

# Factorization for quarkonium production in p+p and p+A collisions

Kazuhiro Watanabe  
Theory Center, Jefferson Lab

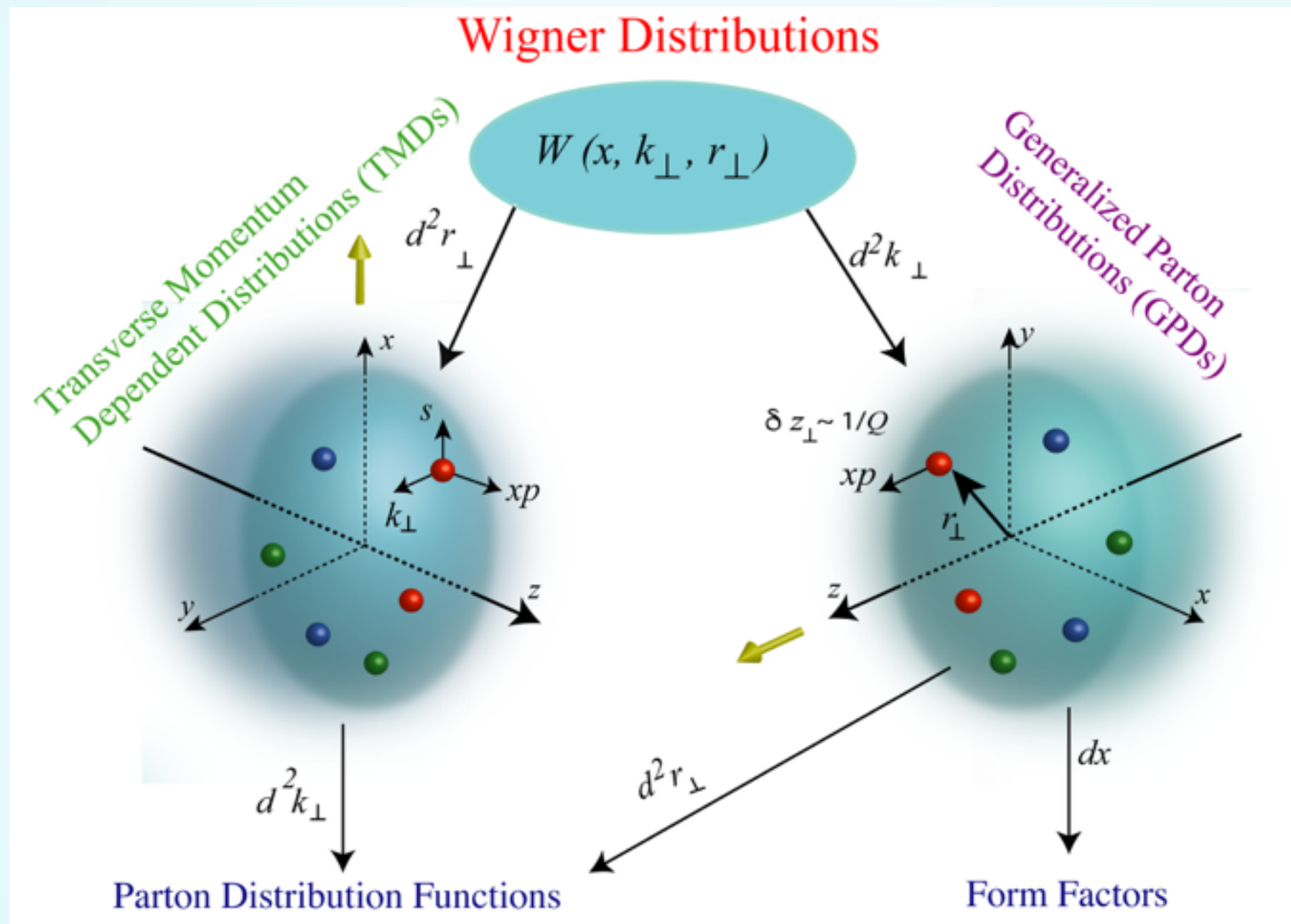
11/15/2018 @ INT

# 3D Tomography: Key physics

4+1D

2+1D

1D

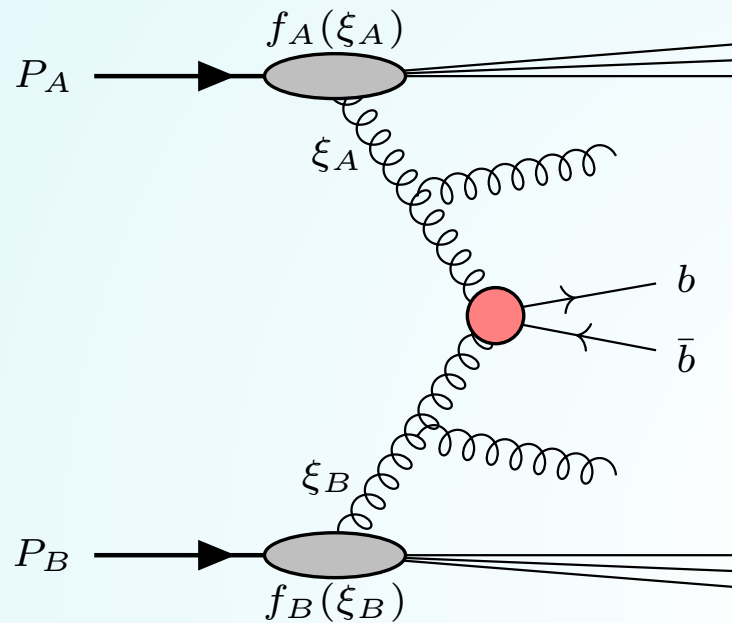


TMDs: 3D parton's confined motion.

GPDs: 3D parton's spatial distribution.

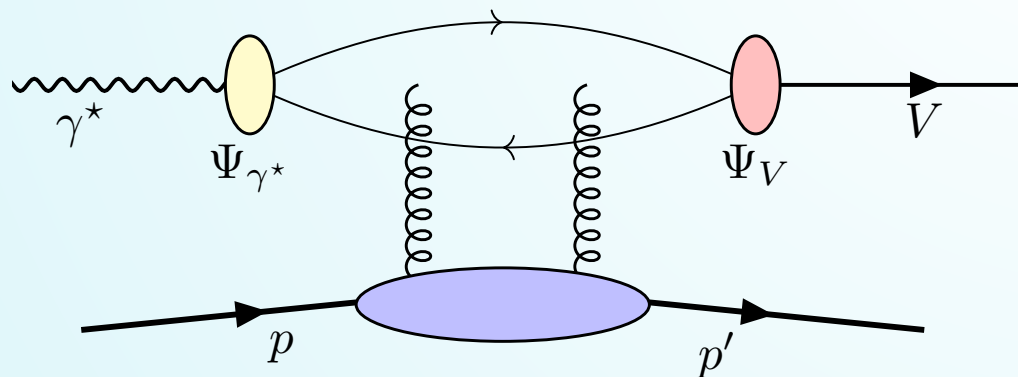
# Access to 3D structures

TMD: Two hard scales required.  $\Lambda_{\text{QCD}} \ll p_{\perp} \ll Q$



- Semi-Inclusive DIS
- Drell-Yan process
- Higgs, Z/W, and **Quarkonium production**

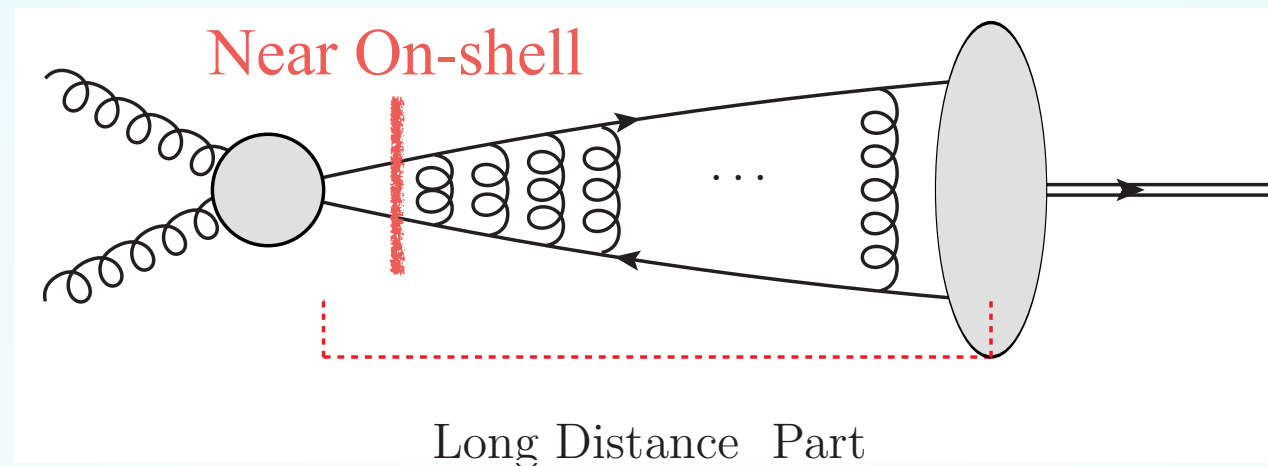
GPD: Non-forward Exclusive process



- Deeply Virtual Compton Scattering
- **Exclusive Vector Meson Production**

Quarkonium is a very interesting probe.

# Modern approaches for quarkonium production



- QQbar production

- Initial parton distribution functions
- Hard scattering part

- Onium production

- Color Singlet Model (CSM): Produced QQbar is in **Color singlet state**.
- Color Evaporation Model (CEM): All color and spin states are summed up.
- Non-Relativistic QCD (NRQCD): **Color octet contribution**, heavy quark velocity expansion.

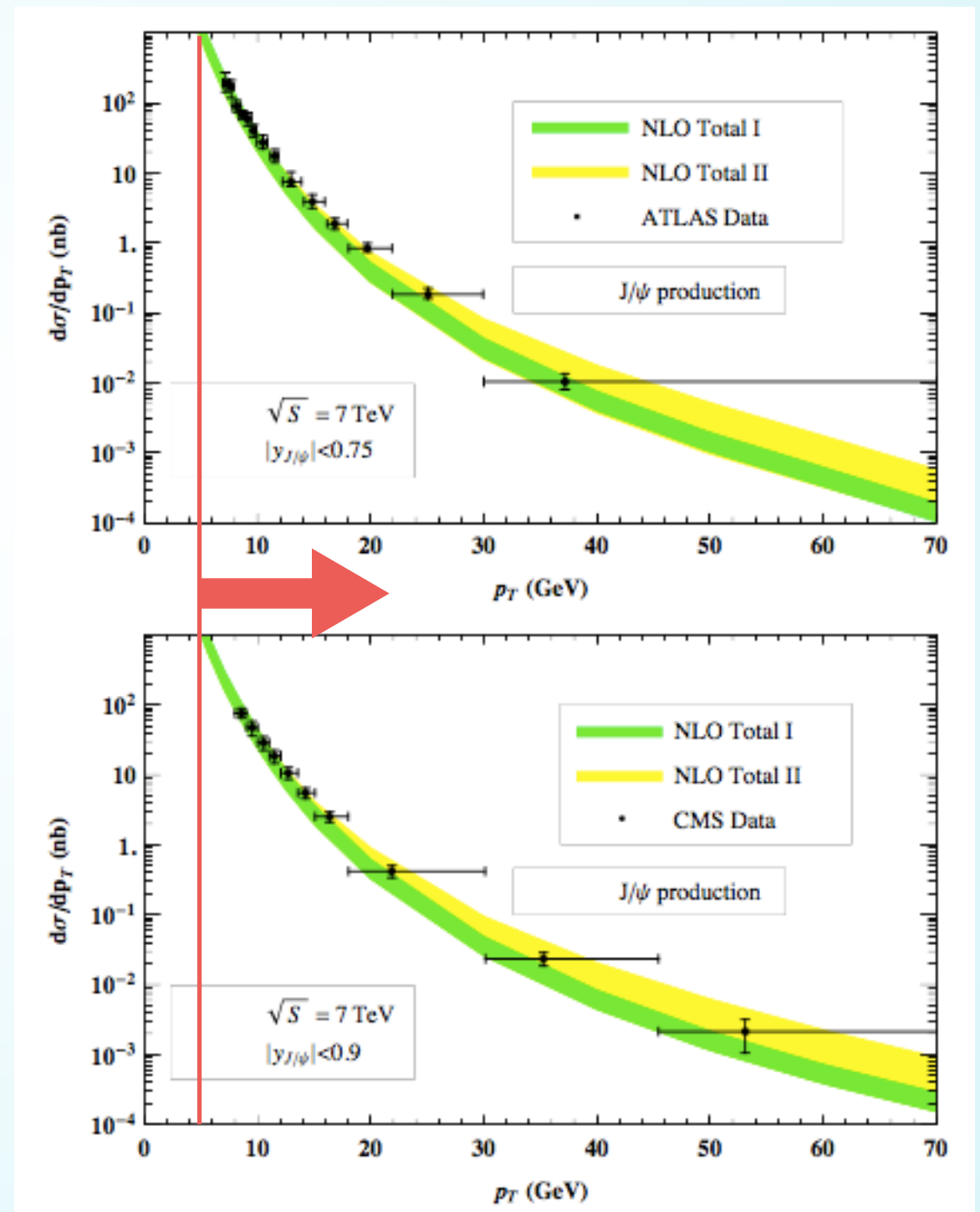


# Success in NRQCD

$$\frac{d\sigma^\psi}{dyd^2p_\perp} = \sum_{\kappa} \frac{d\hat{\sigma}^\kappa}{dyd^2p_\perp} \langle \mathcal{O}_{\kappa}^\psi \rangle$$

- Long Distance Matrix Elements (LDMEs) are fitted by Tevatron data at large  $p_\perp$ .
- NLO, Collinear factorization.
- Fixed order calculation works down to  $p_\perp \sim 5\text{GeV}$  ( $pt \sim M$ ).

Chao, Ma, Shao, Wang and Zhang, PRL108 (2012)



# Issues from 1st week

## THEORY DEVELOPMENTS NEEDED OR PLANNED:

-- Explore applicability of NRQCD methods to exclusive heavy quarkonium photo/electroproduction at EIC. Possible points of interest are (i) tests of universality of the NRQCD matrix elements, (ii) better insight into bound-state structure and production mechanism through  $Q^2$ -dependence.

## CONNECTIONS WITH OTHER FIELDS:

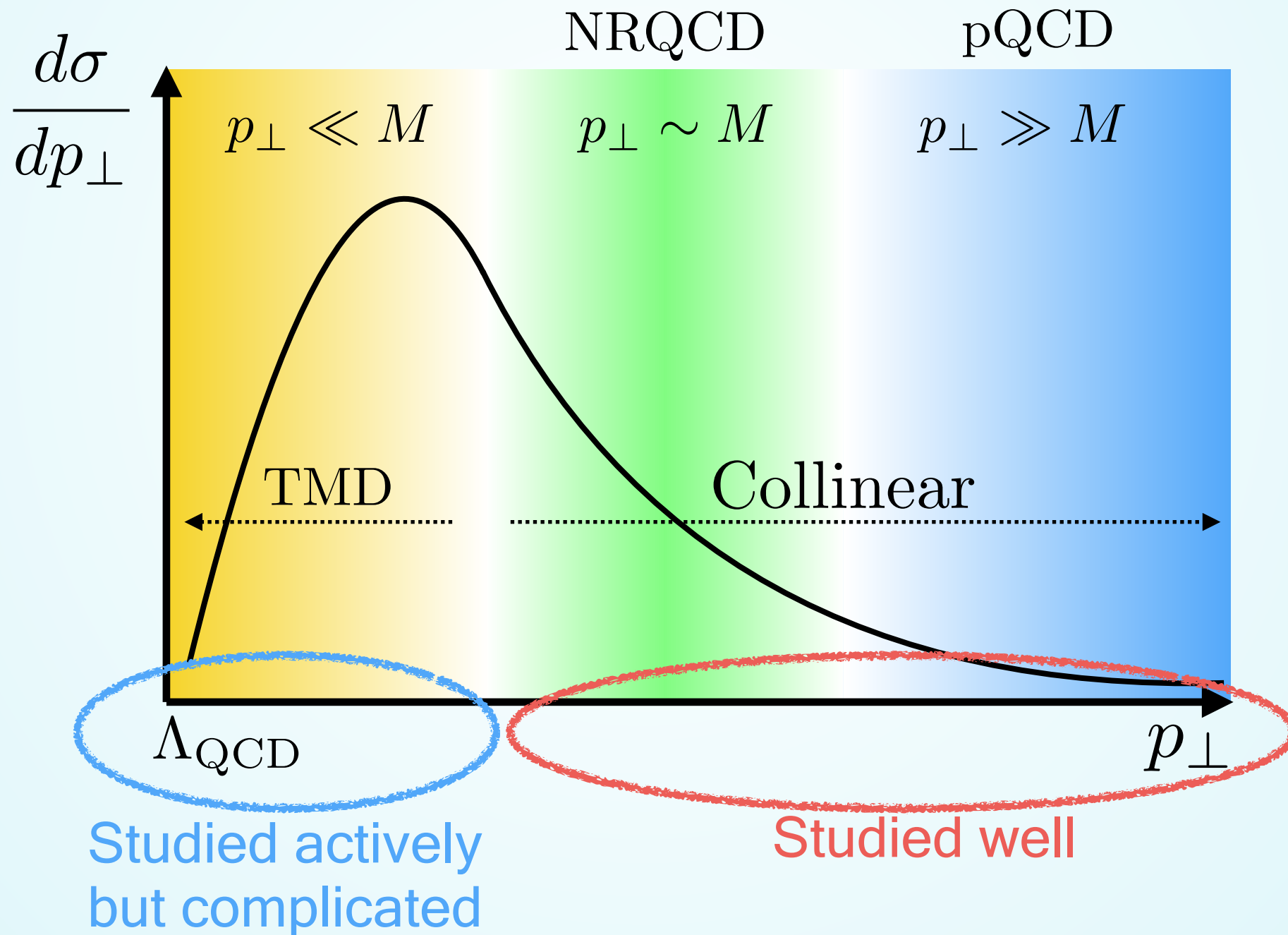
### NONRELATIVISTIC QCD (NRQCD):

-- NRQCD matrix elements used in inclusive heavy quarkonium production in  $pp/pA$  may be used/tested/measured in exclusive production in  $ep/\gamma p$ .

Consider carefully factorization for quarkonium production in  $p+p/p+A$  collisions.

# From TMD to Collinear

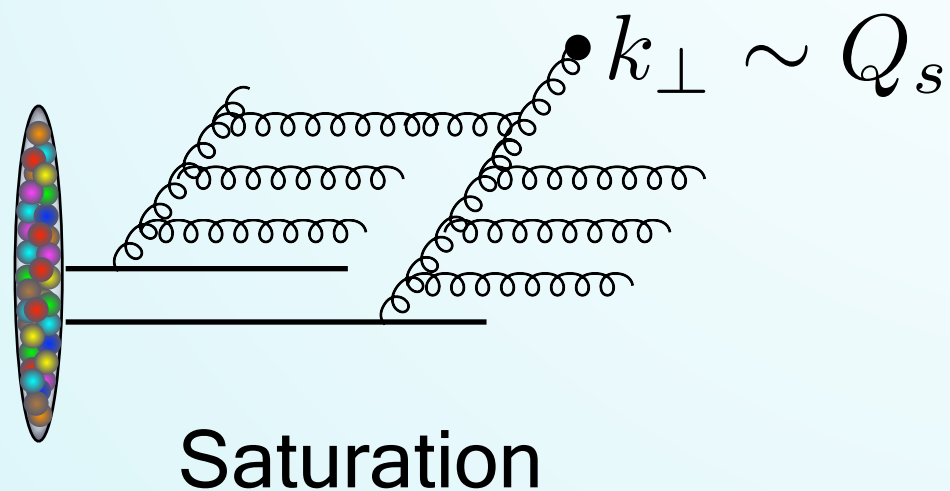
TMD: Transverse Momentum Dependent Framework  
( $kt$ -factorization)



# TMD approach

- CGC framework

- Transverse Momentum Dependent PDF w/  $Q_s$
- Single logs
- JIMWLK or BK eq. (Nonlinear evolution)
- Applicable to only small- $x$



- CSS formalism

- Transverse Momentum Dependent PDF
- Single logs + Double logs
- DGLAP and Collins-Soper eqs. (Linear evolution)
- Applicable to the whole  $x$



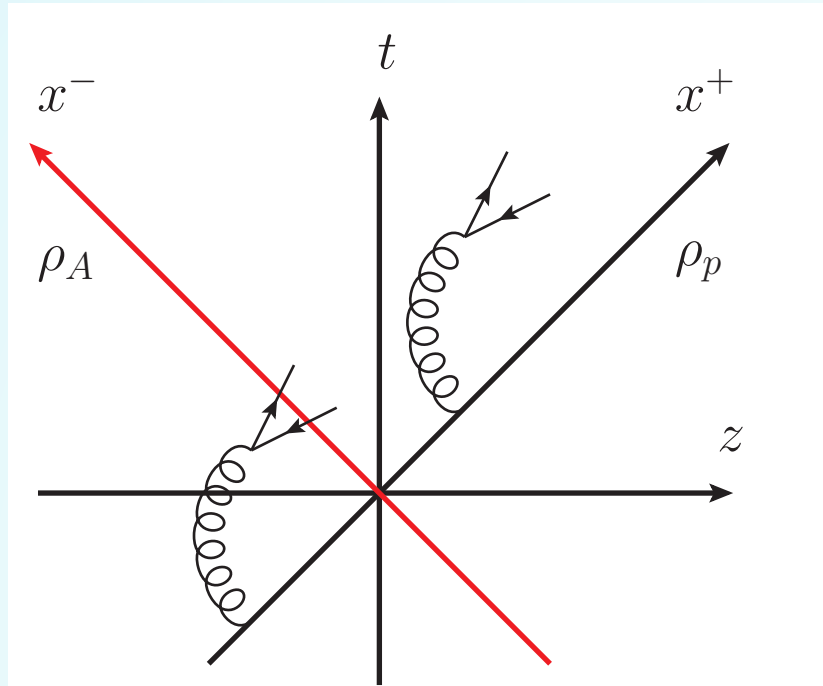
# Outline

1. Quarkonium production in the CGC framework
2. Quarkonium production in the TMD framework
3. Factorization breaking in  $p+A$  collisions

# Quarkonium production in the CGC framework



# QQbar production in dilute-dense system



Blaizot, Gelis, Venugopalan (2004)

Kovchegov, Tuchin (2006)

See also, Kharzeev, Tuchin (2005)

- Proton side: Leading twist
- Nucleus side: All orders resummation
- $\alpha_s^2 A^{1/3} \sim \mathcal{O}(1) \rightarrow$  Multiple scattering

$$M_{s_1 s_2; ij}(p, q) = \frac{g^2}{(2\pi)^4} \int d^2 k_\perp d^2 k_{1\perp} \frac{\rho_p(x_p, k_{1\perp})}{k_{1\perp}^2} \int d^2 x_\perp d^2 y_\perp e^{ik_\perp \cdot x_\perp} e^{i(p_\perp - k_\perp - k_{1\perp}) \cdot y_\perp} \\ \times \bar{u}_{s_1, i} \left( \frac{p}{2} + q \right) [T_g(p, k_{1\perp}) t^b W^{ba}(x_\perp) + T_{q\bar{q}}(p, q, k_{1\perp}, k_\perp) U(x_\perp) t^a U^\dagger(y_\perp)] v_{s_2, j} \left( \frac{p}{2} - q \right)$$

$$\mathcal{O}(\rho_p^1 \rho_A^\infty)$$

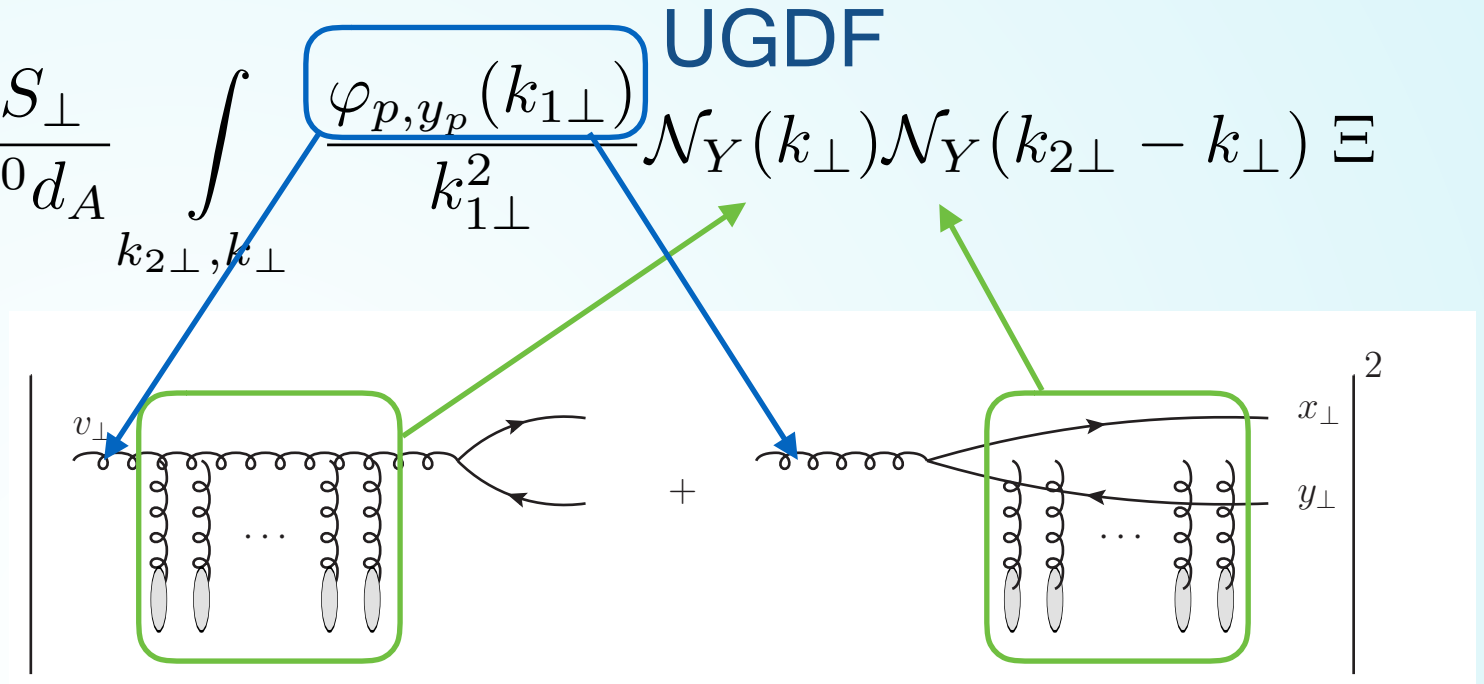
$$U(x_\perp) = \mathcal{P}_+ \exp \left[ ig \int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp) \cdot t \right]$$

$$W(x_\perp) = \mathcal{P}_+ \exp \left[ ig \int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp) \cdot T \right]$$

Wilson lines in the Eikonal approximation

# Onium in the CGC+CEM

$$\frac{d\sigma_{Q\bar{Q}}}{d^2p_{Q\perp}d^2q_{\bar{Q}\perp}dy_Qdy_{\bar{Q}}} = \frac{\alpha_s N_c^2 S_\perp}{2(2\pi)^{10}d_A} \int_{k_{2\perp}, k_\perp} \frac{\varphi_{p,y_p}(k_{1\perp})}{k_{1\perp}^2} \mathcal{N}_Y(k_\perp) \mathcal{N}_Y(k_{2\perp} - k_\perp) \Xi$$



CEM

$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_{c\bar{c} \rightarrow \psi} \int_{2m_c}^{2m_D} dM \frac{d\sigma_{c\bar{c}}}{dM d^2p_\perp dy}$$

Improved CEM

Ma, Vogt, PRD94 (2016)

$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_{c\bar{c} \rightarrow \psi} \int_{m_\psi}^{2m_D} dM \left( \frac{M}{m_\psi} \right)^2 \frac{d\sigma_{c\bar{c}}}{dM d^2p'_\perp dy} \left| p'_\perp = \frac{M}{m_\psi} p_\perp \right.$$

Gluon radiation during hadronization

# Onium in the CGC+NRQCD

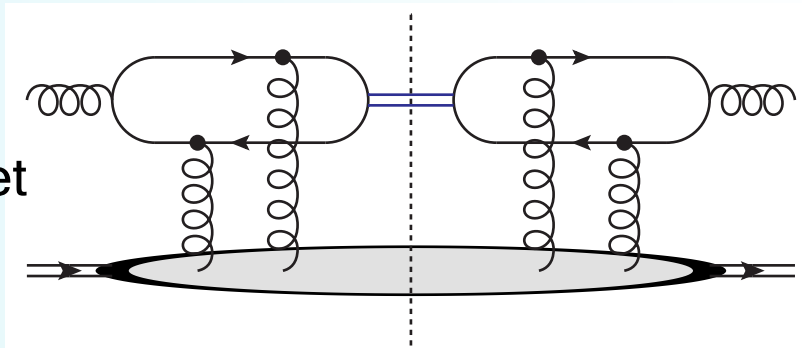
Kang, Ma, Venugopalan, (2013)

$$\frac{d\sigma^\psi}{dydp_\perp^2} = \sum_\kappa \frac{d\hat{\sigma}_{c\bar{c}}^\kappa}{dydp_\perp^2} \times \langle \mathcal{O}_\kappa^\psi \rangle$$

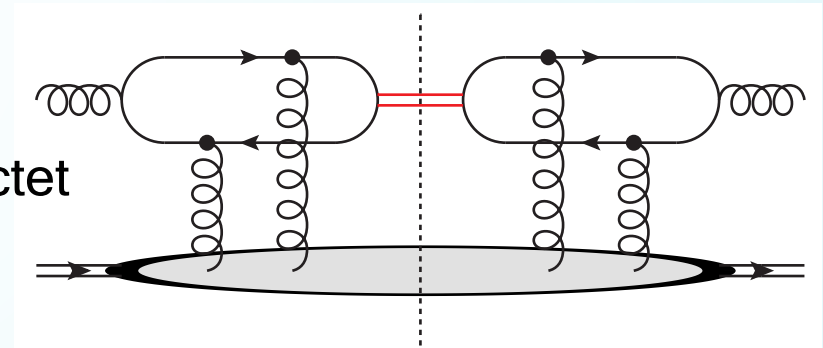
$$\frac{d\sigma_{c\bar{c},CS}^\kappa}{d^2p_\perp dy} = \frac{\alpha_s \pi R_A^2}{(2\pi)^9 d_A} \int_{k_{2\perp}, k_\perp, k'_\perp} \frac{\varphi_{p,y_p}(k_{1\perp})}{k_{1\perp}^2} \mathcal{N}_Y(k_\perp) \mathcal{N}_Y(k'_\perp) \mathcal{N}_Y(k_{2\perp} - k_\perp - k'_\perp) \mathcal{G}_1^\kappa$$

$$\frac{d\sigma_{c\bar{c},CO}^\kappa}{d^2p_\perp dy} = \frac{\alpha_s \pi R_A^2}{(2\pi)^7 d_A} \int_{k_{2\perp}, k_\perp} \frac{\varphi_{p,y_p}(k_{1\perp})}{k_{1\perp}^2} \mathcal{N}_Y(k_\perp) \mathcal{N}_Y(k_{2\perp} - k_\perp) \Gamma_8^\kappa$$

Color Singlet



Color Octet

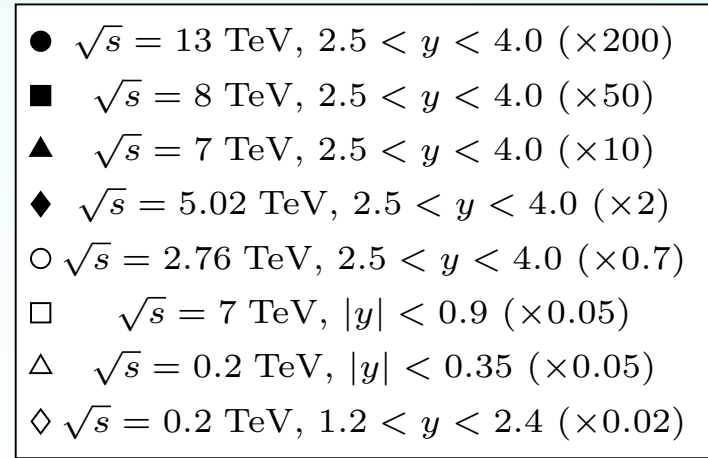
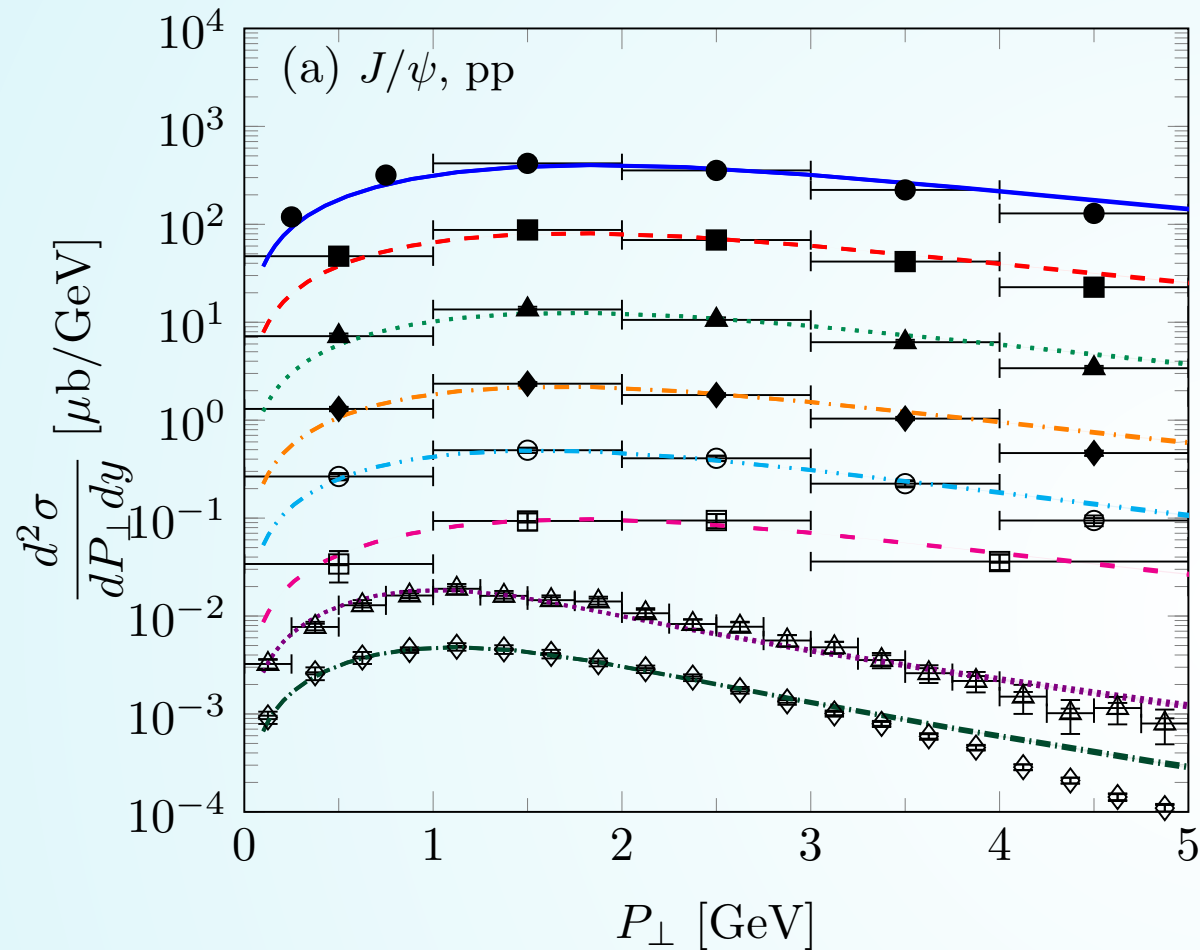


CS channel probes the quadrupole amplitude. However, thanks to large- $N_c$  approximation and quasi-classical approximation, the quadrupole amplitude is simply cubic in  $N_Y$ .

Dominguez, Kharzeev, Levin, Mueller, Tuchin, PLB710, (2012)

# J/psi in the CGC+ICEM

Ma, Venugopalan, KW, Zhang (2018)

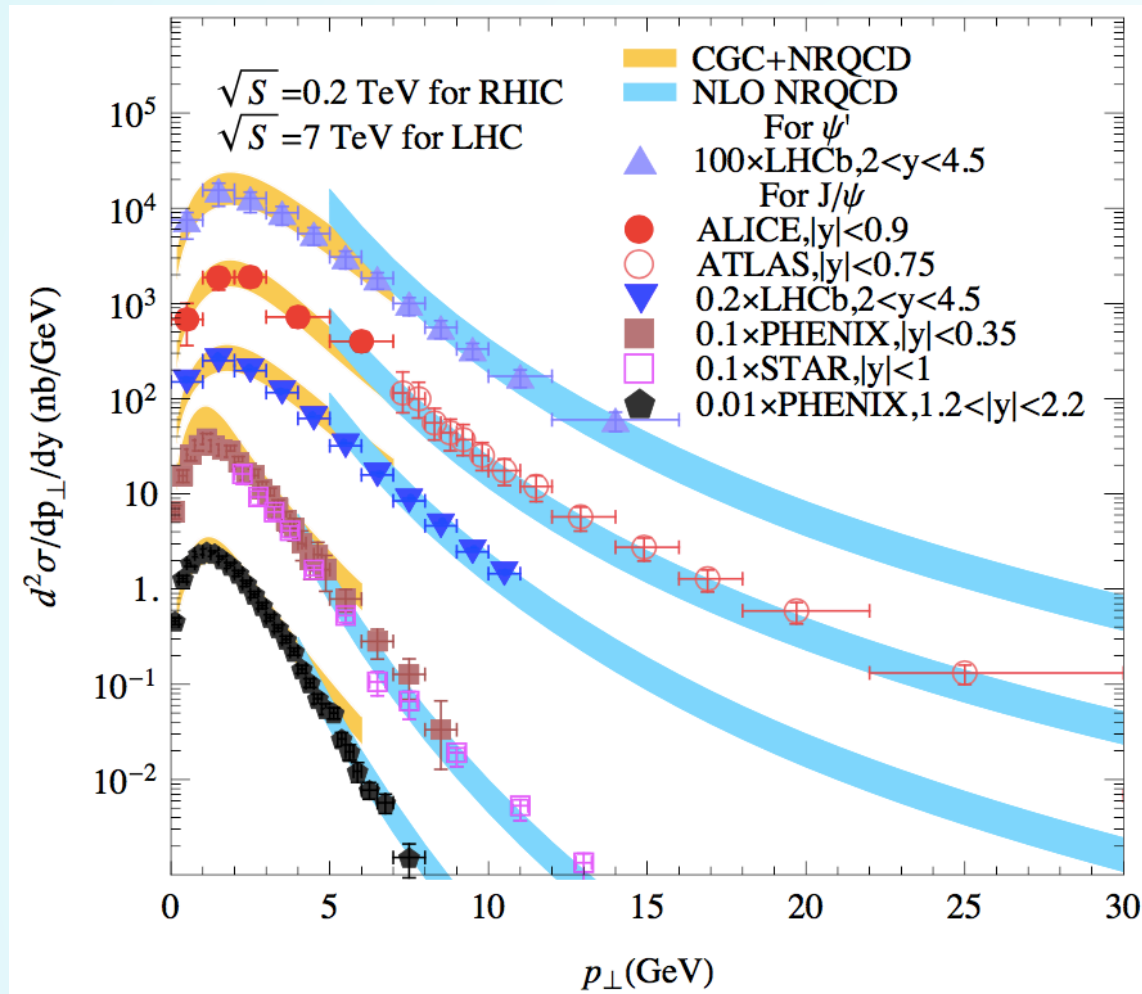


● MV IC + rcBK eq. gives a good parametrization of gluon TMD at small-x.

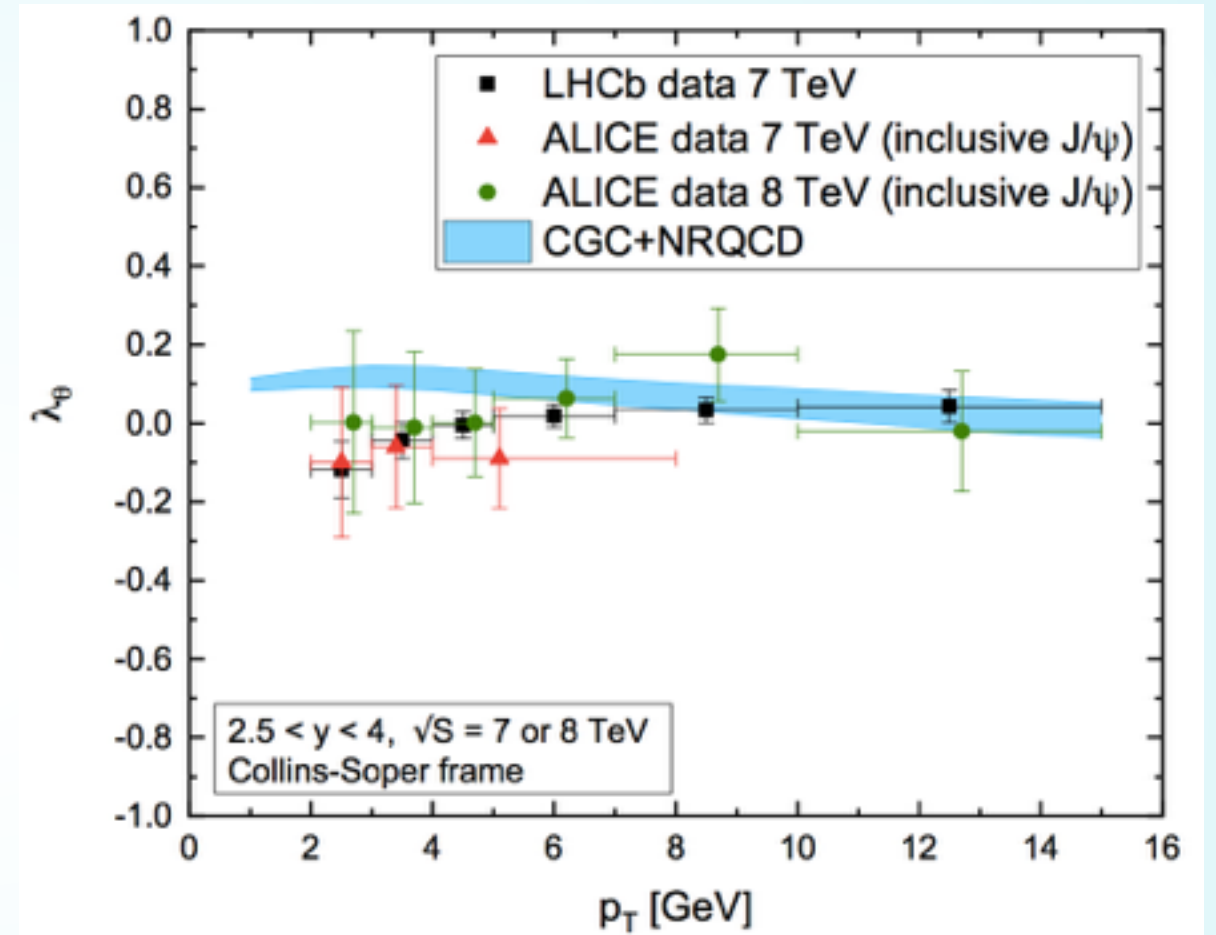
$\sqrt{s}$ [TeV]	$y$ bin	Data points	$F_{J/\psi}(1.3)$	$\chi^2/d.o.f.$
13	$2.5 < y < 4.0$	7	$5.63 \times 10^{-3} \pm 3.56 \times 10^{-4}$	8.7
8	$2.5 < y < 4.0$	6	$5.84 \times 10^{-3} \pm 3.79 \times 10^{-4}$	4.3
7	$ y  < 0.9$	5	$4.52 \times 10^{-3} \pm 2.33 \times 10^{-4}$	0.33
7	$2.5 < y < 4.0$	6	$4.86 \times 10^{-3} \pm 3.01 \times 10^{-4}$	5.3
5.02	$2.5 < y < 4.0$	6	$5.26 \times 10^{-3} \pm 3.35 \times 10^{-4}$	7.0
2.76	$2.5 < y < 4.0$	6	$5.27 \times 10^{-3} \pm 1.84 \times 10^{-4}$	0.88
0.2	$ y  < 0.35$	21	$6.36 \times 10^{-3} \pm 1.94 \times 10^{-4}$	0.93
0.2	$1.2 < y < 2.4$	24	$1.06 \times 10^{-2} \pm 1.30 \times 10^{-4}$	0.35

# J/psi in the CGC+NRQCD

Ma, Venugopalan (2014)



Ma, Stebel, Venugopalan (2018)



Prompt J/psi data at high  $p_T$  at Tevatron;

Chao, Ma, Shao, Wang, Zhang (2012)

$$\langle \mathcal{O}^{J/\psi} [^1S_0^{[8]}] \rangle = 0.089 \pm 0.0098 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi} [^3S_1^{[8]}] \rangle = 0.0030 \pm 0.0012 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi} [^3P_0^{[8]}] \rangle / m_c^2 = 0.0056 \pm 0.0021 \text{ GeV}^3$$



# NRQCD LDMEs

Fits from Tevatron, LHC, HERA, LEP:

	$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV <sup>3</sup>	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ 10 <sup>-2</sup> GeV <sup>3</sup>
Bodwin et al	-	9.9	1.1	0.49
Butenschoen et al	1.32	3.04	0.16	-0.30
Chao et al	1.16	8.9	0.30	0.56
Gong et al	1.16	9.7	-0.46	-0.95

Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).

Butenschoen, Kniehl, PRD84, 051501 (2011).

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).

Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).

In e<sup>+</sup>e<sup>-</sup> scattering:

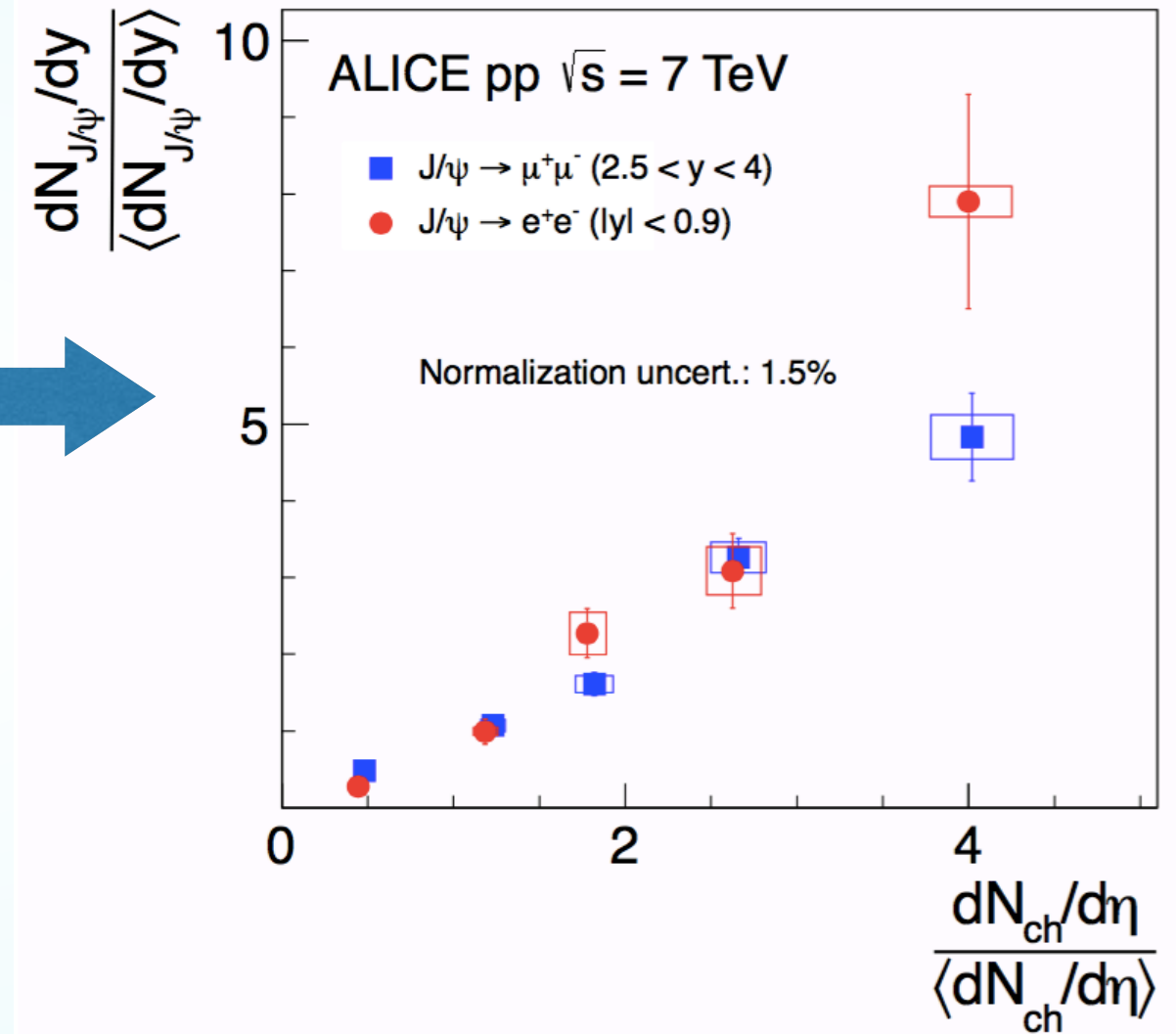
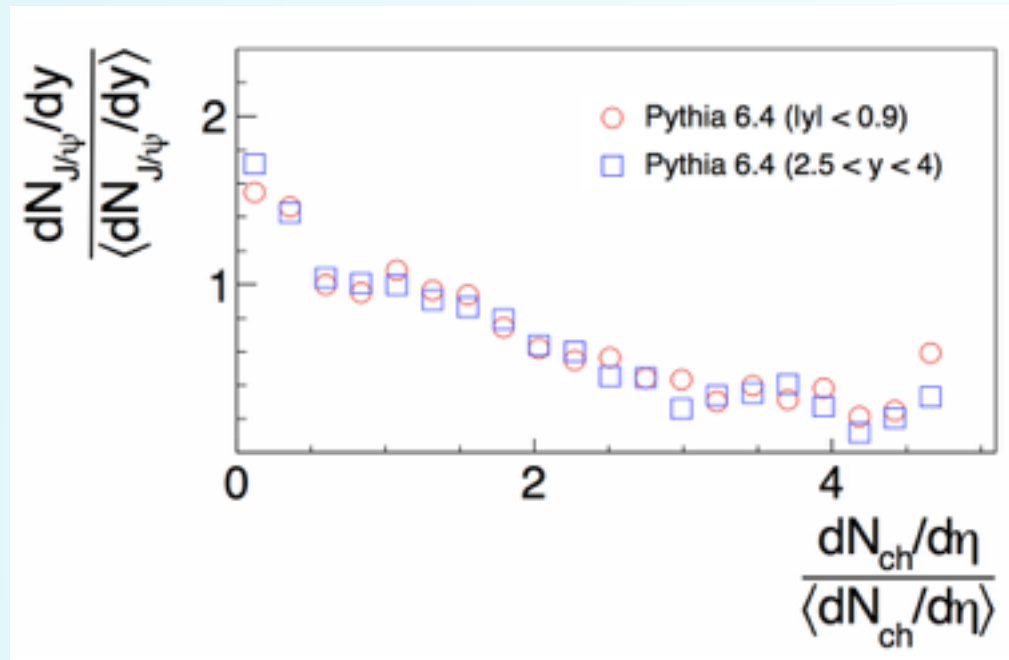
$$\langle \mathcal{O}^{J/\psi}[^1S_0^{[8]}] \rangle + 4.0 \langle \mathcal{O}^{J/\psi}[^3P_0^{[8]}] \rangle / m^2 < 2.0 \pm 0.6 \times 10^{-2} \text{GeV}^3$$

Zhang, Ma, Wang, Chao, PRD81 (2010)



# J/psi vs Nch

ALICE Collaboration,  
PLB712 (2012)



Soft mode vs Hard mode

Can we test universality of LDMEs?

# Quantum Fluctuation

At extreme high energy, gluon field behaves like classical: Small coupling constant and gluon nonlinear interaction.  $\leftrightarrow A \sim 1/g$

$$\frac{dN_{ch}}{d^2b_{\perp} d^2k_{\perp} dy} \sim \langle AA \rangle \sim \frac{f(k_{\perp}/Q_s)}{\alpha_s} \Rightarrow \frac{dN_{ch}}{dy} \sim \frac{S_{\perp} Q_s^2}{\alpha_s}$$

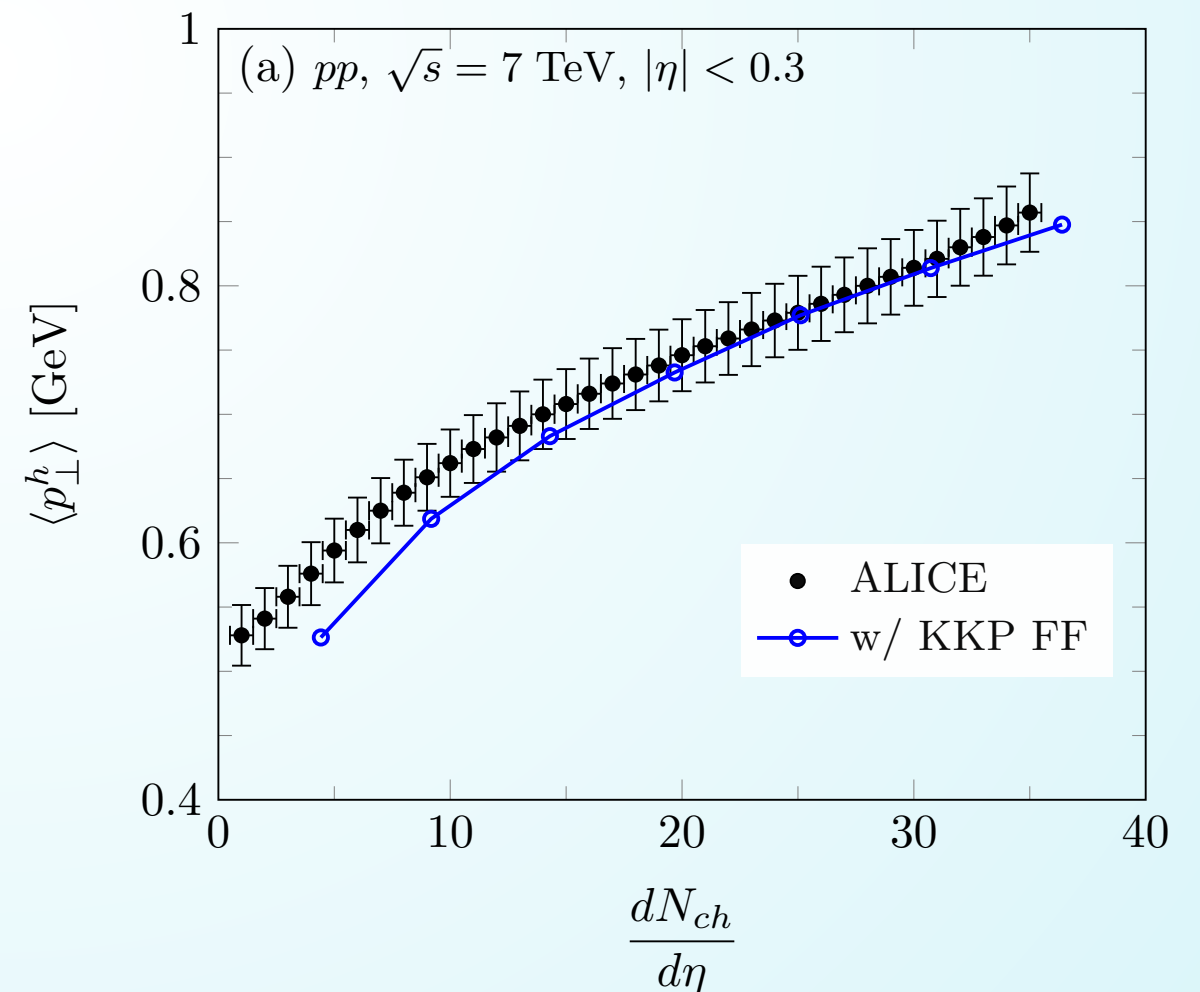
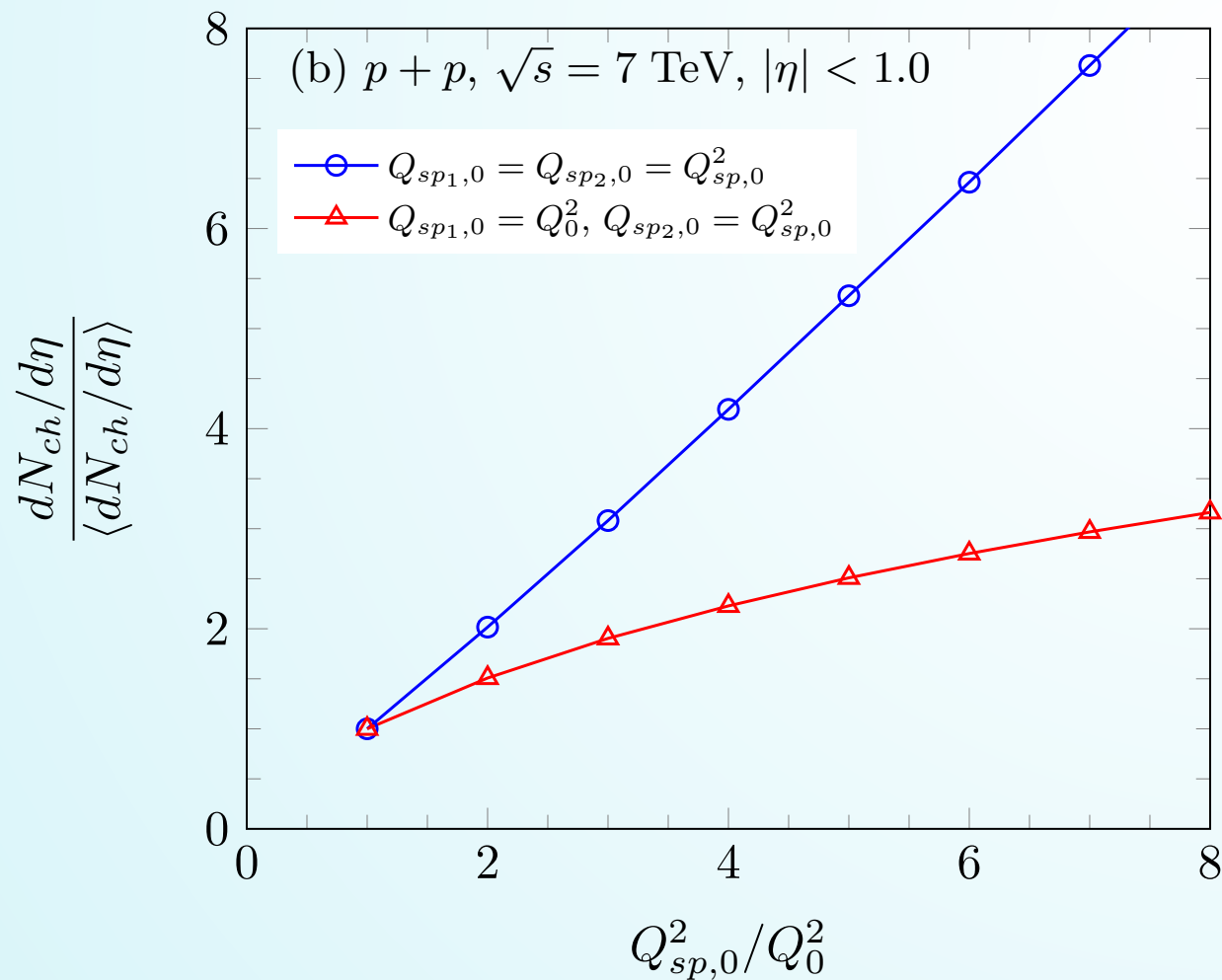
- If  $S_{\perp}$  is fixed,  $Q_s$  can controls  $N_{ch}$ .
- $Q_s$  depends on impact parameter ( $b$ ). Tribedy, Venugopalan (2011)  
Schenke, Tribedy, Venugopalan (2012)
- IP-Sat Model deals with  $b$ -dependent  $Q_s$  and can model NBD of  $N_{ch}$ .
- Spatial configurations of color charge density of parton inside hadron are **nonperturbative** and complicated...

See talk by Mace (11/13)

# Nch in p+p collisions

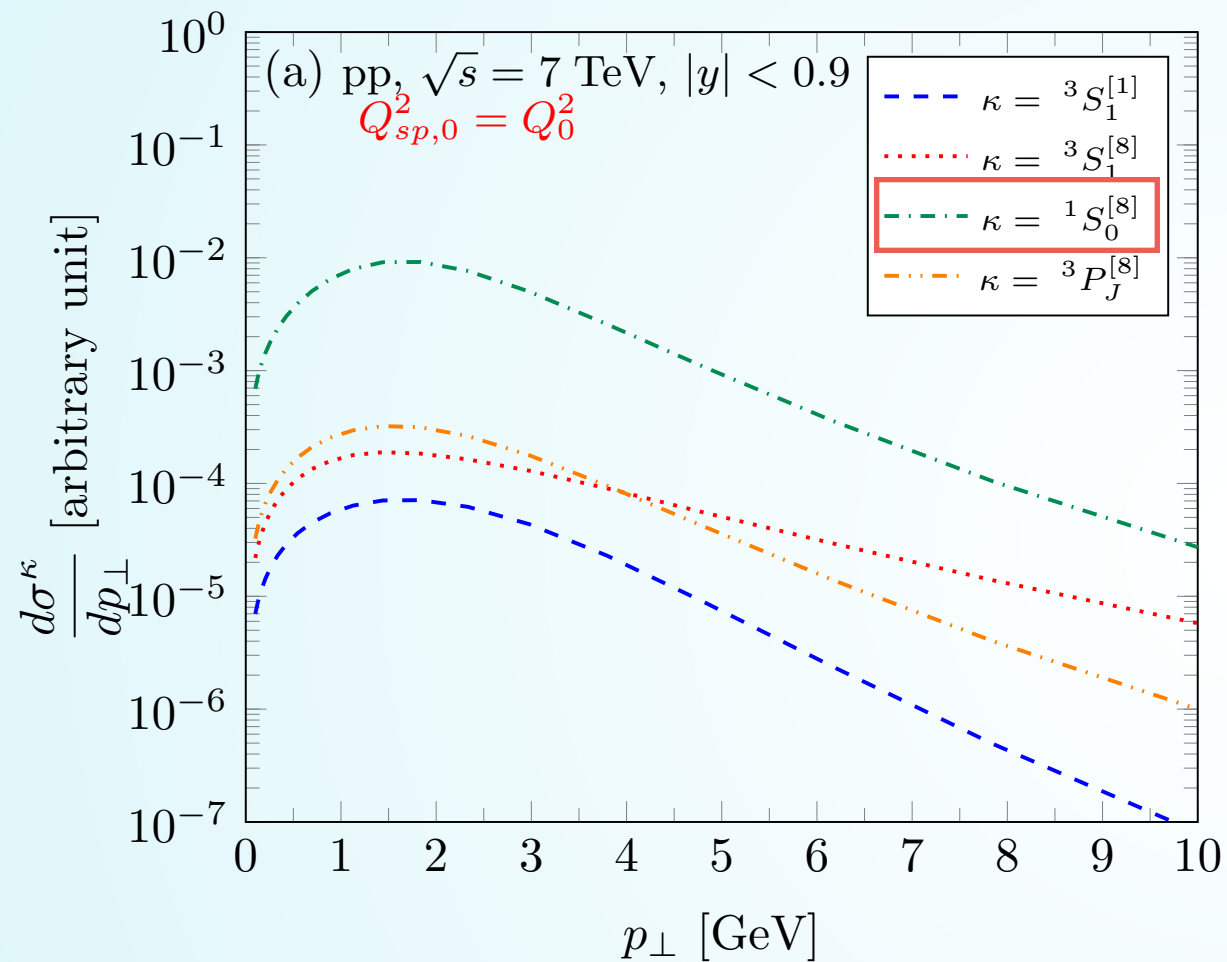
$$\frac{d\sigma_g}{d^2p_{g\perp} dy} = \frac{\alpha_s \hat{K}_b}{(2\pi)^3 \pi^3 C_F} \int \frac{d^2k_{\perp}}{p_{g\perp}^2} \varphi_{p,y_p}(k_{\perp}) \varphi_{A,Y}(p_{g\perp} - k_{\perp})$$

$$\frac{dN_{ch}}{d\eta} = \frac{\hat{K}_{ch}}{\sigma_{inel}} \int d^2p_{\perp} \int_{z_{min}}^1 dz \frac{D_h(z)}{z^2} J_{y \rightarrow \eta} \frac{d\sigma_g}{d^2p_{g\perp} dy}$$

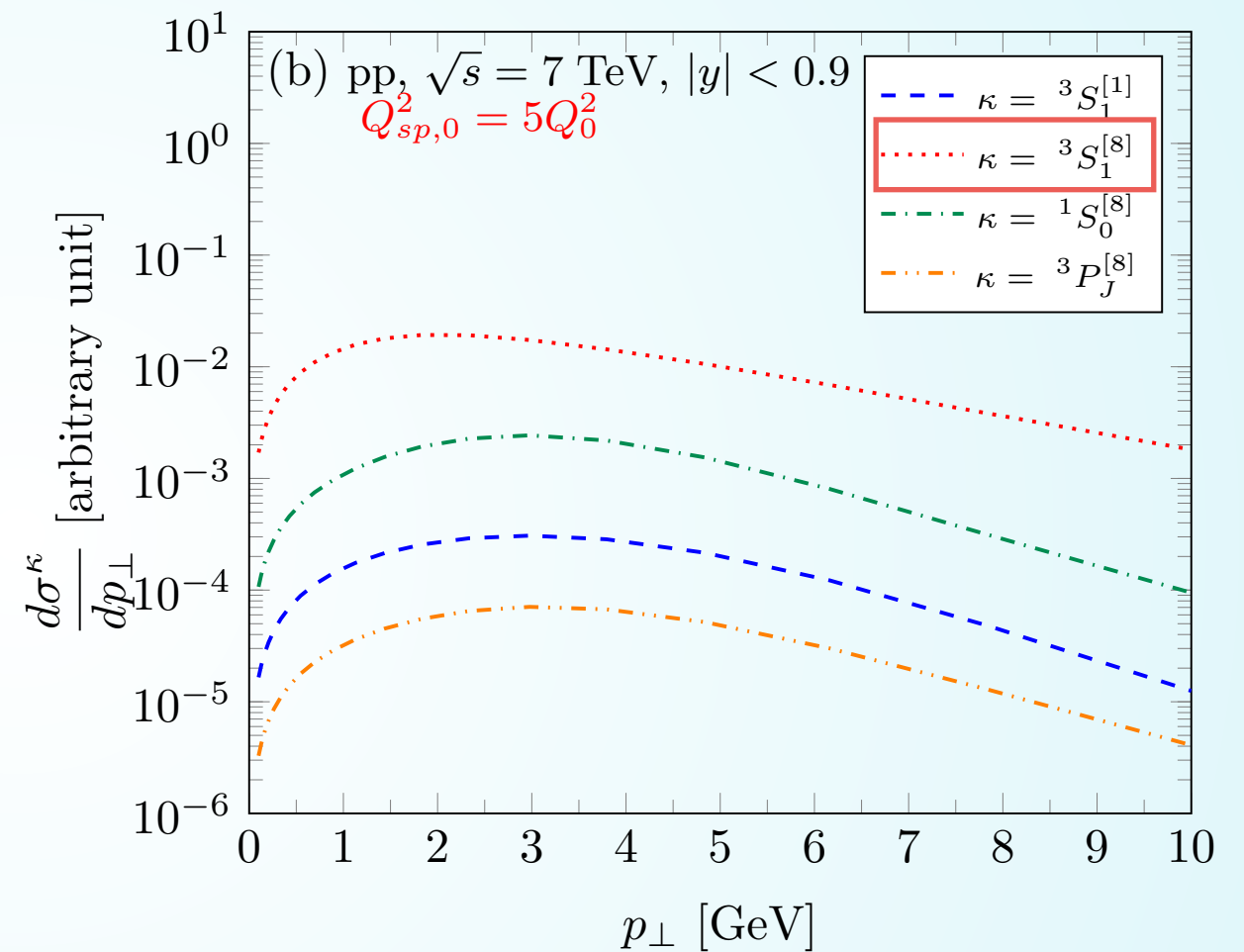


# MB vs High Multiplicity events

Ma, Venugopalan, Tribedy, KW (2018)

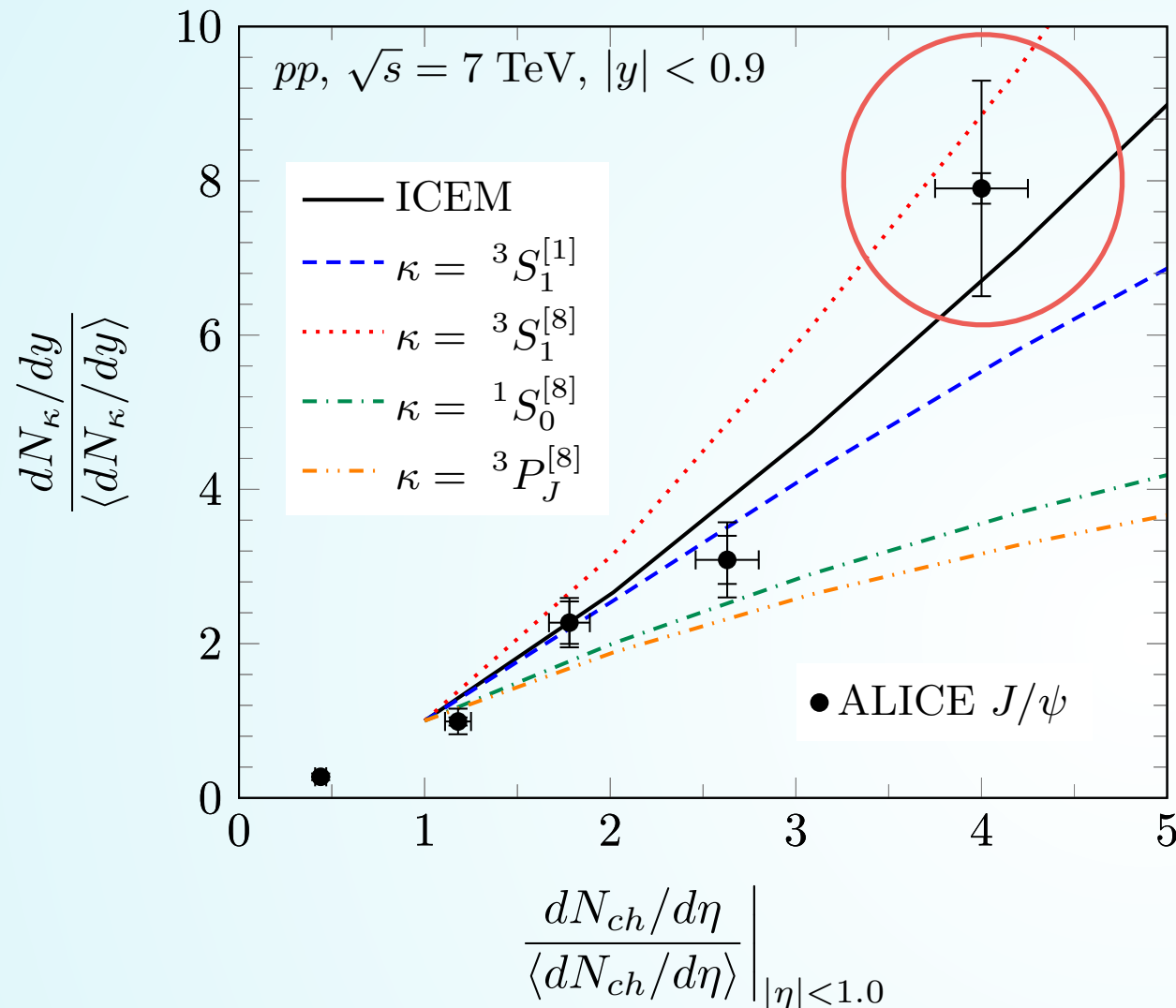


MB events



HM events

# New constraints on the LDMEs



1S0 octet channel should have a larger weight compared to MB.

- Consistent with the universality requirement from BELLE  $e^+e^-$  data:

$$\langle \mathcal{O}^{J/\psi} [{}^1S_0^{[8]}] \rangle + 4.0 \langle \mathcal{O}^{J/\psi} [{}^3P_0^{[8]}] \rangle / m^2 < 2.0 \pm 0.6 \times 10^{-2} \text{ GeV}^3$$

- b-quark decay contribution is not included.

# Short Summary I

- Nice agreement between the CGC expectation and experimental data about  $p_t$  distribution of  $J/\psi$  production.
- $J/\psi$  production in high multiplicity events provides an interesting universality test of LDMEs.
- Other observables:  $J/\psi$  production within jets.

Bain, Makris, Mehen, Dai, Leibovich, PRL119, 032002 (2017)

Kang, Qiu, Ringer, Xing, Zhang, PRL119, 032001 (2017)

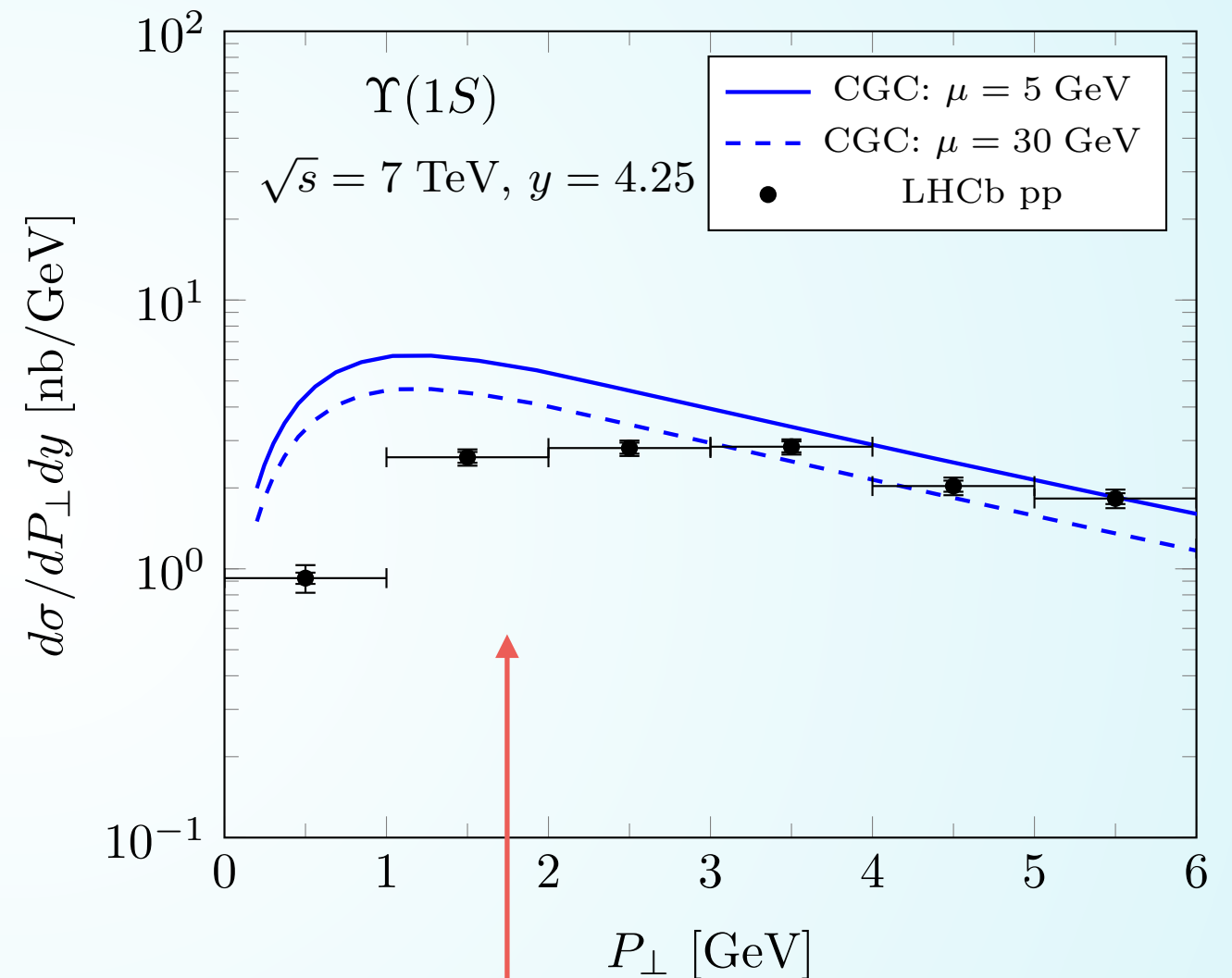
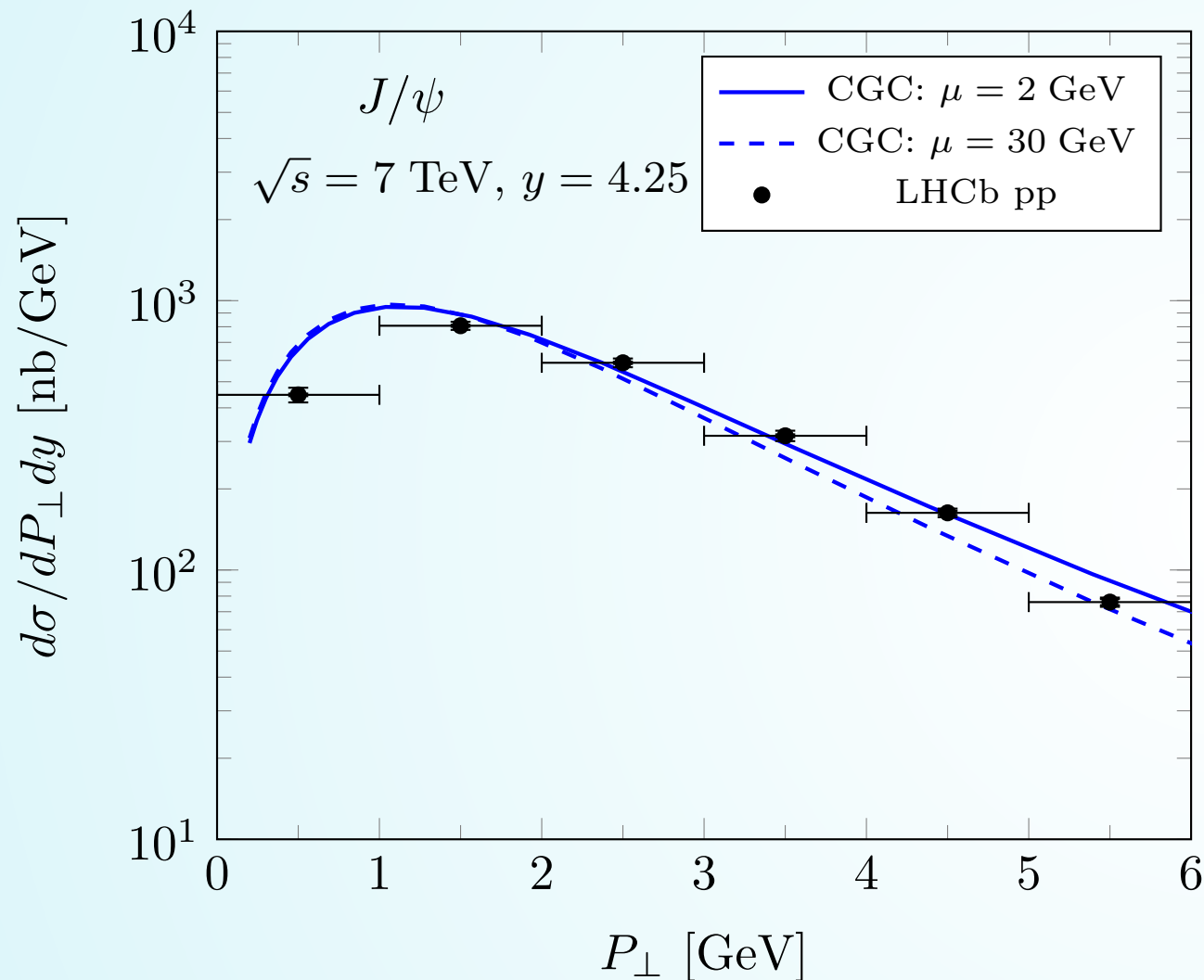


# Quarkonium production in the TMD framework

# J/psi vs Upsilon in the CGC

Fujii, KW (2013)

KW, Xiao (2015)

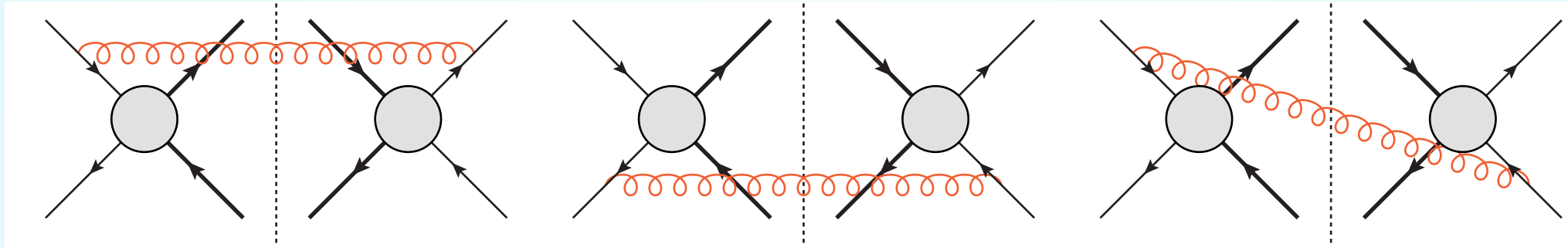


Sudakov resummation effect is important for Upsilon production.

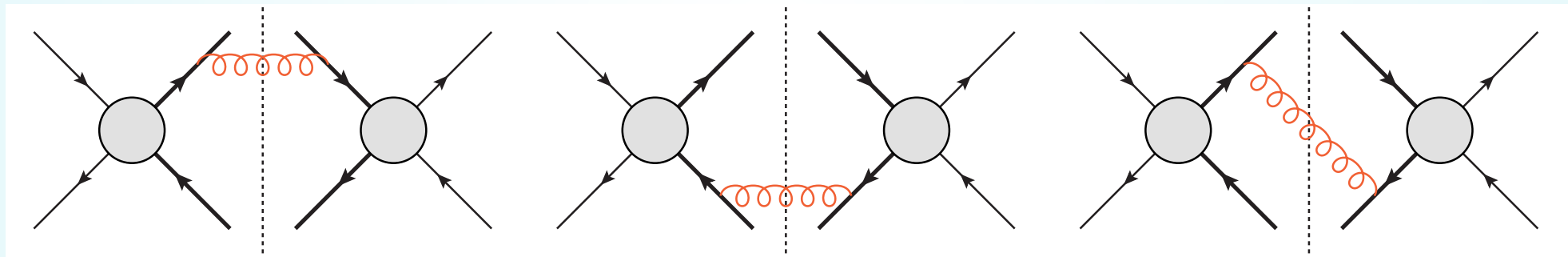
$$\alpha_s \ln^2 \frac{M^2}{p_{\perp}^2} \sim \mathcal{O}(1)$$

# Soft gluon radiation

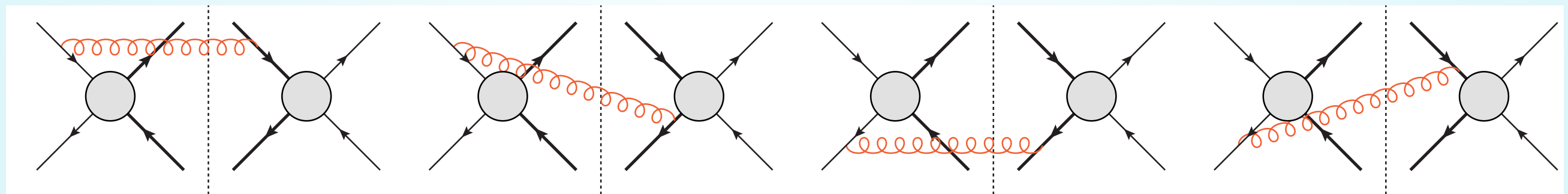
Initial state interaction: Soft and collinear singularities



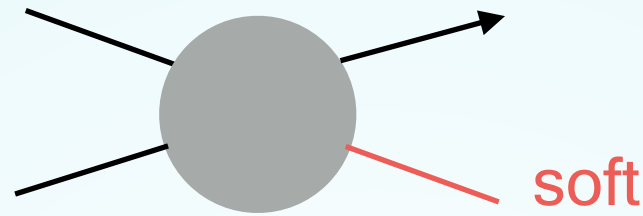
Final state interaction: Soft singularity



Interference: Soft and collinear singularities



# Soft momentum limit: g+g channel



Qiu, KW in progress

$$\frac{\hat{s}}{\pi} \frac{d\hat{\sigma}_\kappa}{d\hat{t}} \delta(\hat{s} + \hat{t} + \hat{u} - M^2)$$

$$\stackrel{p_\perp^2 \ll M^2}{=} \hat{\sigma}_0^\kappa \frac{1}{p_\perp^2} \left[ \frac{3(z_B^2 - z_B + 1)^2 \alpha_s}{\pi^2} \frac{\delta(1 - z_A)}{[1 - z_B]_+} + \frac{3(z_A^2 - z_A + 1)^2 \alpha_s}{\pi^2} \frac{\delta(1 - z_B)}{[1 - z_A]_+} + \frac{3\alpha_s}{\pi^2} \delta(1 - z_A) \delta(1 - z_B) \ln \frac{M^2}{p_\perp^2} \right]$$

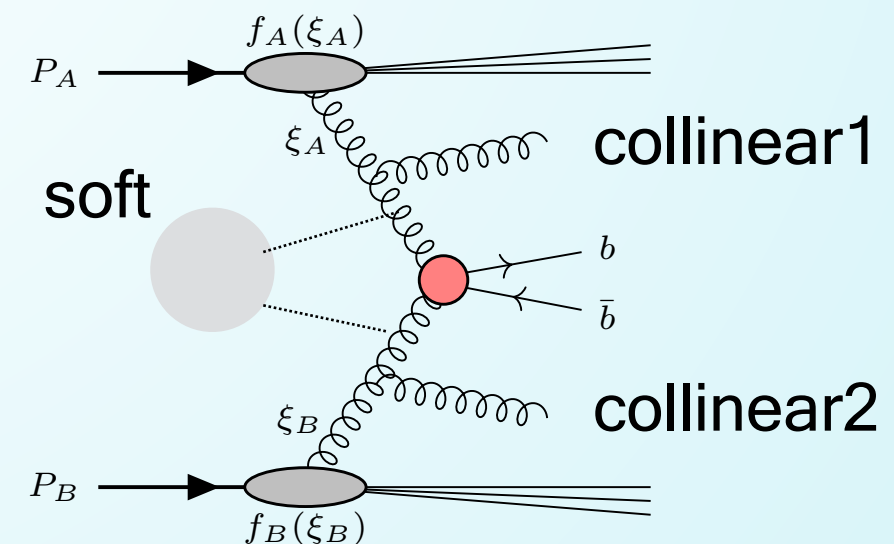
$$= \hat{\sigma}_0^\kappa \frac{C_A \alpha_s}{2\pi^2} \frac{1}{p_\perp^2} \left[ z_B \hat{P}_{gg}^R(z_B) \delta(1 - z_A) + z_A \hat{P}_{gg}^R(z_A) \delta(1 - z_B) + 2\delta(1 - z_A) \delta(1 - z_B) \ln \frac{M^2}{p_\perp^2} \right]$$

$$\frac{d\hat{\sigma}_{gg}^\kappa}{d^2 p_\perp dy} = \hat{\sigma}_{0,gg}^\kappa \frac{C_A \alpha_s}{2\pi^2} \int_{x_A}^1 d\xi_A \int_{x_B}^1 d\xi_B f_A(\xi_A) f_B(\xi_B) \frac{1}{p_\perp^2}$$

$$\times \left[ z_B \hat{P}_{gg}^R(z_B) \delta(1 - z_A) + z_A \hat{P}_{gg}^R(z_A) \delta(1 - z_B) + 2 \ln \frac{M^2}{p_\perp^2} \delta(1 - z_A) \delta(1 - z_B) \right]$$

The same form as Higgs production

Initial state interaction should be important.



# Collins-Soper-Sterman formalism

Collins, Soper, Sterman (1985)

$$\frac{d\sigma^{Q\bar{Q}}}{d^2p_\perp dy} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{ip_\perp \cdot b_\perp} W(M, b_\perp, x_1, x_2) + (d\sigma_{\text{perp}} - d\sigma_{\text{asy}})$$

Resummation

important at high pt

$$W(M, b_\perp, x_1, x_2) = \sum_{ij} d\hat{\sigma}_{\text{LO}}^{ij \rightarrow Q\bar{Q}} W_{ij}(M, b_\perp) e^{-S_{ij}(M, b_\perp)}$$

$$W_{ij}(M, b_\perp) = \sum_{a,b} \int \frac{d\xi}{\xi} \frac{d\xi'}{\xi'} C_{a \rightarrow i} \left( \frac{x_A}{\xi} \right) C_{b \rightarrow j} \left( \frac{x_B}{\xi'} \right) f_{a/A}(\xi, \mu) f_{b/B}(\xi', \mu)$$

collinear pdfs

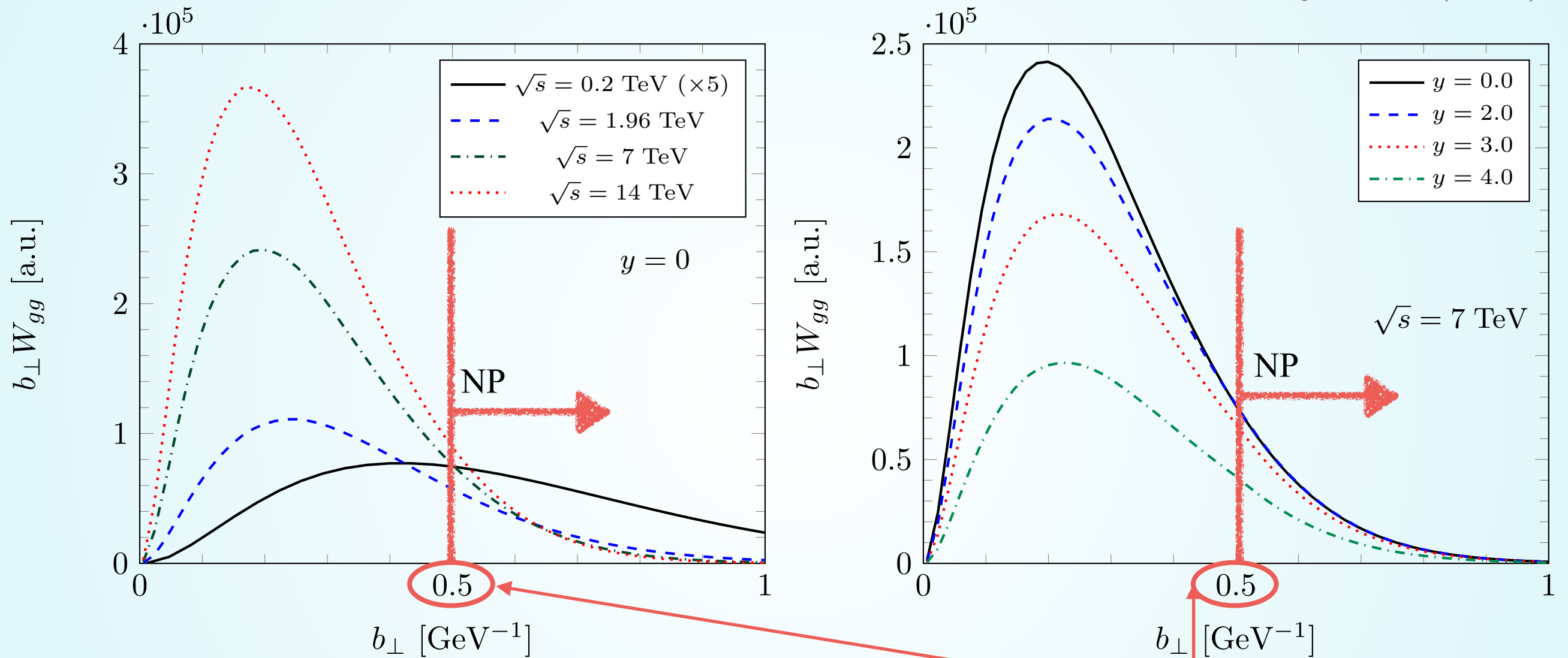
$$S_{ij}(M, b) = \int_{c_0/b^2}^{M^2} \frac{d\mu^2}{\mu^2} \left[ A_{ij} \ln \left( \frac{M^2}{\mu^2} \right) + B_{ij} \right]$$

A, B, C are calculated perturbatively.

universal in gg-channel

# b-space distribution for Upsilon

Qiu, KW (2017)



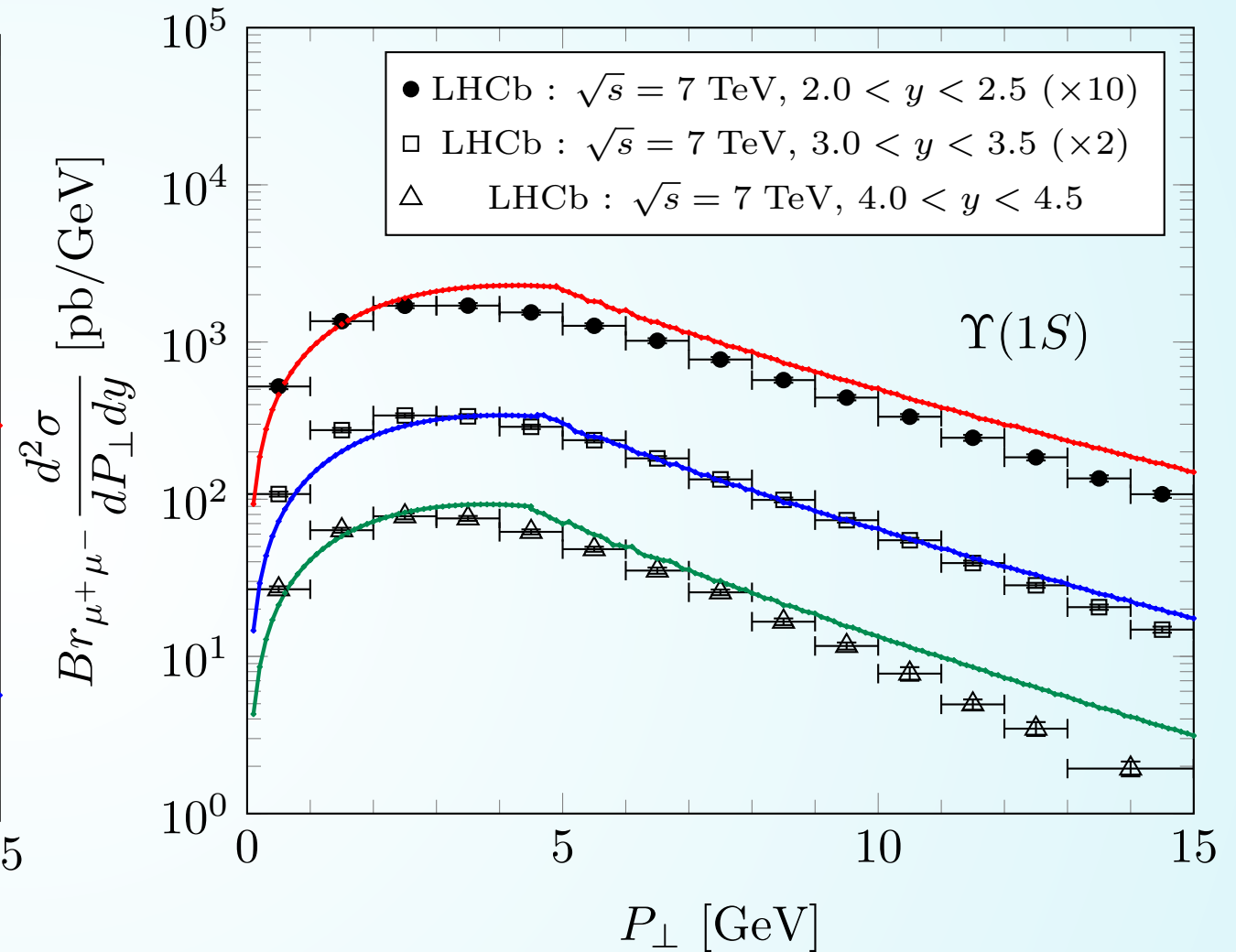
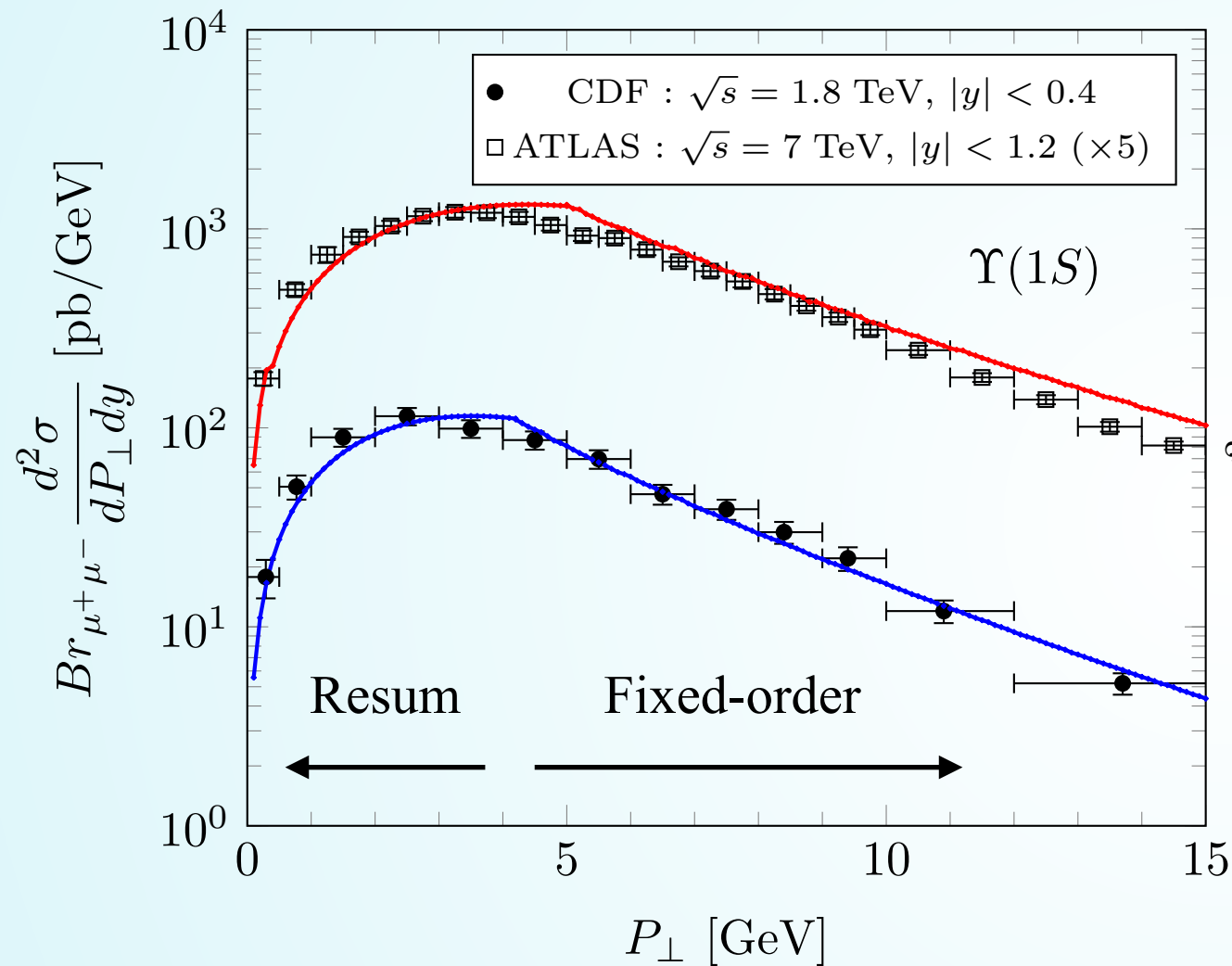
$$W(M, b_{\perp}) = \begin{cases} W^{\text{perp}}(M, b_{\perp}) & b_{\perp} \leq b_{\text{max}} \\ W^{\text{perp}}(M, b_{\text{max}}) F^{\text{NP}}(M, b_{\perp}; b_{\text{max}}) & b_{\perp} > b_{\text{max}} \end{cases}$$

Upsilon: Perturbative shower, J/psi: Nonperturbative shower



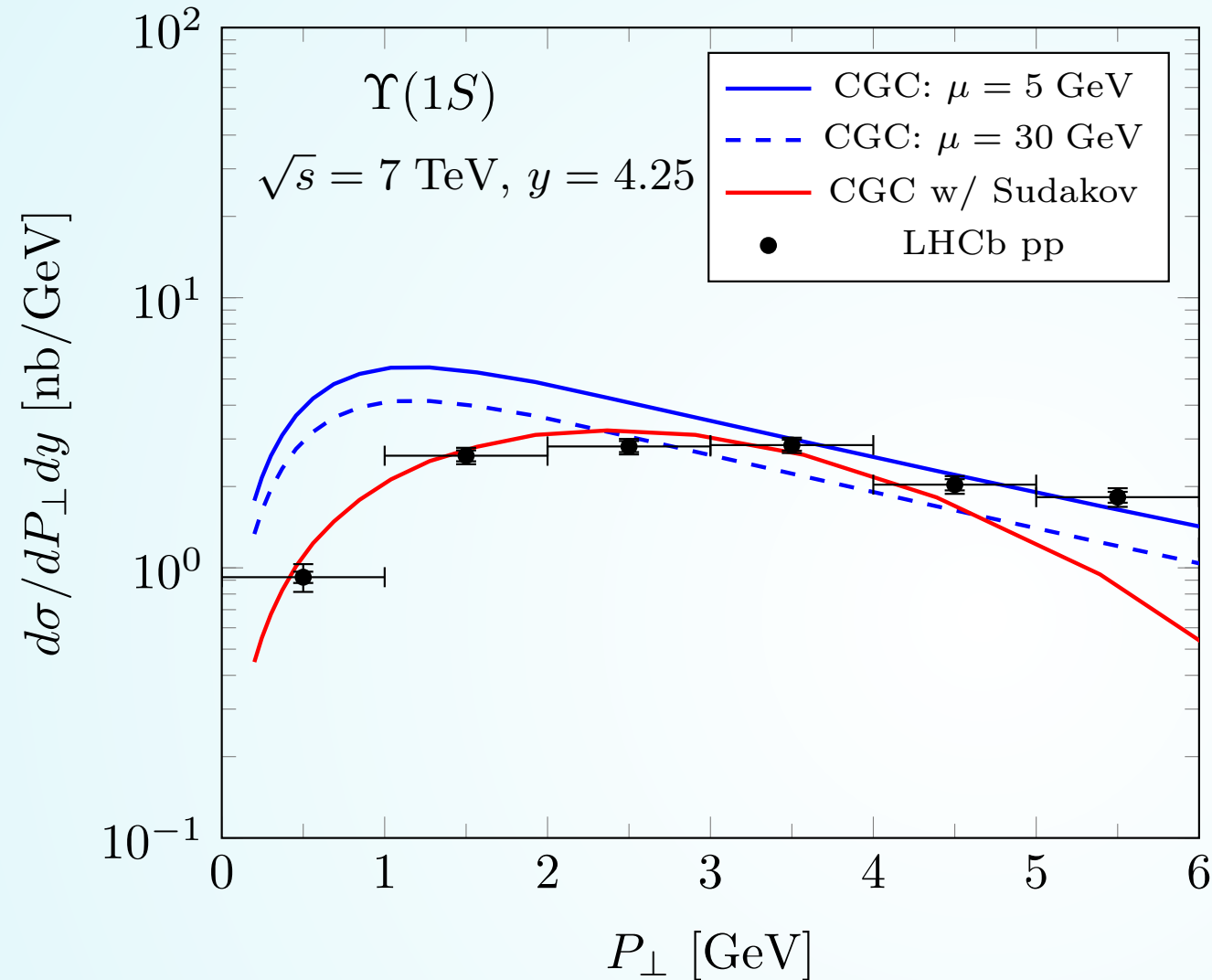
# Results in the CSS formalism

Qiu, KW (2017)



Initial state hard person shower describes data of J/psi production from Tevatron to LHC.

# CGC + Sudakov factor



KW, Xiao (2015)

See also, Mueller, Xiao, Yuan (2013)

$$\frac{d\sigma_{Q\bar{Q}}}{d^2q_{Q\perp} d^2q_{\bar{Q}\perp} dy_Q dy_{\bar{Q}}} = \frac{\alpha_s^2 \bar{S}_\perp}{16\pi^2 C_F} \int d^2l_\perp d^2k_\perp \frac{\Xi_{\text{coll}}(k_{2\perp}, k_\perp - zl_\perp)}{k_{2\perp}^2} F_{\text{TMD}}(l_\perp) \mathcal{N}_{Y_g}(k_\perp) \mathcal{N}_{Y_g}(k_{2\perp} - k_\perp + l_\perp)$$

$$F_{\text{TMD}}(M, l_\perp) = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-ib_\perp \cdot l_\perp} e^{-S_{\text{Sud}}(M, b_\perp)} x_1 G\left(x_1, \mu = \frac{c_0}{b_\perp}\right).$$

# Short Summary II

- Large double logs are essential for Upsilon production.
- Initial state soft gluon radiation can provide double logs. Parton shower  $>$  Saturation effect in  $p+p$  collisions.
- Rigorous calculation of TMD factorization for quarkonium production in the NRQCD is underway.

# Factorization breaking in $p+A$ collisions

# Reminder: Effective factorization

Qiu, Sun, Xiao, Yuan (2013)

Consider

$$\Lambda_{\text{QCD}} \ll p_{\perp} \sim Q_s \ll M$$

If  $Q_s \sim mv \sim Mv/2 \longrightarrow$  v-expansion is unclear

However, at very forward rapidity, Lorentz time dilation gives

$$\frac{1}{mv} \frac{p_{\parallel}}{M} \gg \frac{1}{p_{\perp}} \sim \frac{1}{Q_{sA}} \quad \text{or} \quad y \gg \ln \frac{2mv}{p_{\perp}} \sim \ln \frac{Mv}{Q_{sA}}$$

The QQbar pair's hadronization is effectively frozen when the pair passes through a target.

So, if v is small

$$\frac{1}{mv^2} \frac{p_{\parallel}}{M} \gg \frac{1}{mv} \frac{p_{\parallel}}{M} \gg \frac{1}{p_{\perp}}$$

The effective factorization between the coherent interaction and the hadronization is justified at forward rapidity in CEM and NRQCD.

# Spectators interaction

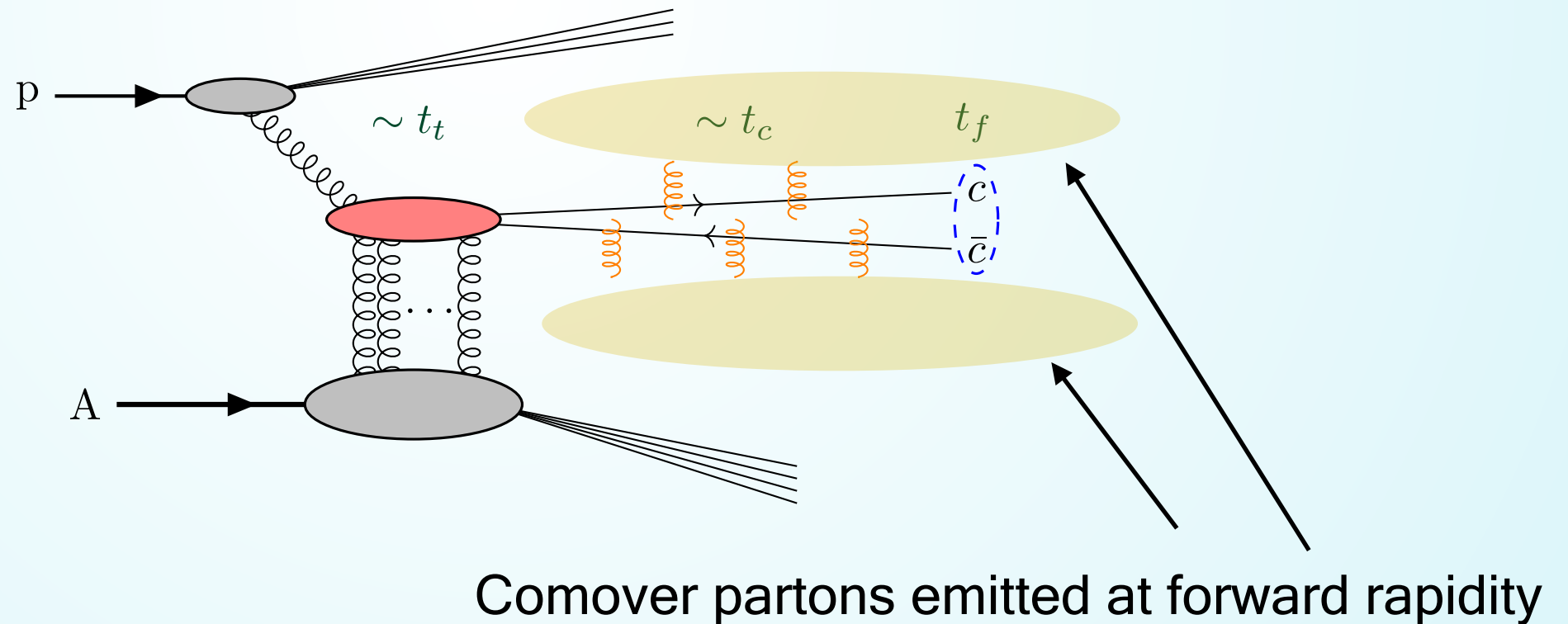
We have assumed that the same statement applies to charmonium.

$$\Lambda_{\text{QCD}} \ll p_{\perp} \sim Q_s \sim M$$

However, we must be careful at low  $p_t$  because soft color exchanges between spectators and  $c\bar{c}$  pair is indispensable.

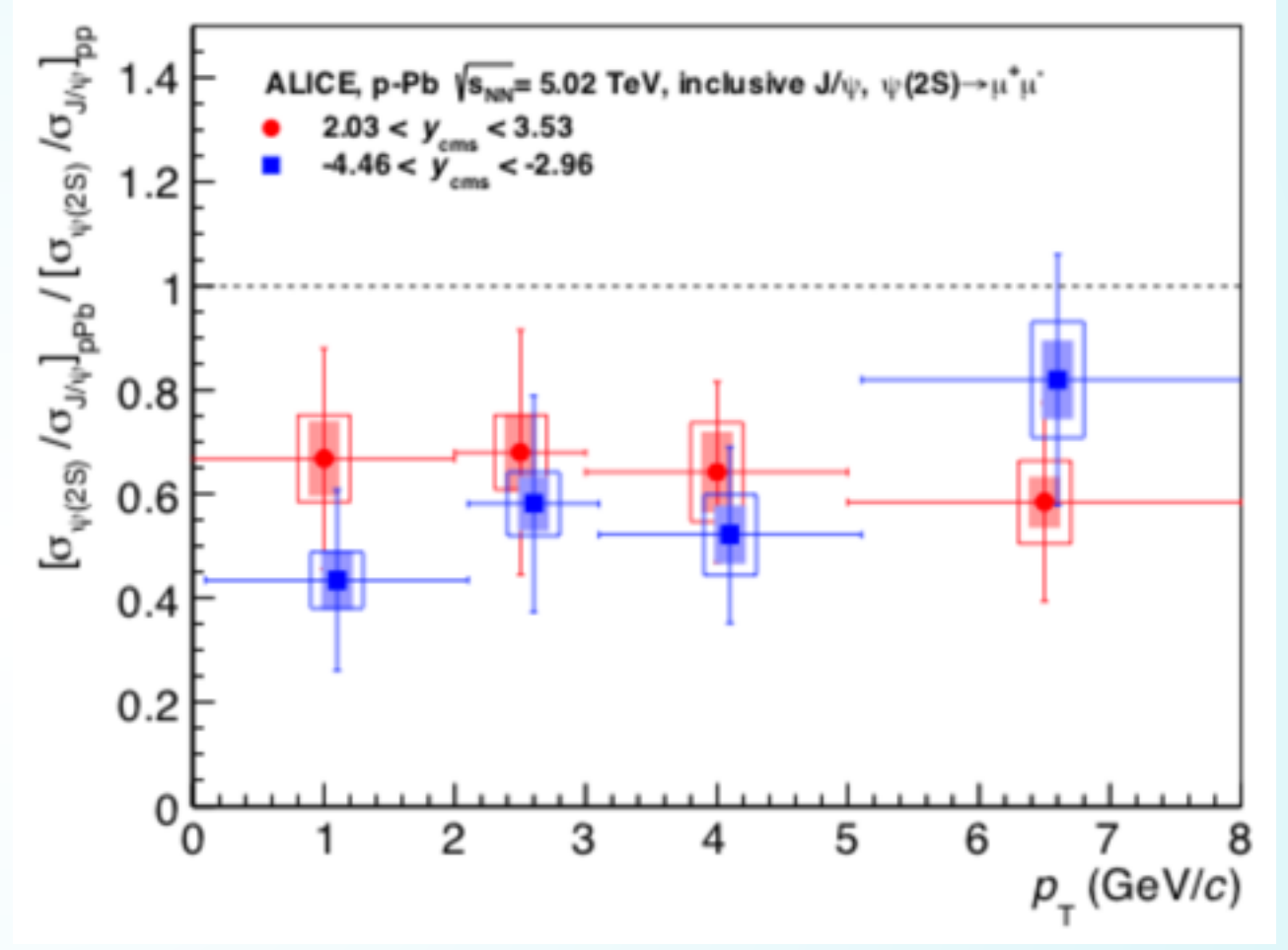
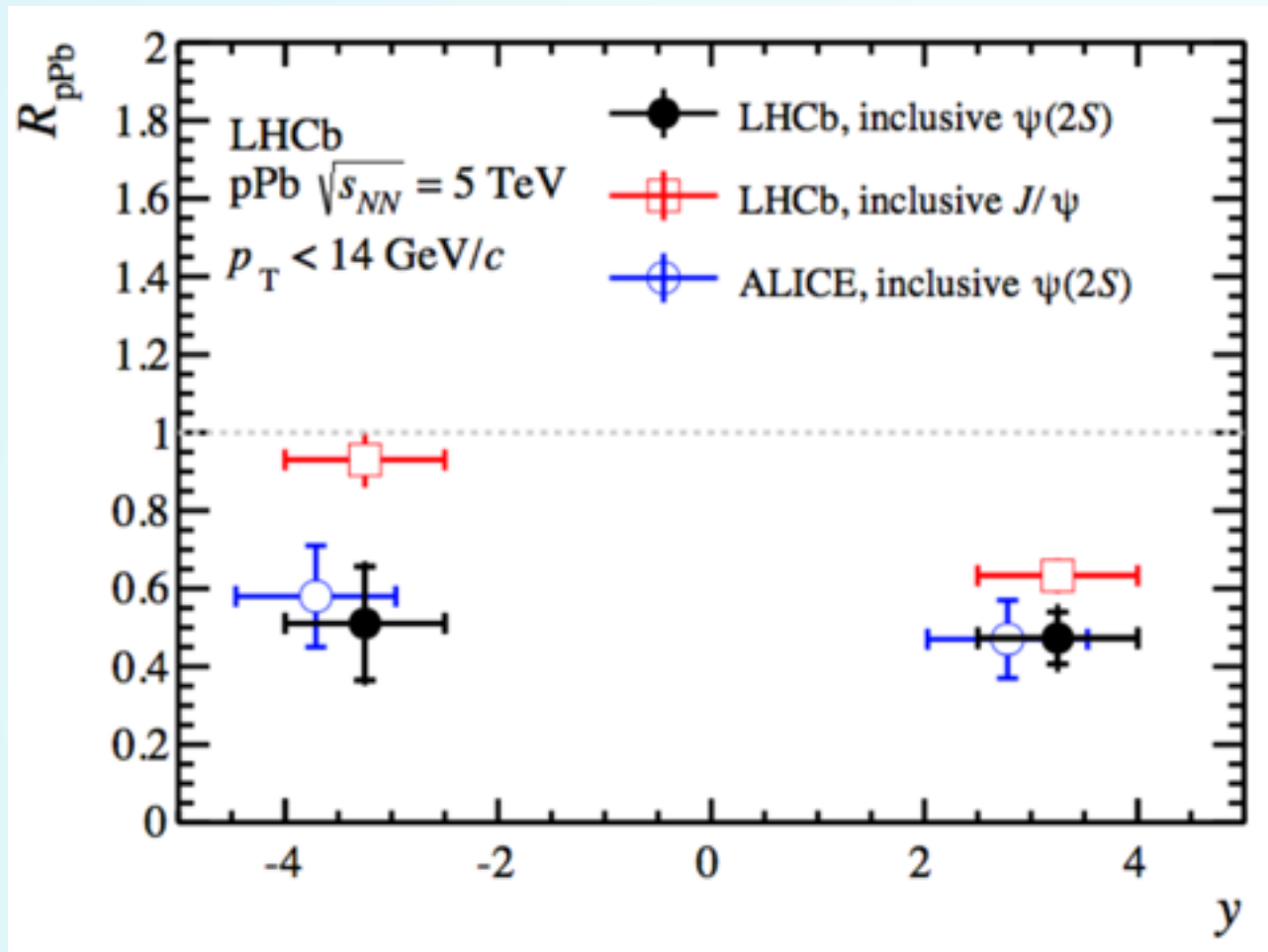
→ Factorization breaking

Brodsky, Mueller (1988)





# Psi(2S) anomalous suppression



The large suppression of Psi(2S) production in p+A at both RHIC and the LHC has widely been interpreted as arising from final state interactions with hadron comovers.

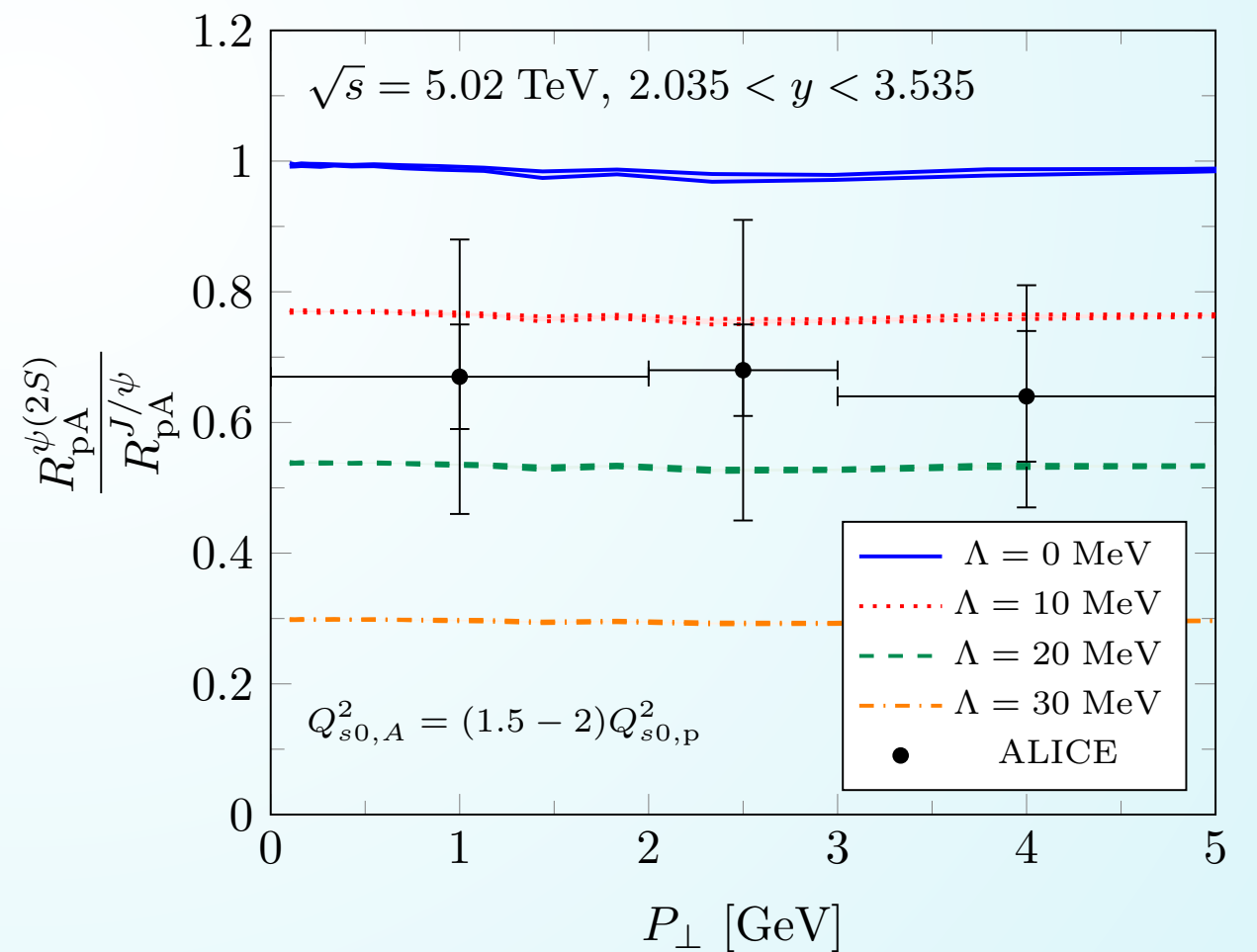
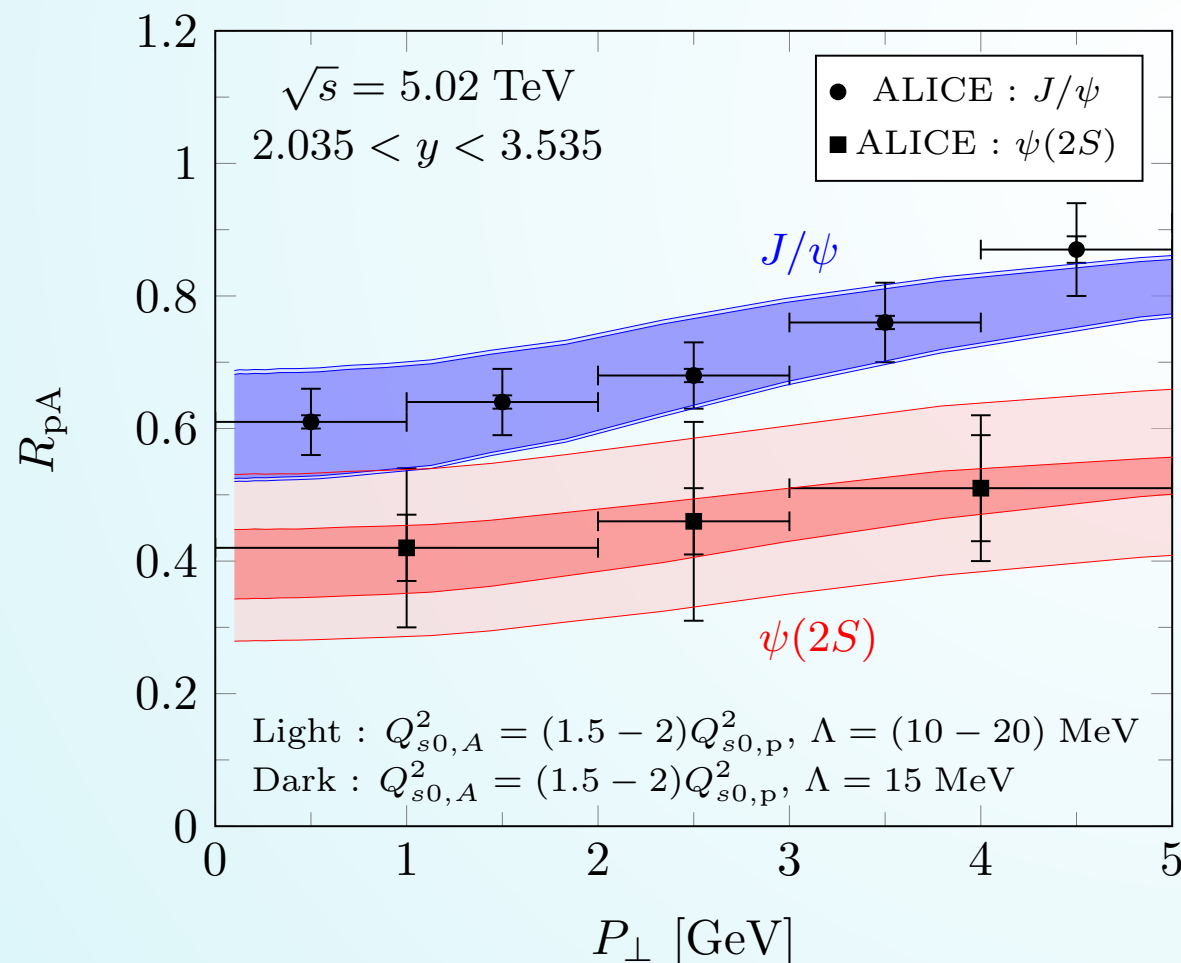
# FSI in the CGC+ICEM

Ma, Venugopalan, KW, Zhang (2017)

Fitted in p+p

$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_{Q\bar{Q}\rightarrow\psi} \int_{m_\psi}^{2m_Q - \Lambda} dM \left(\frac{M}{m_\psi}\right)^2 \frac{d\sigma_{Q\bar{Q}}}{dM d^2p'_\perp dy} \Big|_{p'_\perp = \frac{M}{m_\psi} p_\perp}$$

Assumption: the role of soft color exchanges should be enhanced in p+A collisions.



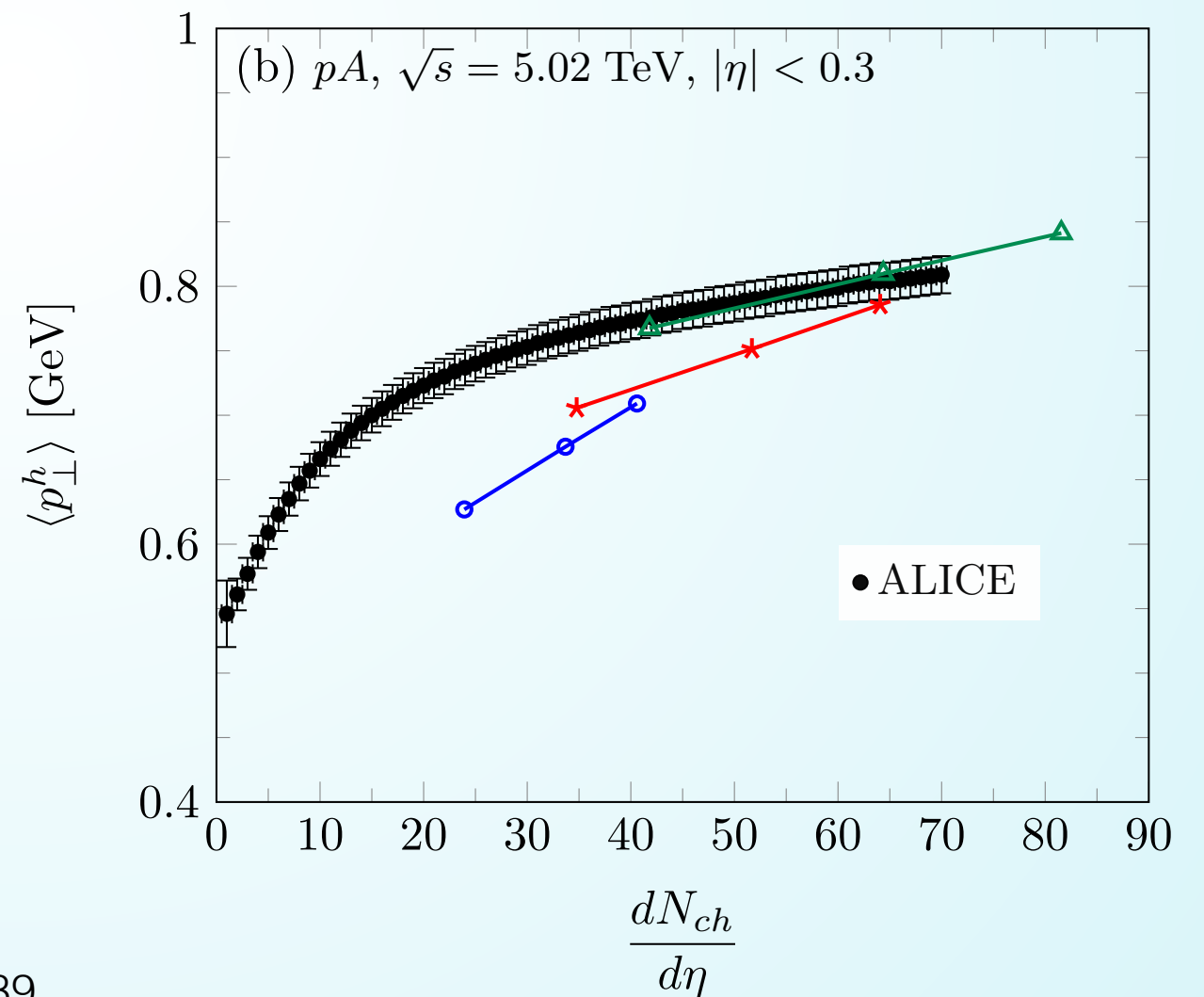
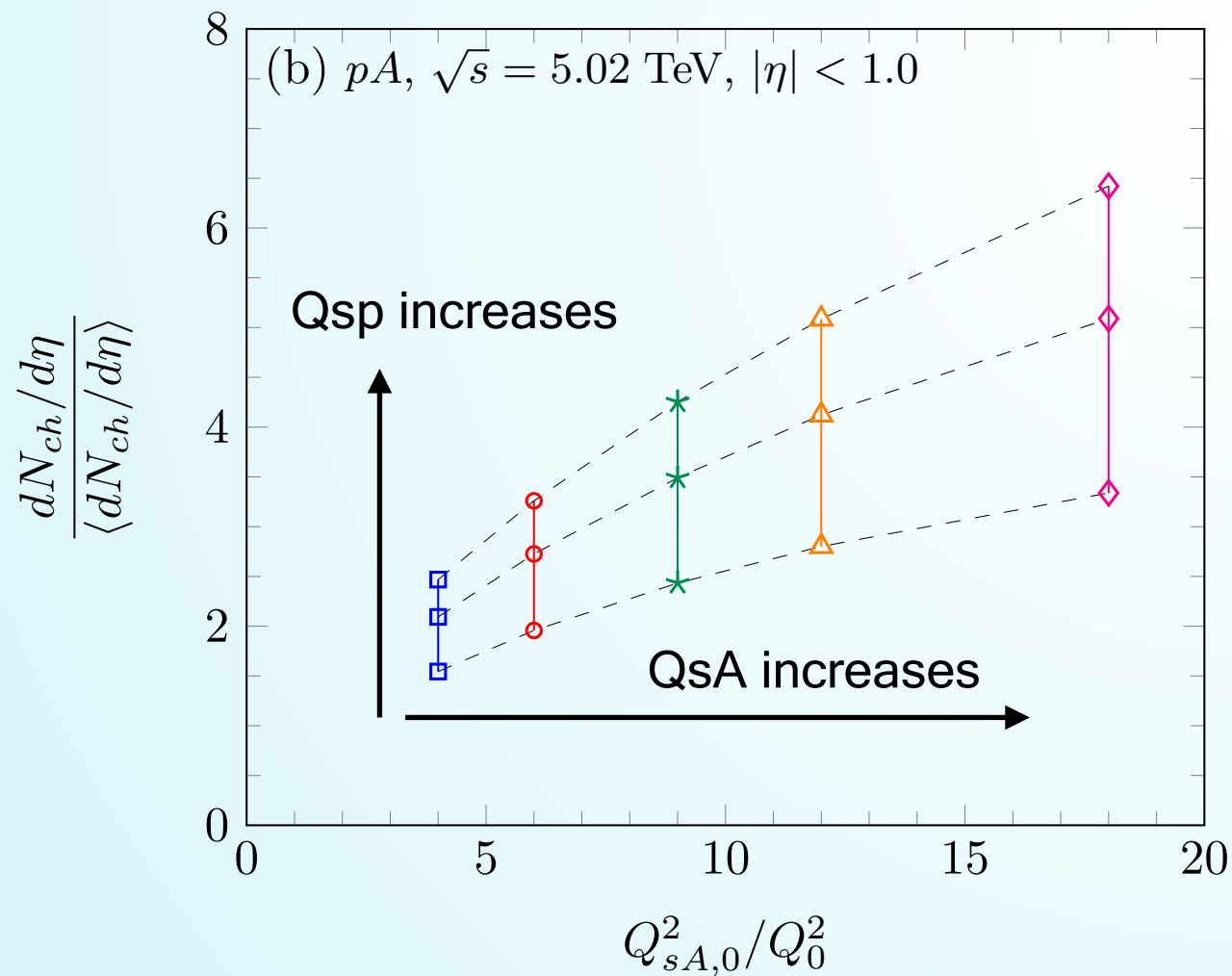
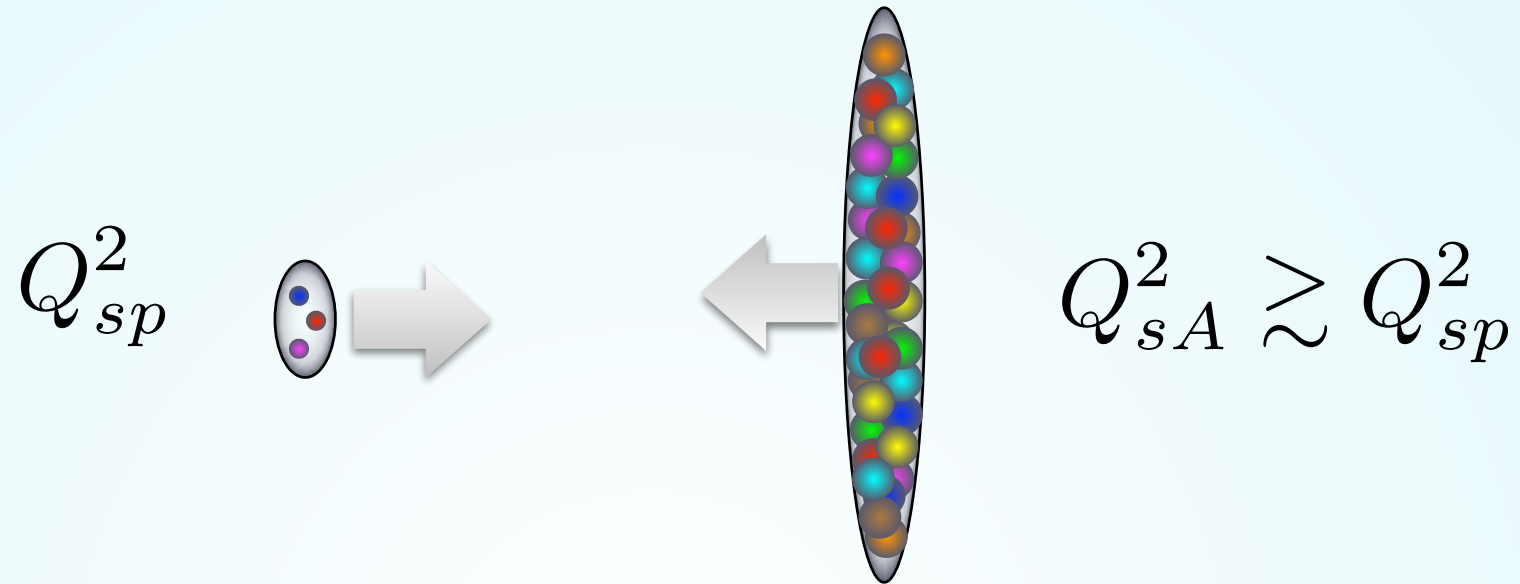
# Short Summary III

- Spectator interaction breaks strongly factorization for  $\Psi(2S)$  production.  $\rightarrow$  **Color Entanglement** between the  $Q\bar{Q}$  pair and spectator partons.
- Need more careful analysis on spectators interaction: higher twist in final state.
- Similar factorization breaking effect could happen in  $e+A$  collisions?

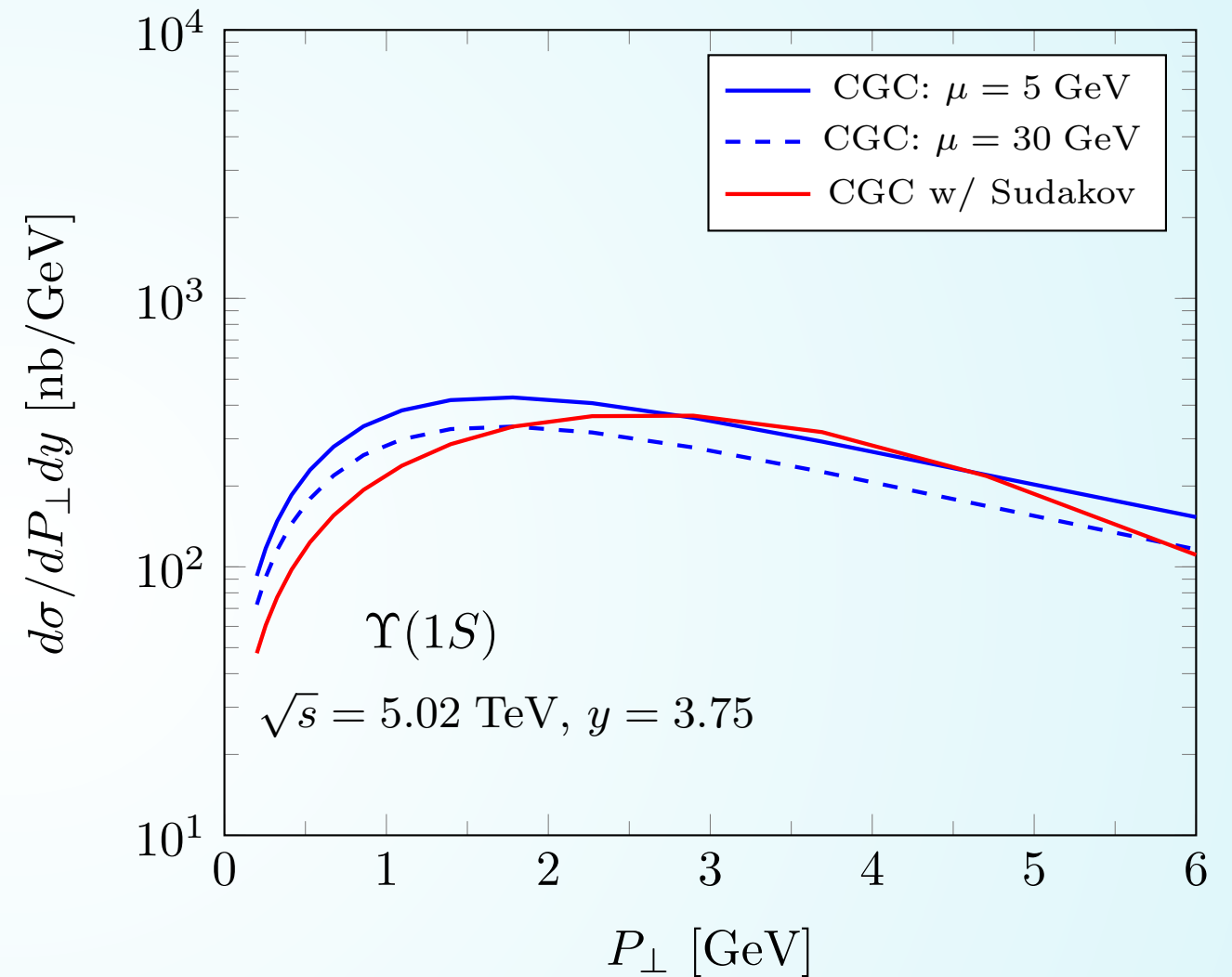
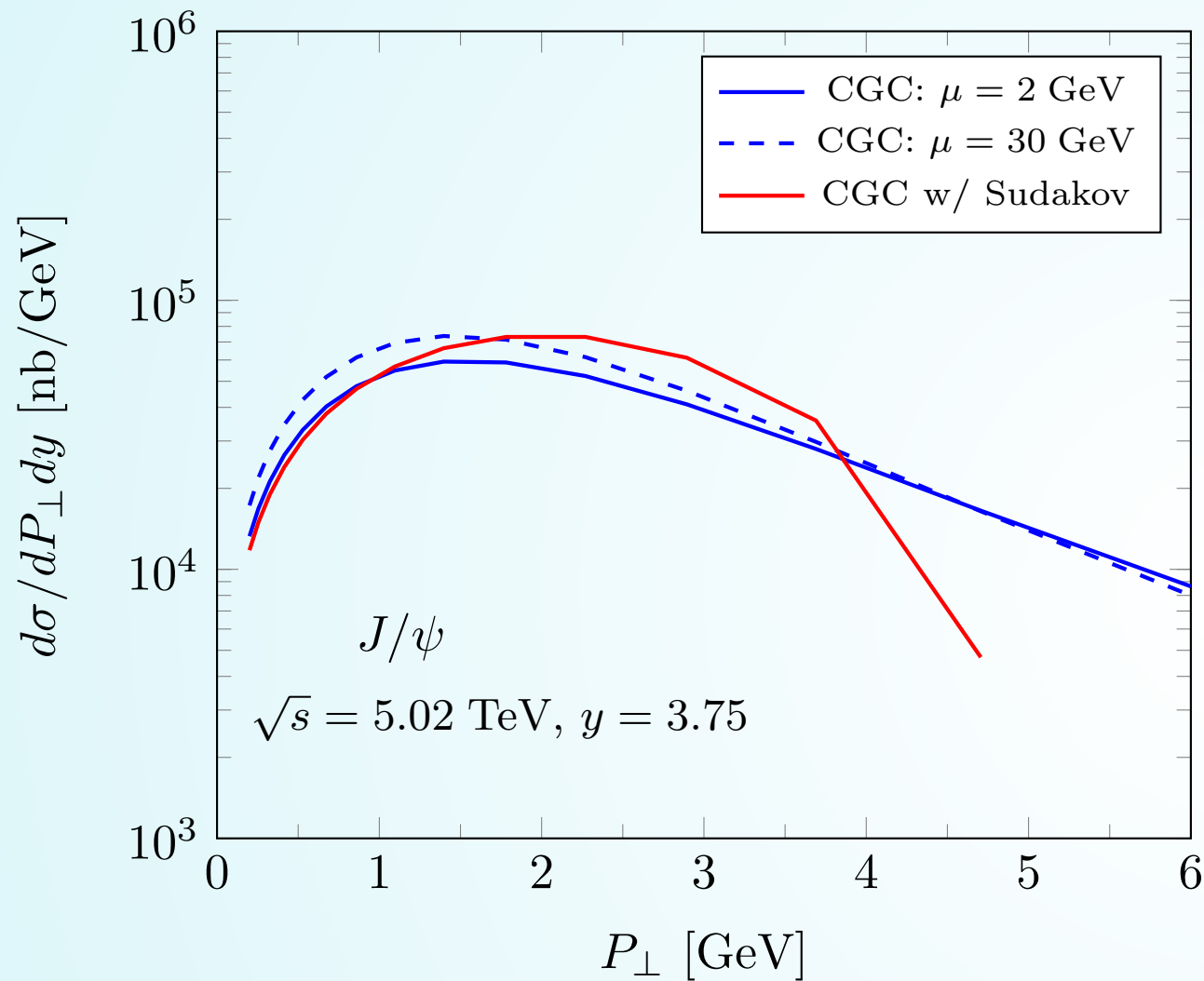
***Thank you!***

# Backup

# Nch in p+A collisions



# Results in p+A collisions



Saturation effect and parton shower effect are comparable in p+A collisions.

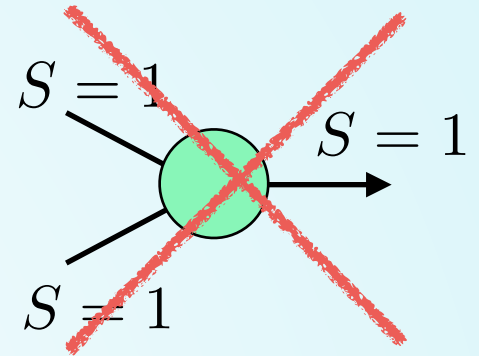


# Singular spike at low pt

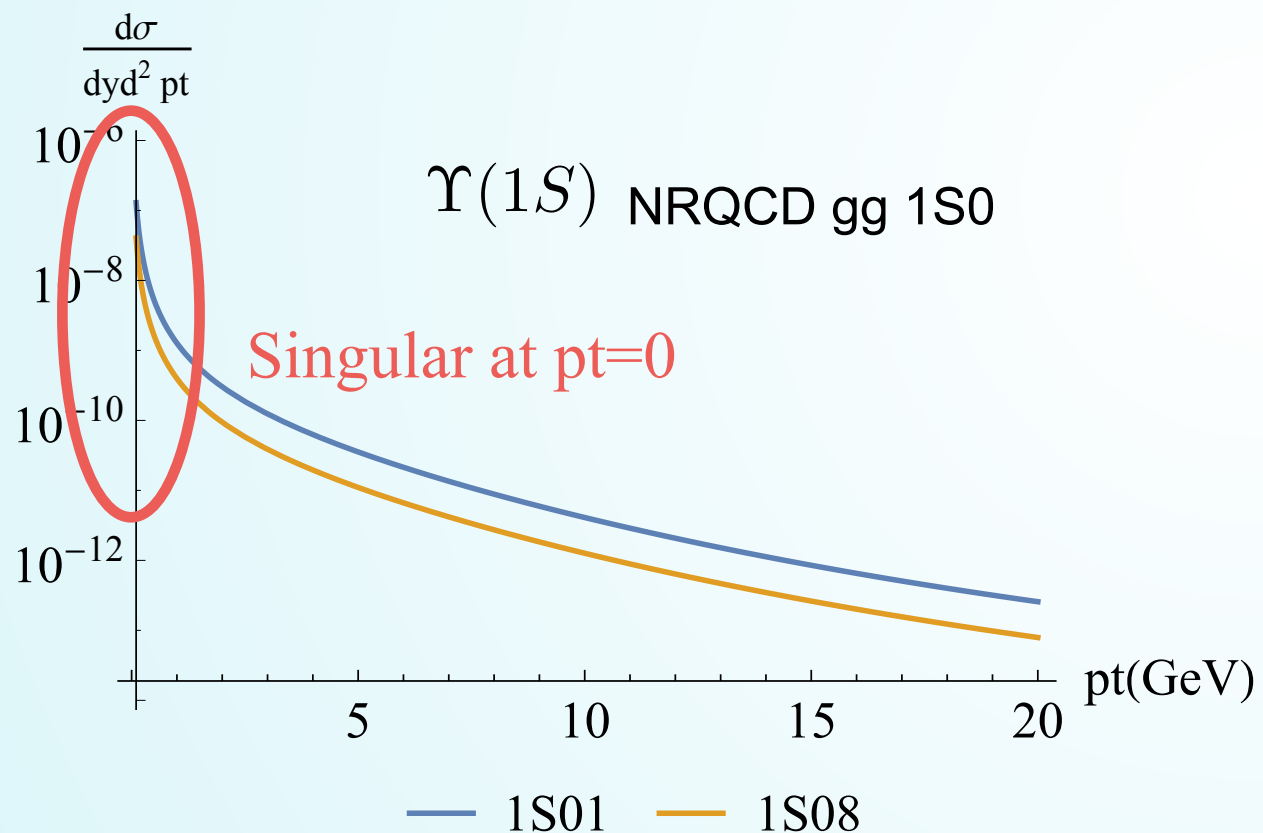
$$\hat{\sigma}_{0,gg}^{1S_0^{[1]}} = \frac{\pi}{M^4} \frac{4}{3} \frac{\pi^2 \alpha_s^2}{M}$$

$$\hat{\sigma}_{0,gg}^{1S_0^{[8]}} = \frac{\pi}{M^4} \frac{5}{12} \frac{\pi^2 \alpha_s^2}{M}$$

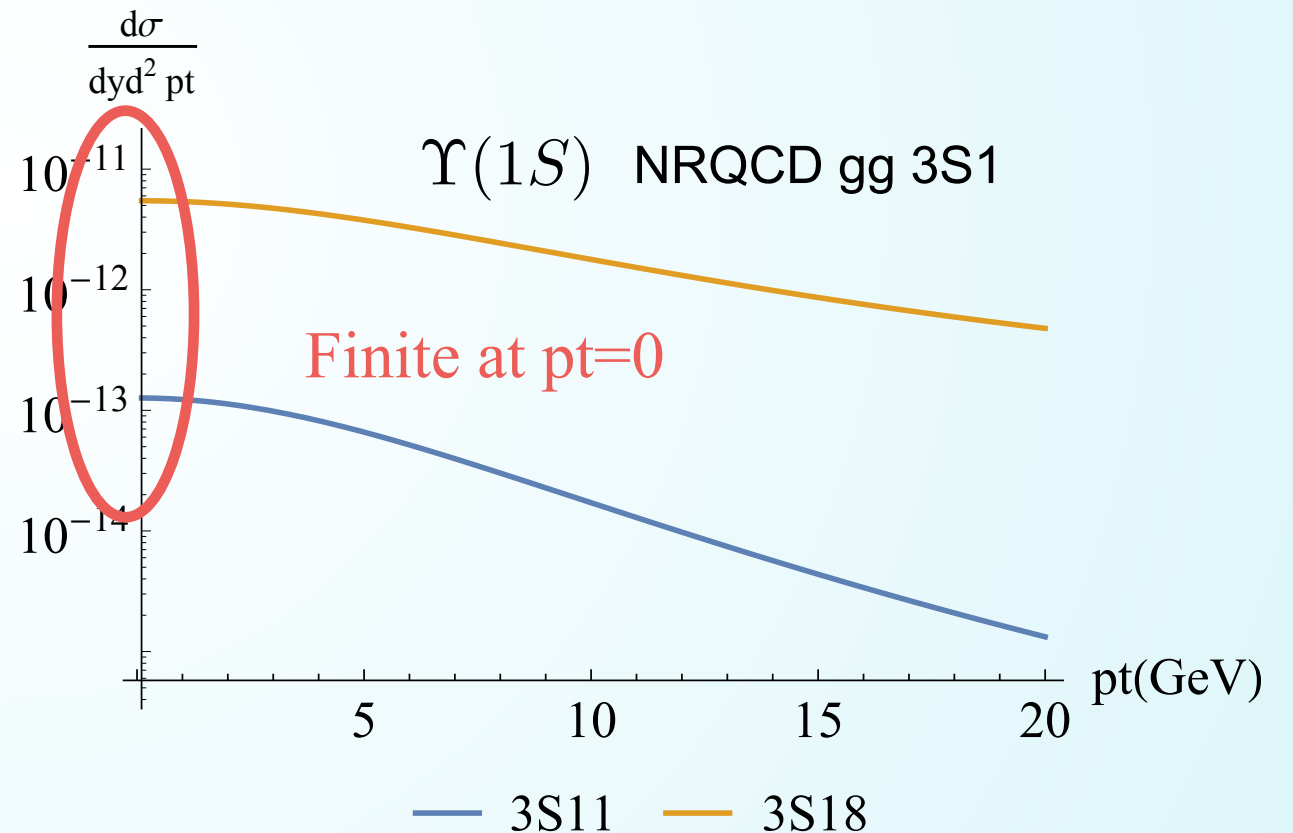
$$\hat{\sigma}_{0,gg}^{3S_1^{[1,8]}} = 0$$



Landau-Yang's theorem



✓ Go to resummation



Go to higher order calculations