Interplay between Reggeon and Photon in pA Collisions at Strong Coupling

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Reference: Phys.Rev. D95 (2017) no.12, 126005
G. Basar, D. Kharzeev, I. Zahed, HUY

High s and low $(-t)$ scatterings: Non-perturbative

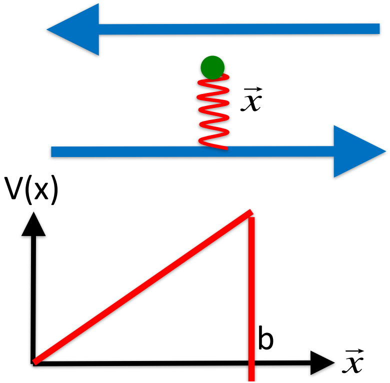
The QCD interactions should penetrate over a large transverse distance, across confining vacuum

It is natural to think of this problem with confining QCD strings

There is a linear potential barrier due to string tension, but when the string end meets the other projectile, there is an immense energy gain from the acceleration by the other projectile.

Tunneling problem for QCD strings that leads to "Stringy Instantons" in Euclidean space with boundary conditions set by the two projectiles

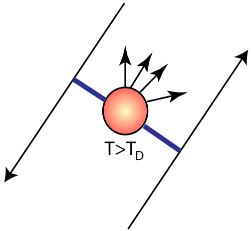
Time dependent quantum tunneling problem



We can interpret the stringy instantons as the instantons for Schwinger mechanism of particle creation, where the rapidity gap χ between the two projectiles plays a role of an effective electric field after a stringy-duality (Basar-Kharzeev-Zahed-HUY)

Compare

$$\exp[-\frac{m^2}{eE}] \quad \text{vs} \quad \exp[-\frac{b^2}{4\alpha'\chi}] \quad (b = \text{impact parameter})$$

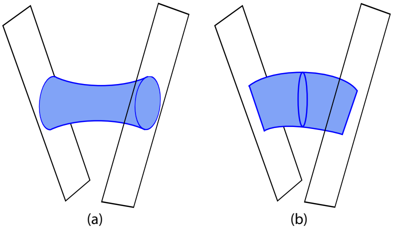


Stringy instantons

Euclidean, semi-classical 2-dimensional world sheets, that are bounded by the world lines of the two projectiles

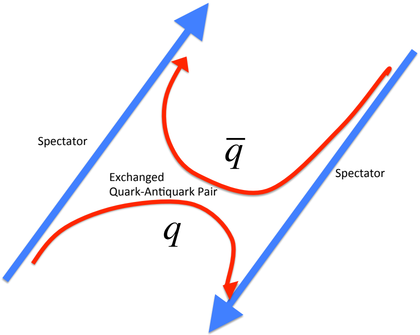
Recall that the string end points are quarks or anti-quarks

For charge neutral Pomerons without quarks, the string should be closed strings



Reggeons

For Reggeons with quark and anti-quark pair exchange, it should be open strings with boundary charged under QED. Its quantum number is that of flavored mesons.



Stringy instantons are known for both Pomerons and Reggeons (Gross-Mende, Schubert, Janik-Peschanski, BKIY)

In weak coupling perturbative QCD, we have BFKL gluon ladders.

Q: Can we describe BFKL with some gluonic instantons, like t'Hooft-Polyakov instanton?

Realization in AdS/CFT

The non-perturbative, strong coupling nature may warrant the application of AdS/CFT. There have been many works (Janik-Peschanski, Sin-Zahed, Brower-Polchinski-Strassler-Tan, Hatta-Iancu-Mueller, Albacete-Kovchegov-Taliotis, Watanabe-Suzuki, BKIY, Mamo-Zahed, and more)

It can be shown that a low $(-t)$ prefers the strings to stay in more IR "bottom" of the AdS.

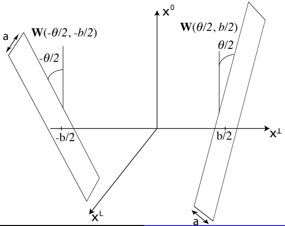
For this regime, relevant for the total cross section of $t \to 0$ limit, the strings essentially stay in 3+1 dimensional "bottom" of the 5 dimensional space-time, and the description becomes identical to the usual 3+1 D QCD strings.

Some useful formulas

The stringy instantons are found in the impact parameter geometry, where *b* is the transverse distance between two projectiles, and $\chi \sim \log s$ is the rapidity gap. In Euclidean space, the χ becomes the angle θ in the longitudinal space between the two projectiles by analytic continuation $\theta = -i\chi$ (Meggiolaro). Recall

$$ds^2 = -d\tau^2 + \tau^2 d\chi^2 \,, \quad ds^2_{Euclidean} = dr^2 + r^2 d\theta^2$$

The action of instanton is the minimal Euclidean area of the worldsheet, in saddle point approximation



After finding $\mathcal{T}(s, \boldsymbol{b}) \sim \frac{i}{s} \exp[-S_{intanton}(\chi, \boldsymbol{b})]$, we can go to $(-t)$ space by

$$\mathcal{T}(s,t) = \int d^2\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{T}(s,\boldsymbol{b})\,, \quad t \equiv -\boldsymbol{q}^2$$

The optical theorem at $t \to 0$ gives the total cross section

$$\sigma_{tot} = \frac{1}{s} \mathrm{Im}\left[\mathcal{T}(s, t=0)\right] = \frac{1}{s} \int d^2\boldsymbol{b}\, \mathrm{Re}\left[e^{-S_{instanton}(\chi,\boldsymbol{b})}\right]$$

QED in Reggeon Physics?

The QED effect is usually neglected due to small α_{EM}, and assumed to NOT modify the QCD Reggeon aplitudes

$$\mathcal{T}_{Reggeon} \sim \frac{1}{s}\exp(-S_{instanton})$$

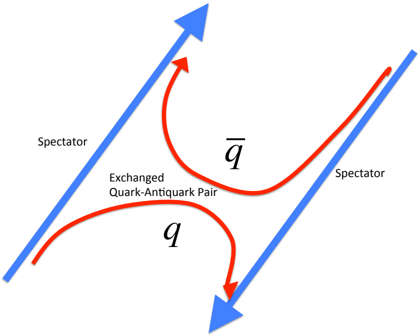
where $\frac{1}{s}$ is from the spinor overlap between *p* and $-p$. The QED amplitude is given by the Wilson lines of EM field produced by projectiles, evaluated along the exchanged quark-antiquark lines (the boundary of the instanton) $\mathcal{T}_{QED} \sim \exp(iq\int A_{EM})$. The total scattering amplitude is $\mathcal{T}_{Reggeon} \cdot \mathcal{T}_{QED}$, which contains an interference effect, when we go to $(-t)$ space from the impact parameter b space.

This seems okay for *pp* or *ep*

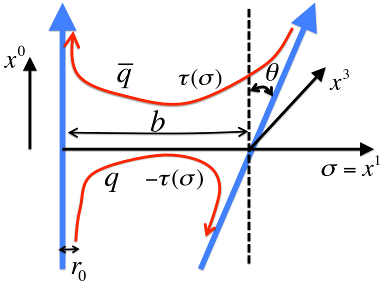
QED in pA with $Z \approx 100$ may be significant

$$Z \cdot \alpha_{EM} \sim 1$$

We will see that QED action needs to be included in the saddle point analysis, and the instantons of the full action $S_{QCD} + S_{QED}$ are very different from the pure Reggeon case. This is not just an interference effect



Short recap of pure Reggeon case (Janik-Peschanski)



The world-sheet is assumed to be in the helicoid surface defined by $x^0 = \tau\cos(\theta(\sigma)), x^3 = \tau\sin(\theta(\sigma)), x^1 = \sigma$ where $\theta(\sigma) \equiv \frac{\theta}{b}\sigma$ and $0 < \sigma < b$. The string boundary is given by the curve $\tau(\sigma)$ so that τ ranges $-\tau(\sigma) < \tau < \tau(\sigma)$ for a given σ

The string action is given by the string area (Nambu-Goto)

$$S_{Reggeon} = \frac{1}{2\pi\alpha'}\int_0^b d\sigma \int_{-\tau(\sigma)}^{\tau(\sigma)} d\tau \sqrt{1+\frac{\theta^2}{b^2}\tau^2} = \frac{1}{2\pi\alpha'}\int_0^b d\sigma \int_{-\tau(\sigma)}^{\tau(\sigma)} d\tau \sqrt{1-\frac{\chi^2}{b^2}\tau^2}$$

The $\tau(\sigma)$ is the variational parameter to find the saddle point (instantons). The equation of motion is

$$1-\frac{\chi^2}{b^2}\tau(\sigma)^2 = 0$$

with a simple solution $\tau(\sigma) = \frac{b}{\chi}$ (constant), and the instanton action is

$$S_{instanton} = \frac{1}{2\pi\alpha'}\int_0^b d\sigma \int_{-\frac{b}{\chi}}^{\frac{b}{\chi}} d\tau \sqrt{1-\frac{\chi^2}{b^2}\tau^2} = \frac{b^2}{4\alpha'\chi}$$

The t-space amplitude from this is

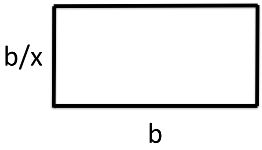
$$\mathcal{T}(s,t) \sim \frac{2i}{s}\int_0^\infty bdb \int_0^{2\pi} d\theta e^{i\sqrt{(-t)}b\cos\theta} e^{-\frac{b^2}{4\alpha'\chi}} \sim i(\log s)s^{\alpha' t-1}$$

which is the Regge behavior.

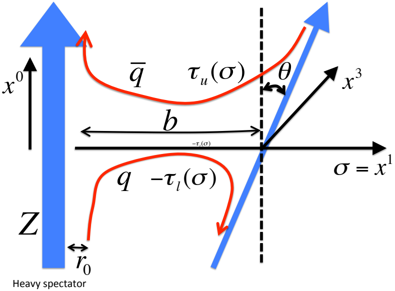
The intercept was shown to arise from 1-loop contribution

$$S_{1-loop} = \frac{D_\perp}{2} \log\det(\partial^2)$$

The string worldsheet has an elongated shape with two sides b **and** $\frac{\pi}{2}\frac{b}{\chi} \ll b$**, and** S_{1-loop} **is given by Casimir scaling** $S_{1-loop} \sim -\frac{\pi D_\perp}{24}\frac{b}{\left(\frac{\pi}{2}\frac{b}{\chi}\right)} = -\frac{D_\perp}{12}\chi$ **and** $\exp(-S_{1-loop}) = s^{\frac{D_\perp}{12}}$**, so that the intercept is** $\alpha_0 = \frac{D_\perp}{12} = 0.25$**.**



Reggeon and QED in pA case



The nucleus of charge Z produces the Euclidean gauge field

$$A_0 = i\frac{Ze}{4\pi r}\,, \quad r = \sqrt{\boldsymbol{x}\cdot\boldsymbol{x}}$$

and the quark-antiquarks boundary represented by $\tau_u(\sigma)$ and $\tau_d(\sigma)$ of charges q_u and $-q_d$ will get a leading order Wilson lines

$$\exp\left[ie\sum_i q_i \int_{C_i} A_\mu dx^\mu\right] \equiv \exp\left[-S_{QED}\right]$$

The expression for S_{QED} is

$$S^{q_u}_{\rm QED} = \frac{q_u Z e^2}{4\pi}\left(\frac{1}{r_0}\int_{\tau_u(0)}^{\infty} d\tau + \int_{\infty}^{\tau_u(b)} \frac{d\tau}{\sqrt{(b+r_0)^2+\tau^2\sin^2\theta}} - \int_0^b d\sigma\, \frac{\left(\frac{d\tau_u(\sigma)}{d\sigma}\right)\cos\left(\frac{\theta\sigma}{b}\right) - \frac{\theta}{b}\tau_u(\sigma)\sin\left(\frac{\theta\sigma}{b}\right)}{\sqrt{(\sigma+r_0)^2+\tau_u^2(\sigma)\sin^2\left(\frac{\theta\sigma}{b}\right)}}\right)$$

and the full instanton equation of motion with $S_{Reggeon} + S_{QED}$ becomes

$$\frac{1}{2\pi\alpha'}\sqrt{1-\frac{\chi^2}{b^2}y(\sigma)} - \frac{q_u Z e^2}{4\pi}\frac{\left(\cosh\left(\frac{\chi\sigma}{b}\right)(\sigma+r_0) - \frac{\chi}{b}\sinh\left(\frac{\chi\sigma}{b}\right)y(\sigma)\right)}{\left((\sigma+r_0)^2 - \sinh^2\left(\frac{\chi\sigma}{b}\right)y(\sigma)\right)^{\frac{3}{2}}} = 0$$

where $y(\sigma) \equiv \tau(\sigma)^2$.

This is the basic equation to solve to study the interplay with QCD Reggeon of string worldsheet and strong QED

There exist two branches of solutions

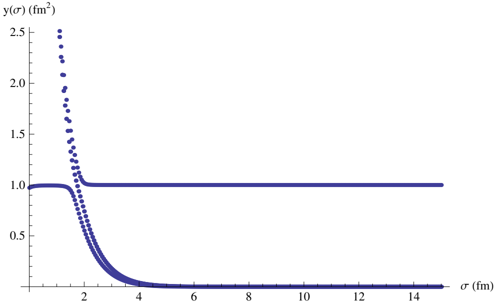


Figure: $\alpha' = 0.036\mathrm{fm}^2$, $r_0 = 1\mathrm{fm}$, $\chi = 15$, $Z = 100$, $\frac{b}{\sqrt{\alpha'}} = 80$

The upper curve is "perturbative QED" branch, that ceases to exist for $\sigma > \sigma_c$. The S_{QED} is purely imaginary.
The lower curve is "non-perturbative" branch, and S_{QED} develops a real part in $\sigma < \sigma_c$, where $\sigma_c \approx 0.23\frac{b}{\chi}$

The $y(\sigma)$ is nearly constant in $\sigma < \sigma_c$ with a value

$$y(0) = \tau(0)^2 = \frac{b^2}{\chi^2}\left(1 - \left(\frac{2\pi\alpha' q_u Z e^2}{4\pi r_0^2}\right)^2\right)$$

Note that $\tau(\sigma)$ becomes imaginary when $\left(\frac{2\pi\alpha' q_u Z e^2}{4\pi r_0^2}\right)^2 > 1$, and the full action becomes purely imaginary: complete QED dominance over QCD

This doesn't happen with our relevant parameters, though

The leading large b behavior of the instanton action is

$$\mathrm{Re}[S_{\mathrm{total}}] \approx \frac{C}{4\pi\alpha'}\left(\sqrt{\bar{y}(0)\left(1-\bar{y}(0)\right)}+\sin^{-1}\left(\sqrt{\bar{y}(0)}\right)\right)\frac{b^2}{\chi^2}$$

$$\equiv \frac{1}{4\pi\alpha'(Z)}\frac{b^2}{\chi^2}$$

Since the string worldsheet shape is square-like, there is no important contribution from 1-loop

From this, we have the t-space amplitude

$$\mathcal{T}(s,t) \sim i(\log s)^2 e^{\alpha'(Z)t(\log s)^2}$$

and the total cross section is

$$\sigma_{tot} \sim \frac{(\log s)^2}{s}$$

Recall the Froissart bound $\sigma_{tot} \leq (\log s)^2$

Numerical plot of $\alpha'(Z)$