Resummed High Energy Non-Linear Evolution at NLO

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B. Ducloué, E. Iancu, A.H. Mueller, E. Iancu, DT, in preparation

Outline

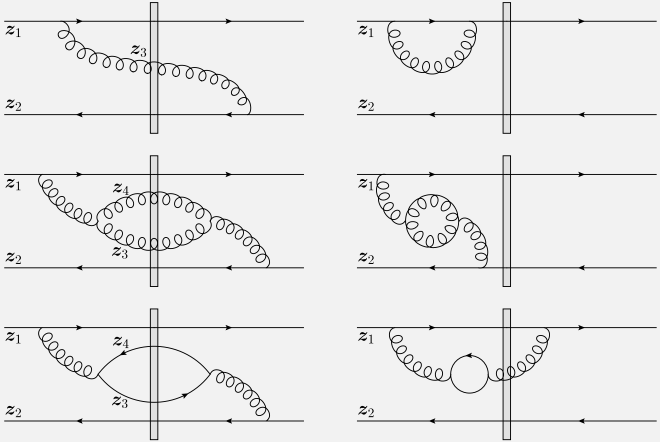
- Large transverse logarithms in NLO BK and instabilities
- Resummation (double logs) and stability
- Issues with resummed evolution in projectile rapidity (Y)
- Comparison of saturation fronts in Y and η
- NLO BK in target rapidity (η)
- Resummed evolution in target rapidity (η) via shifts
- Matching to NLO BK

NLO BK Evolution

LO

NLO N_c

NLO N_f



Evolution of right moving projectile: modes with smaller longitudinal momentum k^+. Soft plus non-soft (in general).

NLO BK Evolution

Proj: (z_1, z_2), q^+, $q^- = 1/2z_{12}^2 q^+$ Targ: Q_0^2, q_0^-, $q_0^+ = Q_0^2/2q_0^-$
NLO evolution for $S_{12} \equiv S(z_1, z_2; Q_0^2; Y)$, with $Y = \ln q^+/q_0^+$:

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{\mathrm{d}^2 z_3\, z_{12}^2}{z_{13}^2 z_{32}^2}\Bigg[1 + \bar{\alpha}_s\Bigg(\underbrace{\bar{b}\ln z_{12}^2\mu^2 - \bar{b}\,\frac{z_{13}^2 - z_{23}^2}{z_{12}^2}\ln\frac{z_{13}^2}{z_{12}^2}}_{\text{RC: choose } \mu = 1/z_{ij\,\mathrm{min}}}$$

$$\underbrace{-\frac{1}{2}\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}}_{\text{TO in Projectile}}\Bigg)\Bigg](S_{13}S_{32} - S_{12})$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\int\frac{\mathrm{d}^2 z_3\,\mathrm{d}^2 z_4}{z_{34}^4}\Bigg(\underbrace{-2 + \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4 z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}}_{\text{Proj \& Targ DGLAP} \rightsquigarrow -11/12}\Bigg)$$

$$\times(S_{13}S_{34}S_{42} - S_{13}S_{32})$$

$$+\mathcal{O}(N_f) + \mathcal{O}(1/N_c^2) + \text{regular terms}$$

Large Transverse Logarithms

Strongly ordered large perturbative dipoles (DLA)

$$z_{12} \ll z_{13} \simeq z_{23} \ll z_{14} \simeq z_{24} \simeq z_{34} \ll 1/Q_s$$

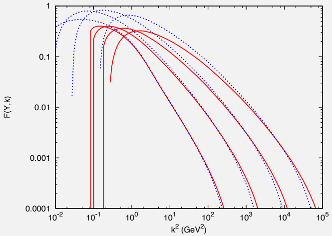
Linearize, large dipoles strong interaction, reals terms dominate

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \mathrm{d}z_{13}^2 \frac{z_{12}^2}{z_{13}^4} \left(1 - \frac{\bar{\alpha}_s}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2}\right) T_{13}$$

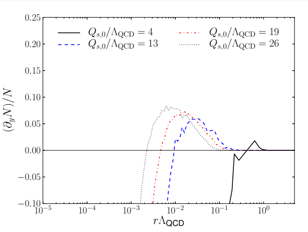
NLO>LO, unstable expansion in coupling

Even single iteration leads to negative solution

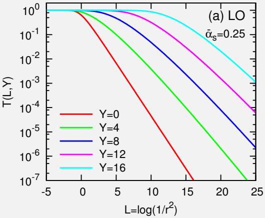
Unstable Numerical Solutions



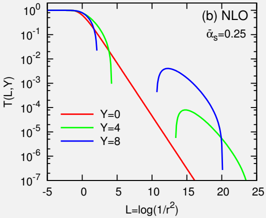
NLO BFKL + Sat Bound



NLO BK

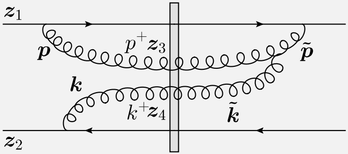


LO BK



LO BK + Double Log

Time Ordering and Resummation in DLA



Hard to soft projectile evolution: $\boldsymbol{k} \ll \boldsymbol{p}$ and $k^+ \ll p^+$
Time ordering non-trivial, requires: $\tau_k \sim k^+ z_4^2 \ll \tau_p \sim p^+ z_3^2$
Resum to all orders in a non-local equation

$$\frac{\mathrm{d}T_{12}(Y, z_{12}^2)}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \Theta\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}\right) T\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}, z_{13}^2\right)$$

Mathematically equivalent to local equation

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}\right)}{\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}} T_{13}$$

Resummation in BK

Match local resummed DLA equation to include BK physics

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2 z_3 z_{12}^2}{z_{13}^2 z_{32}^2} \mathcal{K}_{\mathrm{DLA}} \left(\sqrt{\ln \frac{z_{13}^2}{z_{12}^2} \ln \frac{z_{23}^2}{z_{12}^2}} \right) (S_{13}S_{32} - S_{12})$$

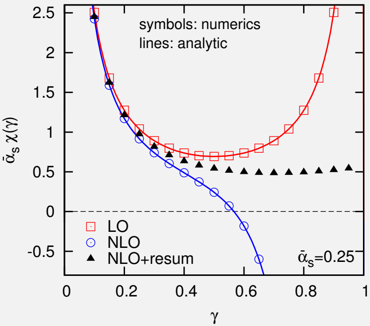
with

$$\mathcal{K}_{\mathrm{DLA}}(\rho) = \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \rho^2}\right)}{\sqrt{\bar{\alpha}_s \rho^2}} = 1 - \frac{\bar{\alpha}_s \rho^2}{2} + \frac{(\bar{\alpha}_s \rho^2)^2}{12} + \cdots$$

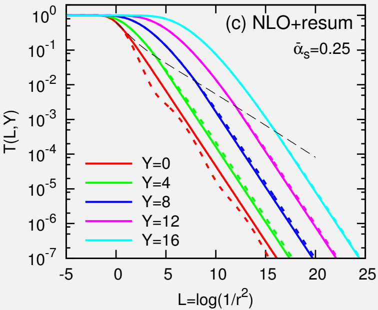
Resums double logarithms to all orders (and nothing more)

LO BK + NLO double log when truncated to $\bar{\alpha}_s^2$

Numerical Solution



Linear on $z_{12}^{2\gamma} \equiv r^{2\gamma}$

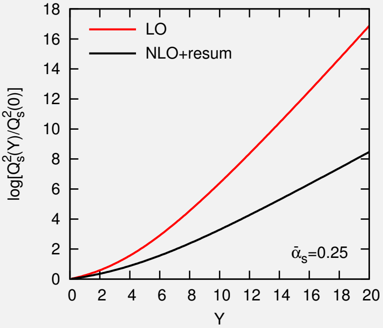


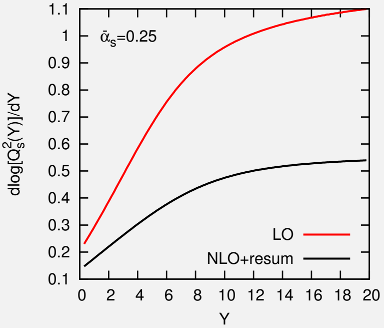
Tail $\sim r^2$

$$\omega_{\rm LO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} + \text{regular} \qquad \omega_{\rm NLO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \text{regular}$$

$$\omega = \frac{\bar{\alpha}_s}{\gamma} + \frac{1}{2}\left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s}\right] + \text{regular}$$

Numerical Solution





Considerable speed reduction, perhaps too much?

Target Rapidity η

Target rapidity determines the kinematics

$$\eta \equiv \ln\frac{1}{x_{\rm Bj}} = \ln\frac{s}{Q^2} = \ln\frac{2q^+q_0^-}{2q^+q^-} = \ln\frac{q_0^-}{q^-}$$
$$= \ln\frac{Q_0^2}{2q_0^+}\frac{2q^+}{Q^2} \to Y - \ln\frac{1}{r^2Q_0^2} \equiv Y - \rho$$

NB:

Could have started from target DGLAP in η

Change variables from (η,ρ) to (Y,ρ)

Get large logarithms in the BK kernel

ISSUES/COMMENTS WITH EVOLUTION IN Y

- Initial condition at $Y = 0$ or boundary at $\eta = 0 \Leftrightarrow Y = \rho$
- Erroneous use of MV or GBW type IC leads to unphysical pushed front
- $\gamma_s \sim 1$, where is BFKL dynamics? λ_s seems too small (compare to DT 03)
- For DIS express final result in terms of $\eta = Y - \rho$ and ρ
  Saturation: target property, need $Q_s^2(\eta)$
- Fronts in Y and η very different for relevant $\bar{\alpha}_s$ values
- Saturation intercept in $Q_s^2(\eta)$ looks unphysical

Starting the Evolution

- Physical initial condition given at $\eta = 0$
- Construct initial condition at unphysical value $Y = 0$

  Can do at level of DLA, e.g. for GBW physical IC

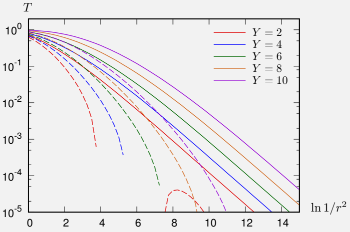
  $$z_{12}^2 Q_0^2 \to z_{12}^2 Q_0^2 \mathrm{J}_0 \left(2\sqrt{\bar{\alpha}_s \ln^2 z_{12}^2 Q_0^2} \right) \quad \text{for} \quad z_{12}^2 Q_0^2 \ll 1$$

  Analytically continued backwards DLA evolution to

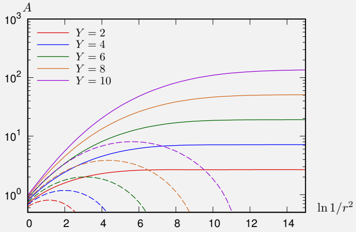
  $\eta = -\rho \Leftrightarrow Y = 0$

  Exponentiate to unitarize
- Evolution will not likely reproduce physical IC at $\eta = 0$

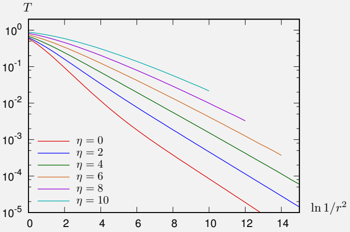
(A Little) Ugly Solution?

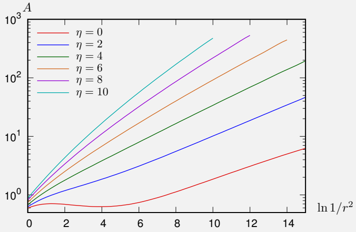


dashed: resummed J_0



$A = T/r^2$, dashed: resummed J_0



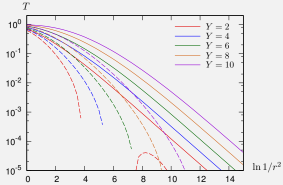


Difficult to trust solution, not able to solve BC problem

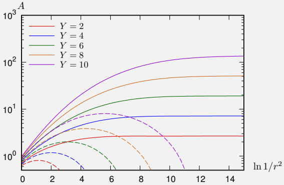
More Beautiful but Mostly Wrong Solution

Try GBW IC at $Y = 0$?

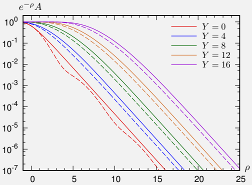
A mixture of GBW and resummed IC?



solid: GBW



$A = T/r^2$, solid: GBW



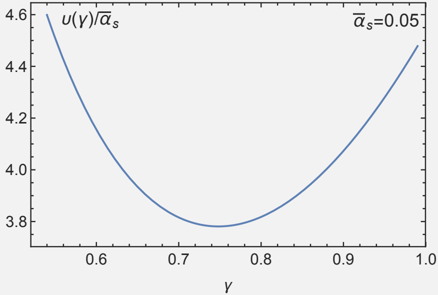
$[(1 + \mathrm{J}_0)/2]$GBW, GBW

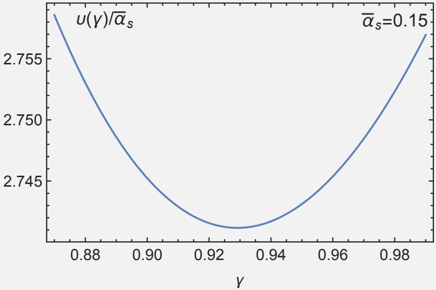
Fronts look nice.

They are unphysical.

Why $\gamma_s = 1$?

Steepness of Initial Condition





Velocity function: $v(\gamma) = \omega(\gamma)/\gamma$

Saturation saddle point: $v'(\gamma_s) = 0$

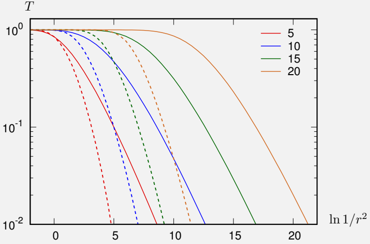
Front speed: $\lambda_s = \mathrm{d}\ln Q_s^2/\mathrm{d}Y = v(\gamma_s) + \mathcal{O}(1/Y)$

For $\bar{\alpha}_s < \bar{\alpha}_s^* \simeq 0.22$, $\gamma_s < \gamma_{\rm ic} = 1$: pulled front ✓

For $\bar{\alpha}_s > \bar{\alpha}_s^* \simeq 0.22$, $\gamma_s > \gamma_{\rm ic} = 1$: pushed front ✗

J_0 makes IC steep enough for all (relevant) values of $\bar{\alpha}_s$

Slope and Speed for the η-Front



Front in $Y \Leftrightarrow$ Front in η, physical

Scaling in Y: $-\ln T = \gamma_s(\rho - \lambda_s Y)$

Change variable $Y = \eta + \rho$

Scaling in η: $-\ln \bar{T} = \bar{\gamma}_s(\rho - \bar{\lambda}_s \eta)$

(valid inside diffusion radius)

$$\boxed{\bar{\gamma}_s = \gamma_s(1 - \lambda_s)} \quad \text{and} \quad \boxed{\bar{\lambda}_s = \frac{\lambda_s}{1 - \lambda_s}}$$

$\lambda_s = \mathcal{O}(\bar{\alpha}_s)$, difference is NLO. In practice it is large.

Physical η-front less steep and faster

Unphysical Intercept in Resummed Evolution

Solve analytically SP condition and/or numerically BK

Determine λ_s and γ_s for Y evolution

Transform to $\bar{\lambda}_s$ and $\bar{\gamma}_s$ for η evolution

$\bar{\alpha}_s$	λ_s	γ_s	$\bar{\lambda}_s$	$\bar{\gamma}_s$
$\to 0$	$4.88\bar{\alpha}_s$	0.628	$4.88\bar{\alpha}_s$	0.628
0.1	$0.313 = 3.13\bar{\alpha}_s$	0.847	$0.456 = 4.56\bar{\alpha}_s$	0.582
0.2	$0.489 = 2.45\bar{\alpha}_s$	0.977	$0.957 = 4.78\bar{\alpha}_s$	0.499
0.3	$0.645 = 2.15\bar{\alpha}_s$	1.250	$1.820 = 6.06\bar{\alpha}_s$	0.444

Resummed evolution is not reliable for $\bar{\alpha}_s \gtrsim 0.1$.

NLO BK in the Target Rapidity η (I)

• Always evolve the projectile: dipoles kernel with projectile coordinates in transverse space

• Variable change $\eta = Y - \rho$ in longitudinal space

$$S(Y,r) = S\big(\eta + \ln(1/r^2 Q_0^2), r\big) \equiv \bar{S}(\eta, r)$$
$$S(Y,z) = S\big(\eta + \ln(1/z^2 Q_0^2) - \underbrace{\ln(r^2/z^2)}_{\delta_z}, z\big) = \bar{S}(\eta - \delta_z, z)$$

Non-local, but at NLO one treats $\delta_z \sim \mathcal{O}(1)$ and expands

$$\bar{S}(\eta - \delta_z, z) = \bar{S}(\eta, z) - \delta_z \frac{\partial \bar{S}(\eta, z)}{\partial \eta}$$

To order of accuracy use LO BK for $\partial \bar{S}(\eta, z)/\partial \eta$

NLO BK in the Target Rapidity η (II)

Shift generates new $\mathcal{O}(\bar{\alpha}_s^2)$ term from the $\mathcal{O}(\bar{\alpha}_s)$ term

Shift does not modify $\mathcal{O}(\bar{\alpha}_s^2)$ terms

Overall, NLO BK in η has the extra term on the r.h.s.

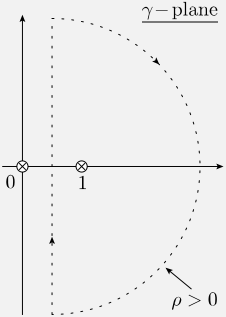
$$\boxed{\frac{\bar{\alpha}_s^2}{2\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{z}\, \mathrm{d}^2\boldsymbol{u}\, (\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{u})^2(\boldsymbol{u}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{u}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\, \bar{S}_{\boldsymbol{xu}} \left[\bar{S}_{\boldsymbol{uz}}\bar{S}_{\boldsymbol{zy}} - \bar{S}_{\boldsymbol{uy}}\right]}$$

Linearizing: cancel double logs of large dipoles and create

$$-\frac{\bar{\alpha}_s^2}{4\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\, (\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \left[\ln \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2} \ln \frac{(\boldsymbol{z}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2}\, \bar{T}_{\boldsymbol{xz}} + \mathrm{symmetric}\right]$$

Large double logarithms for small daughter dipoles

Small Dipoles in η-Evolution. An Instability?



$$\bar{T}(\eta,\rho) = \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}}\, \bar{T}_0(\gamma)\, \mathrm{e}^{\bar{\omega}(\gamma)\eta-\gamma\rho}$$

$$\bar{\omega}(\gamma) = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{\gamma^3} + \mathrm{regular}$$

Bad poles not enclosed for $\rho > 0$

$$\frac{\partial \bar{T}(\eta,r)}{\partial\eta} \simeq \bar{\alpha}_s \int_0^{r^2} \frac{\mathrm{d}z^2}{z^2} \left(1 - \bar{\alpha}_s \ln^2 \frac{r^2}{z^2}\right) \bar{T}(\eta,z) \quad \mathrm{for} \quad z \ll r$$

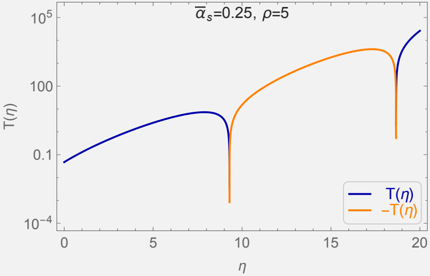
No double logarithms in solution due to color transparency.

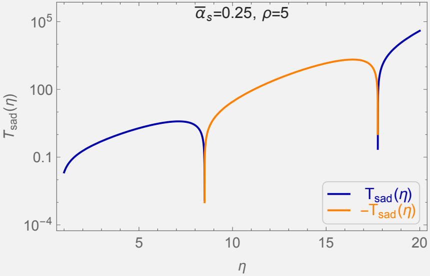
Perspective not complete: Solution develops anomalous dimension $\rightsquigarrow$ large NLO corrections

INSTABILITY IN η-EVOLUTION AT NLO (I)

Fixed $\rho > 0$ and large η:

$\bar{\alpha}_s > \bar{\alpha}_s^{\rm cr} \simeq 0.03 \rightsquigarrow$ two complex conjugate saddle points $\rightsquigarrow$ oscillating solution

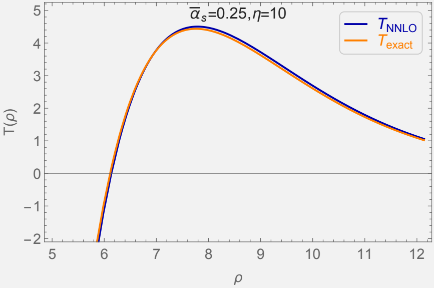


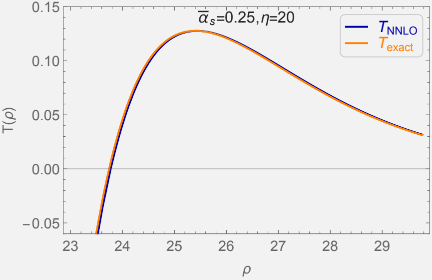


$$\bar{\omega}(\gamma_{\mathbb{P}}) = 0.719 + 0.336\text{i}$$

$$\tau_\eta = \pi/\Im\bar{\omega}(\gamma_{\mathbb{P}}) = 9.35 \ \checkmark$$

INSTABILITY IN η-EVOLUTION AT NLO (II)





Amplitude positive only at very high $\rho > \hat{\rho}(\eta)$

$$\hat{\rho}(\eta) = \bar{\omega}'(\gamma_c)\eta + \mathcal{O}(\eta^{1/3})$$

For $\bar{\alpha}_s = 0.25$, $\bar{\omega}'(\gamma_c) = 2.26 = 9.03\bar{\alpha}_s$.

Larger than LO saturation intercept.

Constraints in Evolution: from Y to η

TO in hard to soft Y-evolution, local or non-local, forbids emission of large daughter dipoles $|\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}| \gtrsim r = |\boldsymbol{x}-\boldsymbol{y}|$

$$\boxed{\ln \frac{q^+}{k^+} > \ln \frac{r_>^2}{r^2}} \quad \text{with} \quad r_> = \max\{|\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\}$$

Make change of variables

$$q^+ = \frac{1}{2r^2 q^-} \quad , \quad k^+ = \frac{1}{2r_<^2 k^-}$$

Constraints in Evolution: from Y to η

TO contraints becomes

$$\ln \frac{k^-}{q^-} > \ln \frac{r_>^2}{r^2} - \ln \frac{r_<^2}{r^2} \Rightarrow \boxed{\ln \frac{k^-}{q^-} > \ln \frac{r_>^2}{r_<^2}}$$

TO in η-evolution forbids emission of small daughter dipoles (or equivalently forbids disparate daughter dipole sizes)

Relevant when scattering small dipole off dense hadron?
Yes, BFKL diffusion $\rightsquigarrow$ not uni-directional evolution

Non-Local and Local Equation in η

Non-local equation respecting TO in η-evolution

$$\frac{\mathrm{d}S_{\boldsymbol{xy}}(\eta)}{\mathrm{d}\eta} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\Theta(\eta-\Delta_1)\Theta(\eta-\Delta_2)$$

$$\big[S_{\boldsymbol{xz}}(\eta-\Delta_1)S_{\boldsymbol{zy}}(\eta-\Delta_2) - S_{\boldsymbol{xy}}(\eta)\big]$$

An initial value problem (modulo details)

$$\Delta_1 = \begin{cases} \ln\dfrac{r^2}{(\boldsymbol{x}-\boldsymbol{z})^2} & \text{when} \quad |\boldsymbol{x}-\boldsymbol{z}| \ll r, \\ 0 & \text{when} \quad \boldsymbol{z}\to\boldsymbol{y}, \\ \to 0 & \text{when} \quad |\boldsymbol{x}-\boldsymbol{z}| \gg r, \\ \geq 0 & \text{otherwise.} \end{cases}$$

Characteristic function

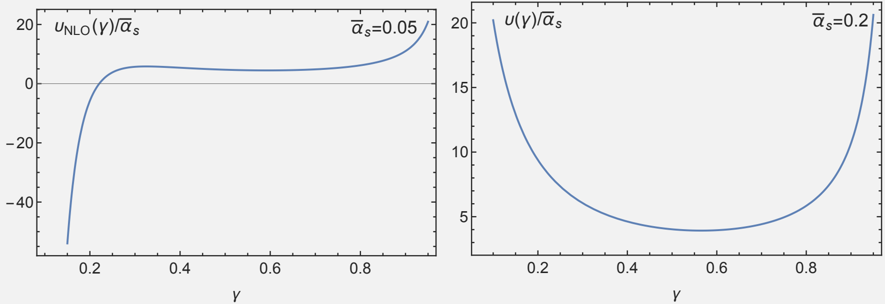
Truncation of shift to NLO: triple pole with $s_0 = Q^2$ ✓

$$\omega_{\rm NLO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{\gamma^3} + \text{regular}$$

Shifted: Finite at $\gamma = 0$

$$\omega = \frac{\bar{\alpha}_s}{1-\gamma} + \underbrace{\frac{1}{2}\left[-\gamma + \sqrt{\gamma^2 + 4\bar{\alpha}_s}\right]}_{\omega_{\rm DLA}} + \text{regular}$$

Velocity Function and Saddle Point



No real saddle point when truncating $\rightsquigarrow$ expect oscillations

Well defined saddle point with resummation

$\omega_{\mathbb{P}} = 1/2 < \gamma_s < \gamma_0 = 0.628$: pulled front with MV-GBW IC

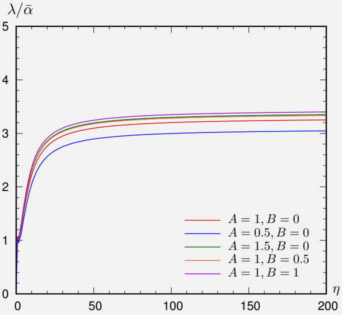
Matching to NLO BK in η

1. Start with shifted equation (matched to LO BK)
2. Subtract $\mathcal{O}(\bar{\alpha}_s^2)$ contribution
3. Add all $\mathcal{O}(\bar{\alpha}_s^2)$ contributions of NLO BK in η

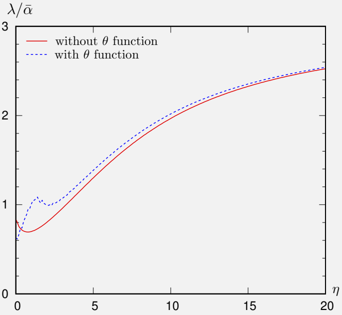
Uncertainty due to details in choosing shift Δ:

- If matched to NLO BK in η, error is $\mathcal{O}(\bar{\alpha}_s^3)$
- If matched to LO BK in η, error is $\mathcal{O}(\bar{\alpha}_s^2)$

Numerical Solution



$\bar{\alpha}_s = 0.3$



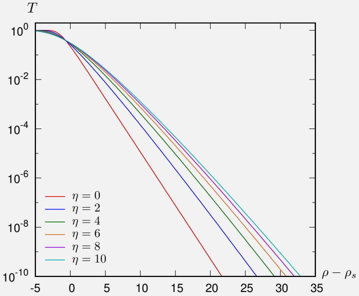
$\bar{\alpha}_s = 0.3$

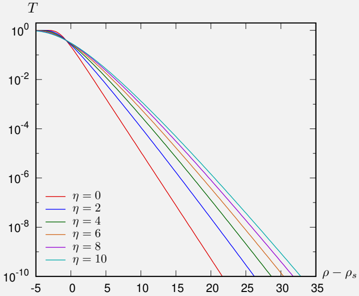
Varying shift according to

$$\Delta_1 = \frac{1-\kappa^2}{A\kappa^4 + B\kappa^2 + 1} \ln \frac{1}{\kappa^2} \quad \text{with} \quad \kappa = \frac{|\boldsymbol{z}-\boldsymbol{x}|}{r}$$

$\mathcal{O}(\bar{\alpha}_s^2)$ variation in λ

Geometric scaling





Fit in regime above Q_s up to diffusion radius:

η	γ_s LO	γ_s TO
5	0.741	0.753
10	0.672	0.661
20	0.642	0.604

Other Corrections

- Resum single logarithms for projectile and target DGLAP
- Running coupling
- Estimate for including remaining regular NLO corrections
  $\delta\omega = \pm\bar{\alpha}_s^2$, find $\delta\lambda_s/\lambda_0 \sim \pm\#\bar{\alpha}_s$ with $\# \simeq 0.35$

Still, what done here is a valid step

- Resummed unstable correction to render it $\mathcal{O}(\bar{\alpha}_s)$
- Definite sign (negative)
- Numerically larger than regular NLO corrections

Conclusions

- Very difficult (and not necessary?) to construct front in projectile rapidity Y
- Physical front is in terms of target rapidity η
- Front in η obtained directly : LO plus resummed via shift
- Front in η is faster and less steep
- Compared to LO: $\delta\lambda_s/\lambda_s \simeq \mathcal{O}(\bar{\alpha}_s)$, roughly the same γ_s
- Can match to full NLO BK evolution