Wigner Distributions: recent developments

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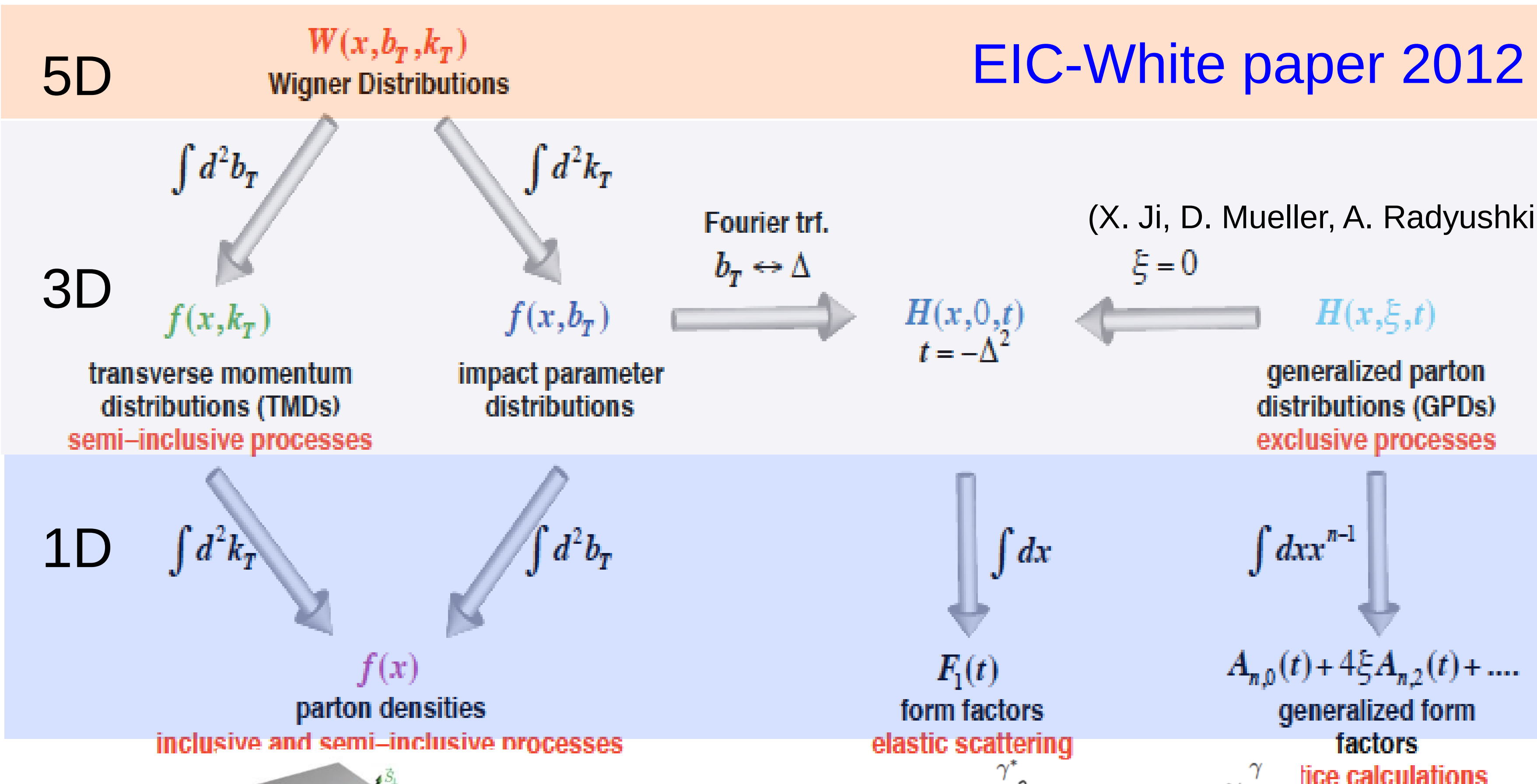
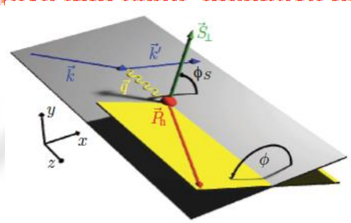
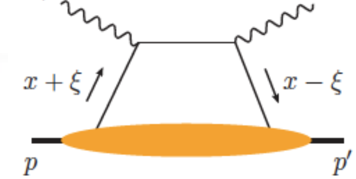
Unified view of the Nucleon

❑ **Wigner distributions (Belitsky, Ji, Yuan)**







Selected References

- Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions, Renaud Boussarie, Yoshitaka Hatta, Bo-Wen Xiao, Feng Yuan, arXiv:1807.08697 [hep-ph].
- Exclusive double quarkonium production and generalized TMDs of gluons, Shohini Bhattacharya, Andreas Metz, Vikash Kumar Ojha, Jeng-Yuan Tsai, Jian Zhou, arXiv:1802.10550 [hep-ph].
- Classical and quantum entropy of parton distributions, Yoshikazu Hagiwara, Yoshitaka Hatta, Bo-Wen Xiao, Feng Yuan, arXiv:1801.00087 [hep-ph].
- Wigner Distributions For Gluons, Jai More, Asmita Mukherjee, Sreeraj Nair, arXiv:1709.00943 [hep-ph].
- Accessing the gluon Wigner distribution in ultraperipheral pA collisions, Yoshikazu Hagiwara, Yoshitaka Hatta, Roman Pasechnik, Marek Tasevsky, Oleg Teryaev, arXiv:1706.01765 [hep-ph].
- Gluon Tomography from Deeply Virtual Compton Scattering at Small-x, Yoshitaka Hatta, Bo-Wen Xiao, Feng Yuan, arXiv:1703.02085 [hep-ph].
- Suppression of maximal linear gluon polarization in angular asymmetries, Daniel Boer, Piet J. Mulders, Jian Zhou, Ya-jin Zhou, arXiv:1702.08195 [hep-ph].
- Generalized TMDs and the exclusive double Drell–Yan process, Shohini Bhattacharya, Andreas Metz, Jian Zhou, arXiv:1702.04387 [hep-ph].
- Elliptic Flow in Small Systems due to Elliptic Gluon Distributions? Yoshikazu Hagiwara, Yoshitaka Hatta, Bo-Wen Xiao, Feng Yuan, arXiv:1701.04254 [hep-ph].
- Gluon orbital angular momentum at small-x, Yoshitaka Hatta, Yuya Nakagawa, Feng Yuan, Yong Zhao, Bowen Xiao, arXiv:1612.02445 [hep-ph].
- Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider, Xiangdong Ji, Feng Yuan, Yong Zhao, arXiv:1612.02438 [hep-ph].
- Elliptic gluon generalized transverse-momentum-dependent distribution inside a large nucleus, Jian Zhou, arXiv:1611.02397 [hep-ph].
- Wigner, Husimi, and generalized transverse momentum dependent distributions in the color glass condensate, Yoshikazu Hagiwara, Yoshitaka Hatta, Takahiro Ueda, arXiv:1609.05773 [hep-ph].
- On the one loop $ {\gamma}^{\left(\ast \right)}\to q\overline{q} $ impact factor and the exclusive diffractive cross sections for the production of two or three jets, R. Boussarie, A.V. Grabovsky, L. Szymanowski, S. Wallon, arXiv:1606.00419 [hep-ph].
- The spin dependent odderon in the diquark model, Lech Szymanowski, Jian Zhou, arXiv:1604.03207 [hep-ph].
- Proper definition and evolution of generalized transverse momentum dependent distributions, Miguel G. Echevarria, Ahmad Idilbi, Koichi Kanazawa, Cédric Lorcé, Andreas Metz, Barbara Pasquini, Marc Schlegel, arXiv:1602.06953 [hep-ph].
- Orbital Angular Momentum and Generalized Transverse Momentum Distribution, Yong Zhao, Keh-Fei Liu, Yibo Yang, arXiv:1506.08832 [hep-ph].
- Probing the Small- x Gluon Tomography in Correlated Hard Diffractive Dijet Production in Deep Inelastic Scattering, Yoshitaka Hatta, Bo-Wen Xiao, Feng Yuan, arXiv:1601.01585 [hep-ph].

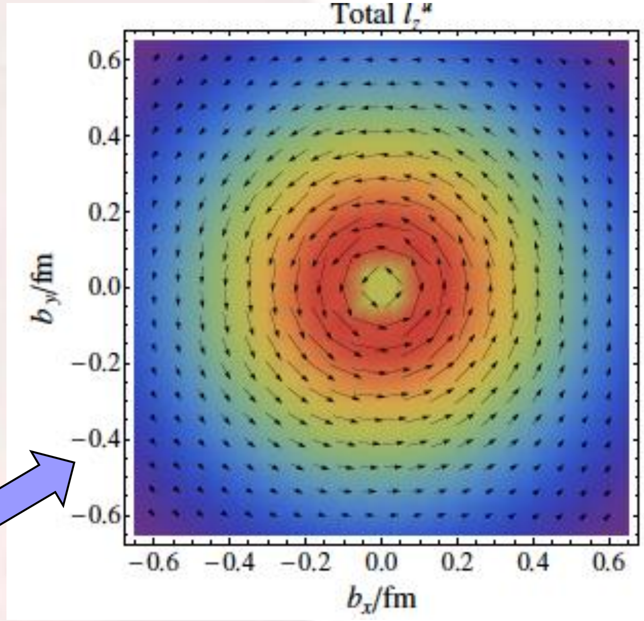
Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto p (x) to get the momentum (probability) density

Quark orbital angular momentum

$$L(x)=\int(\vec{b}_\perp\times\vec{k}_\perp)W(x,\vec{b}_\perp,\vec{k}_\perp)d^2\vec{b}_\perp d^2\vec{k}_\perp$$



Well defined in QCD:

Lorce-Pasquini 2011

Hatta 2011

Lorce-Pasquini-Xiong-Yuan 2011

Ji-Xiong-Yuan 2012

Importance of the gauge links

- The quark operator Ji: PRL91,062001(2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

- Straightline gauge link leads to the OAM in the Ji-sum rule

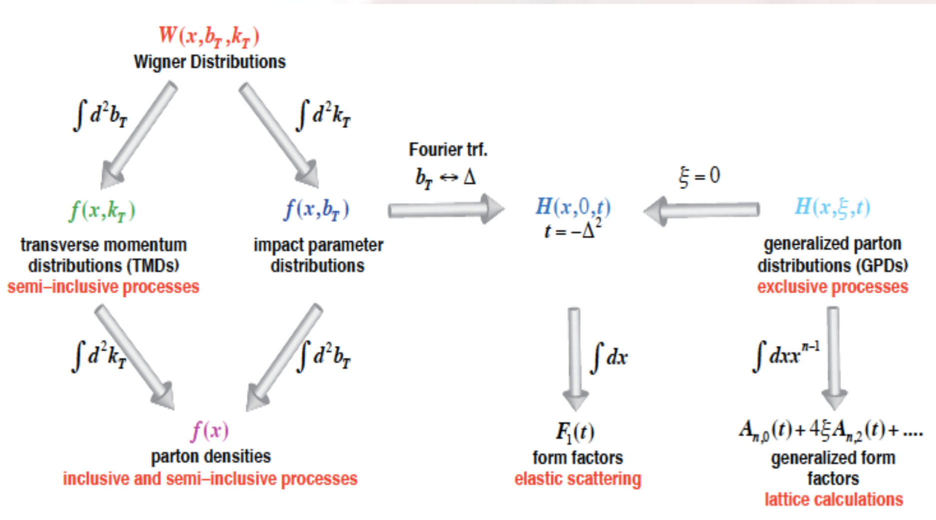
$$\Psi_{FS}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda\xi\cdot A(\lambda\xi)\right)\right]\psi(\xi)$$

- Light-cone gauge link leads to the OAM in Jaffe-Manohar sum rule

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\xi)\right)\right]\psi(\xi)$$

Grand Jewels of Hadron Physics

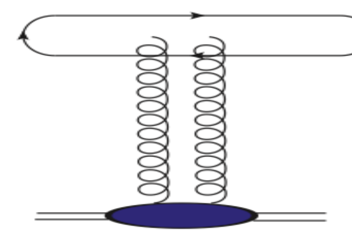
- **Wigner distributions (Belitsky, Ji, Yuan)**



Small-x

Dipole scattering amplitudes

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(R_\perp)\,U^\dagger(R'_\perp)\right]\right\rangle_x$$

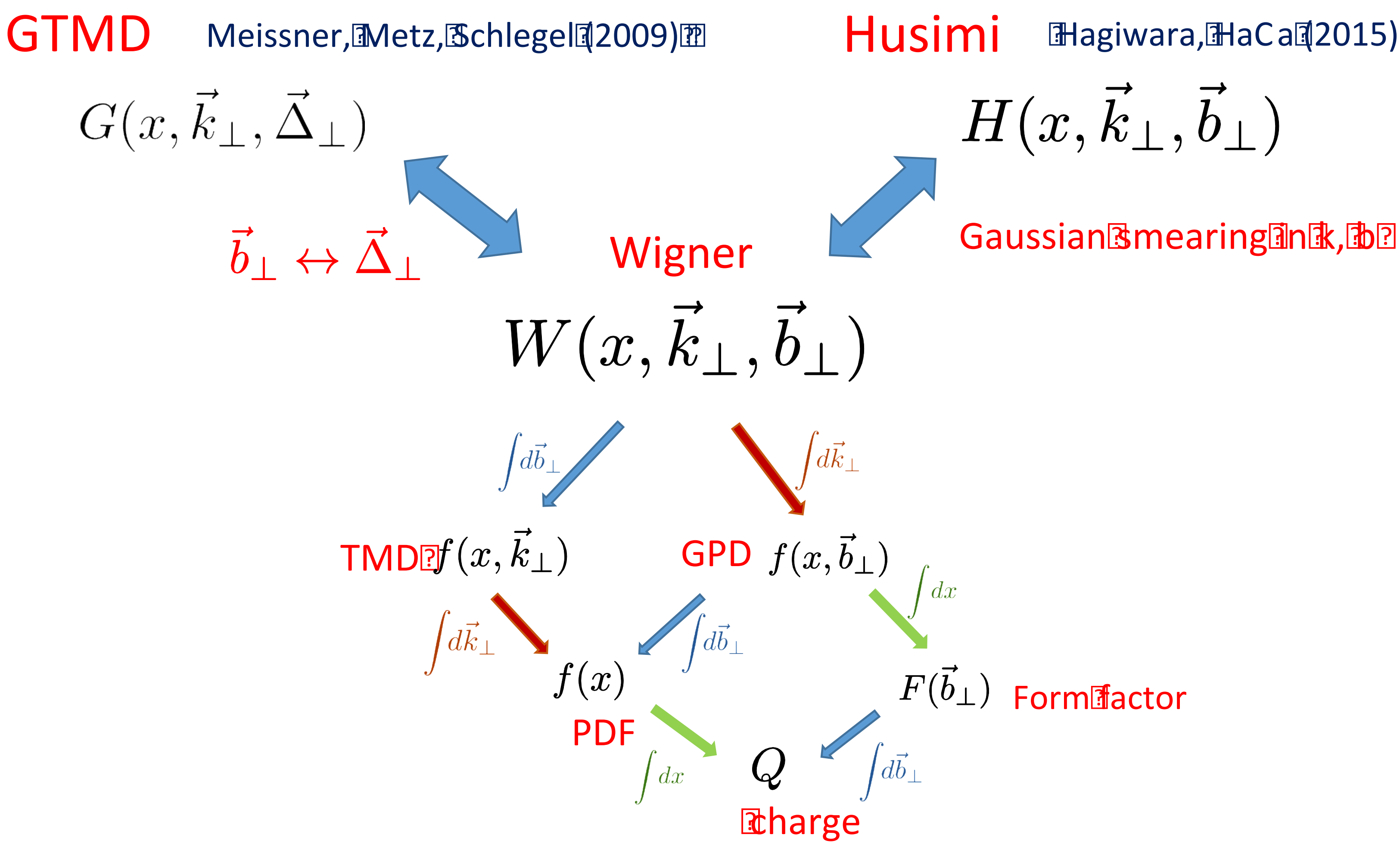


Hatta-Xiao-Yuan,1601.01585
earlier: Mueller, NPB 1999

Recent developments

- Theory
  - New idea, new functions
  - QCD evolution/factorization
- Phenomenology
  - How to measure them
  - What we can learn?

5D tomography: GTMD and Husimi



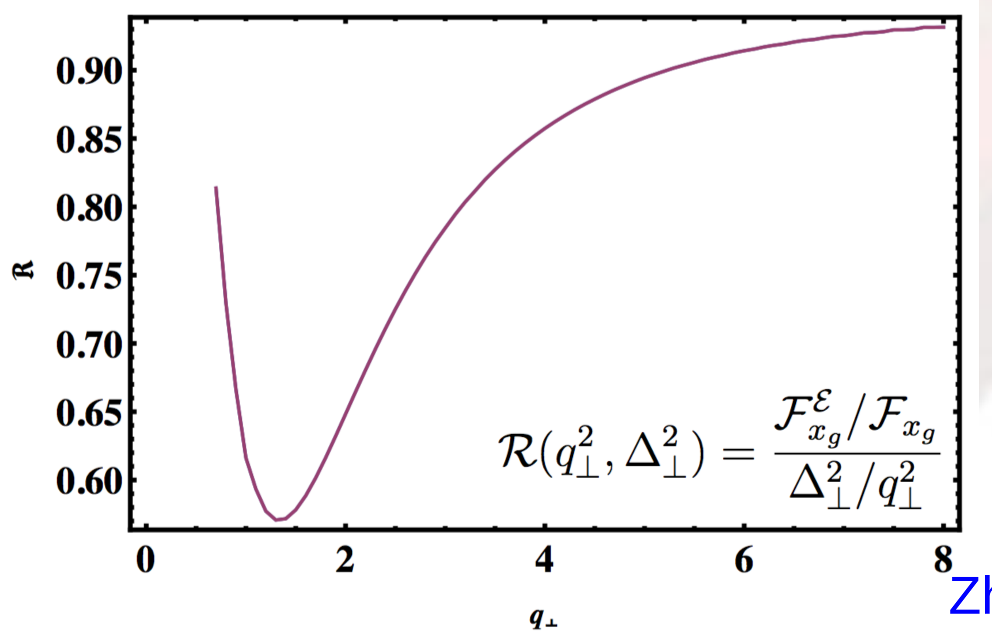
Hatta

Elliptic gluon distribution

Hatta-Xiao-Yuan16
Zhou 16

- Nontrivial correlation between the transverse momentum and impact parameter

$$xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) = x\mathcal{W}_g^T(x,|\vec{q}_\perp|,|\vec{b}_\perp|) + 2\cos(2\phi)x\mathcal{W}_g^\epsilon(x,|\vec{q}_\perp|,|\vec{b}_\perp|)$$



Zhou 16

Evolution/factorization: proper definition

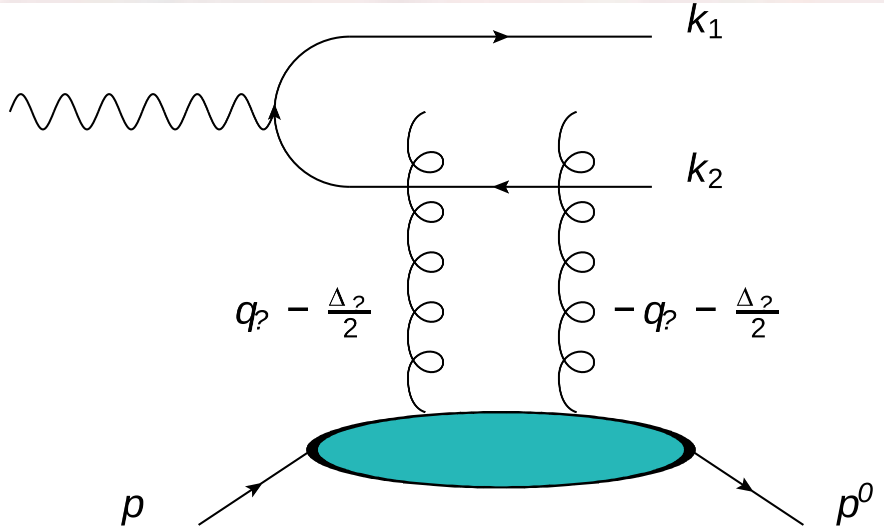
EIKLMPS2016

- The evolution is identical to the TMD evolution!
  - Sudakov double logs should be fine
  - Anomalous dimension?
  - Skewness dependence?

Probing 3D Tomography of Protons at Small-x at EIC

Diffractive back-to-back dijet productions at EIC:

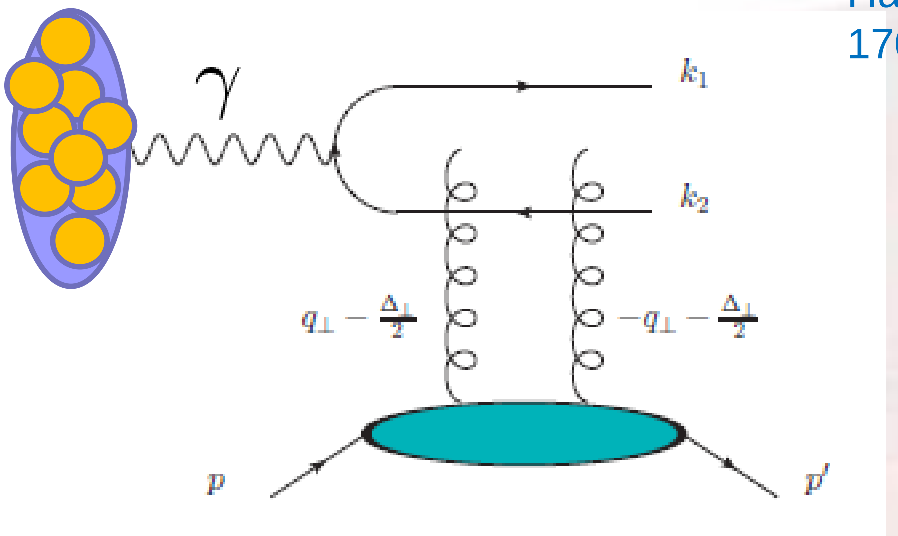
Hatta-Xiao-Yuan,1601.01585
Altinoluk, Armesto, Beuf, Rezaeian (2015)



- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T. By measuring jets momenta, one can approximately access q_T.
- The diffractive dijet cross section is proportional to the square of the Wigner distribution.

Measuring Wigner in ultra-peripheral pA collisions

Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 1706.01765



Q^2 preferably small

Use the Weiszacker-Williams photons in UPC!

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega \frac{dN}{d\omega} \frac{N_c \alpha_{em} (2\pi)^4}{P_\perp^2} \sum_f e_f^2 2z(1-z)(z^2+(1-z)^2)\left(A^2 + 2\cos 2(\phi_P - \phi_\Delta) AB\right)$$

$\propto Z^2$

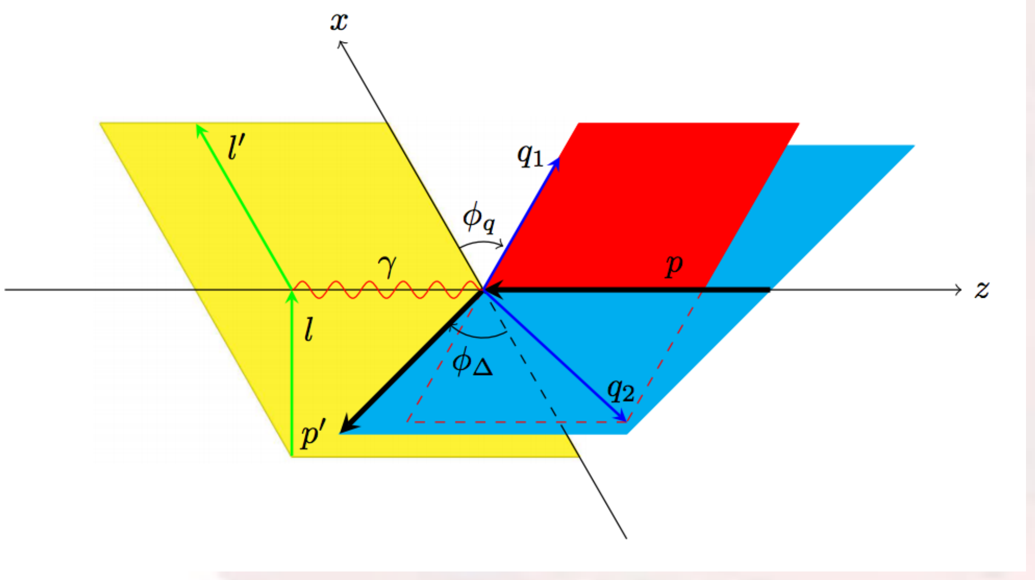
photon flux

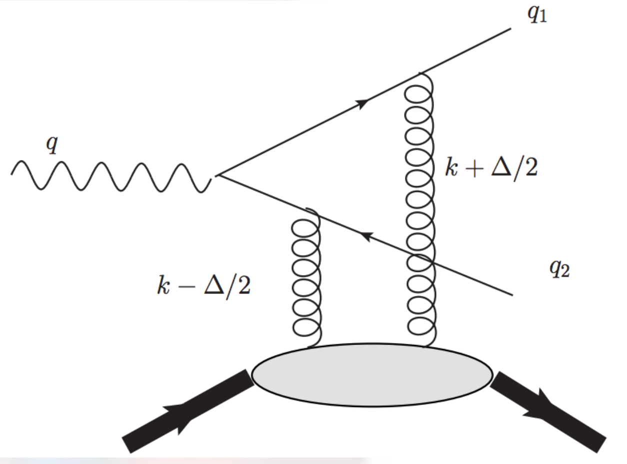
$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp} \frac{\partial}{\partial P_\perp} A(P_\perp, \Delta_\perp).$$

$$S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2} \int^{P_\perp^2} \frac{dP'^2_\perp}{P'^2_\perp} B(P'_\perp, \Delta_\perp)$$

Hatta

Hunting the Gluon Orbital



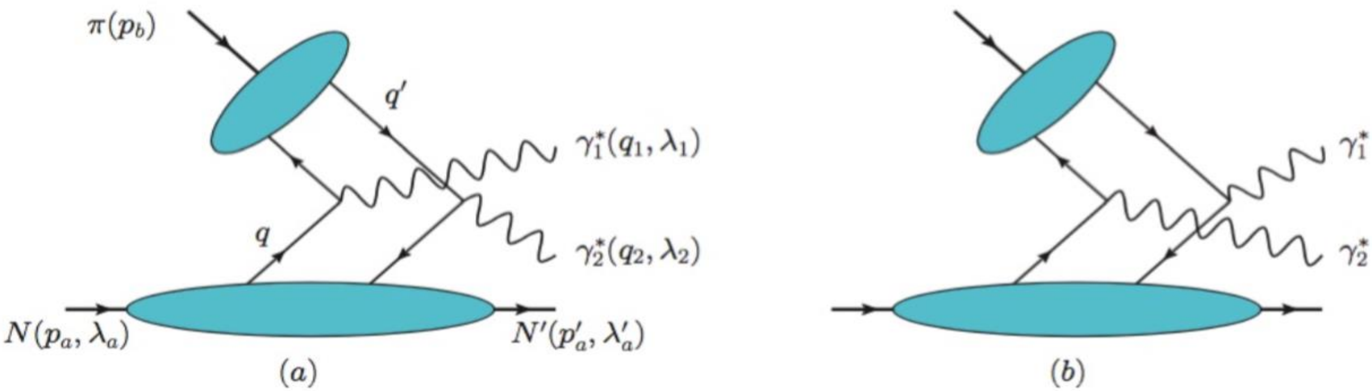


$$A_{\sin(\phi_q-\phi_\Delta)} \propto \frac{(\bar{z}-z)|\vec{q}_\perp||\vec{\Delta}_\perp|}{\vec{q}_\perp^2+\mu^2}\mathcal{L}_g(\xi,t)$$

Ji,Yuan,Zhao, arXiv:1612.02438
Hatta,Nakagawa,Yuan,Zhao, arXiv:1612.02445

Double Drell-Yan

Bhattacharya-Metz-Zhou 2017



- Sensitive to quark GTMDs in the ERBL region
- Spin asymmetries will lead to probe the quark OAM (Jaffe-Manohar)
- Should study the feasibility in existing facilities

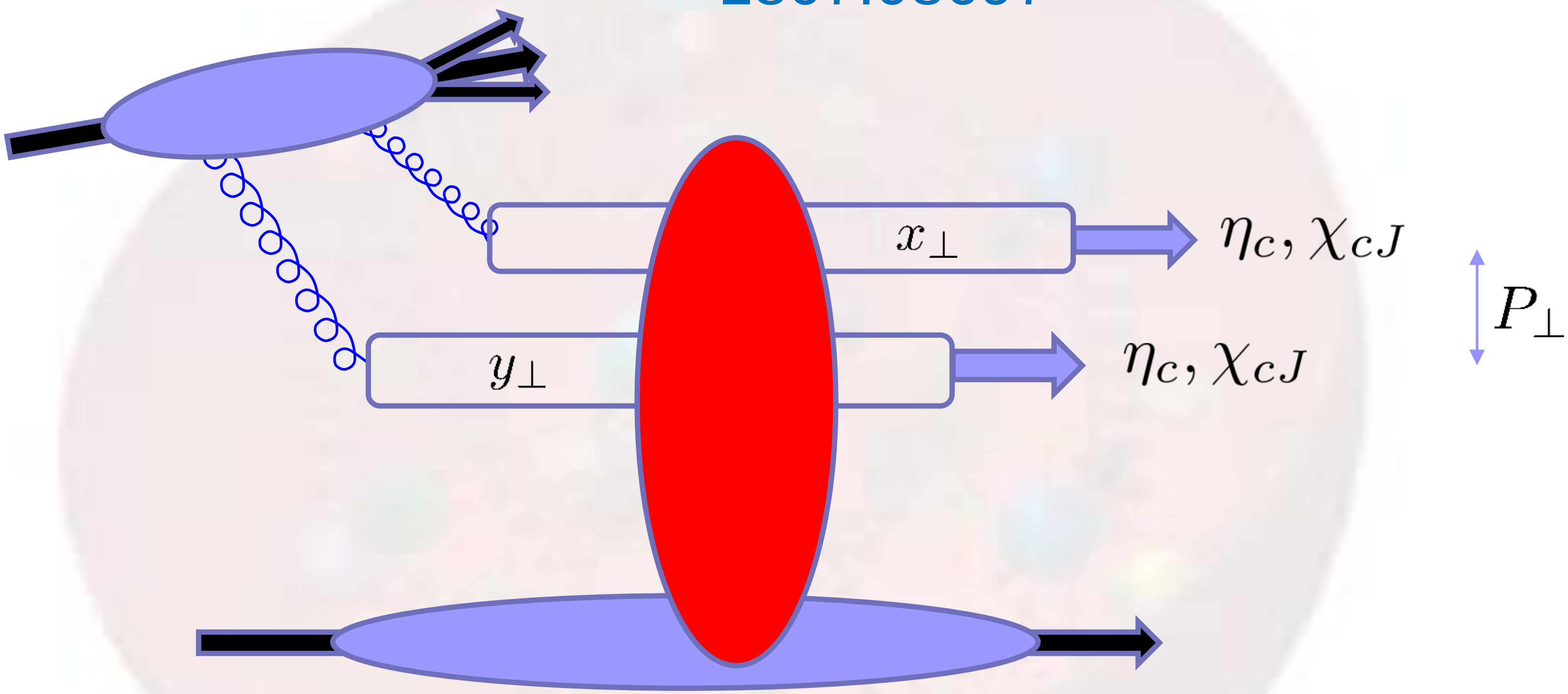
Double Charmonium

Bhattacharya-Metz-Ojha-Tsai-Zhou 18

- Focus on pseudo-scalar pair production from two gluons.
- Similar to the double Drell-Yan process, and sensitive to the gluob OAM for polarized case
- Feasibility in exp.

Probing the gluon WW GTMD in pp

Boussarie, Hatta, Xiao, Yuan, 1807.08697



Diffractive production of a $C=+1$ quarkonium pair

Amplitude proportional to

$$\int d^2(x_\perp - y_\perp) e^{iP_\perp\cdot(x_\perp - y_\perp)} \langle P' | U_x \vec{\partial} U_x^\dagger U_y \vec{\partial} U_y^\dagger | P \rangle$$

Hatta

Boussarie, Hatta, Xiao, Yuan, 1807.08697

Very simple result in the case of χ_{c1}, χ_{c1} production, in the limit $P_\perp \gg \Delta_\perp$

Gluon dPDF

Long distance matrix element (LDME)

$$\frac{d\sigma}{dY_1 dY_2 d^2\Delta_\perp d^2P_\perp} = \frac{x_1 x_2 F(x_1, x_2)}{64 m^{18} N_c^4 (N_c^2 - 1)^2} \alpha_s^4 \langle \mathcal{O}_{\chi_1} \rangle^2 P_\perp^4 \left(G(P_\perp, \Delta_\perp) + \frac{P_\perp^2}{2M^2} G_2 \right)^2$$

WW gluon GTMD

Linearly polarized WW gluon GTMD

No convolution in $P_\perp$!

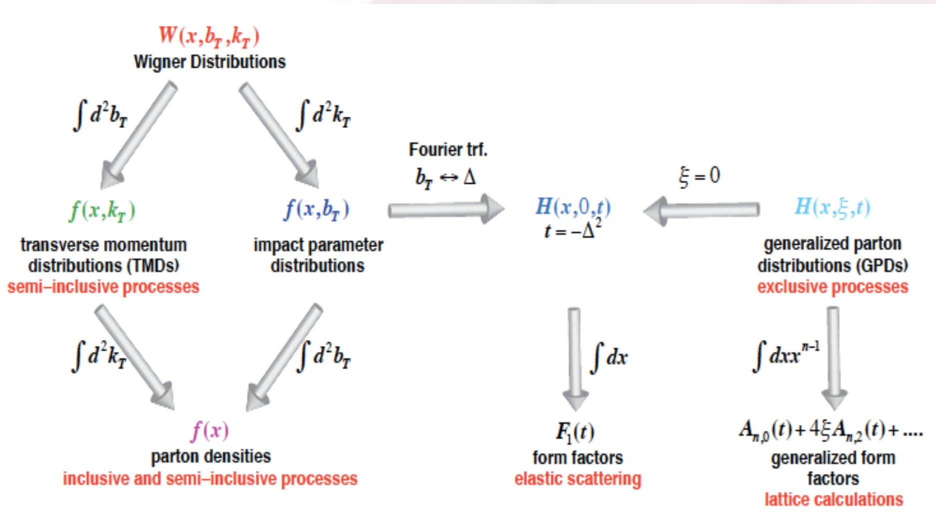
Caveat: only color-singlet production included

Berkeley Lab

Hatta

GTMDs/GPDs at small-x

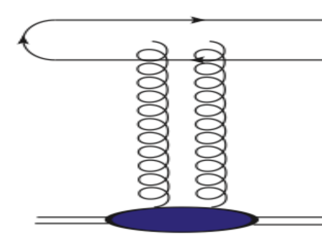
❑ **Wigner distributions (Belitsky, Ji, Yuan)**



Small-x

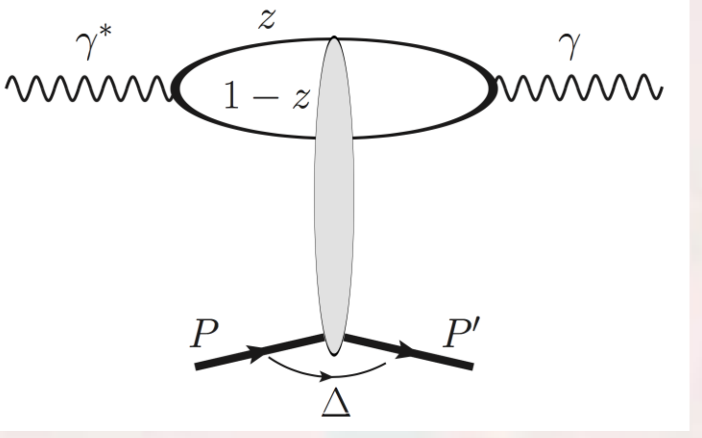
Dipole scattering amplitudes

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U\left(R_\perp\right)U^\dagger\left(R'_\perp\right)\right]\right\rangle_x$$



Hatta-Xiao-Yuan,1601.01585
earlier: Mueller, NPB 1999

DVCS and GPDs at small-x



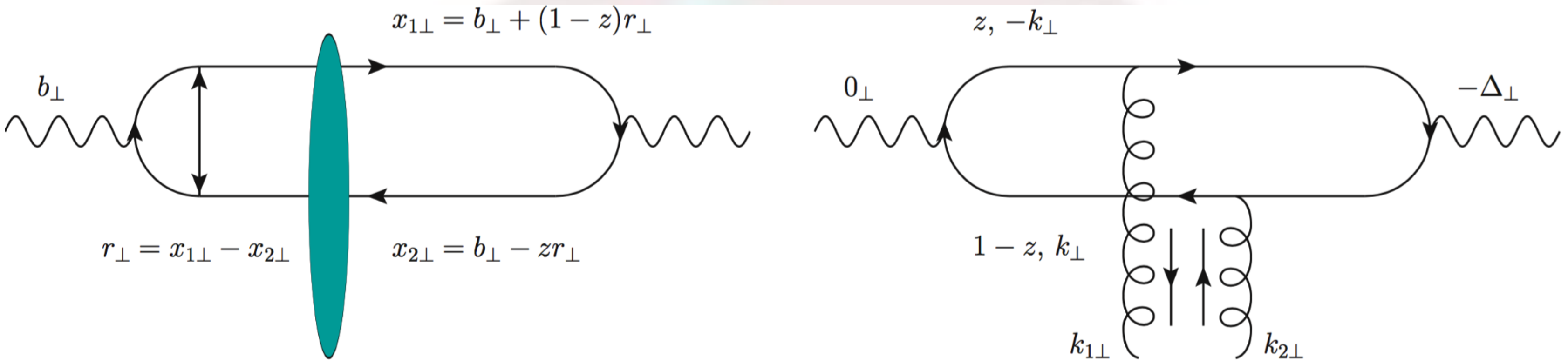
$$\frac{1}{P^+}\int \frac{d\zeta^-}{2\pi} e^{ixP^+\zeta^-} \langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$

$$= \frac{\delta^{ij}}{2} xH_g(x,\Delta_\perp) + \frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i \Delta_\perp^j - \frac{\delta^{ij}\Delta_\perp^2}{2}\right) +$$

Hoodbhoy-Ji 98
Diehl 01

- All other GPDs suppressed at small-x

Dipole formalism



$$F_x(q_\perp, \Delta_\perp) = \int \frac{d^2 r_\perp d^2 b_\perp}{(2\pi)^4} e^{i b_\perp \cdot \Delta_\perp + i r_\perp \cdot q_\perp} S_x\left(b_\perp + \frac{r_\perp}{2}, b_\perp - \frac{r_\perp}{2}\right)$$

- Elliptic gluon distribution (Hatta-Xiao-Yuan 16)

$$F_x(q_\perp, \Delta_\perp) = F_0(|q_\perp|, |\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) F_\epsilon(|q_\perp|, |\Delta_\perp|)$$

GPDs and dipole

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0\,,$$

$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_c M^2}{\alpha_s \Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

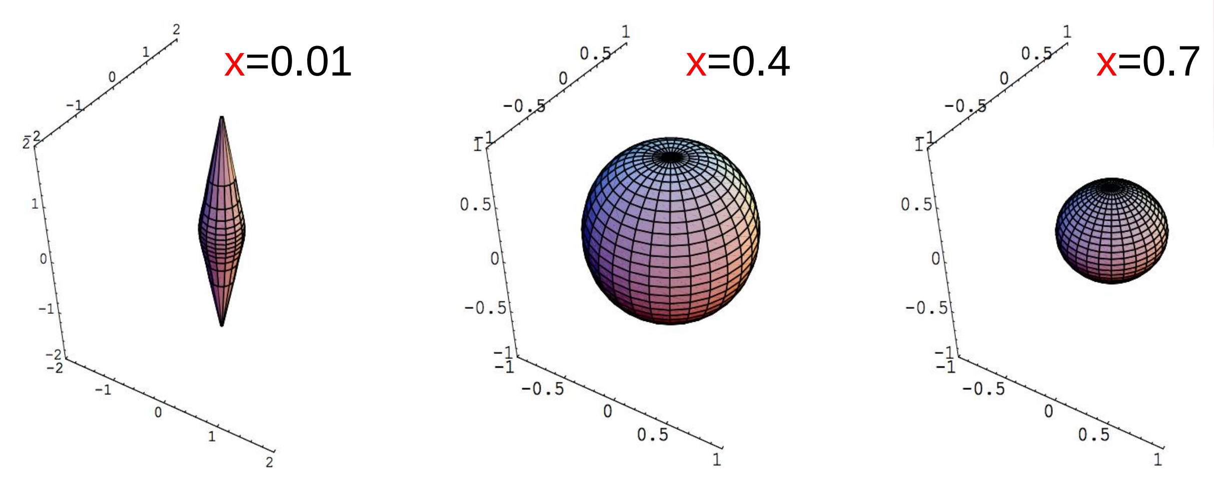
Elliptic gluon distribution

- The cos(2phi) asymmetry in DVCS will provide information on the elliptic gluon distribution at small-x

Hatta-Xiao-Yuan 1703.02085

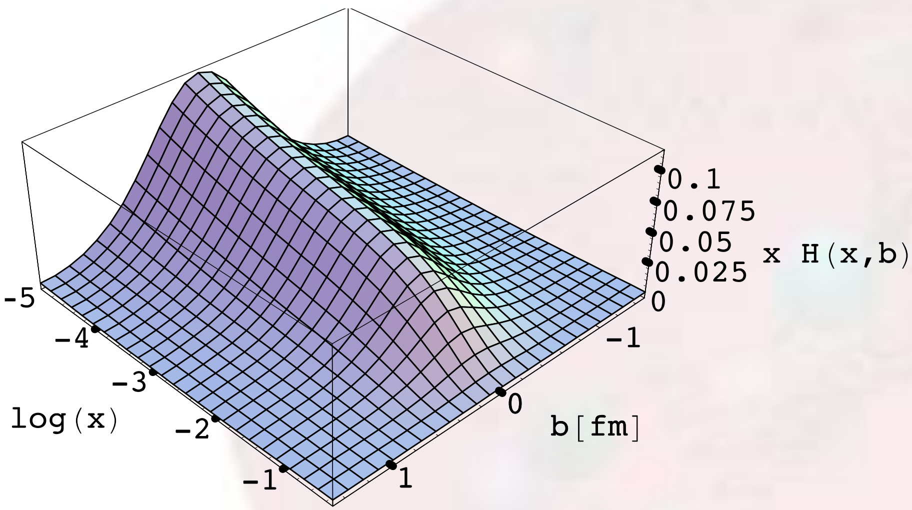
GPDs:
Proton size as function of x

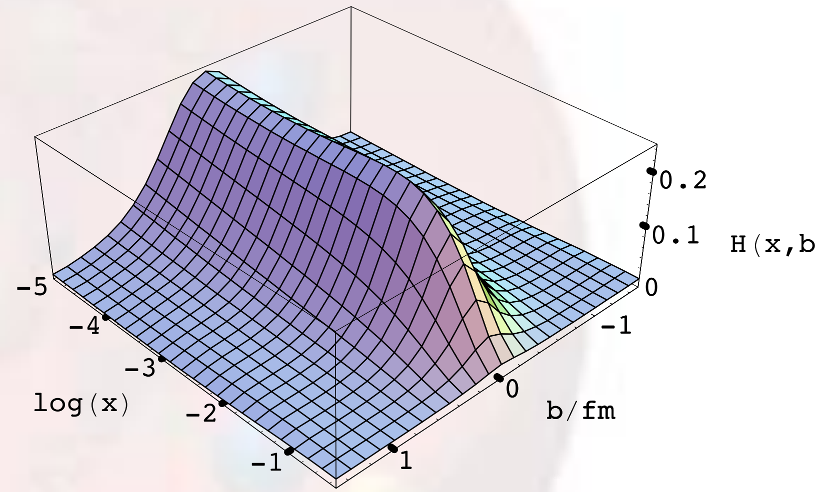
- Moderate x-range: sea/valence quarks
- Charge radii as functions of *x*
  - *x*→1, it vanishes



Ji-Belitsky-Yuan 03

Transverse profile: gluon vs quark





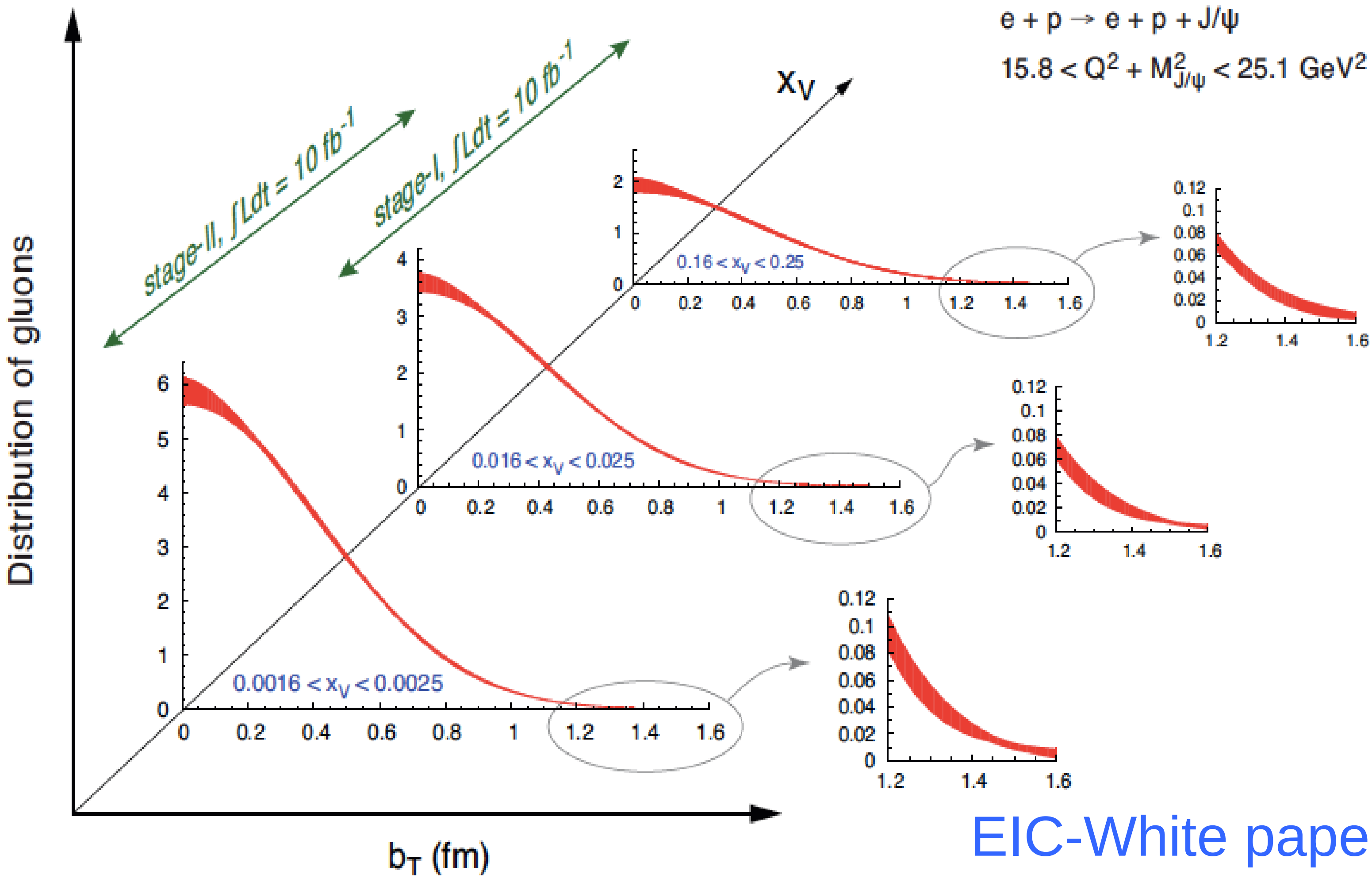
GPD fit to the DVCS data from HERA, Kumerick-D.Mueller, 09,10

GPD fit to the DVCS data from HERA, Kumerick-Mueller, 09,10

HERA data show that gluons lives typically at 0.5~0.6fm, while we know charge radius around 0.9 fm

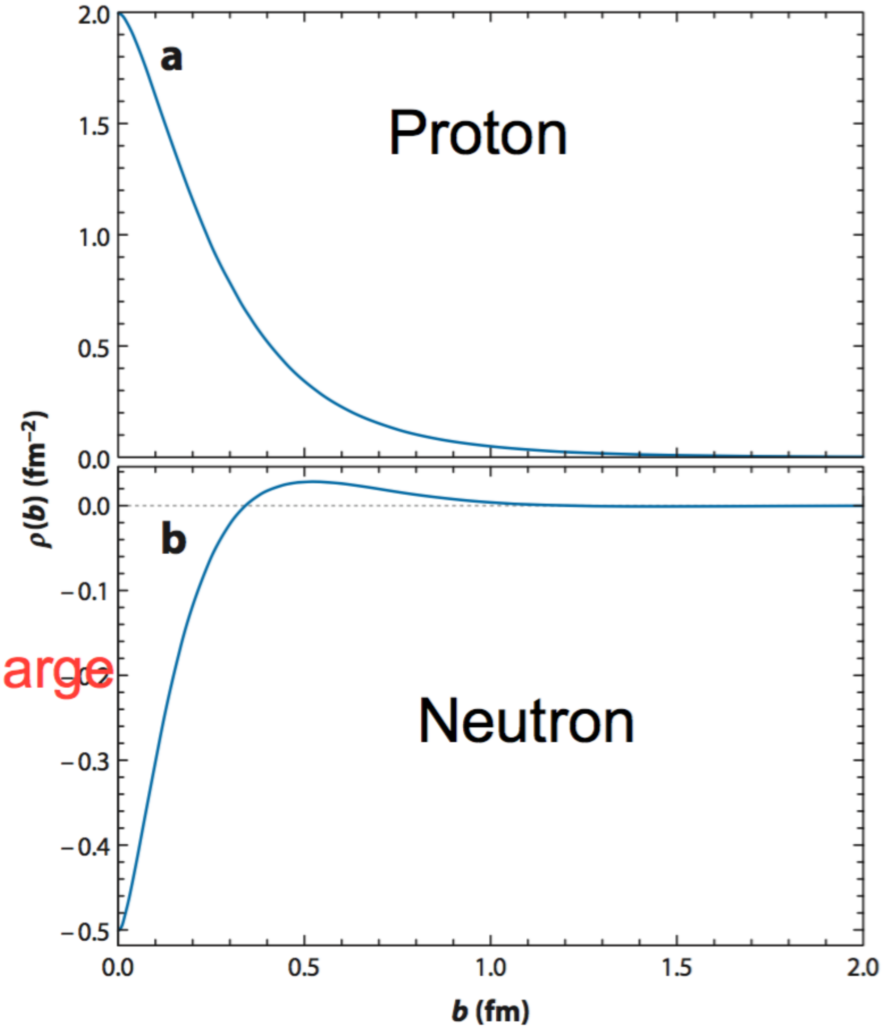
See also, Dumitru-Miller-Venugopalan 2018

Gluon tomography at small x (GPDs)



EIC-White paper 2012
arXiv:1212.1701

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM PRL '07

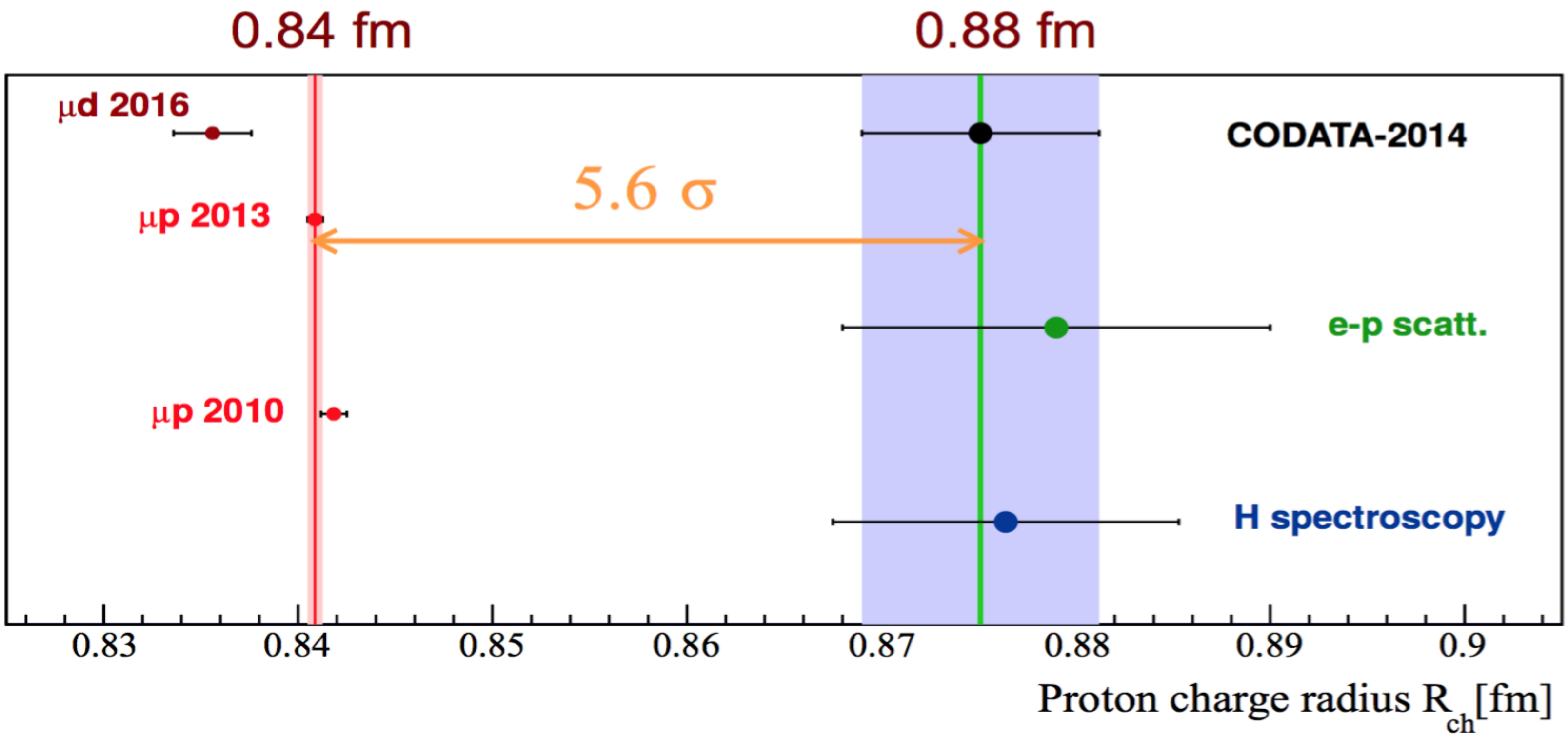
Miller

Size does matter!!

The “Proton Radius Puzzle”

Measuring R_p using electrons: 0.88 fm (+- 0.7%)

using muons: 0.84 fm (+- 0.05%)



μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)

μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

Randolf Pohl

Back-ups

DVCS: Collinear factorization

$$T^{\mu\nu} = i \int d^4 z e^{-iq\cdot z} \langle P' | j^\mu(z/2) j^\nu(-z/2) | P \rangle \equiv g_\perp^{\mu\nu} T_0 + h_\perp^{\mu\nu} T_2$$

$$T_0 = -\sum_q e_q^2 \int dx\, \alpha(x) H_q(x, \xi, \Delta_\perp^2) \ ,$$

$$h_\perp^{\mu\nu} = \frac{2\Delta_\perp^\mu \Delta_\perp^\nu}{\Delta_\perp^2} - g_\perp^{\mu\nu}$$

$$T_2 = \sum_q e_q^2 \frac{\alpha_s}{4\pi} \frac{\Delta_\perp^2}{4M^2} \int dx\, \alpha(x) E_{Tg}(x, \xi, \Delta_\perp^2)$$

$$\alpha(x) = \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon}$$

Hoodbhoy-Ji 98

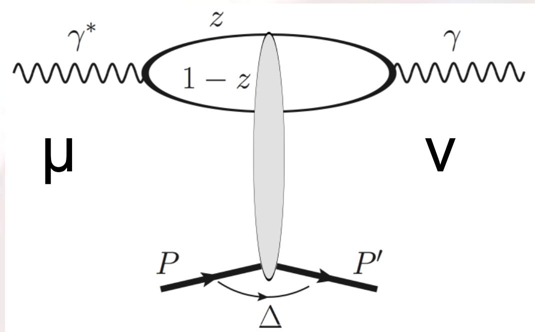
- Imaginary part at xi=x

$$\mathrm{Im}\, T_0 = \frac{\pi}{\xi} \sum_q e_q^2 \left[\xi H_q(\xi, \xi, \Delta_\perp^2) + \xi H_{\bar{q}}(\xi, \xi, \Delta_\perp^2) \right]$$

$$\mathrm{Im}\, T_2 = -\frac{\pi}{\xi} \frac{\alpha_s}{2\pi} \frac{\Delta_\perp^2}{4M^2} \sum_q e_q^2 \xi E_{Tg}(\xi, \xi, \Delta_\perp^2) \ ,$$

Vanishes at LO

DVCS: Helicity-conserved Amp.



$$g_\perp^{\mu\nu} A_0(\Delta_\perp) + h_\perp^{\mu\nu} A_2(\Delta_\perp)$$

$$\int dz d^2q_\perp d^2k_\perp \frac{(z^2+(1-z)^2)k_\perp \cdot (k_\perp + q_\perp)}{(k_\perp+q_\perp)^2(k_\perp^2+\epsilon_q^2)} F_x(q_\perp, \Delta_\perp)$$

$$\epsilon_q^2 = z(1-z)Q^2$$

- Dominant contributions from z~1 or 0,

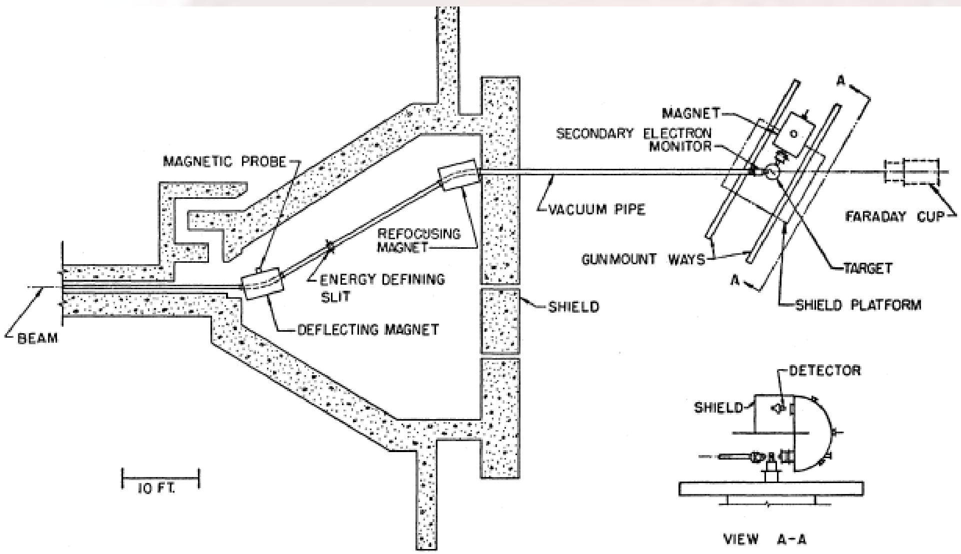
$$\int \frac{d^2k'_\perp}{(2\pi)^2}\frac{1}{k'^2_\perp}\int d^2q_\perp q_\perp^2 F_x(q_\perp,\Delta_\perp) \Rightarrow \int \frac{d^2k'_\perp}{(2\pi)^2}\frac{1}{k'^2_\perp} xH_g(x)$$

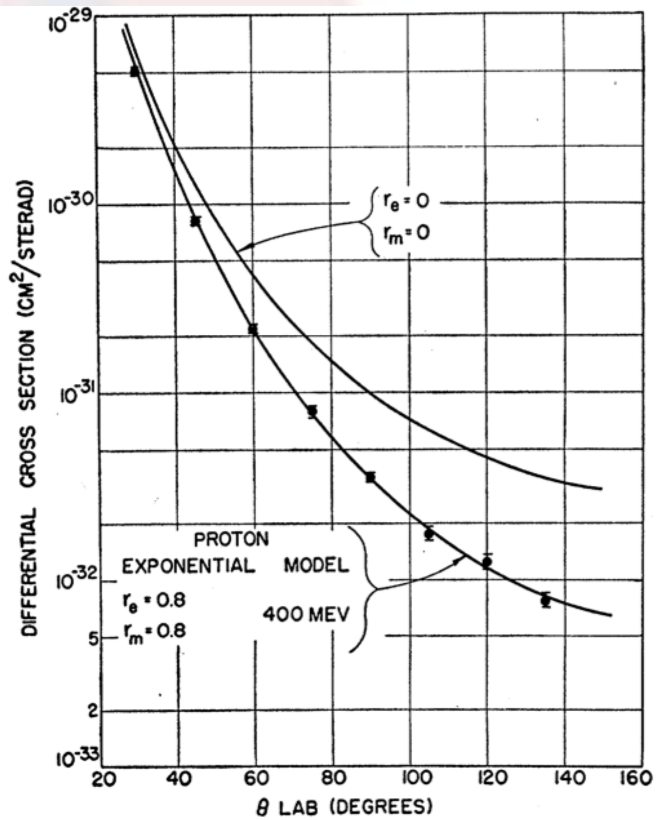
Hatta-Xiao-Yuan 1703.02085

Finite size of nucleon (charge radius)

Hofstadter

- Rutherford scattering with electron





Renewed interest on proton radius:

10/5/2018

RevModPhys.28.214

Size does matter!

- Rutherford formula for pint-like particle

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} = \left(\frac{\alpha}{4E\sin^2(\theta/2)}\right)^2$$

Extended target

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} \times \left(G(Q^2)\right)^2$$

$$G(Q^2) \overset{NR}{=} \int d^3r\, e^{i\vec{Q}\cdot\vec{r}}\, |\psi(r)|^2$$

$$G(Q^2) = 1 - \frac{1}{6}\langle r^2\rangle Q^2 + $$

Helicity-flip amplitude

$$\int dz d^2q_\perp d^2q_{1\perp} \frac{z(1-z)\left[2q_{1\perp}\cdot\Delta_\perp k_\perp\cdot\Delta_\perp - q_{1\perp}\cdot k_\perp\Delta_\perp^2\right]}{q_{1\perp}^2(k_\perp^2+\epsilon_q^2)\Delta_\perp^2} F_x(q_\perp,\Delta_\perp)$$

- In the DVCS limit, Q>>Δ

$$\mathcal{A}_2 = -\sum_q \frac{e_q^2 N_c}{Q^2}\int d^2q_\perp q_\perp^2 F_\epsilon(q_\perp,\Delta_\perp)$$

$$= -\frac{e_q^2\alpha_s\Delta_\perp^2}{4Q^2M^2}E_{Tg}(x,\Delta_\perp)$$

Hatta-Xiao-Yuan 1703.02085

Quark/GPD quark at small-x

- DGLAP splitting dominated by gluon distribution/GPD gluon

$$xq(x) = \frac{\alpha_s}{2\pi}\frac{1}{2}\int_x^1 d\zeta\left(\zeta^2+(1-\zeta)^2\right)x'G(x')\int\frac{dk_\perp^2}{k_\perp^2} \approx xG(x)\frac{\alpha_s}{2\pi}\frac{1}{2}\cdot\frac{2}{3}\int\frac{dk_\perp^2}{k_\perp^2}$$

$$xH_q(x,\xi,\Delta_\perp^2) = \frac{\alpha_s}{2\pi}\frac{1}{2}\int_x^1 d\zeta\frac{\zeta^2+(1-\zeta)^2-\frac{\xi^2}{x^2}\zeta^2}{(1-\frac{\xi^2}{x^2}\zeta^2)^2}x'H_g(x',\xi,\Delta_\perp^2)\int\frac{dk_\perp^2}{k_\perp^2}$$

Ji 97, Radyushkin 97

GPD quark distribution

$$\approx \xi H_g(\xi,\xi)\frac{\alpha_s}{2\pi}\frac{1}{2}\cdot 1\int\frac{dk_\perp^2}{k_\perp^2}$$