Towards QCD corrections to single-spin observables

Werner Vogelsang

Univ. Tübingen

INT, 10/10/2018

Outline:

- $\ell p \to h\,X$: Introduction / Motivation
- NLO calculation for cross section
- Spin asymmetry A_N

In collaboration with
Patriz Hinderer, Marc Schlegel
Patriz Hinderer, Yuji Koike (+ discussion with Jianwei Qiu)

Introduction / Motivation

Single-inclusive scattering:

$$\ell p \;\to\; \mathrm{jet}\, X$$

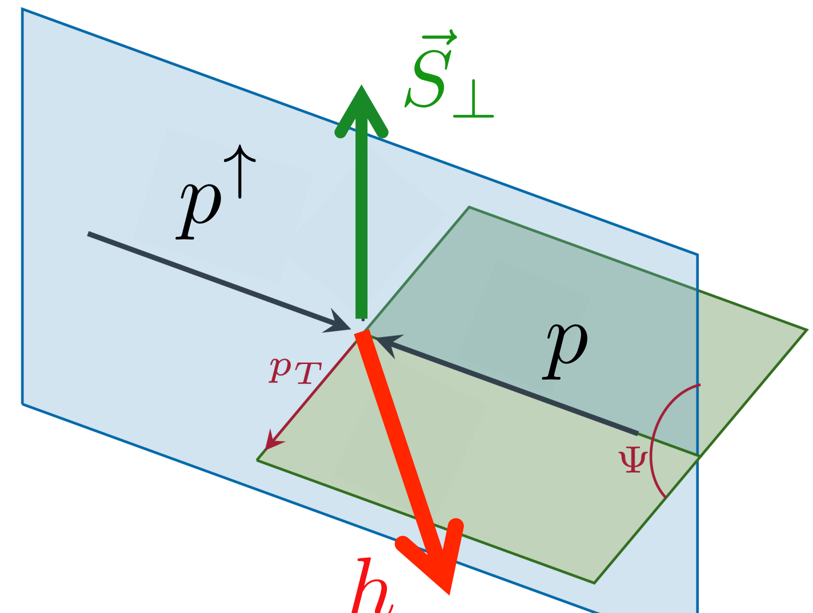
$$\ell p \;\to\; h\, X$$

Single-inclusive scattering: $\ell p \to \text{jet}\, X$

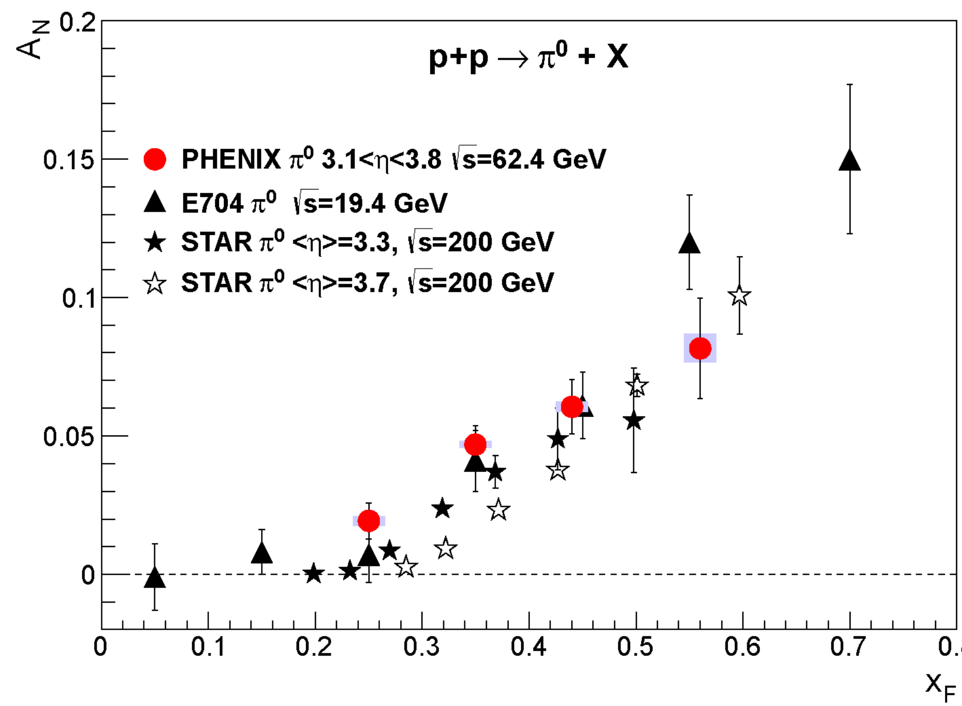
$\ell p \to h\, X$

Why care?

Large single-spin asymmetries in $p^\uparrow p \to h\, X$



$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$



- A_N in $p^{\uparrow}p \to h\,X$ power-suppressed in QCD

- A_N in $p^\uparrow p \to h\,X$ power-suppressed in QCD
- various mechanisms discussed:

★ single hard scale p_T : collinear factorization

twist-3 proton matrix elements, twist-3 fragmentation

Qiu, Sterman; Efremov, Teryaev; Koike et al.; Kouvaris, Qiu, WV, Yuan;
Kanazawa, Koike, Metz, Pitonyak; Kang, Metz, Qiu, Zhou;
Gamberg, Kang, Metz, **Pitonyak**, Prokudin; …

- A_N in $p^{\uparrow}p \to h\,X$ power-suppressed in QCD
- various mechanisms discussed:

★ single hard scale p_T : collinear factorization

twist-3 proton matrix elements, twist-3 fragmentation

Qiu, Sterman; Efremov, Teryaev; Koike et al.; Kouvaris, Qiu, WV, Yuan;
Kanazawa, Koike, Metz, Pitonyak; Kang, Metz, Qiu, Zhou;
Gamberg, Kang, Metz, **Pitonyak**, Prokudin; …

★ a plethora of contributions, even at LO

★ all studies LO so far, no proper evolution

- process $\ell p \to h\, X$:

Anselmino, Boglione, Hansson, Murgia '99; … Koike '00, '02

★ can choose same kinematics as for $p^{\uparrow} p$

★ simpler -- fewer subprocesses, contributions

★ (for jets: no fragmentation)

- process $\ell p \rightarrow h\, X$:

Anselmino, Boglione, Hansson, Murgia '99; … Koike '00, '02

★ can choose same kinematics as for $p^{\uparrow} p$

★ simpler -- fewer subprocesses, contributions

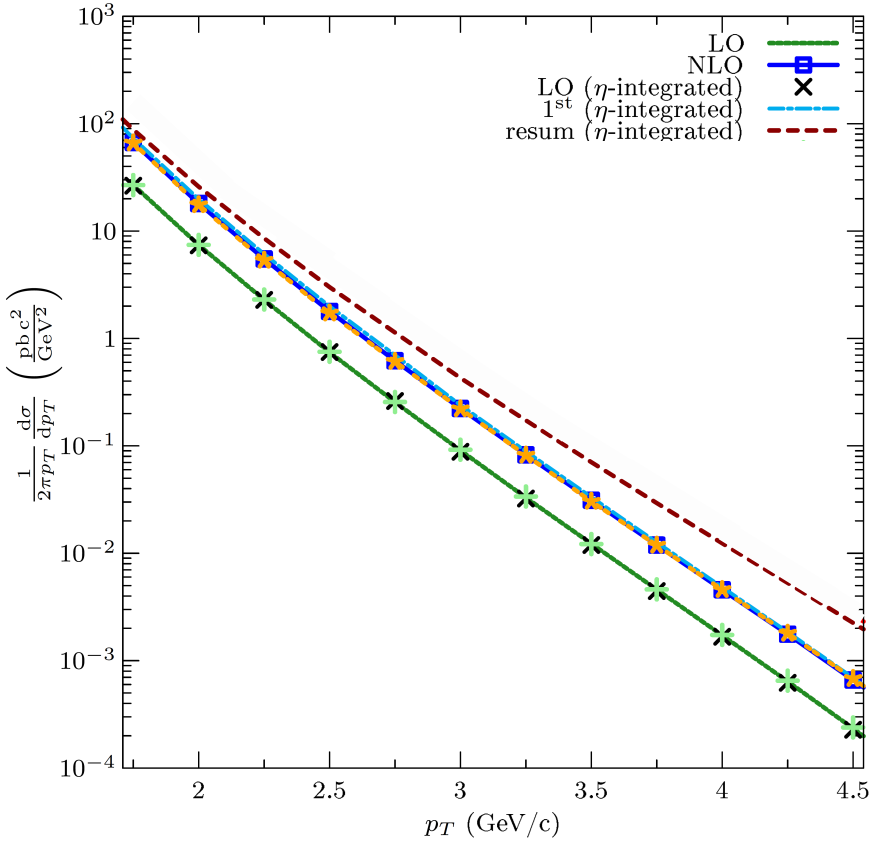
★ (for jets: no fragmentation)

★ can shed light on mechanisms for $p^{\uparrow} p \rightarrow h\, X$

★ may serve as template for theoretical calculations

especially: higher-order QCD corrections

- NLO corrections generally sizable for single-inclusive:



COMPASS $\mu p \to \mu' h X$

(de Florian, Pfeuffer, Schäfer, WV)

NLO cross section for $p\ell \rightarrow h\,X$

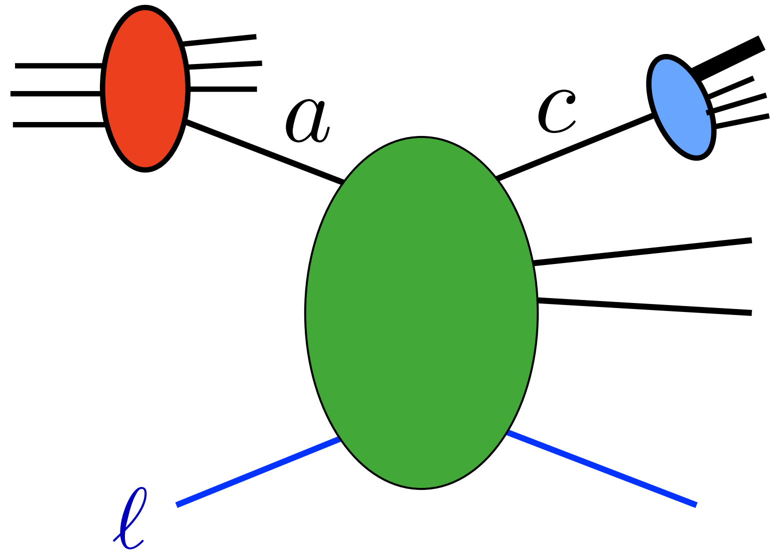
Hinderer, Schlegel, WV

NLO cross section for $p\ell \rightarrow h\,X$

Hinderer, Schlegel, WV

NB: NNLO available now!

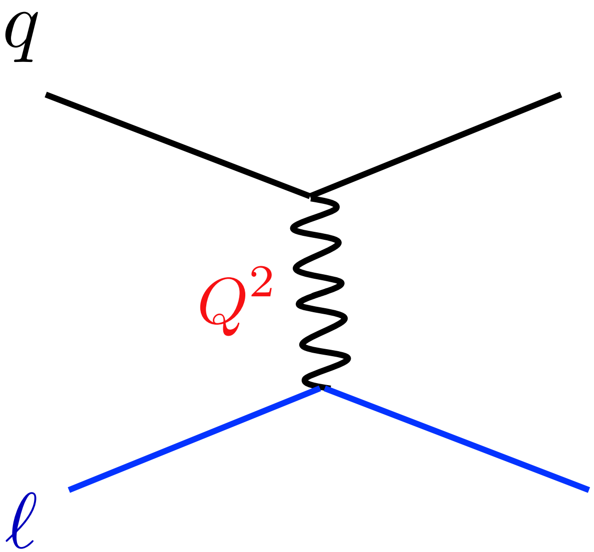
Abelof, Boughezal, Liu, Petriello



$$\frac{E_h d^3\sigma^{p\ell\to hX}}{d^3P_h} = \frac{1}{\pi S}\sum_{a,c}\int\frac{dx}{x}\int\frac{dz}{z^2}\, f_a(x,\mu)\, D_c^h(z,\mu)\, \frac{d^2\hat{\sigma}^{a\ell\to cX}}{v\, dv dw}$$

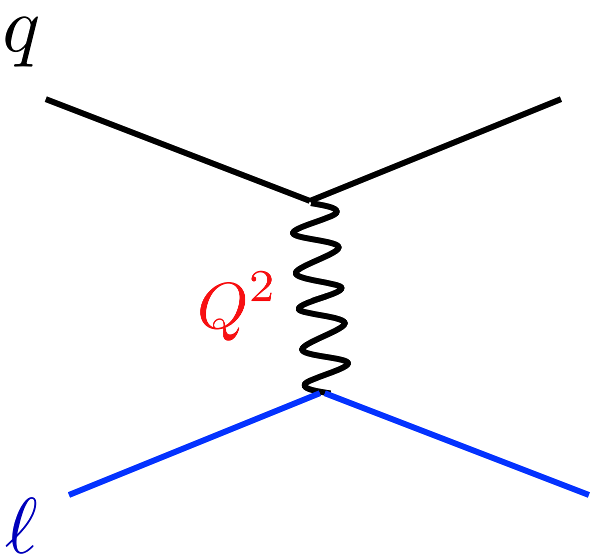
$$v = 1 + \frac{t}{s} \qquad\qquad w = -\frac{u}{t+s}$$

LO:



- always at large Q^2

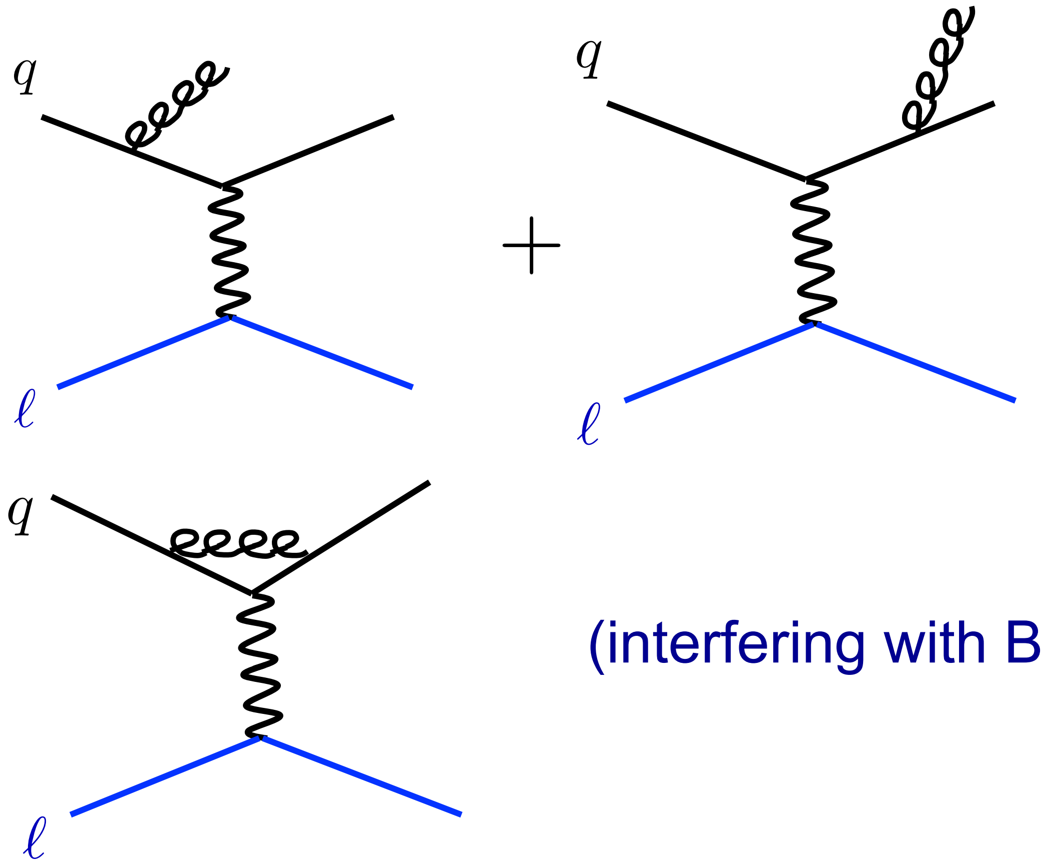
LO:



- always at large Q^2

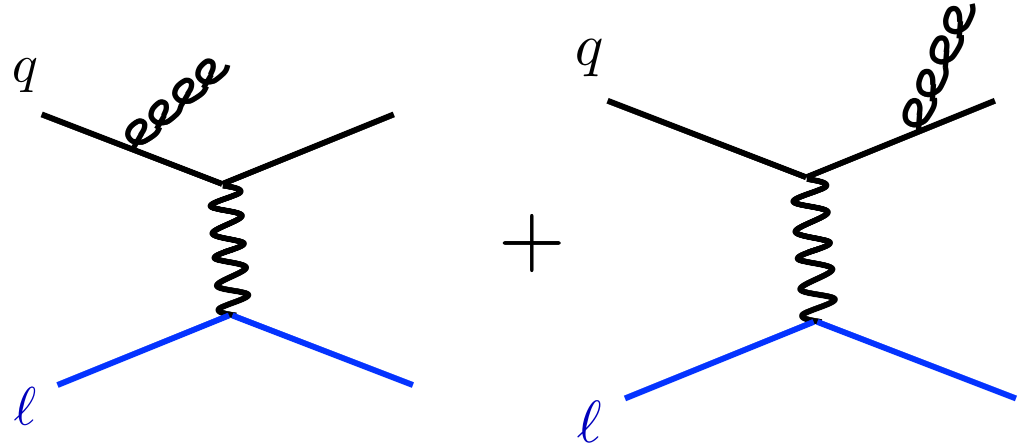
$$\frac{d^2\hat{\sigma}^{q\ell\to q\ell}_{\mathrm{LO}}}{v\,dv dw} \propto \alpha_{\mathrm{em}}^2\,\delta(1-w)\;\frac{1+v^2}{(1-v)^2}$$

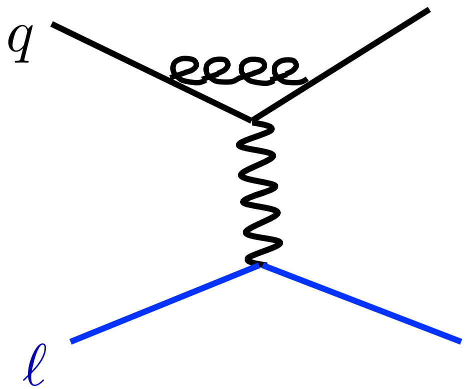
NLO:



(interfering with Born)

NLO:



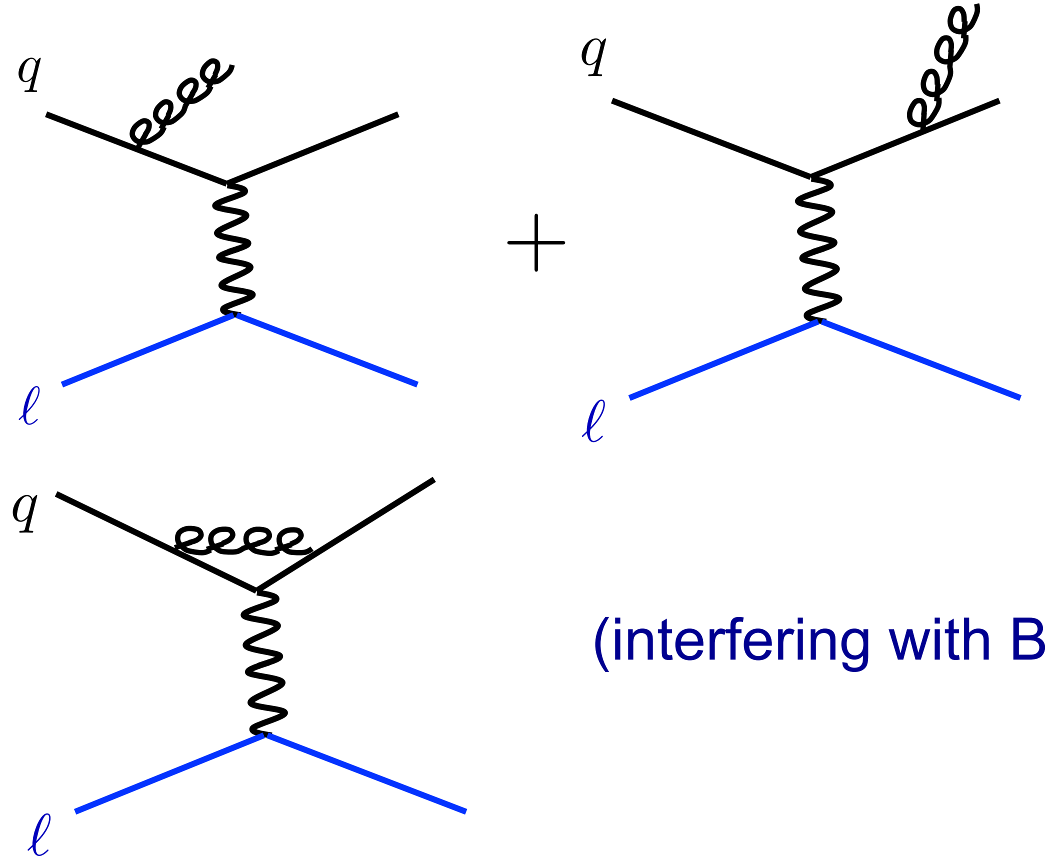


(interfering with Born)

$$\frac{d\hat{\sigma}^{q\to q}_{\rm NLO}}{v\,dv\,dw} \propto \alpha_{\rm em}^2\,\alpha_s \Bigg[A_0^{q\to q}(v)\,\delta(1-w)$$

$$+ A_1^{q\to q}(v)\left(\frac{\ln(1-w)}{1-w}\right)_+ + \frac{1}{(1-w)_+} A_2^{q\to q}(v) + R(v,w) \Bigg]$$

NLO:



(interfering with Born)

$$\frac{d\hat{\sigma}^{q\to q}_{\rm NLO}}{v\,dvdw} \propto \alpha_{\rm em}^2\,\alpha_s \left[A_0^{q\to q}(v)\,\delta(1-w) + A_1^{q\to q}(v)\left(\frac{\ln(1-w)}{1-w}\right)_+ + \frac{1}{(1-w)_+}A_2^{q\to q}(v) + R(v,w)\right]$$

- (plus new channels with initial or fragmenting gluon)

2→3 phase space:

$$v = 1 + \frac{t}{s} \qquad w = -\frac{u}{t+s}$$

2→3 phase space:

$$v = 1 + \frac{t}{s} \qquad w = -\frac{u}{t+s}$$

$$\frac{d^2\Phi_3}{dv\,dw} = \frac{s}{(4\pi)^4\Gamma(1-2\varepsilon)}\left(\frac{4\pi}{s}\right)^{2\varepsilon} v^{1-2\varepsilon}\,(1-v)^{-\varepsilon}\,w^{-\varepsilon}\,(1-w)^{-\varepsilon}$$

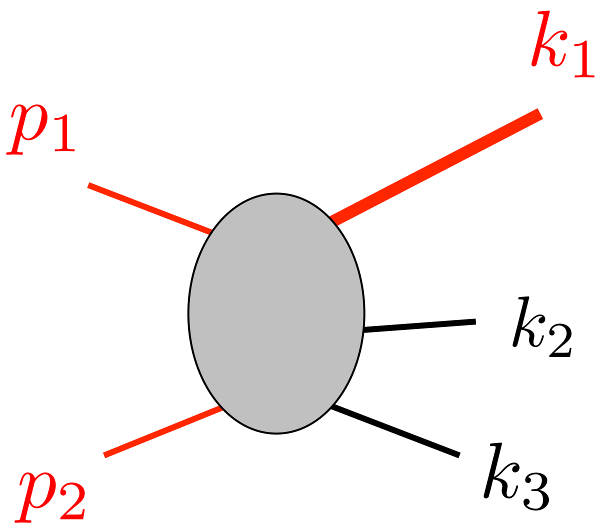
$$\times \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2$$

2→3 phase space:

$$v = 1 + \frac{t}{s} \qquad w = -\frac{u}{t+s}$$

$$\frac{d^2\Phi_3}{dv\,dw} = \frac{s}{(4\pi)^4\Gamma(1-2\varepsilon)}\left(\frac{4\pi}{s}\right)^{2\varepsilon} v^{1-2\varepsilon}\,(1-v)^{-\varepsilon}\,w^{-\varepsilon}\,(1-w)^{-\varepsilon}$$

$$\times \int_0^{\pi} d\theta_1 \int_0^{\pi} d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2$$

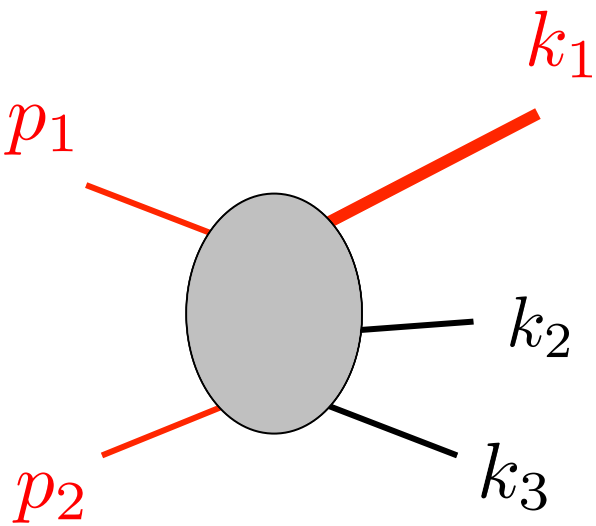


2→3 phase space:

$$v = 1 + \frac{t}{s} \qquad w = -\frac{u}{t+s}$$

$$\frac{d^2\Phi_3}{dv\,dw} = \frac{s}{(4\pi)^4\Gamma(1-2\varepsilon)}\left(\frac{4\pi}{s}\right)^{2\varepsilon} v^{1-2\varepsilon}\,(1-v)^{-\varepsilon}\,w^{-\varepsilon}\,(1-w)^{-\varepsilon}$$

$$\times \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2$$

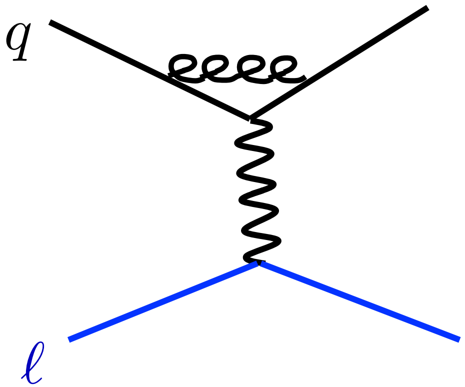


e.g.

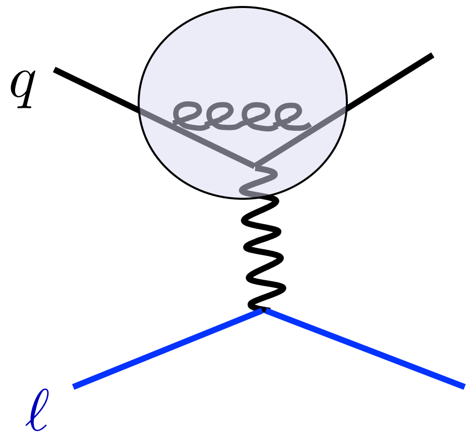
$$(k_2 - p_1)^2 = -\frac{sv}{2}(1-\cos\theta_1)$$

$$(k_1 + k_2)^2 = \frac{s(1-v+vw)}{2}\left(1 + s_\psi \sin\theta_1 \cos\theta_2 - c_\psi \cos\theta_1\right)$$

virtual correction is trivial:



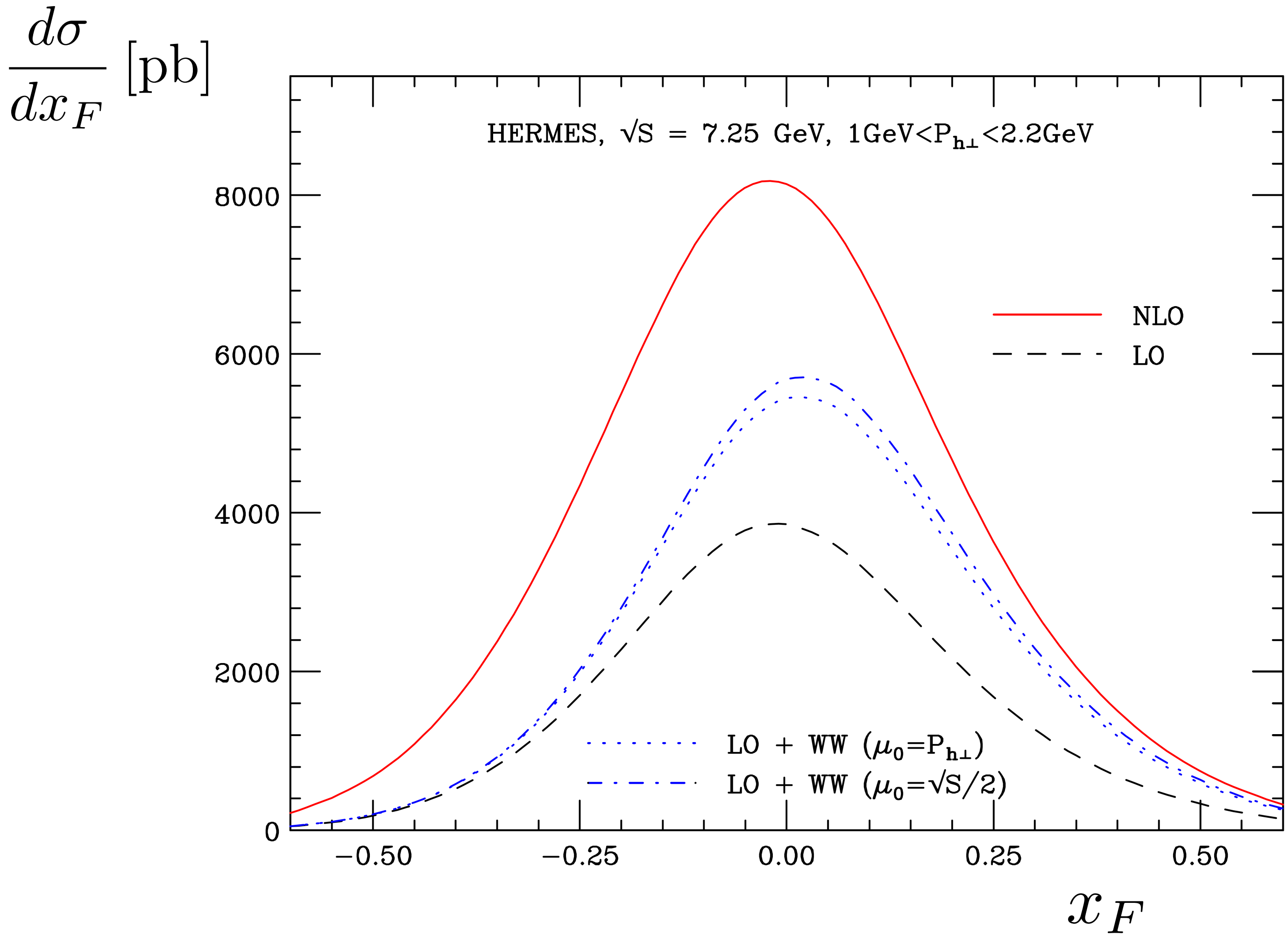
virtual correction is trivial:



$$\Gamma^\mu = \gamma^\mu \left\{ 1 + \frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right] \right\}$$

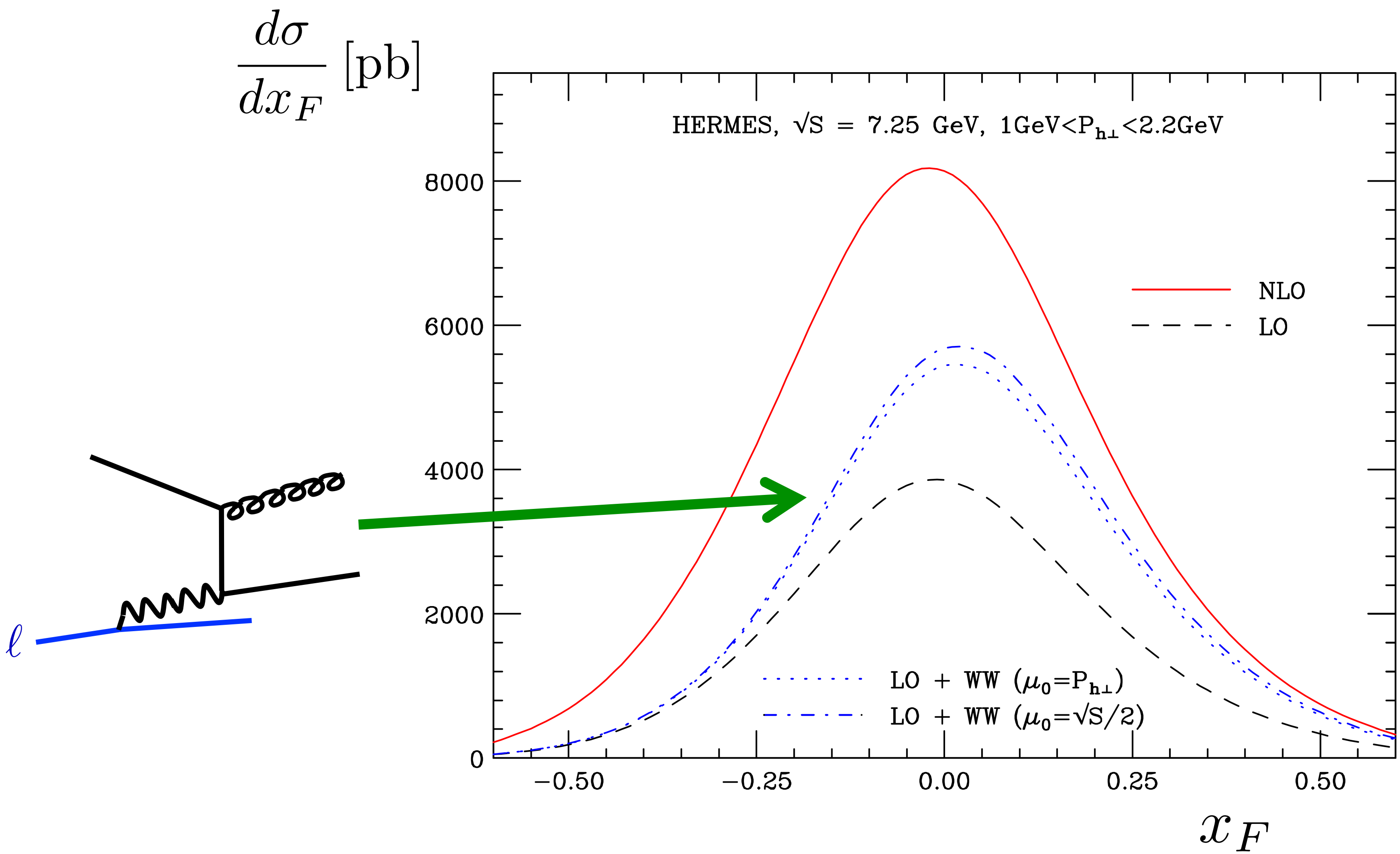
Hinderer, Schlegel, WV

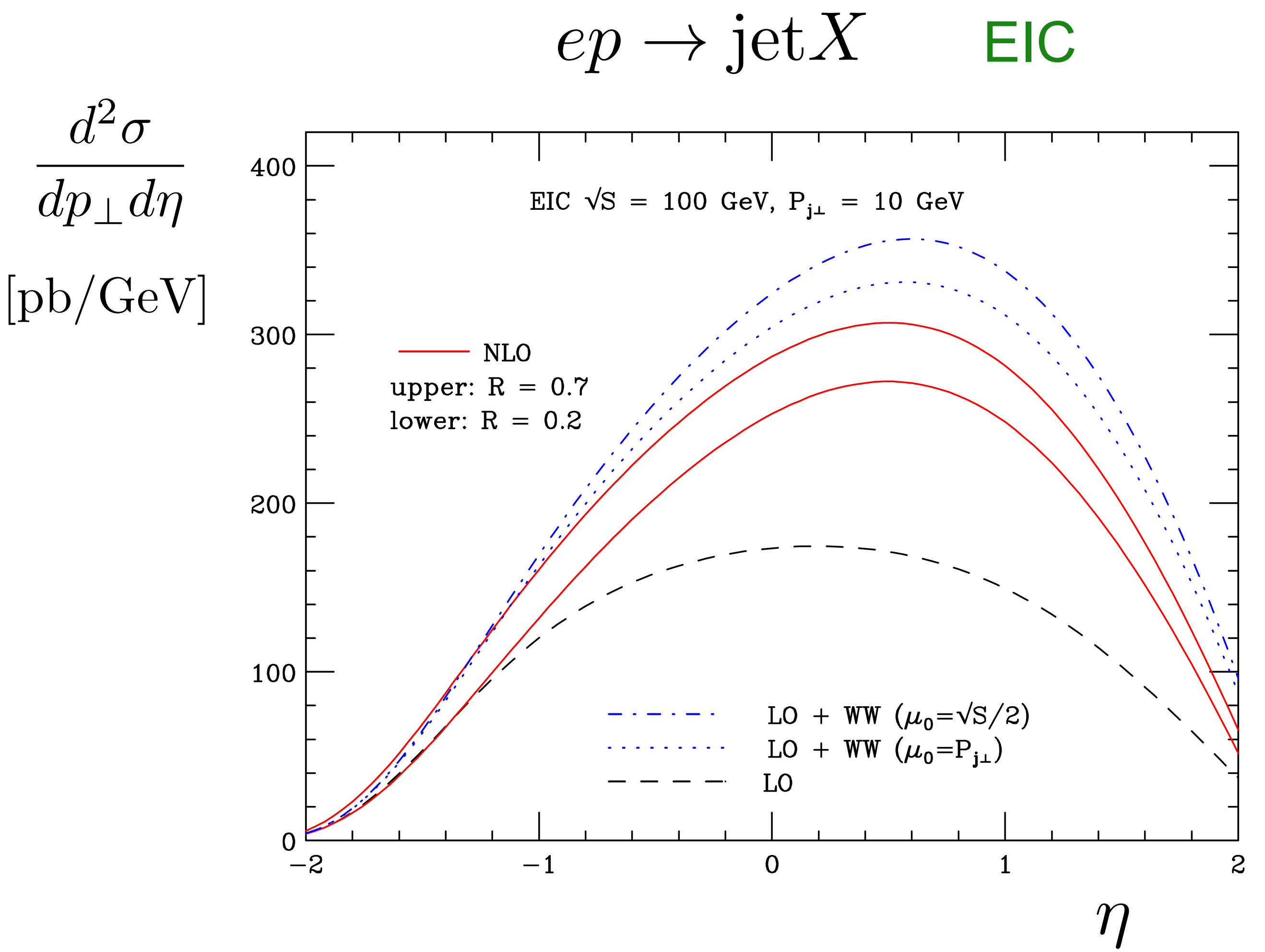
$ep \to \pi^+ X$ HERMES



Hinderer, Schlegel, WV

$ep \to \pi^+ X$ HERMES

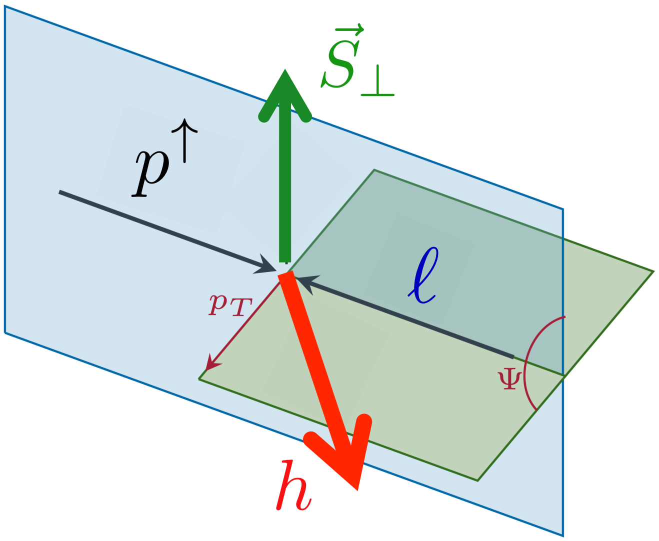




$ep \to \text{jet}X$
EIC
$\frac{d^2\sigma}{dp_\perp d\eta}$
[pb/GeV]
EIC √S = 100 GeV, $P_{j\perp}$ = 10 GeV
NLO
upper: R = 0.7
lower: R = 0.2
LO + WW (μ_0=√S/2)
LO + WW (μ_0=$P_{j\perp}$)
LO
0
100
200
300
400
−2
−1
0
1
2
η

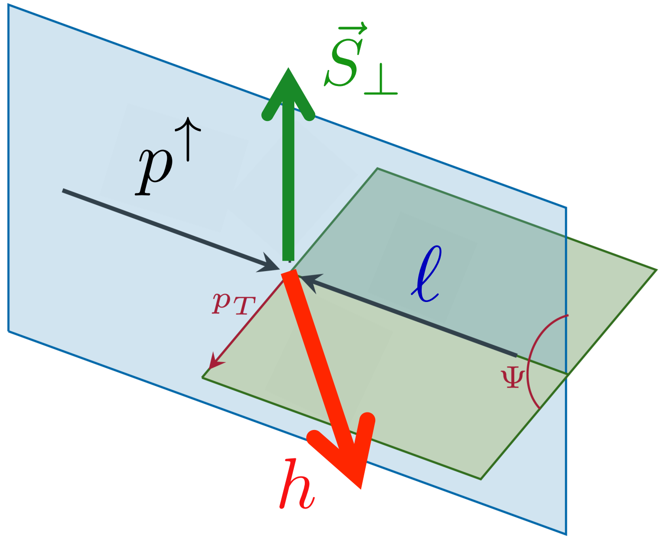
Towards NLO for A_N

$$p^{\uparrow}\ell \to hX$$



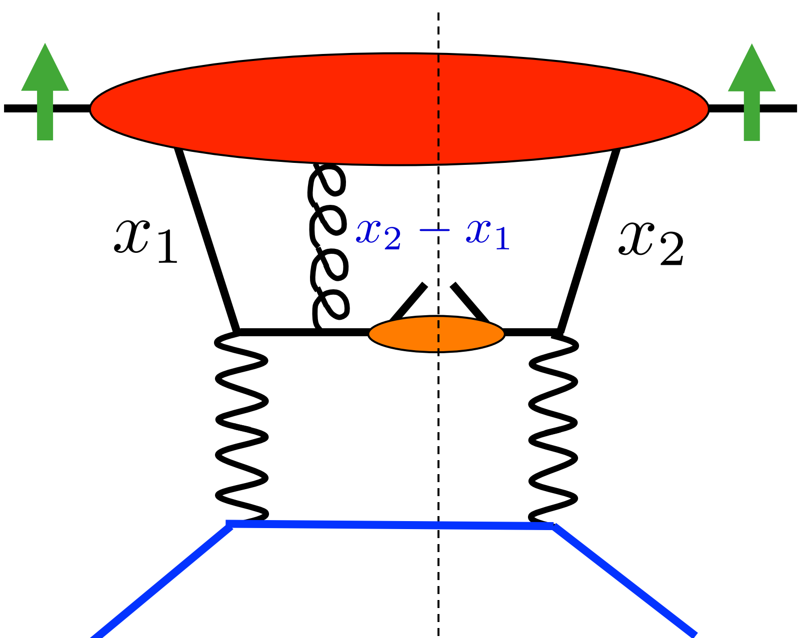
$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$$p^{\uparrow}\ell \to hX$$

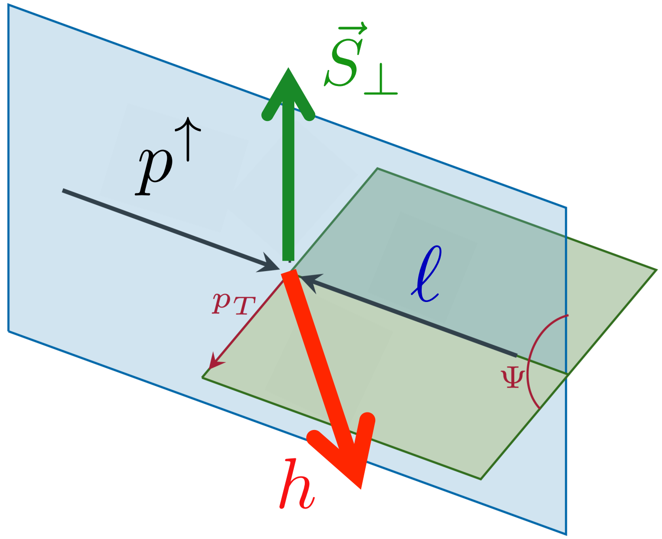


$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

Qiu, Sterman; Kanazawa, Koike;
Kang, Metz, Qiu, Zhou; Gamberg, Kang, Metz, Pitonyak, Prokudin

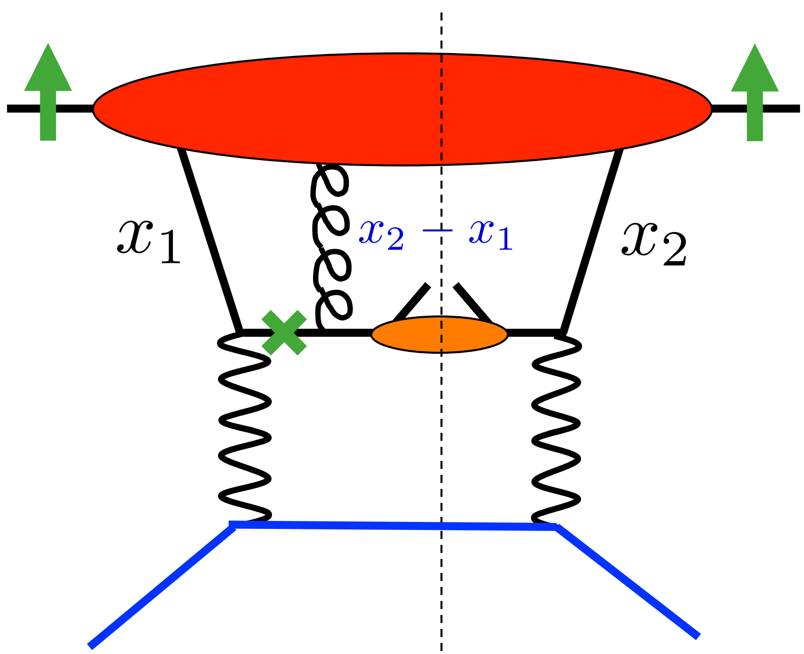


$$p^{\uparrow}\ell \to hX$$



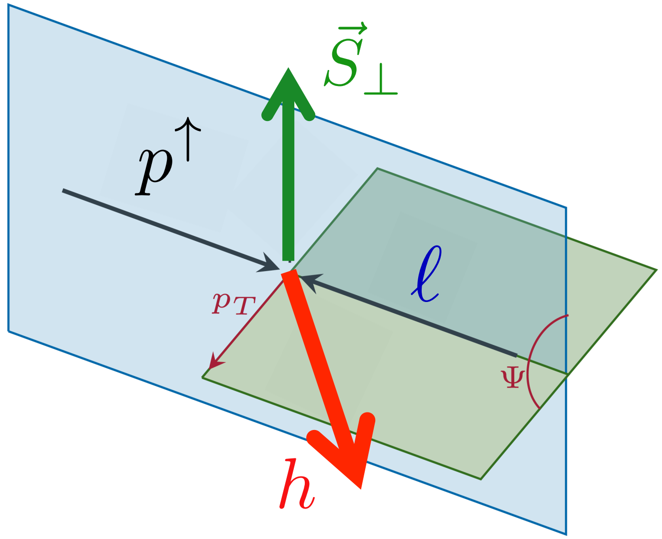
$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

Qiu, Sterman; Kanazawa, Koike;
Kang, Metz, Qiu, Zhou; Gamberg, Kang, Metz, Pitonyak, Prokudin



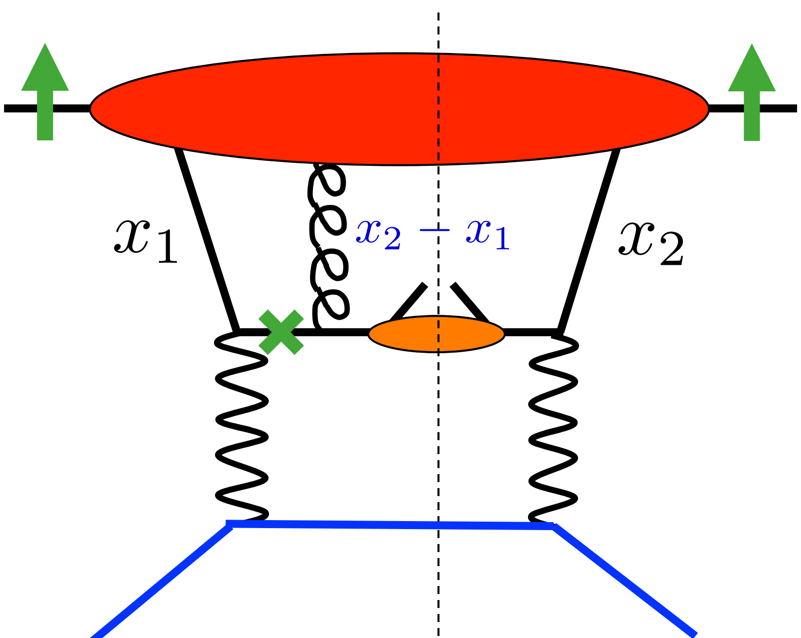
- phase in hard-scattering

$p^{\uparrow}\ell \to hX$

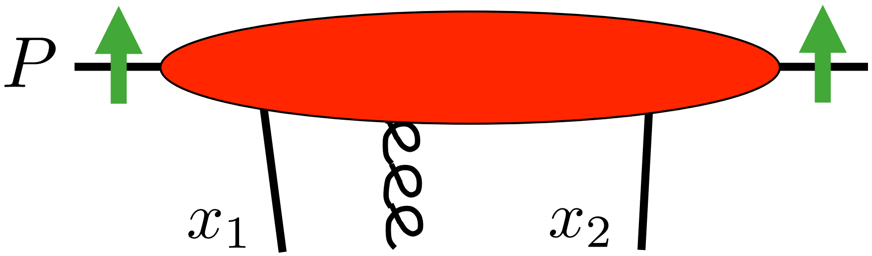


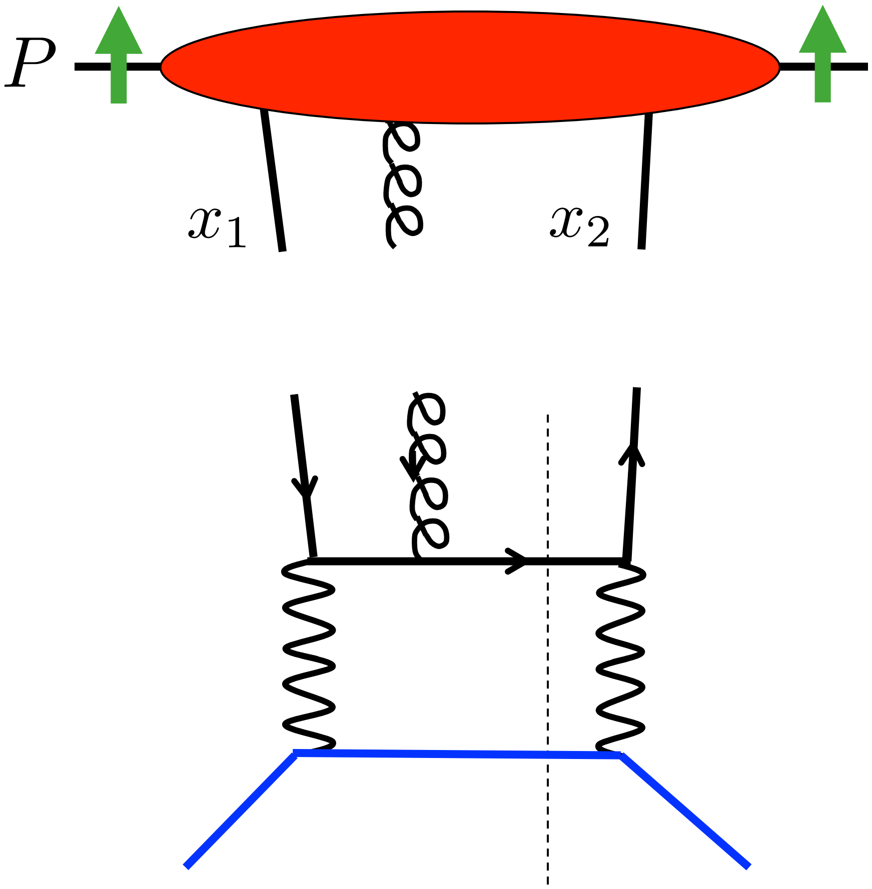
$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

Qiu, Sterman; Kanazawa, Koike;
Kang, Metz, Qiu, Zhou; Gamberg, Kang, Metz, Pitonyak, Prokudin

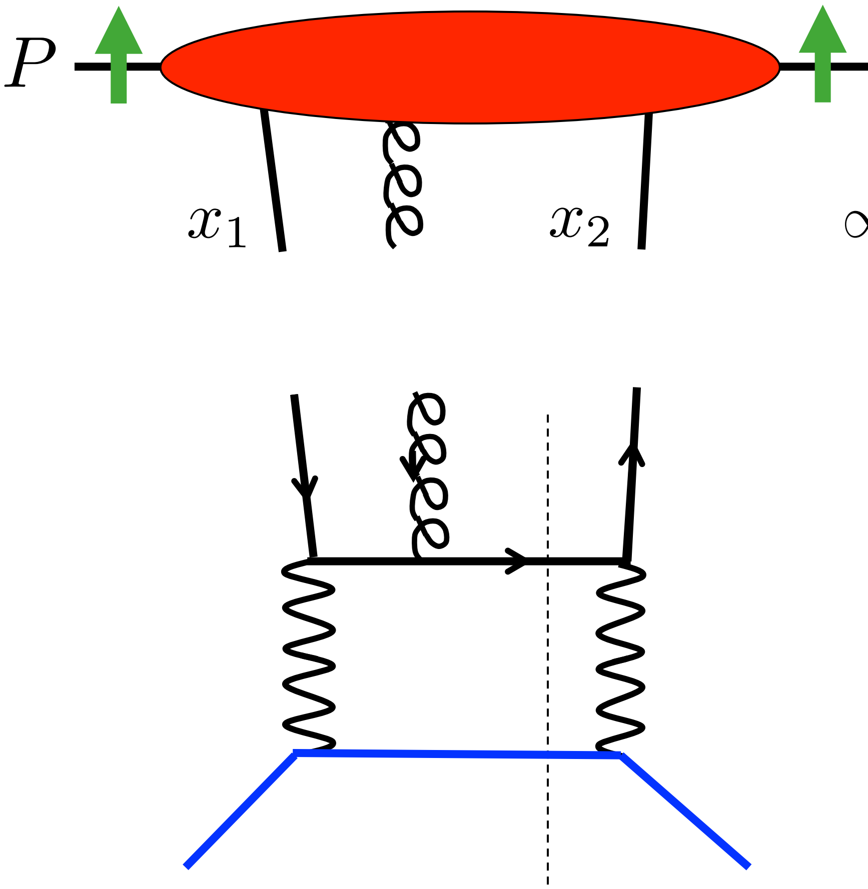


- phase in hard-scattering
- (also fragmentation, not considered here)


P
x_1
x_2

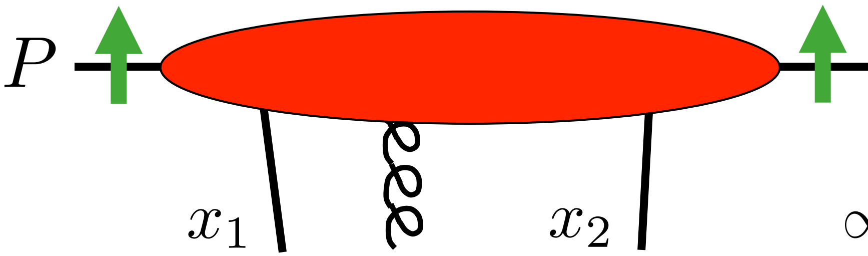


P
x_1
x_2



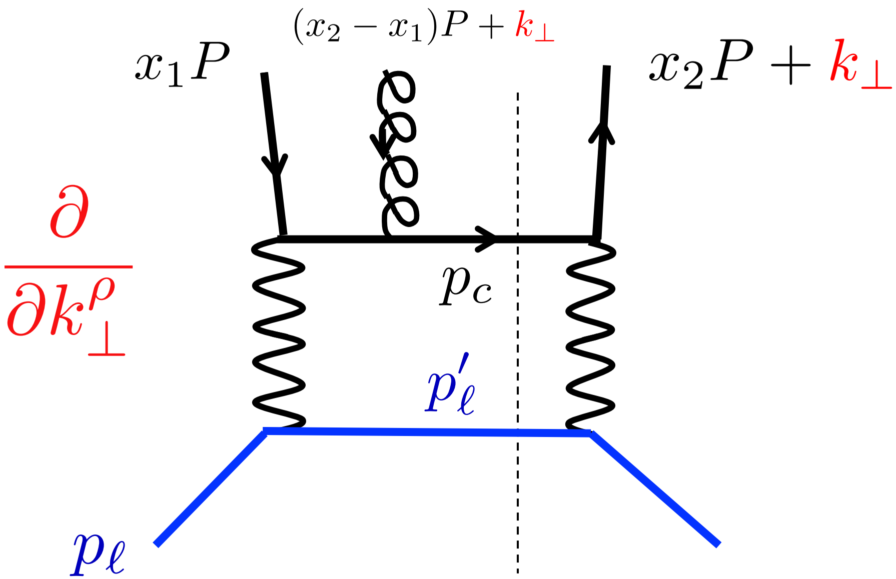
$$G_F(x_1, x_2)$$

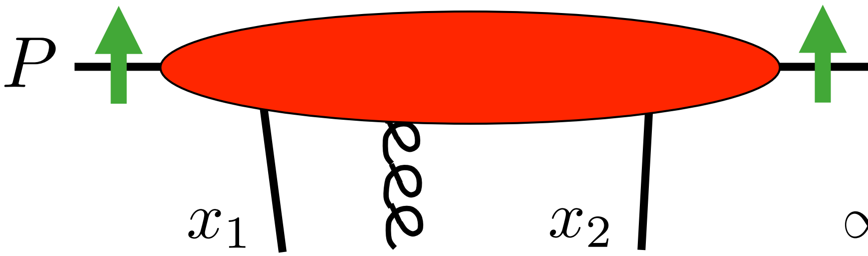
$$\propto \epsilon^{S_\perp \alpha n \bar{n}}\, \mathcal{F}\mathcal{T}\Big[\langle P, S|\, \overline{\psi}\, \gamma^+ F_\alpha{}^+\, \psi\, |P, S\rangle \Big]$$



$$G_F(x_1, x_2)$$

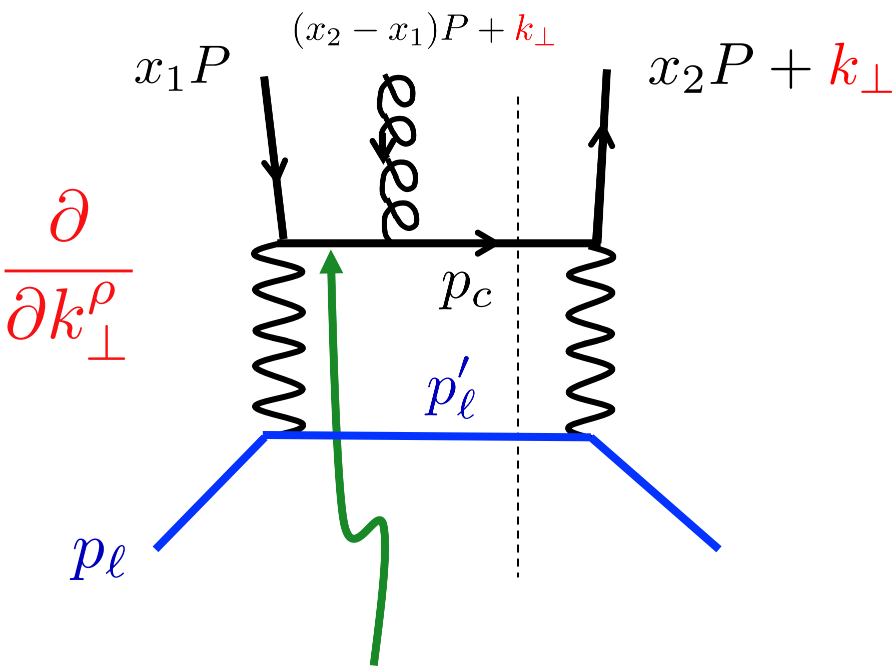
$$\propto\ \epsilon^{S_\perp \alpha n \bar{n}}\, \mathcal{F}\mathcal{T}\Big[\,\langle P, S|\, \overline{\psi}\, \gamma^+\, F_\alpha{}^+\, \psi\, |P, S\rangle\,\Big]$$





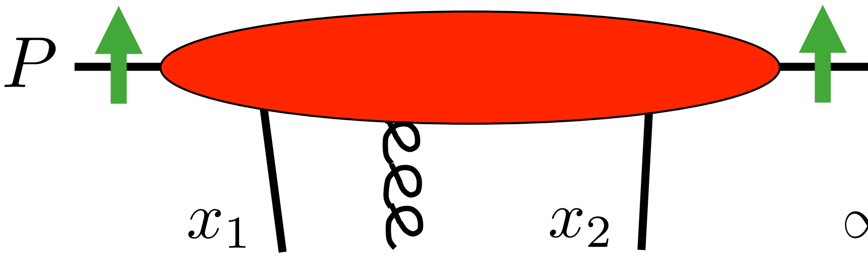
$$G_F(x_1, x_2)$$

$$\propto \epsilon^{S_\perp \alpha n \bar{n}} \, \mathcal{FT}\Big[\langle P, S | \, \overline{\psi} \, \gamma^+ \, F_\alpha{}^+ \, \psi \, | P, S \rangle \Big]$$



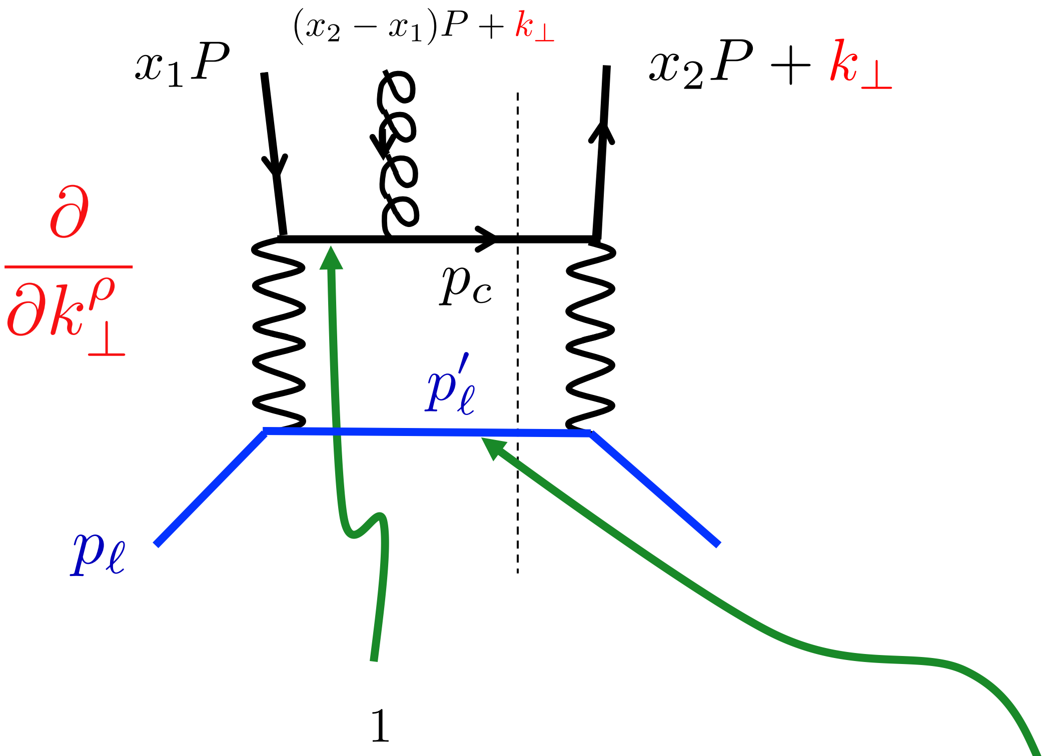
$$\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 + i\varepsilon}$$

$$\to \; -\frac{i\pi}{t} \, \delta \left(x_2 - x_1 - \frac{2 p_c \cdot k_\perp}{t} \right)$$



$$G_F(x_1, x_2)$$

$$\propto \epsilon^{S_\perp \alpha n \bar{n}} \, \mathcal{FT}\Big[\langle P, S | \, \overline{\psi} \, \gamma^+ \, F_\alpha{}^+ \, \psi \, | P, S \rangle \Big]$$

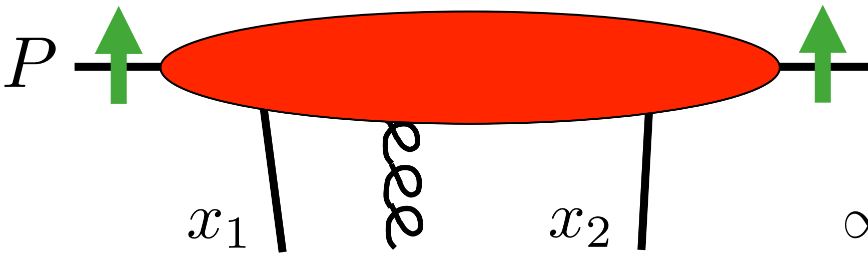


$$\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 + i\varepsilon}$$

$$\to \; -\frac{i\pi}{t} \, \delta\left(x_2 - x_1 - \frac{2 p_c \cdot k_\perp}{t} \right)$$

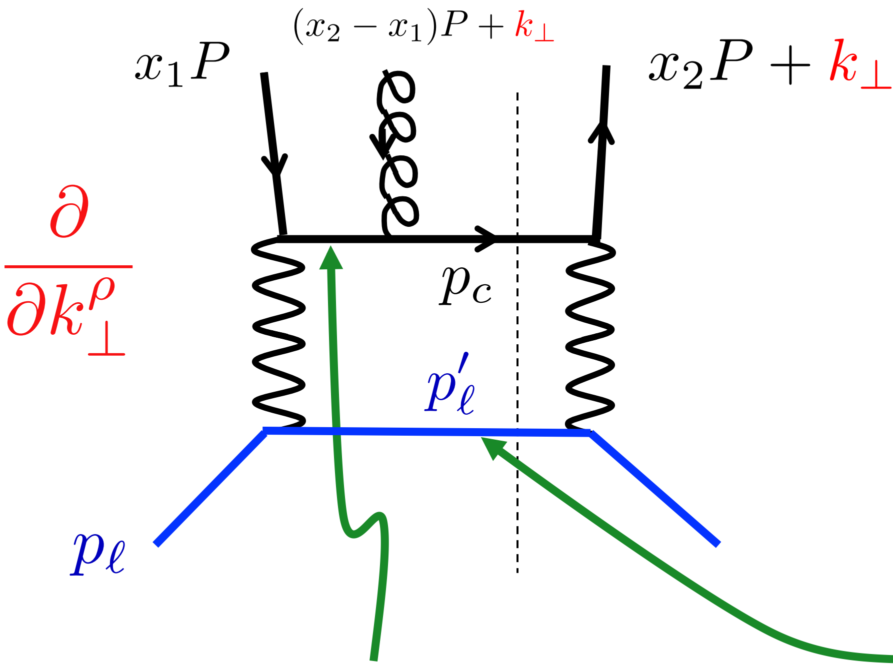
$$\delta\Big((p'_\ell)^2 \Big)$$

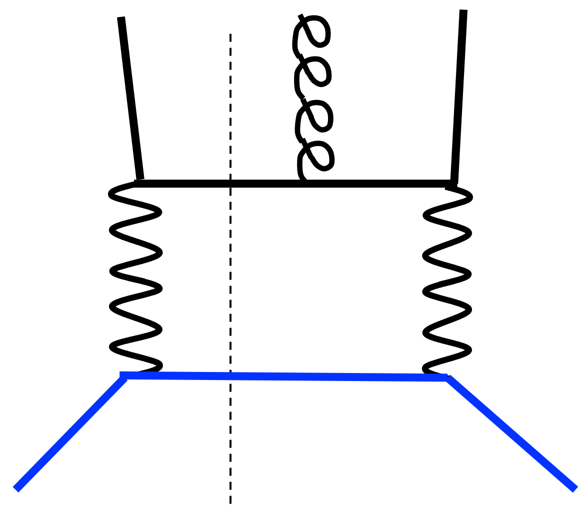
$$\propto \; \delta\left(x_2 - \frac{-u}{s+t} - \frac{2 p_c \cdot k_\perp}{s+t} \right)$$



$$G_F(x_1, x_2)$$

$$\propto \epsilon^{S_\perp \alpha n \bar{n}} \, \mathcal{FT}\Big[\langle P, S | \, \overline{\psi} \, \gamma^+ \, F_\alpha{}^+ \, \psi \, | P, S \rangle \Big]$$





$$\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 + i\varepsilon}$$

$$\to \; -\frac{i\pi}{t} \, \delta\left(x_2 - x_1 - \frac{2 p_c \cdot k_\perp}{t} \right)$$

$$\delta\Big((p'_\ell)^2 \Big)$$

$$\propto \; \delta\left(x_2 - \frac{-u}{s+t} - \frac{2 p_c \cdot k_\perp}{s+t} \right)$$

easiest way of combining:

$$\lim_{k_\perp \to 0} \frac{\partial}{\partial k_\perp^\rho} \int dx_1 \int dx_2 \, T_{a,F}(x_1, x_2) \Big[H_L(x_1, x_2, k_\perp) \, \delta(x_1 - x_2 + v_1 \cdot k_\perp) \delta(x_2 - x - v_2 \cdot k_\perp)$$

$$- H_R(x_1, x_2, k_\perp) \, \delta(x_1 - x_2 + v_1 \cdot k_\perp) \delta(x_1 - x) \Big]$$

easiest way of combining:

$$\lim_{k_\perp \to 0} \frac{\partial}{\partial k_\perp^\rho} \int dx_1 \int dx_2 \, T_{a,F}(x_1, x_2) \Big[H_L(x_1, x_2, k_\perp) \, \delta(x_1 - x_2 + v_1 \cdot k_\perp) \delta(x_2 - x - v_2 \cdot k_\perp)$$

$$- H_R(x_1, x_2, k_\perp) \, \delta(x_1 - x_2 + v_1 \cdot k_\perp) \delta(x_1 - x) \Big]$$

$$= (v_2 - v_1)_\rho \, H_L(x, x, 0) \, \frac{dT_{a,F}(x, x)}{dx}$$

$$+ T_{a,F}(x, x) \lim_{k_\perp \to 0} \frac{\partial}{\partial k_\perp^\rho} \Big[H_L(x + (v_2 - v_1) \cdot k_\perp, x + v_2 \cdot k_\perp, k_\perp) - H_R(x, x + v_1 \cdot k_\perp, k_\perp) \Big]_{k_\perp = 0}$$

eventually: LO formula

Kang, Metz, Qiu, Zhou

$$\frac{E_h d^3 \Delta\sigma^{p\ell\to hX}}{d^3 P_h} = \frac{\epsilon^{S_\perp p_c n \bar{n}}}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} \, D_c^h(z,\mu)$$

$$\times \left[\qquad\qquad -x \frac{d}{dx} G_{F,a}(x,x) \right]$$

$$\times \frac{s}{tu} \frac{d^2 \Delta\hat{\sigma}_{\mathrm{LO}}^{a\ell\to cX}}{v\,dv} \delta(1-w)$$

eventually: LO formula

Kang, Metz, Qiu, Zhou

$$\frac{E_h d^3 \Delta\sigma^{p\ell\to hX}}{d^3 P_h} = \frac{\epsilon^{S_\perp p_c n \bar{n}}}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} D_c^h(z,\mu)$$

$$\times \left[\qquad -x \frac{d}{dx} G_{F,a}(x,x) \right]$$

$$\times \frac{s}{tu} \frac{d^2 \Delta\hat{\sigma}_{\rm LO}^{a\ell\to cX}}{v\, dv} \delta(1-w)$$

"Soft-Gluon pole"

Qiu, Sterman '98

eventually: LO formula

Kang, Metz, Qiu, Zhou

$$\frac{E_h d^3 \Delta\sigma^{p\ell \to hX}}{d^3 P_h} = \frac{\epsilon^{S_\perp p_c n \bar{n}}}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} D_c^h(z,\mu)$$

$$\times \left[G_{F,a}(x,x) - x \frac{d}{dx} G_{F,a}(x,x) \right]$$

$$\times \frac{s}{tu} \frac{d^2 \Delta\hat{\sigma}_{\mathrm{LO}}^{a\ell \to cX}}{v\, dv} \delta(1-w)$$

"Soft-Gluon pole"

Qiu, Sterman '98

Kouvaris, Qiu, WV, Yuan '06

eventually: LO formula

Kang, Metz, Qiu, Zhou

$$\frac{E_h d^3 \Delta\sigma^{p\ell\to hX}}{d^3 P_h} = \frac{\epsilon^{S_\perp p_c n \bar{n}}}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} D_c^h(z,\mu)$$

$$\times \left[G_{F,a}(x,x) - x \frac{d}{dx} G_{F,a}(x,x) \right]$$

$$\times \frac{s}{tu} \frac{d^2 \Delta\hat{\sigma}_{\mathrm{LO}}^{a\ell\to cX}}{v\, dv} \delta(1-w)$$

"Soft-Gluon pole"

Qiu, Sterman '98

Kouvaris, Qiu, WV, Yuan '06

(rigorous proof for massless single-inclusive: Koike, Tanaka '07)

NLO corrections to single-spin observables:

- ★ Drell-Yan weighted asymmetry — Yuan, WV; Chen, Ma, Zhang
- ★ SIDIS weighted asymmetry — Kang, Vitev, Xing; Yoshida

NLO corrections to single-spin observables:

★ Drell-Yan weighted asymmetry — Yuan, WV; Chen, Ma, Zhang

★ SIDIS weighted asymmetry — Kang, Vitev, Xing; Yoshida

do not have $x \frac{d}{dx} G_F(x,x)$ at LO

NLO corrections to single-spin observables:

★ Drell-Yan weighted asymmetry — Yuan, WV; Chen, Ma, Zhang

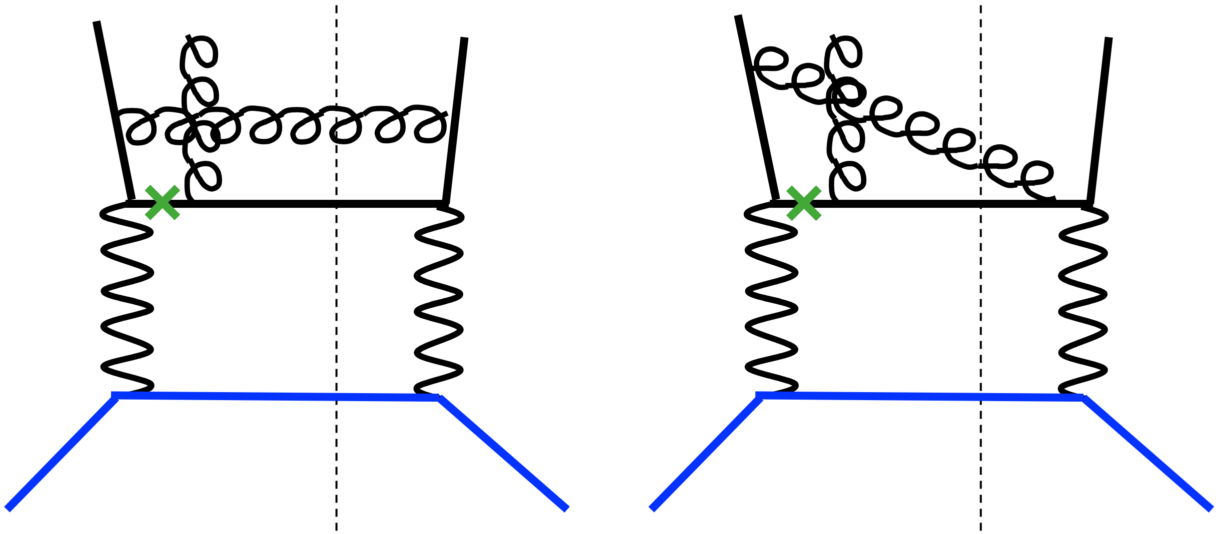
★ SIDIS weighted asymmetry — Kang, Vitev, Xing; Yoshida

do not have $x \frac{d}{dx} G_F(x,x)$ at LO

(close connections to NLO in twist-4 transverse-momentum broadening) — Kang, Wang, Wang, Xing; Kang, Qiu, Wang, Xing

Some features of NLO calculation for $p^{\uparrow}\ell \to hX$:

★ coupling to fragmenting quark line (SGP):

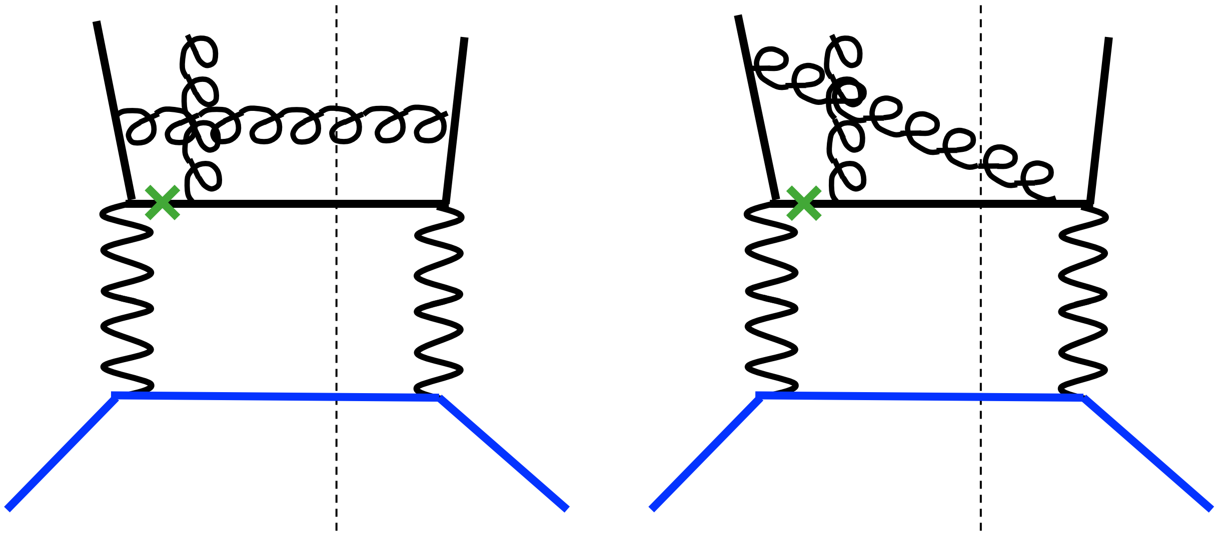


$+ \ldots$

$\propto C_F - \frac{C_A}{2}$

Some features of NLO calculation for $p^{\uparrow}\ell \to hX$:

★ coupling to fragmenting quark line (SGP):



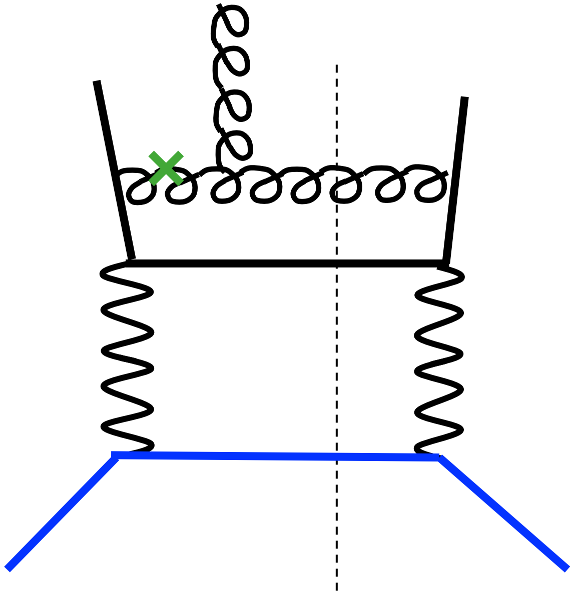
$+ \ldots$

$\propto C_F - \frac{C_A}{2}$

contributions $G_F(x,x)$ and $x\,\frac{d}{dx}G_F(x,x)$

with distributions $\left(\frac{\ln(1-w)}{1-w}\right)_+$ etc.

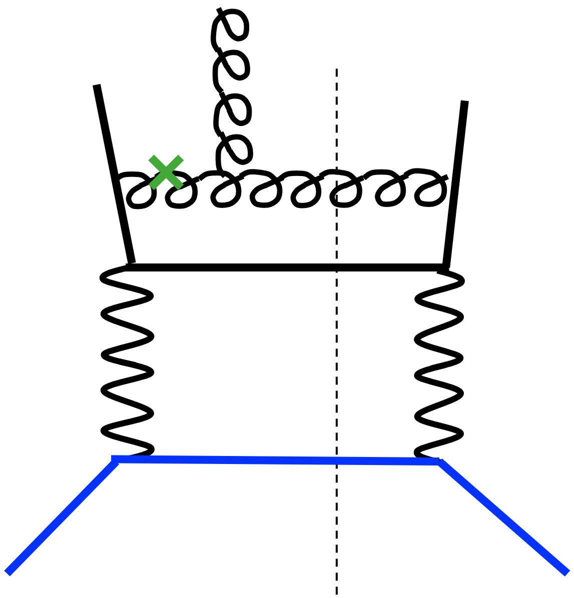
★ coupling to non-fragmenting gluon (SGP):



$+ \ldots$

$\propto \; C_A$

★ coupling to non-fragmenting gluon (SGP):

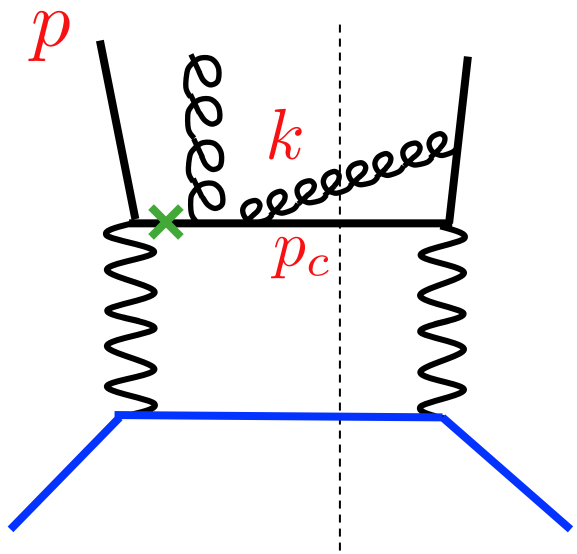


$+ \ldots$

$\propto \; C_A$

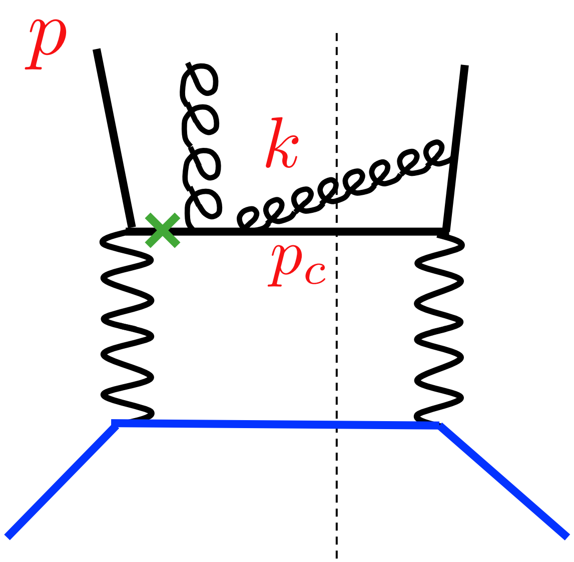
even contributes double poles and
derivative piece $x \, \frac{d}{dx} G_F(x,x)$

★ hard-pole contributions:



$+ \cdots$

★ hard-pole contributions:

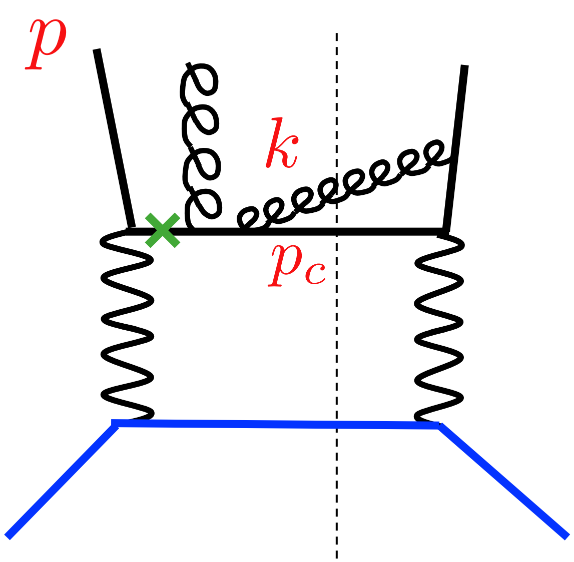


$+ \ldots$

makes x_1, x_2 kinematics-dependent,

$$G_F\Big(x, x(1-\tilde{x})\Big)$$

★ hard-pole contributions:



$+ \ldots$

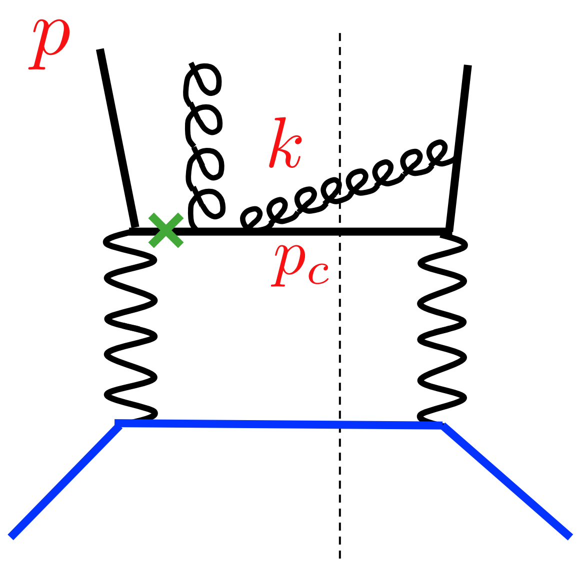
makes x_1, x_2 kinematics-dependent,

$$G_F\Big(x, x(1-\tilde{x})\Big)$$

where

$$\tilde{x} = -\frac{(p_c + k)^2}{(p_c - p)^2 + (k - p)^2}$$

★ hard-pole contributions:



$+ \ldots$

makes x_1, x_2 kinematics-dependent,

$$G_F\Big(x, x(1-\tilde{x})\Big)$$

subject to phase space integration

where

$$\tilde{x} = -\frac{(p_c + k)^2}{(p_c - p)^2 + (k - p)^2}$$

$$\mathcal{HP} = \int d\Phi_3 \, G_F(x, x(1-\tilde{x})) \, \mathcal{M}_{\mathrm{HP}}(p, p_c, k)$$

$$\mathcal{HP} \; = \; \int d\Phi_3 \, G_F(x, x(1-\tilde{x})) \, \mathcal{M}_{\text{HP}}(p, p_c, k)$$

$$= \int d\Phi_3 \, \left[G_F(x,x) - \tilde{x} \, \frac{1}{2} (x G'_F(x,x)) + \sum_{n=2}^{\infty} \frac{(-1)^n \tilde{x}^n}{n!} \left(x^n \, \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \right] \mathcal{M}_{\text{HP}}(p, p_c, k)$$

$$\mathcal{HP} = \int d\Phi_3 \, G_F(x, x(1-\tilde{x})) \, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \int d\Phi_3 \left[G_F(x,x) - \tilde{x}\,\frac{1}{2}(x G'_F(x,x)) + \sum_{n=2}^{\infty} \frac{(-1)^n \tilde{x}^n}{n!} \left(x^n \, \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \right] \mathcal{M}_{\rm HP}(p, p_c, k)$$

combine with SGP pieces

$$\mathcal{HP} = \int d\Phi_3 \, G_F(x, x(1-\tilde{x})) \, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \int d\Phi_3 \left[G_F(x,x) - \tilde{x}\,\frac{1}{2}(xG'_F(x,x)) + \sum_{n=2}^{\infty} \frac{(-1)^n \tilde{x}^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \right] \mathcal{M}_{\rm HP}(p, p_c, k)$$

combine with SGP pieces

now consider

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \int d\Phi_3 \, \tilde{x}^n \, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$\mathcal{HP} = \int d\Phi_3\, G_F(x, x(1-\tilde{x}))\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \int d\Phi_3 \left[\underbrace{G_F(x,x) - \tilde{x}\,\frac{1}{2}(xG'_F(x,x))}_{\text{combine with SGP pieces}} + \sum_{n=2}^{\infty} \frac{(-1)^n \tilde{x}^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \right] \mathcal{M}_{\rm HP}(p, p_c, k)$$

combine with SGP pieces

now consider

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \int d\Phi_3\, \tilde{x}^n\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)!} \left(x^{n+2} \frac{d^{n+2} G_F(x,y)}{dy^{n+2}} \right)_{y=x} \int d\Phi_3\, \tilde{x}^{n+2}\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$\mathcal{HP} = \int d\Phi_3\, G_F(x, x(1-\tilde{x}))\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \int d\Phi_3 \left[G_F(x,x) - \tilde{x}\,\frac{1}{2}(xG'_F(x,x)) + \sum_{n=2}^{\infty} \frac{(-1)^n \tilde{x}^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \right] \mathcal{M}_{\rm HP}(p, p_c, k)$$

combine with SGP pieces

now consider

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \left(x^n \frac{d^n G_F(x,y)}{dy^n} \right)_{y=x} \int d\Phi_3\, \tilde{x}^n\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)!} \left(x^{n+2} \frac{d^{n+2} G_F(x,y)}{dy^{n+2}} \right)_{y=x} \int d\Phi_3\, \tilde{x}^{n+2}\, \mathcal{M}_{\rm HP}(p, p_c, k)$$

integral less singular

obtain

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)!} \left(x^{n+2} \, \frac{d^{n+2} G_F(x,y)}{dy^{n+2}} \right)_{y=x} \int dPS_3 \, \tilde{x}^{n+2} \, \mathcal{M}_{\mathrm{HP}}(p, p_c, k) \, .$$

obtain

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)!} \left(x^{n+2} \frac{d^{n+2} G_F(x,y)}{dy^{n+2}} \right)_{y=x} \int dPS_3 \, \tilde{x}^{n+2} \, \mathcal{M}_{\mathrm{HP}}(p, p_c, k) \, .$$

$$= \frac{1}{\epsilon} f(v,w) \left(G_F(x,0) + \frac{1}{2} (x G'_F(x,x)) - G_F(x,x) \right)$$

$$+ \frac{1}{\epsilon} g_1(v,w) \left(\frac{G_F(x,xw) - G_F(x,x)}{1-w} + \frac{1}{2} (x G'_F(x,x)) \right)$$

$$+ \frac{1}{\epsilon} g_2(v,w) \frac{\partial}{\partial w} \left(\frac{G_F(x,xw) - G_F(x,x)}{1-w} \right)$$

+ finite

obtain

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)!} \left(x^{n+2} \frac{d^{n+2} G_F(x,y)}{dy^{n+2}} \right)_{y=x} \int dPS_3 \, \tilde{x}^{n+2} \, \mathcal{M}_{\rm HP}(p, p_c, k) \, .$$

$$= \frac{1}{\epsilon} f(v,w) \left(G_F(x,0) + \frac{1}{2} (x G'_F(x,x)) - G_F(x,x) \right)$$

$$+ \frac{1}{\epsilon} g_1(v,w) \left(\frac{G_F(x,xw) - G_F(x,x)}{1-w} + \frac{1}{2} (x G'_F(x,x)) \right)$$

$$+ \frac{1}{\epsilon} g_2(v,w) \frac{\partial}{\partial w} \left(\frac{G_F(x,xw) - G_F(x,x)}{1-w} \right)$$

+ finite

★ now have all $1/\epsilon$ pole terms from 2→3 part

★ factorization of collinear singularities:

$$\frac{\partial}{\partial \log \mu^2} G_F(x,x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\Bigg\{ G_F(z,z,\mu^2)\, P_{qq}(\hat{x}) - N_c\, \delta(1-\hat{x})\, G_F(z,z,\mu^2)$$

$$+ \frac{N_c}{2}\left[\frac{1+\hat{x}}{(1-\hat{x})_+} G_F(z,x,\mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+} G_F(z,z,\mu^2)\right]\Bigg\}$$

$$\hat{x} = \frac{x}{z}$$

Braun, Manashov, Pirnay; Kang, Qiu; Yuan, WV; Ma, Wang; Yoshida

★ factorization of collinear singularities:

$$\frac{\partial}{\partial \log \mu^2} G_F(x,x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\Bigg\{ G_F(z,z,\mu^2)\, P_{qq}(\hat{x}) - N_c\,\delta(1-\hat{x})\, G_F(z,z,\mu^2)$$

$$\hat{x} = \frac{x}{z} \qquad + \frac{N_c}{2}\left[\frac{1+\hat{x}}{(1-\hat{x})_+} G_F(z,x,\mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+} G_F(z,z,\mu^2)\right]\Bigg\}$$

Braun, Manashov, Pirnay; Kang, Qiu; Yuan, WV; Ma, Wang; Yoshida

★ need $\frac{\partial}{\partial \log \mu^2}\big(G_F(x,x) - x G'_F(x,x)\big)$

★ factorization of collinear singularities:

$$\frac{\partial}{\partial \log \mu^2} G_F(x,x,\mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \Big\{ G_F(z,z,\mu^2)\, P_{qq}(\hat{x}) - N_c\, \delta(1-\hat{x})\, G_F(z,z,\mu^2)$$

$$+ \frac{N_c}{2} \left[\frac{1+\hat{x}}{(1-\hat{x})_+} G_F(z,x,\mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+} G_F(z,z,\mu^2) \right] \Big\}$$

$$\hat{x} = \frac{x}{z}$$

Braun, Manashov, Pirnay; Kang, Qiu; Yuan, WV; Ma, Wang; Yoshida

★ need $\frac{\partial}{\partial \log \mu^2} \left(G_F(x,x) - x G_F'(x,x) \right)$

★ use

$$x \frac{d}{dx} \left(\int_x^1 \frac{dy}{y} f(y)\, G_F\left(\frac{x}{y}, \frac{x}{y}\right) \right) = -f(x) \underbrace{G_F(1,1)}_{=0} + \int_x^1 \frac{dy}{y} f(y)\, x \frac{d}{dx} G_F\left(\frac{x}{y}, \frac{x}{y}\right)$$

$$= \int_x^1 \frac{dy}{y} f(y) \left(x G_F' \right)\Big|_{x=x/z}$$

★ find:

$$
\begin{aligned}
\frac{\partial}{\partial \log \mu^2}\Big(G_F(x,x) - xG'_F(x,x)\Big) \;=\; & \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\Bigg\{ \Big(G_F(z,z) - zG'_F(z,z)\Big)\, P_{qq}(\hat{x}) \\
& - \frac{N_c}{2}\,\delta(1-\hat{x})\,\Big(G_F(z,z) - zG'_F(z,z)\Big) \\
& - \frac{N_c}{2}\,\frac{G_F(z,z) - G_F(z,x)}{1-\hat{x}} \\
& + \frac{N_c}{2}\,(1+\hat{x})\,x\frac{d}{dx}\left(\frac{G_F(z,z) - G_F(z,x)}{1-\hat{x}}\right)\Bigg\}
\end{aligned}
$$

★ cancelation of singularities for $w \neq 1$ after substantial amount of integration by parts

$$g(w)(xG'_F(x,x)) \ \rightarrow \ \left[-g(1)\,\delta(1-w) + \frac{1}{w}(w^2 g(w))'\right] G_F(x,x)$$

★ cancelation of singularities for $w \neq 1$ after substantial amount of integration by parts

$$g(w)(xG'_F(x,x)) \;\to\; \left[-g(1)\,\delta(1-w) + \frac{1}{w}(w^2 g(w))'\right] G_F(x,x)$$

★ leading terms:

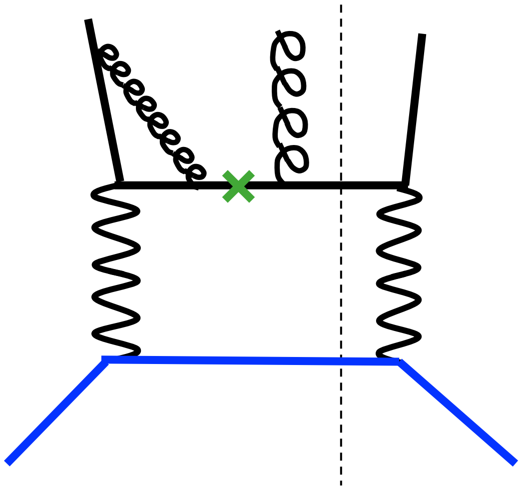
$$\frac{d^2\hat{\sigma}^{q\ell qX}_{\mathrm{NLO}}}{v\,dvdw} = \frac{\alpha^2 e_q^2 C_F}{sv}\left[8\,\frac{1+v^2}{(1-v)^2}\left(\frac{\log(1-w)}{1-w}\right)_+ + \frac{A(v)}{(1-w)_+} + \ldots\right]$$

$$\frac{d^2\Delta\hat{\sigma}^{q\ell qX}_{\mathrm{NLO}}}{v\,dvdw} = \frac{\alpha^2 e_q^2 C_F}{sv}\left[8\,\frac{1+v^2}{(1-v)^2}\left(\frac{\log(1-w)}{1-w}\right)_+ + \frac{\tilde{A}(v)}{(1-w)_+} + \ldots\right]$$

★ final step: virtual corrections $\propto \delta(1-w)$

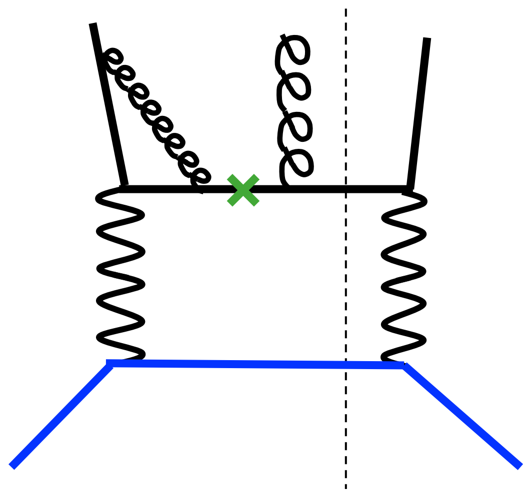
★ final step: virtual corrections $\propto \delta(1-w)$

straightforward pieces:



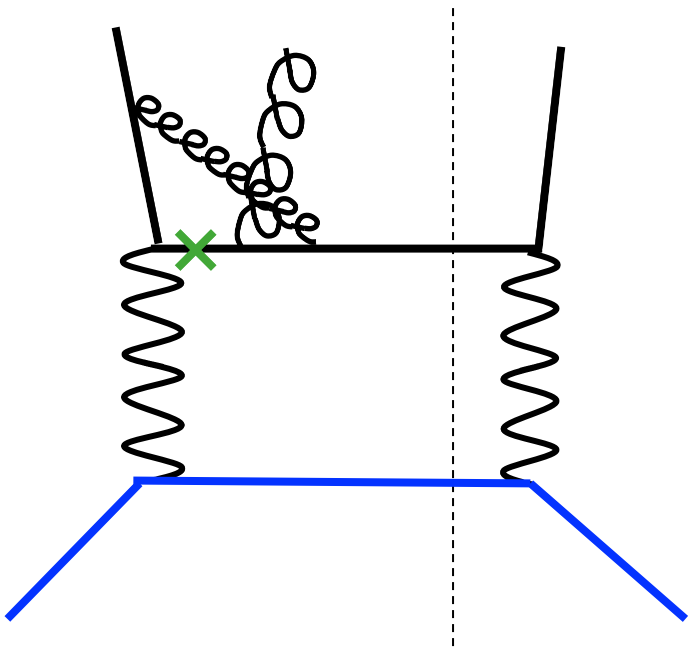
★ final step: virtual corrections $\propto \delta(1-w)$

straightforward pieces:

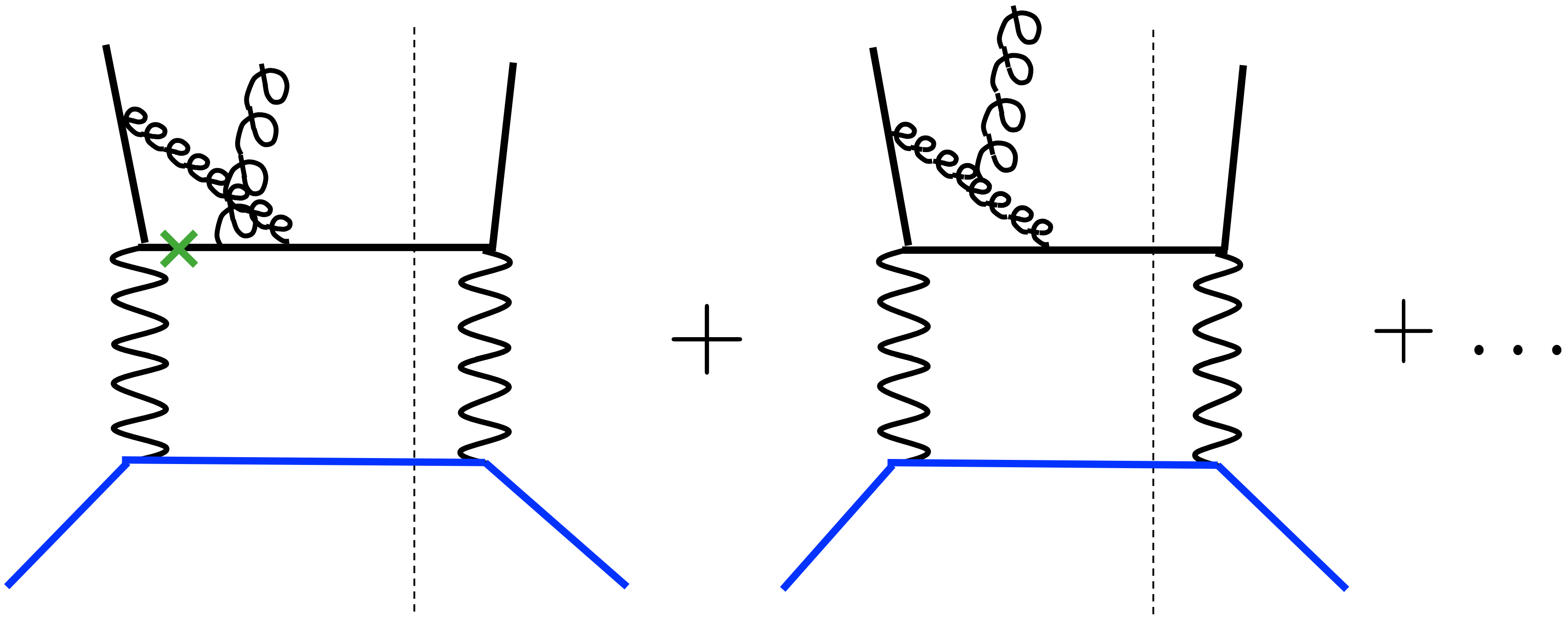


★ do not have LO structure $G_F(x,x) - xG_F'(x,x)$

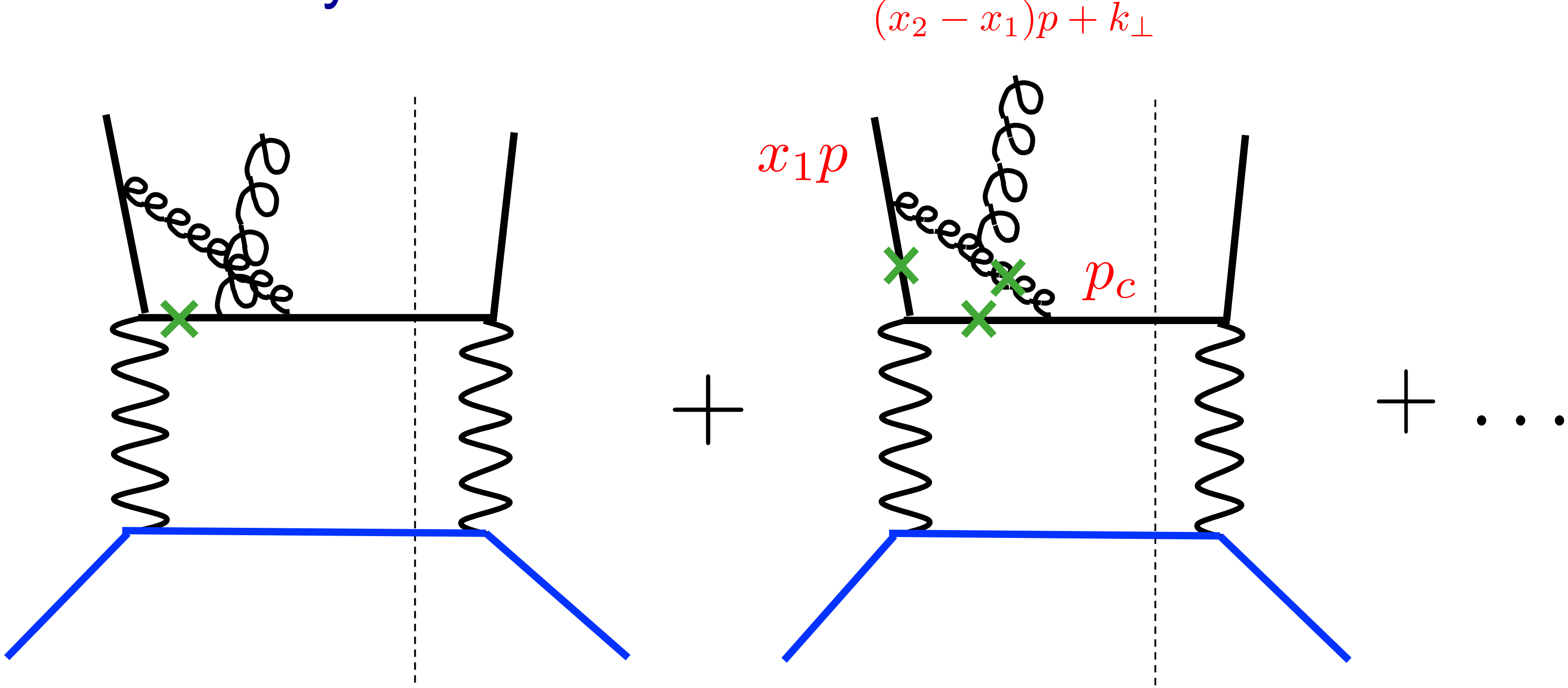
★ more tricky:



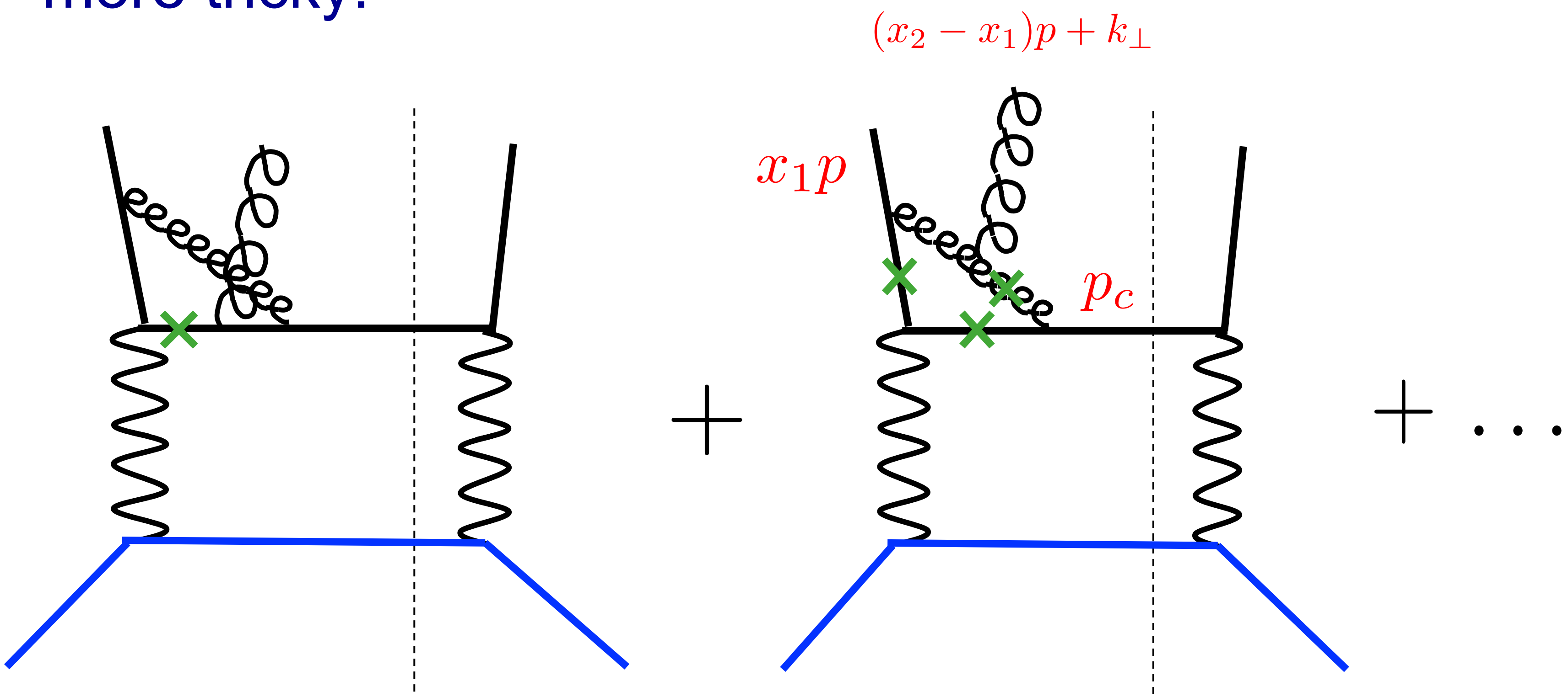
★ more tricky:



★ more tricky:



★ more tricky:



★ not clear whether works out. Problems especially in C_A part

Conclusions:

- much interest in single-inclusive processes

  $\ell p \to h\,X \qquad \ell p \to \text{jet}\,X$

- NLO QCD corrections sizable

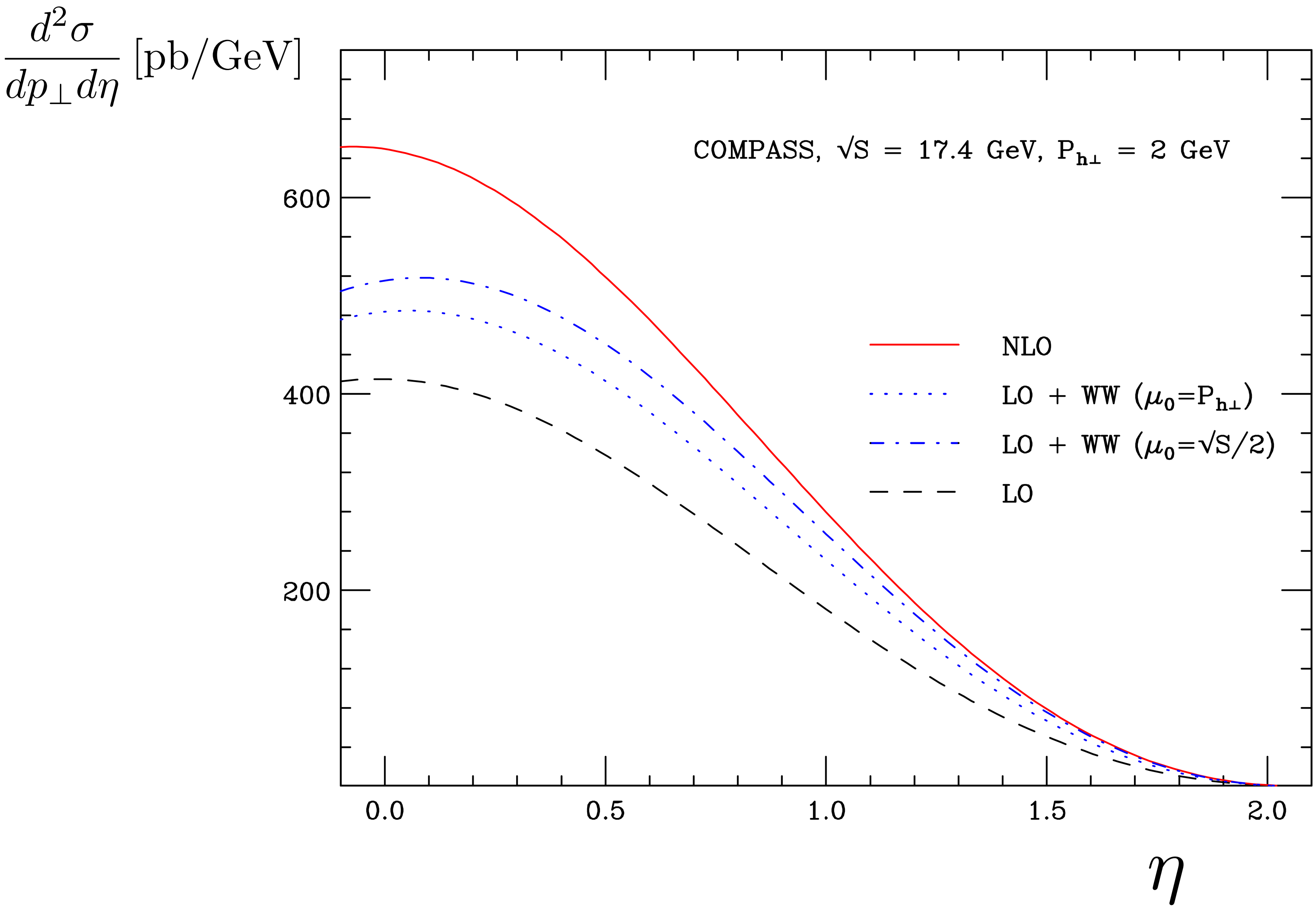
- A_N : $\ell p \to h\,X$ as template for $pp \to hX$

- NLO probably doable, but tedious

- numerical work will be major challenge

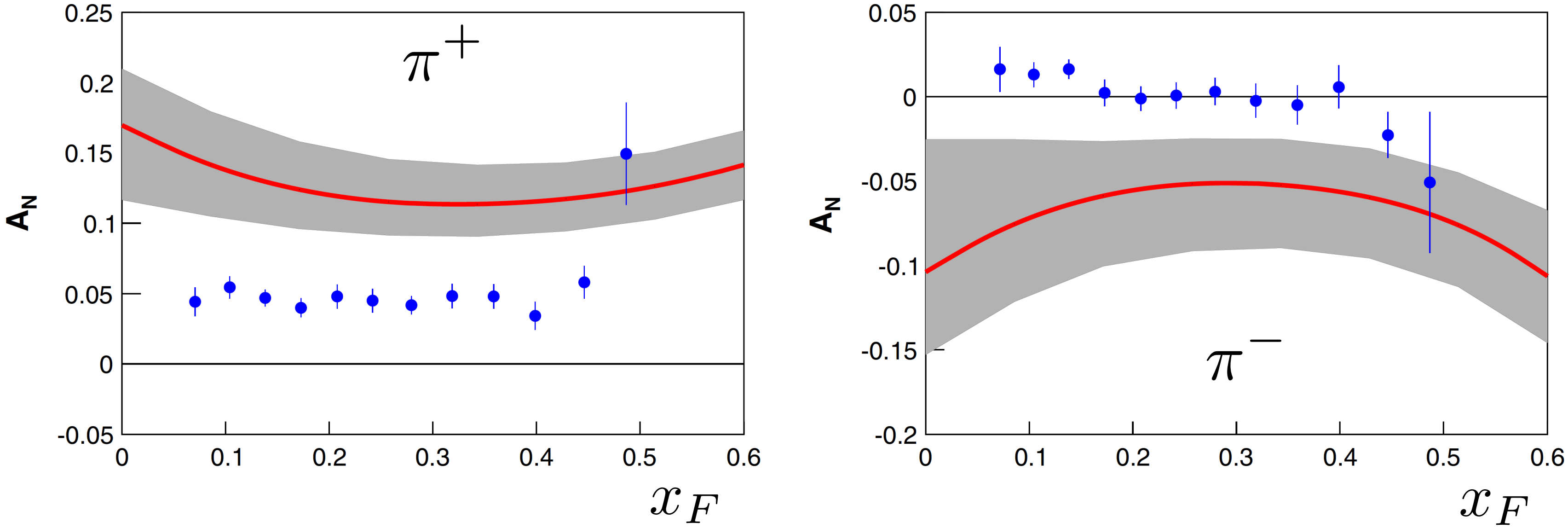
- plus: SFP, gluonic correlation fcts., fragmentation,…

$$\mu p \to \pi^0 X$$ COMPASS



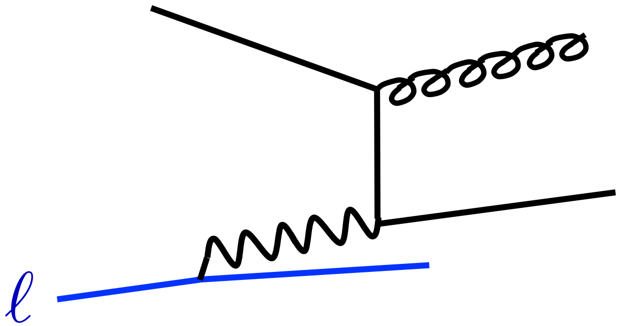
$p^{\uparrow}\ell \to \pi^{\pm}X$

Gamberg, Kang, Metz, Pitonyak, Prokudin



HERMES data $\sqrt{S} = 7.25\ \mathrm{GeV},\ p_\perp \sim 1\ \mathrm{GeV}$

New feature at NLO:



- almost real photon (Weizsäcker-Williams)
- enhanced as m_ℓ small

- structure is

$$f^{\ell\to\gamma} \otimes \hat{\sigma}^{\gamma q\to qg}$$

- if dominant, could give “easy access” to NLO for single-spin

- computed in two ways:

  ★ keep lepton massive, but drop terms that vanish as $m_\ell \to 0$

  ★ compute with massless lepton, subtract collinear singularity in $\overline{\text{MS}}$.
  Define “photon parton distribution” in lepton:

$$f_{\text{bare}}^{\ell\to\gamma}(y,\mu) = \frac{\alpha}{2\pi}\, P_{\gamma\ell}(y)\left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{y^2 m_\ell^2}\right) - 1\right]$$

- both results identical and lead to

$$\frac{E_h d^3\sigma^{p\ell\to hX}}{d^3P_h} = \frac{1}{\pi S}\sum_{a,c}\int\frac{dx}{x}\int\frac{dz}{z^2}\, f_a(x,\mu)\, D_c^h(z,\mu)\left[\frac{d^2\hat{\sigma}_{\text{LO}}^{a\ell\to cX}}{v\,dv\,dw} + \frac{\alpha_s}{\pi}\,\frac{d^2\hat{\sigma}_{\text{NLO}}^{a\ell\to cX}(\mu_0)}{v\,dv\,dw}\right.$$

$$\left. + f^{\ell\to\gamma}(z_\gamma,\mu_0)\,\frac{\alpha_s}{\pi}\,\frac{d^2\hat{\sigma}_{\text{LO}}^{a\gamma\to cX}}{v\,dv\,dw}\right]$$