TMDs at small-x

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Probing the Weizsacker-Williams gluon distribution at EIC

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Ref: Dominguez, Xiao, Yuan, arXiv:1009.2141

Nice things about transverse momentum distributions (TMDs)

- Universality and a universal language
  - DIS/Drell-Yan
  - eA/pA/AA(?), small-x wave functions of nucleus
- QCD dynamics
  - TMD evolution
  - Small-x evolution

Among recent developments

- Spin-dependent TMD gluon at small-x
  - Related to the spin-dependent odderon, Boer-Echevarria-Mulders-Zhou, PRL 2016
  - Gluon/quark helicity distributions, Kovchegov-Pitonyak-Sievert, 2016, 2017, 2018
- Subleading power corrections in the TMD gluon/quark distributions
  - Balitsky-Tarasov, 2017, 2018
- Sudakov resummation for small-x TMDs
  - Mueller-Xiao-Yuan, PRL110, 082301 (2013); Xiao-Yuan-Zhou, NPB921, 104 (2017); Zhou 2018
  - Balitsky-Tarasov, JHEP1510,017 (2015)

TMDs: Conventional gluon distribution

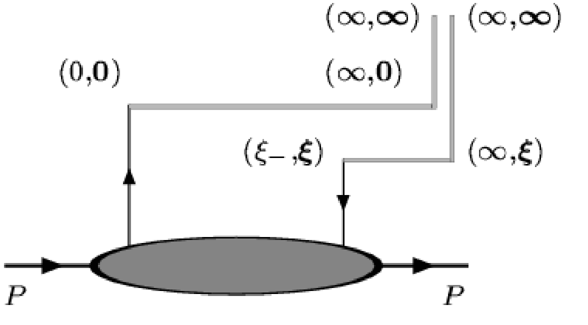
- Collins-Soper, 1981

$$xG^{(1)}(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \times \langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle$$

  - Gauge link in the adjoint representation

$$\mathcal{L}_\xi = \mathcal{P}\exp\{-ig\int_{\xi^-}^\infty d\zeta^- A^+(\zeta,\xi_\perp)\}$$

$$\mathcal{P}\exp\{-ig\int_{\xi_\perp}^\infty d\zeta_\perp\cdot A_\perp(\zeta^-=\infty,\zeta_\perp)\}$$



Physical interpretation

- Choosing light-cone gauge, with certain boundary condition (either one, but not the principal value) $A_\perp(\zeta^- = \infty) = 0$
- Gauge link contributions can be dropped
- Number density interpretation, and can be calculated from the wave functions of nucleus
  - McLerran-Venugopalan
  - Kovchegov-Mueller

Classic YM theory

- McLerran-Venugopalan

$$xG^{(1)}(x,k_\perp) = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\int\frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_s^2}{4}}\right)$$

  - See also, Kovchegov-Mueller
  - We can reproduce this gluon distribution using the TMD definition with gauge link contribution, following BJY 02, BHPS 02
  - WW gluon distribution is the conventional one

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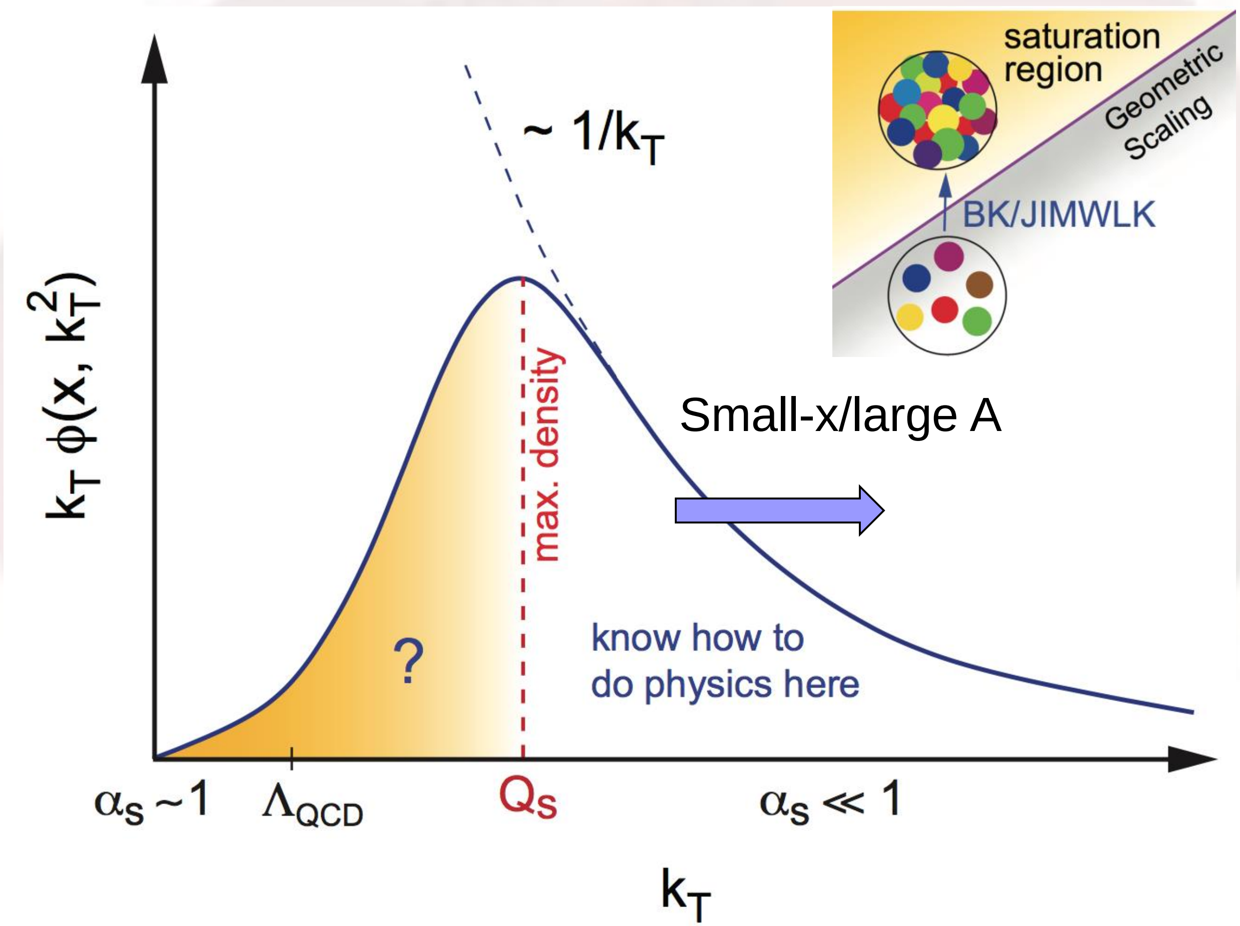
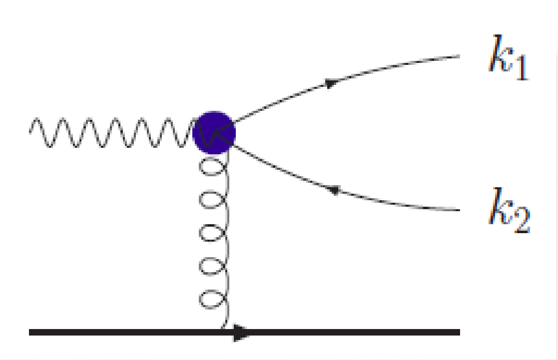
Saturation at small-x/large A



DIS dijet probes **WW** gluons

$$\gamma_T^* A \to q(k_1) + \bar{q}(k_2) + X$$



- Hard interaction includes the gluon attachments to both quark and antiquark
- The q_t dependence is the gluon distribution w/o gauge link contribution at this order

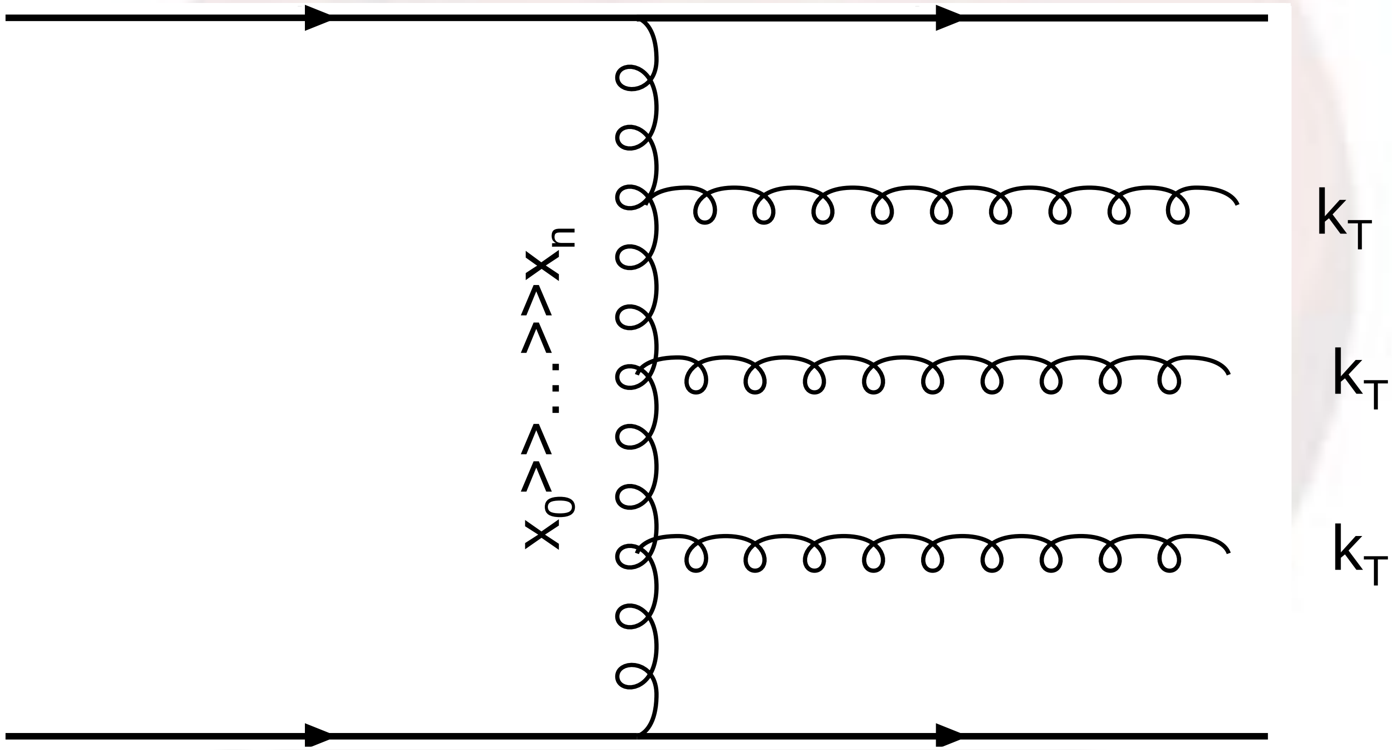
Dominguez-Marquet-Xiao-Yuan 2011

QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014
Xiao-Yuan-Zhou 2016

High energy scattering



BFKL: $\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{K} \bigotimes \mathcal{F}$ Un-integrated gluon distribution

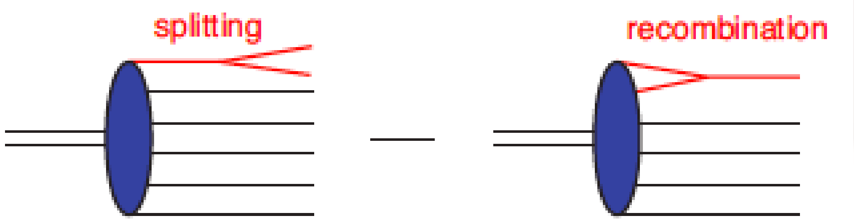
Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

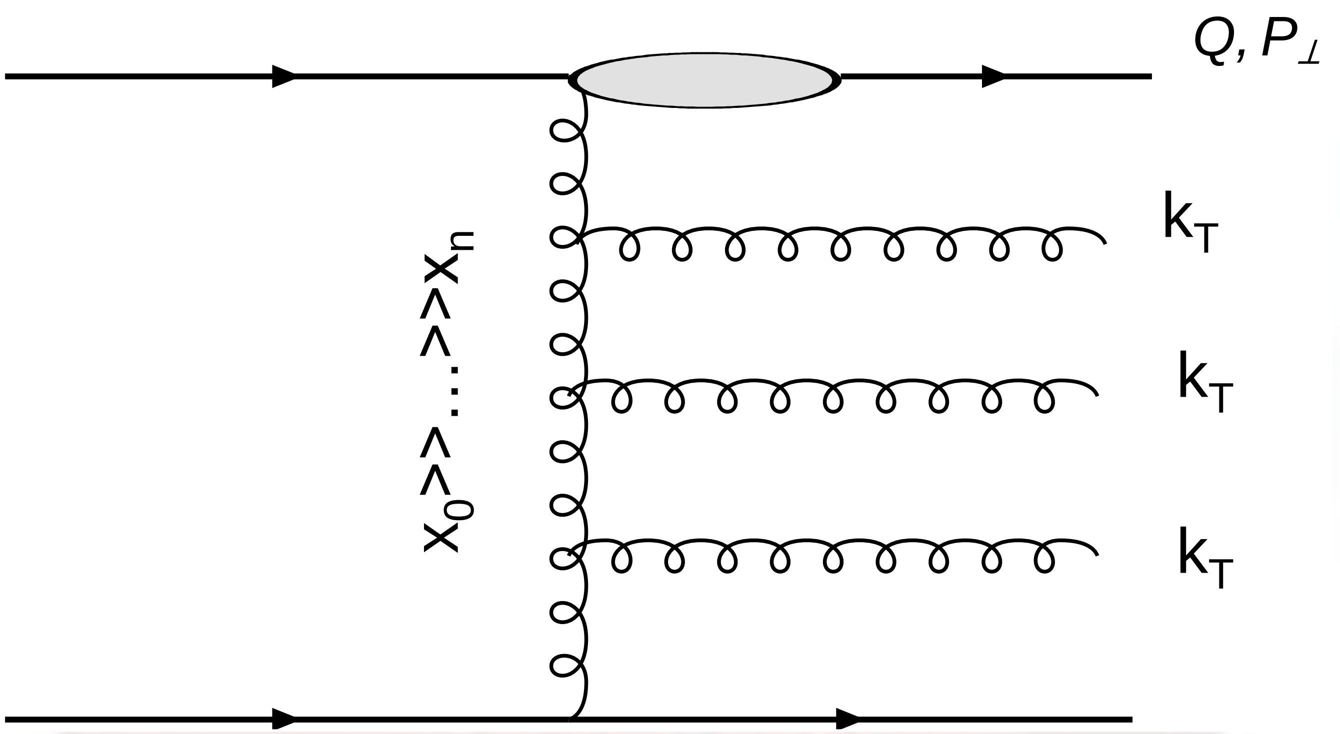
$$\frac{\partial N(x,r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x,r_T)$$

- Balitsky-Kovchegov: Non-linear term, 98

$$\frac{\partial N(x,r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x,r_T) - \alpha_s \left[N(x,r_T)\right]^2.$$

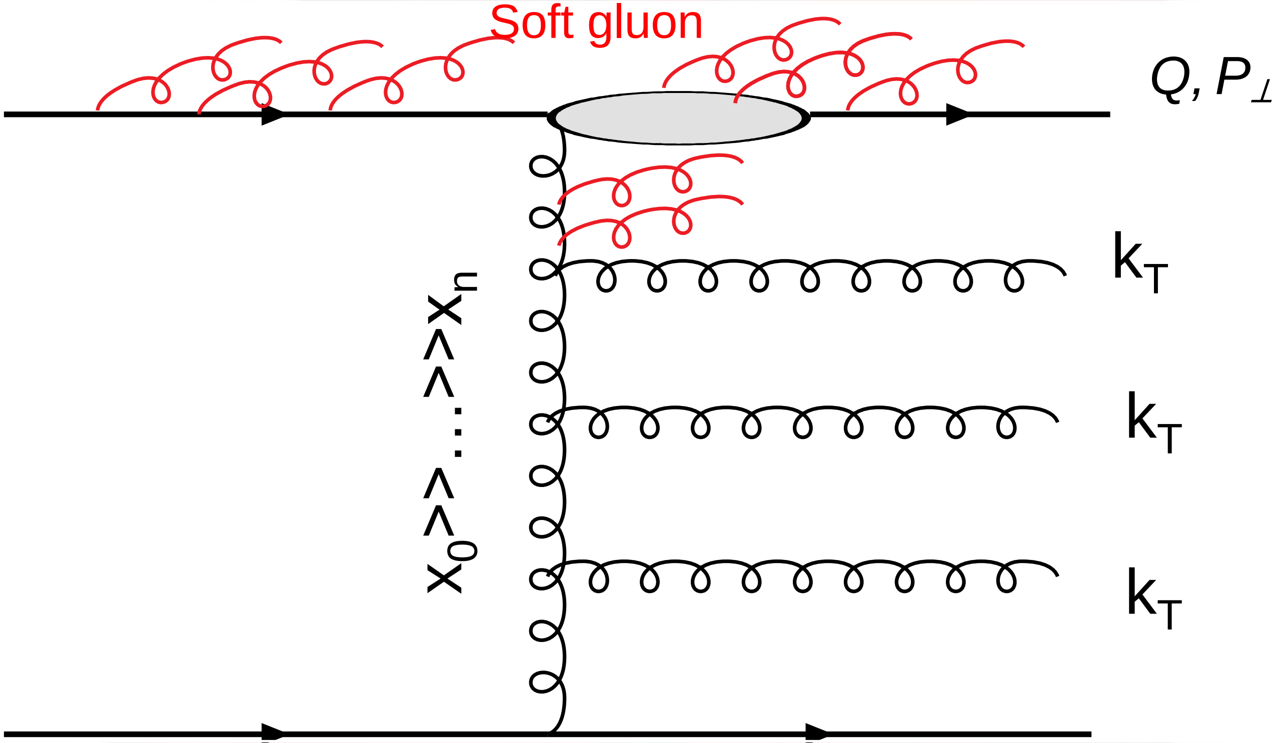


Hard processes at small-x



- Manifest dependence on un-integrated gluon distributions
  - Dominguiz-Marquet-Xiao-Yuan, 2010

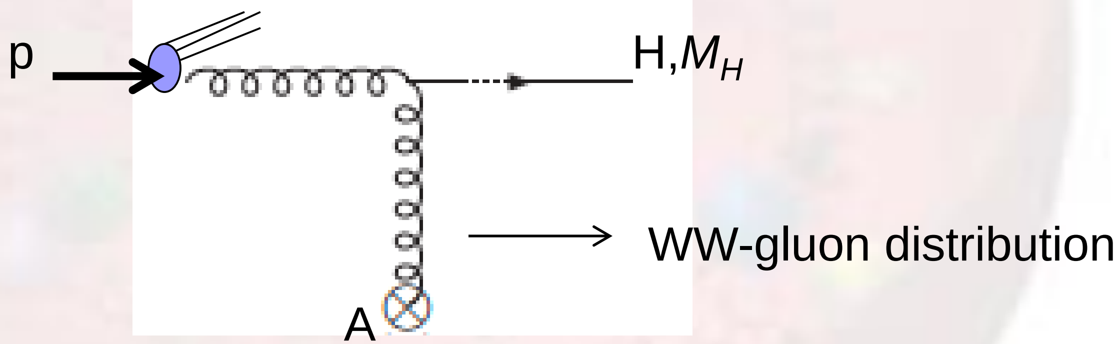
Additional dynamics comes in



- BFKL vs Sudakov resummations (LL)

Sudakov resummation at small-x

- Take massive scalar particle production *p+A->H+X* as an example to demonstrate the double logarithms, and resummation



$$\frac{d\sigma^{(\mathrm{LO})}}{dy d^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$$

Sudakov leading double logs+small-x logs in hard processes

- Each incoming parton contributes to a half of the associated color factor in Sudakov
  - Initial gluon radiation, aka, TMDs

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot(x_\perp - y_\perp)}\widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Sudakov

$$\Rightarrow H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot R_\perp} e^{-S_{sud}(P_\perp, R_\perp)}\widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Mueller-Xiao-Yuan 2013

Sudakov vs BFKL (BK)

- Start with the factorized TMDs, with full operator definitions
- Calculate the high order corrections in dipole formalism
  - With proper subtraction
- Solve the TMD evolution with BK-evolved dipole (quadrupole) amplitude

QCD Evolution: Soft vs Collinear gluons

- Radiated gluon momentum

$$k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \ ,$$

- Soft gluon, α~β<<1
- Collinear gluon, α~1, β<<1
- Small-x collinear gluon, 1-β<<1, α→0
  - Rapidity divergence

Subtracted TMD at small-x

$$f_g^{(sub.)}(x, r_\perp, \mu_F, \zeta_c) = f_g^{unsub.}(x, r_\perp)\sqrt{\frac{S^{\bar{n},v}(r_\perp)}{S^{n,\bar{n}}(r_\perp)S^{n,v}(r_\perp)}}$$

WW-gluon
Dipole gluon

Subtract the endpoint
Singularity (Collins 2011)

$$\zeta_c^2 = x^2(2v\cdot P)^2/v^2$$

- TMD evolution follows Collins 2011
  - with resummation, doesn't depend on scheme
  - Beta_0 term missing though
- Small-x evolution follows the relevant BK-evolution, respectively
  - Dipole: BK
  - WW: DMMX

Final results

$$xG^{(1)}(x, k_\perp, \zeta_c = \mu_F = Q)$$

Hard scale entering TMD
Factorization, e.g., Higgs

$$-\frac{2}{\alpha_S}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-S_{sud}(Q^2, r_\perp^2)}$$
$$\times \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp) \ ,$$

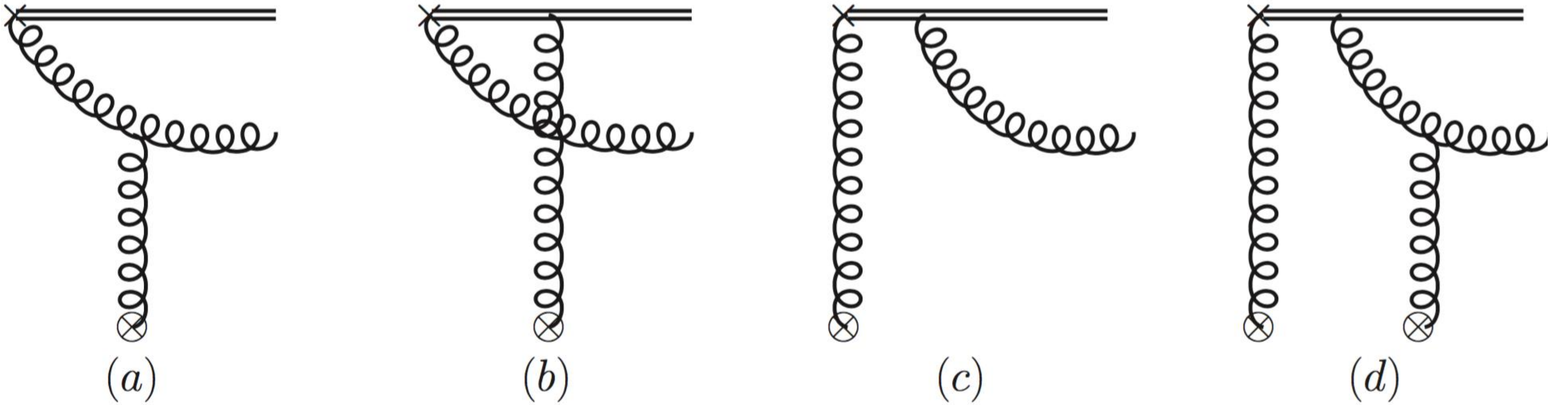
Small-x evolution

Pert. corrections

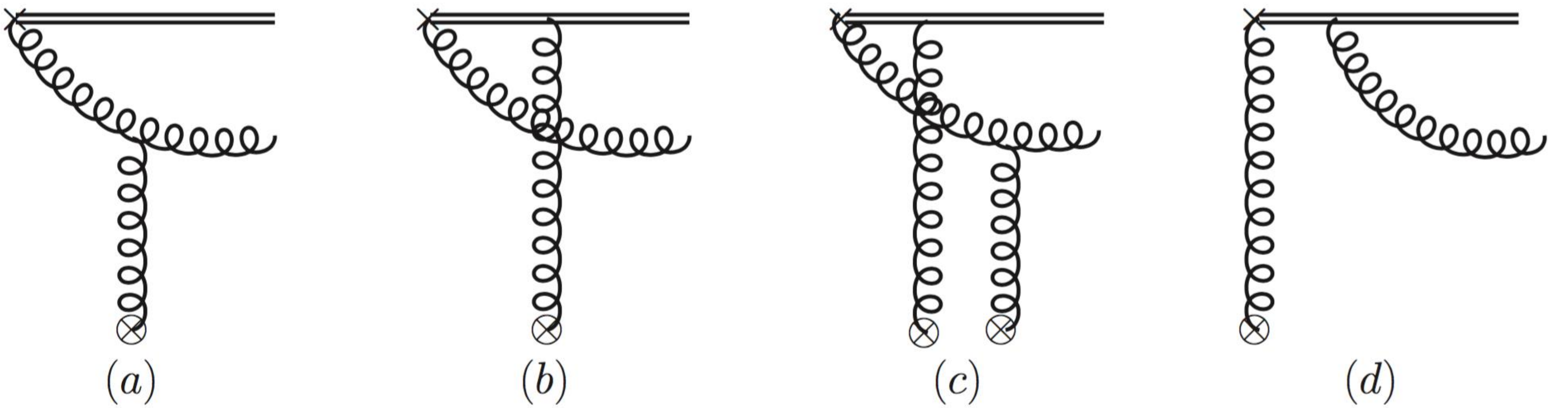
Sudakov resum.

One-loop examples

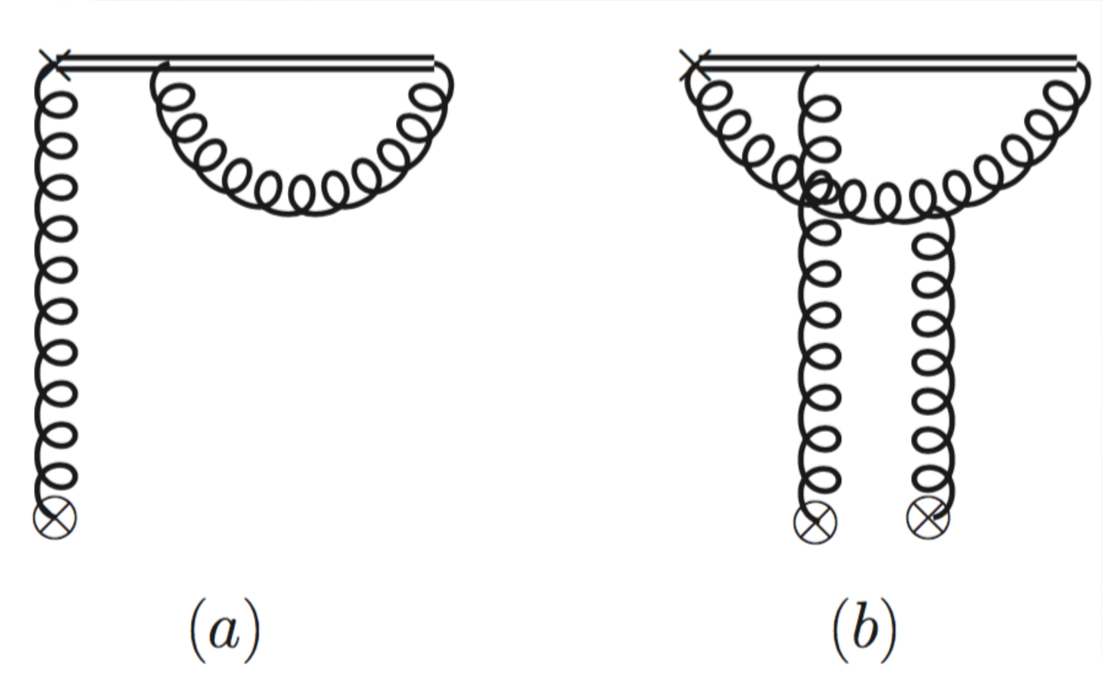
Gauge link goes to $-\infty$



Gauge link goes to $+\infty$



Virtual is the same



One-loop result

- WW-gluon (universal)

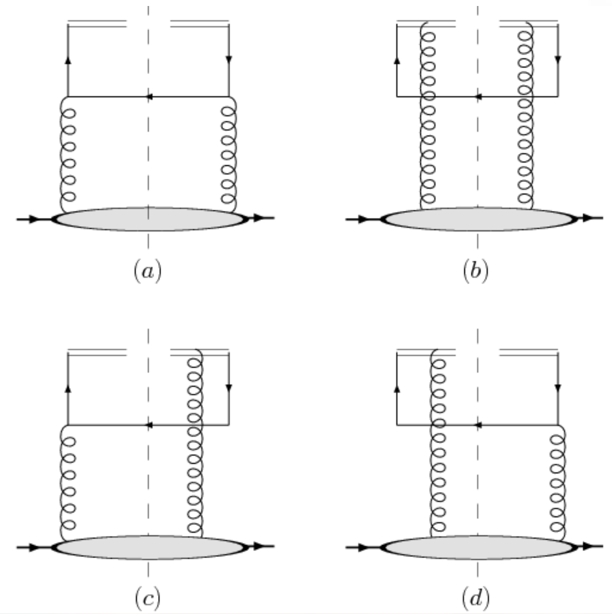
$$xG^{(WW)}_{(-\infty)}(x,r_\perp)|^{(1)} =$$

Sudakov double logs

$$\frac{\alpha_s}{2\pi}C_A\left\{\left(\frac{-2}{\alpha_s}\right)\mathcal{F}^{(WW)}(r_\perp)\left[\frac{1}{2}\left(\ln\frac{\zeta_c^2}{\mu^2}\right)^2-\frac{1}{2}\left(\ln\frac{\zeta_c^2 r_\perp^2}{c_0^2}\right)^2\right]\right.$$
$$\left.+\ln\left(\frac{1}{x}\right)\left(\frac{-2}{\alpha_s}\right)\int \mathbf{K}_{\mathrm{DMMX}}\otimes\mathcal{F}^{(WW)}(x_g,r_\perp)\right\} ,$$

Small-x logs (BK-type of evolution)

TMD quark at small-x



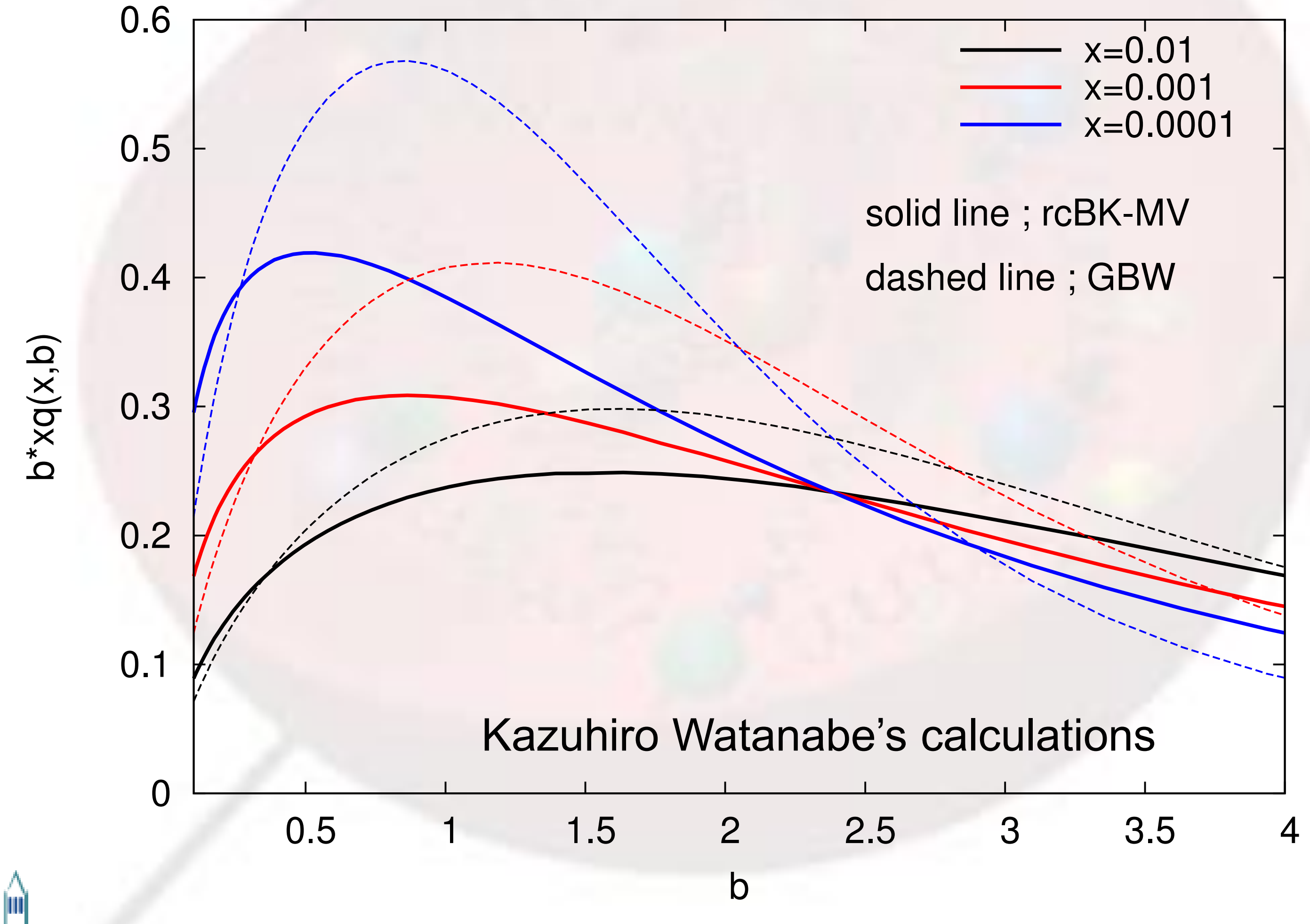
McLerran-Venugopalan 98

$$q(x, k_\perp) = \frac{N_c}{8\pi^4} \int \frac{dx'}{x'^2} \int d^2b d^2q_\perp F(q_\perp, x') A(q_\perp, k_\perp)$$

- Can be calculated from the dipole amplitude, and can be applied to DIS and Drell-Yan processes
  - Universality

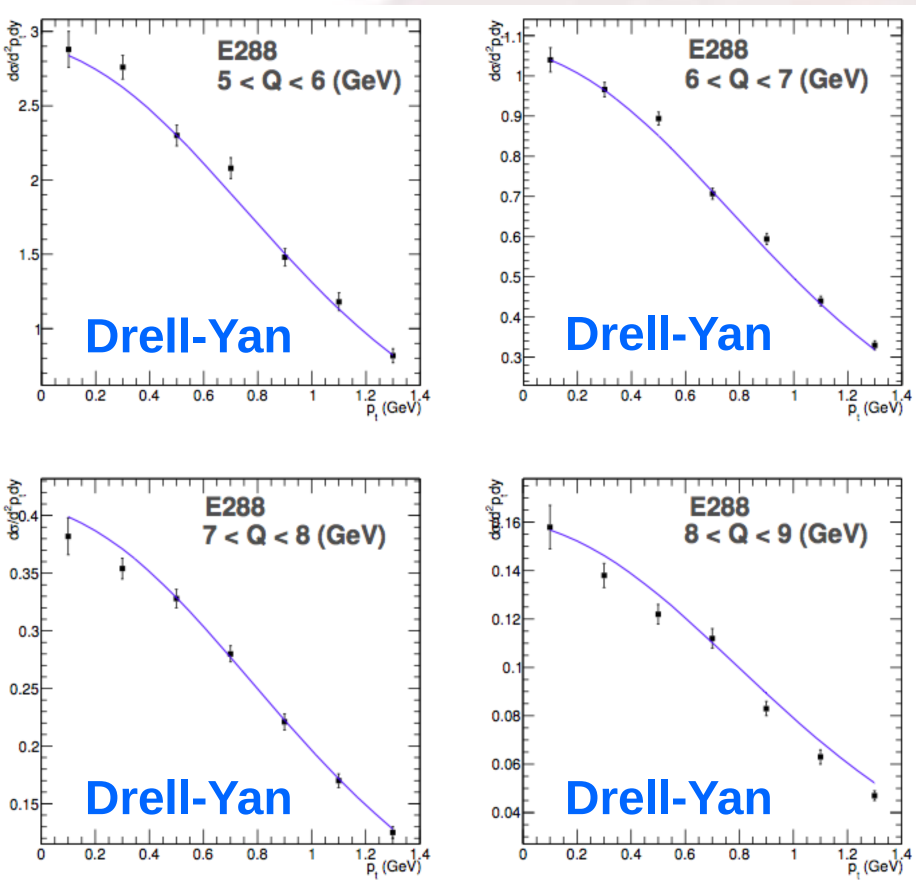
Marquet-Xiao-Yuan 2009

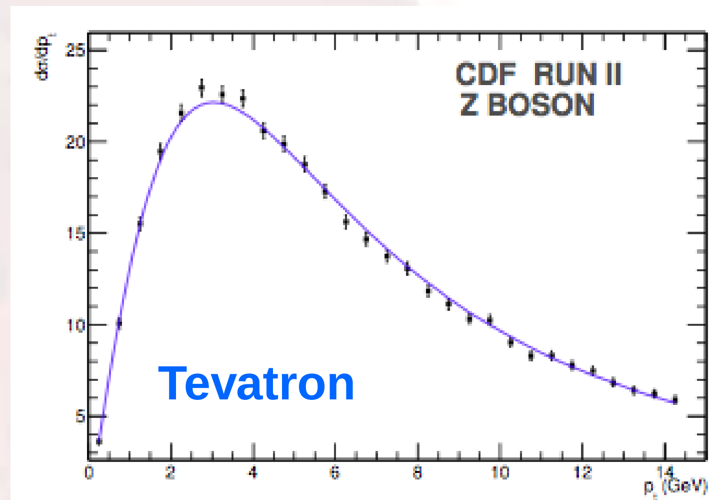
TMD quark at small-x

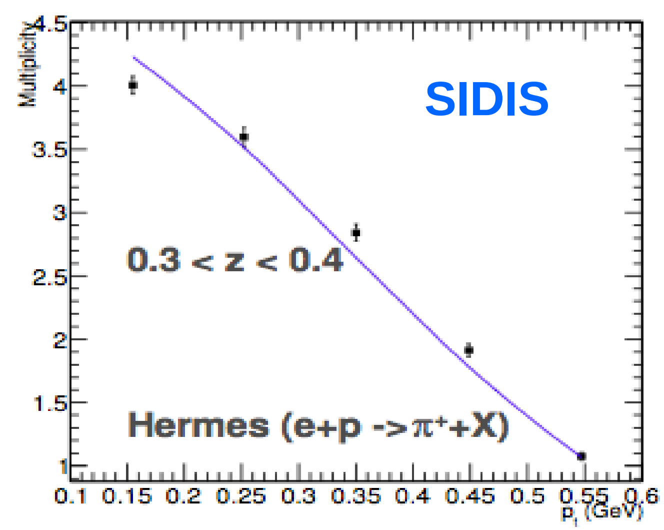


What we know the TMD quarks (not small-x)

Sun-Issacson-Yuan-Yuan, 2014

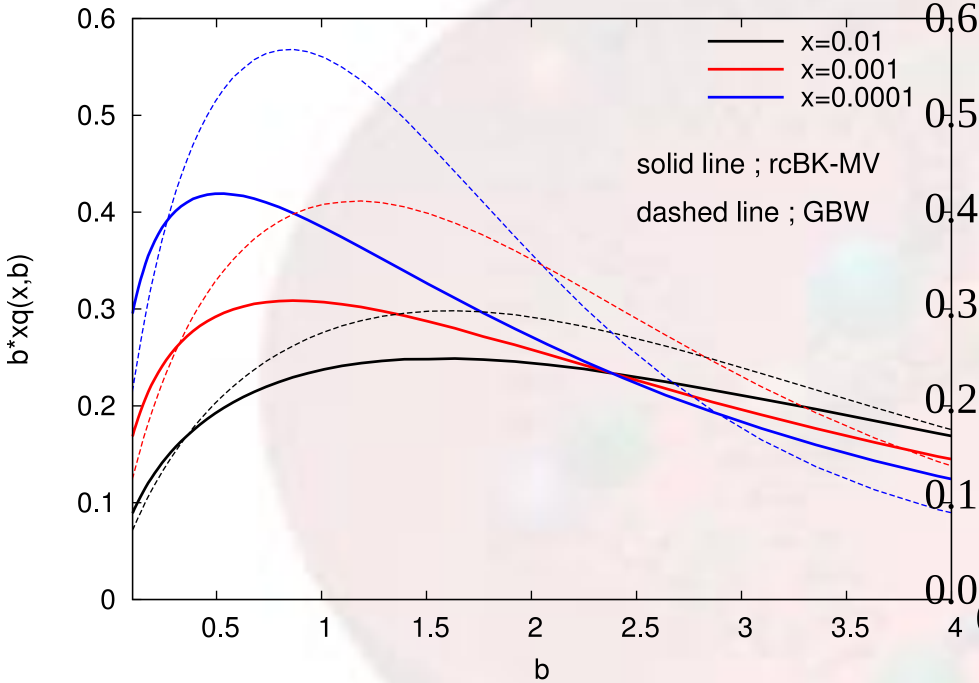


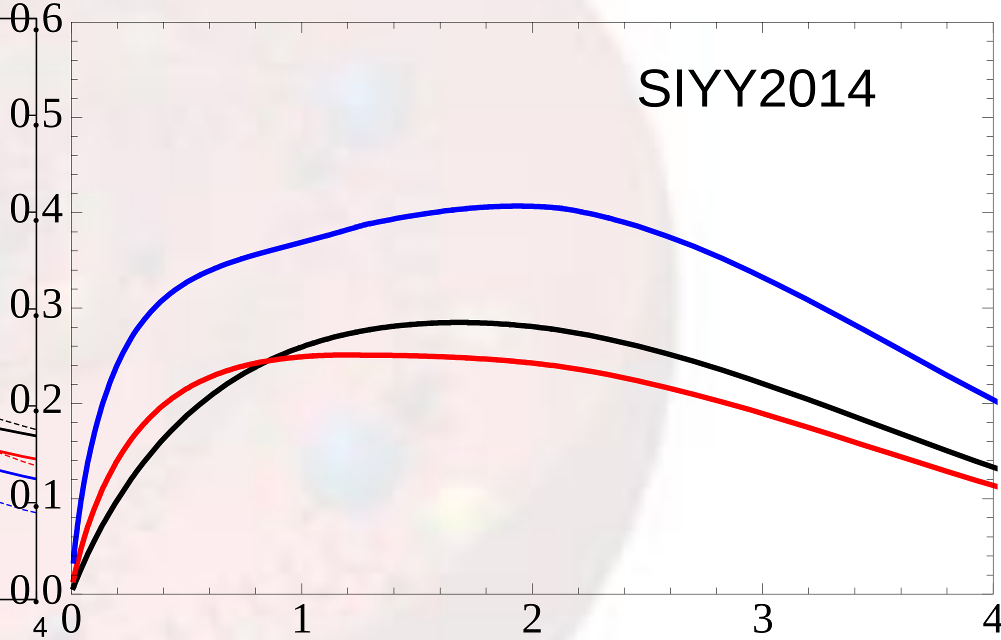




See also, BLNY 2002

TMD quark at small-x: CGC vs Collinear

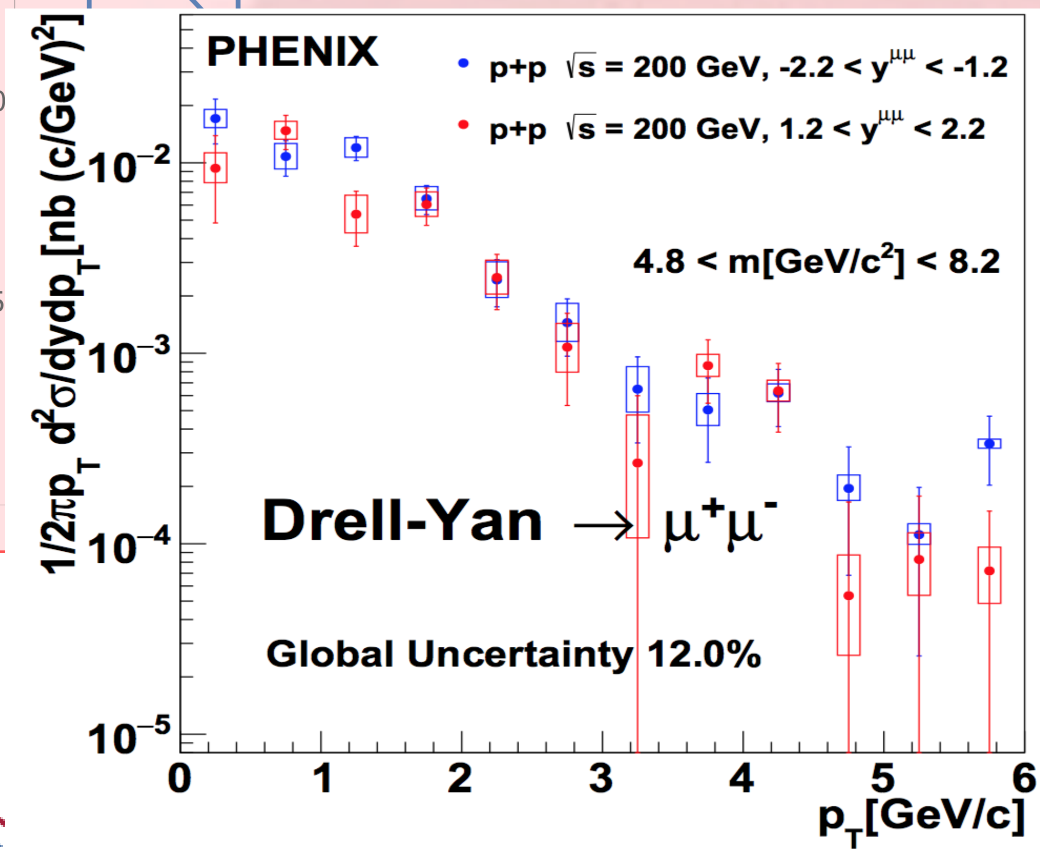




- Realistic comparison will shed light on the TMD quarks at small-x (work in progress)

We need more data at small-x

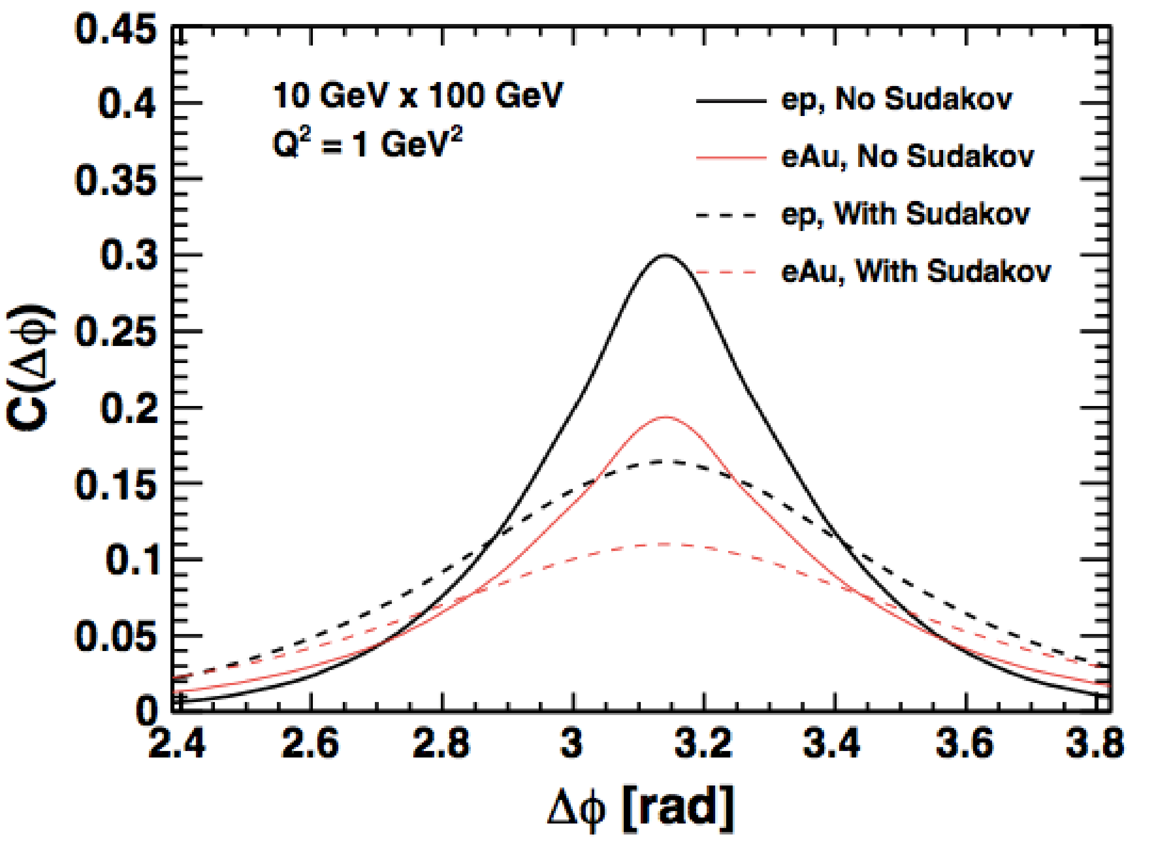
Drell-Yan from PHENIX 1805.02448



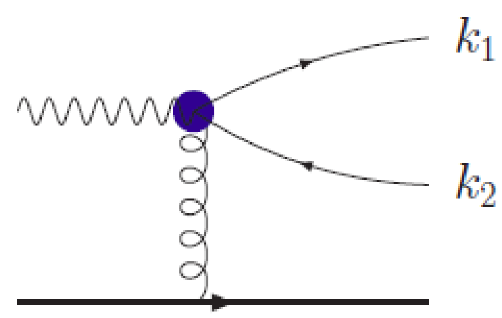
LHCb, pp and pA:
Drell-Yan and Upsilon

EIC:
SIDIS and di-hadron

Sudakov Resummation

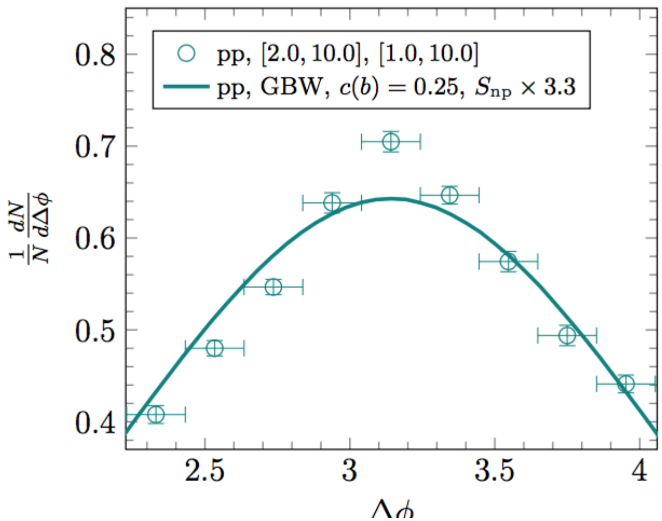


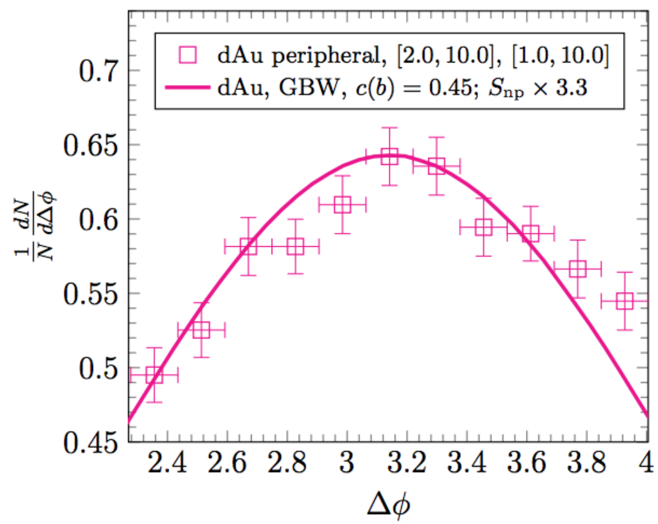
Di-hadron azimuthal Correlations at the Electron-ion Collider

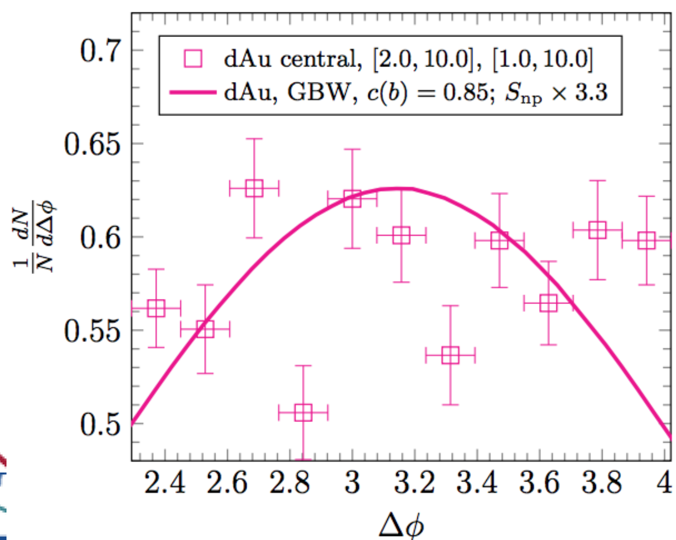


Zheng, Aschenauer,Lee,Xiao, Phys.Rev. D89 (2014) 074037

Compare to RHIC Data







Saturation and Sudakov resummation in a single formula to describe both pp and dAu,

Stasto-Wei-Xiao-Yuan, 1805.0571

Conclusions

- Theory developments since the last INT program provide solid ground to study TMDs at small-x
- Looking forward to new data from RHIC/LHC, and of course, EIC