The fake plateau problem and normality checks in the direct method for two baryon systems

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For
HAL QCD Collaboration

Hadrons to Atomic nuclei HAL from Lattice QCD

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I. Fundamental issues - Fake plateau problem -

II. Operator dependences - A sign of fake plateau I -

III. Normality (sanity) check - A sign of fake plateau II -

I. Fundamental issue

- The fake plateau problem -

T. Iritani et al. (HAL QCD), JHEP1610(2016)101 (arXiv:1607.06371)

The fake plateau problem

Modeling $m_\pi \simeq 500\ \mathrm{MeV},\ m_N \simeq 2\ \mathrm{GeV},\ L \simeq 4\ \mathrm{fm}$

$$R(t) = e^{-\Delta E t}\left(1 + b\ e^{-\delta E_{\mathrm{el}} t} + c\ e^{-\delta E_{\mathrm{inel}} t}\right)$$

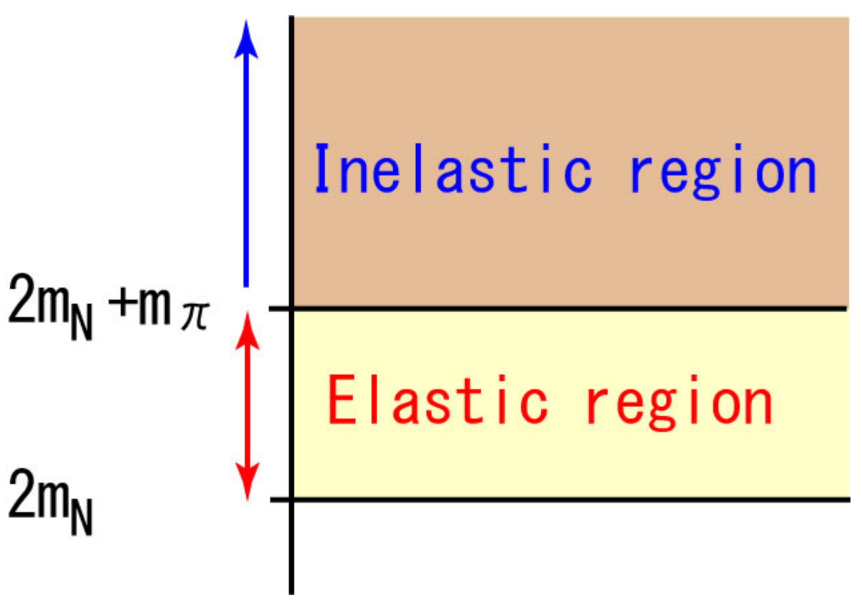
$\delta E_{\mathrm{el}} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

$\delta E_{\mathrm{el}} = 50\ \mathrm{MeV}\ \mathrm{at}\ L \simeq 4\ \mathrm{fm}$ $\frac{1}{\delta E_{\mathrm{el}}} \simeq 4\ \mathrm{fm}$

$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

$\delta E_{\mathrm{inel}} = 500\ \mathrm{MeV}$ the inelastic energy from heavy pions

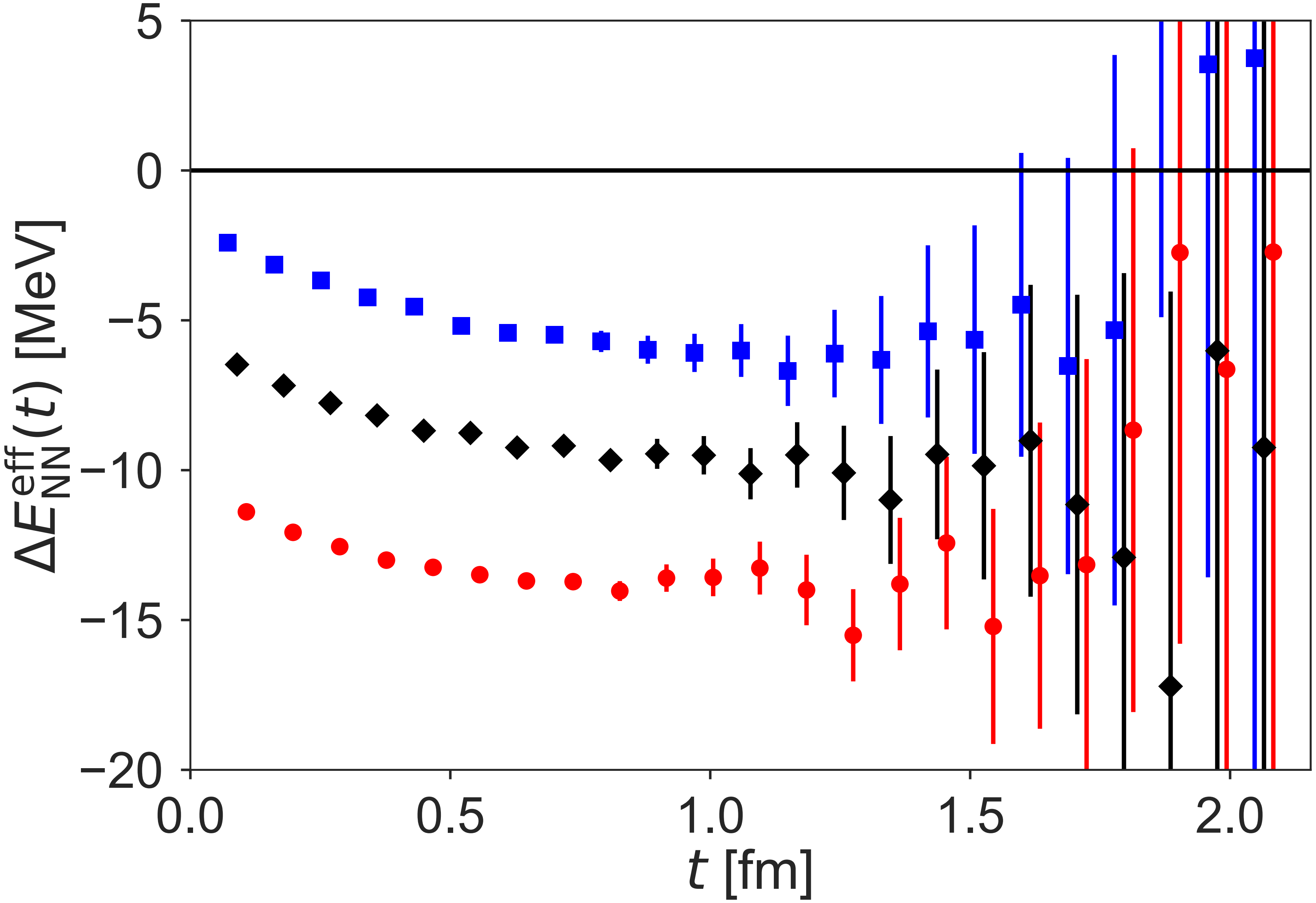
$c = 0.01$ 1% contamination $\frac{1}{\delta E_{\mathrm{inel}}} \simeq 0.4\ \mathrm{fm}$



+ add errors and fluctuations increasing with t

The fake plateau problem

$$\Delta E^{\rm eff}(t) = -\frac{1}{a}\log\frac{R(t+a)}{R(t)}$$



“Plateaux” at t ~ 1 fm but some are fake.

One can not tell which is correct by its plateau behavior at small t.

Response by Z. Davoudi (arXiv:1711.02020 [hep-lat])

``What is misleading about this conclusion is that while for generic interpolating operators ("sources" or "sinks") an O (1) overlap to all states in the volume is plausible (thus enforcing the estimates given above), with physically-motivated source and/ or sink operators, the exponential degradation of the signal for the ground state can be compensated by a large overlap factor to the ground state, pushing the start of the single-exponential region in the correlation functions to much earlier times than the naive estimates."

Our comments

Since the plateau behavior at small t can not tell whether it is true, ``optimizing" operator for the early plateau does not necessarily give a truly optimized operator.

To optimize operator can easily produce the fake plateau, which may fool you.

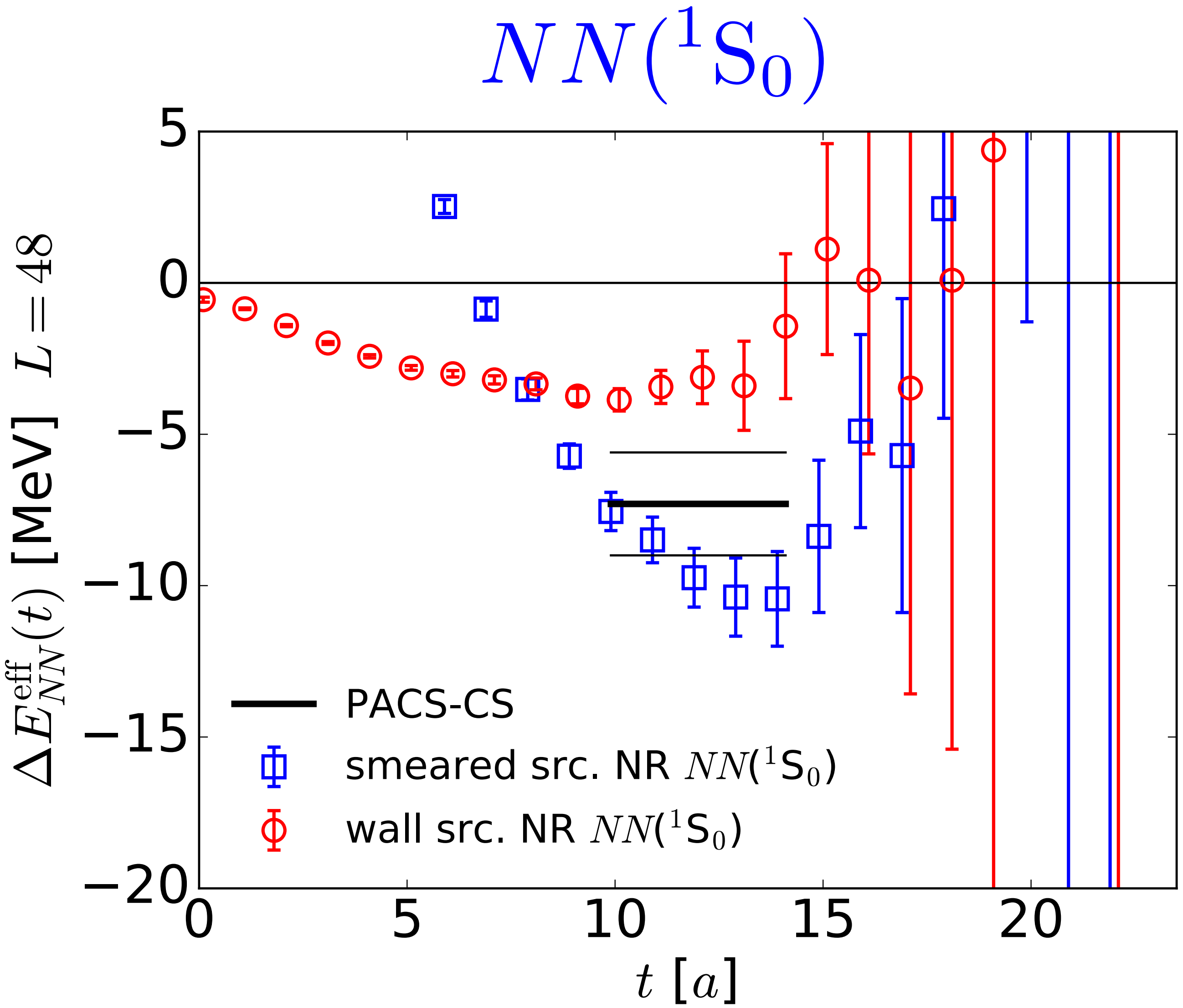
The ``plateau" is NOT enough.

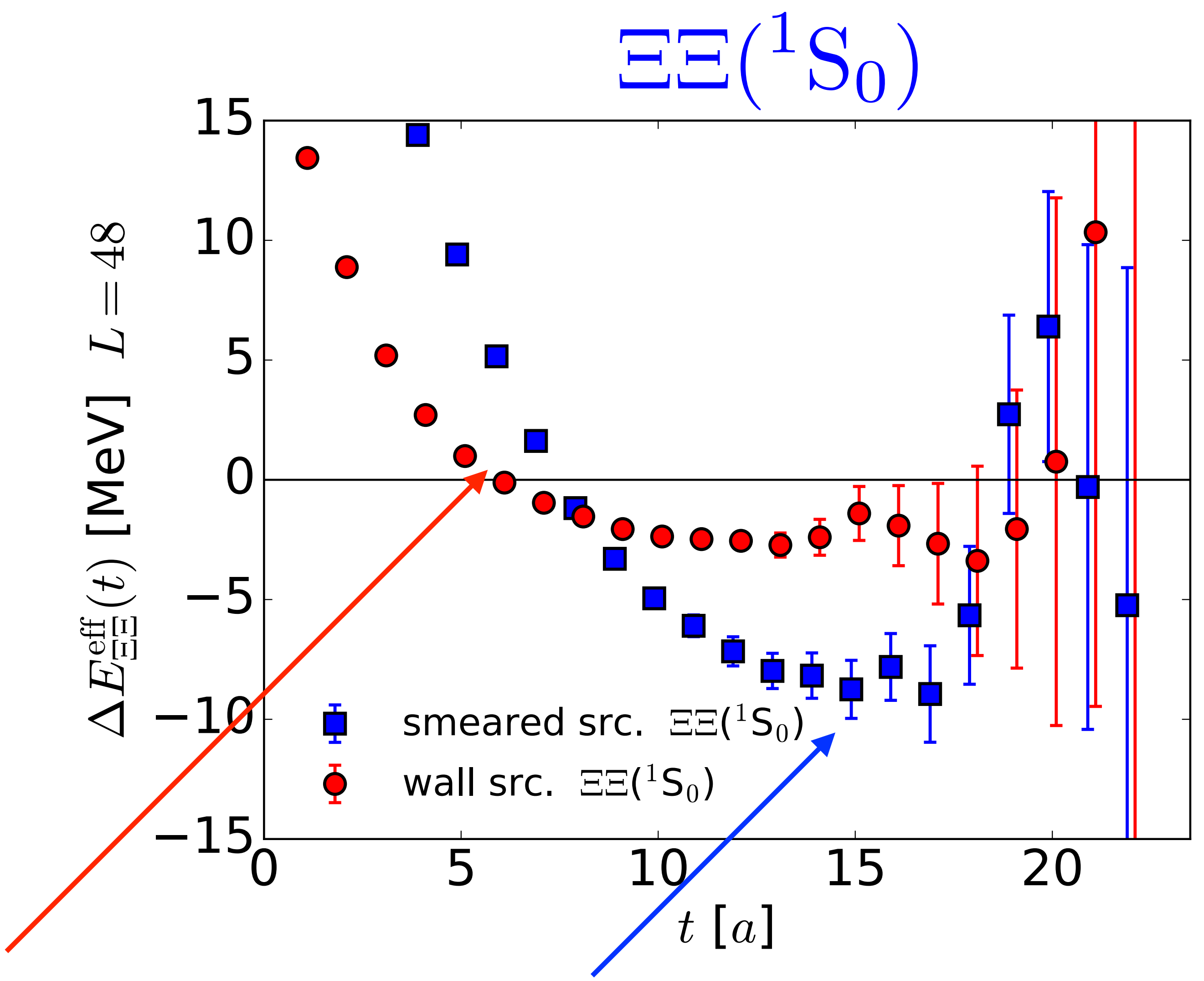
II. Operator dependences

- A sign of the fake plateau I -

T. Iritani et al. (HAL QCD), JHEP1610(2016)101 (arXiv:1607.06371)

Source operator dependence



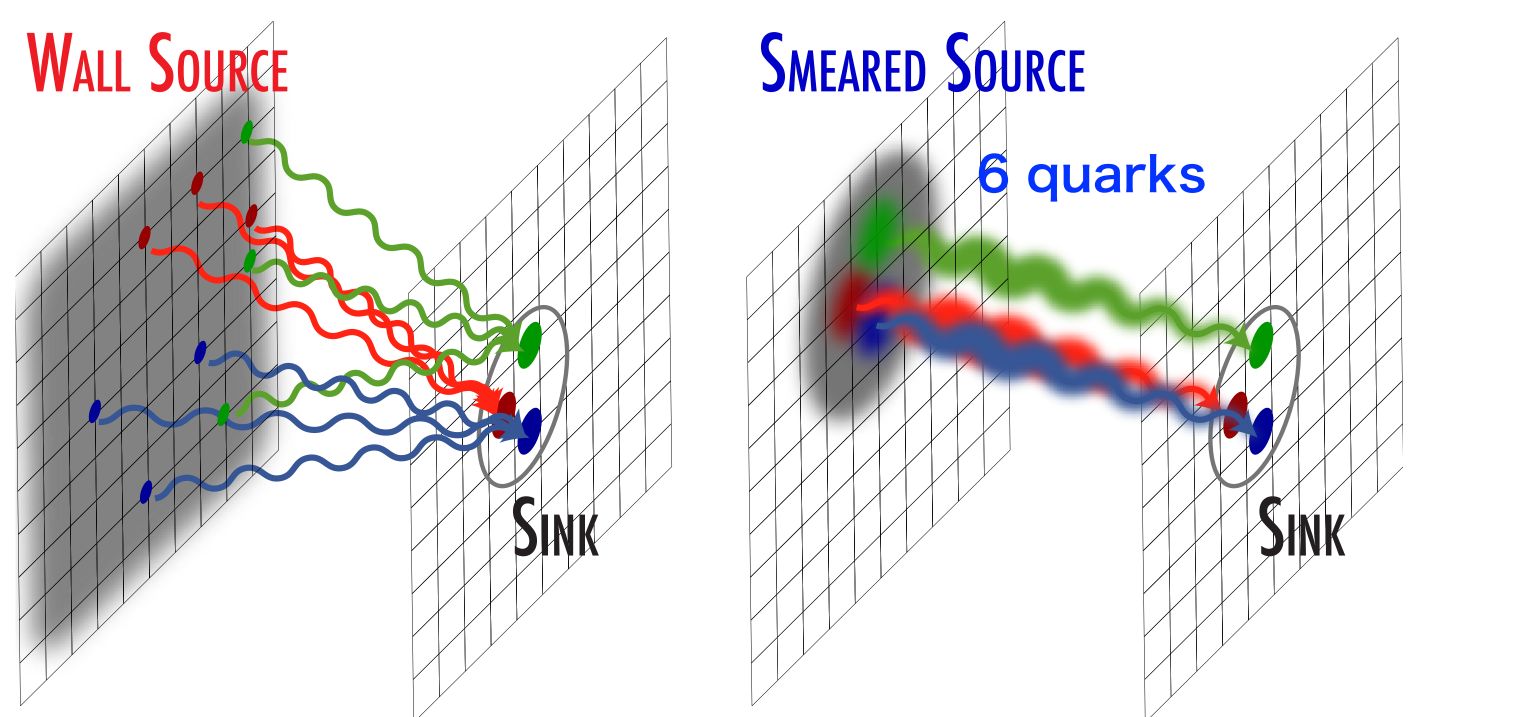


2+1 flavor QCD

$a = 0.09$ fm ($a^{-1} = 2.2$ GeV)

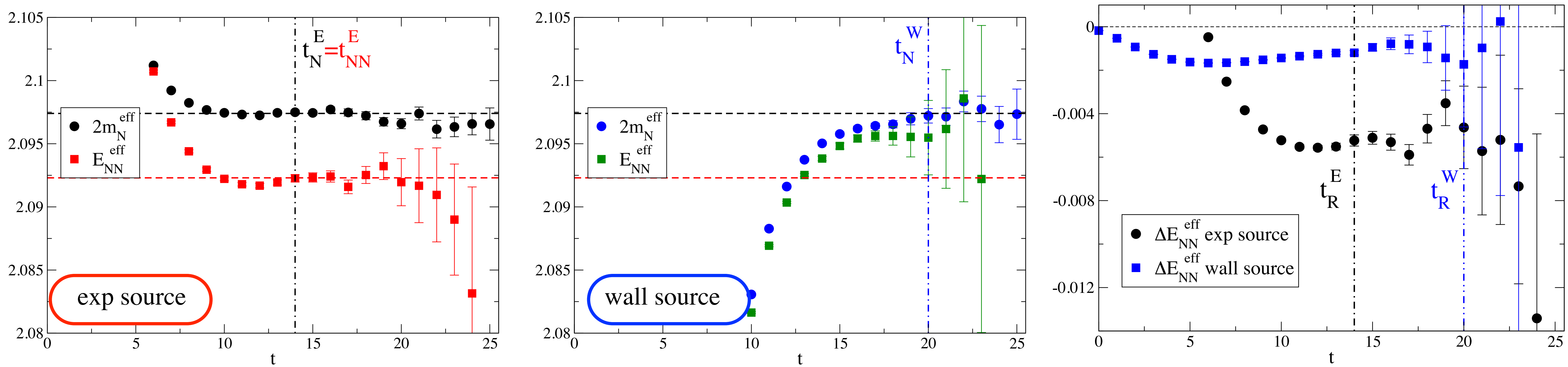
$m_\pi = 510$ MeV, $m_N = 1320$ MeV

same gauge configurations of YIKU 2012



Response by YIK2017 (arXiv:1710.08066 [hep-lat])

$N_f = 0,\ m_\pi \simeq 800\ \text{MeV},\ a^{-1} \simeq 1.54\ \text{GeV}$



The plateau of wall source at small $t < t_R = \max(t_N, t_{NN})$ is unreliable. Both agree at $t > t_R$.

No operator dependence

Our comments

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

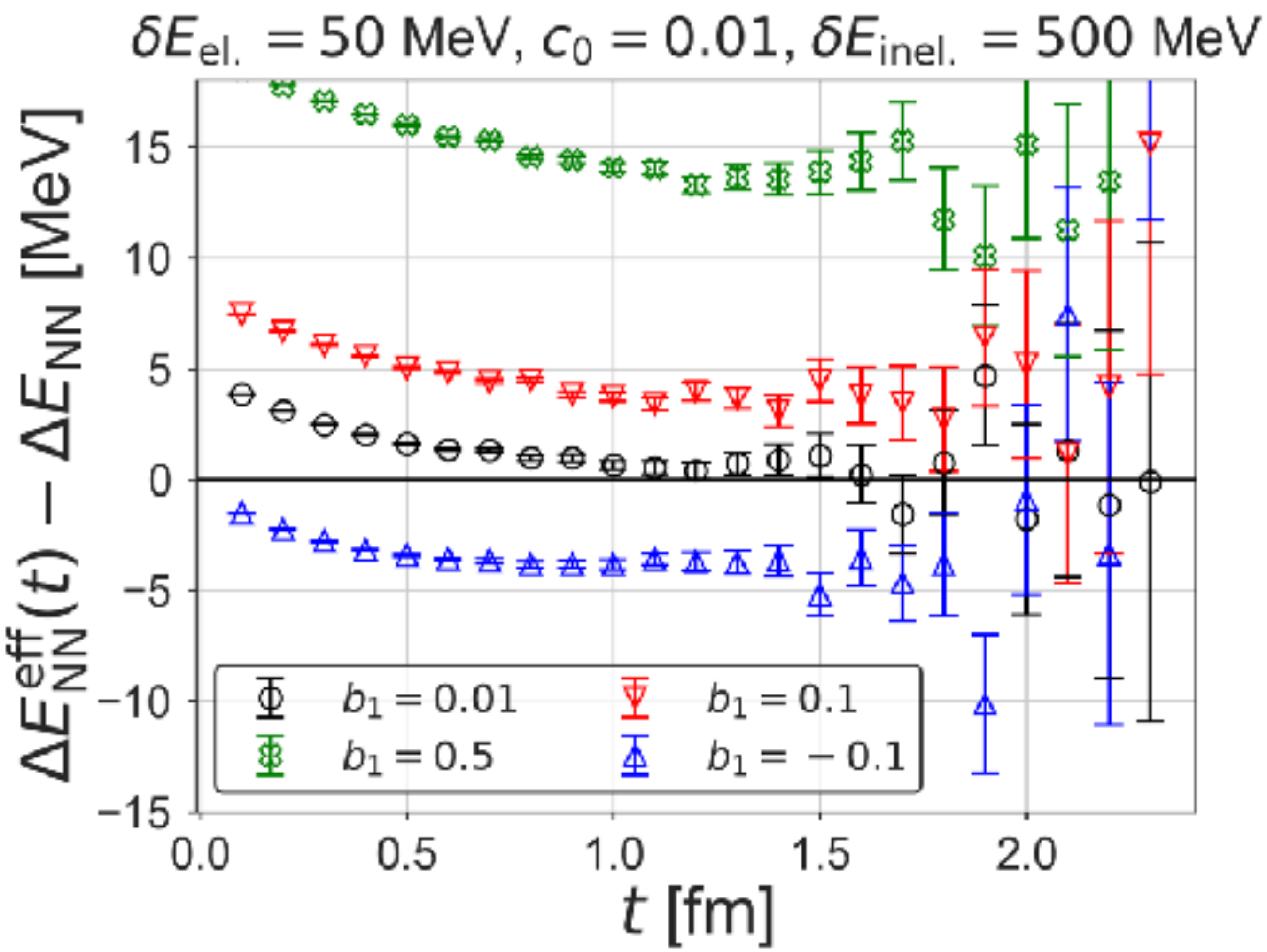
The fake plateaux are caused by b, not by c.

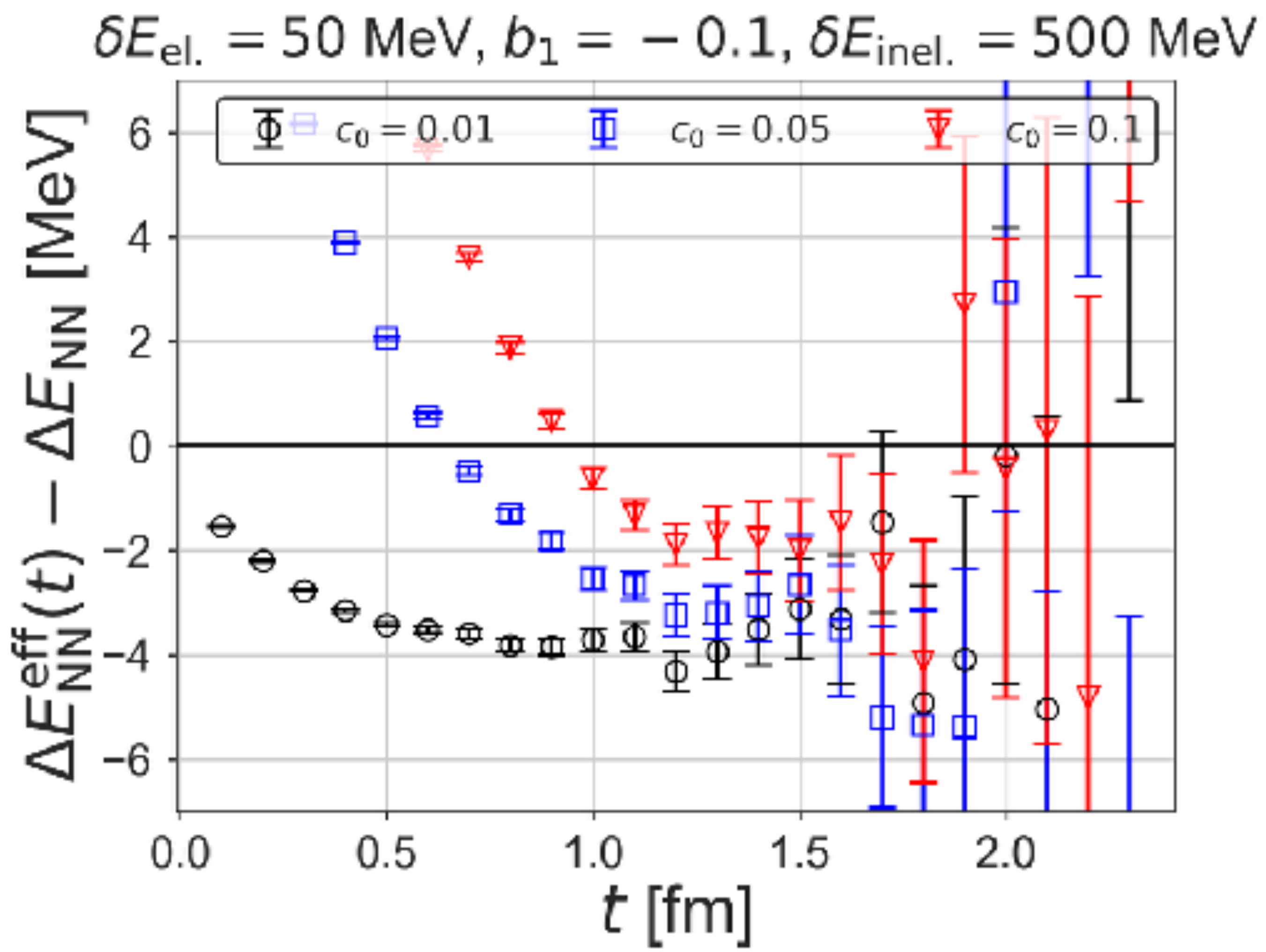
Thus small c_N and c_{NN} are neither necessary nor sufficient condition.

Indeed c_R can be small by the cancellation between c_N and c_{NN}.

The fake plateaux can appear for both sources.

The ``plateau” is still NOT enough.





b controls the value of the fake plateau.

c controls the convergence to the fake plateau.

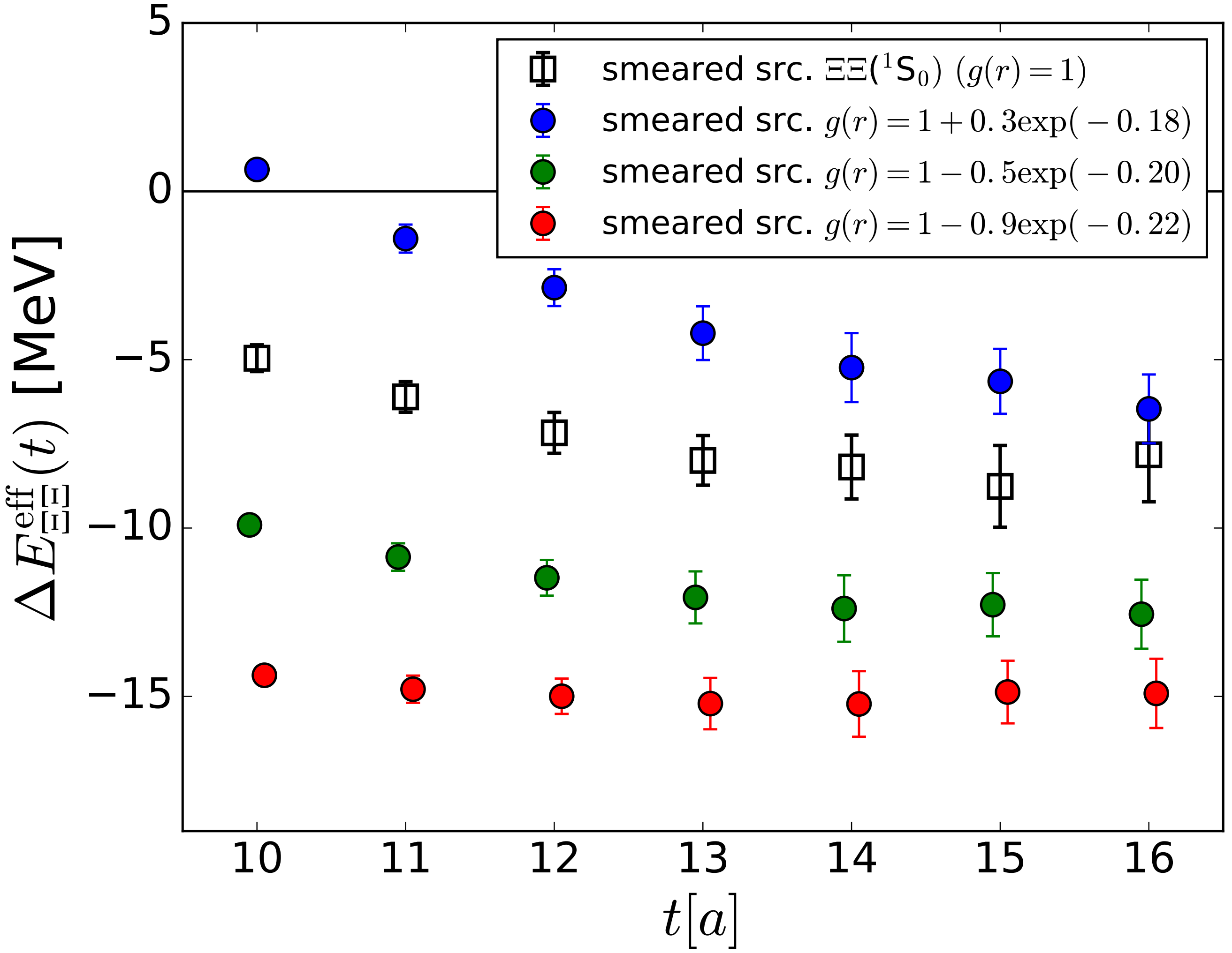
The wall source seems to have smaller c in the ratio due to the cancellation.

Sink operator dependence

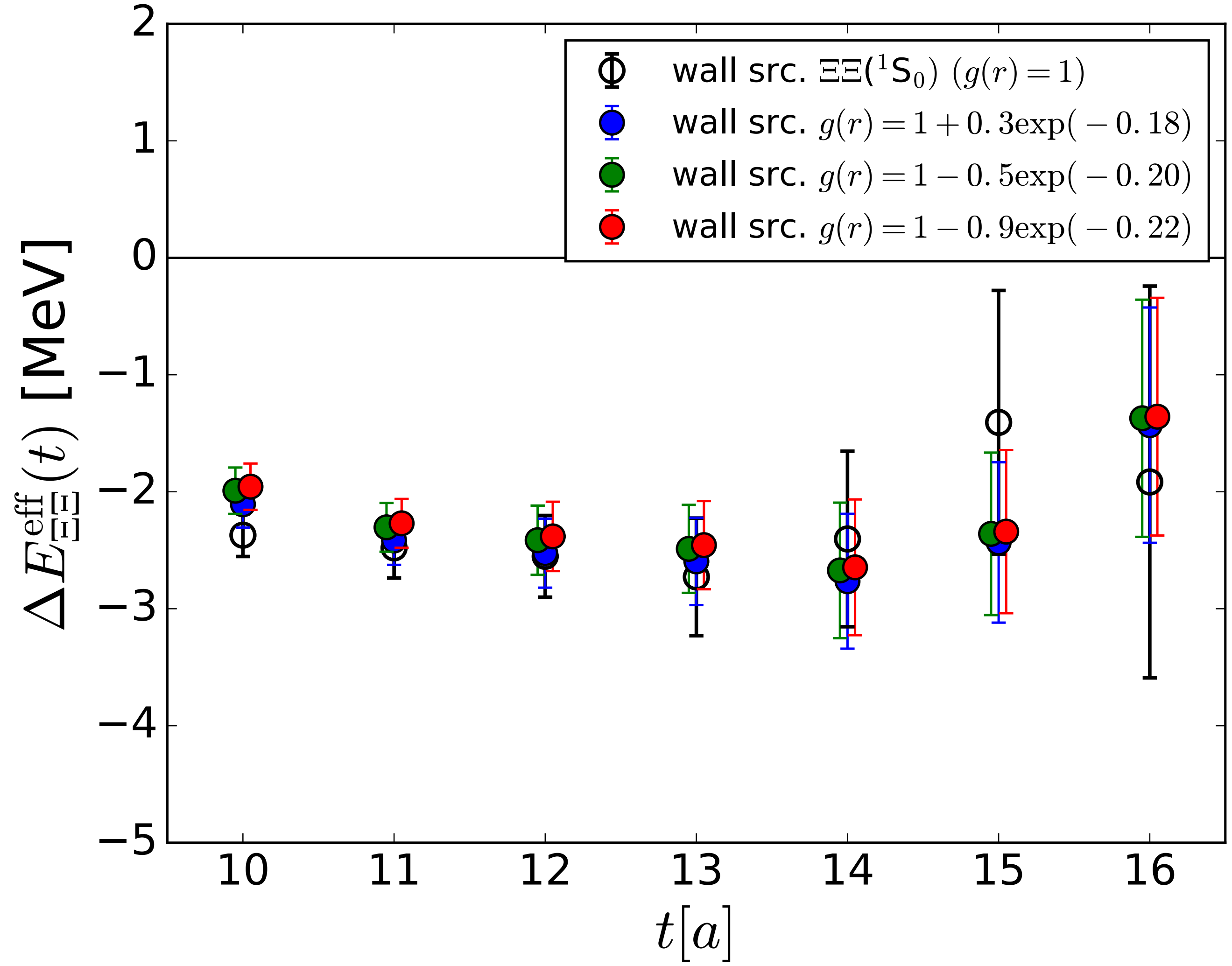
$$G(t-t_0) = \sum_{\mathbf{x},\mathbf{y}} g(|\mathbf{x}-\mathbf{y}|)\langle O(\mathbf{x},t)O(\mathbf{y},t)\mathcal{J}_{OO}(t_0)\rangle$$

$$g(r) = 1 + A\exp(-Br)$$

Smeared source



Wall source



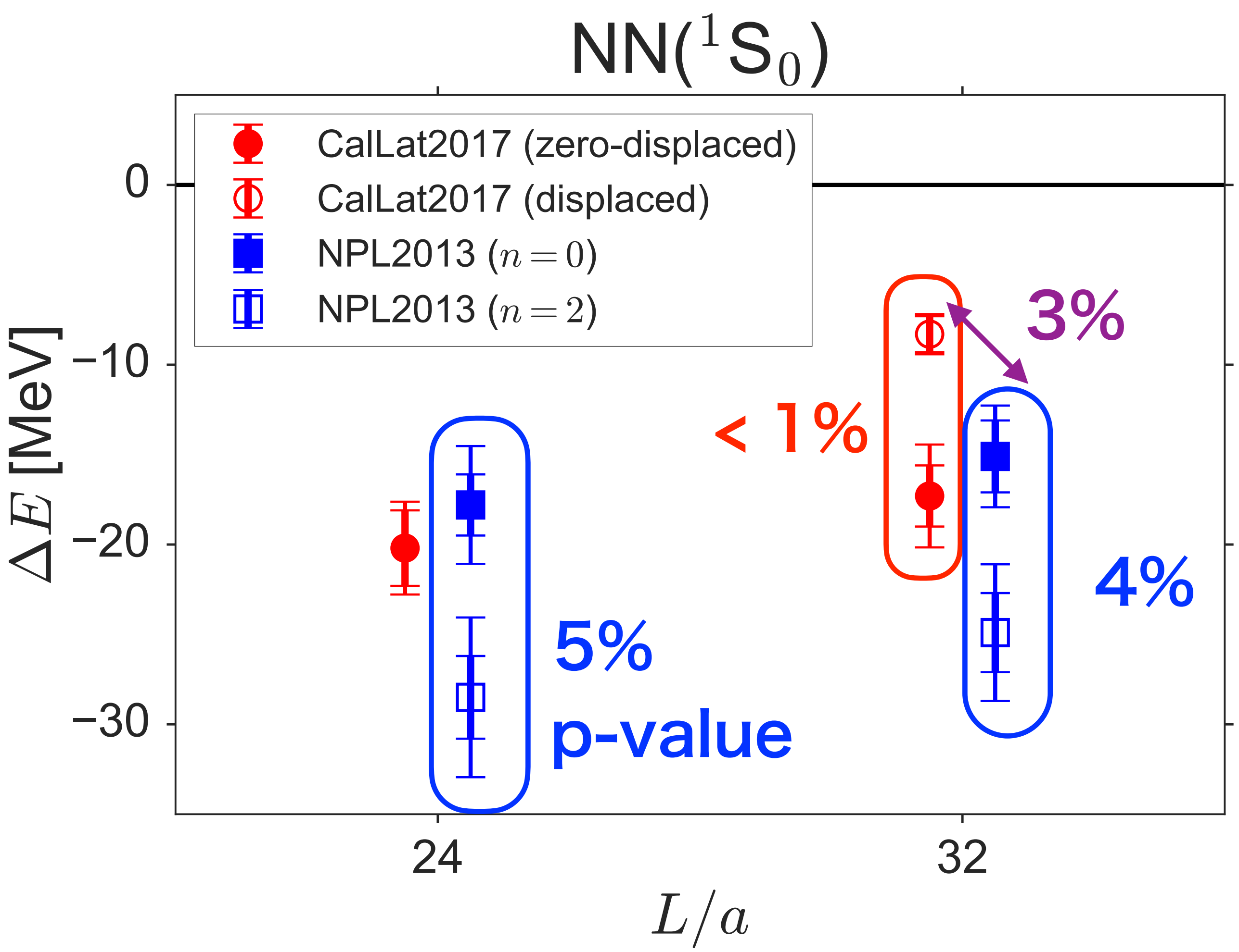
The plateaux of the smeared source are sensitive to the sink operator.

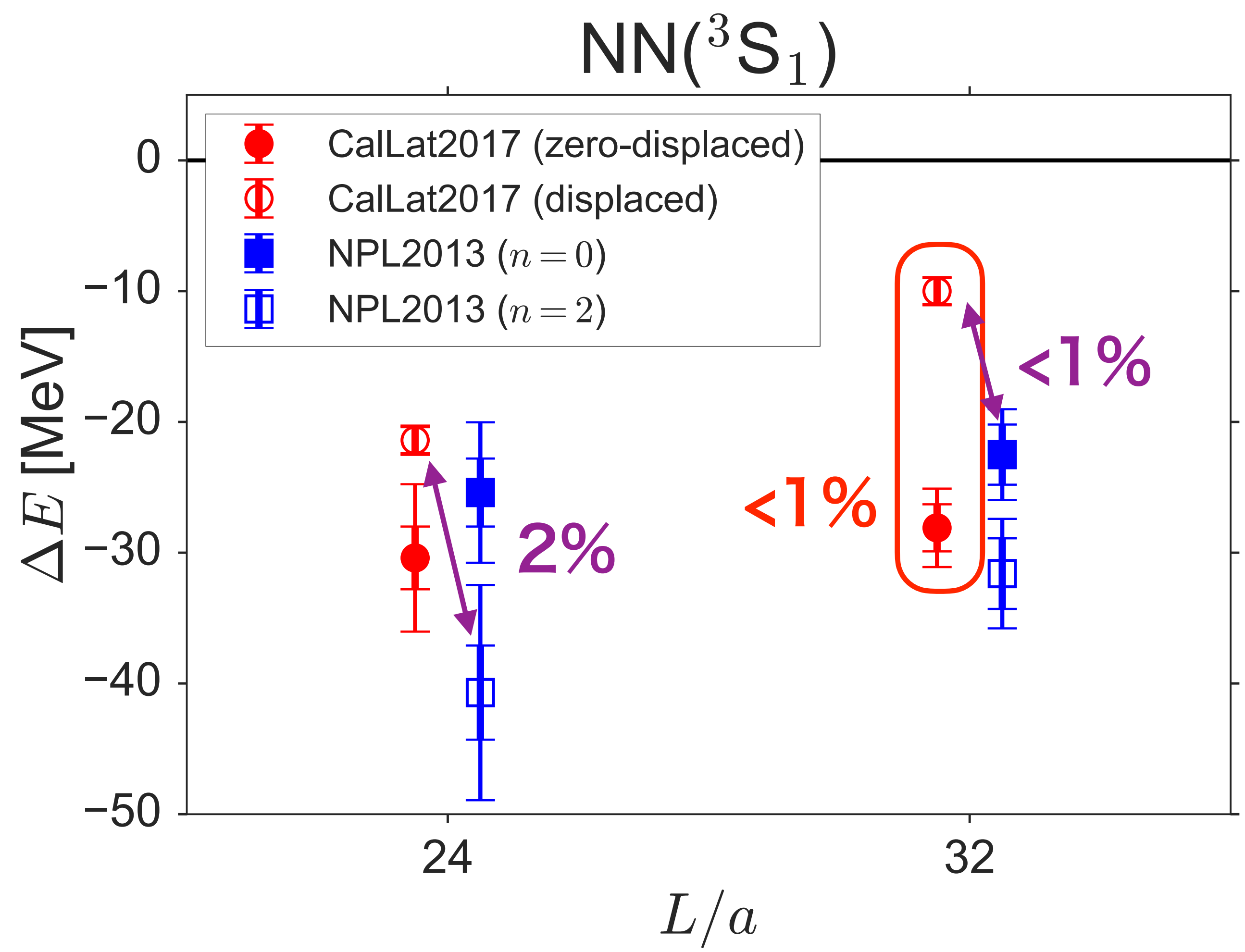
The wall source is insensitive.

The plateaux from smeared source are fake.

The ``plateau” is NOT enough.

More on operator dependences



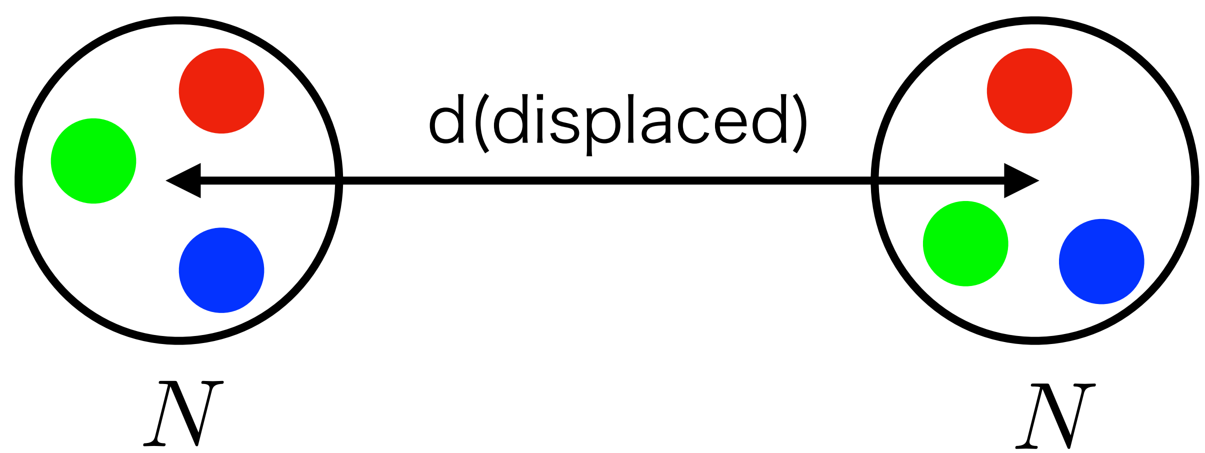


NPL2013: smeared sources

bost momentum: $p = 2\pi n/L$

$d = 0$

CalLat2017: smeared sources



Source Operator dependence

NPL2013/CalLat2017 used the same gauge configurations.

Response by NPL (arXiv:1705.09239 [hep-lat])

``Unfortunately the figure includes a second state from Ref. [14] that the authors of Ref. [14] explicitly indicate is not the ground state, and reporting it as such is a critical error on which many of the invalid arguments of HAL are based.”

Statement in CalLat2017(PLB765(2017)285)

``The state closer to threshold (and additionally, the negative energy state near threshold in the 1S0 channel) has strong overlap onto the non-local NN interpolating field, and has not been found in previous works.”

Our comments

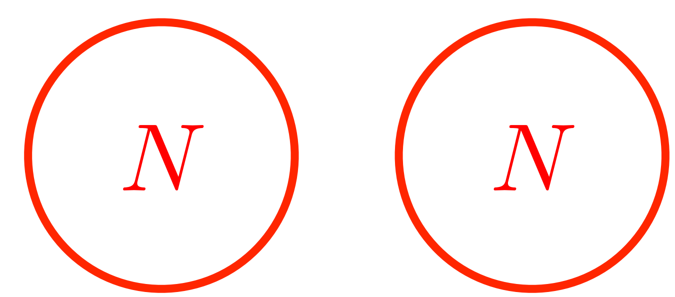
This kind of excuses might be justified only if (1) we know that there must exist two (or more) states, and (2) these states are very different.

Ex.(A) string breaking. The Wilson loop operator may have a poor overlap to two heavy-light meson state.

Ex.(B) rho resonance: rho meson operator may have a poor overlap to two pions.

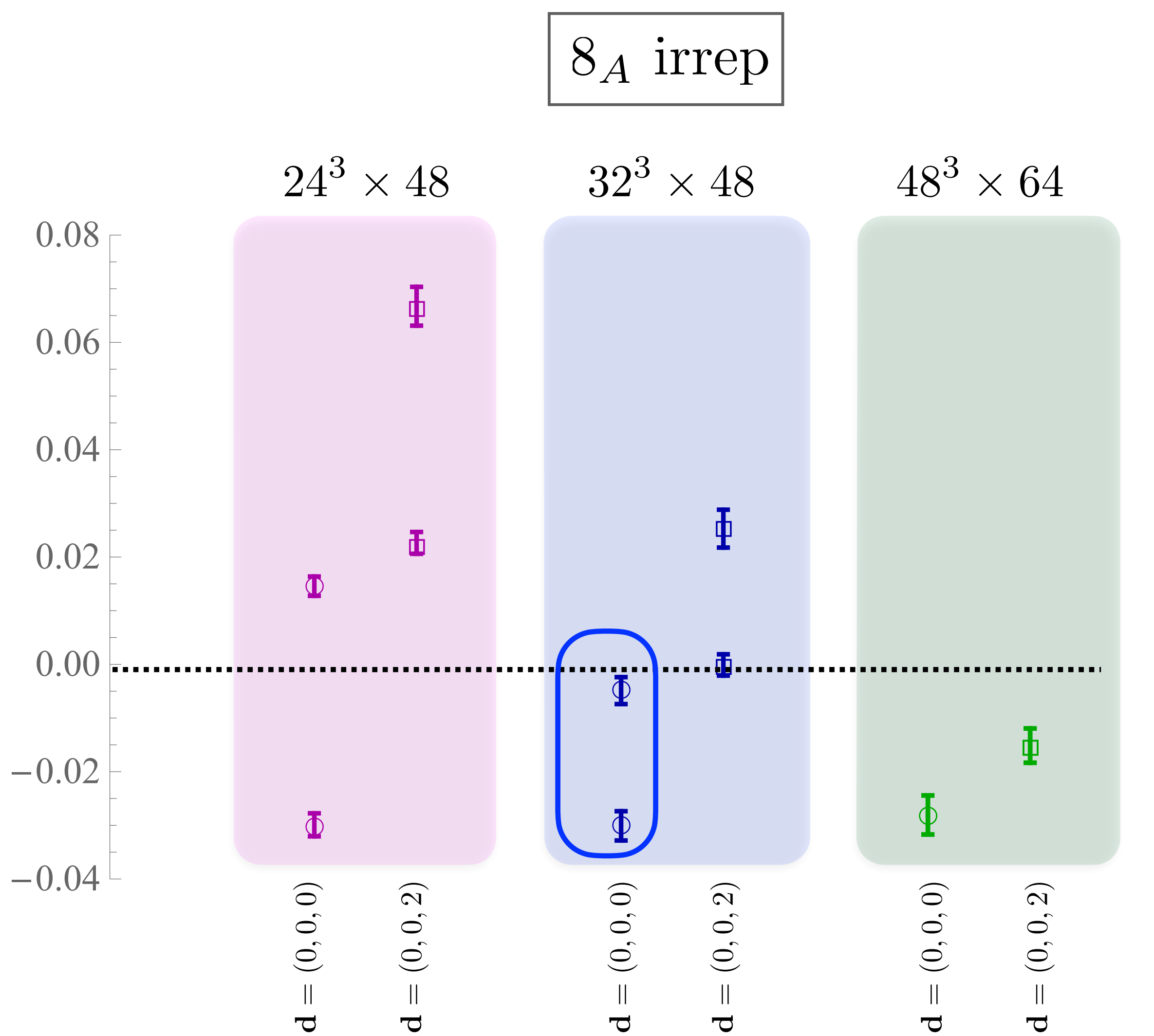
Even such cases, the variational analysis including the two types of operators is required for a reliable calculation.

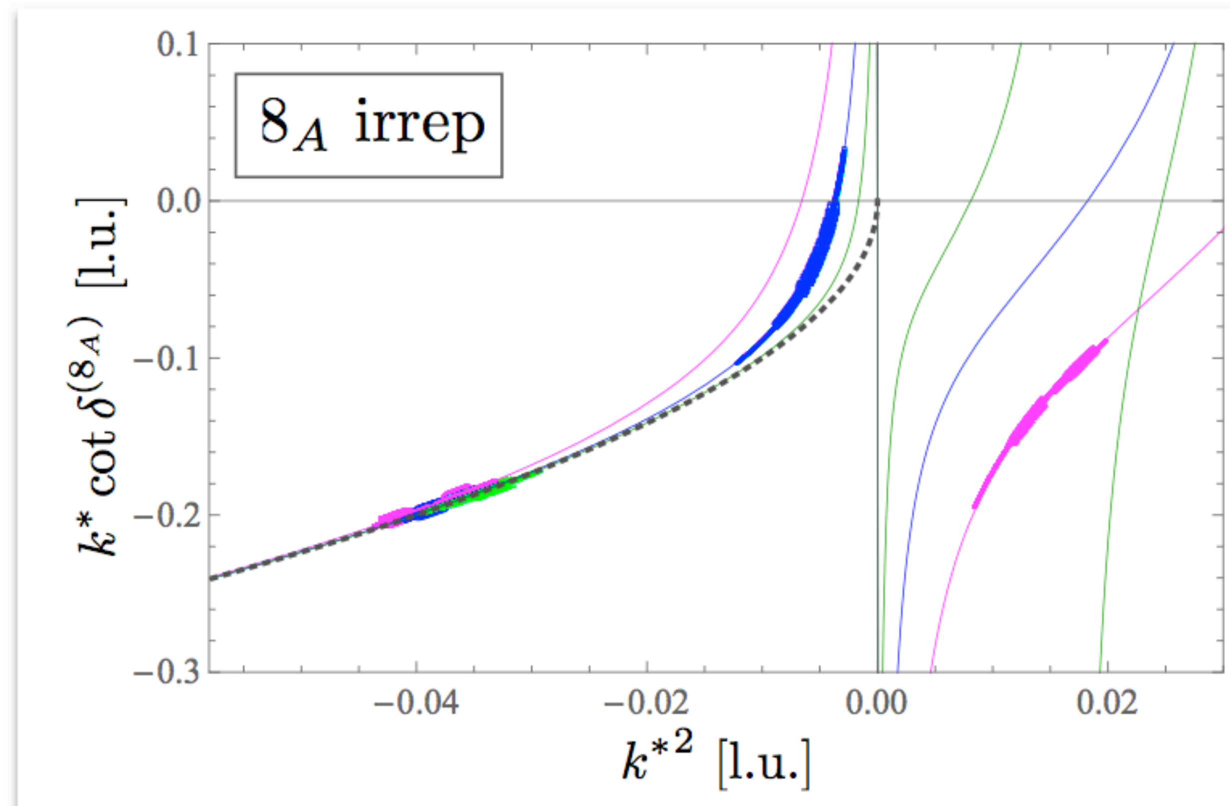
However it is not legitimate to use such an excuse to deny the operator dependence for the NN case without any explicit evidences.



A similar symptom ?

NPL(arXiv:1706.06550)





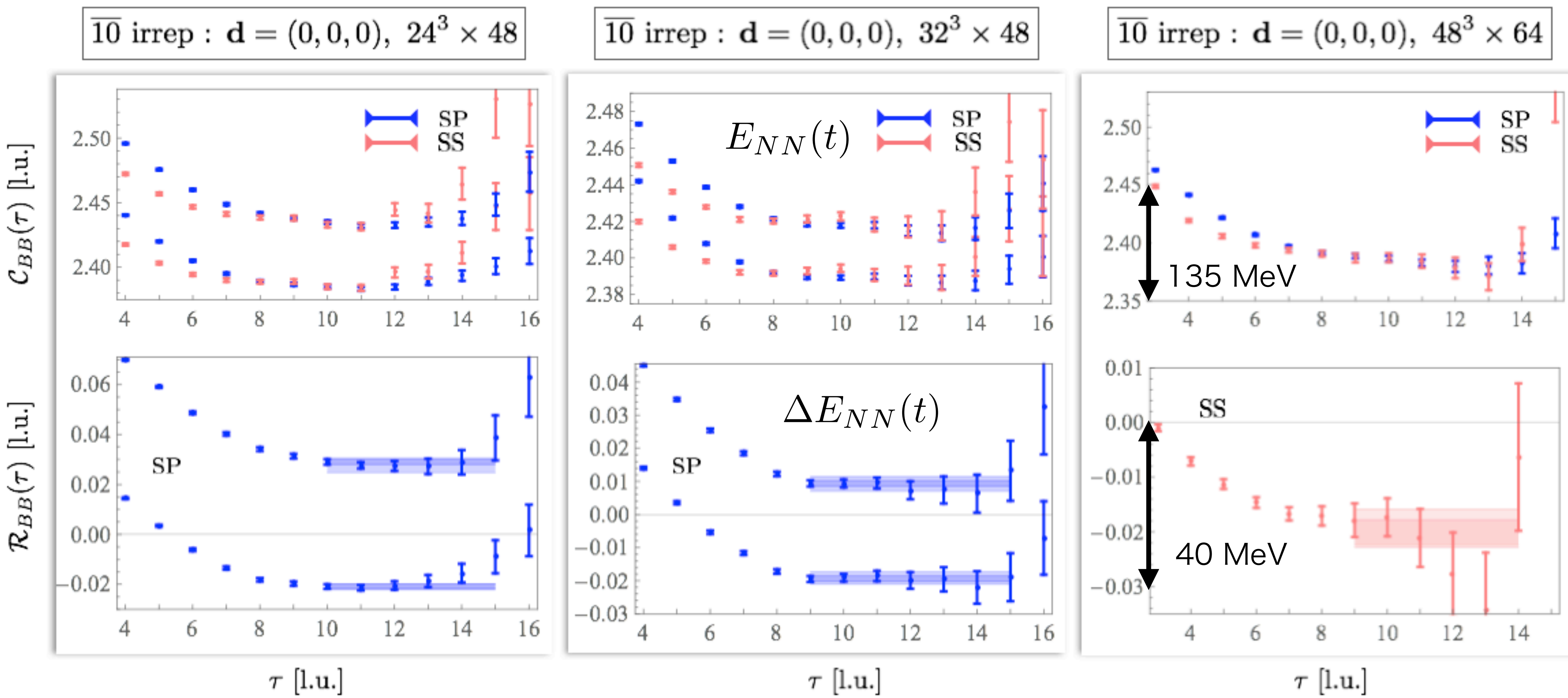
There would be two bound states if plateaux were correct.

Otherwise there is the operator dependence in this channel.

Sink operator independence

NPL(arXiv:1706.06550)

$NN(^3\mathrm{S}_1)$

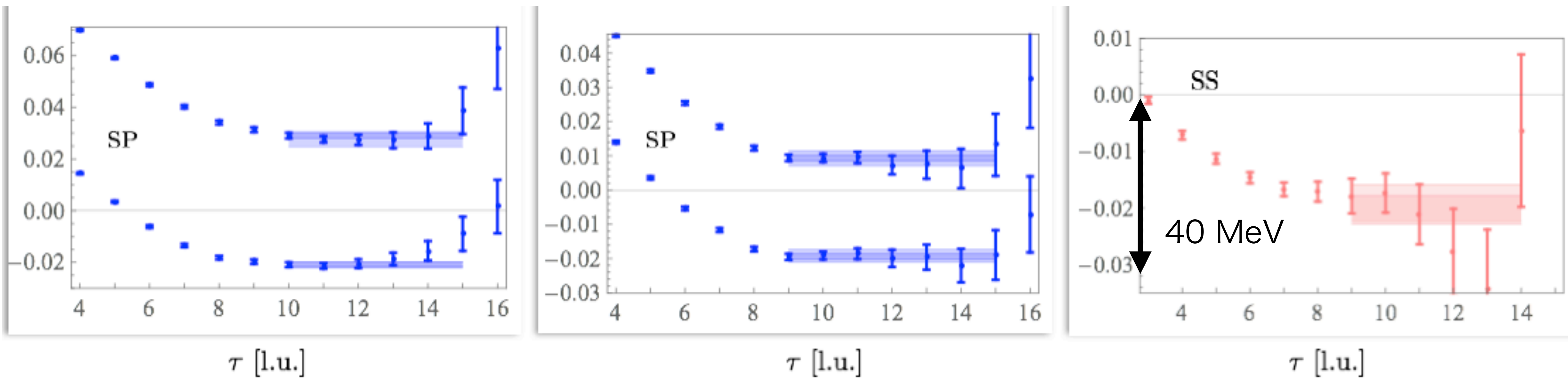


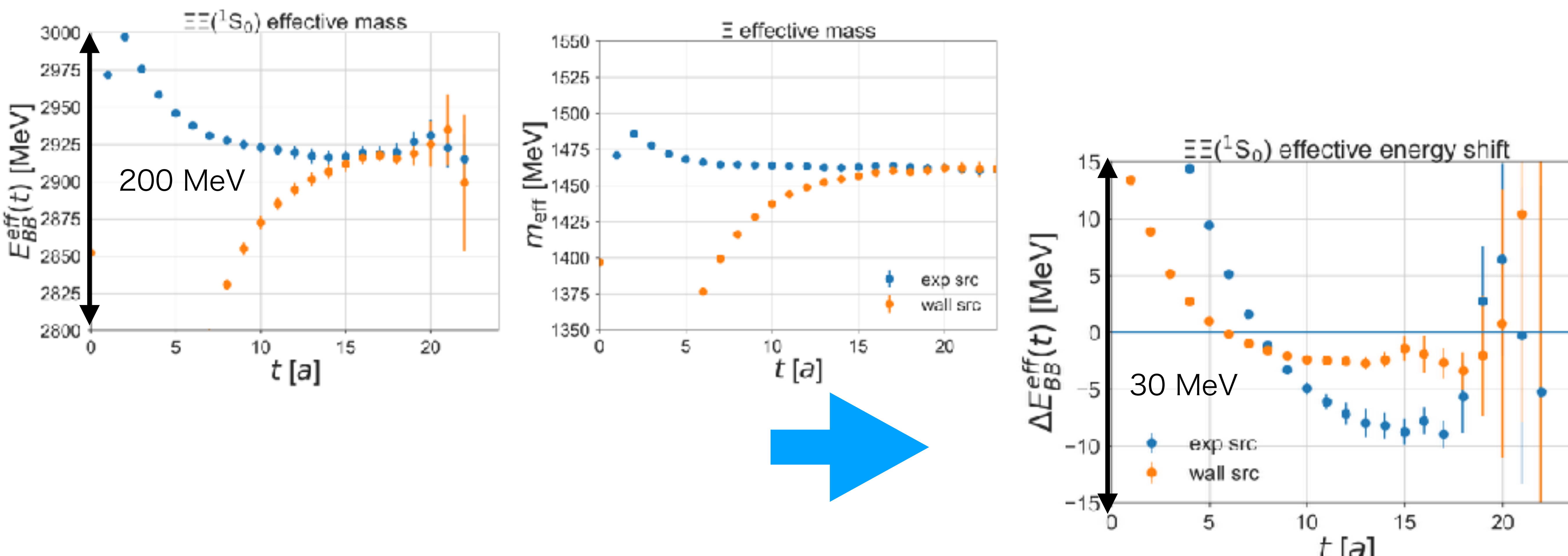
Smeared source - Point Sink (SP) Smeared source - Smeared Sink (SS)

$E_{NN}(t)$ from SP and SS roughly agree in large scale.

Sink operator independence

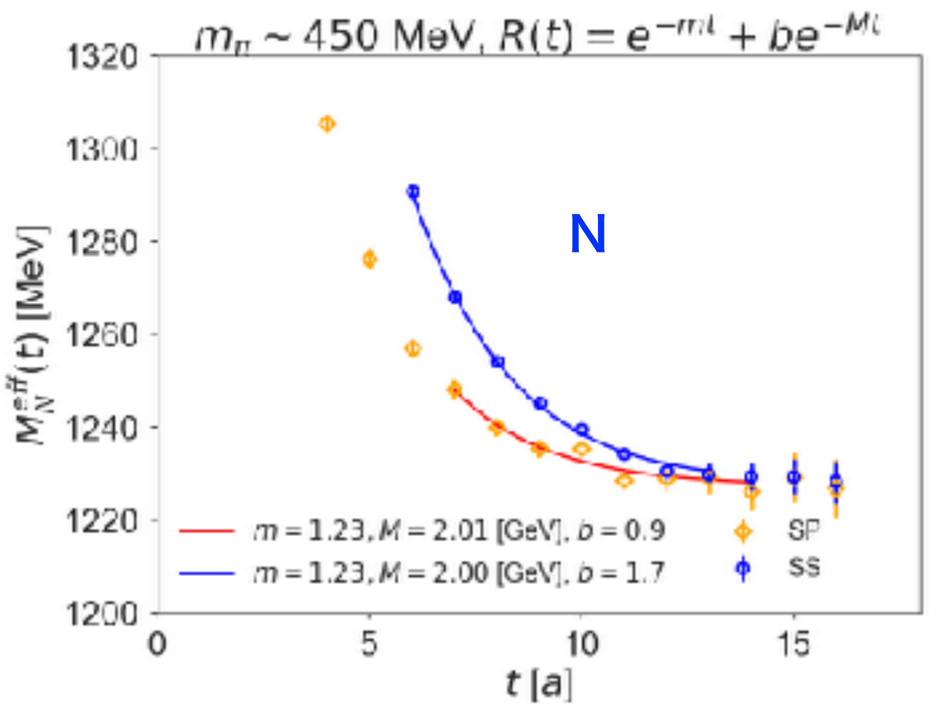
Need to check ΔE_{NN} from SP and SS agree in finer scale.



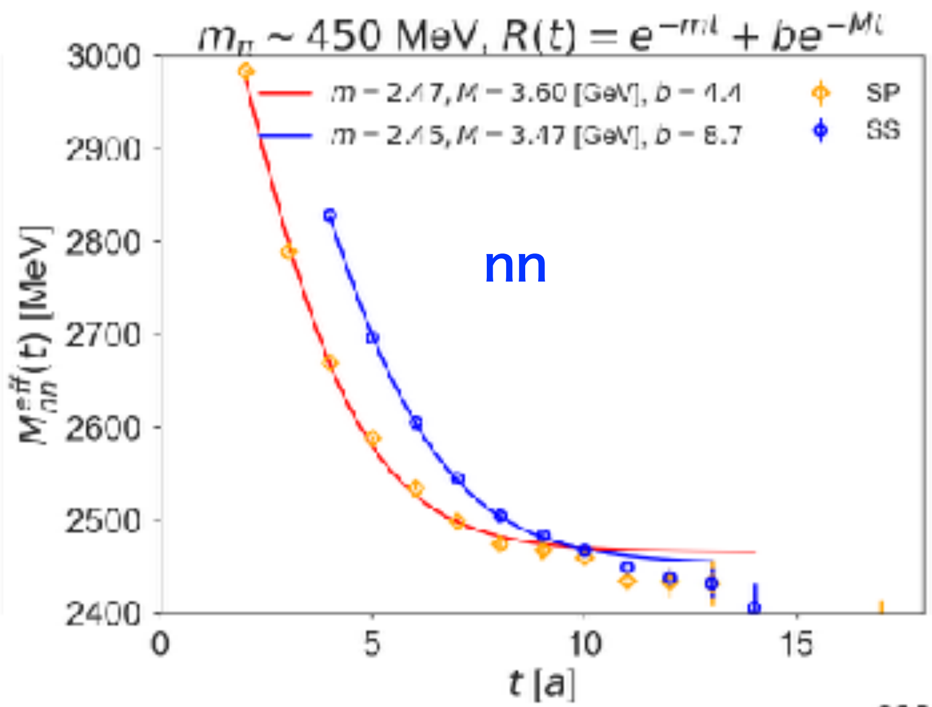


Two pole fitting

NPL(PRD96(2017)094512)

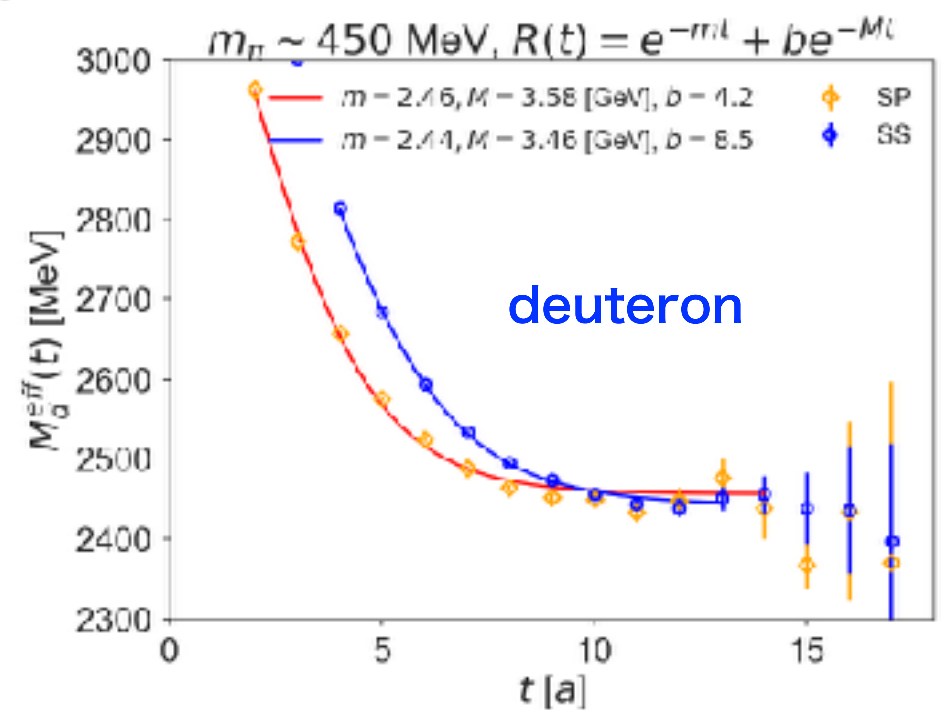


$\Delta M_N \simeq 0.8$ GeV



$\Delta M_{nn} \simeq 1.0$ GeV

$\Delta M_d \simeq 1.0$ GeV



Our comments

$$R(t) = e^{-\Delta E t}\left(1 + b\ e^{-\delta E_{\rm el} t} + c\ e^{-\delta E_{\rm inel} t}\right)$$

This method may identify the inelastic states, but can not disentangle the ground state from elastic excited states.

III. Normality (sanity) check

- A sign of the fake plateau II-

T. Iritani et al. (HAL QCD), PRD96 (2017)034521 (arXiv:1703.07210)

sanity -> normality

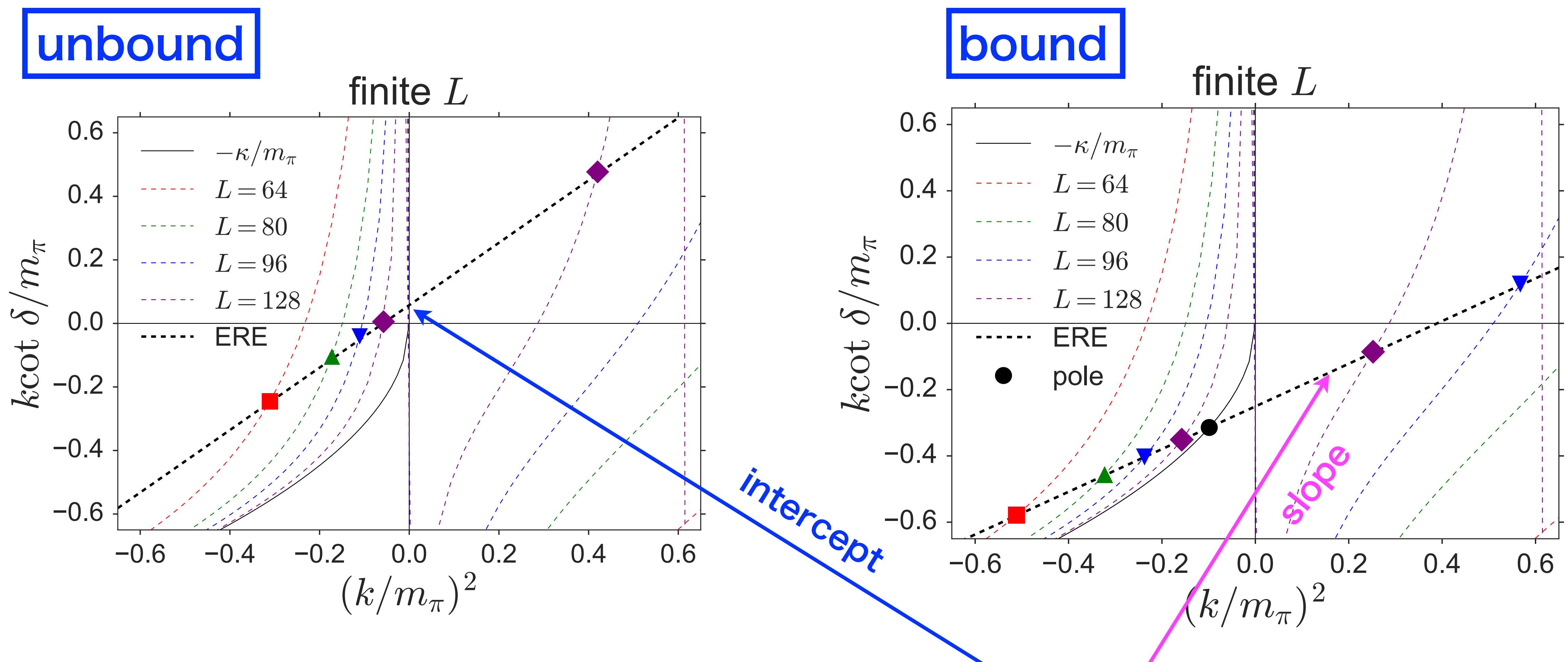
sanity $\not\fallingdotseq$ consistency

Finite volume formula

The operator dependence is a sign of the fake plateau, but an extra work is required. We need a simpler method to see a sign of the problem.

Finite volume test

$$\Delta E = 2\sqrt{k^2 + m_N^2} - 2m_N, \quad q = \frac{kL}{2\pi} \quad \Rightarrow \quad k\cot\delta(k) = \frac{1}{\pi L}\sum_{\vec{n}\in\mathbb{Z}^3}\frac{1}{\vec{n}^2 - q^2}$$



Effective Range Expansion (ERE)

$$k\cot\delta(k) = \frac{1}{a} + \frac{1}{2}rk^2 + \cdots$$

Normality check

(i) Consistency: $k\cot\delta(k)$ must be consistent between $k^2 < 0$ and $k^2 > 0$.

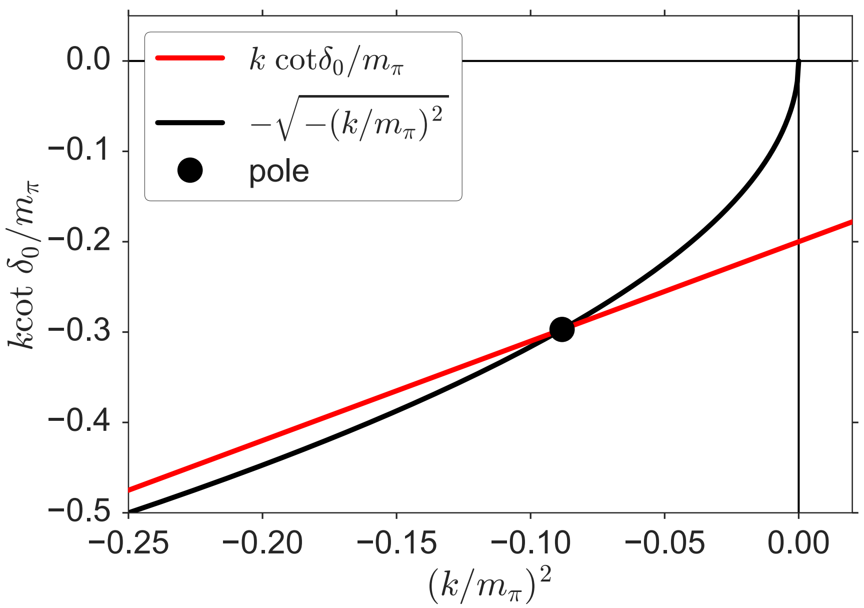
(ii) non-singular behavior: $k\cot\delta(k)$ should be non-singular.

a singular behavior requires a reasonable explanation.

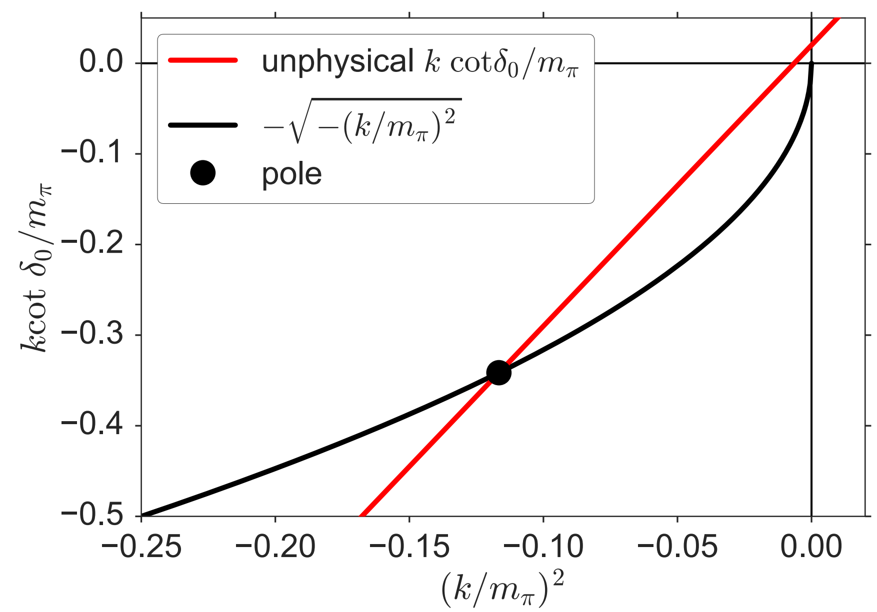
(iii) physical pole condition: $k\cot\delta(k)$ must satisfy

bound state condition

$$\frac{d}{d\,k^2}\left[k\cot\delta(k) - (-\sqrt{-k^2})\right]\Big|_{k^2=-\kappa_b^2} < 0$$



physical

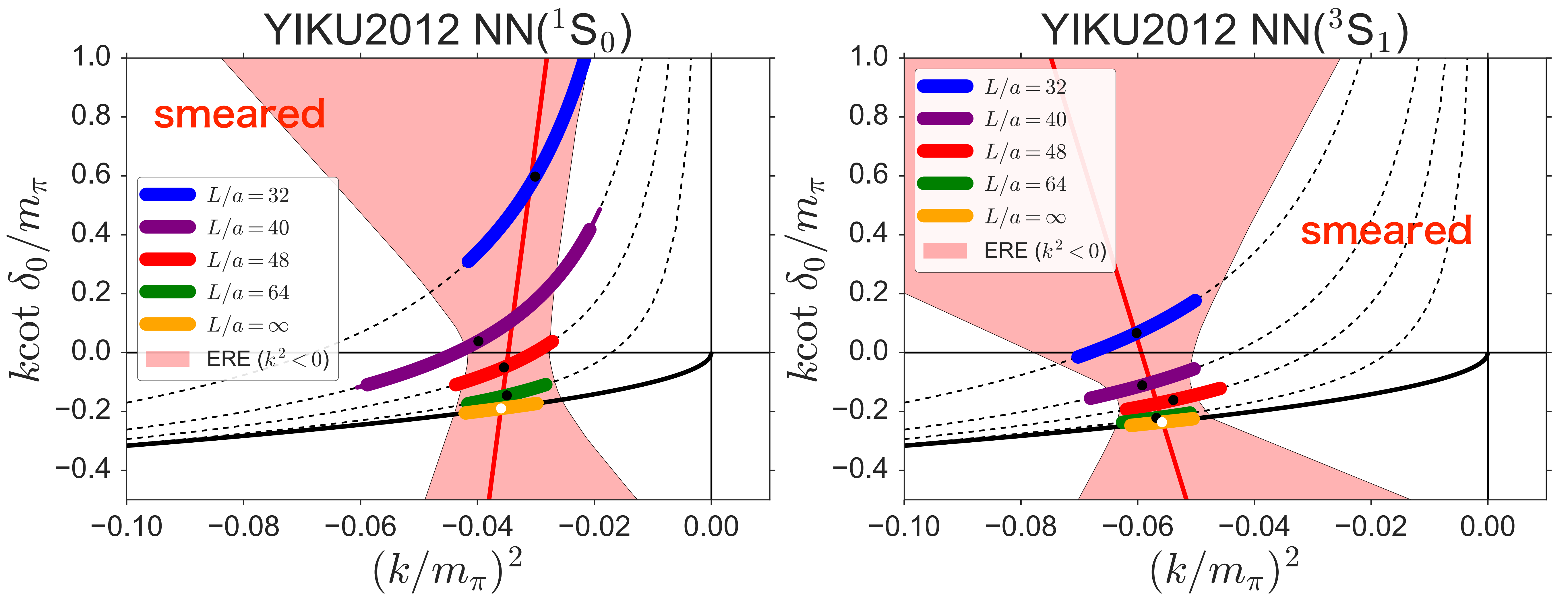


unphysical

It is necessary but not sufficient to pass the normality check.
Data may not be correct even if they pass the check.

YIKU2012(PRD86(2012)074514)

$m_\pi = 0.51$ GeV, $L = 2.9 - 5.8$ fm



(ii)singular behaviors

We have already seen operator dependences on these data.

➡ Fake plateaux

Response by YIK2017 (arXiv:1710.08066 [hep-lat])

``In the comparison between the expectation in the ideal case and the lattice data, there could be several sources of systematic errors, such as finite lattice spacing and finite volume effects, which may deform the two-nucleon interaction."

``It is noted that even if there is a finite volume effect in ΔE_{NN} , which cannot be treated by the finite volume method [14 , 15], we consider that the signal of the existence of the bound state is meaningful in our calculation, because we discuss the existence in the infinite volume limit, so that our result does not contain the finite volume effect."

Our comments

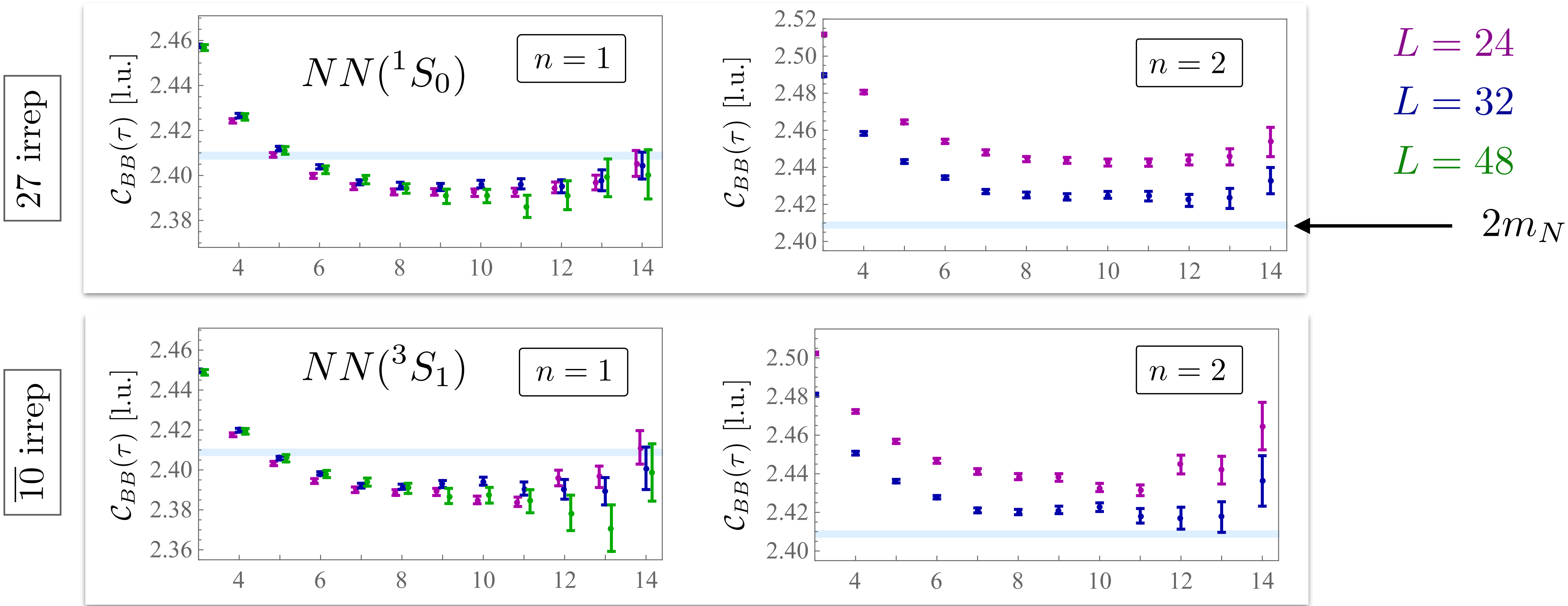
If data are distorted so badly by the finite lattice spacing/the finite volume, the result can not be regarded as the QCD prediction.

The infinite volume extrapolation even from almost volume independent data can not be trusted, if the finite volume formula is not applicable.

Volume independence

Claim by NPL (arXiv:1705.09239,1706.06550)

$N_f = 3,\ m_\pi \simeq 810$ MeV, $a \simeq 0.15$ fm



Volume independent plateaux should NOT be fake.

Our reply

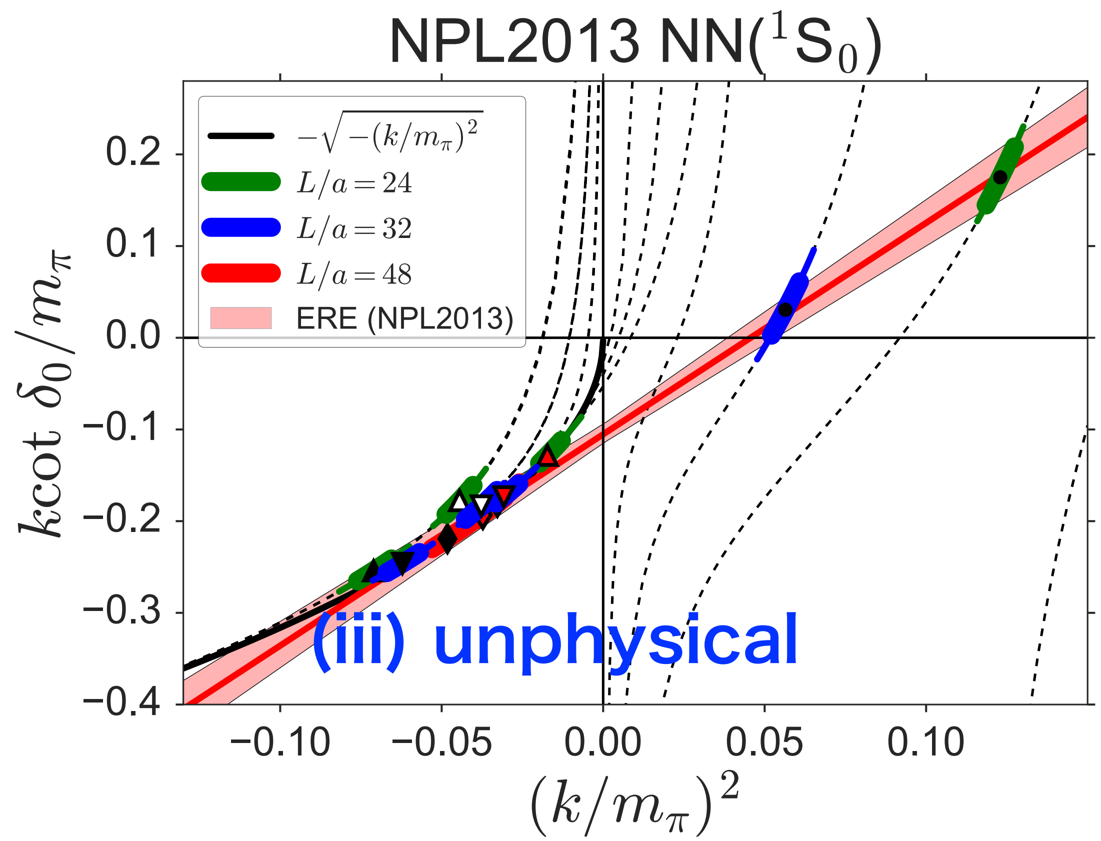
(1) There is no theoretical argument to support this.

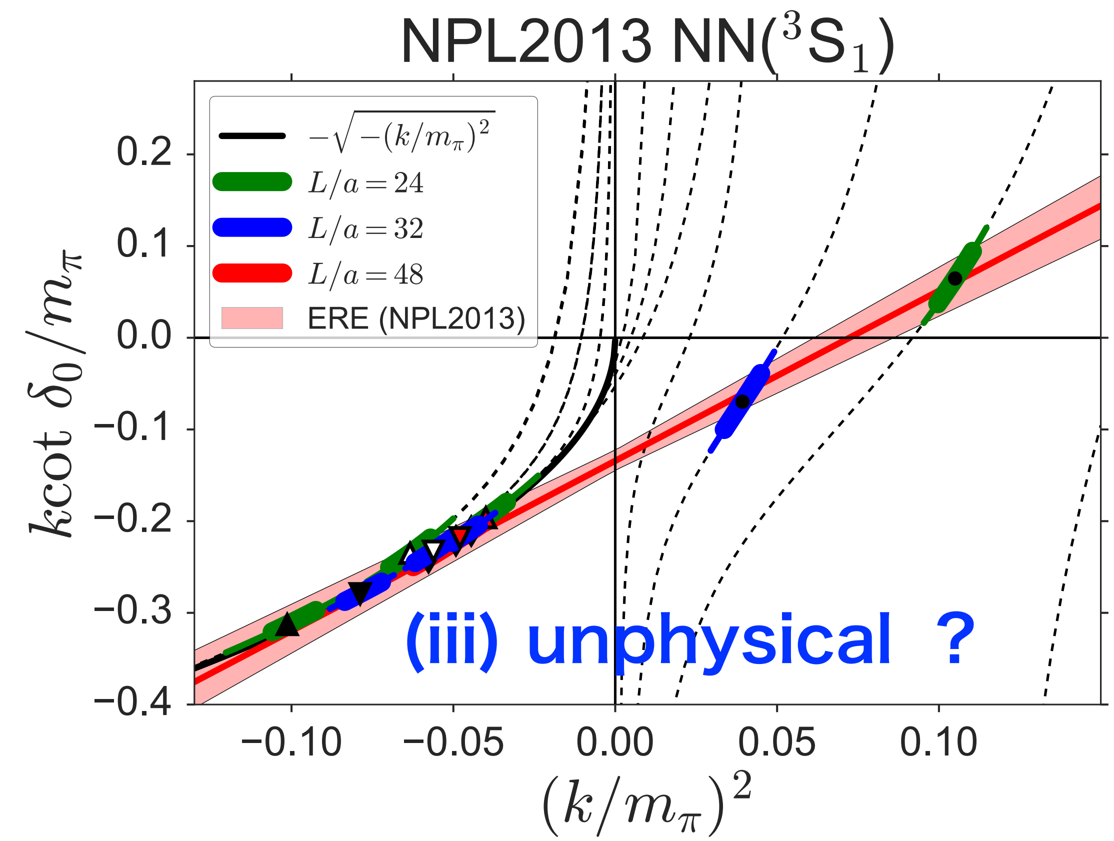
(2) This argument assumes an existence of the bound state, which however needs to be confirmed. (assumption = conclusion.)

(3) Volume independent plateaux of YIKU2012 lead to singular behavior.

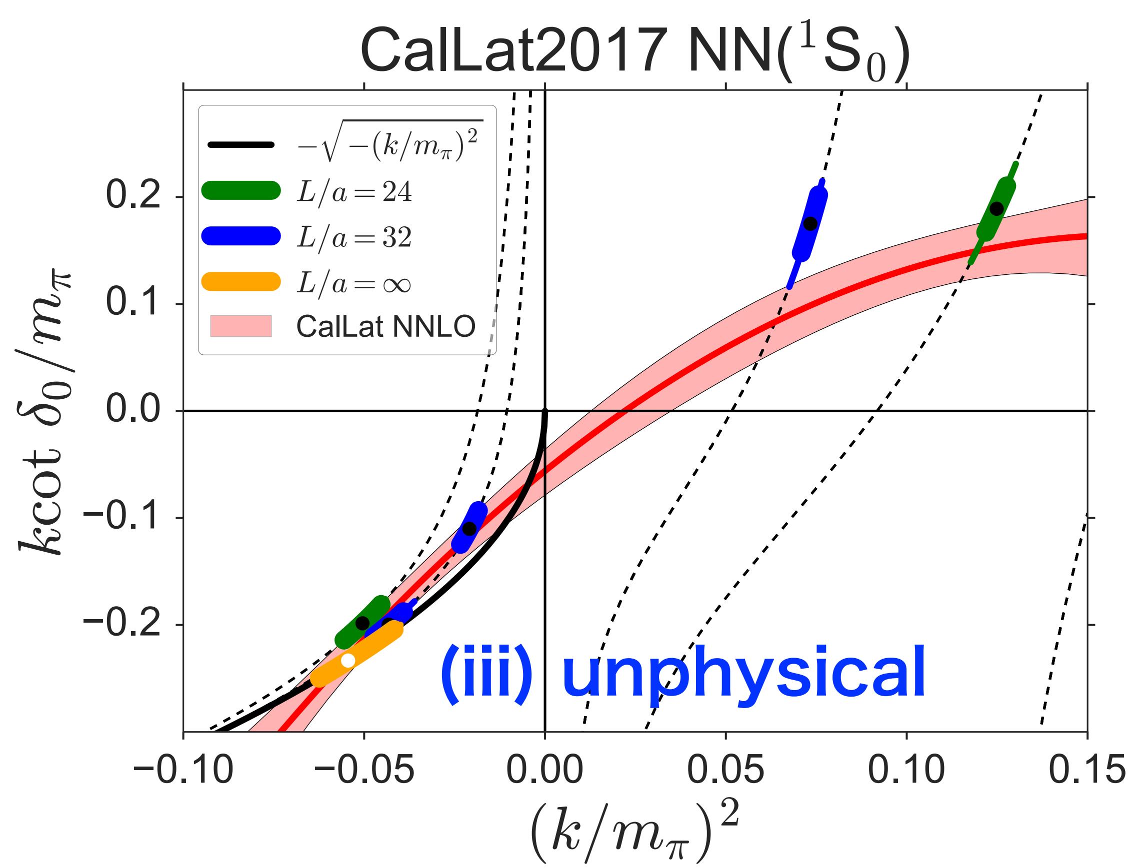
NPL2013 (PRC88(2013)024003)

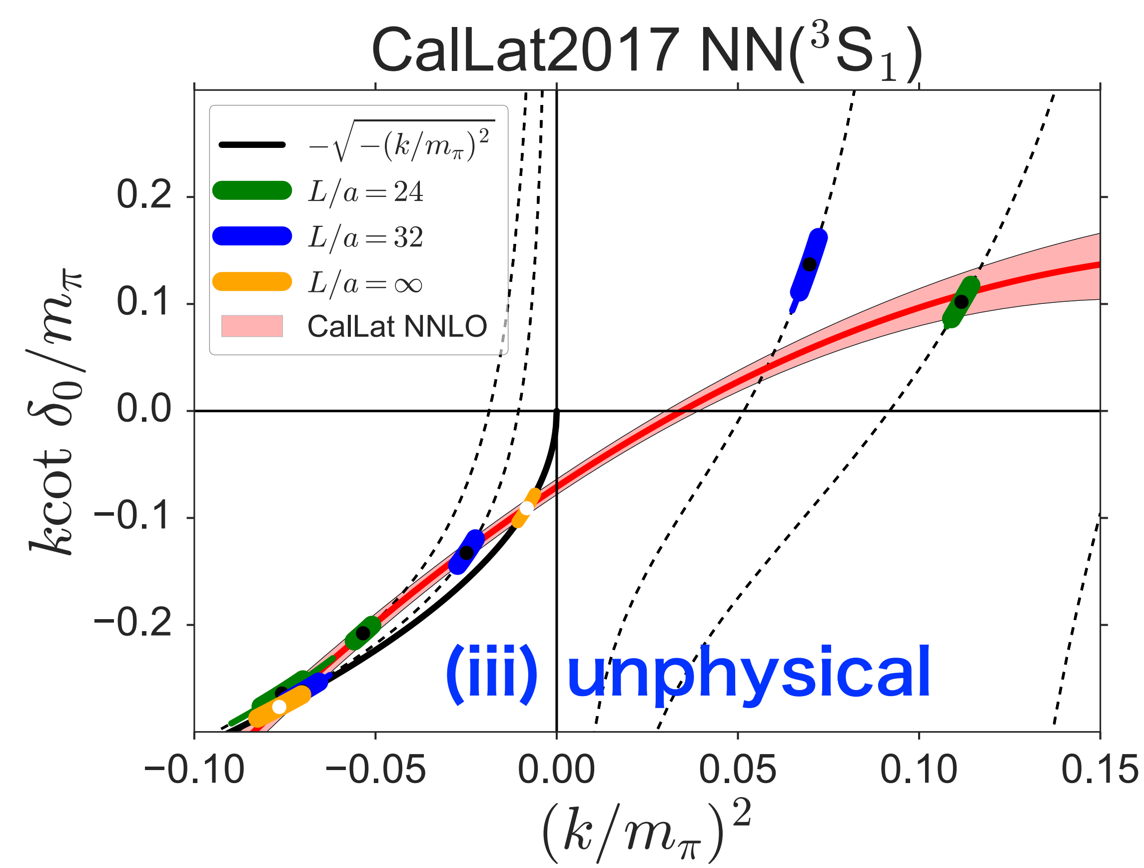
$N_f = 3,\ m_\pi \simeq 810$ MeV, $a \simeq 0.15$ fm





CalLat2017 (PLB765(2017)285)

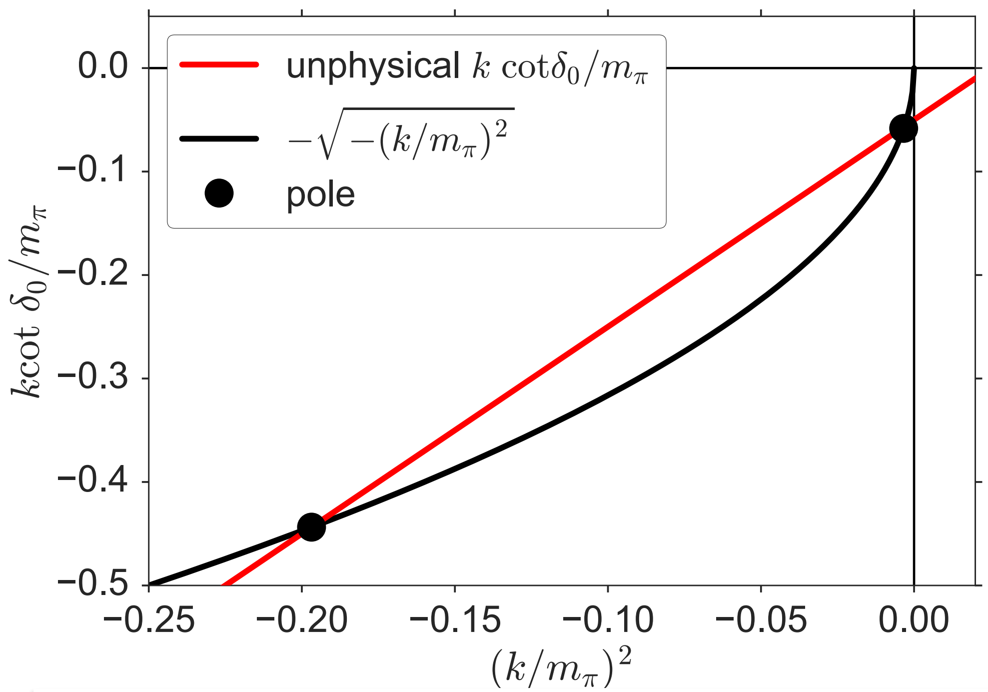




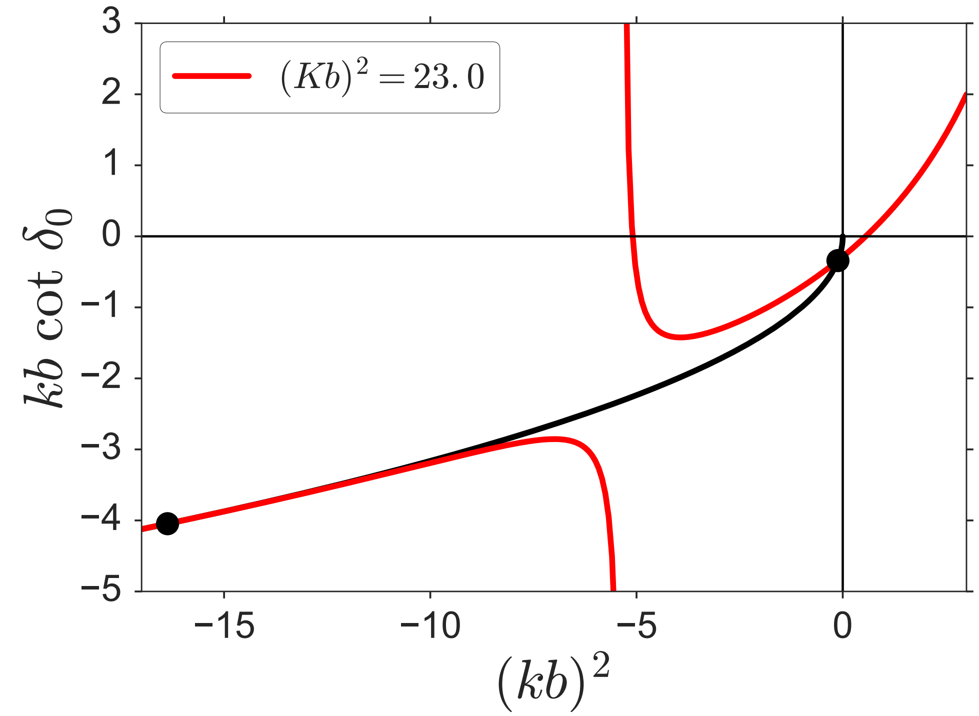
ERE line crosses the bound state condition twice.

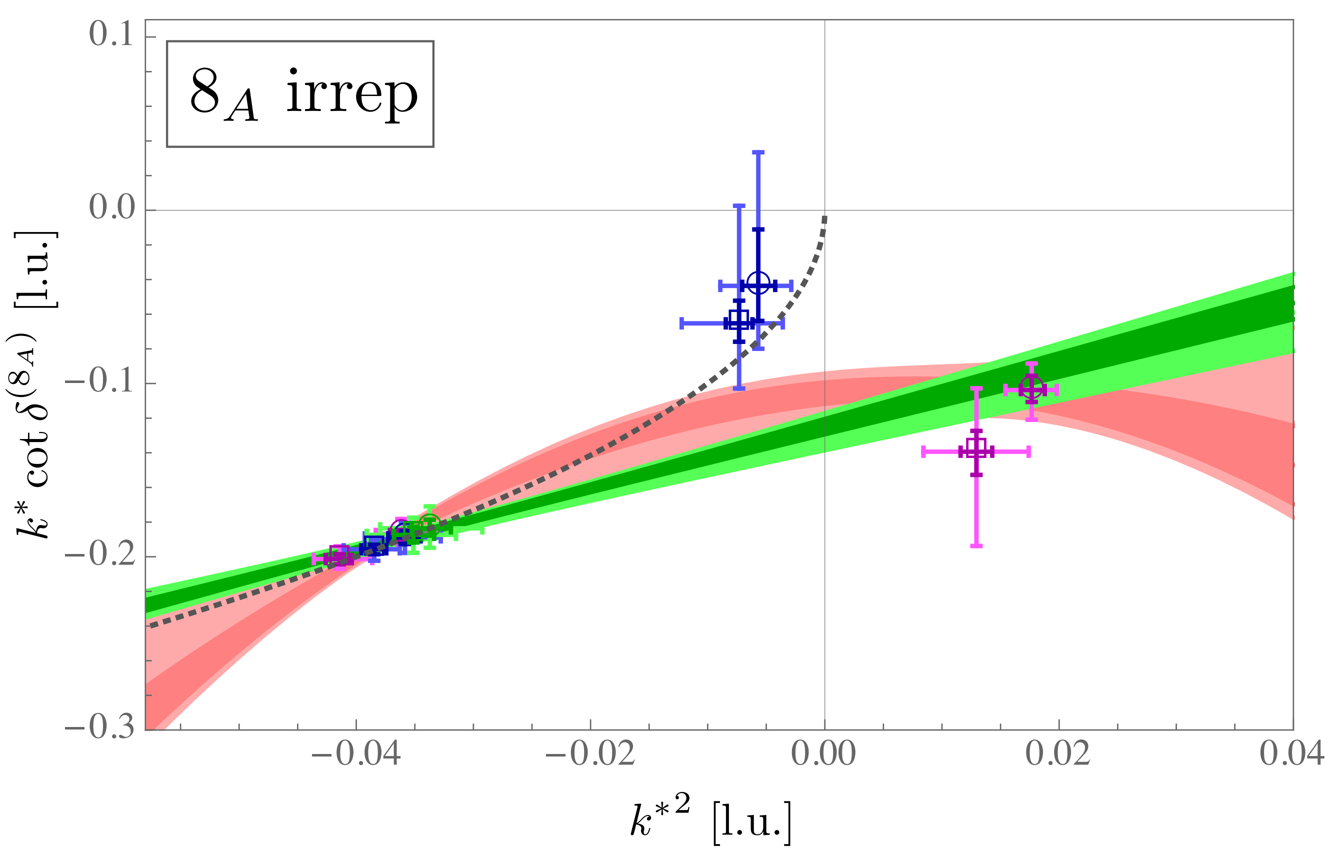
If data were correct,

unphysical



correct analysis



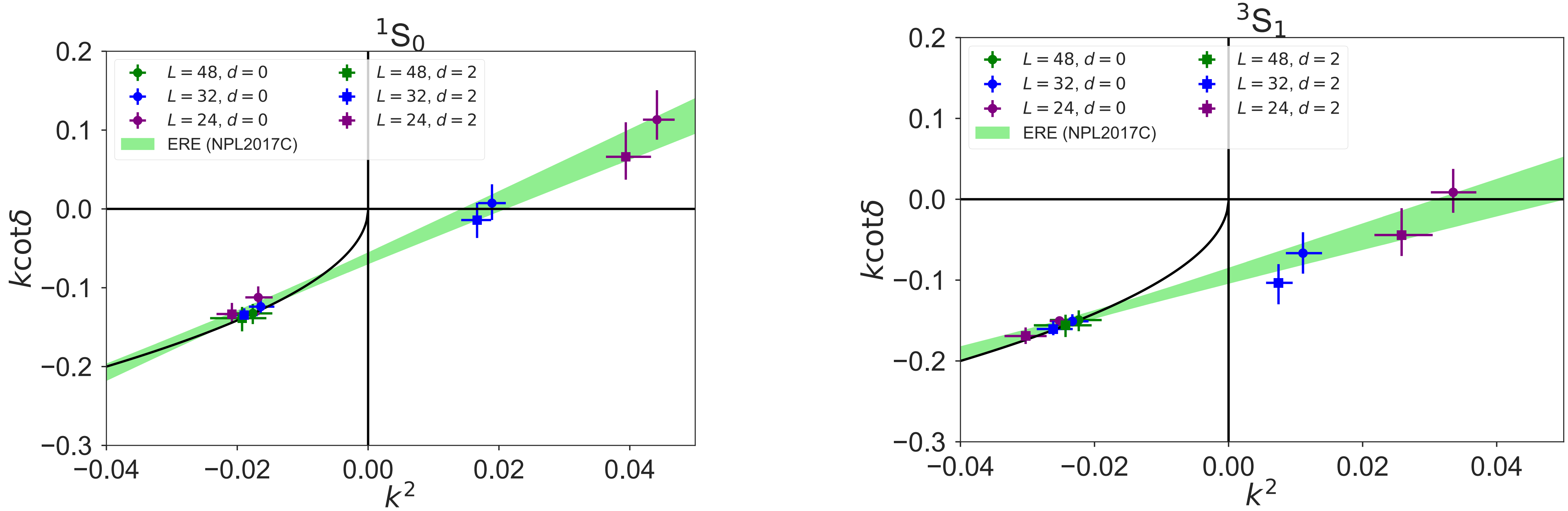


A similar problem in other channel.

NPL2017(arXiv:1706.06550)

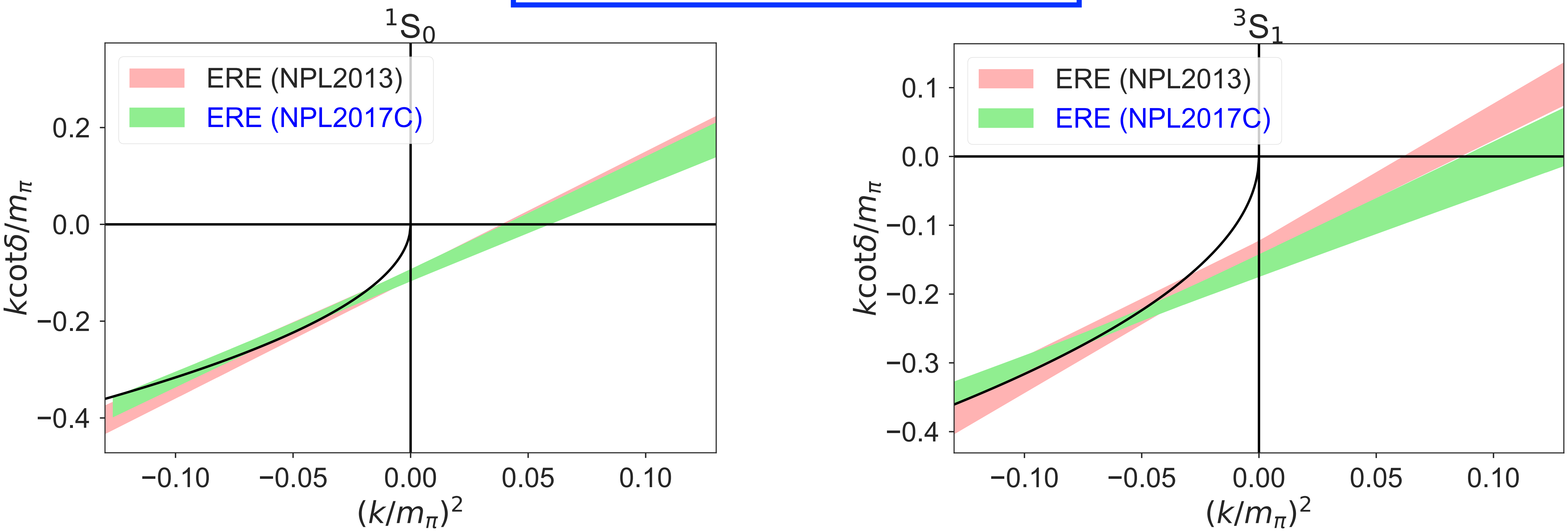
Response by NPL2017C(arXiv:1705.09239)

Data in NPL2013 pass the normality check.



Our reply 1:

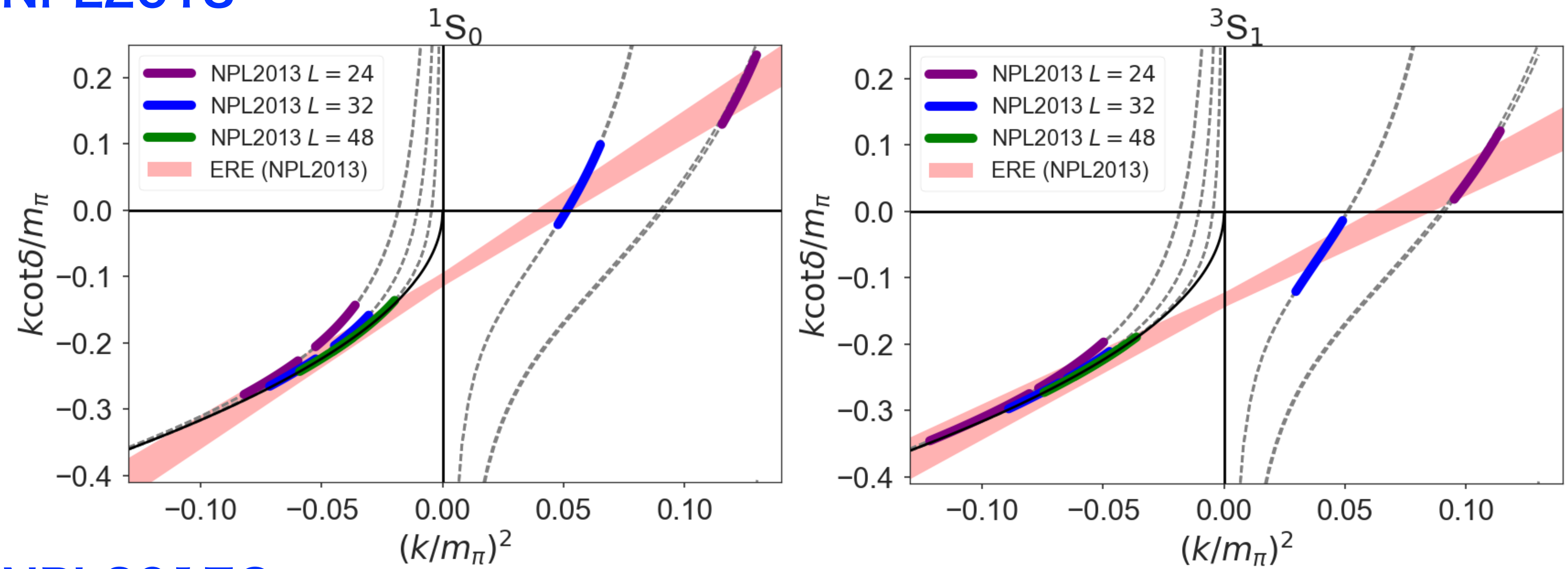
ERE fit lines are different.



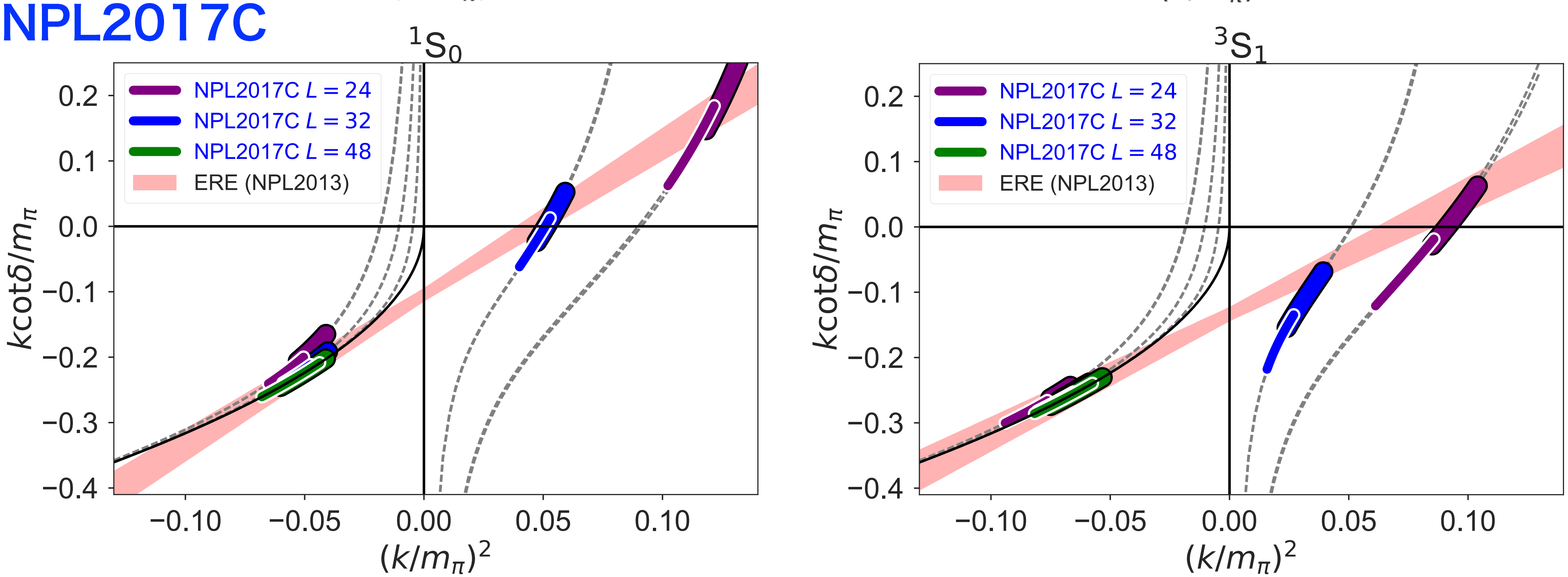
Our reply 2:

Reanalysis was made without mention(arXiv: ver1).*
Reanalysis was mentioned in footnote/caption (arXiv: ver2). **

NPL2013



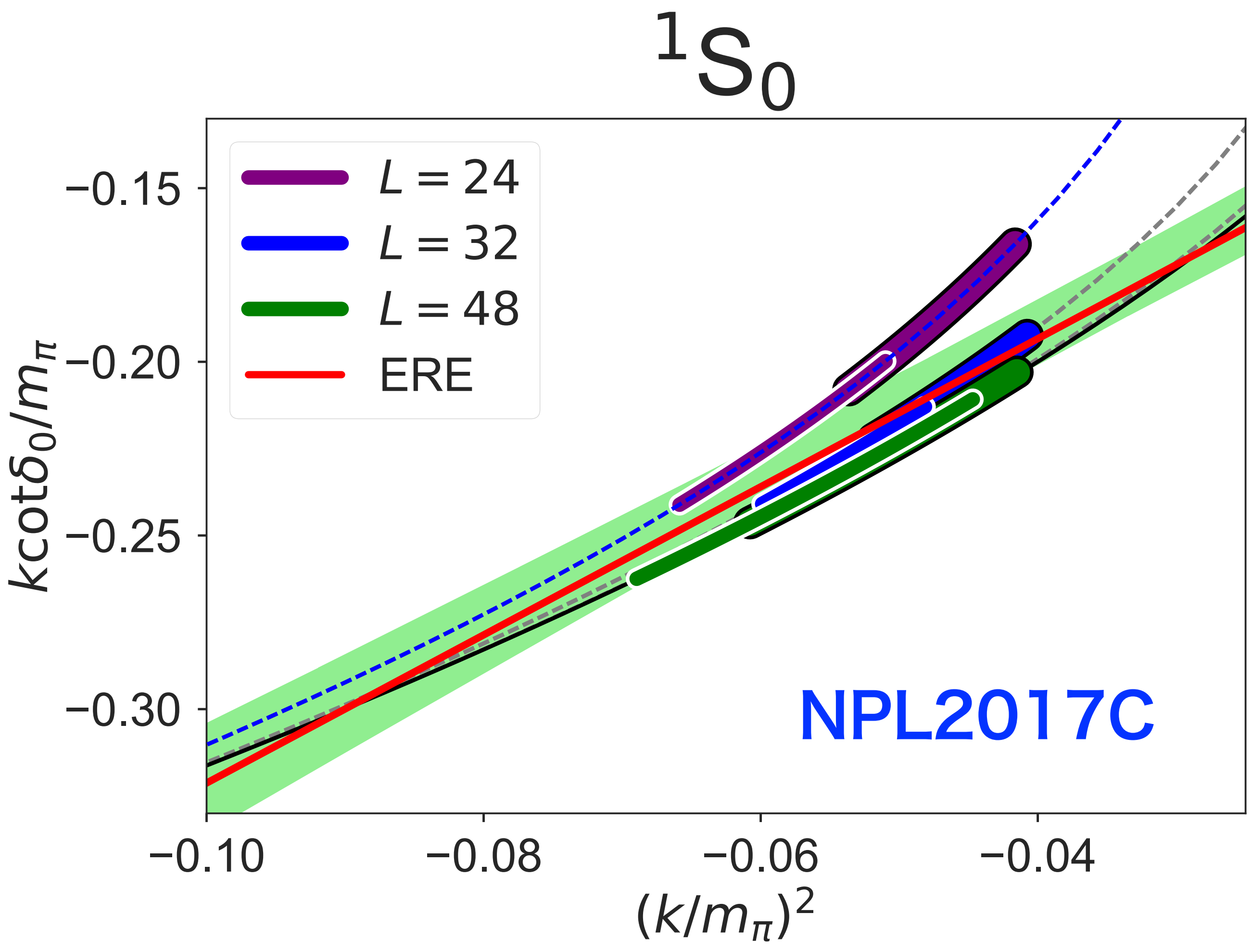
NPL2017C



*Reanalysis posted subsequently in 1706.06550.

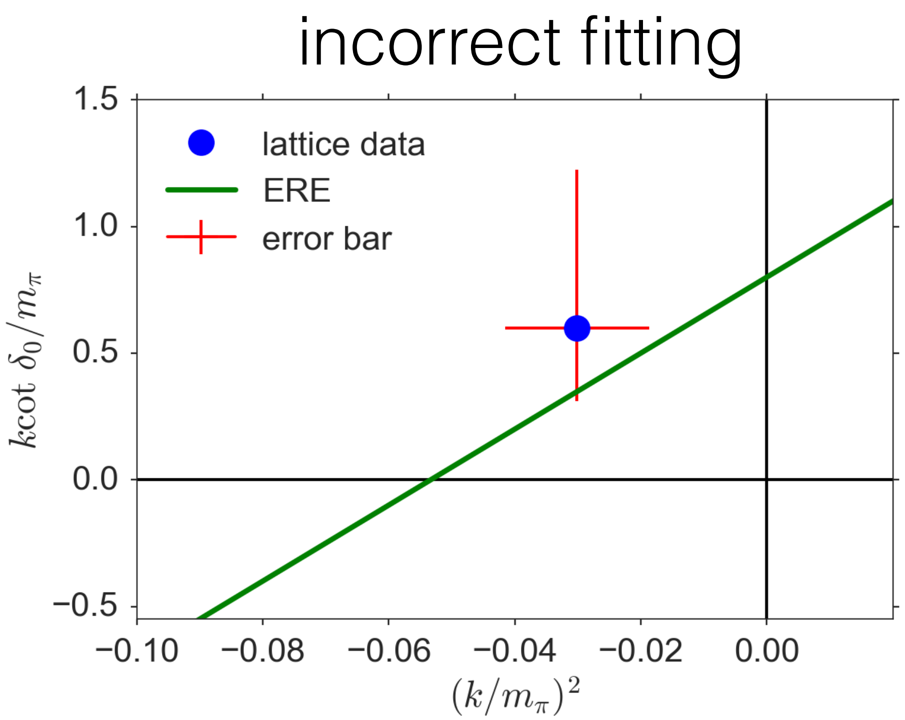
**After this workshop, reanalysis was mentioned in the main text (arXiv: ver3).

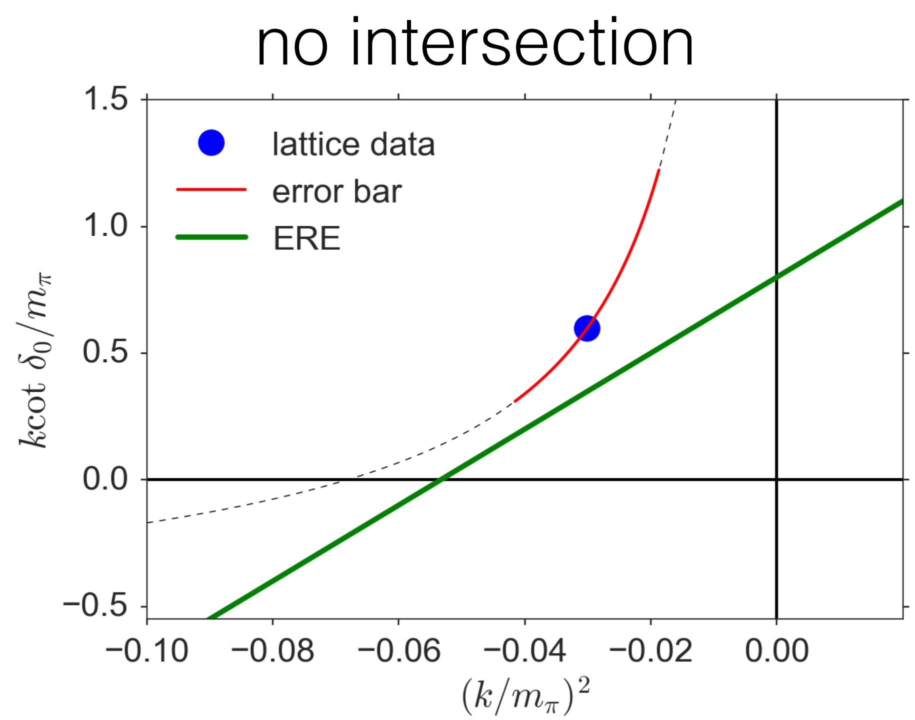
Our reply 3: Their ERE fits ignored correlations.



The central value of this ERE has no intersection with the finite volume formula at L=24.

The χ^2 can not be calculated.





ERE Fits with the finite volume constraint must be employed.

Statement by Z. Davoudi (arXiv:1711.02020 [hep-lat])

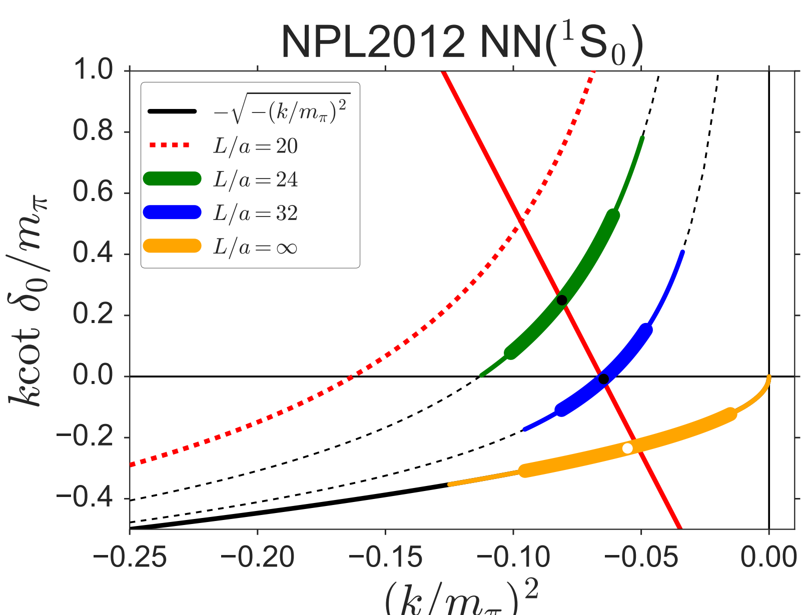
``Consequently the conclusions presented in Ref. [56] concerning other studies must be fully examined before a definite statement can be made regarding the state of the results in literature for multi-nucleon systems.”

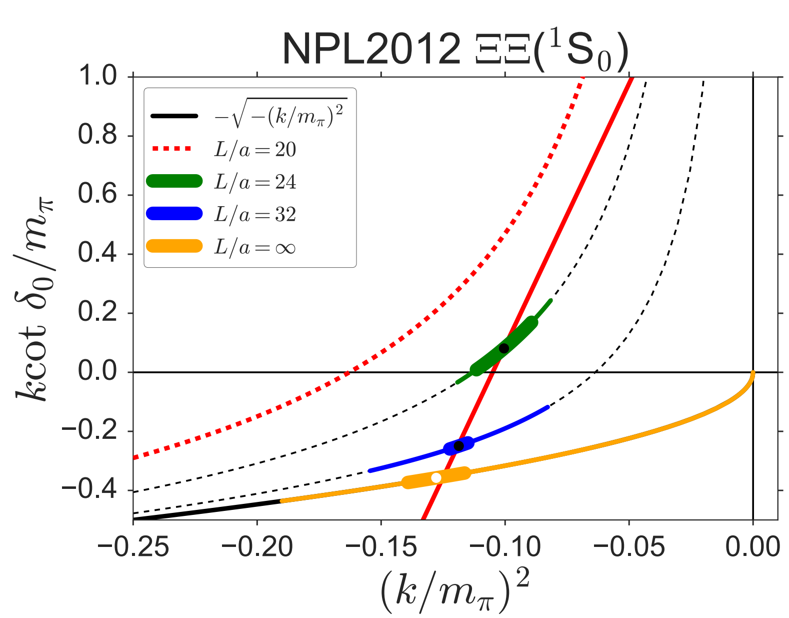
``Nonetheless, these checks (some to be taken with more caution) are quite useful in establishing either the validity of LQCD determination of the finite-volume spectra or the assumptions made about the low-energy parametrization of the scattering amplitude in a given hadronic channel and given the values of the quark masses of the calculation.”

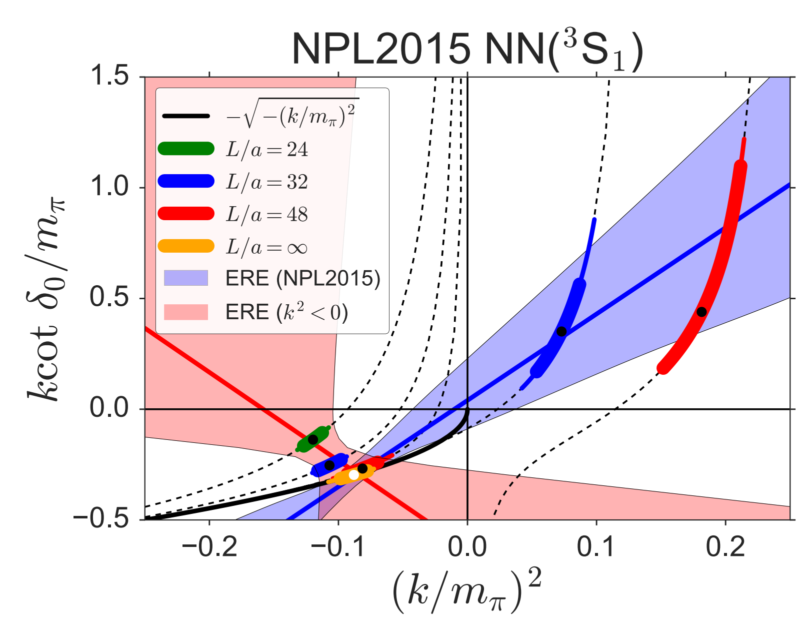
Our comments:

It is necessary but not sufficient to pass the normality check.
Data may not be correct even if they pass the check.
One therefore can not establish the validity.

We would like to know the status of other studies than NPL2013.

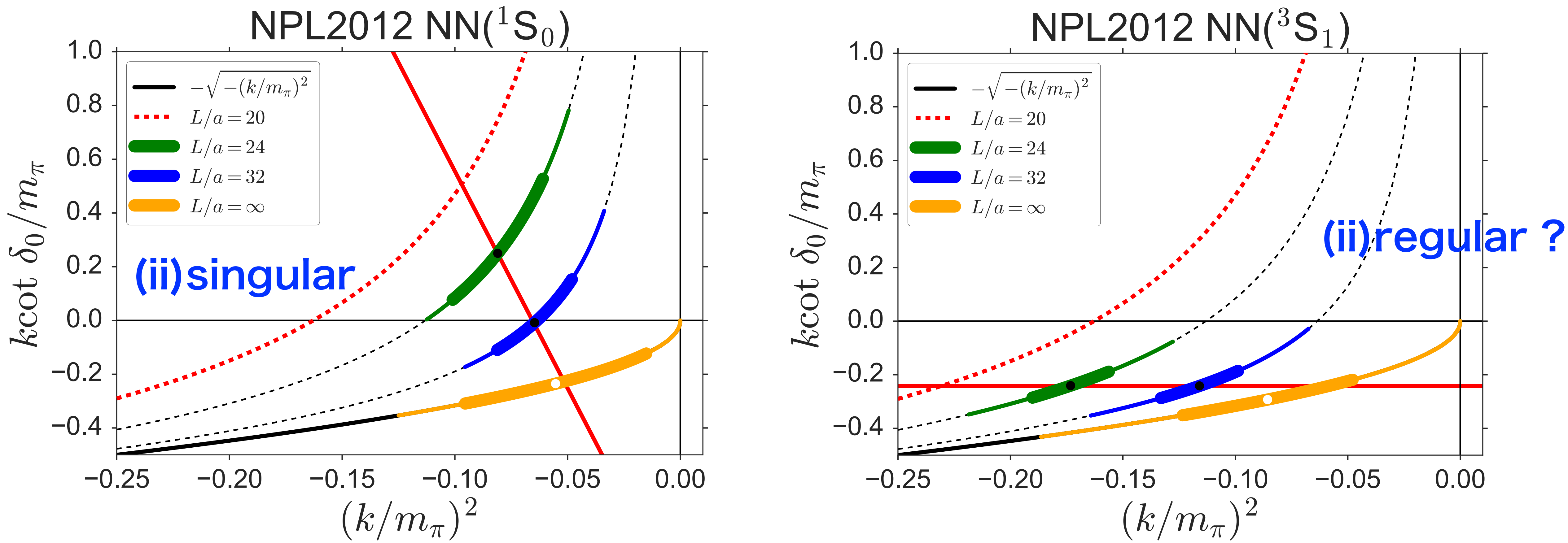




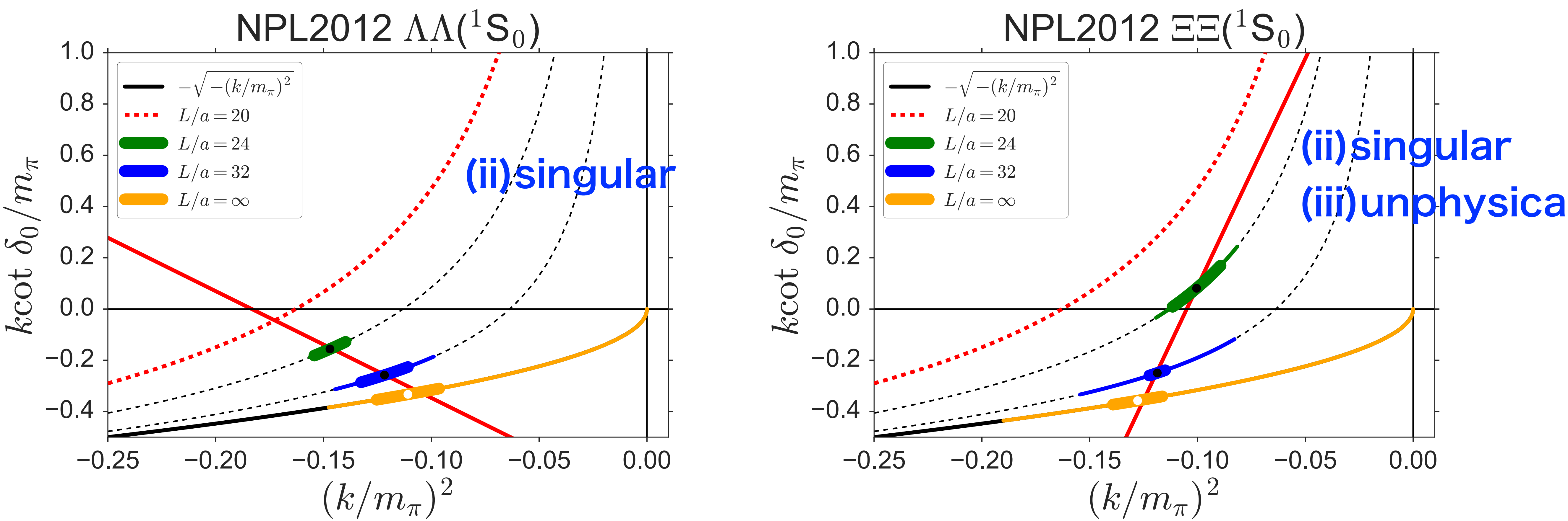


NPL 2012 (PRD85(2012)054511)

$N_f = 2+1$, $m_\pi \simeq 390$ MeV, $a_s \simeq 0.12$(aniso) fm



L=20 data are outside of above figures (inconsistent)



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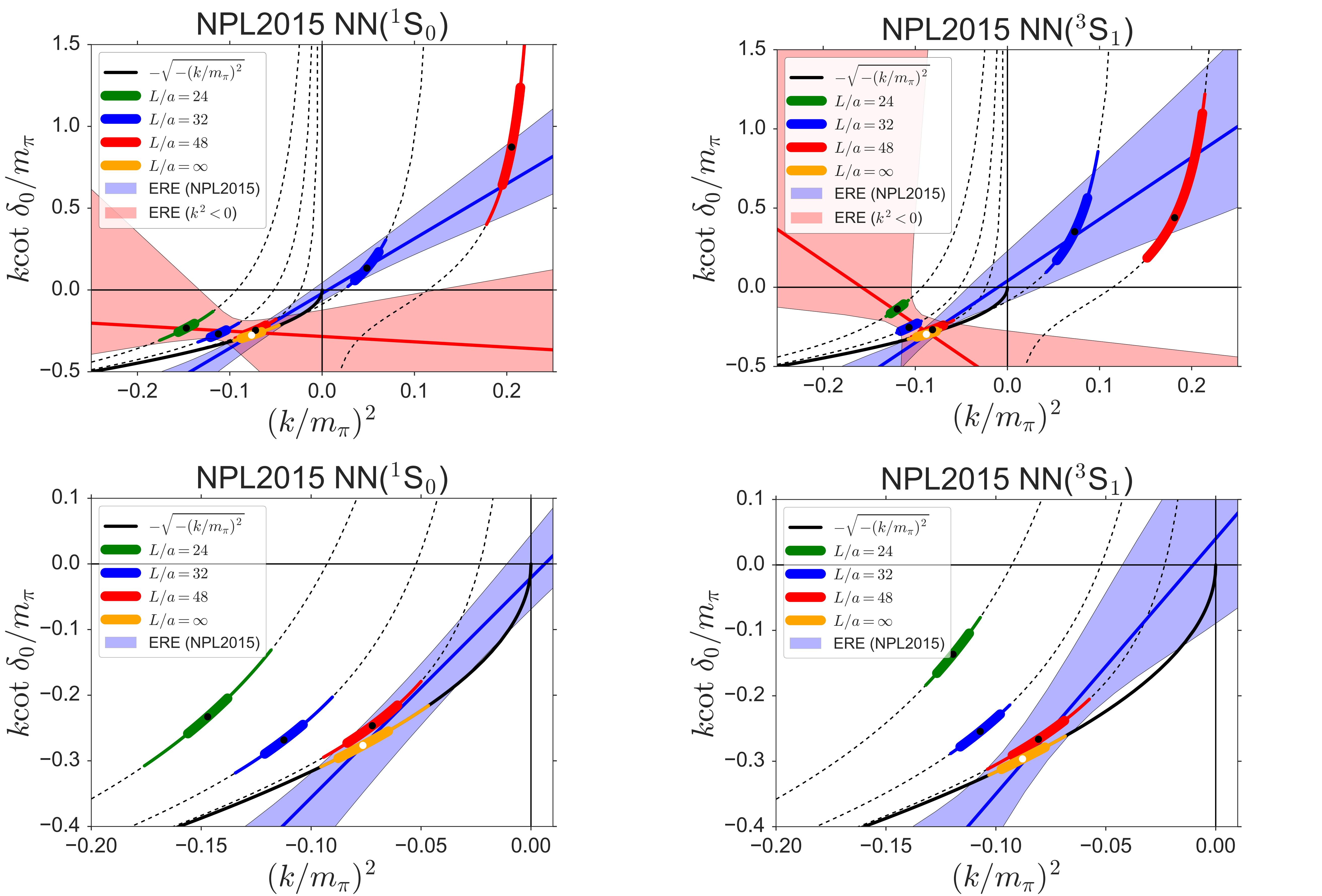
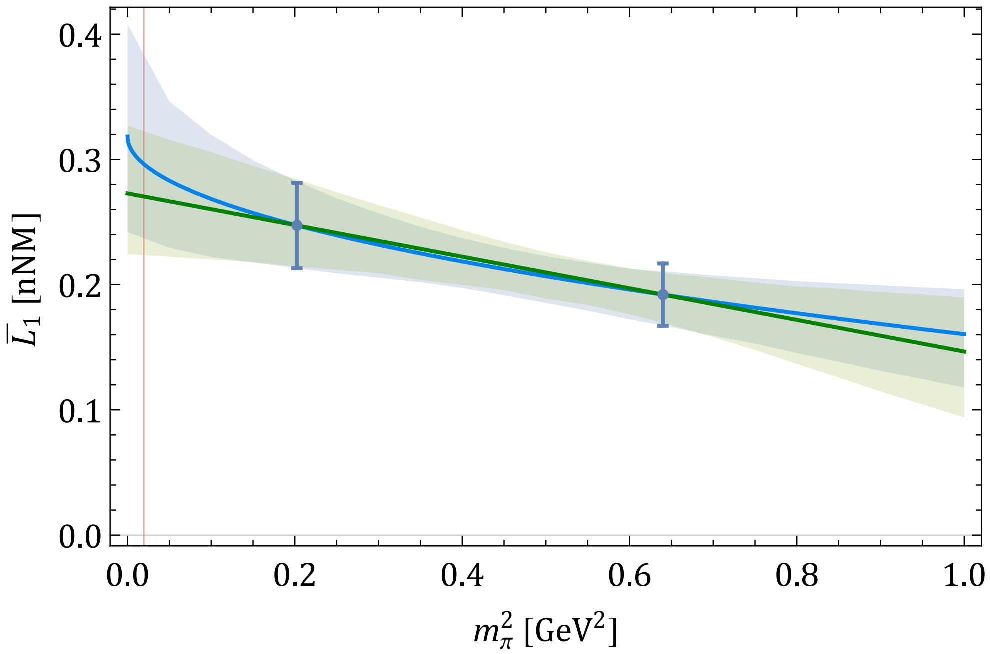
$N_f = 2+1$, $m_\pi \simeq 450$ MeV, $a \simeq 0.12$ fm

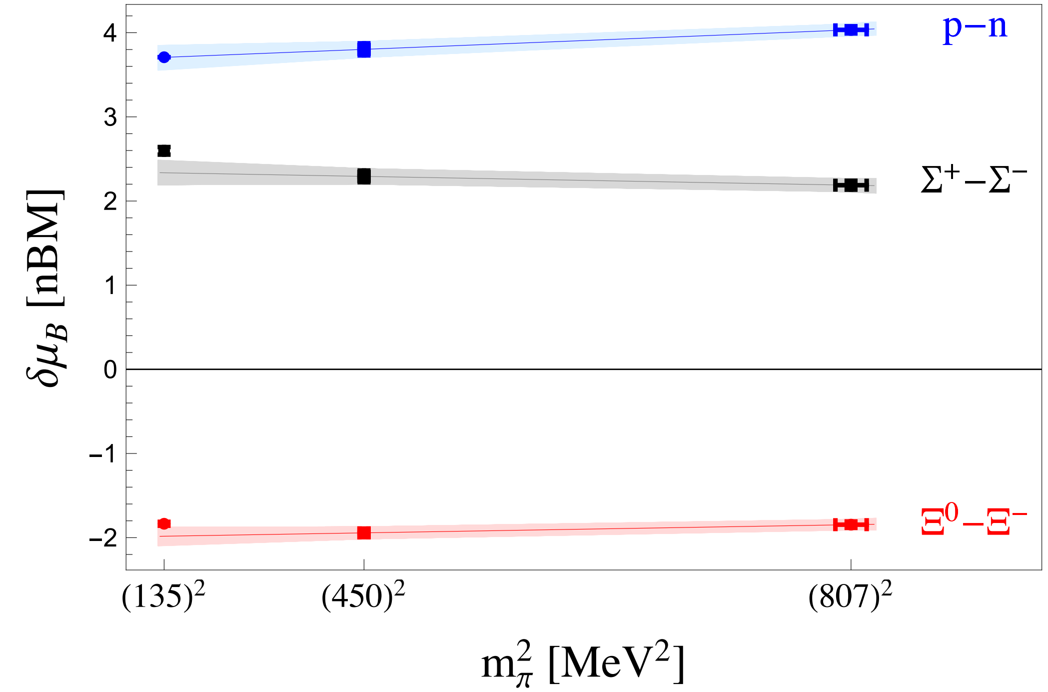


(i)inconsistency or (iii) unphysical pole

Chiral extrapolation[PRL115.132001]

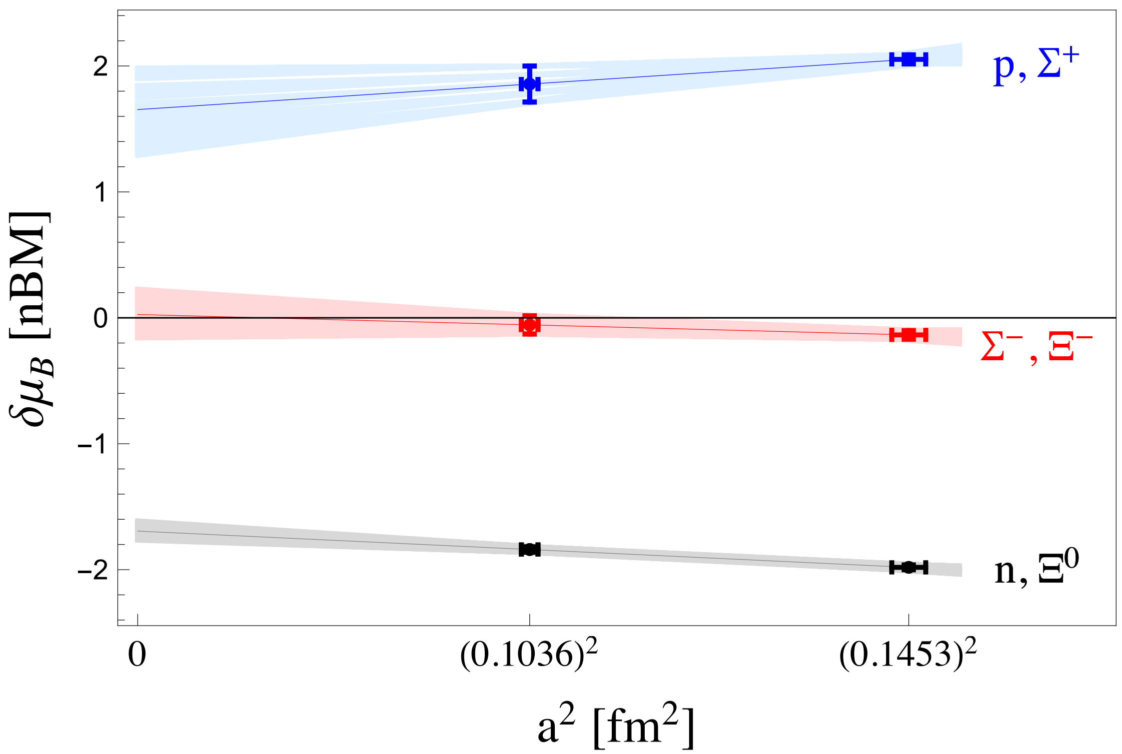
$\beta = 6.1,\ a \simeq 0.11$ fm, $m_\pi \simeq 450,\ 806$ MeV

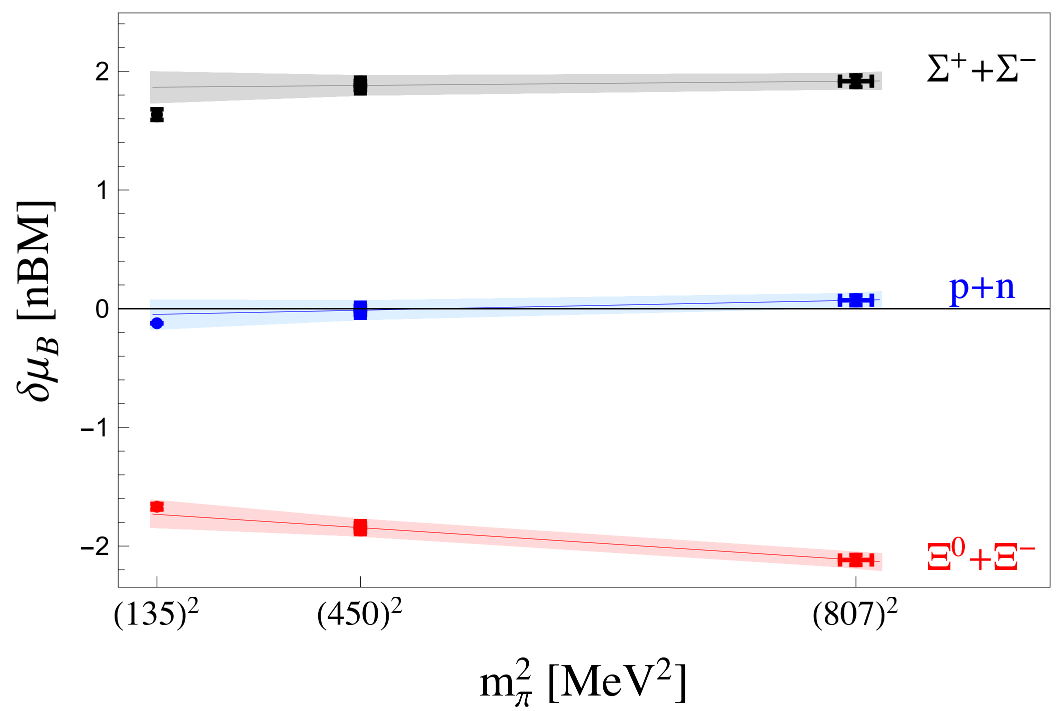




Chiral & continuum extrapolations [PRD95.114513]

a) $m_\pi \simeq 806$ MeV, $\beta = 6.1$, $a \simeq 0.15$ fm
b) $m_\pi \simeq 767$ MeV, $\beta = 6.3$, $a \simeq 0.10$ fm
c) $m_\pi \simeq 450$ MeV, $\beta = 6.1$, $a \simeq 0.12$ fm





Conclusion

The plateau is not enough for multi-baryon systems.

The GEVP is called for.