Lattice Calculation of GPD with Large-Momentum Effective Theory

INT Program INT-18-3,
"Probing Nucleons and Nuclei in High Energy Collisions"
Seattle, Oct. 1-Nov. 16, 2018

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October 5, 2018

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work in preparation.

Outline

1. Large-momentum effective theory
   - Systematic approach to calculate parton physics
   - Prominent applications
2. Factorization formula for the quasi GPD
   - Operator product expansion
   - Correct form of matching coefficient
3. Renormalization and matching for lattice calculation
   - Nonperturbative renormalization on the lattice
   - Matching the quasi-GPD in the RI/MOM scheme

Introduction

The Electron-Ion-Collider (EIC) goals:

- Gluon distributions, small-x physics
- Sea quark distributions
- Gluon spin, parton orbital angular momentum
- 3-D tomography of the nucleon. TMDs, GPDs, Wigner distributions (or GTMDs).
- ......

Lattice QCD calculation of EIC physics is both timely and necessary:

- Kinematic regions and flavor structures not available at experiments;
- Useful information for global analysis of less known quantities, such as transversity PDFs, TMDs, GPDs, Wigner distributions, etc;
- Interplay between theory and experiment in the EIC era.

Parton physics from large-momentum effective theory

Large-momentum effective theory (LaMET) is a systematic approach to calculate parton physics on the light-cone from lattice QCD [1]:

- Parton physics is related to (correlation) operators on the light-cone whose matrix elements cannot be directly calculated on a Euclidean lattice;
- LaMET relates a designed time-independent (or equal-time) quasi-observable in a large-momentum nucleon state to the desired parton observable through a factorization formula where the momentum is the large scale in the power counting;
- The quasi-observable can be directly calculated on the lattice, and LaMET is used to extract the parton observable from it.

[1] Ji, PRL 2013, Sci. China Phys. Mech. Astro., 2014

Example: Collinear PDFs

- Unpolarized quark PDF:

$$q_i(x,\mu) \equiv \int \frac{d\xi^-}{4\pi}\, e^{-ixP^+\xi^-} \langle P|\bar{\psi}_i(\xi^-)\gamma^+ W(\xi^-,0)\psi_i(0)|P\rangle,$$

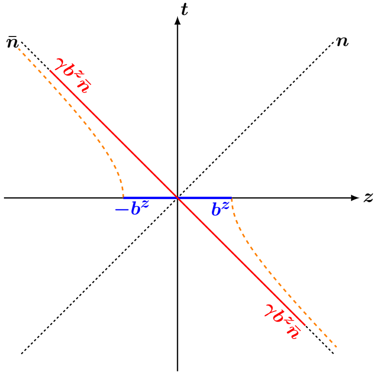
$$W(\xi^-,0) = P\exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right).$$

- Unpolarized quark quasi-PDF:

$$\tilde{q}_i(x,P^z,a^{-1}) \equiv \int \frac{dz}{4\pi}\, e^{ixP^z z} \langle P|\bar{\psi}_i(z)\Gamma W_z(z,0)\psi_i(0)|P\rangle,$$

$$W_z(z,0) = P\exp\left(ig\int_0^{z} dz' A^z(z')\right),$$

$$\Gamma = \gamma^t \text{ or } \gamma^z .$$



Systematic procedure to calculate the PDF

Factorization formula:

$$\tilde{q}_i^X(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|} \, C_{ij}^X\left(\frac{x}{y}, \frac{\tilde{\mu}}{\mu}, \frac{\mu}{|y|P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right),$$

Procedure of calculation:

[2] Lin et al. (LP^3), PRD 2017; Alexandrou et al. (ETMC), NPB 2017.

[3] Chen et al. (LP^3), NPB 2016

[4] Xiong, Ji, Zhang and Y.Z., PRD 2014; I. Stewart and Y.Z., PRD 2017; Y.-S. Liu et al. (LP^3), 2018.

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[4] Xiong, Ji, Zhang and Y.Z., PRD 2014; I. Stewart and Y.Z., PRD 2017; Y.-S. Liu et al. (LP^3), 2018.

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Procedure of calculation:

1. (Nonperturbative) renormalization of the lattice matrix element in a particular scheme "X" [2];
2. Continuum (and infinite volume) limit;
3. Subtraction of mass corrections and higher-twist corrections [3];
4. Perturbative matching to obtain the PDF [4].

[2] Lin et al. (LP^3), PRD 2017; Alexandrou et al. (ETMC), NPB 2017.

[3] Chen et al. (LP^3), NPB 2016

[4] Xiong, Ji, Zhang and Y.Z., PRD 2014; I. Stewart and Y.Z., PRD 2017; Y.-S. Liu et al. (LP^3), 2018.

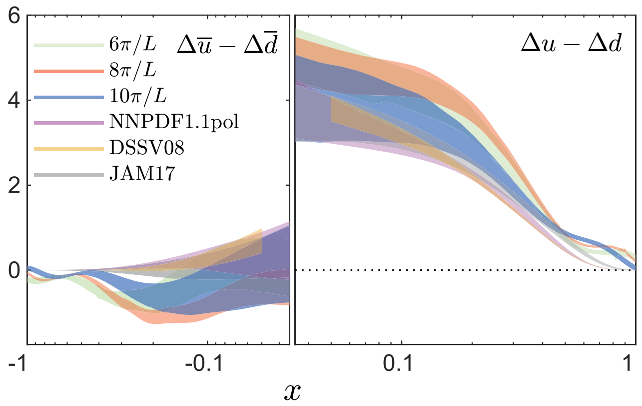
Prominent applications

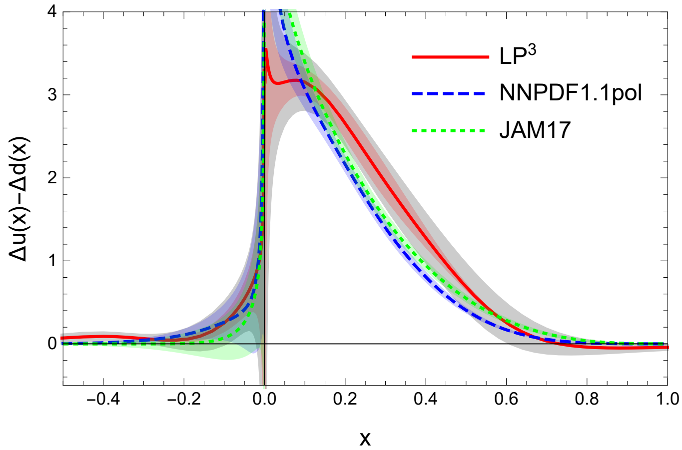
- Gluon helicity contribution to the proton spin [5]
- Unpolarized iso-vector quark PDF of proton [6]
- Iso-vector quark helicity PDF of proton
- Iso-vector quark transversity PDF of proton
- Meson distribution amplitudes of proton
- Iso-vector quark PDF of pion

[5] Y.-B. Yang et al.(χQCD), PRL 2017.

[6] See works by LP^3, ETMC collaborations.

Iso-vector quark helicity PDF of proton





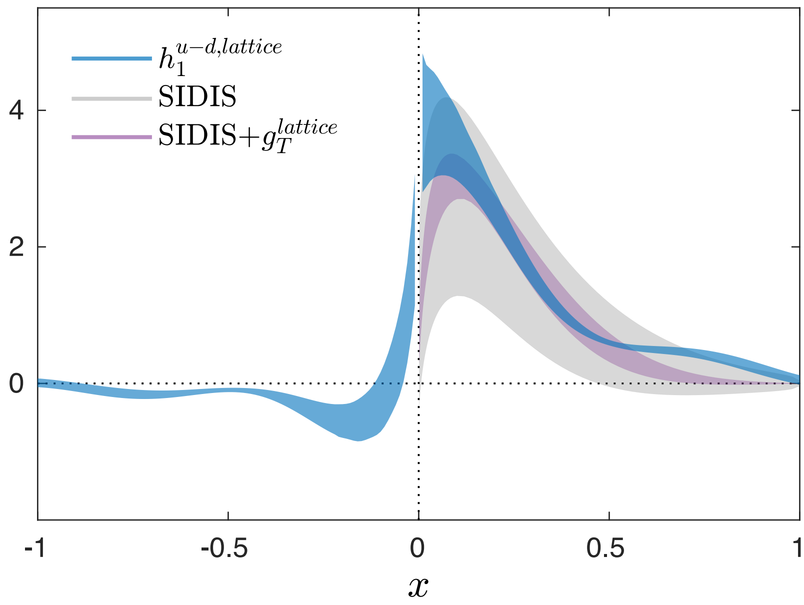
C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, (ETMC), PRL 2018

Dynamical N_f=2+1+1 twisted mass fermions, a=0.09fm, L=4.8fm, $m_\pi \sim$130 MeV, P^z=1.4 GeV, μ=2 GeV.

H.-W. Lin, J.W. Chen, X. Ji, L. Jin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, and Y.Z., (LP^3), 1807.07431

Clover valence fermions on N_f=2+1+1 flavors of HISQ generated by MILC, a=0.09fm, L=5.8fm, $m_\pi \sim$135 MeV, P^z=3.0 GeV, μ=3.0 GeV.

Iso-vector quark transversity PDF of proton



C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, (ETMC), 1807.00232

Dynamical N_f=2+1+1 twisted mass fermions, a=0.09fm, L=4.5fm, m_π ~130 MeV, P^z=1.4 GeV, $\mu = \sqrt{2}$ GeV.

GPD and quasi-GPD

- Unpolarized GPD:

$$F_i(x,\xi,t,\mu) \equiv \int \frac{d\xi^-}{4\pi}\, e^{-ix\bar{P}^+\xi^-} \langle P' | \bar{\psi}_i(\frac{\xi^-}{2})\gamma^+ W\left(\frac{\xi^-}{2}, -\frac{\xi^-}{2}\right)\psi_i(-\frac{\xi^-}{2}) | P\rangle,$$

$$\xi \equiv -\frac{P'^+ - P^+}{P'^+ + P^+}, \quad t = \Delta^2 \equiv (P' - P)^2, \quad \bar{P} = \frac{P + P'}{2}.$$

- Unpolarized quasi-GPD:

$$\tilde{F}_i(x, \bar{P}^z, \tilde{\xi}, t, a^{-1}) \equiv \int \frac{dz}{4\pi}\, e^{ix\bar{P}^z z} \langle P' | \bar{\psi}_i(\frac{z}{2})\Gamma W_z\left(\frac{z}{2}, -\frac{z}{2}\right)\psi_i(-\frac{z}{2}) | P\rangle,$$

$$\tilde{\xi} = -\frac{P'^z - P^z}{P'^z + P^z} \approx -\frac{P'^+ - P^+}{P'^+ + P^+} = \xi\,,$$

$$\Gamma = \gamma^t \text{ or } \gamma^z\,.$$

Factorization formula for the quasi-GPD?

Similar to the quasi-PDF, one can anticipate that the factorization formula takes the form,

$$\tilde{F}_i^X(x, \bar{P}^z, \xi, t, \tilde{\mu}) \\ = \int_{-1}^{+1} \frac{dy}{|y|} C_{ij}^X\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\tilde{\mu}}{\mu}, \frac{\mu}{|y|\bar{P}^z}\right) F_j(y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right),$$

- The one-loop matching coefficient was calculated for bare quasi-GPD and GPD in a transverse-momentum cutoff regularization scheme [7];
- Rigorous derivation of the factorization formula not done yet, and it is nontrivial to determine the dependence of C^X on ξ and $y\bar{P}^z$ [8];
- Transverse-momentum cutoff regularization scheme is not suitable for nonperturbative renormalization on the lattice.

[7] Ji, Schäfer, Xiong and Zhang, PRD 2015

[8] Izubuchi, Ji, Jin, Stewart and Zhao, PRD 2018.

Derivation of the factorization formula with operator product expansion

Consider the nonlocal bilinear operator that defines the quasi-PDF and quasi-GPD,

$$\tilde{O}_\Gamma(z) = \bar{\psi}(\frac{z}{2})\Gamma W_z\left(\frac{z}{2}, -\frac{z}{2}\right)\psi(-\frac{z}{2}),$$

In coordinate space, $\tilde{O}_\Gamma(z)$ can be multiplicatively renormalized [9]

$$\tilde{O}_\Gamma(z, \mu) = Z_{\psi,z}\, e^{\delta m|z|}\tilde{O}_\Gamma(z, \epsilon).$$

- δm subtracts the linear power divergences (if it exist);
- $Z_{\psi,z}$ renormalizes the logarithmic divergences.

[9] Ji, Zhang and Zhao, PRL 2018; Ishikawa, Ma, Qiu and Yoshida, PRD 2017; Green, Jansen and Steffens, PRL 2018.

Derivation of the factorization formula for quasi-GPD

Operator product expansion (OPE) of $\tilde{O}_\Gamma(z,\mu)$ (in the $\overline{\rm MS}$ scheme) in the limit of $|z| \to 0$:

$$\tilde{O}_{e\cdot\gamma}(z,\mu) = \sum_{n=0}^{\infty}\Bigg[C_n(\mu^2 z^2)\frac{(-iz)^n}{n!}e_{\mu_1}\cdots e_{\mu_n}O_1^{\mu_0\mu_1\cdots\mu_n}(\mu)$$
$$+C'_n(\mu^2 z^2)\frac{(-iz)^n}{n!}e_{\mu_1}\cdots e_{\mu_n}O_2^{\mu_0\mu_1\cdots\mu_n}(\mu) + \text{higher-twist operators}\Bigg],$$

where $e^\mu = (0,0,0,1)$, and the local twist-two operators are

$$O_1^{\mu_0\mu_1\dots\mu_n}(\mu) = Z^{qq}_{n+1}\bar{\psi}\gamma^{\{\mu_0}iD^{\mu_1}\cdots iD^{\mu_n\}}\psi\,,$$
$$O_2^{\mu_0\mu_1\dots\mu_n}(\mu) = Z^{qg}_{n+1}F^{\{\mu_0\rho}iD^{\mu_1}\cdots iD^{\mu_{n-1}}F_\rho{}^{\mu_n\}}\,,$$

with $Z^{ij}_{n+1} = Z^{ij}_{n+1}(\mu,\epsilon)$, and $\{\cdots\}$ means symmetrized and traceless. Using the above OPE, one can prove the factorization formula of the quasi-PDF [10]. Note that the above OPE is for the forward case only.

[10] Ma and Qiu, PRL 2018; Izubuchi, Ji, Jin, Stewart and Zhao, PRD 2018.

OPE for the off-forward case

For the off-forward case, the twist-two operators will mix with twist-two operators with total derivatives [11],

$$\mu^2\frac{d}{d\mu^2}\left[\bar{\psi}\gamma^{\{\mu_0}i\overleftrightarrow{D}^{\mu_1}\dots i\overleftrightarrow{D}^{\mu_n\}}\psi\right]$$
$$=\sum_{m=0,\text{even}}^{n}\Gamma_{nm}(\alpha_s(\mu))\left[i\bar{\partial}^{\{\mu_1}\dots i\bar{\partial}^{\mu_m}\bar{\psi}\gamma^{\mu_0}i\overleftrightarrow{D}^{\mu_{m+1}}\dots i\overleftrightarrow{D}^{\mu_n\}}\psi\right],$$

where the anomalous dimension Γ_{nm} is an upper-triangle matrix. At leading-log, the above equations can be diagonalized by the conformal operators with the leading eigen vector [12],

$$O_1^{n,0}=(ie\cdot\bar{\partial})^n\bar{\psi}(e\cdot\gamma)C_n^{3/2}\left(\frac{ie\cdot\overleftrightarrow{D}}{ie\cdot\overleftarrow{\partial}+ie\cdot\overrightarrow{\partial}}\right)\psi-\text{trace}\,,$$

where $C_n^{3/2}$ is the Geigenbauer polynomial.

[11] M. Diehl, Phys. Rept. 2003

[12] Efremov and Radyushkin, Theor. Math. Phys. 1980

OPE for the off-forward case

At higher orders, we can always diagonalize an upper-triangle matrix, and can write the eigen-vectors in more general forms

$$O_1^{n,m} = (ie\cdot\bar\partial)^n \bar\psi (e\cdot\gamma) C_{n,m}\left(\frac{ie\cdot\overleftrightarrow{D}}{ie\cdot\overleftarrow{\partial}+ie\cdot\overrightarrow{\partial}}\right)\psi - \text{trace}\,,$$

We can re-express the OPE (for the non-singlet case) in terms of the comformal operators which do not mix under renormalization:

$$\tilde{O}_{e\cdot\gamma}(z,\mu) = \sum_{n=0}^{\infty}\left[C_n(\mu^2z^2)\frac{(-iz)^n}{n!}\sum_{m=0,\text{even}}^{n}\lambda_{n,m}O_1^{n,m}(\mu) + \text{higher-twist}\right]\,,$$

where $\lambda_{n,0}=1$, $\lambda_{n,m\neq 0} = O(\alpha_s)$ or 0? (do not affect the conclusion.)

When evaluated in a large-momentum nucleon state, the comformal moments are

$$\begin{aligned}\langle P|O_1^{n,m}(\mu)|P\rangle =& 2a_{n+1}^m(\mu)\left[(e\cdot\bar P)^{n+1} - \text{trace}\right]\,,\\ a_{n+1}^m(\mu) =& \xi^n\int_{-1}^{1}dy\; C_{n,m}\left(\frac{y}{\xi}\right)F(y,\xi,t,\mu)\,. \end{aligned}\qquad (1)$$

OPE for the off-forward case

The full off-forward matrix element of $\tilde{O}_\Gamma(z,\mu)$ is

$$\langle P|\tilde{O}_{\gamma^z}(z,\mu)|P\rangle = 2\bar{P}^z \sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(iz\bar{P}^z)^n}{n!} \sum_{m=0,\text{even}}^{n} \lambda_{n,m}\xi^n \times \int_{-1}^{1} dy\, C_{n,m}\left(\frac{y}{\xi}\right) F(y,\xi,t,\mu) + O(\frac{\bar{P}^2}{\bar{P}_z^2}, z^2\Lambda_{\text{QCD}}^2)\,,$$

Fourier transform to x-space,

$$\begin{aligned}\tilde{F}(x,\bar{P}^z,\xi,t,\mu) &= \int \frac{dz}{4\pi} e^{ixz\bar{P}^z} \langle P|\tilde{O}_{\gamma^z}(z,\mu)|P\rangle \\ &= \int \frac{dz\bar{P}^z}{2\pi} e^{ixz\bar{P}^z} \sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(iz\bar{P}^z)^n}{n!} \sum_{m=0,\text{even}}^{n} \lambda_{n,m}\xi^n \\ &\quad \times \int_{-1}^{1} dy\, C_{n,m}\left(\frac{y}{\xi}\right) F(y,\xi,t,\mu) + O(\frac{\bar{P}^2}{\bar{P}_z^2}, \frac{\Lambda_{\text{QCD}}^2}{x^2\bar{P}_z^2})\,,\end{aligned}$$

Correct form of matching coefficient

$$\tilde{F}(x,\bar{P}^z,\xi,t,\mu)=\int\frac{dz\bar{P}^z\xi}{2\pi|\xi|}e^{i\frac{x}{\xi}z\bar{P}^z\xi}\sum_{n=0}^{\infty}C_n\left(\mu^2\frac{(z\bar{P}^z\xi)^2}{\xi^2\bar{P}_z^2}\right)\frac{(iz\bar{P}^z\xi)^n}{n!}\sum_{m=0,\text{even}}^{n}\lambda_{n,m}$$

$$\times\int_{-1}^{1}dy\,C_{n,m}\left(\frac{y}{\xi}\right)F(y,\xi,t,\mu)+O(\frac{\bar{P}^2}{\bar{P}_z^2},\frac{\Lambda_{\text{QCD}}^2}{x^2\bar{P}_z^2}),$$

$$=\int_{-1}^{1}\frac{dy}{|\xi|}\,\mathcal{C}\left(\frac{x}{\xi},\frac{y}{\xi},\frac{\mu^2}{\xi^2\bar{P}_z^2}\right)F(y,\xi,t,\mu)+O(\frac{\bar{P}^2}{\bar{P}_z^2},\frac{\Lambda_{\text{QCD}}^2}{x^2\bar{P}_z^2}),$$

Or, by using the identity that

$$\xi^nC_{n,m}\left(\frac{y}{\xi}\right)=y^nC_{n,[\frac{n}{2}]-m}\left(\frac{\xi}{y}\right),$$

we have

$$\tilde{F}(x,\bar{P}^z,\xi,t,\mu)=\int_{-1}^{1}\frac{dy}{|y|}\,\mathcal{C}\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu^2}{y^2\bar{P}_z^2}\right)F(y,\xi,t,\mu)+O(\frac{\bar{P}^2}{\bar{P}_z^2},\frac{\Lambda_{\text{QCD}}^2}{x^2\bar{P}_z^2}).$$

Special limits

Both forms are equivalent and similar to the evolution kernel of GPD.

- Distribution amplitude limit $\xi \to 1$ [13]:

$$\tilde{F}(x, \bar{P}^z, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|}\, \mathcal{C}\left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu^2}{\xi^2 \bar{P}_z^2}\right) F(y, \xi, t, \mu) + O\left(\frac{\bar{P}^2}{\bar{P}_z^2}, z^2\Lambda_{\rm QCD}^2\right),$$

$$\lim_{\xi\to 1} \tilde{F}(x, \bar{P}^z, \xi, t, \mu) = \int_{-1}^{1} dy\, \mathcal{C}\left(x, y, \frac{\mu^2}{\bar{P}_z^2}\right) F(y, \xi, t, \mu) + O\left(\frac{\bar{P}^2}{\bar{P}_z^2}, z^2\Lambda_{\rm QCD}^2\right), \tag{2}$$

- PDF in the forward limit $\xi \to 0$:

$$\tilde{F}(x, \bar{P}^z, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, \mathcal{C}\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu^2}{y^2 \bar{P}_z^2}\right) F(y, \xi, t, \mu) + O\left(\frac{\bar{P}^2}{\bar{P}_z^2}, z^2\Lambda_{\rm QCD}^2\right).$$

$$\tilde{F}(x, \bar{P}^z, 0, 0, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, \mathcal{C}\left(\frac{x}{y}, 0, \frac{\mu^2}{y^2 \bar{P}_z^2}\right) F(y, 0, 0, \mu) + O\left(\frac{\bar{P}^2}{\bar{P}_z^2}, z^2\Lambda_{\rm QCD}^2\right).$$

[13] J. Xu, Q.-A. Zhang and S. Zhao, PRD 2018

Nonperturbative renormalization on the lattice

The renormalization of the operator does not depend on the external state, so the renormalization factor for the quasi-GPD is the same as the quasi-PDF case.

Regularization-independent momentum subtraction (RI/MOM) scheme: for a quark state that is far off-shell $p^2 \gg \Lambda_{\rm QCD}^2$, the renormalization factor is obtained by imposing the subtraction condition [14]

$$
\begin{aligned}
&Z_{\rm OM}^{-1}(z, a^{-1}, p_z^R, \mu_R)\langle p|\tilde{Q}_\Gamma(z, a^{-1})|p\rangle\Big|_{p^2=\mu_R^2, p_z=p_z^R} \\
=&\langle p|\tilde{Q}_\Gamma(z, a^{-1})|p\rangle_{\rm tree}\,.
\end{aligned}
$$

$Z_{\rm OM}$ can be nonperturbatively calculated on the lattice, and used to renormalize the nucleon matrix elements of quasi-GPD.

[14] Constantinou et al., PRD 2017; Stewart and Y.Z., PRD 2017; C. Alexandrou et al. (ETMC), NPB 2017; H.-W. Lin et al. (LP^3), PRD 2017; Y.-S. Liu et al. (LP^3), 2018

Matching the quasi-GPD in the RI/MOM scheme

The renormalized quasi-GPD should be independent of the UV regulator, so it must be the same in dimensional regularization (dim reg) $d = 4 - 2\epsilon$. Therefore, the matching coefficient can be easily calculated with dim reg.

At one-loop,

$$Z_{\rm OM}^{-1}(z, \epsilon, p_z^R, \mu_R) = 1 - \int_{-\infty}^{\infty} dx \left[e^{-i(x-1)zp_z^R} - 1 \right] \tilde{q}^{(1)}(x, p_z^R, \epsilon, \mu_R^2) \,,$$

where $\tilde{q}^{(1)}(x, p_z^R, \mu_R^2)$ is the one-loop correction to the quark quasi-PDF, which is already calculated [15].
Next we calculate the quasi-GPD in an on-shell quark state (for the purpose of matching), and renormalize it with $Z_{\rm OM}$.

[15] Stewart and Y.Z., PRD 2017; Y.-S. Liu et al. (LP^3), 2018

Matching the quasi-GPD in the RI/MOM scheme

In coordinate space, the renormalized quasi-GPD is

$$\langle p'|\tilde{O}^R_\Gamma(z,\epsilon)|p\rangle = Z^{-1}_{\rm OM}(z,\epsilon,p^R_z,\mu_R)\langle p'|\tilde{O}_\Gamma(z,\epsilon)|p\rangle\,.$$

Knowing that the bare quasi-GPD can be matched onto the GPD up to an $\overline{\rm MS}$ renormalization,

$$\begin{aligned}\langle p'|\tilde{O}_\Gamma(z,\epsilon)|p\rangle =& Z_{\overline{\rm MS}}(\epsilon)\int dx\ e^{-ixzp^z}\tilde{F}(x,p^z,\xi,t,\mu)\\ =& Z_{\overline{\rm MS}}(\epsilon)\int dx\ e^{-ixzp^z}\int_{-1}^{1}\frac{dy}{|y|}\,\mathcal{C}\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu^2}{y^2\bar{P}_z^2}\right)F(y,\xi,t,\mu)\,,\end{aligned}$$

we can easily derive the relationship between the $\overline{\rm MS}$ and RI/MOM matching coefficients at one-loop order [16],

$$\begin{aligned}\mathcal{C}\left(\frac{x}{y},\frac{\xi}{y},\frac{y\bar{P}^z}{p^R_z},\frac{\mu_R}{p^R_z},\frac{\mu^2}{y^2\bar{P}_z^2}\right) =& \mathcal{C}\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu^2}{y^2\bar{P}_z^2}\right)-\frac{y\bar{P}^z}{p^R_z}\tilde{q}^{(1)}\left(1+\frac{P^z}{p^R_z}(x-y),p^R_z,\epsilon,\mu_R^2\right)\\ &+Z_{\overline{\rm MS}}(\epsilon)+\int dx'\ \tilde{q}^{(1)}(x',p^R_z,\epsilon,\mu_R^2)\,.\end{aligned}$$

[16] Y.-S. Liu, W. Wang, J. Xu, J.-H. Zhang, Q.-A. Zhang, S. Zhao and Y.Z., work in preparation.

Matching the quasi-GPD in the RI/MOM scheme

Discussion:

- The matching correction is supposed to cancel out all the dependence on the intermediate scales p_z^R, μ_R, as well as $\bar{P}^z$, up to power corrections;
- Remant depedence on p_z^R and μ_R could be higher-order perturbative effects;
- The $\overline{\mathrm{MS}}$ matching coefficient is independent of the infrared regulator, so one can choose either massless or massive quarks for convenience of calculation;
- The $\overline{\mathrm{MS}}$ matching coefficient is very similar to the transverse-momentum scheme matching. The latter can serve as a cross check.

Summary

- We use OPE to rigorously derive the factorization formula for the quasi-GPD;
- The quasi-GPD can be renormalized the same way as the quasi-PDF;
- Perturbative matching for the RI/MOM quasi-GPD has been derived at one-loop order.