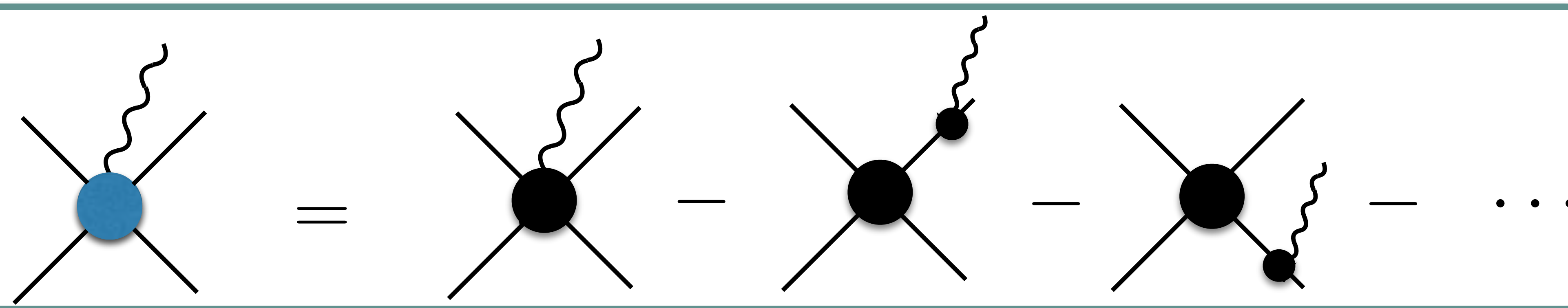
Workflow

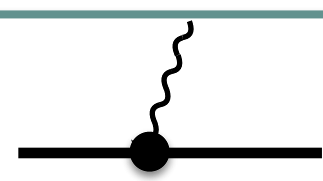


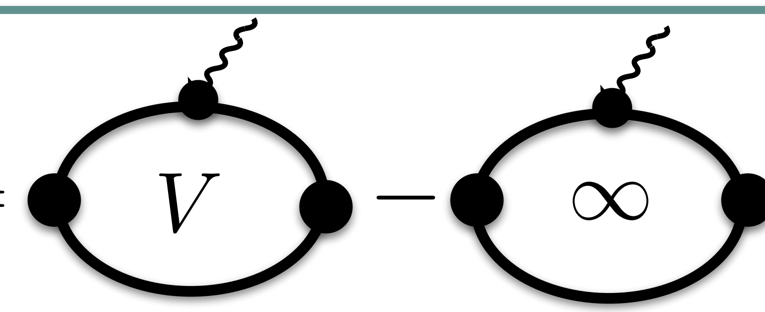
$2+\mathcal{J}\rightarrow 2$

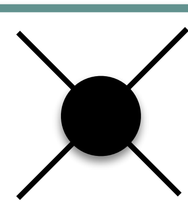
$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

$$W_{L,\mathrm{df}} = W_{\mathrm{df}} + MG(L,w)M$$

$W_{\mathrm{df}} =$ 

$w =$ 

$G(L,w) =$ 

$M =$ 

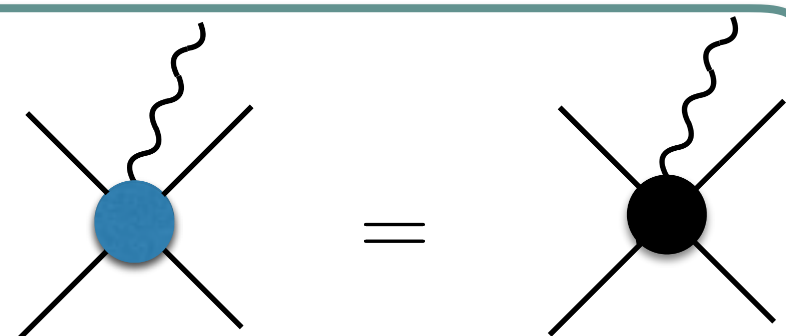
$2+\mathcal{J}\rightarrow 2$

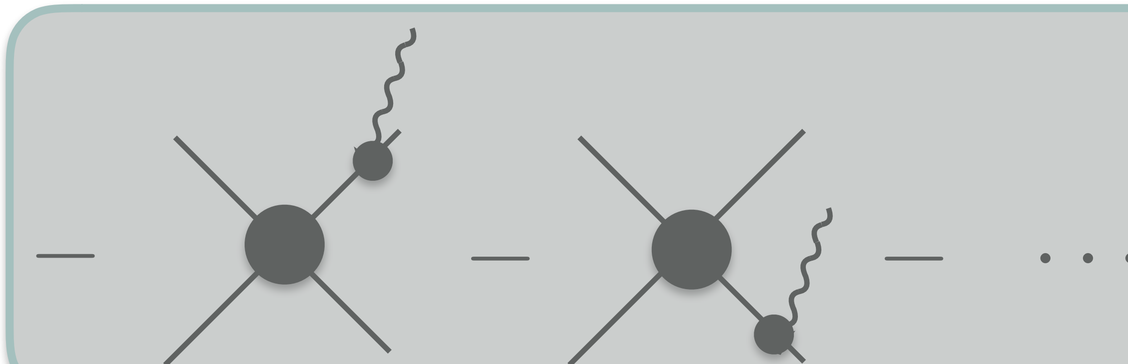
$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

$$W_{L,\mathrm{df}} = W_{\mathrm{df}} + MG(L,w)M$$

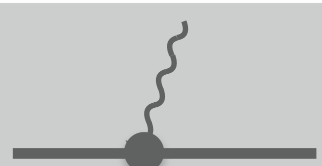
Naive Lellouch & Lüscher relation

$W_{\mathrm{df}} =$

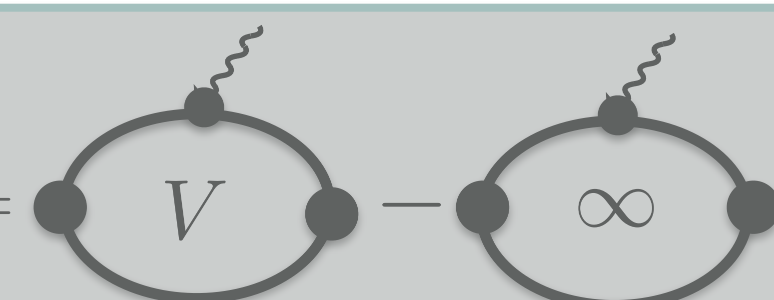




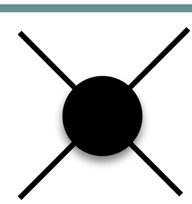
$w =$



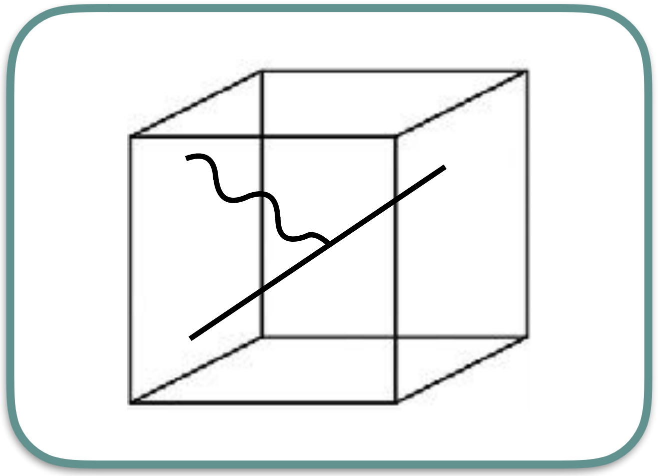
$G(L,w) =$

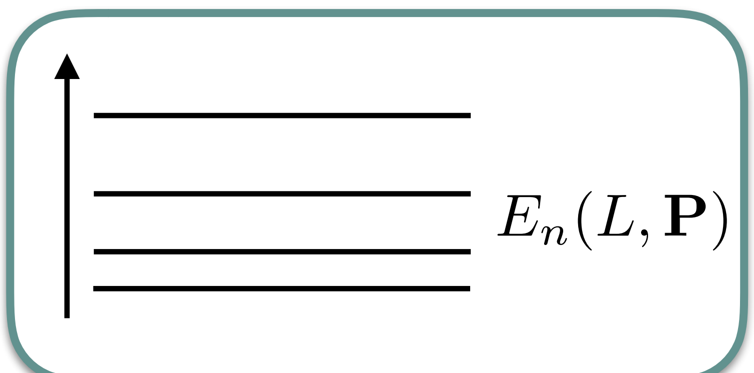


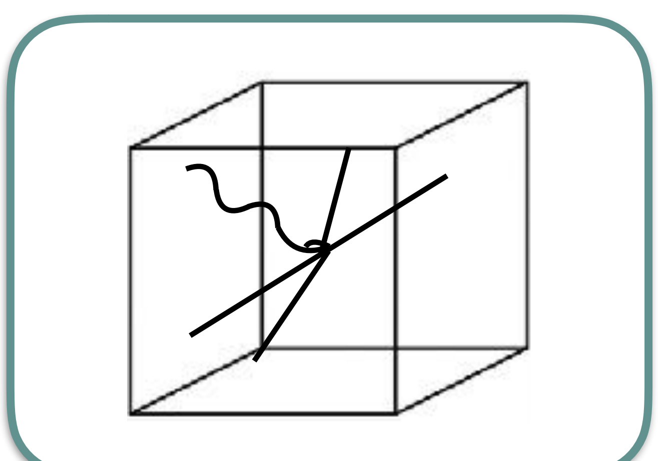
$M =$



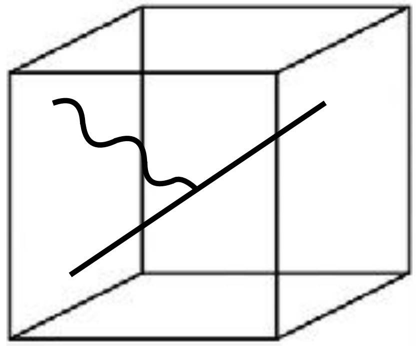
Ingredients of the calculation







Pion form factor

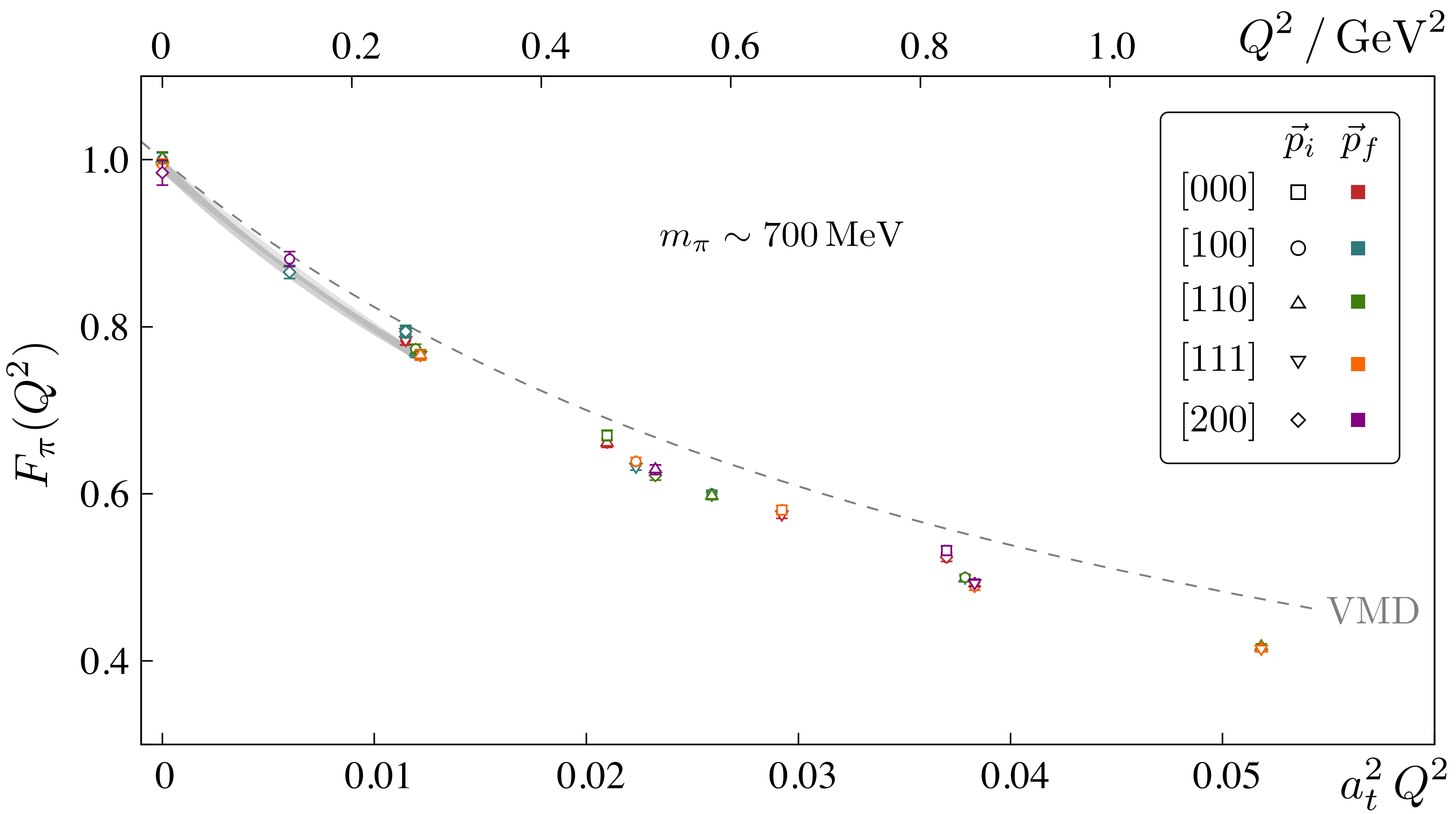


$$\langle \pi(p_f)|\mathcal{J}^\mu|\pi(p_i)\rangle =$$ Lorentz, T-reversal and P-invariance

$$= \left[(p_i + p_f)^\mu + (p_i - p_f)^\mu \frac{(m_{\pi'}^2 - m_\pi^2)}{Q^2}\right] F_{\pi\pi}(Q^2)$$

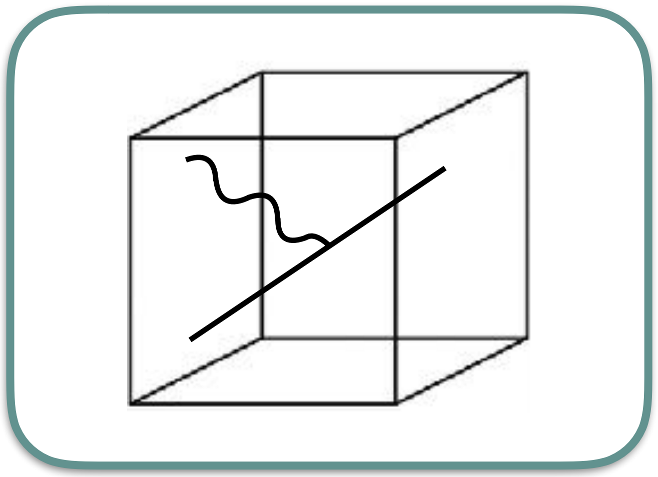
- Only one form factor
  - Obtained from the lattice

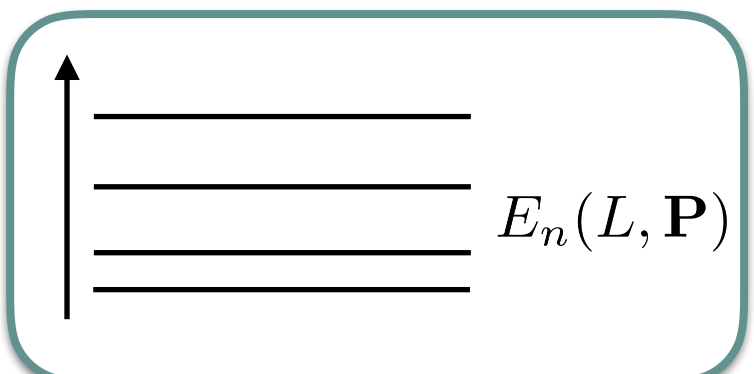
Pion form factor

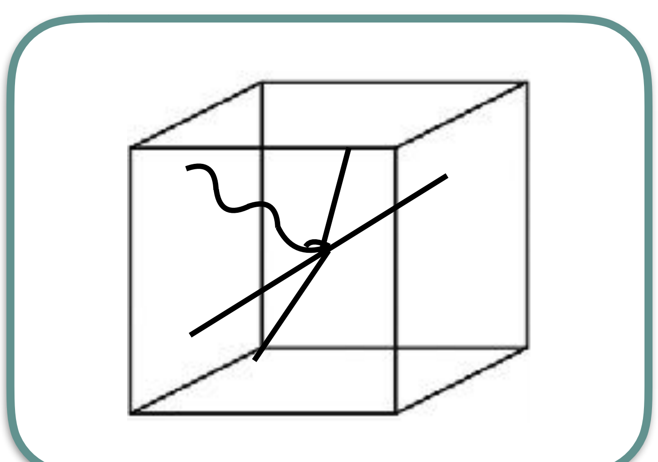


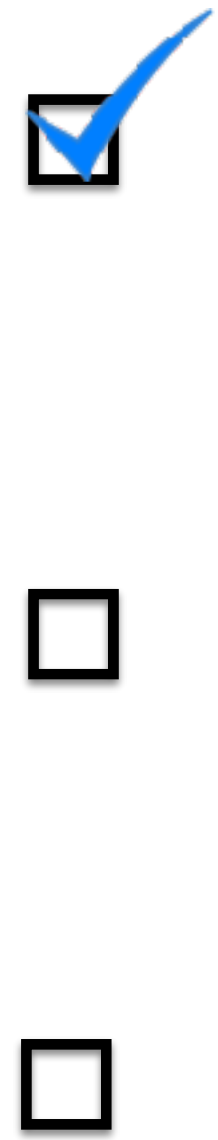
- Shultz, Dudek, and Edwards PRD (2015)

Ingredients of the calculation



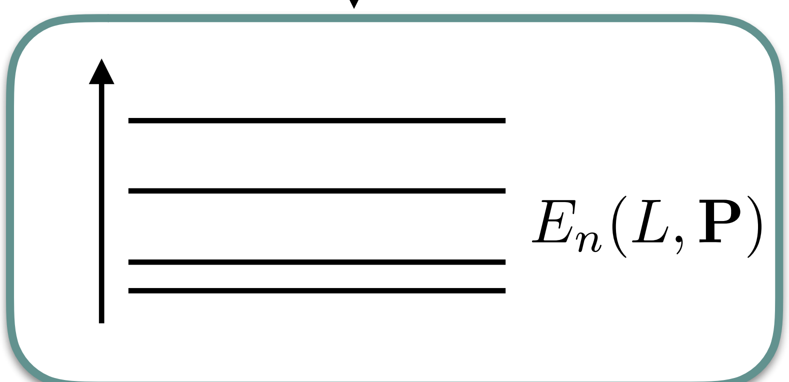


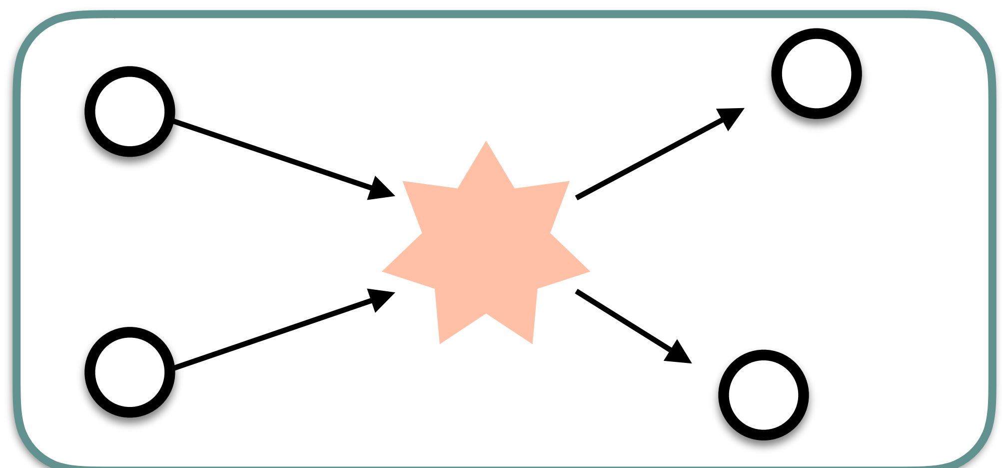




2→2

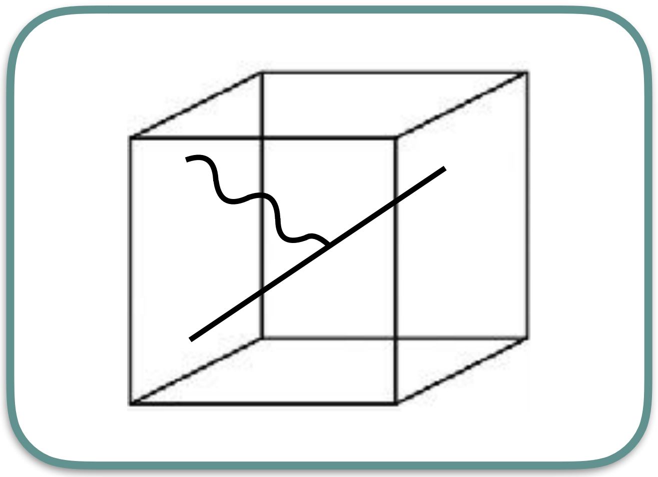
$$C^{2pt.}_{2\to2} = \langle 0|\mathcal{O}_f(\delta t)\mathcal{O}_i^\dagger(0)|0\rangle_L = \sum_n Z_{n,f} Z^\star_{n,i} e^{-E_n \delta t}$$

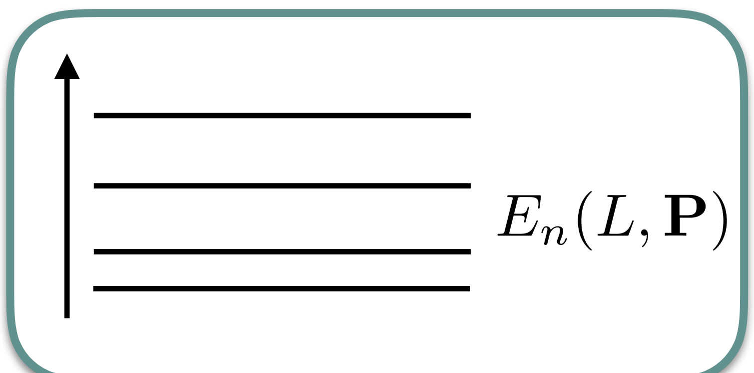


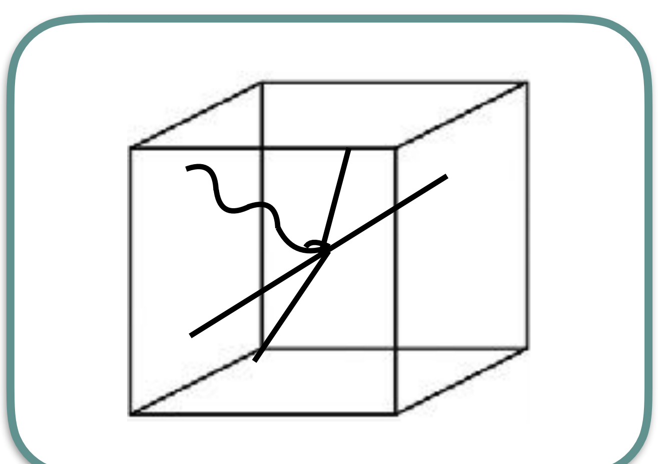


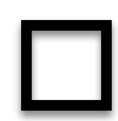
- Lüscher formalism

Ingredients of the calculation



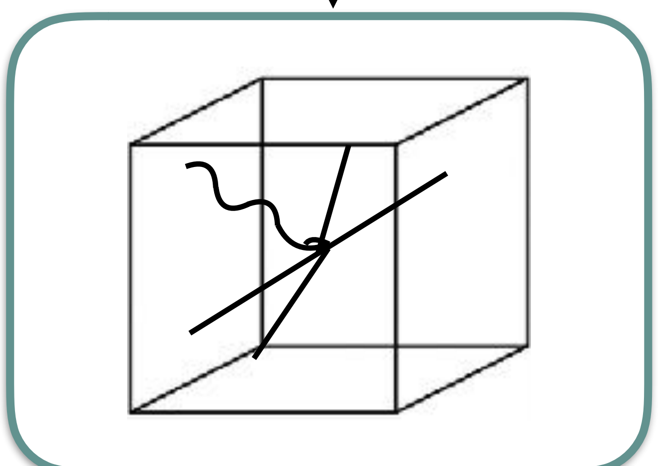


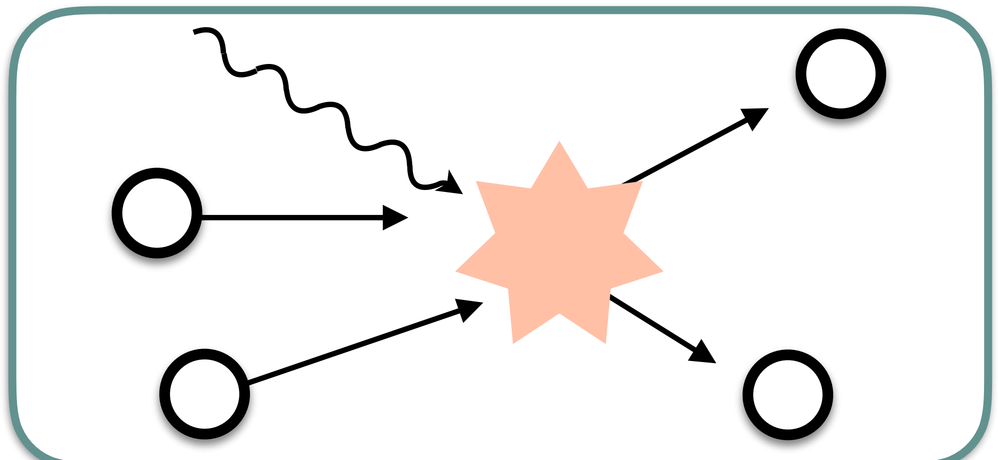




$2 + \mathcal{J} \to 2$

$$C^{3pt.}_{2+\mathcal{J}\to 2} = \langle 0|\mathcal{O}_f(\delta t)\mathcal{J}(t)\mathcal{O}^\dagger_i(0)|0\rangle_L = \sum_{n,n'} Z_{n,f} Z^\star_{n',i} e^{-(\delta t - t)E_n} e^{-tE'_n} \langle n|\mathcal{J}|n'\rangle_L$$



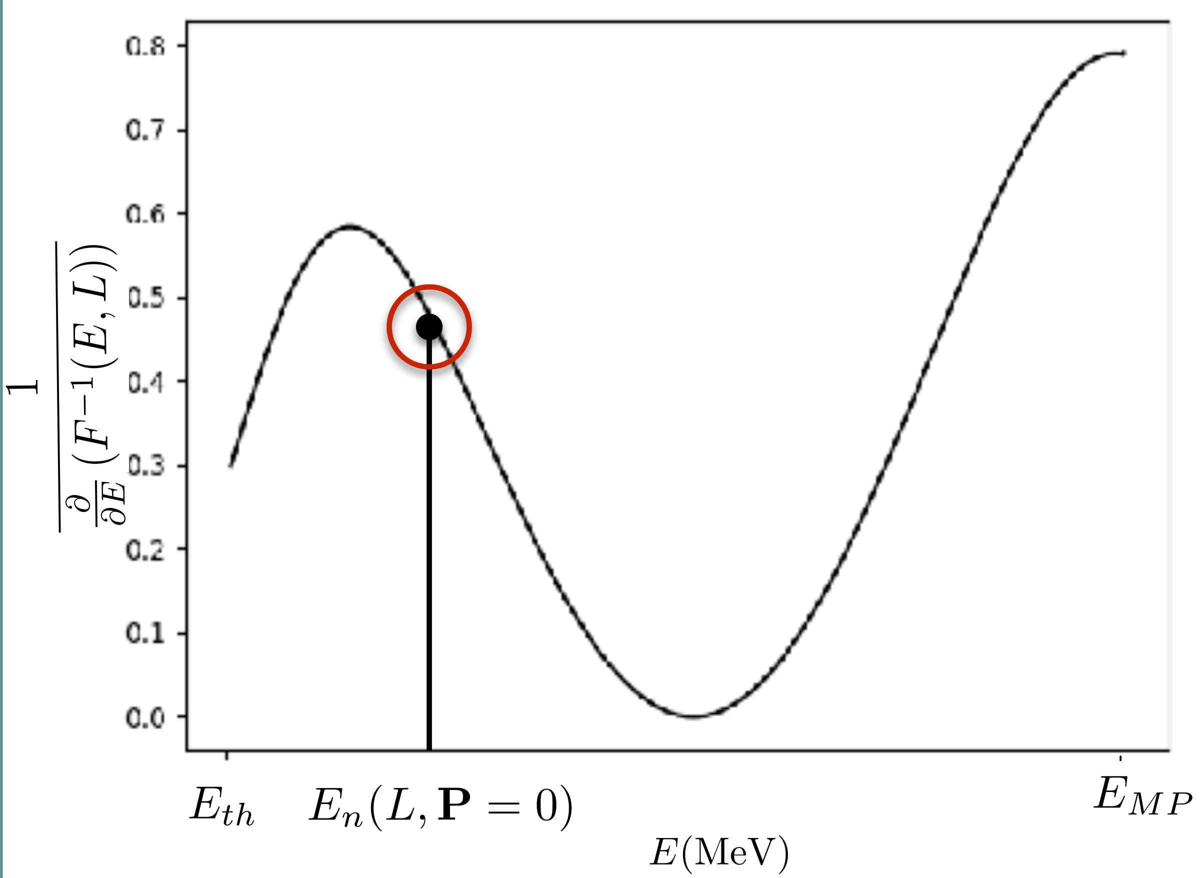


- Briceño & Hansen (2016)

Kinematic functions

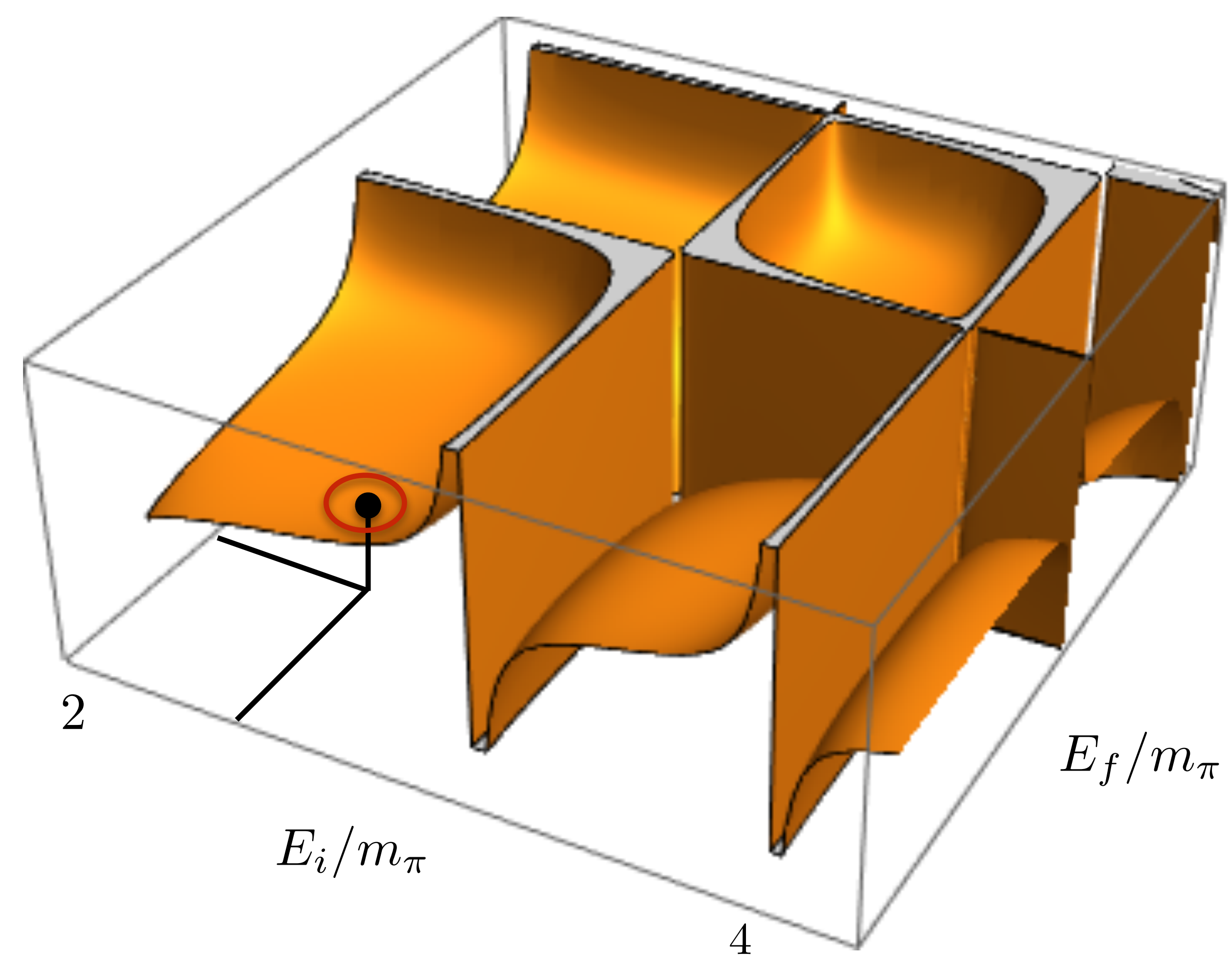
$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E}\left(F^{-1}(E,L)\right) + M(E))}\Bigg|_{E=E_L}$$

$$G(E_i, E_f, L) = \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int d\mathbf{k}\right](\cdots\cdots)$$



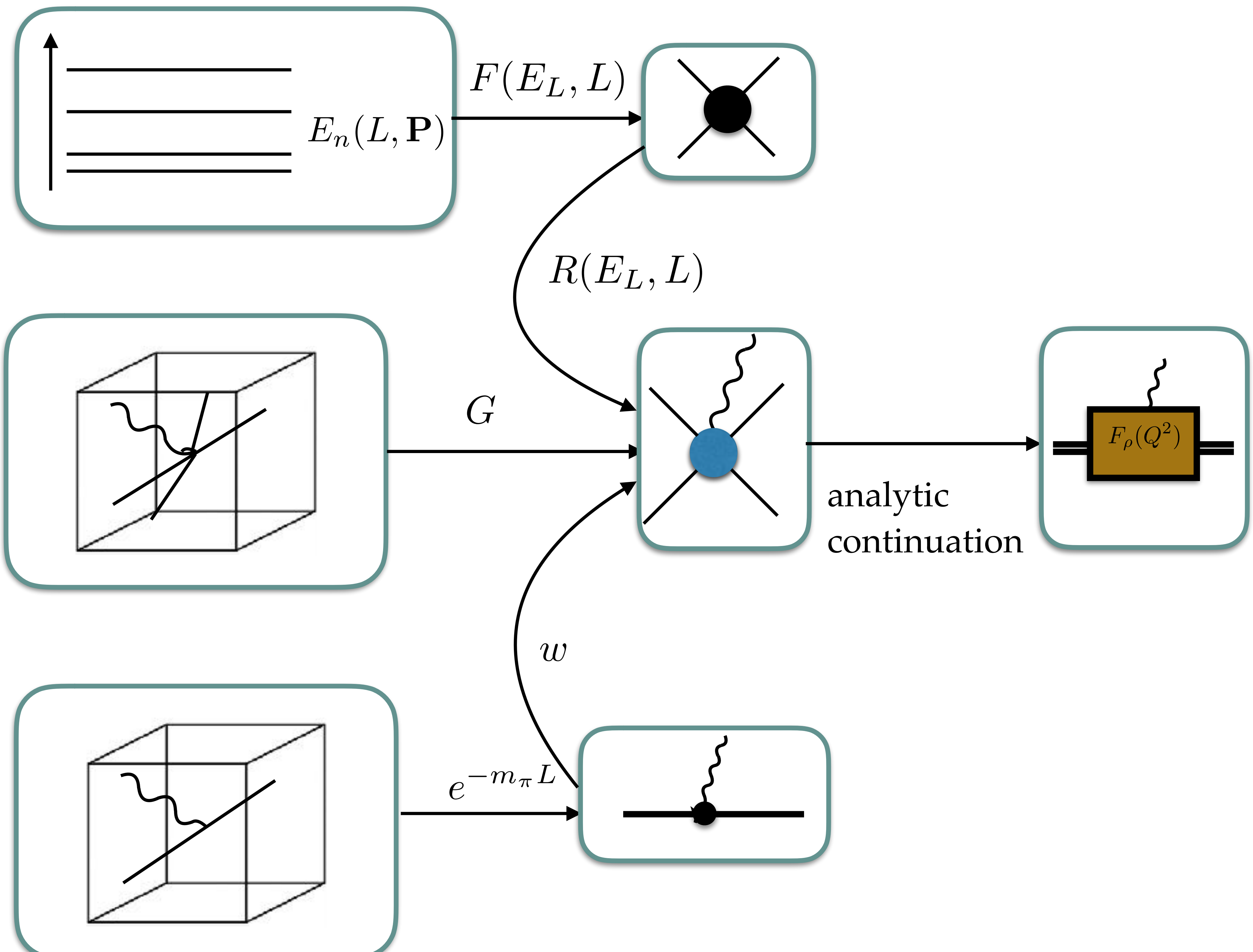
E_{th} = threshold energy

E_{MP} = multiparticle states energy



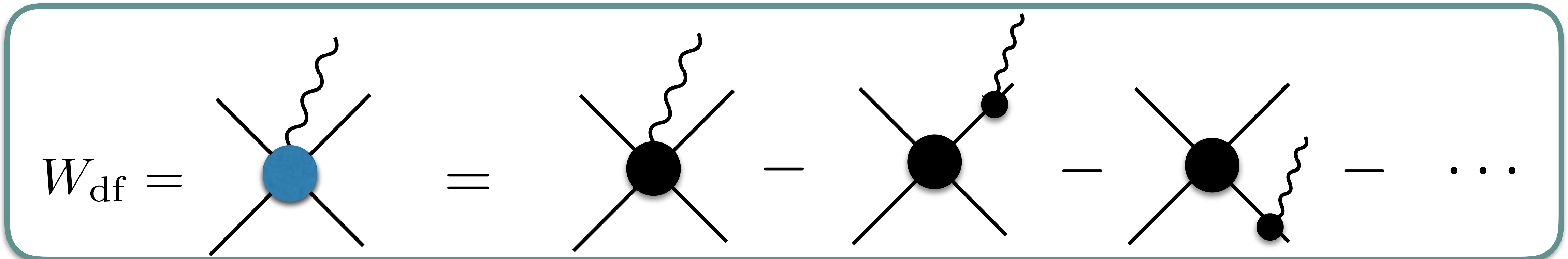
Singularities at free particle energies

Workflow

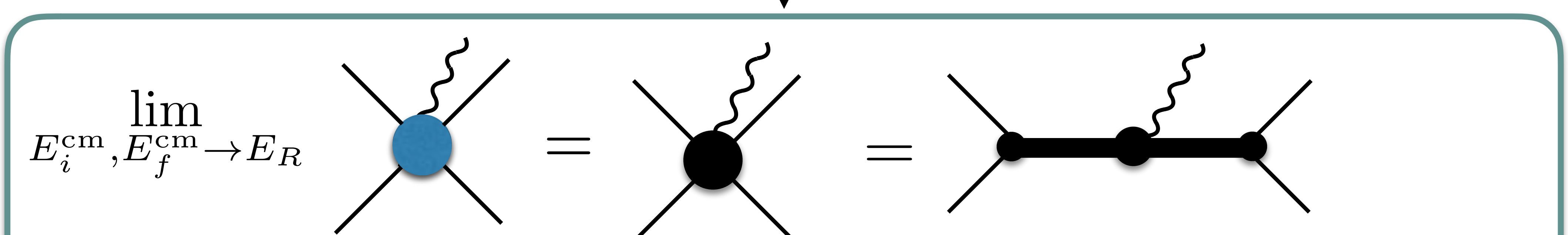


$E_n(L, \mathbf{P})$
$F(E_L, L)$
$R(E_L, L)$
G
w
$e^{-m_\pi L}$
$F_\rho(Q^2)$
analytic continuation

A detail



Kinematic singularities not showing up in this limit



In the resonance region

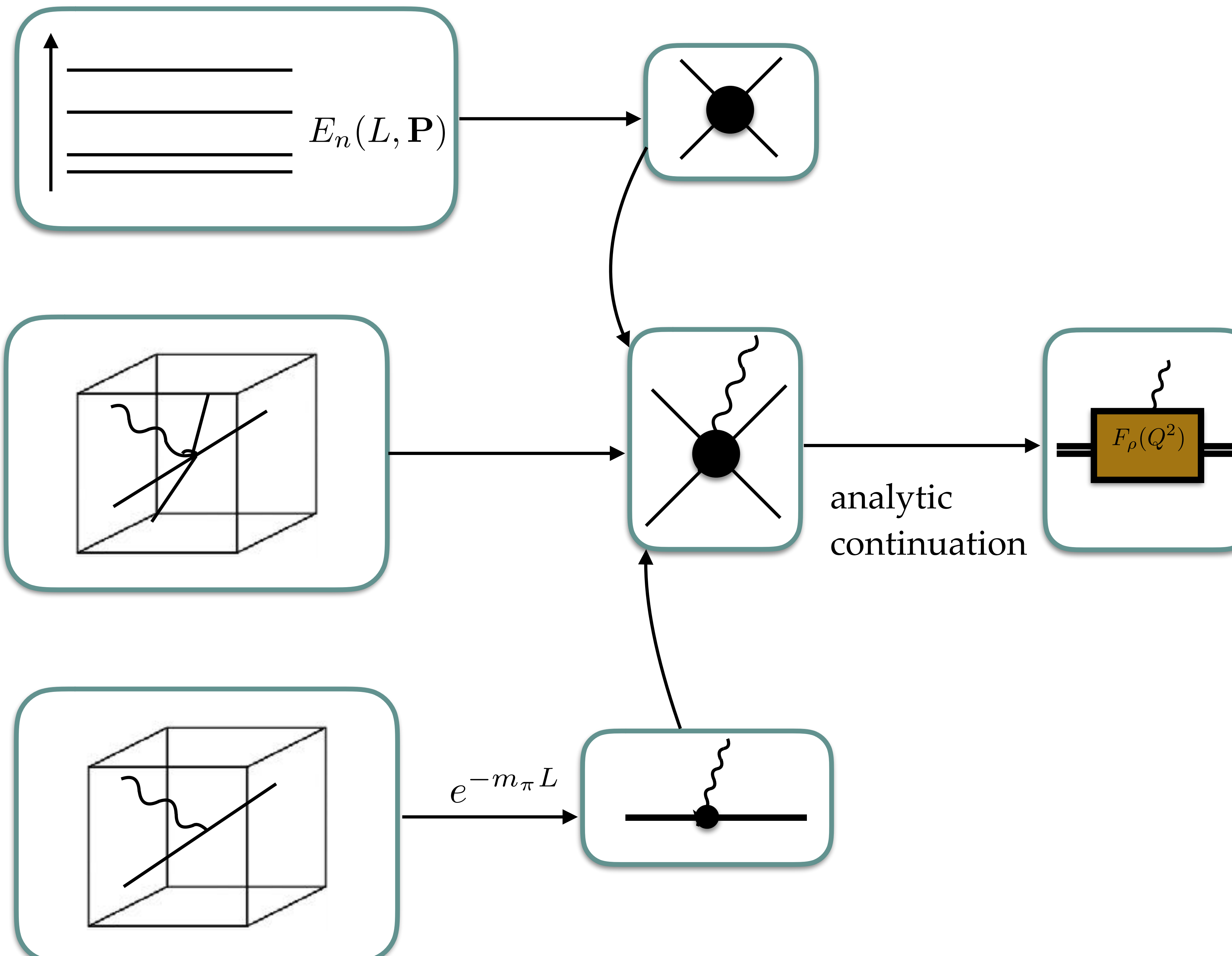
Some challenges

- A spin 1 particle between non-degenerate states has four form factors
- There is not a one-to-one mapping between matrix elements and amplitudes
    - Solved problem for spectrum analysis
- Analytical continuation of the amplitudes

Thank you!

Backup slides

Workflow 101



Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\rm FV} \to \langle 2|\mathcal{J}|2\rangle_{\infty}$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves…..

- From $\langle 2|\mathcal{J}|2\rangle_{\infty}$ how do we get the four form factors?
  - Analytic continuation