News on helicity PDFs

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INT, 10/17/2018

Selected topics on helicity PDFs

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Outline:

- DSSV update – not just yet!
- T-odd helicity observable at EIC
- Spin content of the proton at NNLO and beyond

Towards DSSV update

- goal is to be as global as possible. For example, for RHIC Δg probes:

STAR 1-JET RUN9 FWD (11 PTS)
STAR 1-JET RUN9 MID (11 PTS)

STAR 1-JET 510 GEV (12 PTS)

STAR 1-JET RUN6 (9 PTS)

STAR 1-JET RUN5 (10 PTS)

PHENIX PI0 510GeV (4 PTS)

PHENIX PI0 510GeV (14 PTS)

PHENIX PI0 RUN9 (12 PTS)

PHENIX PI0 62GEV (5 PTS)

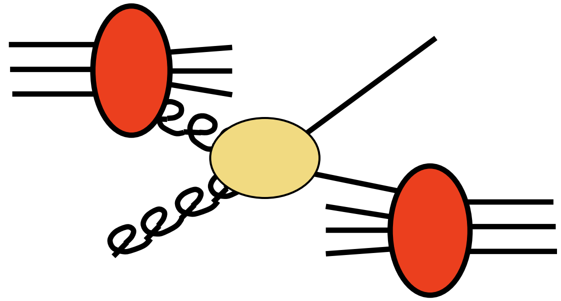
PHENIX PI0 RUN6 (10 PTS)

PHENIX PI0 RUN5 (10 PTS)

"consolidation"

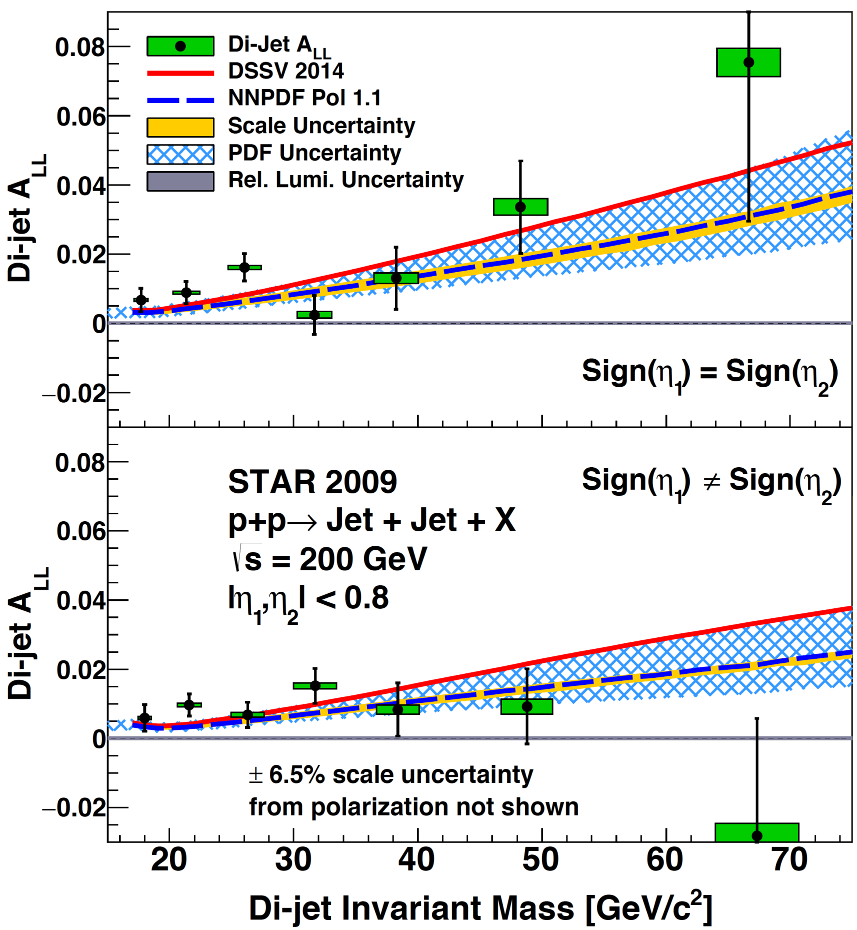
+ STAR dijets

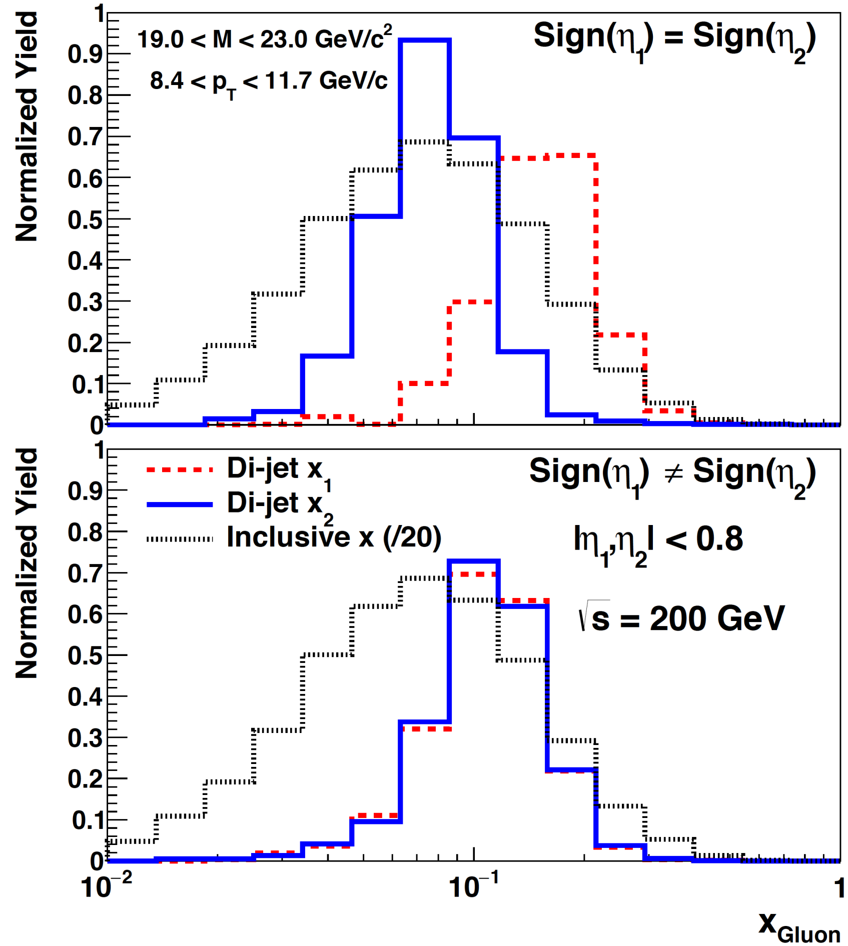
- STAR dijets:

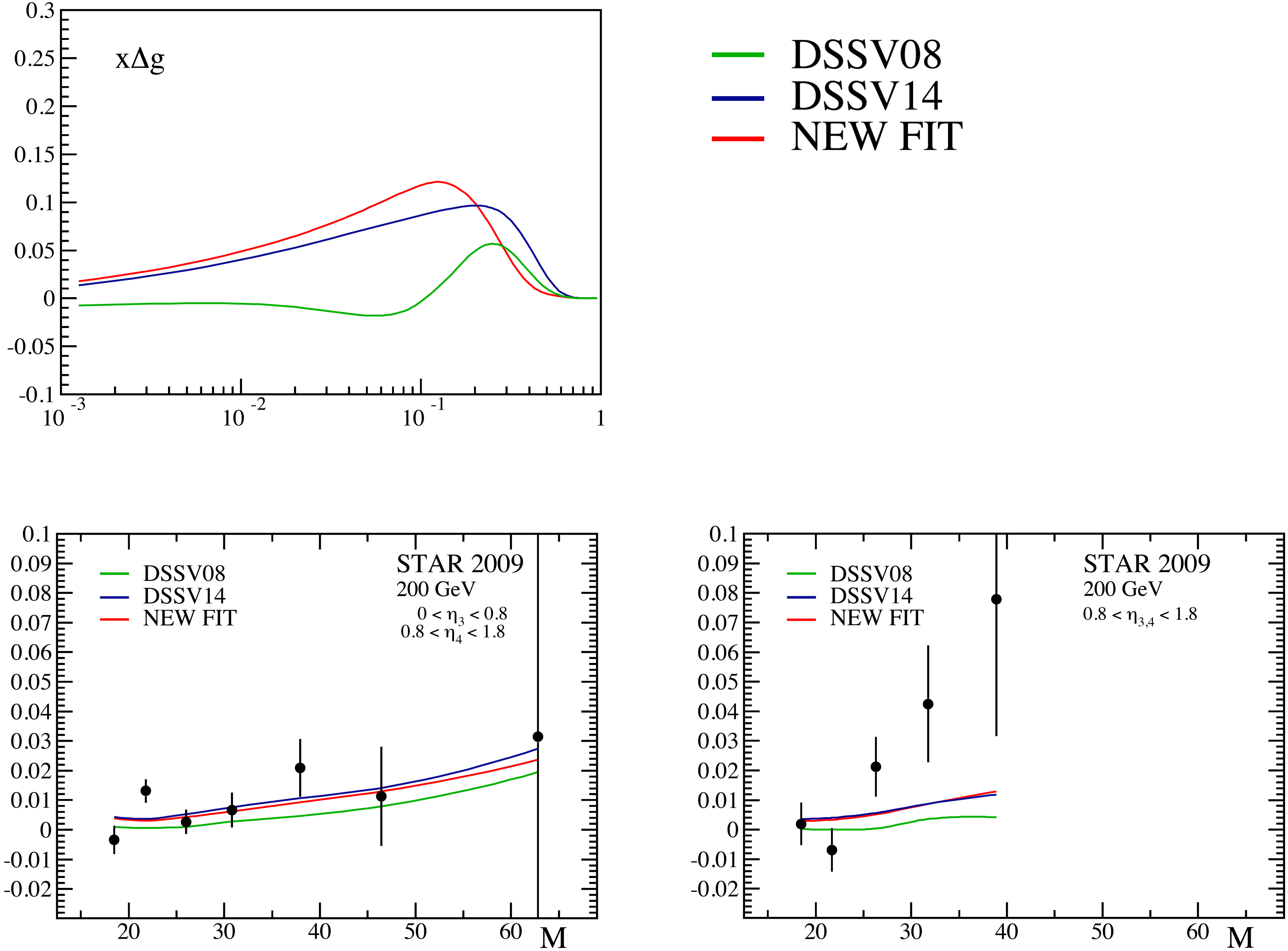


$$x_1 \sim \frac{M}{\sqrt{s}} e^{(\eta_1+\eta_2)/2}$$

$$x_2 \sim \frac{M}{\sqrt{s}} e^{-(\eta_1+\eta_2)/2}$$

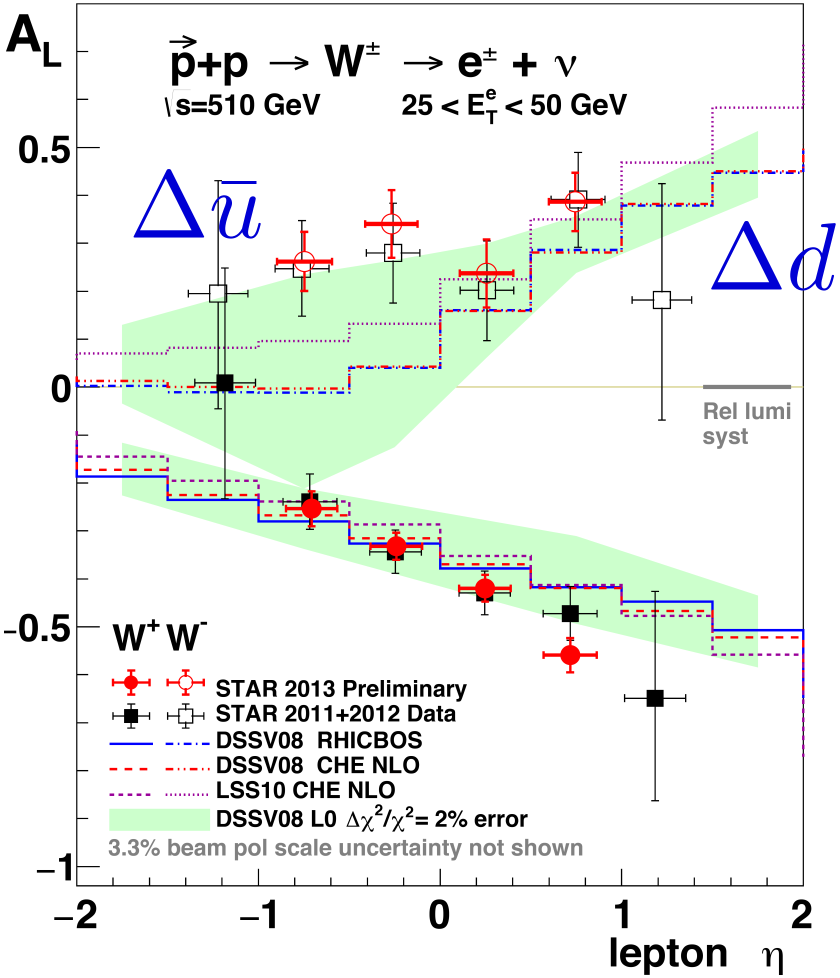




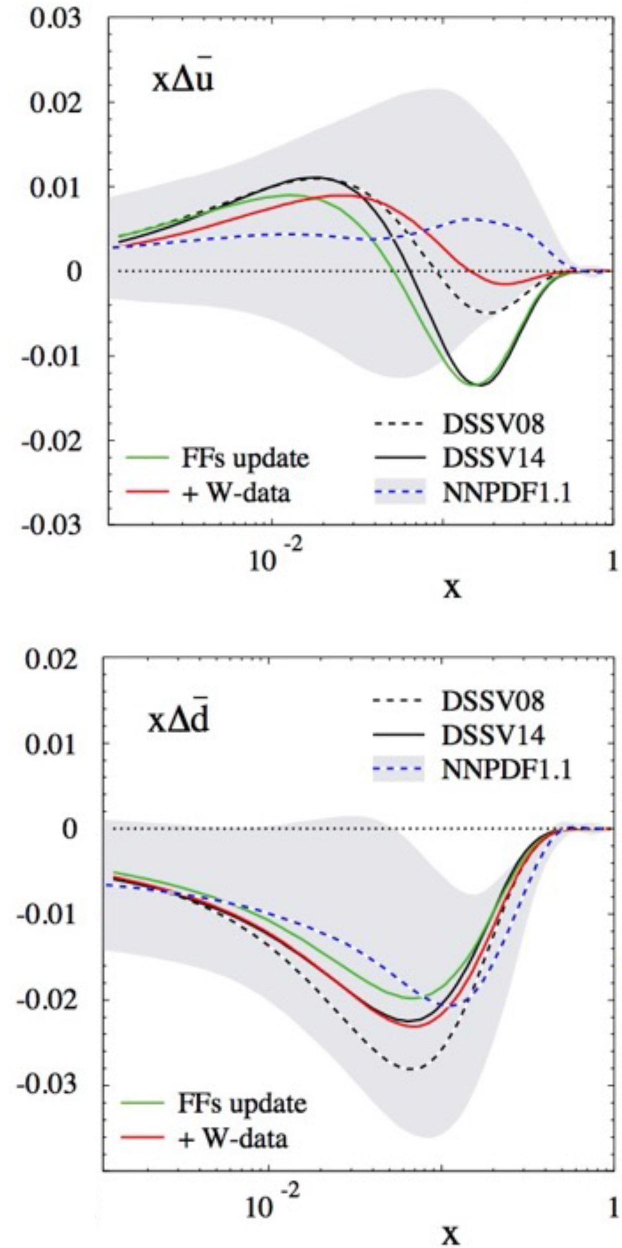


xΔg
DSSV08
DSSV14
NEW FIT
STAR 2009
200 GeV
0 < η3 < 0.8
0.8 < η4 < 1.8
0.8 < η3,4 < 1.8
M

STAR



A_L
p+p → W± → e± + ν
√s=510 GeV
25 < E_T^e < 50 GeV
Δū
Δd
Rel lumi syst
W+ W-
STAR 2013 Preliminary
STAR 2011+2012 Data
DSSV08 RHICBOS
DSSV08 CHE NLO
LSS10 CHE NLO
DSSV08 L0 Δχ²/χ²= 2% error
3.3% beam pol scale uncertainty not shown
lepton η

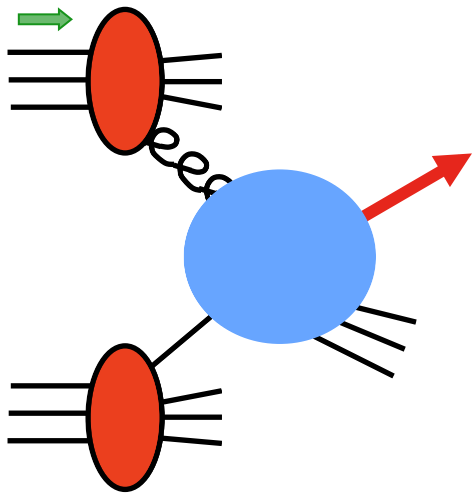


xΔū
DSSV08
DSSV14
NNPDF1.1
FFs update
+ W-data
x
xΔd̄
DSSV08
DSSV14
NNPDF1.1
FFs update
+ W-data
x

T-odd helicity observable at EIC

M. Aicher, A. Schäfer, WV

- A_L for single-inclusive process:

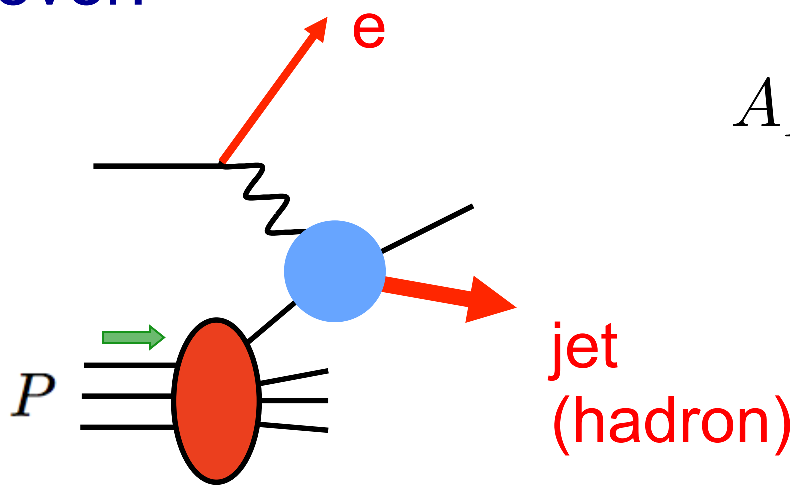


$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$A_L \neq 0$ requires parity violation

(only $\vec{p}_T \cdot \vec{S}_L$ available)

- however:

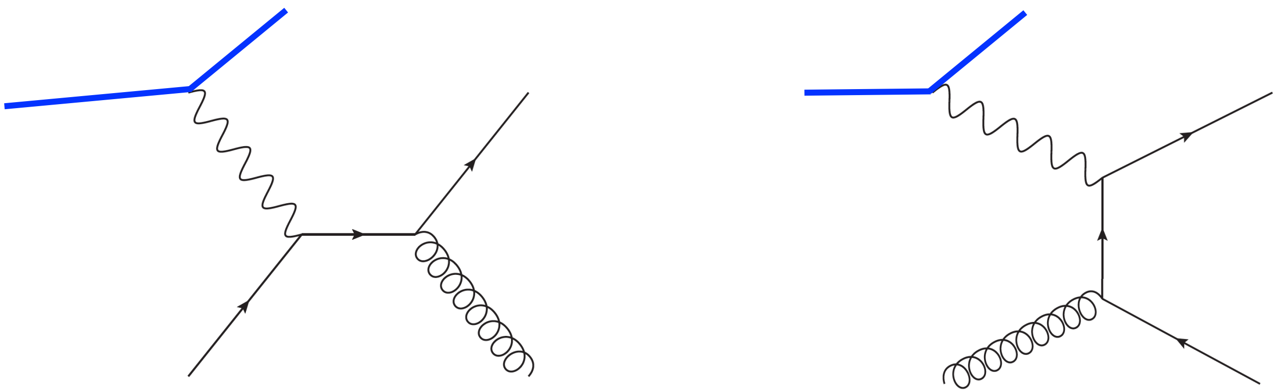


$$A_L \sim \vec{S}_L \cdot \left(\vec{p}_T^{\,e^-} \times \vec{p}_T^{\,\mathrm{jet}} \right)$$

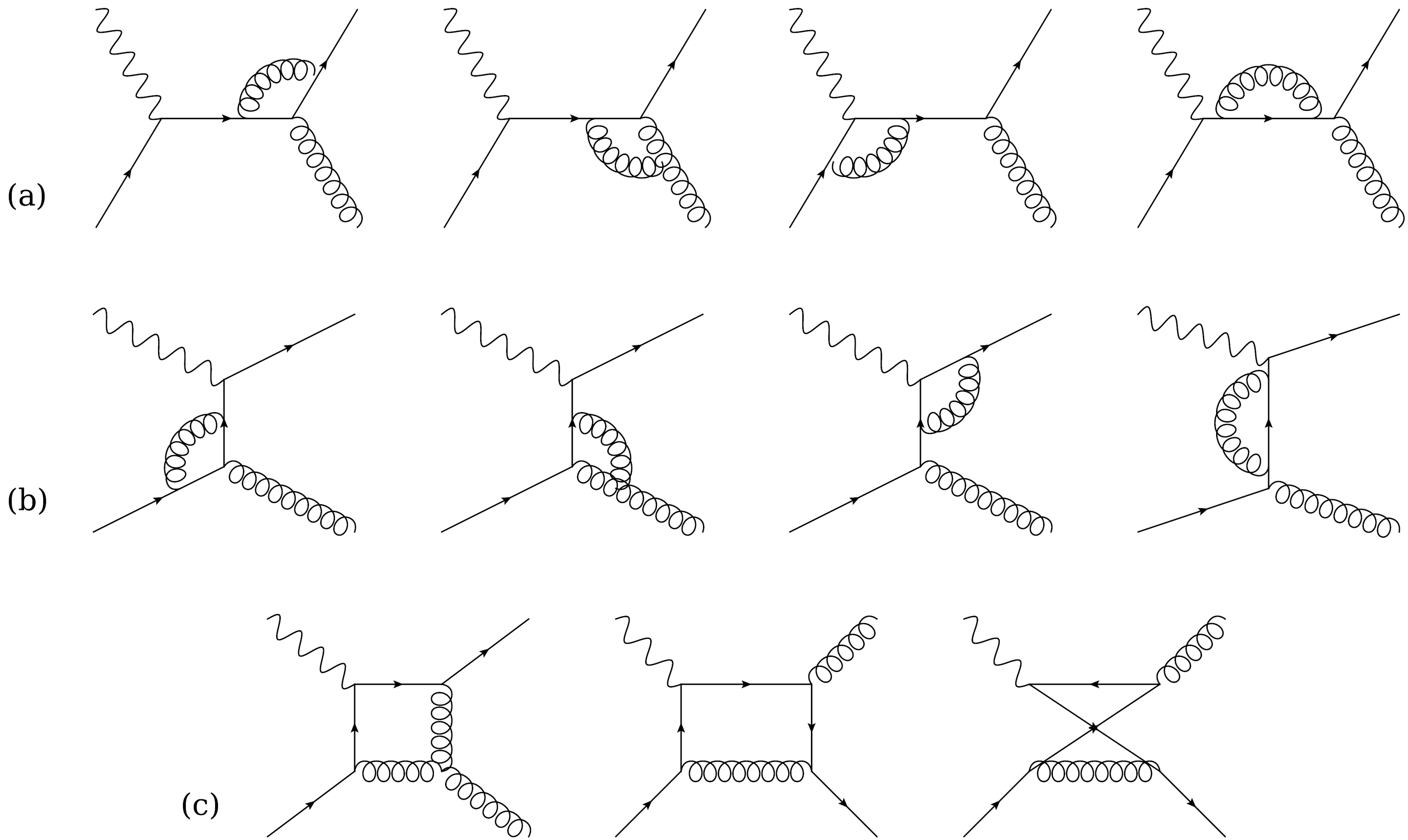
P even
"T-odd"

$$A_L = \mathcal{A} \sin\phi + \mathcal{B} \sin(2\phi)$$

- Born diagrams for $ep \to ehX$:



- T-odd effects need absorptive part
  → loop corrections


(a)
(b)
(c)

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}x\ \mathrm{d}Q^2\ \mathrm{d}z\ \mathrm{d}\kappa^2\ \mathrm{d}\phi} = \frac{\pi\alpha^2 y^2}{4Q^4 z} L^{\mu\nu}\Delta W_{\mu\nu} \qquad \kappa^2 = \frac{P_T'^2}{Q^2}$$

$$L_{\mu\nu} = 2\left(k_\mu k'_\nu + k_\nu k'_\mu + \frac{q^2}{2} g_{\mu\nu}\right)$$

$$\Delta W_{\mu\nu} = \sum_{a,b}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\int_z^1 \frac{\mathrm{d}\eta}{\eta^2} D_a^H(\eta) e_{ab}\Delta H_{\mu\nu}^{ab}\Delta f_b^p(\xi)$$

- for partonic process $\gamma^*(q) + b(p) \to a(p') + X$:

$$\Delta H_{\mu\nu}^{ab} = \int d\mathcal{PS}\ \left(|\mathcal{M}^+|^2_{\mu\nu}(b\to a) - |\mathcal{M}^-|^2_{\mu\nu}(b\to a)\right)$$

- obtain loop results by crossing 1-loop corrections for

$$\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to q + \bar{q} + g$$

Körner, Melic, Merebashvili

$$L^{\mu\nu}\Delta W_{\mu\nu} = \sum_{a,b}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\int_z^1 \frac{\mathrm{d}\eta}{\eta^2} D_a^H(\eta)\Delta f_b^p(\xi)\frac{2\hat{z}e_{ab}}{(2\pi)^3 y^2}\left(\mathcal{A}^{ab}\sin\phi + \mathcal{B}^{ab}\sin(2\phi)\right)$$

$$\times\,\delta\left(\hat{\kappa}^2 - \frac{1-\hat{x}}{\hat{x}}\hat{z}(1-\hat{z})\right)$$

$$\hat{x} = \frac{Q^2}{2p\cdot q} = \frac{x}{\xi} \qquad \hat{z} = \frac{p\cdot p'}{p\cdot q} = \frac{z}{\eta} \qquad \hat{\kappa} = \frac{p'_T}{Q} = \frac{\kappa}{\eta}$$

where

$$\mathcal{A}^{ab} = \sqrt{1-y}(2-y)i\frac{\hat{\kappa}}{2\hat{x}}\left[\frac{1}{\hat{x}}\Delta h_8^{ab} + \left(\hat{z} + \frac{\hat{\kappa}^2}{\hat{z}}\right)\Delta h_9^{ab}\right]$$

$$\mathcal{B}^{ab} = -(1-y)i\frac{\hat{\kappa}^2}{\hat{x}}\Delta h_9^{ab}$$

$$\Delta h_8^{qq} = -\,8i\pi\alpha_s^2(Q)\frac{\hat{x}^3(1-\hat{x}-\hat{z})}{(1-\hat{x})(1-\hat{z})}$$

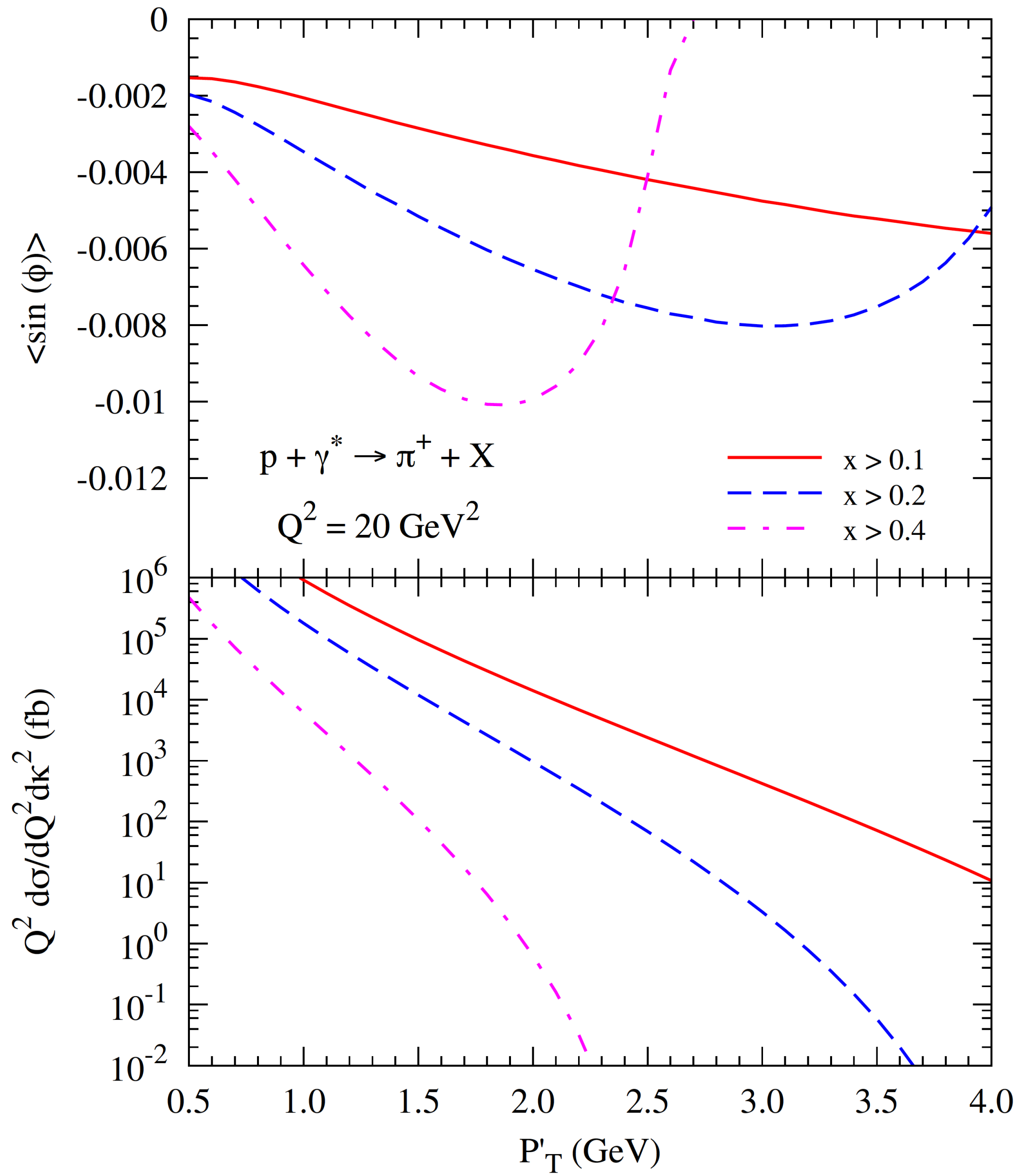
$$\times\left[\frac{1}{2}C_F C_A + C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{3-\hat{z}}{1-\hat{z}} + \ln(\hat{z})\frac{2}{(1-\hat{z})^2}\right)\right]$$

$$\Delta h_9^{qq} = 8i\pi\alpha_s^2(Q)\frac{\hat{x}^3}{(1-\hat{x})(1-\hat{z})}$$

$$\times\left[\frac{3}{2}C_F C_A + C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{1-3\hat{z}}{1-\hat{z}} + \ln(\hat{z})\frac{2(1-2\hat{z})}{(1-\hat{z})^2}\right)\right]$$

Phenomenology for EIC:

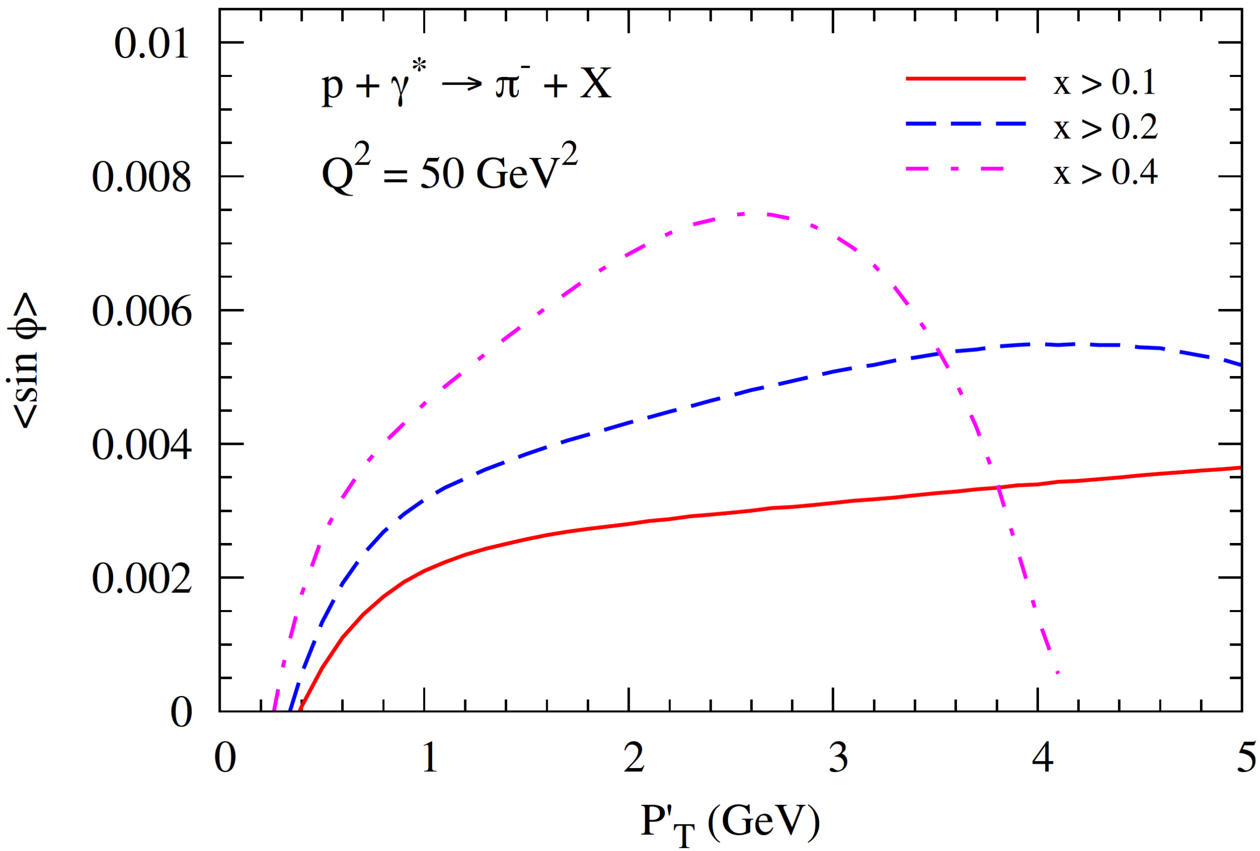
$$\langle \sin(n\phi) \rangle \equiv \frac{\int \mathrm{d}x\mathrm{d}z\mathrm{d}\phi \, \sin(n\phi) \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}z\mathrm{d}\kappa^2\mathrm{d}\phi}}{\int \mathrm{d}x\mathrm{d}z\mathrm{d}\phi \frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}z\mathrm{d}\kappa^2\mathrm{d}\phi}}$$



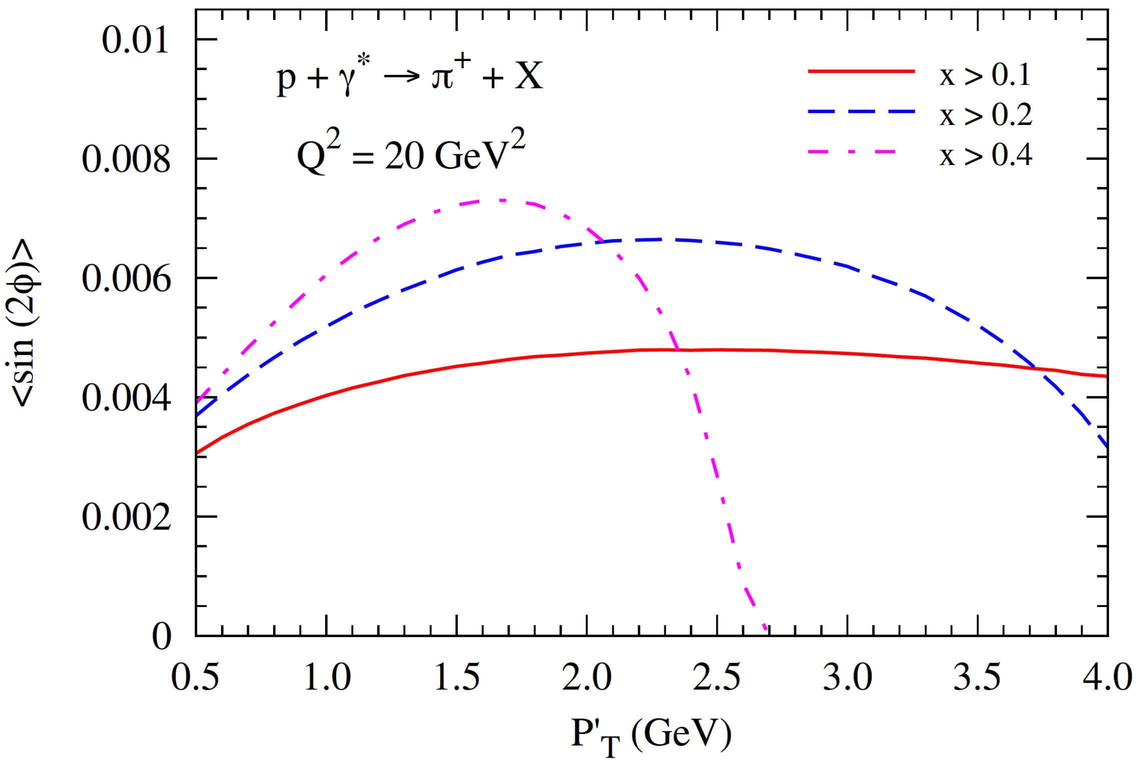
DSSV

mostly up-quark and valence dominated

little Q^2 dependence of $\langle \sin\Phi \rangle$



p + γ* → π⁻ + X
Q² = 50 GeV²
x > 0.1
x > 0.2
x > 0.4
<sin φ>
P'T (GeV)
0.01
0.008
0.006
0.004
0.002
0
0
1
2
3
4
5

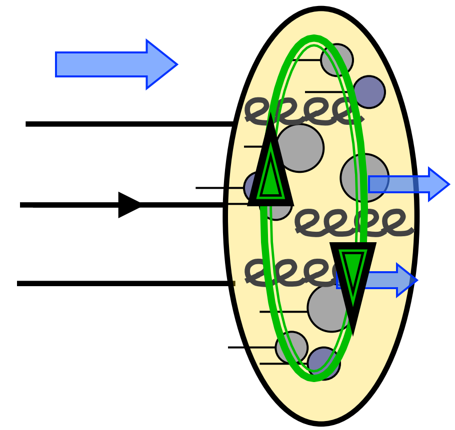


$p + \gamma^* \rightarrow \pi^+ + X$
$Q^2 = 20\ GeV^2$
x > 0.1
x > 0.2
x > 0.4
<sin (2ϕ)>
0.01
0.008
0.006
0.004
0.002
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
P'T (GeV)

Spin content of the proton at NNLO and beyond

D. de Florian, WV

Jaffe-Manohar sum rule:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_g(Q^2)$$

Define: $$\Delta f_a(Q^2) \equiv \int_0^1 dx\, \Delta f_a(x, Q^2)$$

$$\Delta\Sigma(Q^2) = \sum_{i=1}^{N_f} \Big(\Delta f_{q_i}(Q^2) + \Delta f_{\bar{q}_i}(Q^2) \Big)$$

$$\Delta G(Q^2) = \Delta f_g(Q^2)$$

Evolution equations:

$$\frac{d\Delta f_a(Q^2)}{d\ln Q^2} = \sum_b \Delta P_{ab}\big(\alpha_s(Q^2)\big)\ \Delta f_b(Q^2)$$

$$\Delta P_{ab} \ = \ a_s \Delta P^{(0)}_{ab} + a_s^2 \Delta P^{(1)}_{ab} + a_s^3 \Delta P^{(2)}_{ab} + \mathcal{O}\big(a_s^4\big)$$

Ahmed,Ross
Altarelli,Parisi

Mertig, van Neerven
WV

Moch, (Rogal),
Vermaseren,Vogt

Orbital angular momentum: evolution only known to LO,
but evolution of $L_q(Q^2) + L_g(Q^2)$ constrained by sum rule

$$a_s \equiv \frac{\alpha_s}{4\pi} \qquad \frac{d\ln a_s(Q^2)}{d\ln Q^2} \equiv \frac{\beta(a_s)}{a_s} = -\beta_0 a_s - \beta_1 a_s^2 - \beta_2 a_s^3 + \mathcal{O}\big(a_s^4\big)$$

$$\Delta P_{q_i q_k} = \Delta P_{\bar{q}_i \bar{q}_k} \equiv \delta_{ik} \, \Delta P^V_{qq} + \Delta P^S_{qq}$$

$$\Delta P_{q_i \bar{q}_k} = \Delta P_{\bar{q}_i q_k} \equiv \delta_{ik} \, \Delta P^V_{q\bar{q}} + \Delta P^S_{q\bar{q}}$$

Flavor singlet sector:

$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} = \begin{pmatrix} \Delta P_{\Sigma\Sigma} & 2N_f \, \Delta P_{qg} \\ \Delta P_{gq} & \Delta P_{gg} \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix}$$

where

$$\Delta P_{\Sigma\Sigma} \equiv \underbrace{\Delta P^V_{qq} + \Delta P^V_{q\bar{q}}}_{\text{vanishes to all orders}} + N_f \left(\Delta P^S_{qq} + \Delta P^S_{q\bar{q}} \right)$$

vanishes to all orders

What's known (in $\overline{\text{MS}}$)

Mertig, van Neerven; WV;
Moch, (Rogal),Vermaseren,Vogt

$$a_s \equiv \frac{\alpha_s}{4\pi}$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \Bigg\{ a_s \begin{pmatrix} 0 & 0 \\ 3C_F & \beta_0 \end{pmatrix} + a_s^2 \begin{pmatrix} -6N_f\,C_f & 0 \\ C_F\left(\frac{71}{3}C_A - 9C_F - \frac{2}{3}N_f\right) & \beta_1 \end{pmatrix} + a_s^3 \begin{pmatrix} -2N_f C_F\left(\frac{71}{3}C_A - 9C_F - \frac{2}{3}N_f\right) & 0 \\ \# & \beta_2 \end{pmatrix} \Bigg\} \begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

Pattern:

$$\Delta P_{qg} \equiv 0 \qquad \Delta P_{gg} \equiv -\frac{\beta(a_s)}{a_s}$$

$$\Delta P_{\Sigma\Sigma} \equiv -2N_f\,a_s\,\Delta P_{gq}$$

Not really surprising:

Altarelli, Lampe '90

$$\Delta\Sigma \;\propto\; \langle P,S\,|\,\overline{\psi}\,\gamma^{\mu}\,\gamma^{5}\,\psi\,|\,P,S\rangle$$

$$\partial_\mu j_5^\mu \;=\; 2N_f\,a_s\,\mathrm{Tr}\big[F_{\mu\nu}\tilde{F}^{\mu\nu}\big]$$

$$\equiv\; 2N_f\,a_s\,\partial_\mu\Big\{\underbrace{\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\Big[A^\nu\Big(F^{\rho\sigma}-\frac{2}{3}A^\rho\,A^\sigma\Big)\Big]}_{\equiv\,K^\mu}\Big\}$$

$$\Rightarrow\;\; j_5^\mu - 2N_f\,a_s\,K^\mu \quad \text{conserved}$$

$$\longrightarrow\;\; \frac{d}{d\ln Q^2}\Big(\Delta\Sigma + 2N_f a_s \Delta G\Big) \;=\; 0$$

Recall

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma\\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f\Delta P_{qg}\\ \Delta P_{gq} & \Delta P_{gg}\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta G\end{pmatrix}$$

$$\Rightarrow \quad \frac{d}{d\ln Q^2}\Big(\Delta\Sigma(Q^2) + 2N_f a_s(Q^2)\Delta G(Q^2)\Big)$$

$$= \Big(\Delta P_{\Sigma\Sigma} + 2N_f a_s\,\Delta P_{gq}\Big)\Delta\Sigma$$

$$+ \Big(2N_f\Delta P_{qg} + \Delta P_{gg} + \frac{\beta(\alpha_s)}{a_s}\Big)2N_f a_s\Delta G$$

Thus

$$\Delta P_{qg} \equiv 0 \qquad \Delta P_{gg} \equiv -\frac{\beta(a_s)}{a_s}$$

$$\Delta P_{\Sigma\Sigma} \equiv -2N_f\, a_s\,\Delta P_{gq}$$

$$\Delta P_{\Sigma\Sigma} \equiv -2N_f\, a_s\, \Delta P_{gq}$$

from known $\Delta P_{gq}^{(2)}$ obtain

$$\Delta P_{\Sigma\Sigma}^{(3)} = -2N_f\, C_F \left[\frac{1607}{12} C_A^2 - \frac{461}{4} C_F C_A + \frac{63}{2} C_F^2 + \left(\frac{41}{3} - 72\zeta_3 \right) C_A N_f - \left(\frac{107}{2} - 72\zeta_3 \right) C_F N_f - \frac{13}{3} N_f^2 \right]$$

single anomalous dimension controls all evolution:

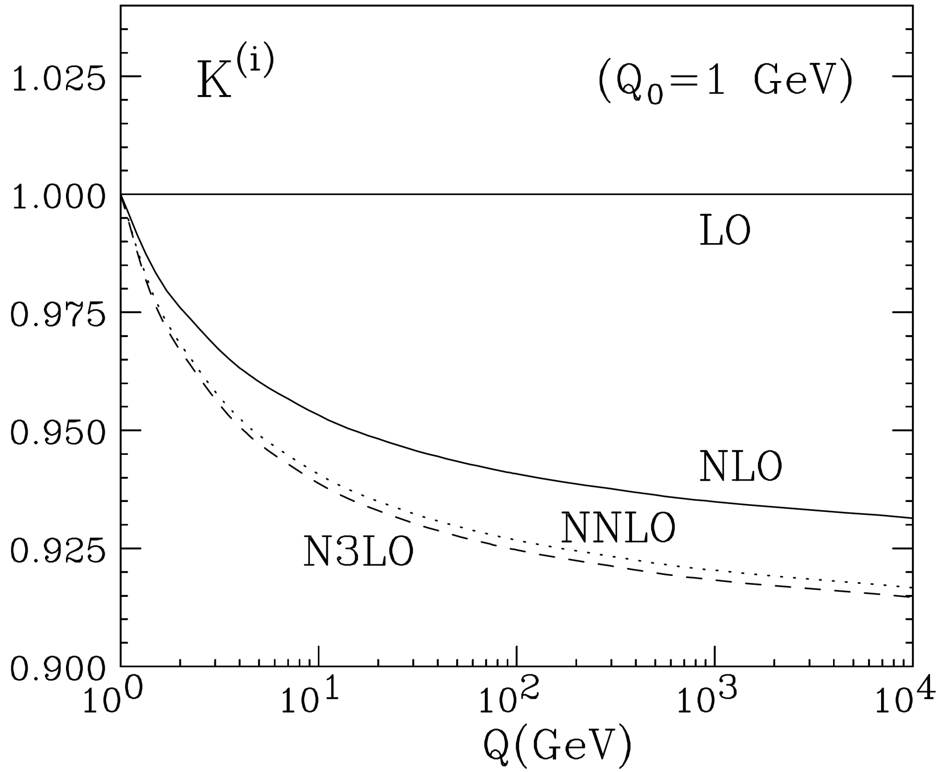
$$\frac{d\Delta\Sigma(Q^2)}{d\ln Q^2} = \Delta P_{\Sigma\Sigma}\big(a_s(Q^2)\big)\, \Delta\Sigma(Q^2)$$

$$\frac{d\Big[a_s(Q^2)\, \Delta G(Q^2)\Big]}{d\ln Q^2} = -\frac{\Delta P_{\Sigma\Sigma}\big(a_s(Q^2)\big)}{2N_f}\, \Delta\Sigma(Q^2)$$

straightforward to solve analytically

$$\frac{\Sigma(Q^2)}{\Sigma(Q_0^2)} = \exp[0] \times \exp\left[-\frac{a_Q - a_0}{\beta_0}\,\delta P^{(1)\Sigma}\right] \times \exp\left[\frac{a_Q^2 - a_0^2}{2\beta_0^2}\,\left(\beta_1\,\delta P^{(1)\Sigma} - \beta_0\,\delta P^{(2)\Sigma}\right)\right]$$

$$\times \exp\left[\frac{a_Q^3 - a_0^3}{3\beta_0^3}\left(-\beta_1^2\,\delta P^{(1)\Sigma} + \beta_0\beta_2\,\delta P^{(1)\Sigma} + \beta_0\beta_1\,\delta P^{(2)\Sigma} - \beta_0^2\,\delta P^{(3)\Sigma}\right)\right]$$



see also Altenbuchinger, Hägler, Weise, Henley

gluon spin:

$$a_s(Q^2)\,\Delta g(Q^2) = a_s(Q_0^2)\,\Delta g(Q_0^2) + \Delta\Sigma(Q_0^2)\; F\Big(a_s(Q^2), a_s(Q_0^2)\Big)$$

$$F(a_s,a_0)^{\mathrm{LO}} = -(a_s - a_0)\frac{\Delta P_{gq}^{(0)}}{\beta_0}$$

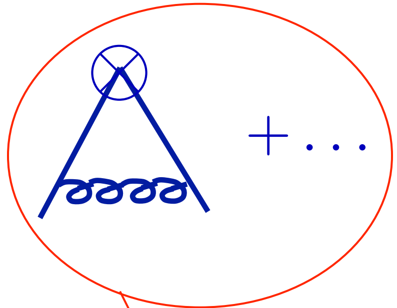
$$F(a_s,a_0)^{\mathrm{NLO}} = \frac{(a_s^2 - a_0^2)}{2\beta_0^2}(\beta_1\,\Delta P_{gq}^{(0)} - \beta_0\,\Delta P_{gq}^{(1)}) + \frac{(a_s - a_0)^2}{2\beta_0^2}\,\Delta P_{gq}^{(0)}\,\Delta P^{(1)\Sigma}$$

$$F(a_s,a_0)^{\mathrm{NNLO}} = \frac{(a_s^3 - a_0^3)}{3\beta_0^3}(\beta_0\beta_2\,\Delta P_{gq}^{(0)} + \beta_0\beta_1\,\Delta P_{gq}^{(1)} - \beta_1^2\,\Delta P_{gq}^{(0)} - \beta_0^2\,\Delta P_{gq}^{(2)})$$
$$+ \frac{(a_s - a_0)^2}{6\beta_0^3}[-3(a_0 + a_s)\beta_1\,\Delta P_{gq}^{(0)}\,\Delta P^{(1)\Sigma} + (a_0 + 2a_s)\beta_0\,\Delta P_{gq}^{(1)}\,\Delta P^{(1)\Sigma}$$
$$+ (2a_0 + a_s)\,\Delta P_{gq}^{(0)}(\beta_0\,\Delta P^{(2)\Sigma} - \,(\Delta P^{(1)\Sigma})^2)]$$

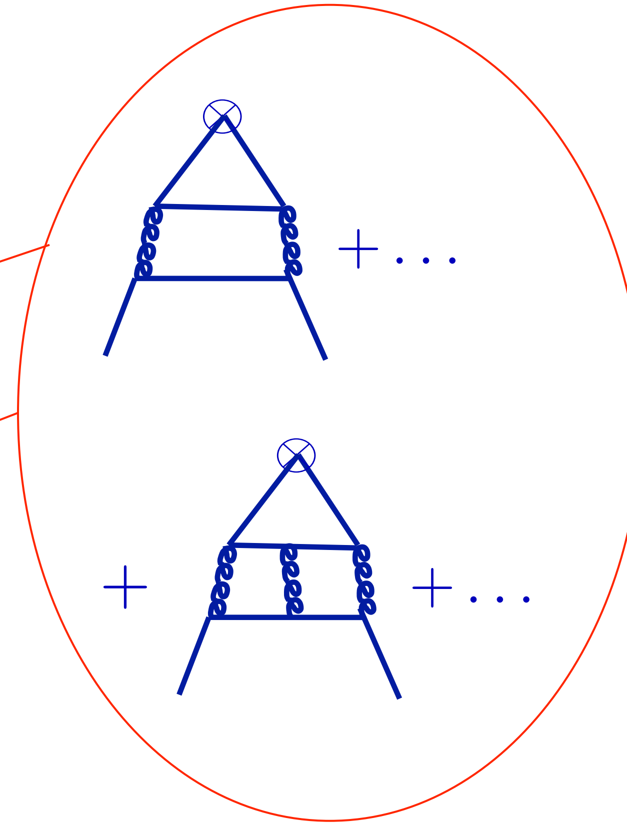
$\Delta G(Q^2)$ will rise as $1/\alpha_s(Q^2)\ldots$

…unless “fine-tuned” input $\Delta G(Q_0^2) \sim -0.1$

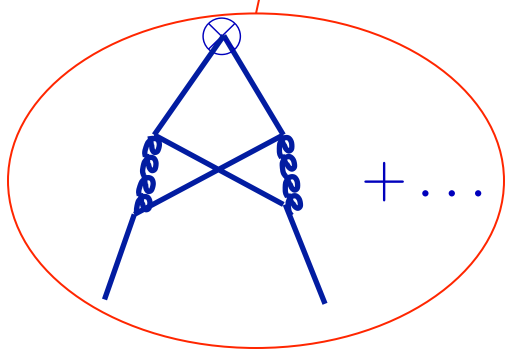
Non-singlet:



$$\Delta P_{q_i q_k} = \Delta P_{\bar{q}_i \bar{q}_k} = \delta_{ik}\,\Delta P^V_{qq} + \Delta P^S_{qq}$$



$$\Delta P_{q_i \bar{q}_k} = \Delta P_{\bar{q}_i q_k} = \delta_{ik}\,\Delta P^V_{q\bar{q}} + \Delta P^S_{q\bar{q}}$$

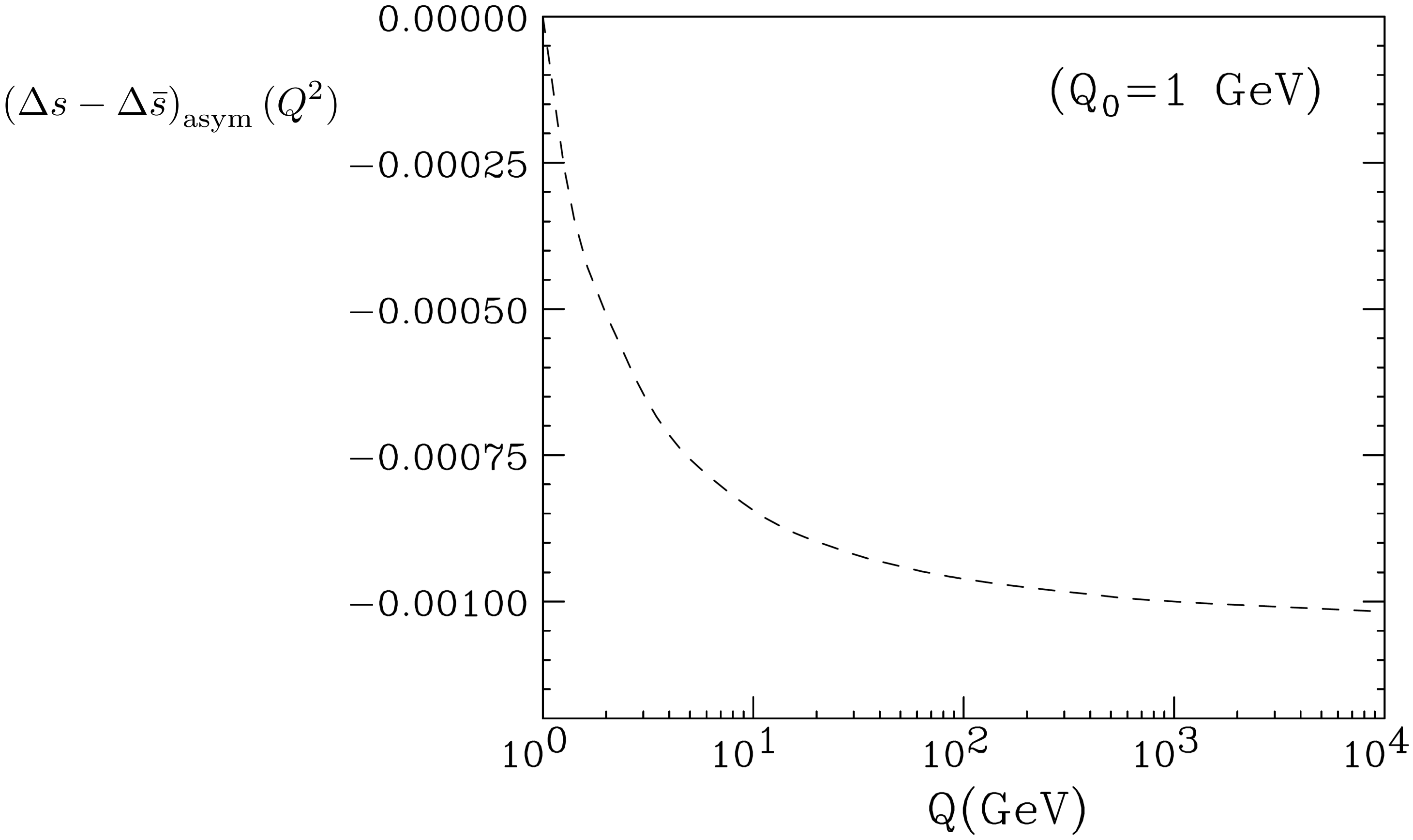


$P^S_{qq} \neq P^S_{q\bar{q}}$ generates strangeness asymmetry $s \neq \bar{s}$

Catani, de Florian, Rodgrigo, WV

Polarized case: de Florian, WV

$$(\Delta s - \Delta\bar{s})_{\rm asym}\,(Q^2) = -\frac{\Delta P_{qq}^{(2)S} - \Delta P_{q\bar{q}}^{(2)S}}{2\beta_0}\,\left(a_Q^2 - a_0^2\right)\,(\Delta u_V + \Delta d_V)\,(Q^2)$$



Conclusions:

- DSSV upgrade in progress
- T-odd observables opportunity at EIC
- Evolution of proton spin content at higher orders