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Radiative Processes and Jet Modification at the EIC

*INT program 18-3, Workshop week 4
EIC Symposium "Probing Nucleons and Nuclei in High Energy Collisions"
INT, Seattle WA, October 22-26, 2018*

Outline of the talk

- New interest in particle and jet production in SIDIS
- Status of in-medium calculations in SIDIS, what is missing
- In-medium radiative corrections calculations
- New techniques, for particle and jet production in nuclei
- Event shapes at EIC
- Conclusions

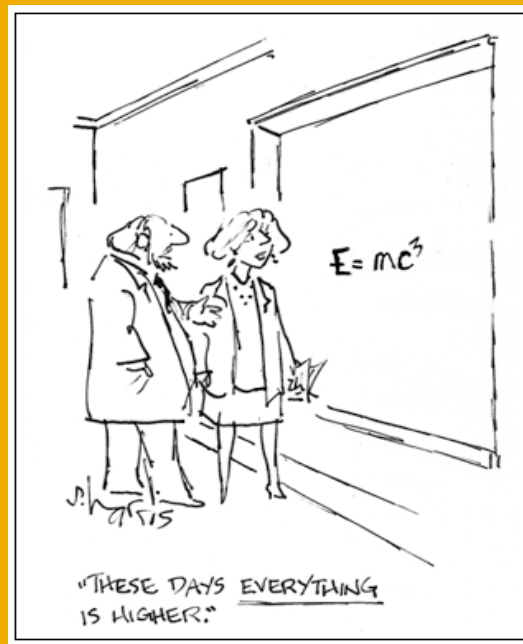


Thanks to the organizers for the opportunity to discuss this physics

Much of the credit for this work goes to my collaborators:

Z. Kang, F. Ringer, [M. Sievert](#), [B. Yoon](#)

Introduction



A new era of high-energy nuclear physics at the EIC

LRP recommendations

- We recommend a high-energy high-luminosity polarized EIC as the highest priority for new facility construction following the completion of FRIB

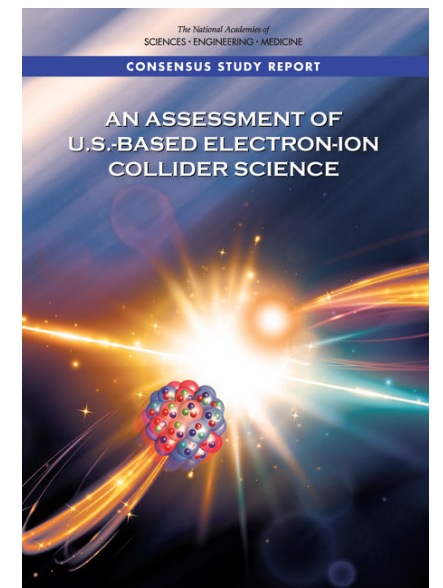
NAS: Extensive list of findings

- EIC essential for US leadership in nuclear physics and accelerator design
- Physics at EIC has very close connections to solid state and atomic physics, high energy physics, astrophysics and computing
- ...
- To realize fully the scientific opportunities an EIC would enable, a theory program will be required to predict and interpret the experimental results within the context of QCD and, furthermore, to glean the fundamental insights into QCD that an EIC can reveal.

Critical gaps in the EIC program

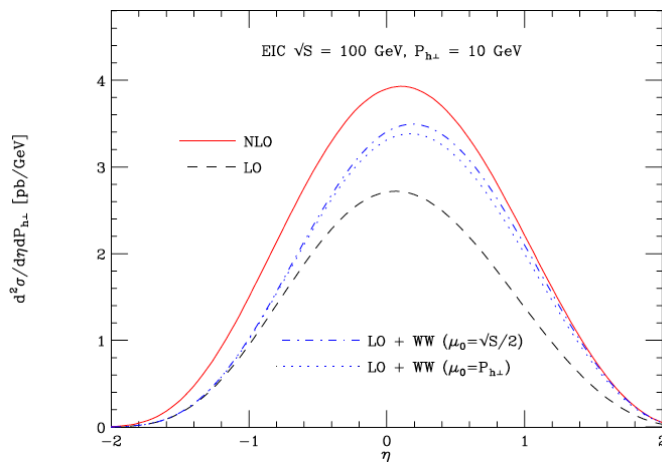


The 2015 LONG RANGE PLAN for NUCLEAR SCIENCE

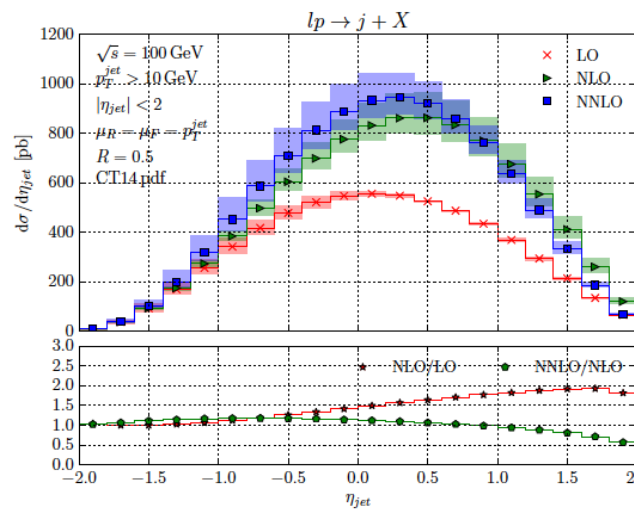


Hadron and jet production at the EIC

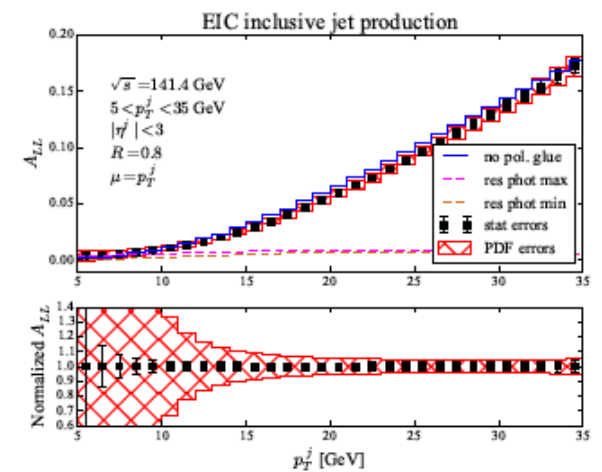
SIDIS has played a key role in pushing the boundaries of QCD, nucleon structure, the TMD approach, and QCD in reactions with nuclei



P. Hinderer et al. (2015)

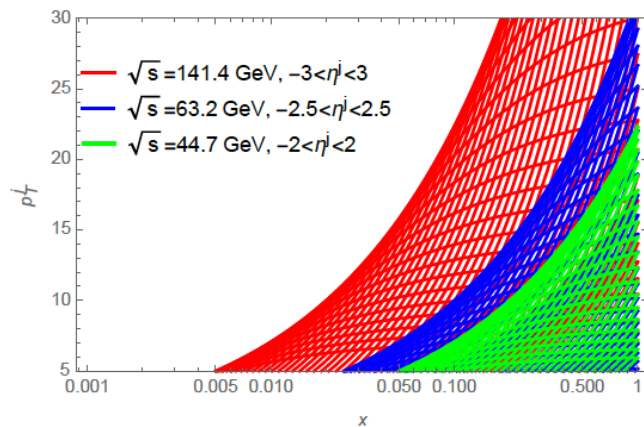


G. Abelf et al. (2016)



$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-} - d\sigma^{-+} + d\sigma^{--}}{d\sigma^{++} + d\sigma^{+-} + d\sigma^{-+} + d\sigma^{--}}$$

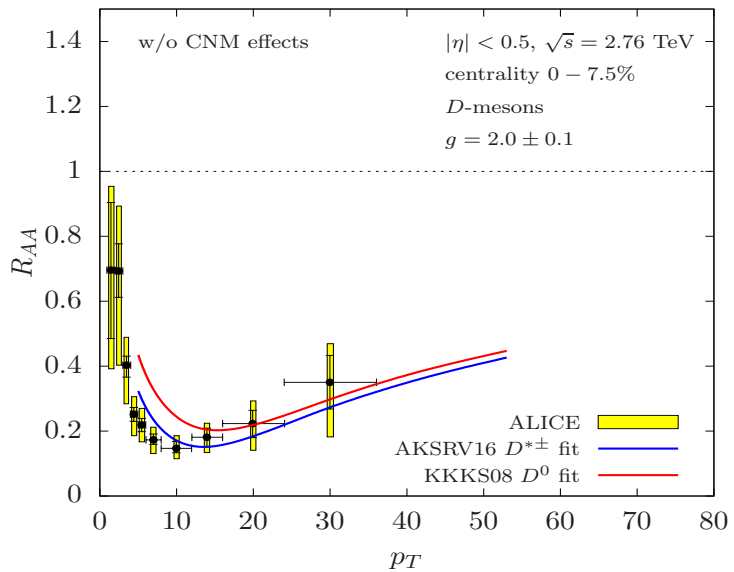
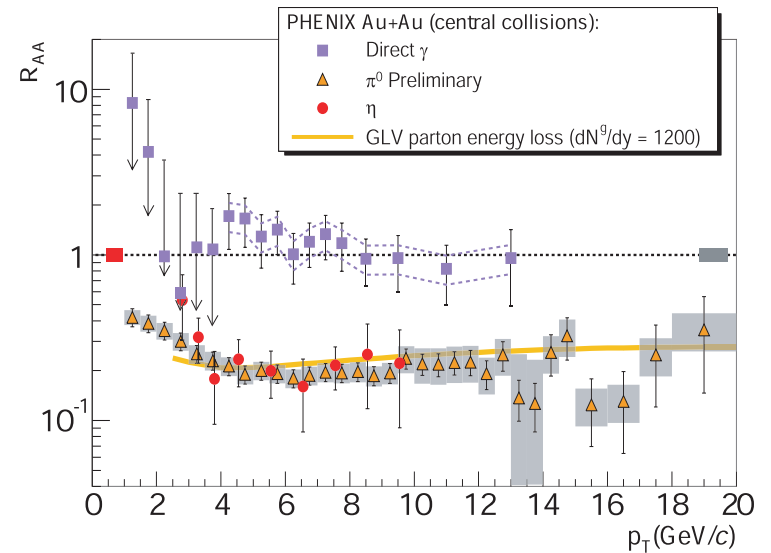
R. Boughezal et al. (2018)



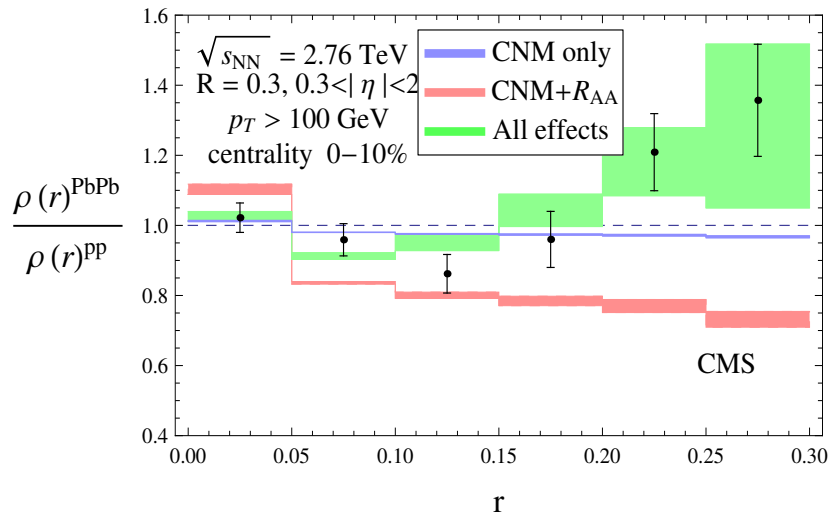
- There is a renewed interest in precision calculations of hadron and jet production at the EIC – wide range of applications

Modification of hadron and jet observables in nuclear matter

- In heavy ion collisions medium-modified parton showers are the cornerstone of high- p_T physics. These are the most significant effects and are not related to nPDFs and small- x physics



Heavy flavor suppression, b-jets, di-b jets

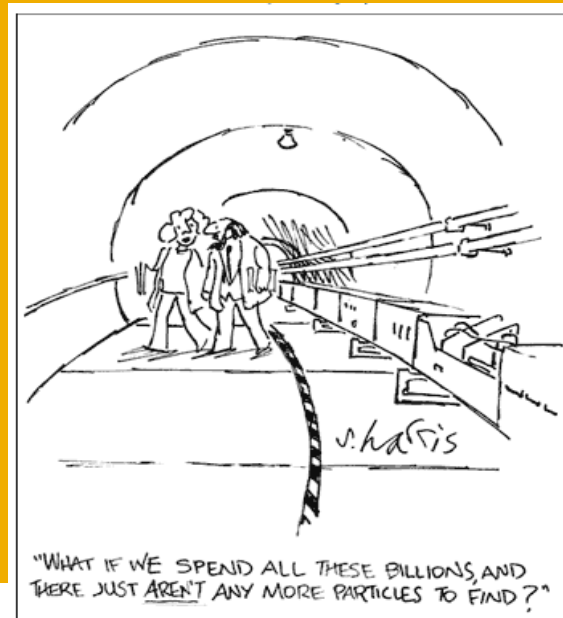


Jet substructure modification, shapes, frag. functions, splitting functions

Inclusive hadron suppression, hadron correlations

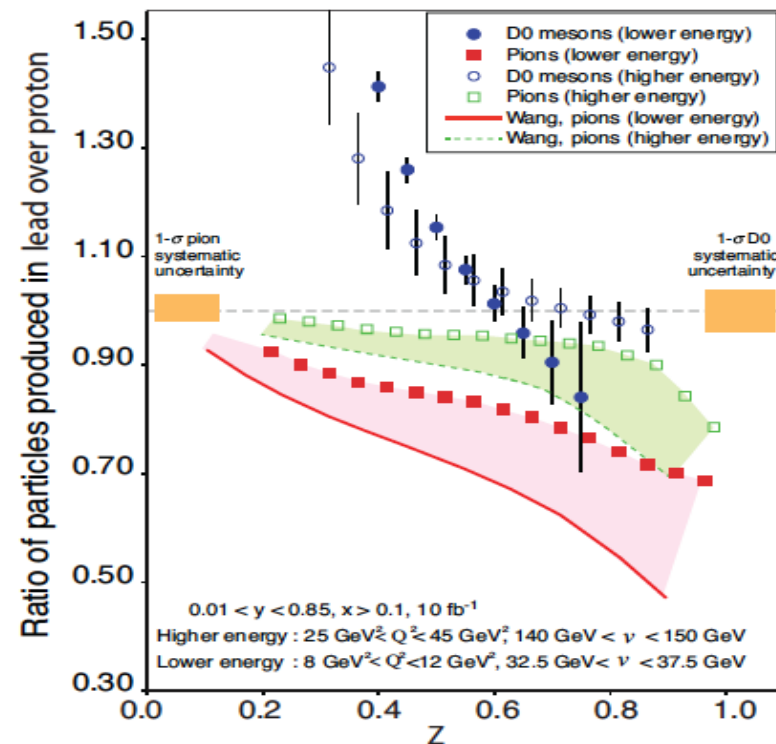
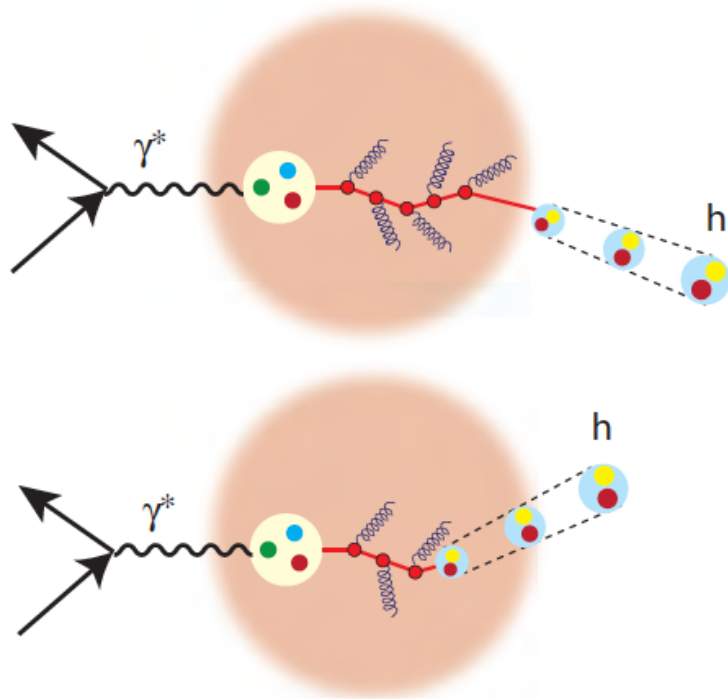
Jet suppression, enhanced dijet asymmetries, γ, Z^0 -tagged jets

Status of in-medium modification in DIS



Status of calculations of particles and jets in large nuclei

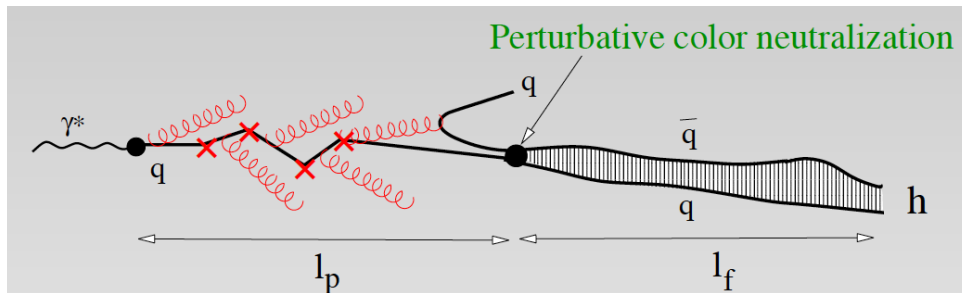
Jets and heavy flavor physics is **seriously underdeveloped**. Realized by EIC working group



- The two topics mentioned are: (1) the possibility of hadronization in nuclei and (2) energy loss in nuclear matter. Circa 2000 physics.

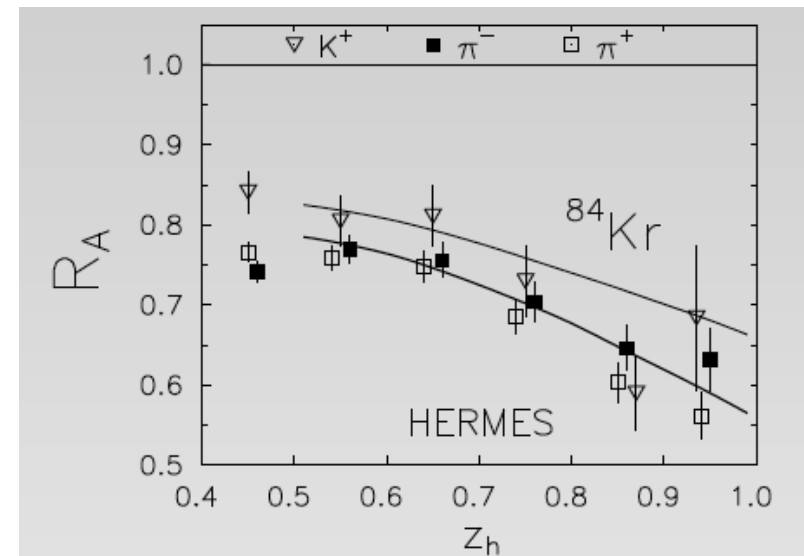
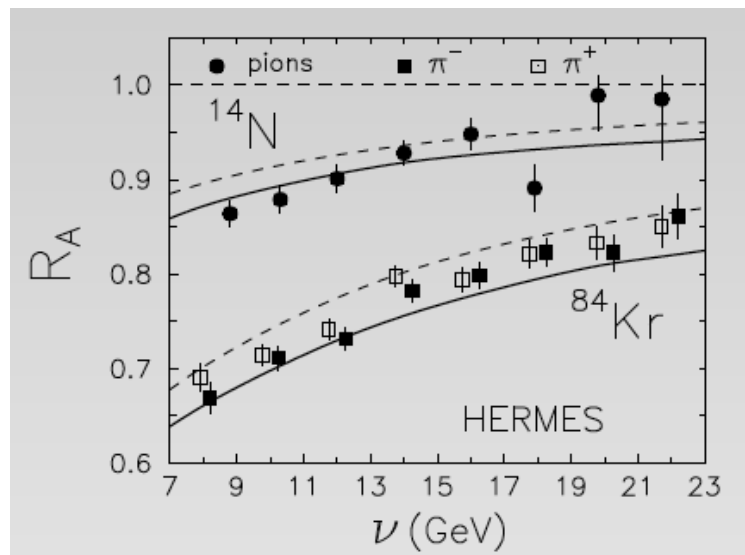
In-medium hadronization

A model based on pre-hadron and hadron formation times



B. Kopeliovich et al. (2003)

$$R_A^h(z, \nu) = \left(\frac{N^h(z, \nu)}{N^e(\nu)} \Big|_A \right) / \left(\frac{N^h(z, \nu)}{N^e(\nu)} \Big|_D \right)$$



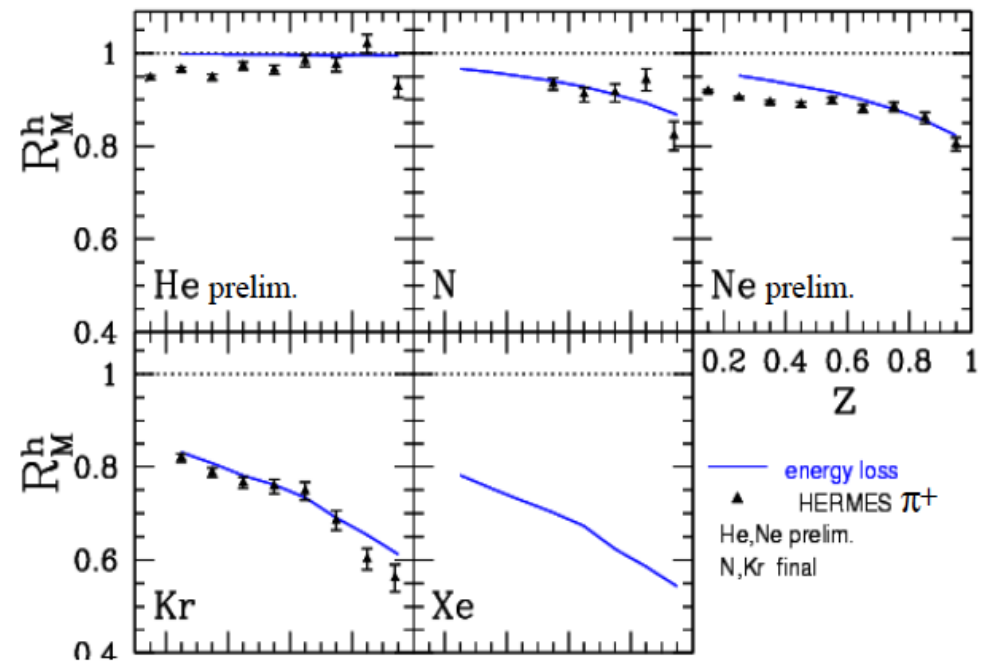
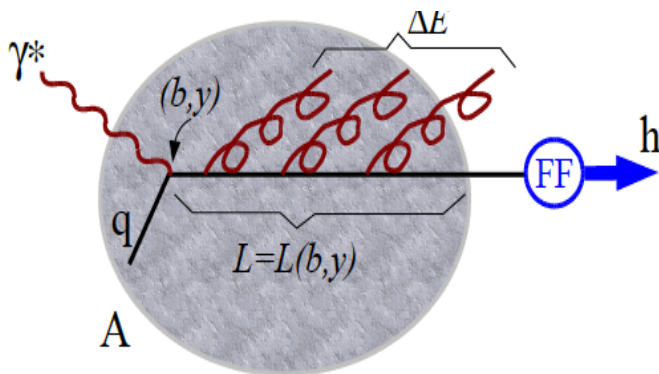
- Can provide some insight in the particle species dependence of the attenuation. Formation times might be underestimated

Semi-inclusive hadron suppression

- Energy loss-based approach compared to Hermes data

$$R_A^h(z, \nu) = \left(\frac{N^h(z, \nu)|_A}{N^e(\nu)} \right) / \left(\frac{N^h(z, \nu)|_D}{N^e(\nu)} \right)$$

$$= \left(\frac{\sum e_q^2 q(x) \tilde{D}_q^h(z)}{\sum e_q^2 q(x)} \right) \Big|_A / \left(\frac{\sum e_q^2 q(x) D_q^h(z)}{\sum e_q^2 q(x)} \right) \Big|_D$$



F. Arleo et al. (2003)

- \hat{q} obtained 0.7 GeV²/fm (very large, typical of infinite media approaches)

Hybrid approach to hadron attenuation at the EIC

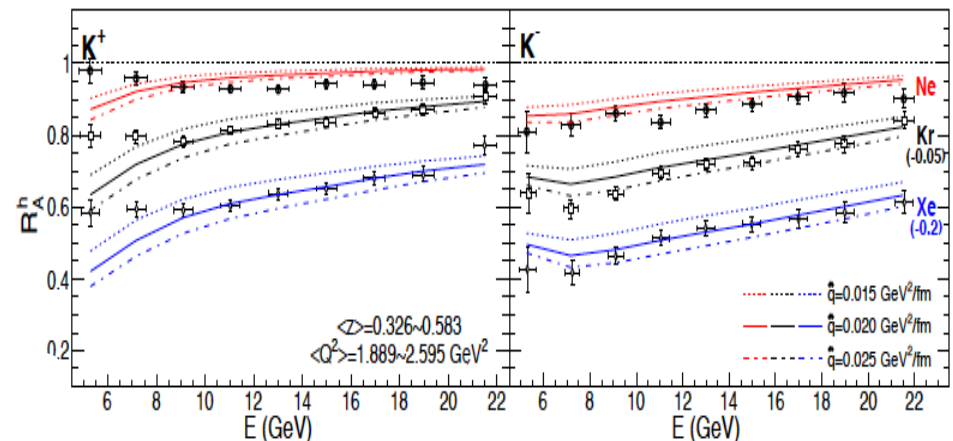
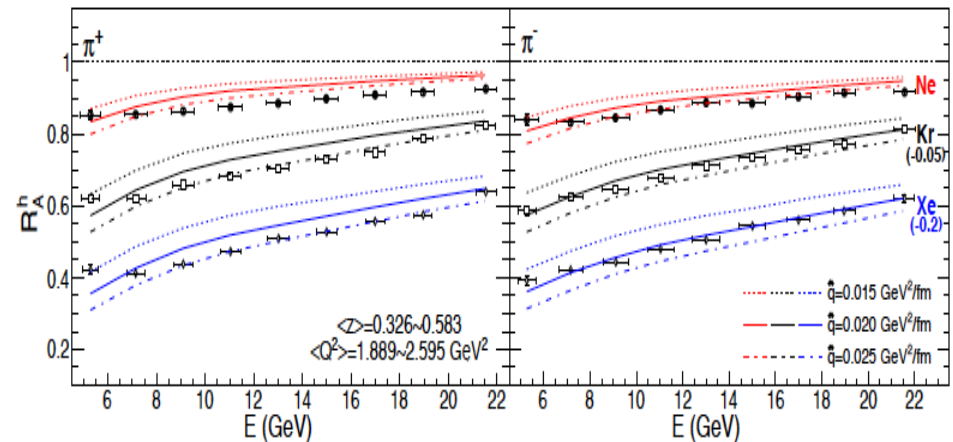
Using E-loss Calculations

N. Chang et al. (2014)

$$\tilde{D}_{h/g}(z, Q_0^2) = \int_0^1 d\epsilon P_g(\epsilon, Q_0^2) \frac{1}{1-\epsilon} D_{h/g}\left(\frac{z}{1-\epsilon}, Q_0^2\right) + \int_0^1 d\epsilon G^g(\epsilon, Q_0^2) \frac{1}{\epsilon} D_{h/g}\left(\frac{z}{\epsilon}, Q_0^2\right),$$

$$\tilde{D}_{h/q}(z, Q_0^2) = \int_0^1 d\epsilon P_q(\epsilon, Q_0^2) \frac{1}{1-\epsilon} D_{h/q}\left(\frac{z}{1-\epsilon}, Q_0^2\right) + \int_0^1 d\epsilon G^q(\epsilon, Q_0^2) \frac{1}{\epsilon} D_{h/q}\left(\frac{z}{\epsilon}, Q_0^2\right).$$

Energy loss modified functions used as initial conditions for evolution

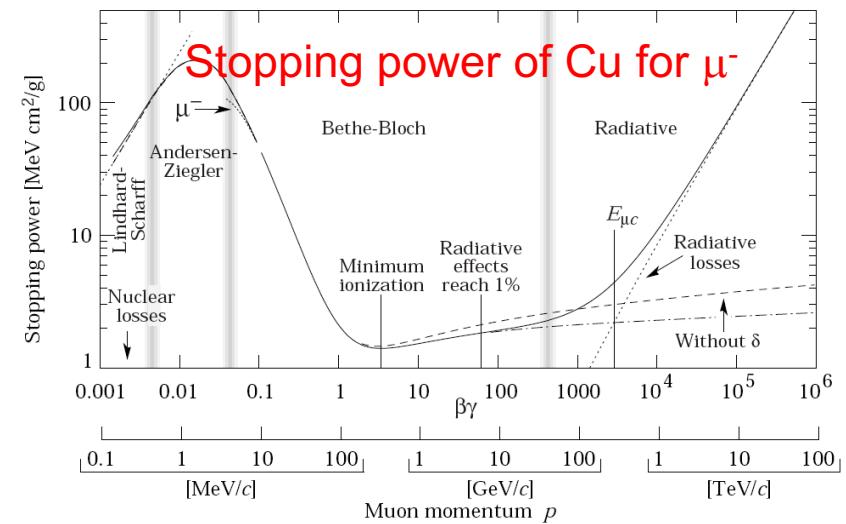


- A quite small $\hat{q} = 0.02 \text{ GeV}^2 / \text{fm}$. Again factor of 30 discrepancy in the transport properties of cold nuclear matter

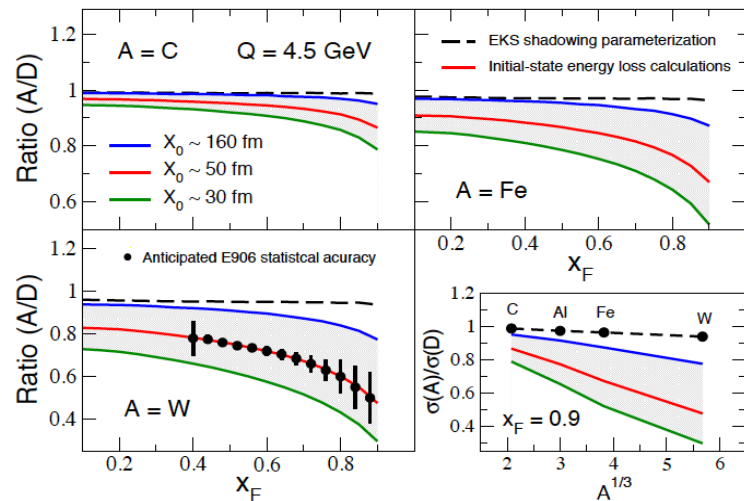
Missing opportunities

- **Serious unresolved discrepancies** in the extraction of transport properties of large nuclei. Especially in this area – circa 2000 physics

- Stopping power of matter for charged particles is a **fundamental probe of its properties**



PDG (2008)

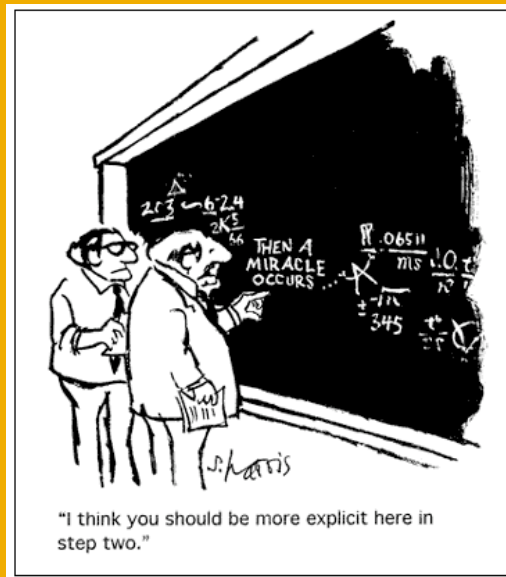


R. Neufeld et al. (2010)

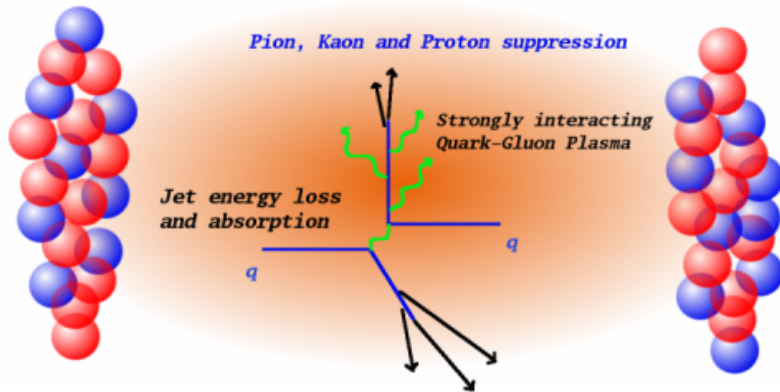
In QED $X_0(\text{min}) \sim \text{mm}$, in nuclei 10 orders of magnitude smaller!

- A whole class of new observables is **missing** – jets and jet substructure
- The measurements and theory can only be done at the EIC

Status of in-medium radiative corrections



Infinite medium approaches



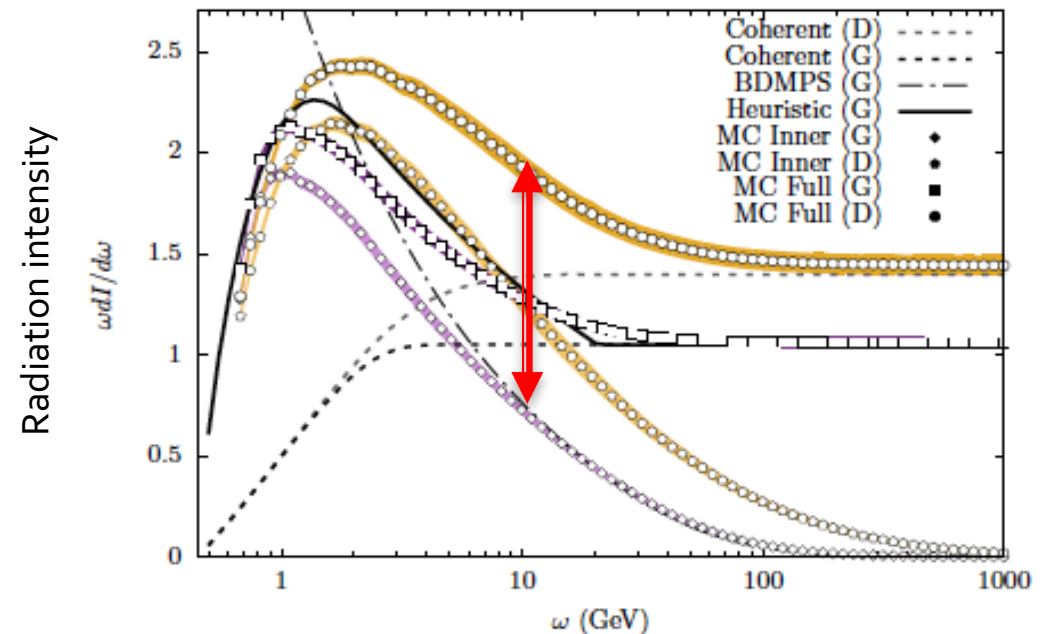
M. Gyulassy et al . (1993)

- The infinitely thick medium approaches – they simplify the problem to resum the interactions

Realistic QGP (or nuclei for that matter) - $q \sim 2.5$ scatterings, $g \sim 5$.

R. Baier et al . (1996)

- Known asymptotic limits are not recovered since initial and final state radiation is missed
- If phenomenologically applied, leads to an order of magnitude too large transport parameters for nuclear matter

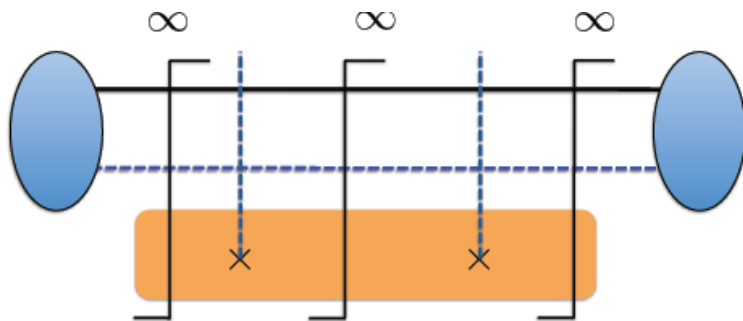


X. Feal et al . (2018)

What will be really useful to see if those diagrams can be added still in in some semi analytic fashion

Thin and finite media

- Finite opacity approaches – builds expansion in the correlation between the scattering centers (called opacity expansion)



Classes of diagrams (single Born, double Born). Reaction Operator

M. Gyulassy et al. (2000)

X. Guo et al. (2001)

I. Vitev (2007)

- Result for soft gluon emission to any order in opacity

$$x \frac{dN^{(n)}}{dx d^2\mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \frac{1}{n!} \left(\frac{L}{\lambda_g(1)} \right)^n \int \prod_{i=1}^n \left\{ d\mathbf{q}_i \left(\frac{\lambda_g(1)}{\lambda_g(i)} \right) (\bar{v}_i^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)) \right\} \text{Medium properties}$$

$$\times \left[-2 \mathbf{C}_{(1, \dots, n)} \cdot \sum_{m=1}^n \mathbf{B}_{(m+1, \dots, n)(m, \dots, n)} \left(\cos \left(\sum_{k=2}^m \omega_{(k, \dots, n)} \Delta z_k \right) - \cos \left(\sum_{k=1}^m \omega_{(k, \dots, n)} \Delta z_k \right) \right) \right]$$

propagators
Interference phases

We can: (1) study the expansion order by order; (2) look at media of finite and small lengths; (3) take various analytic limits and obtain the results

Full in medium splitting

- Full massless and massive in-medium splitting functions now available to first order in opacity

G. Ovanesyan et al . (2011)

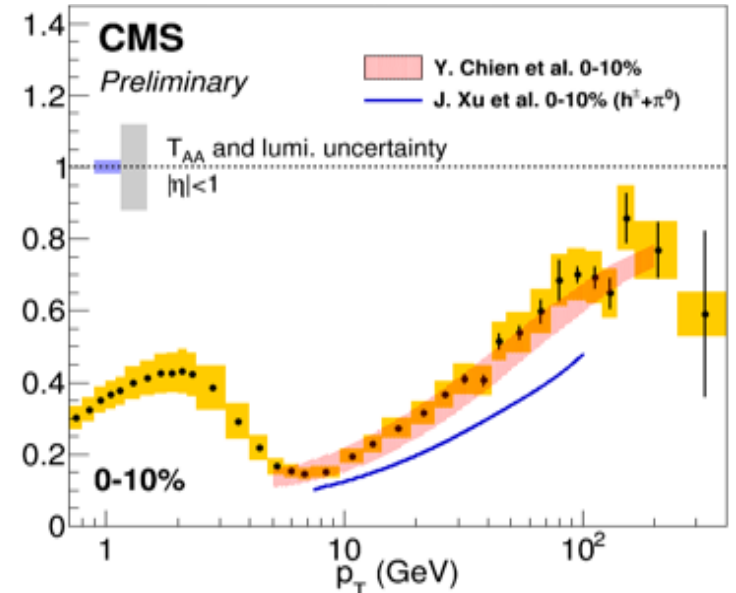
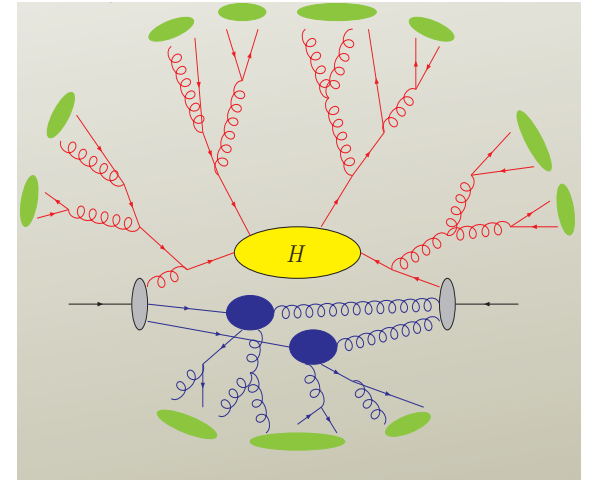
- SCET-based effective theories created to solve this problem

F. Ringer et al . (2016)

Representative example

$$\begin{aligned} \left(\frac{dN^{\text{med}}}{dx d^2k_{\perp}} \right)_{Q \rightarrow Qg} &= \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2q_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{med}}}{d^2q_{\perp}} \left\{ \left(\frac{1+(1-x)^2}{x} \right) \left[\frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right. \right. \\ &\times \left(\frac{B_{\perp}}{B_{\perp}^2 + \nu^2} - \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \cdot \left(2 \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \right. \\ &- \left. \left. \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) + \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \cdot \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) \right. \\ &+ \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \cdot \left(\frac{D_{\perp}}{D_{\perp}^2 + \nu^2} - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \right) (1 - \cos[\Omega_4\Delta z]) - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \cdot \frac{D_{\perp}}{D_{\perp}^2 + \nu^2} (1 - \cos[\Omega_5\Delta z]) \\ &+ \left. \left. \frac{1}{N_c^2} \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \cdot \left(\frac{A_{\perp}}{A_{\perp}^2 + \nu^2} - \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \right] \right. \\ &+ \left. x^3 m^2 \left[\frac{1}{B_{\perp}^2 + \nu^2} \cdot \left(\frac{1}{B_{\perp}^2 + \nu^2} - \frac{1}{C_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \dots \right] \right\} \end{aligned}$$

- For the first time we were able to do is higher order and resummed calculations



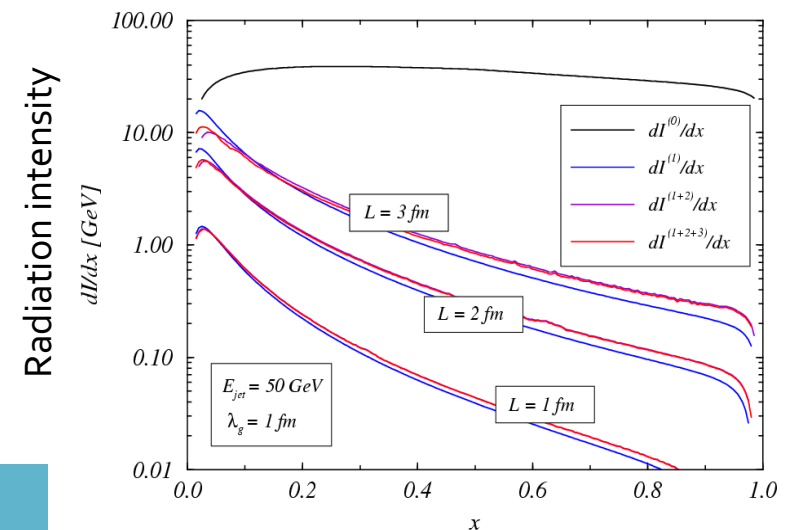
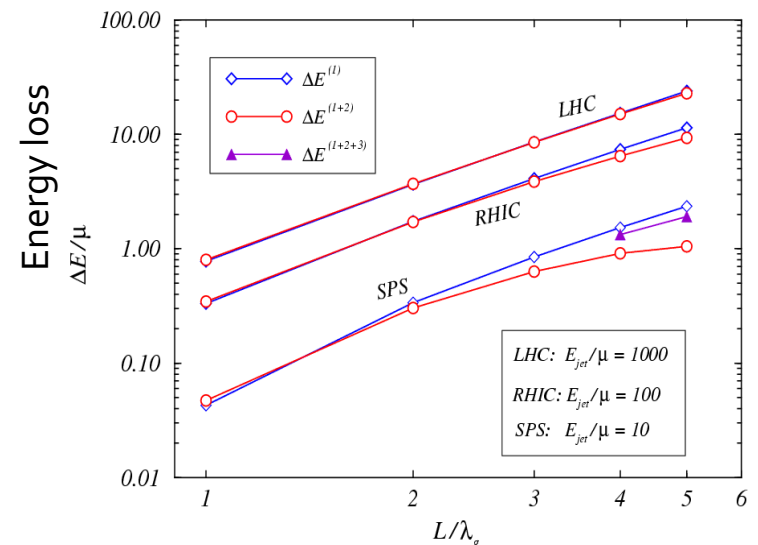
Z. Kang et al . (2015)

Higher order opacity corrections

- Practically all phenomenology is done to first order in opacity. **What are the higher order in opacity corrections?**
- The calculation was done using schematic geometry, no expansion. Qualitative guidance
- The opacity series converges quite fast for L/λ up to 5 - 6 scatterings. Converges faster at higher energies
- For integral quantities like energy loss the correction is as low as 10%. Hence the very good phenomenology
- For more differential quantities – intensity, angular spectra – 30 – 50 %

It was also done to higher orders in opacity (~9) in the soft gluon emission limit

M. Gyulassy et al. (2000)



S. Wicks (2009)

Theoretical framework

- The theoretical framework is completely general – it is applicable for both cold nuclear matter and the QGP.
- This is achieved by isolating the medium in transport parameters and universal gluon-mediated interactions

$$\mathcal{L}_{opac.} = \mathcal{L}_{QCD} + \mathcal{L}_{ext}^{qG} + \mathcal{L}_{ext}^{gG} + \mathcal{L}_{G.F.} + \dots$$

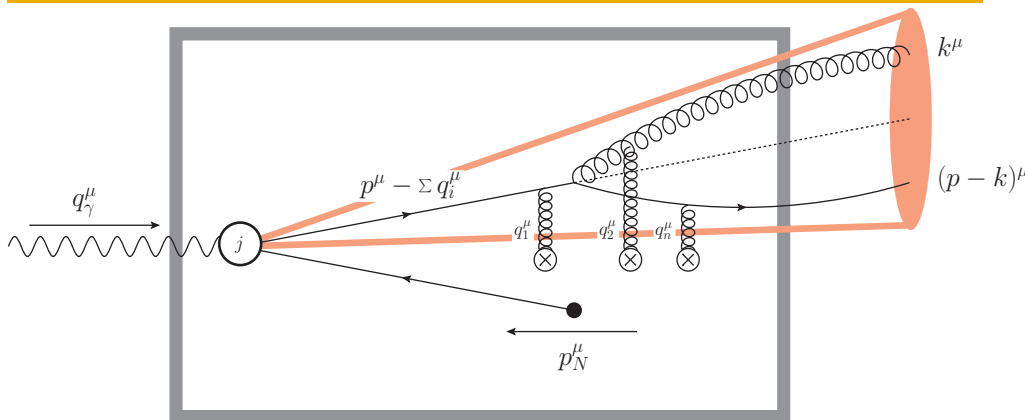
$$v(q_T^2) \rightarrow \frac{-g_{eff}^2}{q_T^2 + \mu^2} \quad \frac{d\sigma^{el}}{d^2q} = \frac{1}{(2\pi)^2} \frac{C_F}{2N_c} [v(q_T^2)]^2$$

In deep inelastic scattering (DIS) the lowest order processes involve prompt quark. Even at NLO the prompt gluon jet contribution is small

F. Ringer et al. (2018)

$$\frac{1}{p_N^-} \ll l_f^+ \sim \lambda^+ \sim L^+$$

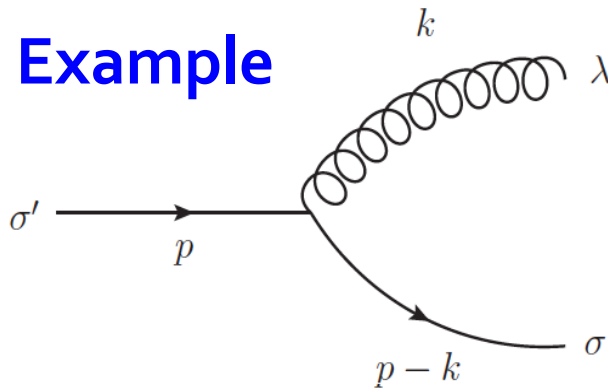
$$\mathcal{O}\left(\frac{1}{Q^2}\right)$$



- The limit we are interested in
- We neglect collisional energy losses

Lightcone wave functions and parton branchings

Example



- The technique of lightcone wavefunctions

$$\begin{aligned} \psi(x, \underline{k} - x\underline{p}) &\equiv \frac{1}{2p^+} \frac{1}{p^- - (p-k)^- - k^-} \bar{U}_\sigma(p-k) [-g\not{\epsilon}_\lambda^*(k)] U_{\sigma'}(p) \\ &= \frac{gx(1-x)}{(k-xp)_T^2 + x^2m^2} \left\{ \frac{2-x}{x\sqrt{1-x}} (\underline{\epsilon}_\lambda^* \cdot (\underline{k} - x\underline{p})) [\mathbb{1}]_{\sigma\sigma'} + \frac{\lambda}{\sqrt{1-x}} (\underline{\epsilon}_\lambda^* \cdot (\underline{k} - x\underline{p})) [\tau_3]_{\sigma\sigma'} \right. \\ &\quad \left. + \frac{imx}{\sqrt{1-x}} \underline{\epsilon}_\lambda^* \times [\underline{\tau}_\perp]_{\sigma\sigma'} \right\}. \end{aligned}$$

$$\langle \psi(x, \underline{\kappa}) \psi^*(x, \underline{\kappa}') \rangle \equiv \sum_{\lambda=\pm 1} \frac{1}{2} \text{tr} \left[\psi(x, \underline{\kappa}) \psi^*(x, \underline{\kappa}') \right]$$

Useful to express in Pauli matrixes

$$= \frac{8\pi\alpha_s(1-x)}{[\kappa_T^2 + x^2m^2][\kappa_T'^2 + x^2m^2]} \left[(\underline{\kappa} \cdot \underline{\kappa}') [1 + (1-x)^2] + x^4m^2 \right]$$

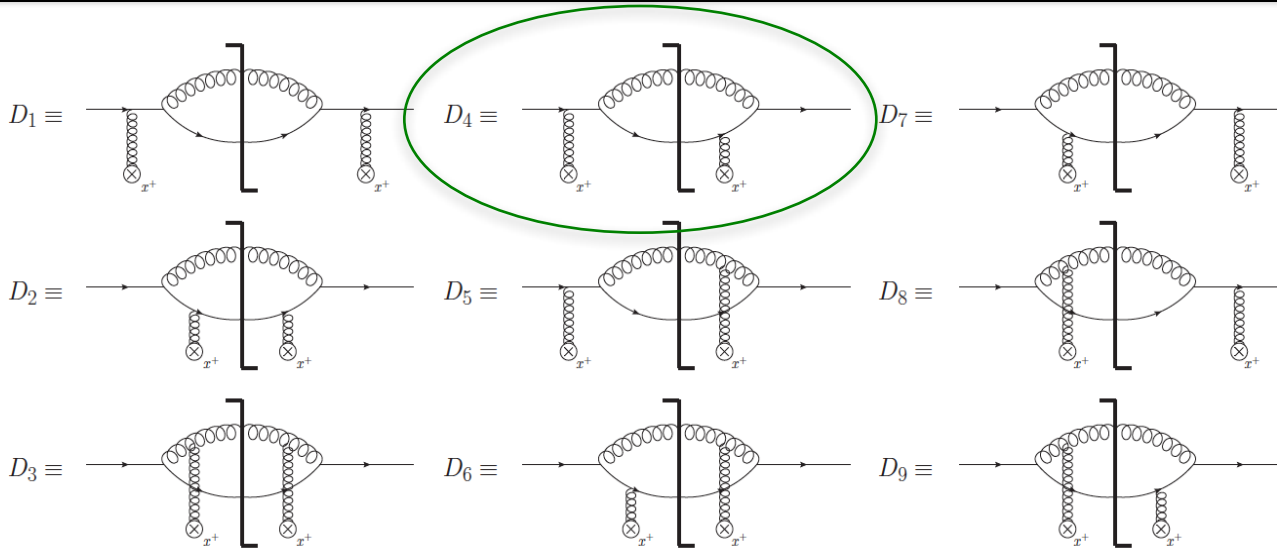
c.f. F. Ringer et al. (2016)

Branchings depending on the intrinsic momentum of the splitting $\underline{\kappa} = \underline{k} - x\underline{p}$.

$$xp^+ \frac{dN}{d^2k dx d^2p dp^+} \Big|_{\mathcal{O}(\chi^0)} = \frac{\alpha_s C_F}{2\pi^2} \frac{(k-xp)_T^2 [1 + (1-x)^2] + x^4m^2}{[(k-xp)_T^2 + x^2m^2]^2} \times \left(p^+ \frac{dN_0}{d^2p dp^+} \right)$$

- Certain advantages – can provide in “one shot” both massive and massless splitting functions
- Have checked that results agree for massless and massive DGLAP splittings

Opacity expansion building blocks – direct terms



- Interaction in the amplitude **and** the conjugate amplitude (Direct or single Born diagrams)

- Propagators hide in the wavefunctions

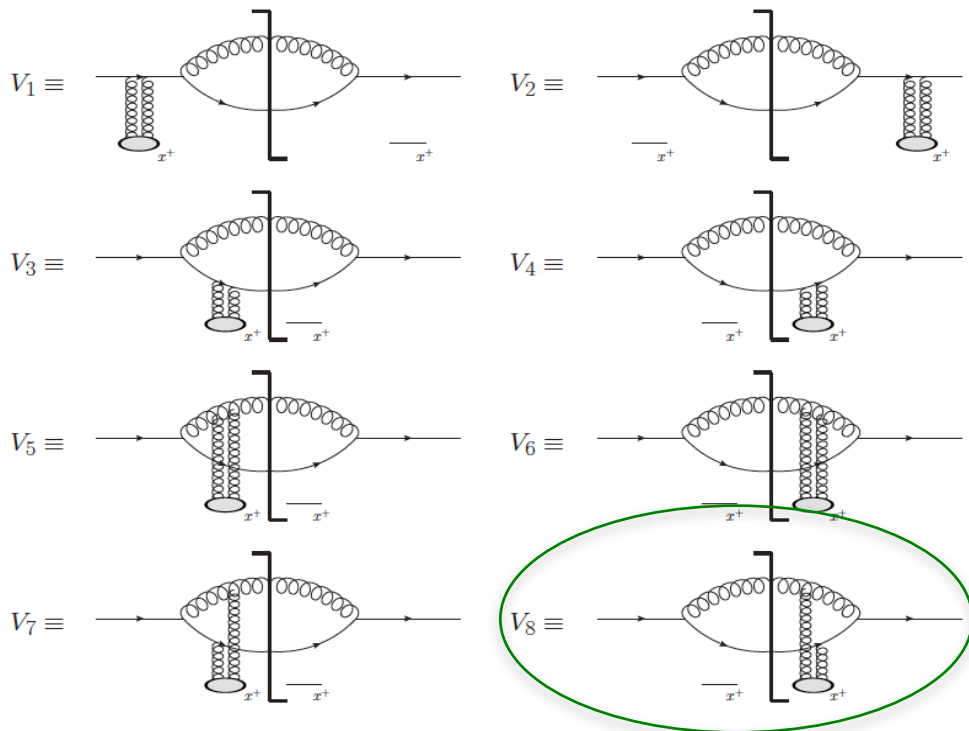
Representative forward cut diagram

$$D_4 = \left[\frac{-1}{2N_c C_F} e^{+i[\Delta E^-(\underline{k}-\underline{x}p) - \Delta E^-(\underline{k}-\underline{x}p+\underline{x}q)]z^+} \right] \psi(x, \underline{k} - \underline{x}p) \left[0 - e^{-i\Delta E^-(\underline{k}-\underline{x}p)z^+} \right] \\ \times \left[e^{+i\Delta E^-(\underline{k}-\underline{x}p+\underline{x}q)z^+} - e^{+i\Delta E^-(\underline{k}-\underline{x}p+\underline{x}q)x_0^+} \right] \psi^*(x, \underline{k} - \underline{x}p + \underline{x}q),$$

- Virtuallity changes enter the interference phases and are related to the propagators

$$p^- - k^- - (p - k)^- = \Delta E^-(\underline{k} - \underline{x}p)$$

Opacity expansion building blocks – virtual terms



- Interaction in the amplitude or the conjugate amplitude (Virtual or double Born diagrams)

Agree with the full splitting functions of

G. Ovanesyan et al . (2011)

F. Ringer et al . (2016)

And energy loss of

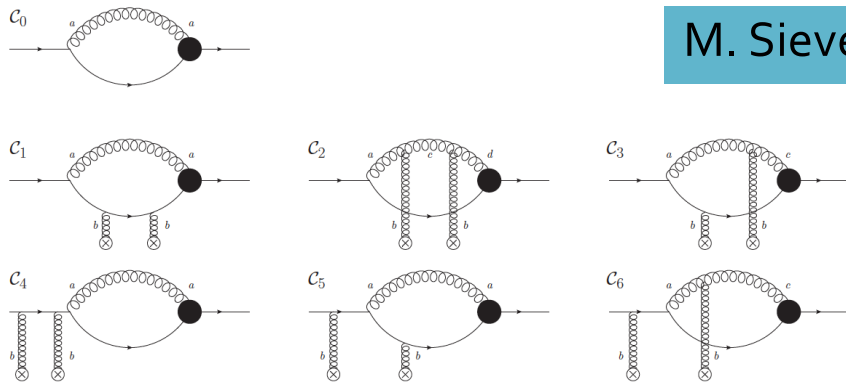
M. Gyulassy et al . (2001)

- A more interesting diagram- Double born can contribute to virtuality changes

$$V_8 = \left[\frac{N_c}{2C_F} e^{i[\Delta E^-(\underline{k}-\underline{x}\underline{p})-\Delta E^-(\underline{k}-\underline{x}\underline{p}-\underline{q})]z^+} \right] \psi(x, \underline{k} - \underline{x}\underline{p}) \left[0 - e^{-i\Delta E^-(\underline{k}-\underline{x}\underline{p})x_0^+} \right] \\ \times \left[e^{+i\Delta E^-(\underline{k}-\underline{x}\underline{p}-\underline{q})z^+} - e^{+i\Delta E^-(\underline{k}-\underline{x}\underline{p}-\underline{q})x_0^+} \right] \psi^*(x, \underline{k} - \underline{x}\underline{p} - \underline{q}) .$$

Parton branching to any order in opacity

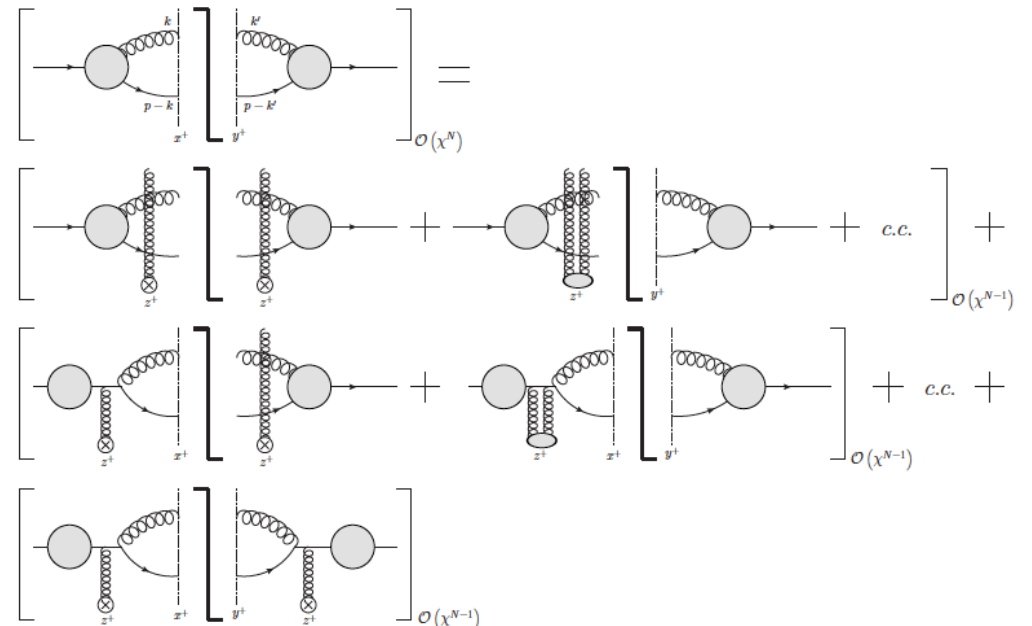
- Treating color (one complication in QCD).



M. Sievert et al. (2018)

- Color is not entangled, homogeneous structure and multiplicative factors that can be algebraically treated
- Finally, relative to the splitting vertex we classify the as
- Initial/Initial, Initial/Final, Final/Initial and Final/Final

$$\begin{aligned}
 C_1 &= \frac{1}{N_c C_F} \text{tr}[t^b t^b t^a M^a] = C_0, \\
 C_2 &= \frac{1}{N_c C_F} f^{acb} f^{cdb} \text{tr}[t^a M^d] = -\frac{N_c}{C_F} C_0, \\
 C_3 &= \frac{1}{N_c C_F} f^{acb} \text{tr}[t^b t^a M^c] = \frac{i N_c}{2 C_F} C_0, \\
 C_4 &= \frac{1}{N_c C_F} \text{tr}[t^a t^b t^b M^a] = C_0, \\
 C_5 &= \frac{1}{N_c C_F} \text{tr}[t^b t^a t^b M^a] = \frac{-1}{2 N_c C_F} C_0, \\
 C_6 &= \frac{1}{N_c C_F} f^{acb} \text{tr}[t^a t^b M^c] = \frac{-i N_c}{2 C_F} C_0.
 \end{aligned}$$



Master equation

- Upper triangular structure. Suggests specific strategy how to solve it.
Calculated: initial conditions, kernels, and wrote a Mathematica code to solve it.

$$\begin{bmatrix} f_{F/F}^{(N)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N)}(\underline{k}', \underline{p}; x^+, y^+) \\ f_{F/I}^{(N)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{I/I}^{(N)}(\underline{p}; x^+, y^+) \end{bmatrix} = \int_{x_0^+}^{\min[x^+, y^+, R^+]} \frac{dz^+}{\lambda^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \begin{bmatrix} \mathcal{K}_1 & \mathcal{K}_2 & \mathcal{K}_3 & \mathcal{K}_4 \\ 0 & \mathcal{K}_5 & 0 & \mathcal{K}_6 \\ 0 & 0 & \mathcal{K}_7 & \mathcal{K}_8 \\ 0 & 0 & 0 & \mathcal{K}_9 \end{bmatrix} \begin{bmatrix} f_{F/F}^{(N-1)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N-1)}(\underline{k}', \underline{p}; x^+, y^+) \\ f_{F/I}^{(N-1)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{I/I}^{(N-1)}(\underline{p}; x^+, y^+) \end{bmatrix}$$

Simplest kernel

$$\mathcal{K}_9 = \left[e^{-\underline{q} \cdot \underline{\nabla}_p} e^{+(z^+ - x^+) \partial_{x^+}} e^{+(z^+ - y^+) \partial_{y^+}} \right] + \left[-\frac{1}{2} \right] \left[e^{+(z^+ - x^+) \partial_{x^+}} + e^{+(z^+ - y^+) \partial_{y^+}} \right]$$

Most complicated kernel

$$\begin{aligned} \mathcal{K}_1 = & \left[e^{i[\Delta E^-(\underline{k} - x\underline{p} + x\underline{q}) - \Delta E^-(\underline{k} - x\underline{p})]z^+} e^{i[\Delta E^-(\underline{k}' - x\underline{p}) - \Delta E^-(\underline{k}' - x\underline{p} + x\underline{q})]z^+} \right] \left[e^{-\underline{q} \cdot \underline{\nabla}_p} e^{+(z^+ - x^+) \partial_{x^+}} e^{+(z^+ - y^+) \partial_{y^+}} \right] \\ & + \left[\frac{N_c}{C_F} e^{i[\Delta E^-(\underline{k} - x\underline{p} - (1-x)\underline{q}) - \Delta E^-(\underline{k} - x\underline{p})]z^+} e^{i[\Delta E^-(\underline{k}' - x\underline{p}) - \Delta E^-(\underline{k}' - x\underline{p} - (1-x)\underline{q})]z^+} \right] \\ & \times \left[e^{-\underline{q} \cdot \underline{\nabla}_k} e^{-\underline{q} \cdot \underline{\nabla}_{k'}} e^{-\underline{q} \cdot \underline{\nabla}_p} e^{+(z^+ - x^+) \partial_{x^+}} e^{+(z^+ - y^+) \partial_{y^+}} \right] \quad + 8 \text{ more lines} \end{aligned}$$

Explicit solution to second order in opacity

- Present the first exact result to this order (including the ability to discuss broad or narrow sources)

$$xp^+ \frac{dN}{d^2k dx d^2p dp^+} \Big|_{\mathcal{O}(\chi^2)} = \frac{C_F}{2(2\pi)^3(1-x)} \int_{x_0^+}^{R^+} \frac{dz_2^+}{\lambda^+} \int_{x_0^+}^{z_2^+} \frac{dz_1^+}{\lambda^+} \int \frac{d^2q_1}{\sigma_{el}} \frac{d^2q_2}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q_1} \frac{d\sigma^{el}}{d^2q_2} \times \left\{ \left(p^+ \frac{dN_0}{d^2p dp^+} \right) \mathcal{N}_1 \right. \\ \left. + \left(p^+ \frac{dN_0}{d^2(p-q_1) dp^+} \right) \mathcal{N}_2 + \left(p^+ \frac{dN_0}{d^2(p-q_2) dp^+} \right) \mathcal{N}_3 + \left(p^+ \frac{dN_0}{d^2(p-q_1-q_2) dp^+} \right) \mathcal{N}_4 \right\}$$

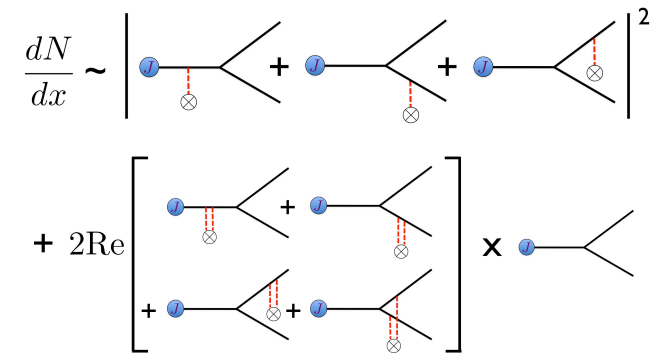
$\mathcal{N}_1 =$

$$|\psi(\underline{k} - x\underline{p})|^2 \left[\frac{(C_F + N_c)^2}{C_F^2} - \frac{N_c(C_F + N_c)}{C_F^2} \cos(\delta z_1 \Delta E^-(\underline{k} - x\underline{p})) + \frac{N_c^2}{2C_F^2} \cos(\delta z_2 \Delta E^-(\underline{k} - x\underline{p})) \right. \\ \left. - \frac{N_c(2C_F + N_c)}{2C_F^2} \cos((\delta z_1 + \delta z_2) \Delta E^-(\underline{k} - x\underline{p})) \right] \quad + 9 \text{ more pages}$$

- For broad sources and in the soft gluon limit we have checked that the result reduces to the GLV second order in opacity

Generalizing the result to all in-medium splittings (4)

- Note – all splittings have the same topology. **Same - structure, interference phases, propagators**
Different - mass dependence, wavefunctions, color (which also affects transport coefficients)



$$\langle \psi(x, \underline{\kappa}) \psi^*(x, \underline{\kappa}') \rangle = \frac{8\pi\alpha_s f(x)}{[\kappa_T^2 + \nu^2 m^2][\kappa_T'^2 + \nu^2 m^2]} \left[g(x) (\underline{\kappa} \cdot \underline{\kappa}') + \nu^4 m^2 \right]$$

$$\Delta E^-(\underline{\kappa}) = -\frac{\kappa_T^2 + \nu^2 m^2}{2x(1-x)p^+}$$

- Master table that gives all ingredients

	d_1	d_2	d_3	d_4	d_5	d_6	v_1	v_3	v_5	v_7	λ_R^+	C_0	ν	$f(x)$	$g(x)$
G/q	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	x	$1-x$	$1 + (1-x)^2$
q/q	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	$1-x$	x	$1 + x^2$
q/G	1	$\frac{C_F}{N_c}$	$\frac{C_F}{N_c}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2N_c^2}$	$-\frac{1}{2}$	$-\frac{C_F}{2N_c}$	$\frac{-C_F}{2N_c}$	$\frac{-1}{2N_c^2}$	λ_G^+	$\frac{1}{2}$	1	$x(1-x)$	$x^2 + (1-x)^2$
G/G	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	λ_G^+	N_c	0	$1 + x^4 + (1-x)^4$	1

We have now solved the problem for all splitting functions and are working on the manuscript

M. Sievert et al. (2019)

Numerical results

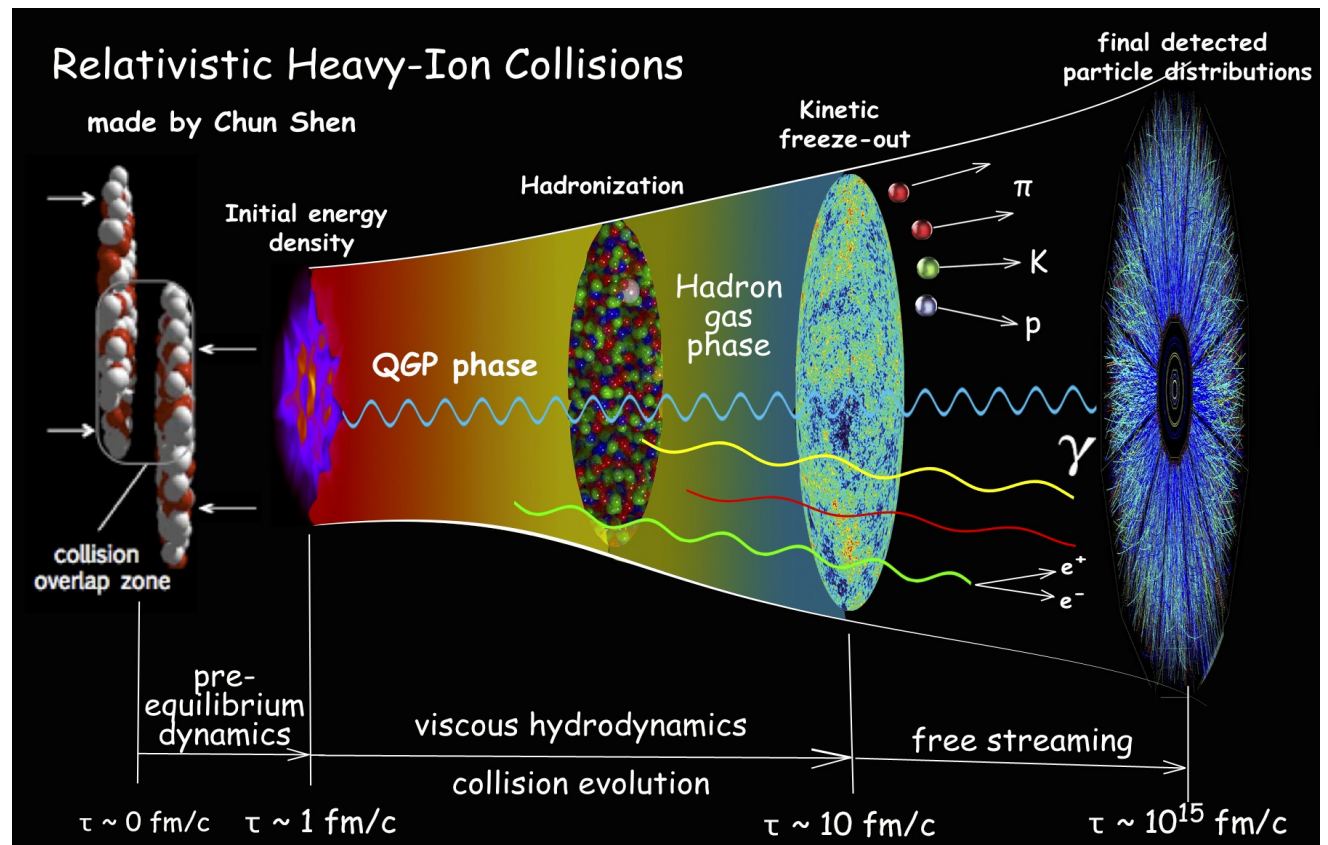


For immediate application

- Numerics can be challenging due to lengthy equations and multi-dimensional integration
- Implementation for the case of QGP (simplified Bjorken expansion)

Lashoff-Regas et al. (2014)

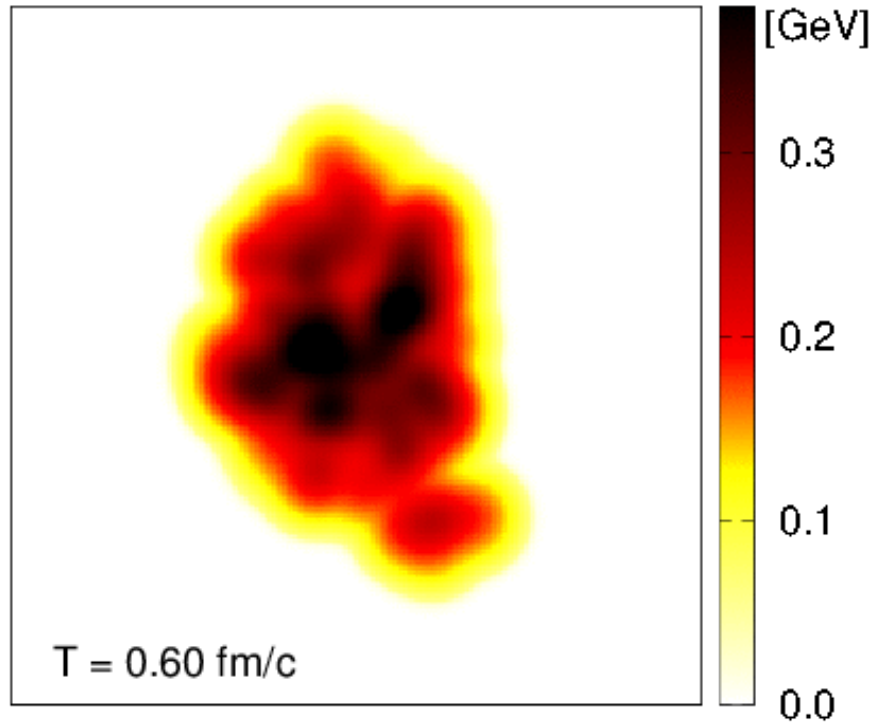
- Still, code needed 3 days for a set of splittings



iEBE-VISHNU package

- Hydro + hadron cascade simulator for relativistic heavy-ion collisions
- Developed by Chun Shen and collaborators

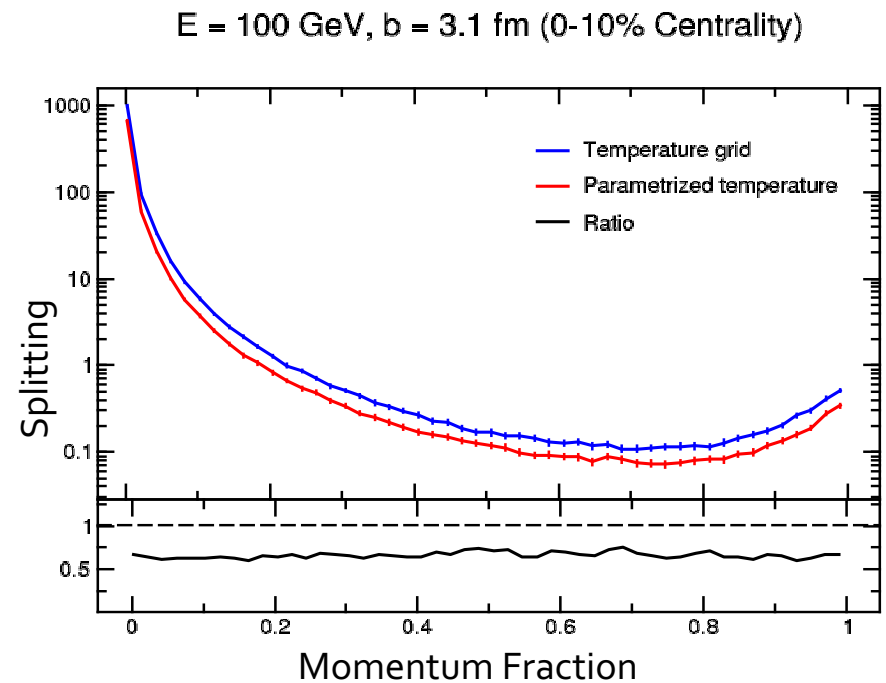
Improvements in physics



Evolution of medium temperature in Au+Au

- (2+1)D viscous hydrodynamics
- Incorporate modern equations-of-state (e.g. the HotQCD EoS)

- Significant difference found in the splitting functions on model vs **hydro simulation** QGP medium
- Code takes **6 times more time**



Code optimization

Refactoring

- Code is **restructured** (in C++) and shortened (**24K → 8K lines**). **20x speed improvement**

Effective incorporation of simulated QGP medium

- Reduced overhead for calling QGP medium grid function. **2x speed improvement**

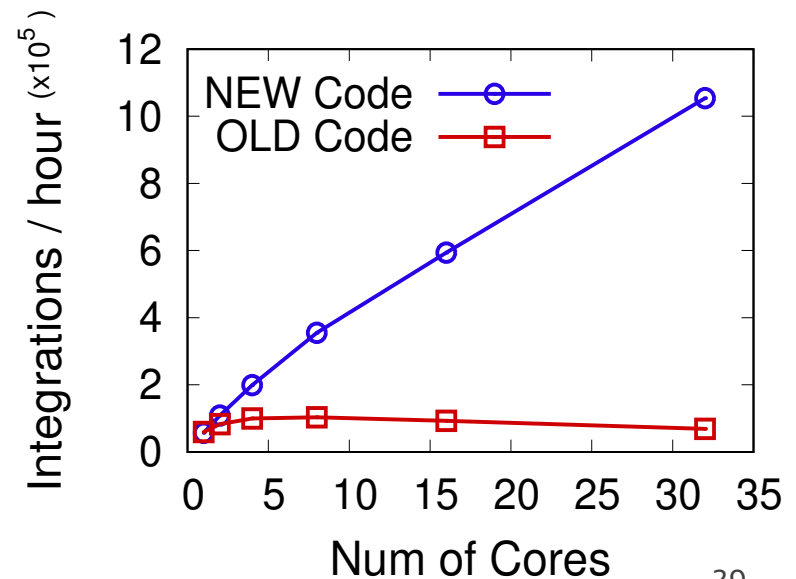
Efficient on-node parallelization

- New parallelization shows much better scaling **10x speed improvement**

Overall improvement:
18 days → 1 hour

```
if (split_id==1) //Quark-->Quark, Gluon
{
  if (int_id==1)
  {
    Vegas(NDIM, NCOMP, Integrand, gqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, NSTART, NINCREASE, NBATCH,
    GRIDNO, STATEFILE,
    &neval, &fail, integral, error, prob);
  }
  if (int_id==2)
  {
    Suave(NDIM, NCOMP, Integrand, gqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose | LAST, SEED,
    MINEVAL, MAXEVAL, NNEW, FLATNESS,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
  }
  if (int_id==3)
  {
    Divonne(NDIM, NCOMP, Integrand, gqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, KEY1, KEY2, KEY3, MAXPASS,
    BORDER, MAXHISLO, MINDEVIATION,
    NGIVEN, LDGIVEN, NULL, NEXTRA, NULL,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
  }
  if (int_id==4)
  {
    Cuhre(NDIM, NCOMP, Integrand, gqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose | LAST,
    MINEVAL, MAXEVAL, KEY,
    STATEFILE,
    &nregions, &neval, &fail, integral, error, prob);
  }
}
if (split_id==2) //Gluon-->Gluon, Gluon
{
  if (int_id==1)
  {
    Vegas(NDIM, NCOMP, Integrand, gqgnocuts, USERDATA,
    EPSREL, EPSABS, verbose, SEED,
    MINEVAL, MAXEVAL, NSTART, NINCREASE, NBATCH,
    GRIDNO, STATEFILE,
    &neval, &fail, integral, error, prob);
  }
}
```

```
if (cut_id == 1)
{
  switch(split_id)
  {
    case 1:
      func = &Integrand.gqgnocuts; break;
    case 2:
      func = &Integrand.gqgnocuts; break;
    case 3:
      func = &Integrand.gqgnocuts; break;
    case 4:
      func = &Integrand.gqgnocuts; break;
    default:
      printf("Error: Unknown split_id %d\n", split_id);
      exit(0);
  } // switch(split_id)
} // cut_id == 1
```

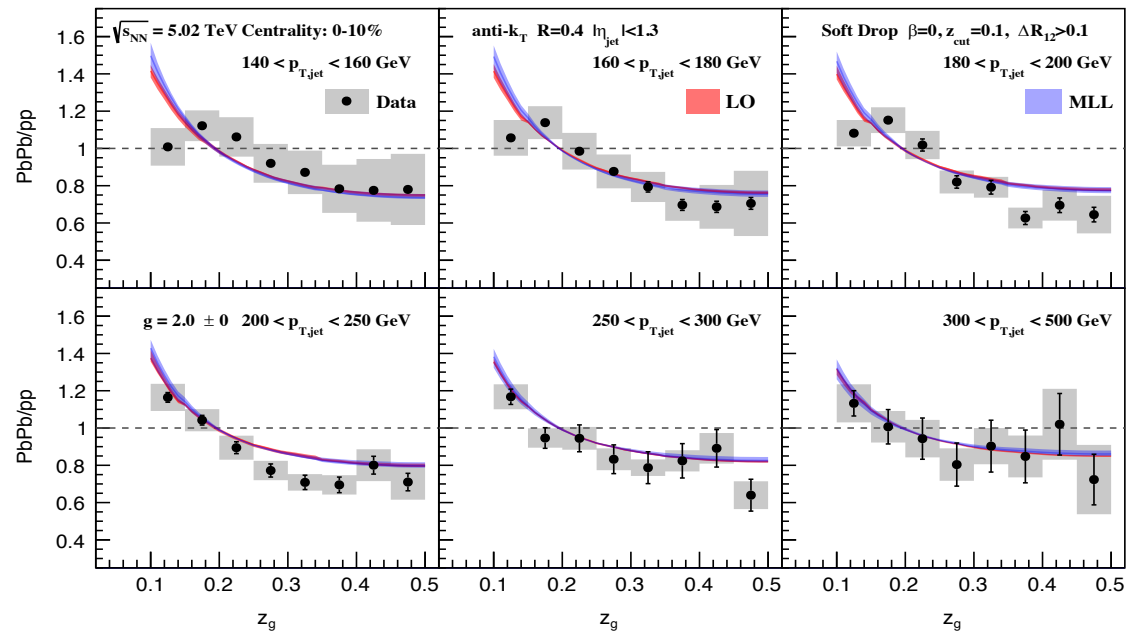


What are the results

The more accurate splitting functions have been implemented in phenomenology. They have been measured directly measured by experiment

$$z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R_0} \right)^\beta$$

H. Li et al. (2018)



Difficulties to second order in opacity

- Arise from larger number of evaluations – 10 pages; additional 3 dimensional integration
- Expect factor of ~ 10 slower. Still hope to get splitting function grids within a day

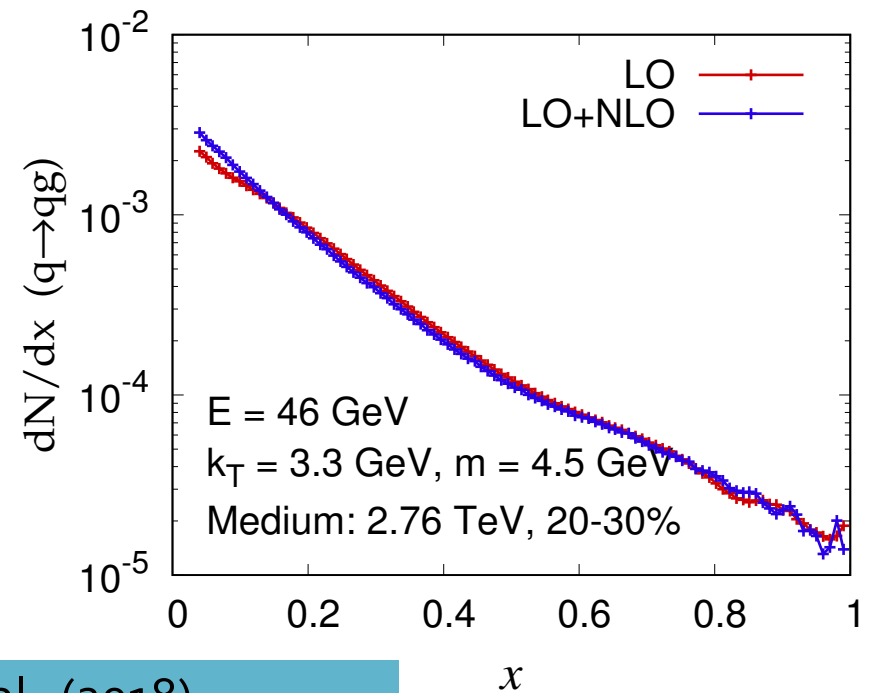
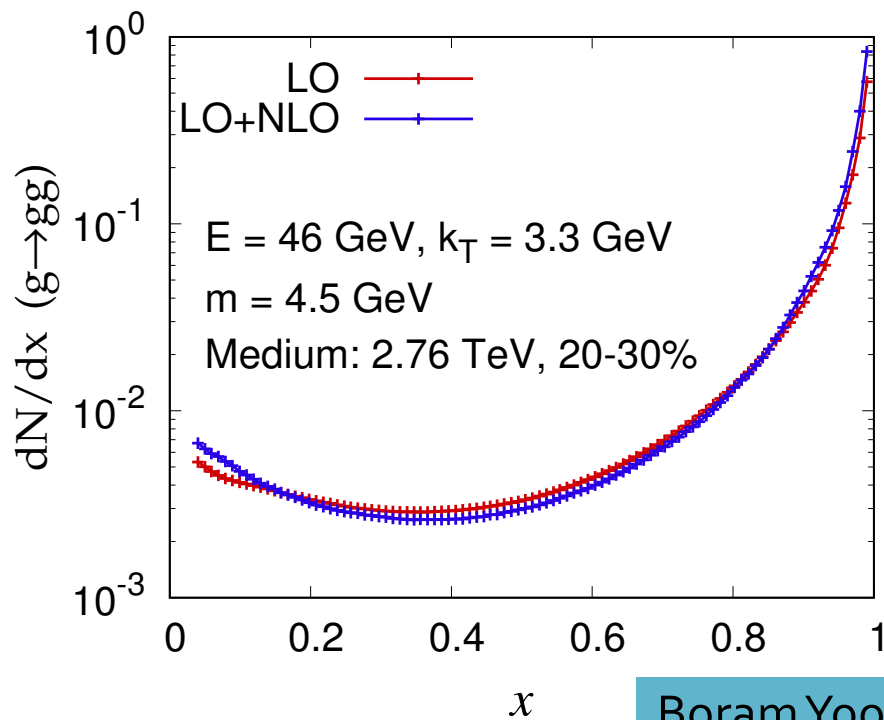
Porting to code

- Results are directly exported from Mathematica to C++

Second order in opacity corrections to parton splitting

Note on notation in figures: LO – first order in opacity, NLO second order in opacity. What is labeled dN/dx is dN/dxd^2k_T at fixed k_T (3.3 GeV). The fully differential splitting functions where corrections are expected to be largest

- Preliminary results show $O(1\sim 50)\%$ NLO corrections
- Evaluation of the NLO corrections is computationally very expensive



Boram Yoon et al. (2018)

Full QCD evolution approach

- Based on DGLAP evolution with with SCET_G medium-induced splitting kernels

Z. Kang et al. (2014)

$$\begin{aligned}\frac{dD_{h/q}(z, Q)}{d \ln Q} &= \frac{\alpha_s(Q)}{\pi} \int_z^1 \frac{dz'}{z'} \left[P_{q \rightarrow qq}^{\text{med}}(z', Q; \beta) D_{h/q}\left(\frac{z}{z'}, Q\right) \right. \\ &\quad \left. + P_{q \rightarrow gq}^{\text{med}}(z', Q; \beta) D_{h/g}\left(\frac{z}{z'}, Q\right) \right], \\ \frac{dD_{h/g}(z, Q)}{d \ln Q} &= \frac{\alpha_s(Q)}{\pi} \int_z^1 \frac{dz'}{z'} \left[P_{g \rightarrow gg}^{\text{med}}(z', Q; \beta) D_{h/g}\left(\frac{z}{z'}, Q\right) \right. \\ &\quad \left. + P_{g \rightarrow q\bar{q}}^{\text{med}}(z', Q; \beta) \sum_q D_{h/q}\left(\frac{z}{z'}, Q\right) \right].\end{aligned}$$

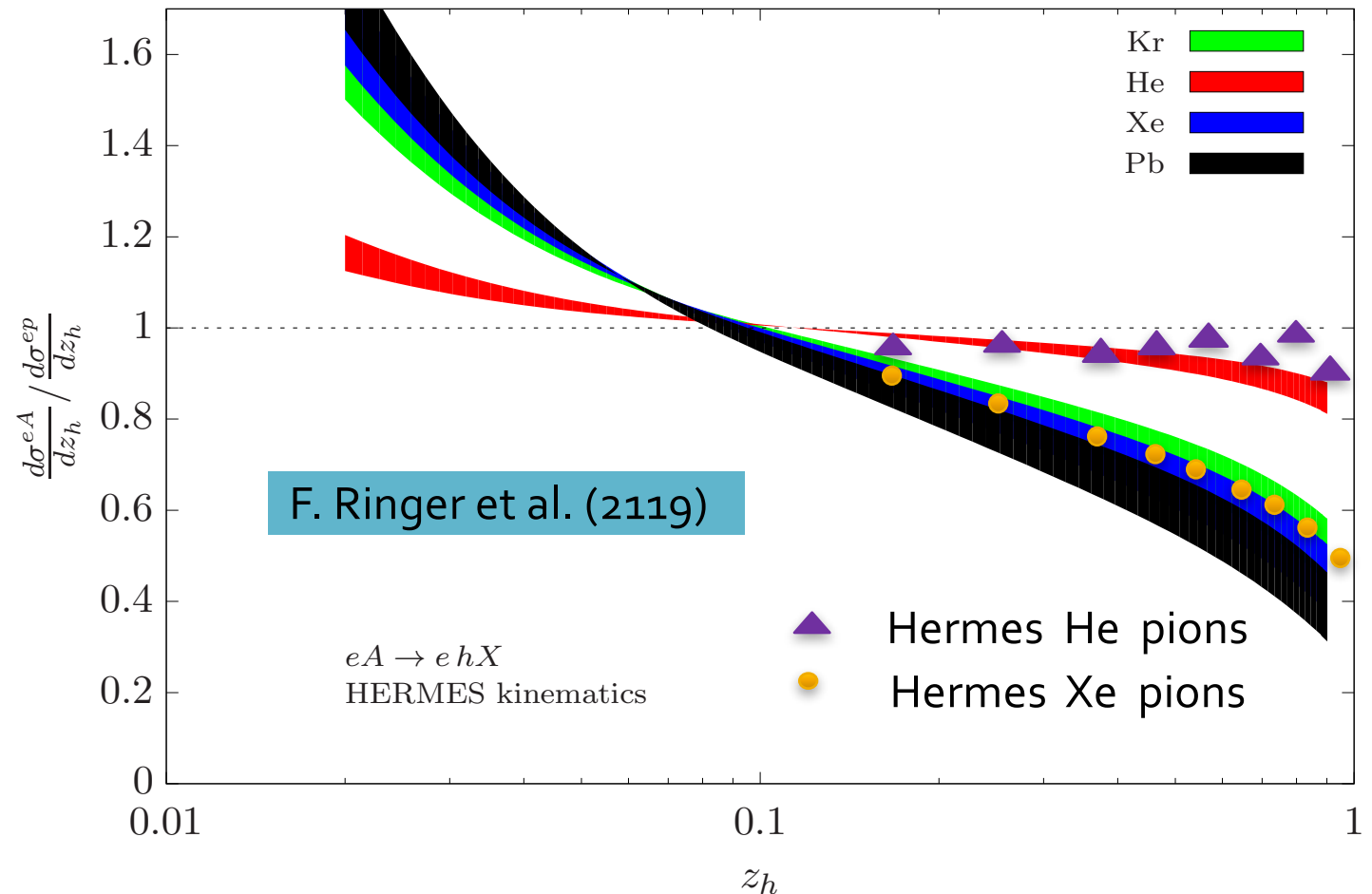
- The approach was shown to give good description

Did not fit HERMES data. Instead we used nuclear transport properties constrained from the Cronin effect, coherent power corrections, and initial-state energy loss

- We looked directly at the modification of the fragmentation function – expect it to be larger than in Kr and Xe at HERMES
- Form the transport properties of CNM (power corrections, CNM e-loss in DY): quarks $\xi^2 A^{1/3} \sim Q_s^2 \sim 0.7 \text{ GeV}^2$ gluons $\xi^2 A^{1/3} \sim Q_s^2 \sim 1.5 \text{ GeV}^2$

Validation against Hermes data

- Description of light pions. On the upper edge of the theory uncertainty bands
- For heavy particle one has to be careful when $E \sim m$



NLO calculation

$$E_h \frac{d^3 \sigma^{\ell N \rightarrow hX}}{d^3 P_h} = \frac{1}{S} \sum_{i,f} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dz}{z^2} f^{i/N}(x, \mu) \times D^{h/f}(z, \mu) \hat{\sigma}^{i \rightarrow f}(s, t, u, \mu)$$

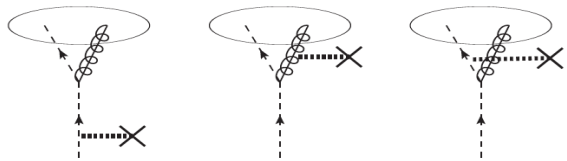
Results for jet fragmentation at the EIC in E+A

- The distribution of hadrons inside jets: **semi-inclusive fragmenting jet functions**

$$\frac{d\sigma^{pp \rightarrow (\text{jet } h) X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_a(x_a, \mu) \otimes f_b(x_b, \mu) \otimes$$

$$H_{ab}^c(\eta, p_T/z, x_a, x_b, \mu) \otimes \mathcal{G}_c^h(z, z_h, \omega_J R, \mu)$$

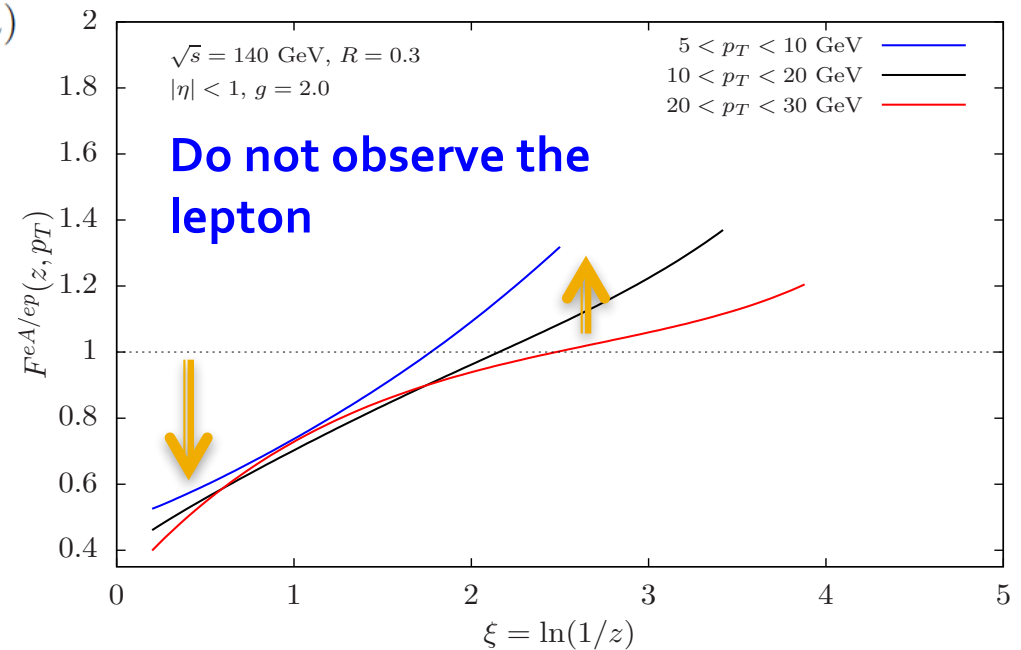
- Derivation in the presence of a medium



$$\mathcal{G}_q^{q,(1)}(z, z_h, \omega R, \mu) = D_q(z_h) \left[\int_{z(1-z)p_T R}^{\mu} P_{qq}(z, q_{\perp}) \right]_+$$

$$+ \delta(1-z) \left[\int_{\mu_0}^{z_h(1-z_h)p_T R} dq_{\perp} P_{qq}(z_h, q_{\perp}) \right]_+ \otimes D_q(z_h)$$

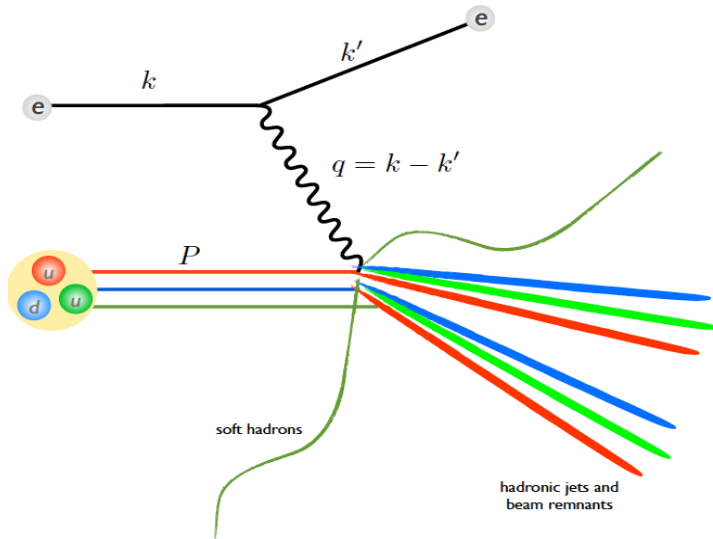
Very preliminary EIC results



For "historic" reasons: $\xi = \ln(1/z_h)$

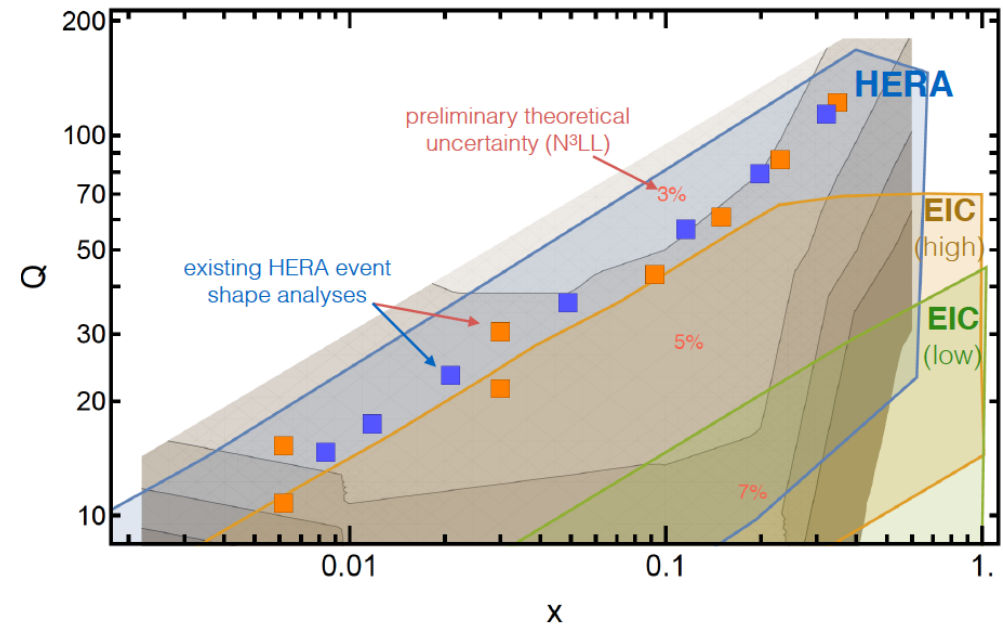
Event shapes in SIDIS

Tensions in the extraction of the strong coupling constant



- Develop precision theories for event shape observables in DIS. Constrain nucleon structure. Extract α_s
- 1-jettiness (1jet and 1 beam)

Determination	Data and procedure	Reference
0.1175 ± 0.0025	ALEPH 3-jet rate (NNLO+MChad)	[25]
0.1199 ± 0.0059	JADE 3-jet rate (NNLO+NLL+MChad)	[29]
0.1224 ± 0.0039	ALEPH event shapes (NNLO+NLL+MChad)	[30]
0.1172 ± 0.0051	JADE event shapes (NNLO+NLL+MChad)	[31]
0.1189 ± 0.0041	OPAL event shapes (NNLO+NLL+MChad)	[32]
$0.1164^{+0.0028}_{-0.0026}$	Thrust (NNLO+NLL+anlhad)	[33]
$0.1134^{+0.0031}_{-0.0025}$	Thrust (NNLO+NNLL+anlhad)	[34]
0.1135 ± 0.0011	Thrust (SCET NNLO+N ³ LL+anlhad)	[8]
0.1123 ± 0.0015	C-parameter (SCET NNLO+N ³ LL+anlhad)	[35]



D. Kang et al. (2016)

Conclusions

- There are tremendous opportunities for jet physics in ep and eA collisions that are not fully explored at the EIC
- On the experimental side one can determine the transport properties of large nuclei, radiation lengths that are the shortest in nature. A multi-year physics program
- On the formal side we now have the technique to calculate the full (beyond soft gluon emission) in-medium splitting functions to any order in opacity. Explicit results and numerical implementation.
- We also have new theoretical techniques (some inspired by SCET) based on factorization and evolution, semi-inclusive jets functions, semi-inclusive fragmenting jet functions. Can validate against existing HERMES data, but the emphasis is on jet physics
- Working on numerical tools (implementation) for jet simulations in reactions with nuclei at the EIC

Transparency or lack thereof

We haven't see transparency
but we have seen quenching

