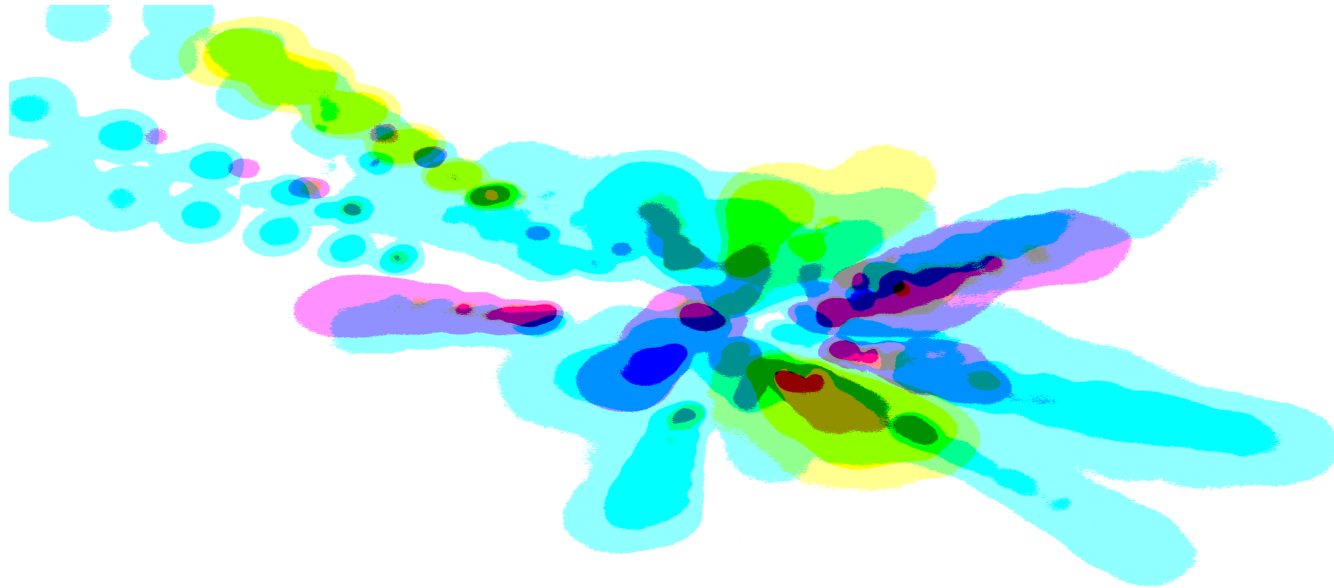


Peering into the infrared with colored glass

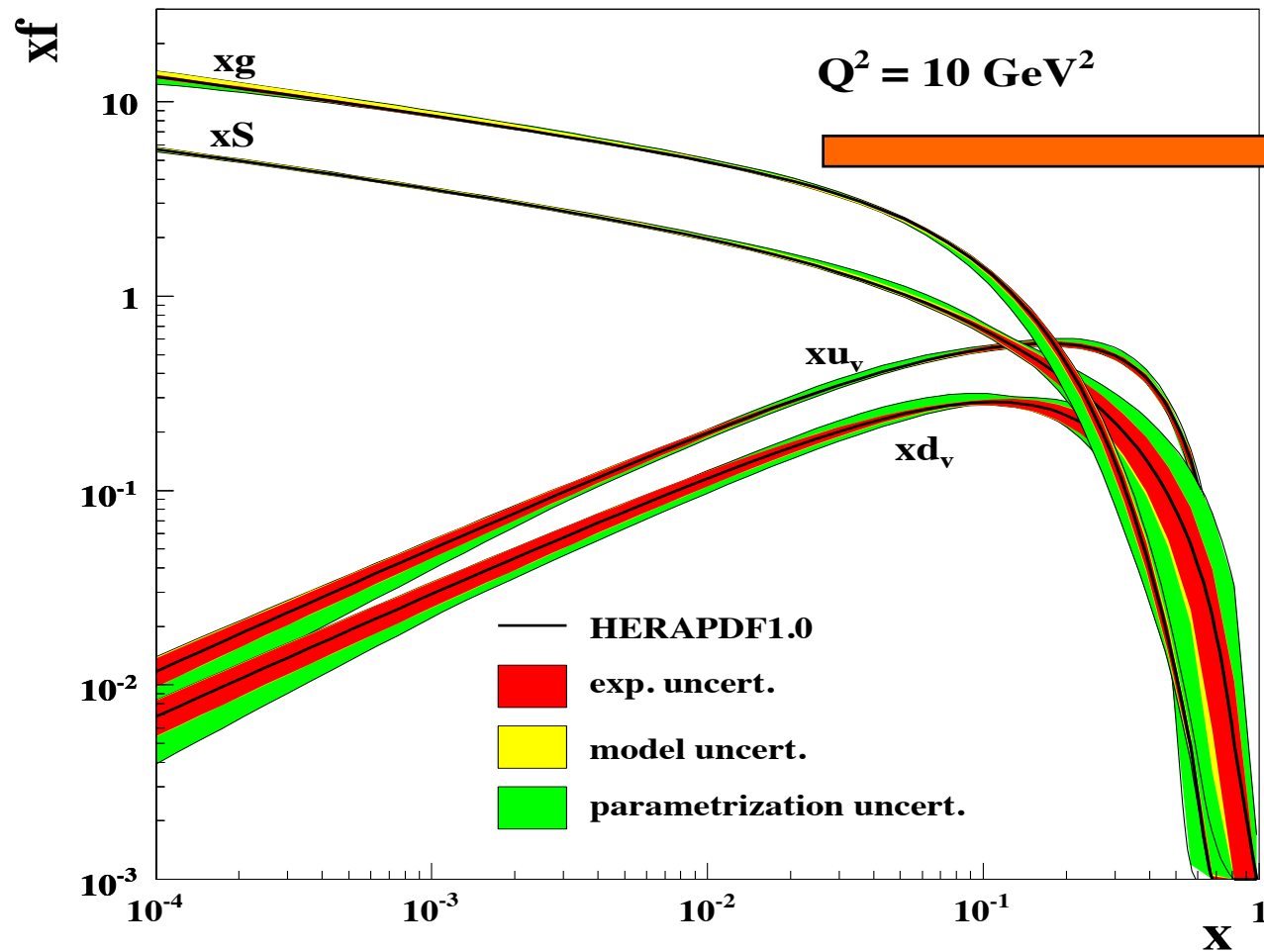


Raju Venugopalan
Brookhaven National Laboratory

Outline of talk

- ❑ **Gluon saturation leads to weakly coupled albeit strongly correlated glue in the Regge limit**
- ❑ **The CGC: a theory framework for gluon saturation**
- ❑ **Light from the CGC: the structure of higher order computations from a specific example**
- ❑ **Color memory and the infrared circle**

The proton: a complex many-body system

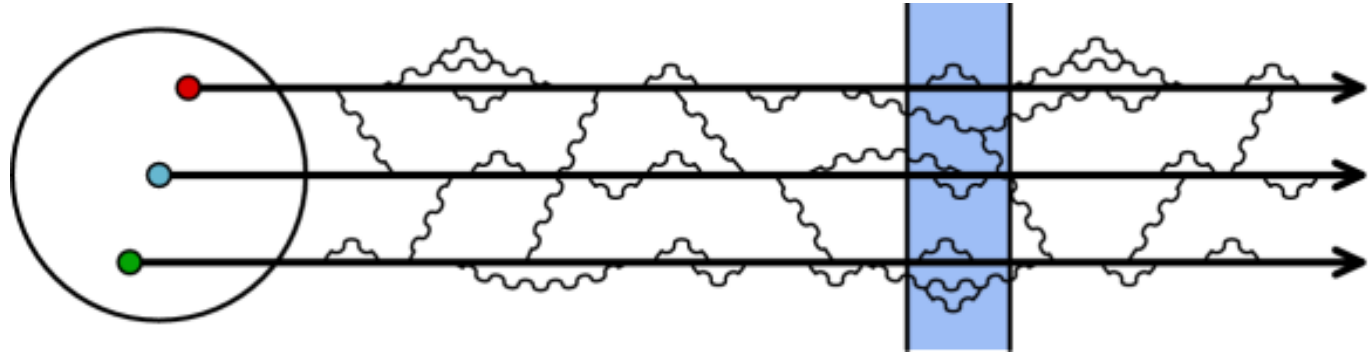


A key lesson from the HERA DIS collider:

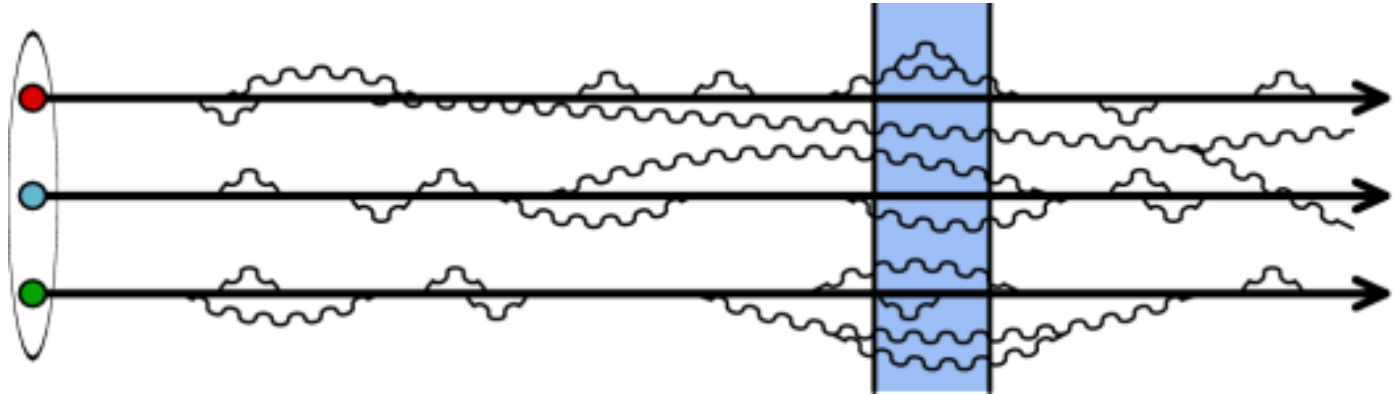
gluons and sea quarks dominate the proton wave-function at high energies

Boosting the proton uncovers many-body structure

Low Energy
(or large x)



High Energy
(or small x)



Wee parton fluctuations time dilated on strong interaction time scales

Long lived gluons can radiate further small x gluons...

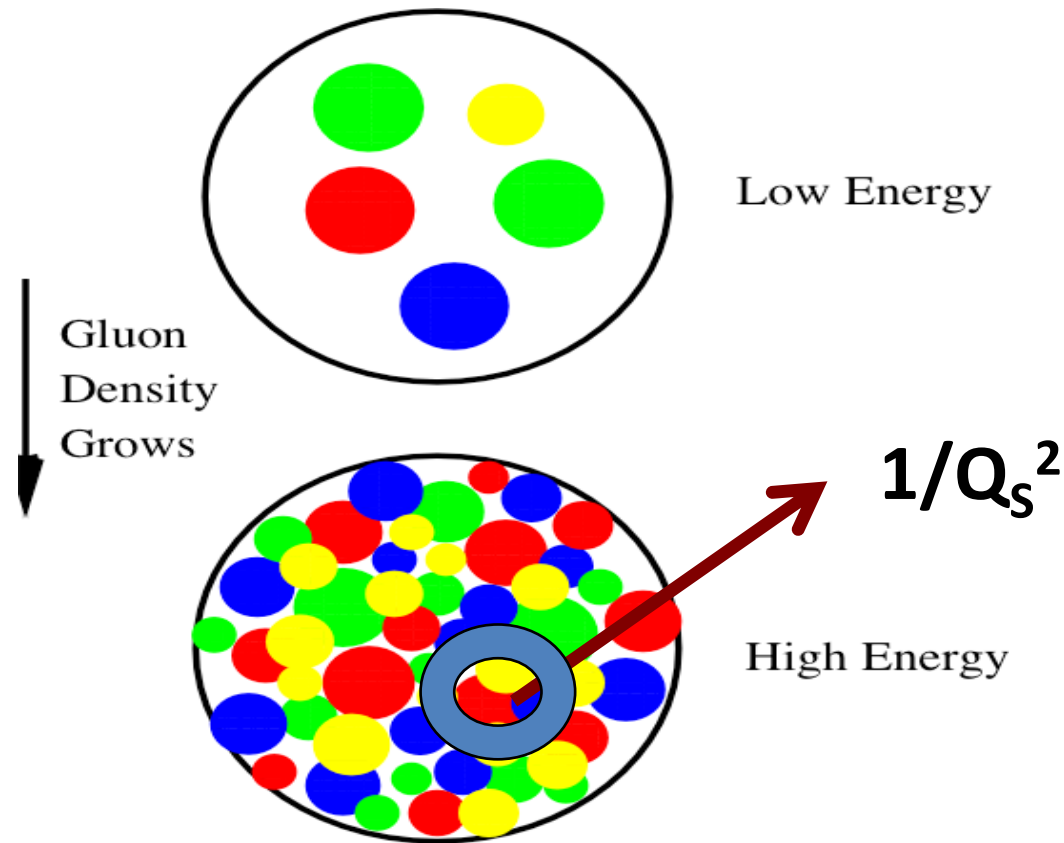
Is the proton a runaway popcorn machine at high energies ?

The runaway proton...



Nature does not like this!

The boosted proton viewed head-on

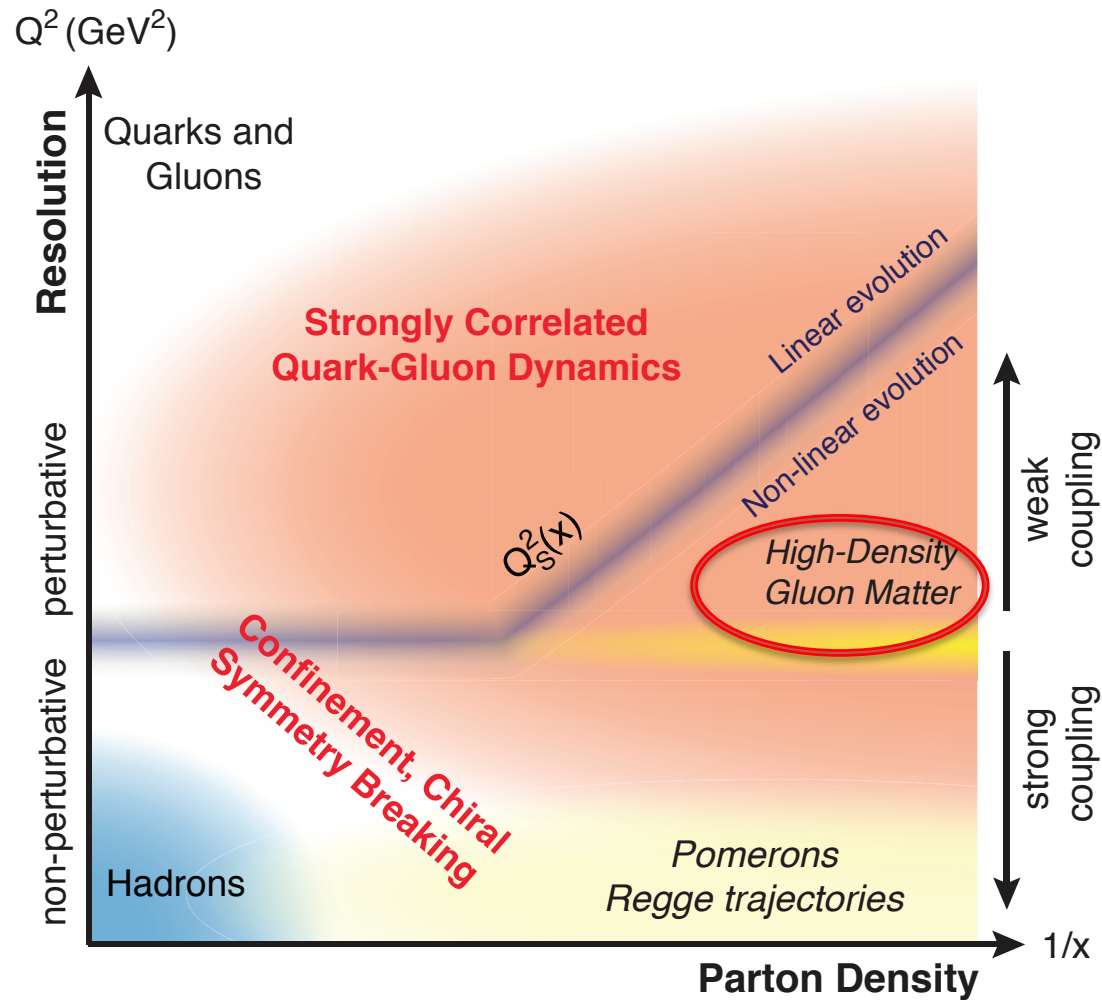


When occupancies become large $\sim 1/\alpha_s$, gluons resist further close packing -- recombining and screening their color charges -- leading to **gluon saturation**

Characterized by an emergent semi-hard scale $Q_s \gg \Lambda_{QCD}$ in the Regge limit...

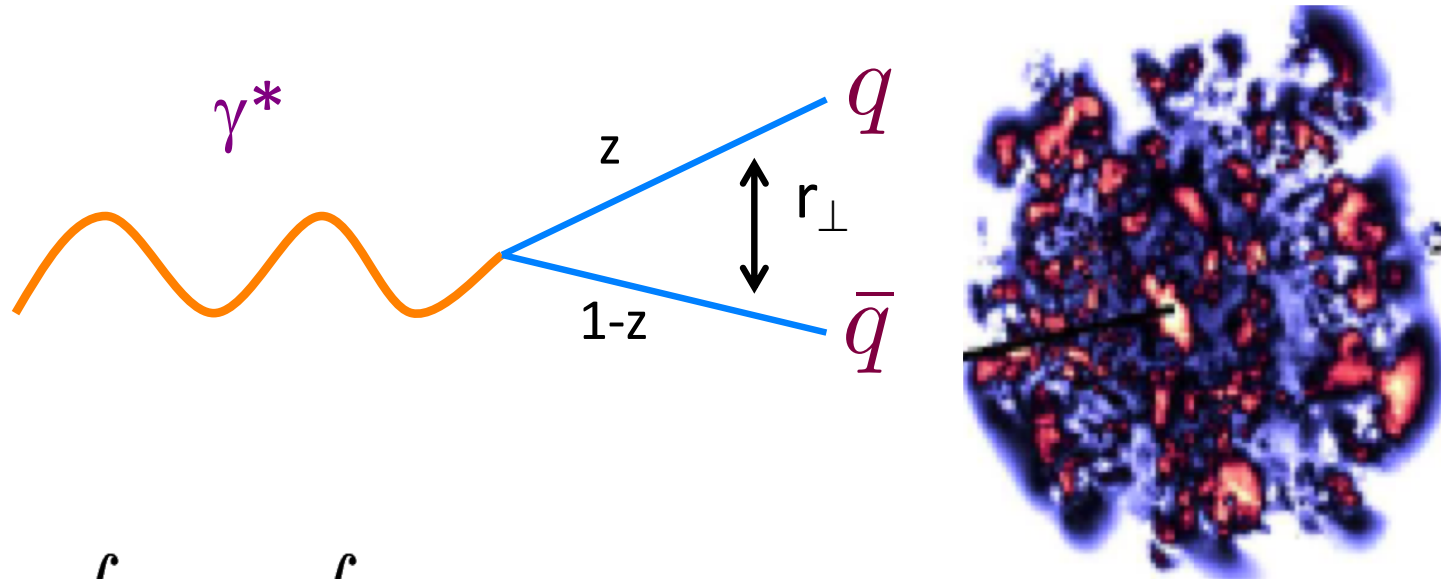
a weak coupling window into the infrared!

Saturation in the QCD landscape



Unique and controlled *dynamical* exploration of a fully nonlinear regime of quantum field theory

Saturation: dipole model formulation in DIS



$$\sigma_{T,L}^{\gamma^*,P} = \int d^2 r_\perp \int dz |\psi_{T,L}(r_\perp, z, Q^2)|^2 \sigma_{q,\bar{q},P}(r_\perp, x)$$

Golec-Biernat Wusthoff model

$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[1 - \exp\left(-r_\perp^2 Q_s^2(x)\right) \right]$$

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x} \right)^\lambda$$

Parameters:

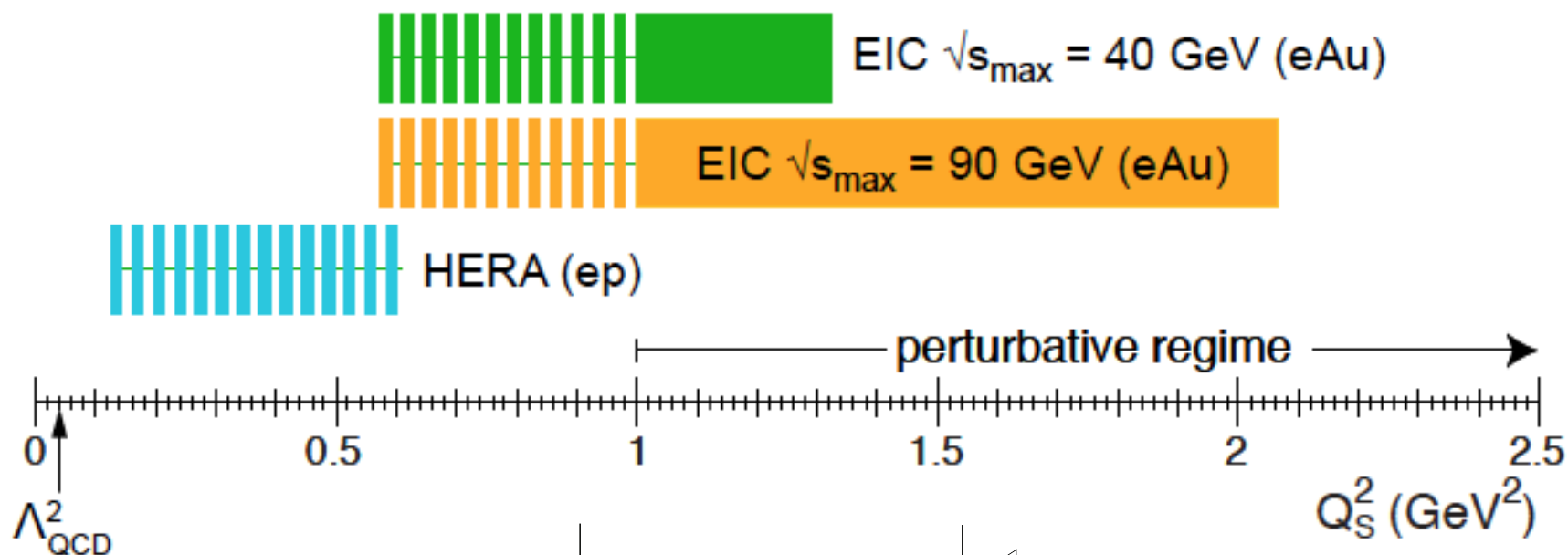
$$Q_0 = 1 \text{ GeV}; \lambda = 0.3;$$

$$x_0 = 3 * 10^{-4}; \sigma_0 = 23 \text{ mb}$$

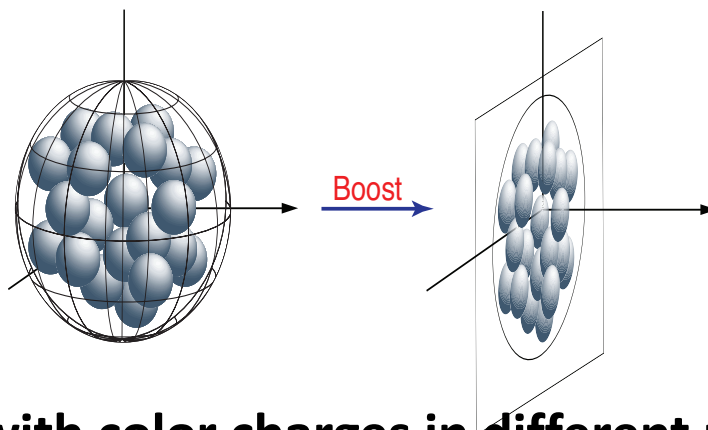
**Sophisticated dipole models
give excellent fits to all HERA small x data**

Saturation scale from dipole model fits to DIS data

$x \leq 0.01$

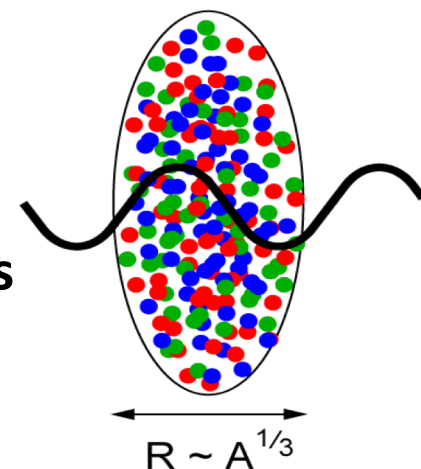


Big nuclear oomph:

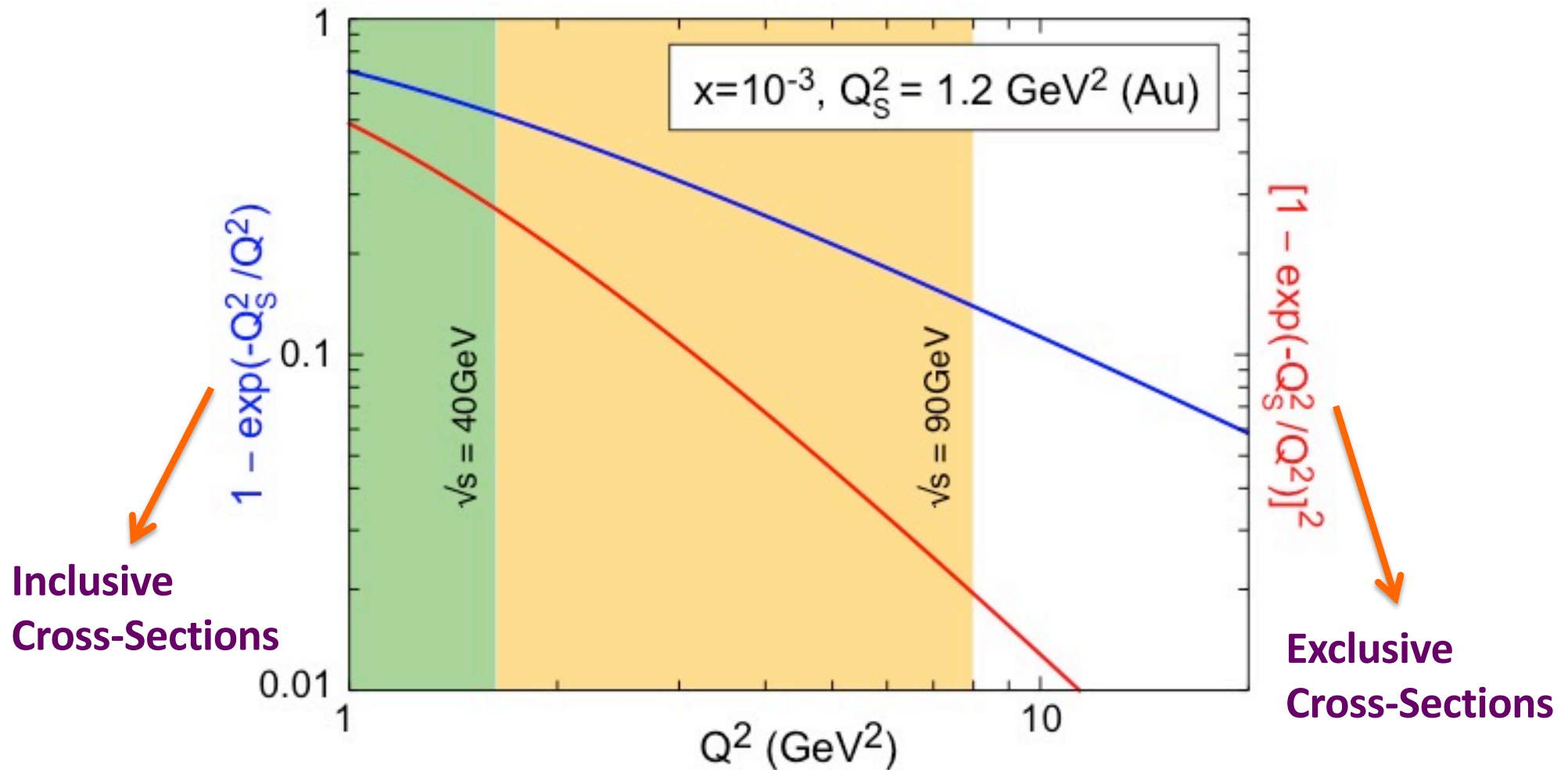


dipole couples coherently with color charges in different nucleons

in path of its scattering: $Q_s^2 \sim A^{1/3}$



Nonlinear response of saturated matter to probes



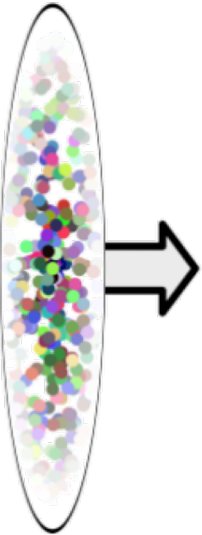
Varying both x and Q^2 essential to see nonlinear response of saturated gluons
- a clear manifestation of the fully nonlinear character of QCD

For further discussion of some potentially striking systematics in DIS off nuclei,
see e.g., Mantysaari and RV, PLB781 (2018) 664

Theory framework: the Color Glass Condensate

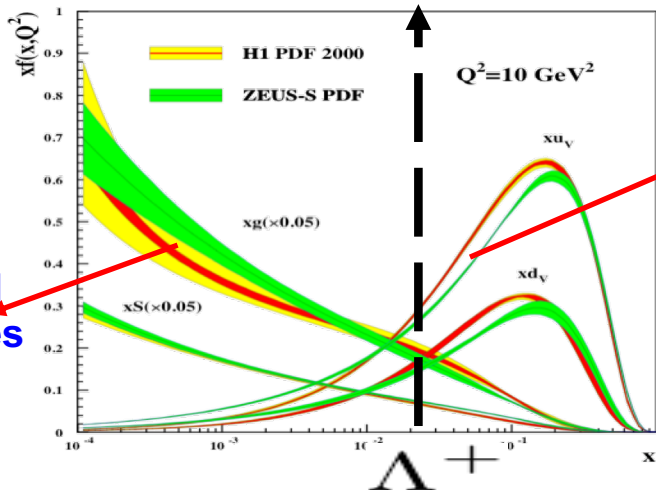


The nuclear wavefunction at high energies



$$|A\rangle = |qqq\dots q\rangle + \dots + |qqq\dots q\underbrace{gg\dots gg}_{\text{gluons}}\rangle$$

Higher Fock components dominate
multiparticle production-
construct Effective Field Theory



Dynamical Wee modes

Valence modes are static sources for wee modes

Born--Oppenheimer LC separation natural for EFT.

RG eqns describe evolution of wavefunction with energy

Effective Field Theory on Light Front

Susskind
Bardacki-Halpern

Poincare group on LF



isomorphism

Galilean sub-group
of 2D Quantum Mechanics

Eg., LF dispersion relation

$$P^- = \frac{P_\perp^2}{2P^+}$$

Energy \swarrow \searrow Momentum
Mass

Large x (P^+) modes: static LF (color) sources ρ^a
Small x ($k^+ \ll P^+$) modes: dynamical fields A_μ^a

McLerran, RV

CGC: Coarse grained many body EFT on LF

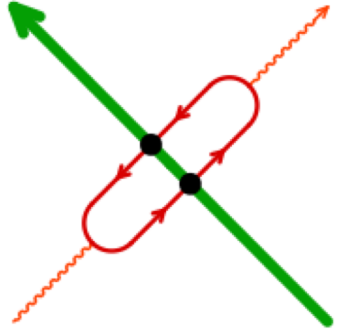
$$\langle P | \mathcal{O} | P \rangle \longrightarrow \int [d\rho^a] [dA^{\mu,a}] W_{\Lambda^+}[\rho] e^{iS_{\Lambda^+}[\rho, A]} \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$ non-pert. gauge invariant “density matrix”
defined at initial scale Λ_0^+

RG equations describe evolution of W with x

JIMWLK, BK

Inclusive DIS: dipole evolution



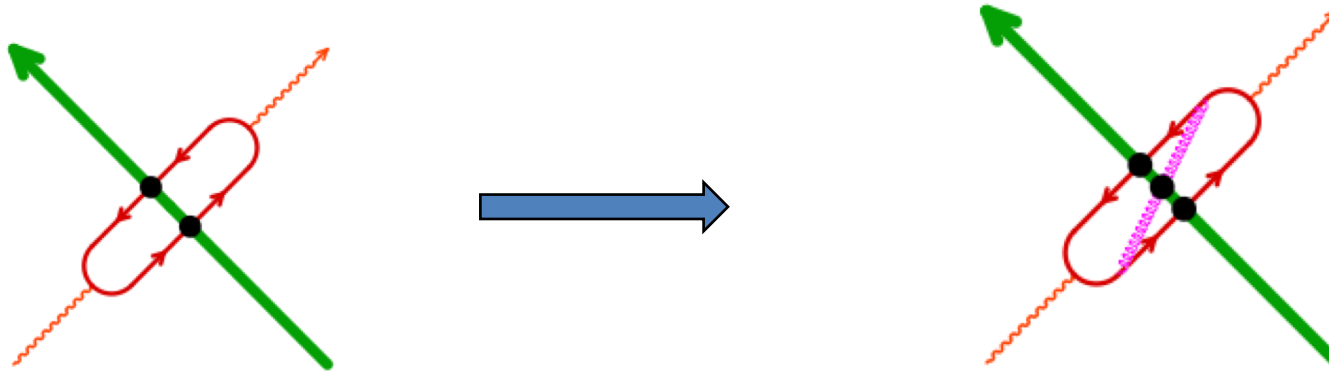
$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_{\perp} |\psi(z, r_{\perp})|^2 \sigma_{\text{dipole}}(x, r_{\perp})$$

$$\sigma_{\text{dipole}}(x, r_{\perp}) = 2 \int d^2 b \int [D\rho] W_{\Lambda^+}[\rho] T\left(b + \frac{r_{\perp}}{2}, b - \frac{r_{\perp}}{2}\right)$$



$$1 - \frac{1}{N_c} \text{Tr} \left(V \left(b + \frac{r_{\perp}}{2} \right) V^{\dagger} \left(b - \frac{r_{\perp}}{2} \right) \right)$$

Inclusive DIS: dipole evolution



B-JIMWLK eqn. for dipole correlator

$$\frac{\partial}{\partial Y} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y = -\frac{\alpha_S N_c}{2\pi^2} \int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \text{Tr}(V_x V_y^\dagger) - \frac{1}{N_c} \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y$$

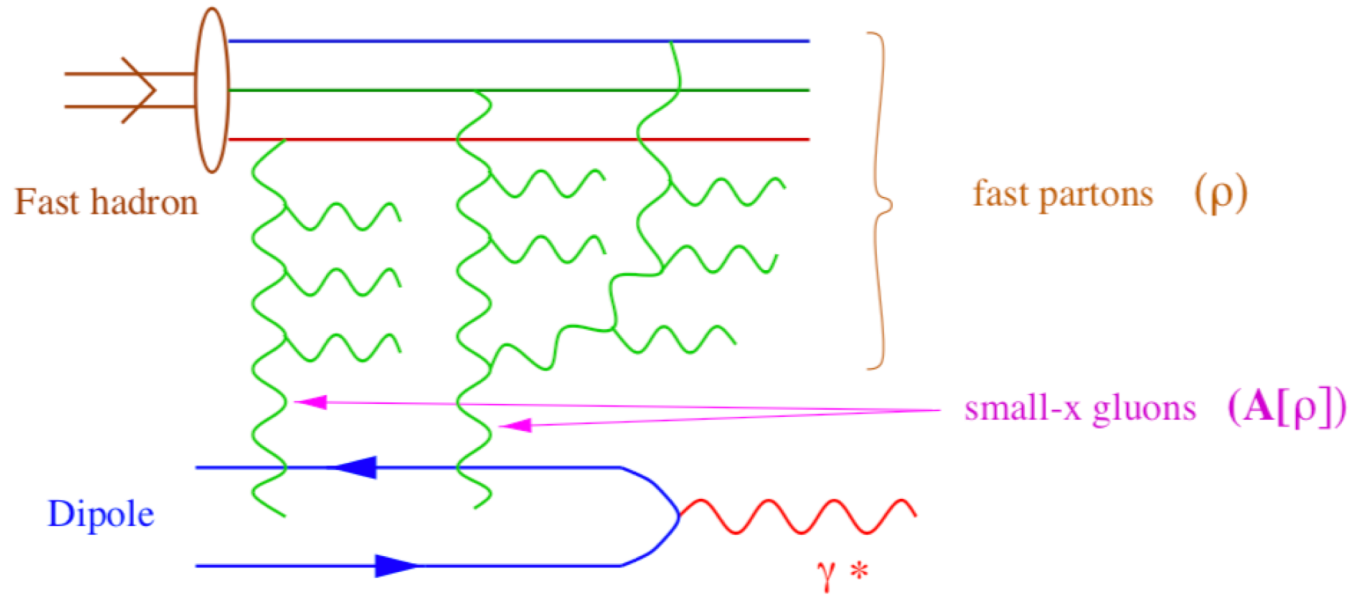
Dipole factorization:

$$\langle \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y \longrightarrow \langle \text{Tr}(V_x V_z^\dagger) \rangle_Y \langle \text{Tr}(V_z V_y^\dagger) \rangle_Y \quad N_c \rightarrow \infty$$

**Resulting closed form eqn. for a large nucleus is the Balitsky-Kovchegov eqn.
Widely used in phenomenological applications**

The BFKL equation is the low density $V \approx 1 - ig\rho/\nabla_t^2$ limit of the BK equation

CGC Effective Theory: B-JIMWLK hierarchy of many-body correlators



$$\frac{\partial}{\partial Y} \langle \mathcal{O}[\rho] \rangle_Y = \frac{1}{2} \left\langle \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi_{x,y}^{ab} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho] \right\rangle_Y$$



“time”

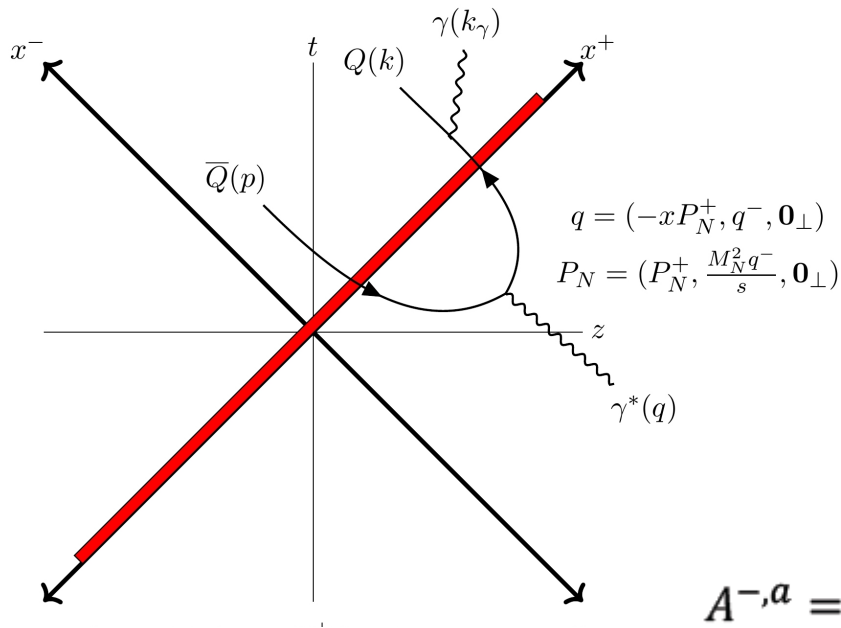


“diffusion coefficient”

Diffusion of the fuzz of “wee” partons in the functional space of colored fields

Can be represented as a Langevin equation that can be solved numerically to “leading logs in x” accuracy.

Lighting up the CGC: Inclusive photon+dijet production

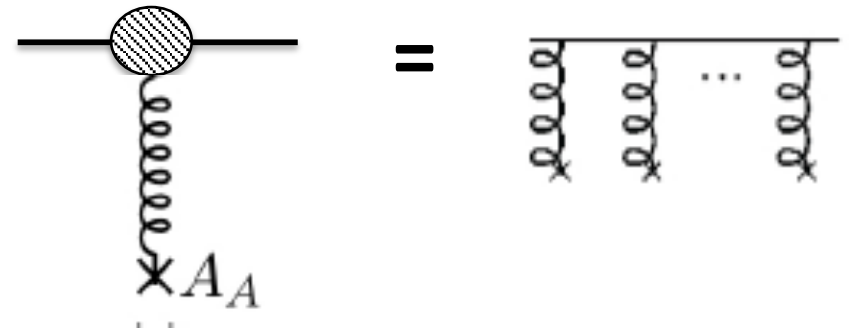
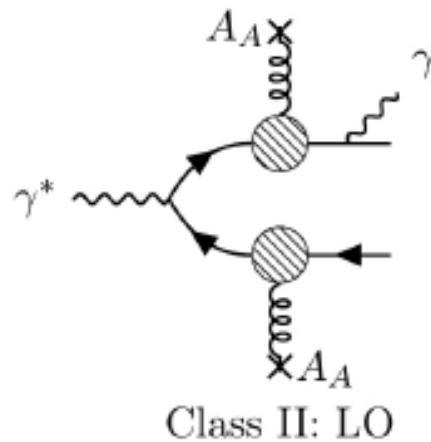
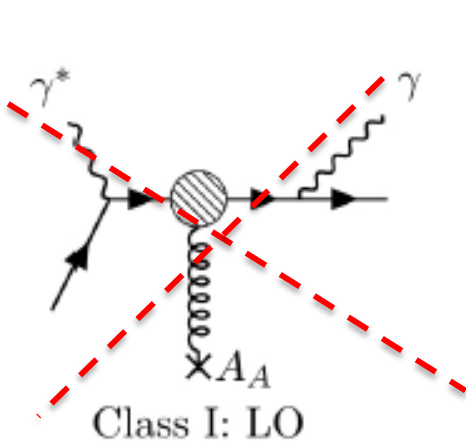


Right moving nucleus with momentum P_N^+ is Lorentz contracted in x^- direction

Glue fields satisfy Yang-Mills eqns.

$$[D_\mu, F^{\mu\nu}](x) = g\delta^{\nu+}\delta(x^-)\rho_A(x_\perp)$$

$$A^{-,a} = 0, F_{ij}^a = 0 \text{ with } A^{+,a}, A^{i,a} \text{ static (independent of } x^+)$$



$$\tilde{U}(\mathbf{x}_\perp) = \mathcal{P}_- \exp \left[-ig^2 \int_{-\infty}^{+\infty} dz^- \frac{1}{\nabla_\perp^2} \rho_A^a(z^-, \mathbf{x}_\perp) t^a \right]$$

Suppressed at small x

Inclusive photon+dijet production in DIS at LO

$$e(\tilde{l}) + A(P) \longrightarrow e(\tilde{l}') + Q(k) + \bar{Q}(p) + \gamma(k_\gamma) + X$$

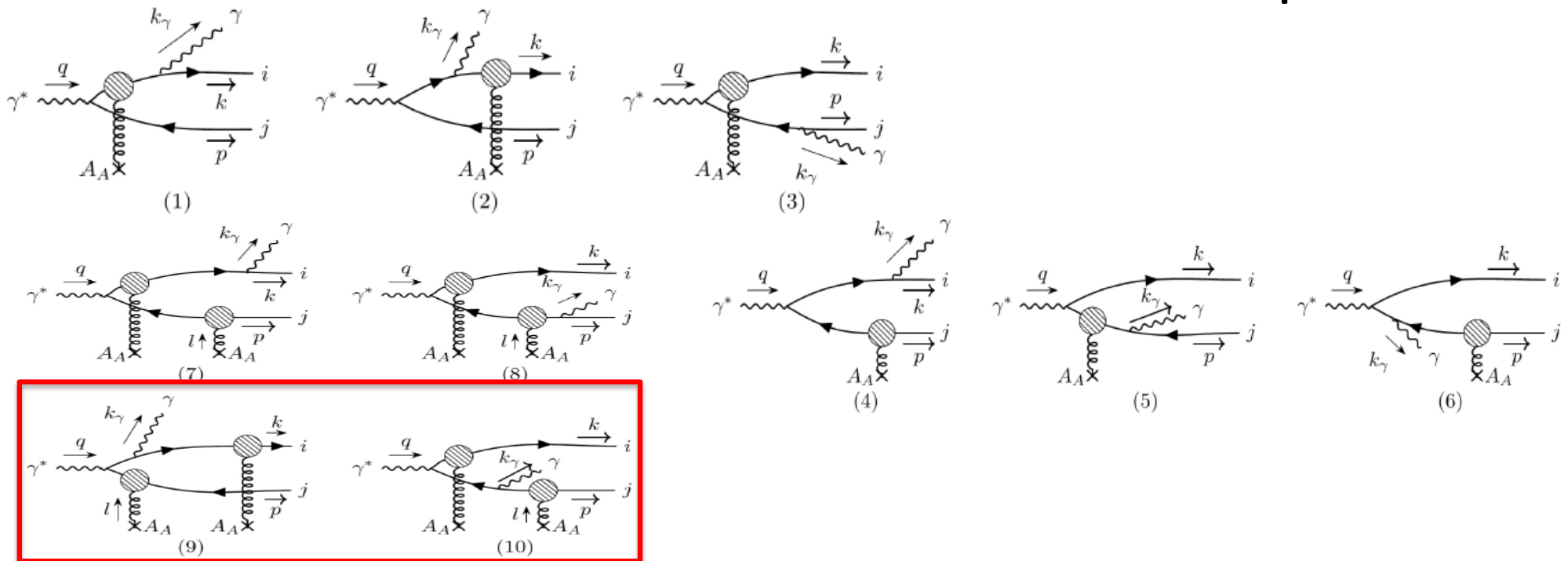
Roy, RV; JHEP 1805 (2018) 013

$$\frac{d\sigma}{dx dQ^2} = \frac{2\pi y^2}{64\pi^3 Q^2} \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3k_\gamma}{(2\pi)^3 2E_{k_\gamma}} \frac{1}{2q^-} \left(\frac{1}{2} \sum_{\text{spins}, \lambda} \langle |\tilde{\mathcal{M}}|^2 \rangle_{Y_A} \right) (2\pi) \delta(P^- - q^-)$$

$$\frac{1}{2} \sum_{\text{spins}, \lambda} \langle |\tilde{\mathcal{M}}|^2 \rangle_{Y_A} = L_{\mu\nu} X^{\mu\nu}$$

$L_{\mu\nu}$ is well known-the lepton tensor

$X^{\mu\nu}$ - the hadron tensor for inclusive photon +dijet production is what we compute



DIS inclusive cross-section at LO

Roy, RV; JHEP 1805 (2018) 013

$$\frac{d\sigma}{dx dQ^2 d^2\mathbf{k}_{\gamma\perp} d\eta_{k_\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2} \frac{1}{2q^-} \int_0^{+\infty} \frac{dk^-}{k^-} \int_0^{+\infty} \frac{dp^-}{p^-} \int_{\mathbf{k}_\perp, \mathbf{p}_\perp} L^{\mu\nu} \tilde{X}_{\mu\nu} (2\pi) \delta(P^- - q^-)$$

$$L^{\mu\nu} = \frac{2e^2}{Q^4} \left[(\tilde{l}^\mu \tilde{l}'^\nu + \tilde{l}'^\nu \tilde{l}^\mu) - \frac{Q^2}{2} g^{\mu\nu} \right]$$

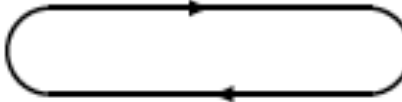
includes Dirac trace

$$\tilde{X}_{\mu\nu} = \int_{\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{l}_\perp, \mathbf{l}'_\perp} e^{-i(\mathbf{P}_\perp - \mathbf{l}_\perp) \cdot \mathbf{x}_\perp - i\mathbf{l}_\perp \cdot \mathbf{y}_\perp + i(\mathbf{P}_\perp - \mathbf{l}'_\perp) \cdot \mathbf{x}'_\perp + i\mathbf{l}'_\perp \cdot \mathbf{y}'_\perp} \tau_{\mu\nu}^{q\bar{q}, q\bar{q}}(\mathbf{l}_\perp, \mathbf{l}'_\perp | \mathbf{P}_\perp) \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$$

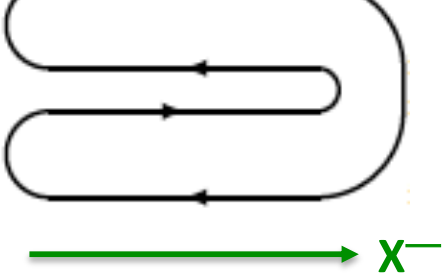
All the nonperturbative info about strongly correlated gluons is in

$$\Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) = 1 - D(\mathbf{x}_\perp, \mathbf{y}_\perp) - D(\mathbf{y}'_\perp, \mathbf{x}'_\perp) + Q(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$$

Dipoles: $D(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$



Quadrupoles: $Q(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(y'_\perp) \tilde{U}^\dagger(x'_\perp) \tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$



Interesting limits

When $k_\gamma \rightarrow 0$, the amplitude satisfies the Low-Burnett-Kroll theorem:

$$\mathcal{M}_\mu(\mathbf{q}, \mathbf{k}, \mathbf{p}, \mathbf{k}_\gamma) \rightarrow -(eq_f)\epsilon_\alpha^*(\mathbf{k}_\gamma, \lambda) \left(\frac{p^\alpha}{p \cdot k_\gamma} - \frac{k^\alpha}{k \cdot k_\gamma} \right) \mathcal{M}_\mu^{NR}(\mathbf{q}, \mathbf{k}, \mathbf{p})$$

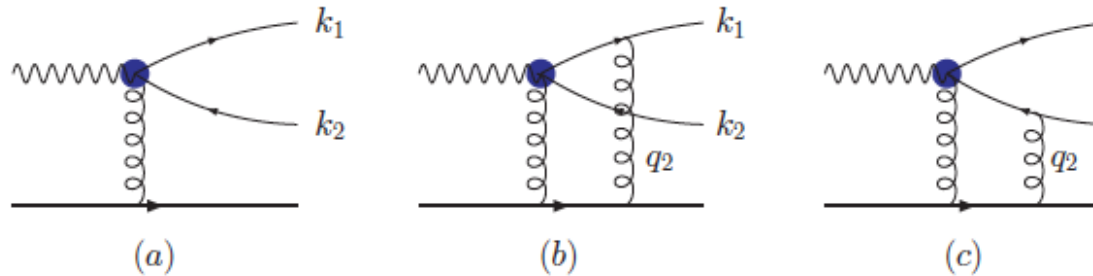
Polarization vector

×

Vectorial structure depending only on momenta of emitted particles

×

Non-radiative DIS amplitude

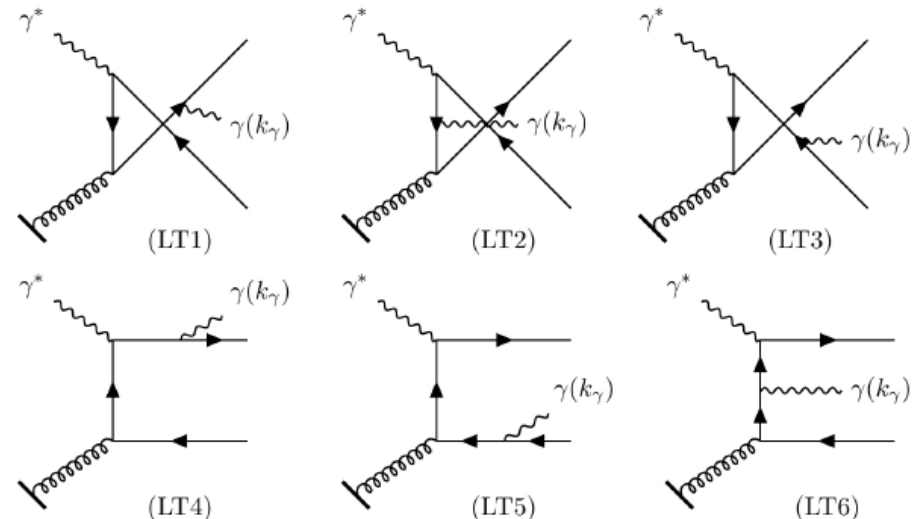


Recover results in soft photon limit for di-jet production
 - sensitive to the gluon Weizsäcker-Williams distribution
 for large pair momenta

Dominguez, Marquet, Xiao, Yuan,
 PRD83 (2011)105005

Recover in DIS
 kt-factorization & collinear factorization
 small x limits
 (sensitivity to leading twist gluon distribution)

Aurenche et al., Z. Phys. C24, 309 (1984)



Structure of higher order computations: Shockwave propagators

Convenient to work in the **wrong** light cone gauge $A^- = 0$ for this problem
(Gauge links in pdf definitions are unity in the **right** LC gauge $A^+ = 0$)

Dressed quark and gluon propagators: remarkably simple forms in $A^- = 0$ gauge

$$S(p, p') = (2\pi)^4 \delta^{(4)}(p - p') S_0(p) + S_0(p) \mathcal{T}(p, p') S_0(p')$$

$$G^{\mu\nu; ab}(p, p') = (2\pi)^4 \delta^{(4)}(p - p') G_0^{\mu\nu; ab} + G_0^{\mu\rho; ac} \mathcal{T}_{\rho\sigma; cd} G_0^{\sigma\nu; db}(p')$$

McLerran, RV; hep-ph/9402335

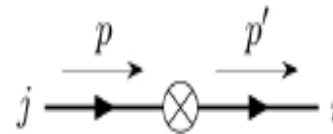
Ayala, Jalilian-Marian, McLerran, RV;
hep-ph/9508302

McLerran, RV; hep-ph/9809427

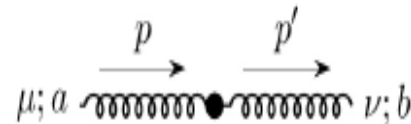
Balitsky, Belitsky, hep-ph/0110158

Structure of vertices identical
to quark-quark-reggion and
gluon-gluon-reggeon in
Lipatov's Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov,
Prygarin, arXiv:1708.05183
Hentschinski, arXiv:1802.06755

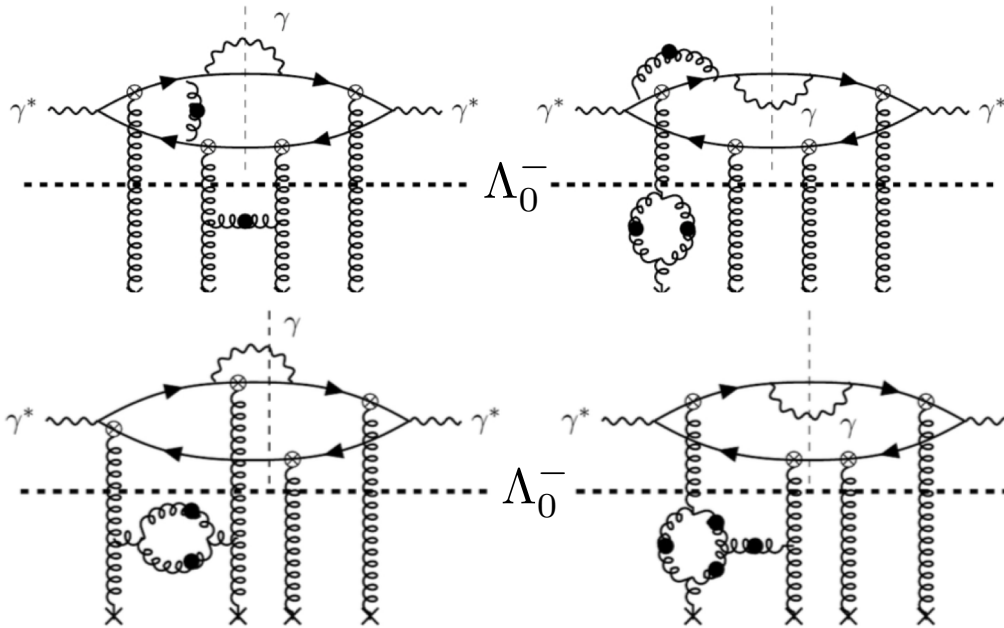


$$\mathcal{T}_{ij}(p, p') = (2\pi) \delta(p^- - p'^-) \gamma^- \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} \tilde{U}^{\text{sign}(p^-)}(\mathbf{z}_\perp)_{ij}$$



$$\mathcal{T}_{\mu\nu; ab}(p, p') = -2\pi \delta(p^- - p'^-) \times (2p^-) g_{\mu\nu} \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} U_{ab}^{\text{sign}(p^-)}(\mathbf{z}_\perp)$$

DIS inclusive photo+dijet production at NLO+NLLx



Roy, RV: in preparation

Formally NNLO: but collect NLO pieces in the photon+dijet impact factor + leading log pieces $\alpha_s \log(\Lambda_1^- / \Lambda_0^-)$

Formally NNLO: but collect LO pieces in the photon+dijet impact factor + NLL leading log pieces $\alpha_s^2 \log(\Lambda_1^- / \Lambda_0^-)$

$$\begin{aligned}
 \langle d\sigma_{NLO+NLLx} \rangle &= \int [\mathcal{D}\rho_A] \left\{ W^{NLLx}[\rho_A] d\hat{\sigma}_{LO}[\rho_A] + W^{LLx}[\rho_A] d\hat{\sigma}_{NLO}[\rho_A] \right\} \\
 &= \int [\mathcal{D}\rho_A] \left(W^{NLLx}[\rho_A] \left\{ d\hat{\sigma}_{LO}[\rho_A] + \boxed{d\hat{\sigma}_{NLO}[\rho_A]} \right\} + O(\alpha_s^3 \ln(\Lambda_1^- / \Lambda_0^-)) \right)
 \end{aligned}$$

$\ln(\Lambda_1^- / \Lambda_0^-) (\mathcal{H}_{LO} + \mathcal{H}_{NLO})$
NLO photon+dijet impact factor

Leading order JIMWLK Hamiltonian computed 20 years ago: 1997-2001

NLO JIMWLK Hamiltonian: 2013-2016

Balitsky, Chirilli, arXiv:1309.7644, Grabovsky, arXiv:1307.5414

Caron-Huot, arXiv:1309.6521, Kovner, Lublinsky, Mulian, arXiv:1310.0378, Lublinsky, Mulian, arXiv:1610.03453

The NLO inclusive photon impact factor

Several computations exist for inclusive DIS – subtleties in choice of scheme, etc.

Balitsky,Chirilli, arXiv:1009.4729

Beuf, arXiv:1606.00777, 1708.06557

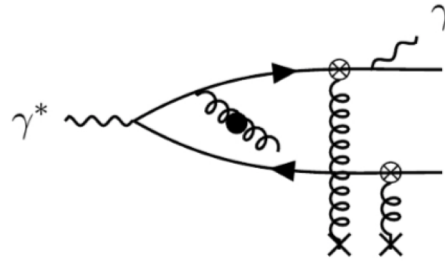
Hanninen, Lappi, Paatelainen, 1711.08207

Dijet: Boussarie, Grabovsky, Szymanowski, Wallon,1606.00419

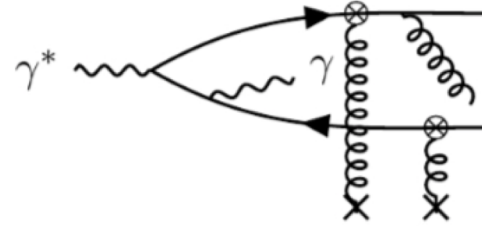
First computation discussed here of photon+dijet:

Roy, RV, in preparation

I) Real contributions:
(20 × 20 diagrams)



Gluon rescatters (or not) along with quarks

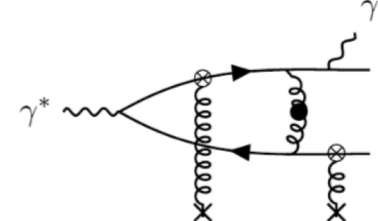
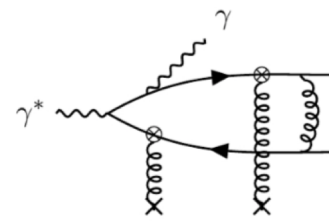
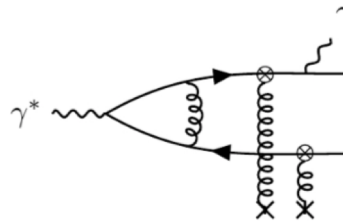


Gluon emitted after rescattering

II) Interference contributions:

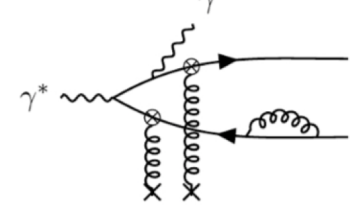
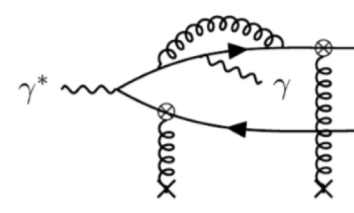
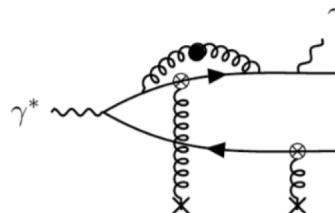
A) Vertex corrections

●●● and 21 more permutations



B) Self-energy corrections:

●●● and likewise, 33 more



Coda: A nontrivial derivation of JIMWLK evolution

coda : something that serves to round out, conclude, or summarize and usually has its own interest.

Roy, RV, in preparation

$$\text{Recall } X_{\mu\nu}^{\text{LO}} = C_{\mu\nu}^{\text{LO}} \otimes \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-) \quad \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-) = 1 - D_{xy} - D_{y'x'} + Q_{y'x';xy}$$

In the "soft gluon limit" that generates logs in x, our NLO hadron tensor gives

$$\begin{aligned} X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln(\Lambda_1^- / \Lambda_0^-) & \left[\frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) D_{xy} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} - \frac{\alpha_S N_c}{(2\pi)^2} \mathcal{K}_1(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{y}'_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) Q_{xy;y'x'} \right. \\ & - \frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) D_{xz} D_{zy} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} + \frac{\alpha_S N_c}{(2\pi)^2} \left(\left\{ \mathcal{A}(\mathbf{x}_\perp, \mathbf{y}'_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) D_{xx'} D_{y'y} + \mathbf{x}_\perp \leftrightarrow \mathbf{y}'_\perp \right\} \right. \\ & \left. \left. + \left\{ \mathcal{K}_2(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) D_{xz} Q_{zy;y'x'} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} + \left\{ \mathcal{K}_2(\mathbf{x}'_\perp, \mathbf{x}_\perp, \mathbf{y}'_\perp; \mathbf{z}_\perp) D_{zx'} Q_{y'z;xy'} + \left(\frac{\mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp}{\mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp} \right) \right\} \right) \right], \end{aligned}$$

Nontrivial combinations of dipole and quadrupole operators

Dominguez,Mueller,Munier,Xiao, PLB705(2011)106

Remarkably, this can be reexpressed as $X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln \left(\frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-)$

This immediately leads to the JIMWLK RG equation for the rapidity evolution of many-body gluon correlators

$$W_{\Lambda_1^-}[\rho_A] = \left(1 + \ln \left(\frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \right) W_{\Lambda_0^-}[\rho_A]$$

Color Memory in the CGC

Ball,Pate,Raclariu,Strominger,RV,
arXiv:0805.12224

$$A_i = 0 \qquad A_i = \frac{-1}{ig} U \partial_i U^\dagger$$

$$x^- = 0$$

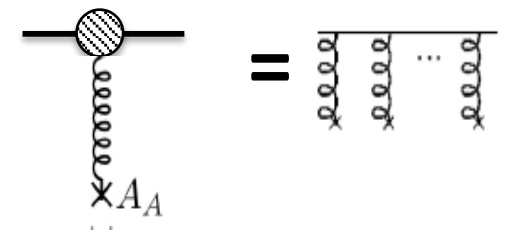
In the CGC, the solutions of the YM-eqns can be represented as two pure gauges separated by a discontinuity at $x^- = 0$ corresponding to the shockwave and A_i satisfies

$$D_i \frac{dA^{i,a}}{dy} = g\rho^a(x_t, y) \quad \text{with } y = \text{Ln}(x^-/x_0^-)$$

with the solution

$$U = P \exp \left(i \int_y^\infty dy' \frac{\rho(x_t, y')}{\nabla_t^2} \right)$$

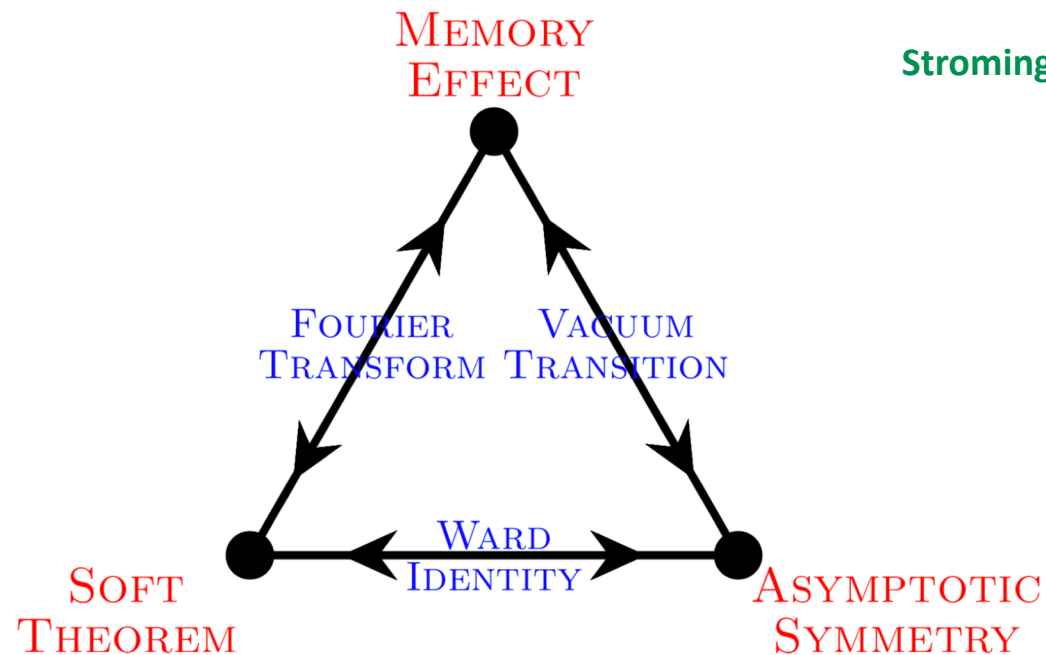
This U is precisely the *color memory* effect corresponding to a color rotation and $p_T \sim Q_s$ kick experienced by a quark-antiquark pair traversing the shock wave – discussed previously as a property of YM fields by Pate, Raclariu and Strominger (1707.08016)



Its presence is ubiquitous in DIS final states

Memory and asymptotic symmetries

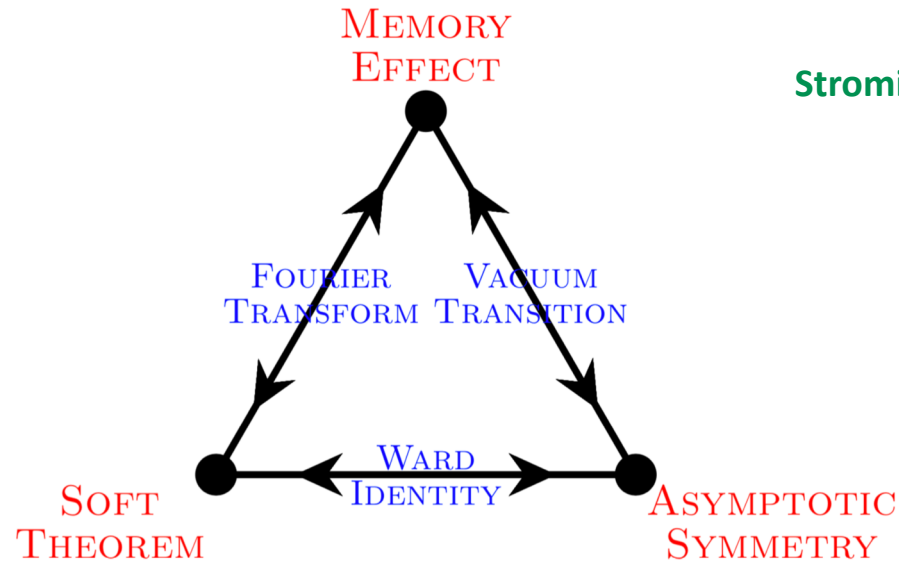
Strominger, arXiv:1703.05448



Conjectured to be very general property of the infrared in gauge theories & gravity
In gravity, the symmetries are the BMS symmetry and the corresponding gravitational memory leads to a physical displacement of inertial detectors measurable by LIGO

Memory and asymptotic symmetries

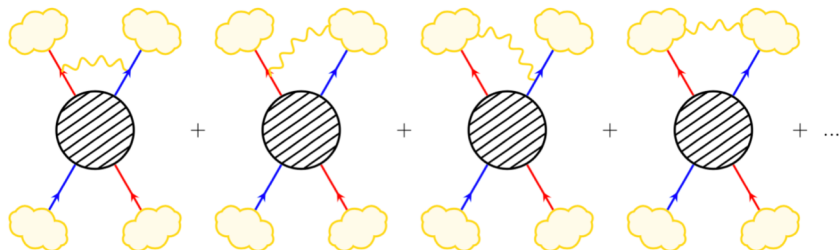
Strominger, arXiv:1703.05448



In QED, these correspond to a symmetry group of an infinite number of conserved charges at $x^- = \pm\infty$ on the celestial sphere (stereographic projection of transverse plane) and these satisfy a conservation law with the S-matrix

$$\langle \text{out} | (Q_\epsilon^+ \mathcal{S} - \mathcal{S} Q_\epsilon^-) | \text{in} \rangle = 0$$

This is equivalent to the soft photon theorem and also imposes a condition that the S-matrix be dressed by a soft factor that renders it infrared finite

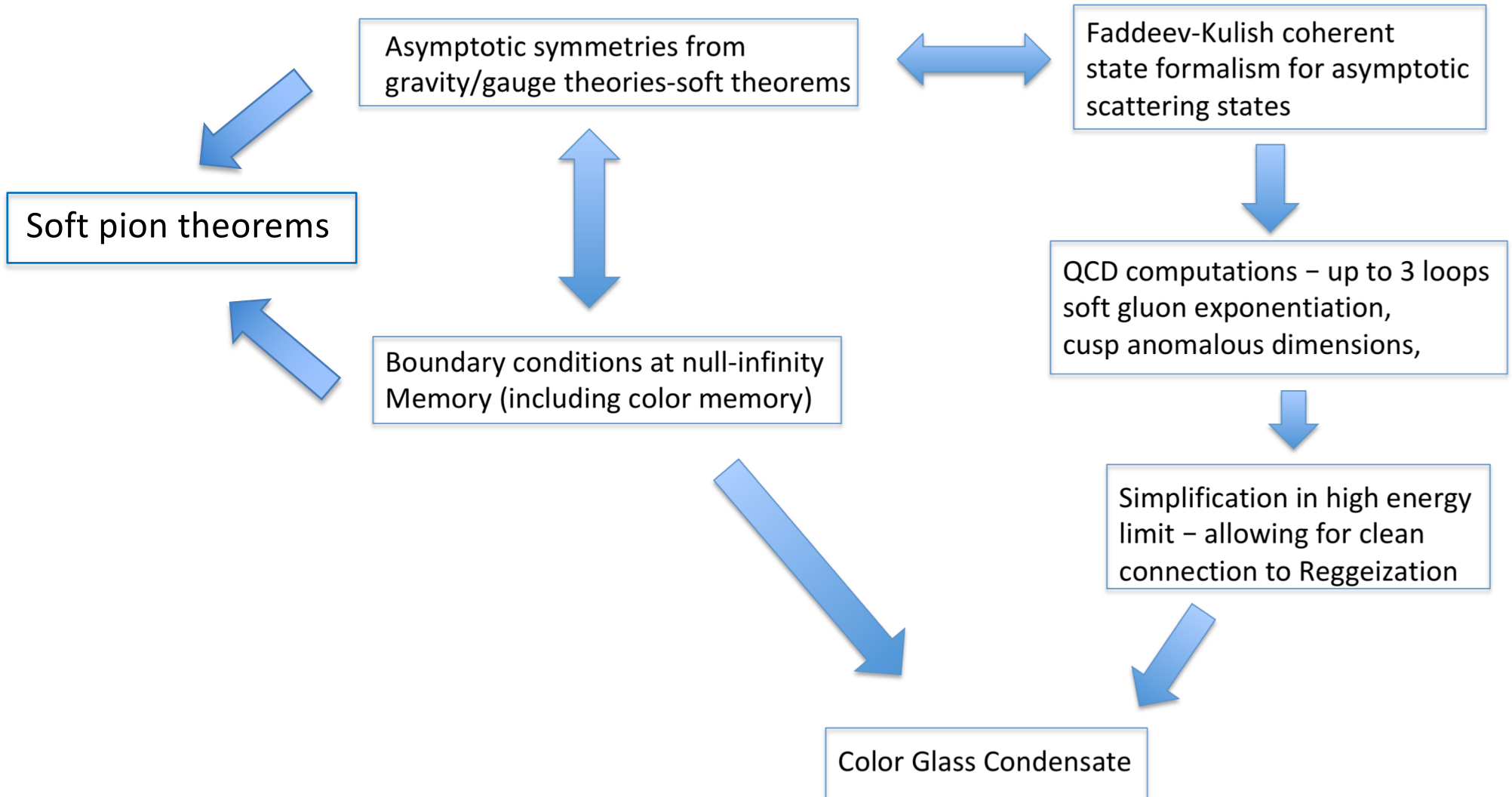


Equivalent to Faddeev-Kulish (!1970)
coherent state representation of S-matrix

Kapec, Perry, Raclariu, Strominger, arXiv:1705.043011

On light front, for FK rep. see More, Misra, 1206.3097

Bold conjecture: QCD in the infrared



Thank you for your attention!

Classical field of a large nucleus

$$\langle AA \rangle_\rho = \int [d\rho] A_{\text{cl.}}(\rho) A_{\text{cl.}}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus, $A \gg 1$,

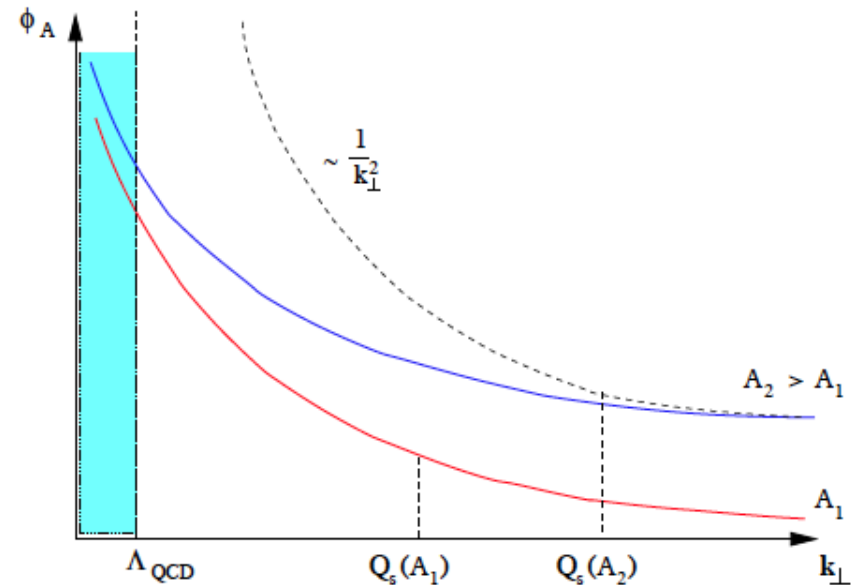
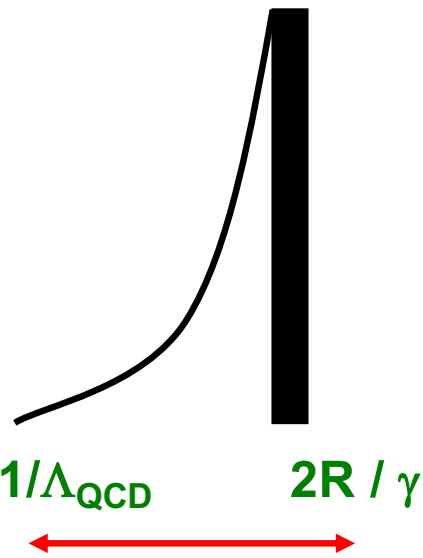
$$W_{\Lambda^+} = \exp \left(- \int d^2 x_\perp \left[\frac{\rho^a \rho^a}{2\mu_A^2} - \frac{d_{abc} \rho^a \rho^b \rho^c}{\kappa_A} \right] \right)$$

“Pomeron” excitations

“Odderon” excitations

McLerran, RV
Kovchegov
Jeon, RV

A_{cl} from $\longrightarrow (D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+} \delta(x^-) \rho^a(x_\perp)$



Wee parton
dist. :

$$\frac{1}{\Lambda_{\text{QCD}}} e^{-\lambda \Delta Y / 2}$$

determined from RG evolution

The NLO inclusive photon impact factor

Inclusive photon cross-section: NLO-real * NLO*real + LO * NLO virtual

Wilson line factor	Real emission	Virtual: Vertex	Virtual: Self-energy
$\frac{N_c^2}{2} \left(1 - D_{xz} D_{zy} - D_{y'z} D_{zx'} + D_{y'y} D_{xx'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(1)*} T_R^{(1)}$		
$C_F N_c \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(2)} + T_R^{(3)*} T_R^{(3)}$	$T_{LO}^* T_V^{(3)} + c.c$	$T_{LO}^* T_S^{(3)} + c.c$
$\frac{N_c^2}{2} [(1 - D_{xy})(1 - D_{y'x'})] - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(3)} + c.c$	$T_{LO}^* T_V^{(4)} + c.c$	
$\frac{N_c^2}{2} \left(1 + (Q_{zy;y'x'} - D_{zy}) D_{xz} - D_{y'x'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(2)*} T_R^{(1)}$	$T_{LO}^* T_V^{(1)}$	$T_{LO}^* T_S^{(1)}$
$\frac{N_c^2}{2} \left(1 + (Q_{xy;y'z} - D_{y'z}) D_{zx'} - D_{xy} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(1)*} T_R^{(2)}$	$T_V^{(1)*} T_{LO}$	$T_S^{(1)*} T_{LO}$
$\frac{N_c^2}{2} \left(1 + (Q_{y'x';xz} - D_{xz}) D_{zy} - D_{y'x'} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(3)*} T_R^{(1)}$	$T_{LO}^* T_V^{(2)}$	$T_{LO}^* T_S^{(2)}$
$\frac{N_c^2}{2} \left(1 + (Q_{xy;zx'} - D_{zx'}) D_{y'z} - D_{xy} \right) - \frac{1}{2} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$	$T_R^{(1)*} T_R^{(3)}$	$T_V^{(2)*} T_{LO}$	$T_S^{(2)*} T_{LO}$

For each Wilson line structure, collinear divergences cancel between real and Interference contributions

Rapidity and UV divergent pieces: these can be absorbed, in a subtraction scheme, into the NLLx JIMWLK expressions.