Power corrections to TMD factorization

Andrey Tarasov

BROOKHAVEN
NATIONAL LABORATORY

Fifth Joint Meeting
of the Nuclear Physics Divisions
of the APS and the JPS
第5回 日米物理学会 合同核物理分科会

OCTOBER 23 – 27, 2018

Hilton Waikoloa Village,
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GENERAL INFORMATION

The Fall Meeting will reprise the highly successful first four joint meetings of the Division of Nuclear Physics (DNP) of the American Physical Society (APS) and the experimental and theoretical nuclear divisions of the Physical Society of Japan (JPS). The previous bilateral meetings in 2001, 2005, 2009, and 2014 were a resounding success, drawing between 800 and 1200 participants from Japan and North America. This year's meeting will be held once again at the Hilton Waikoloa Village on the Big Island of Hawaii on the dates of 23-27 October 2018.

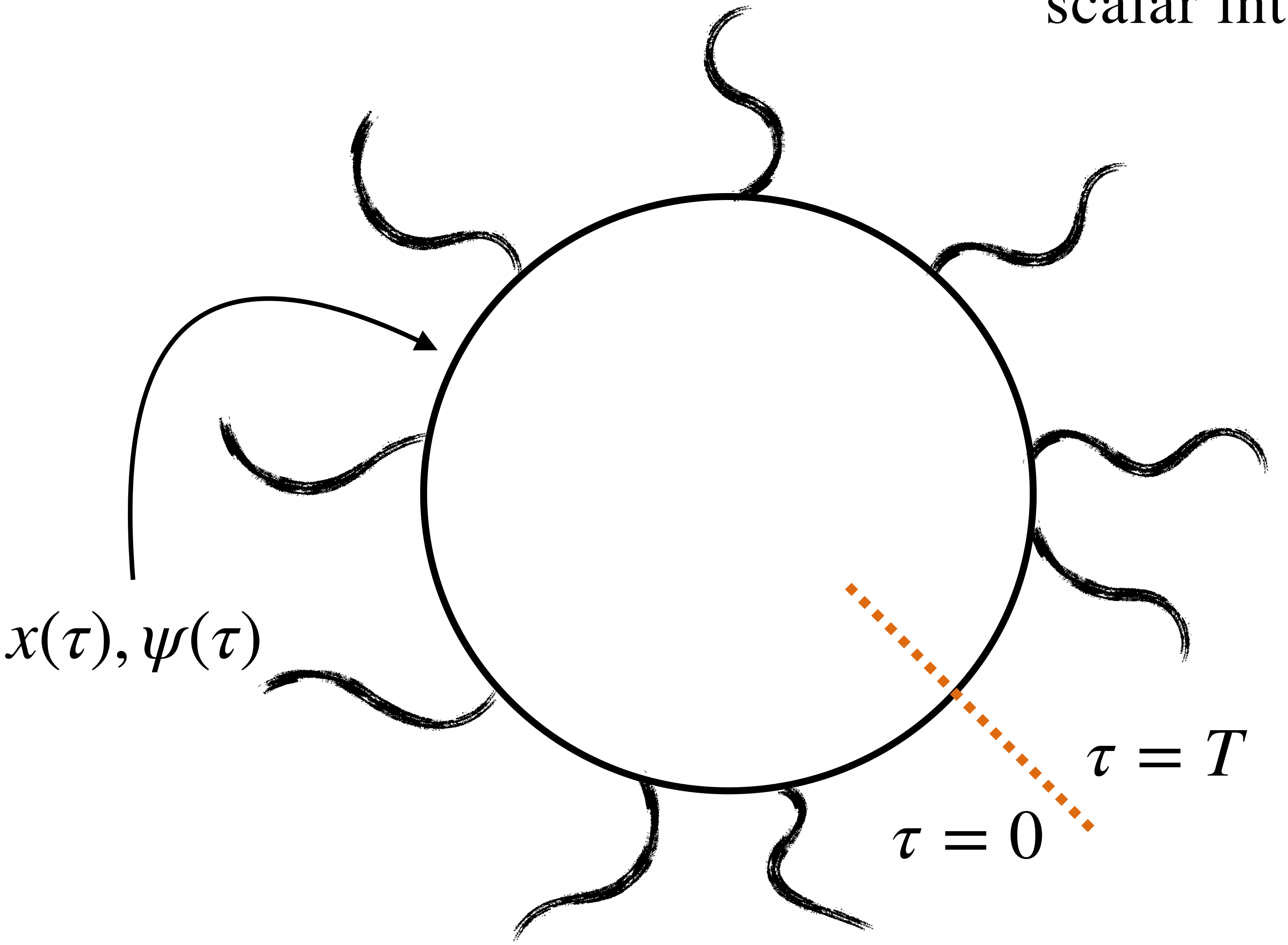
World-lines

Effective way to describe a particle in a background field

$$\Gamma[A] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}e^{-m^2T}\int \mathscr{D}x\int \mathscr{D}\psi\,\exp\Big[-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\dot{\psi}+ieA\dot{x}-ie\psi F\psi\Big)\Big]$$

scalar interaction

interaction through $\sigma^{\mu\nu}$



Z. Bern & D.A. Kosower 88

E. D’Hoker & D. G. Gagne 96

M. J. Strassler 92

World-lines

Eur. Phys. J. C 26, 445–455 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2002-01068-3

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PHYSICAL JOURNAL C

World-line techniques for resumming gluon radiative corrections at the cross-section level

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Received: 10 June 2002 /

Abstract. We employ the Polyakov world-line path-integral version of QCD to identify and resum at leading perturbative order enhanced radiative gluon contributions to the Drell–Yan type ($q\bar{q}$ pair annihilation) cross-sections. We emphasize that this is the first time that world-line techniques are applied to cross-section calculations.

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4 Resummation of enhanced contributions from virtual gluons

The family of world-line paths to which the considerations in the previous section refer was used in order to deal with all (virtual) single-gluon exchanges, consistent with the simple geometrical configuration of two constant four-velocities making a fixed angle γ between them (in Euclidean formulation). Among these gluons there will be "hard" ones (upper limit Q) and "soft" ones (lower limit set by $\bar{\lambda}$). What is debited to the former and what to the latter group of gluons is, of course, relative. It is precisely the role of the renormalization scale μ, entering through the need to face UV divergences arising even for the restricted family of paths, to provide the dividing line. The

with Γ_{cusp} to be read off from (20)–(22) and (12):

$$\Gamma_{\text{cusp}}(\alpha_{\text{s}}) = \frac{\alpha_{\text{s}}}{\pi} C_{\text{F}} + \mathcal{O}(\alpha_{\text{s}}^2). \tag{30}$$

From the second leg of (29), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} \ln U_{C,\text{cusp}} = -\int_{\bar{\lambda}^2}^{\mu^2} \frac{\mathrm{d}t}{2t} \Gamma_{\text{cusp}}\left[\alpha_{\text{s}}(t)\right], \tag{31}$$

which, in turn, gives

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} \ln \hat{U}_C = -\int_{\mu^2}^{Q^2} \frac{\mathrm{d}t}{2t} \Gamma_{\text{cusp}}\left[\alpha_{\text{s}}(t)\right] + \Gamma\left[\alpha_{\text{s}}(Q^2)\right],$$

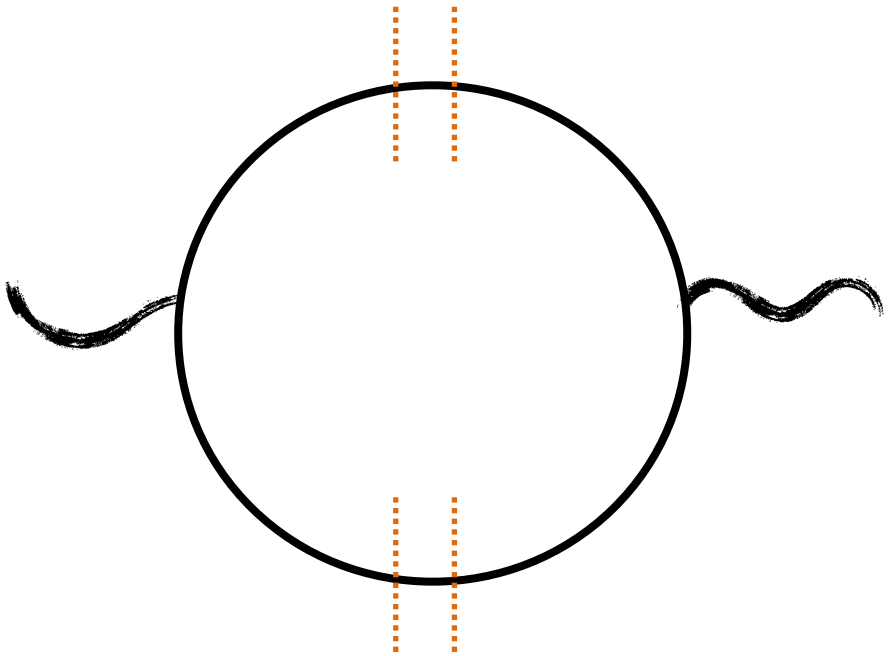
Structure functions at small-x

$$\Gamma[A] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}e^{-m^2T}\int \mathscr{D}x\int \mathscr{D}\psi\,\exp\Big[-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\dot{\psi}+ieA\dot{x}-ie\psi F\psi\Big)\Big]$$

Wilson lines

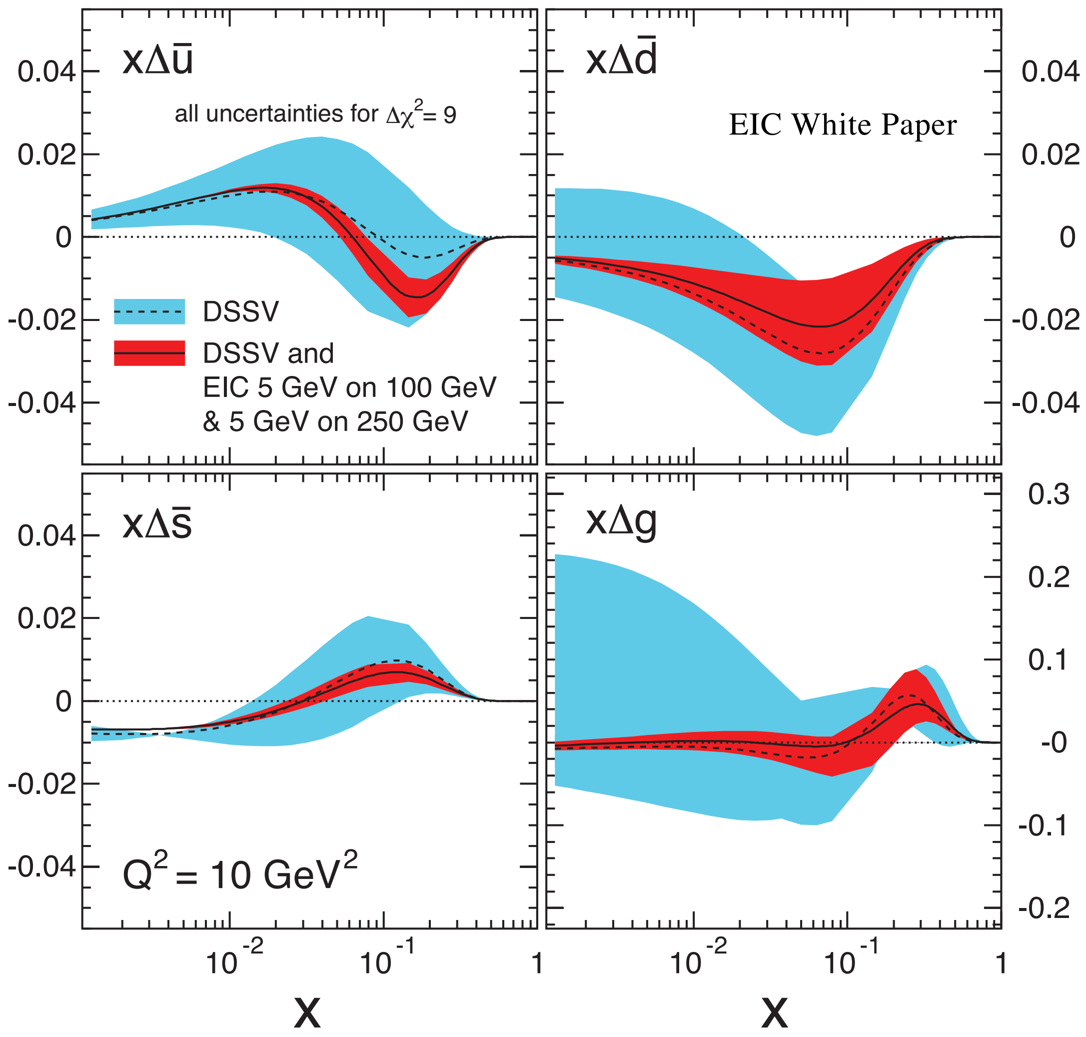
Generates anti-symmetric
part of the hadronic tensor

$W^{\mu\nu} \sim$

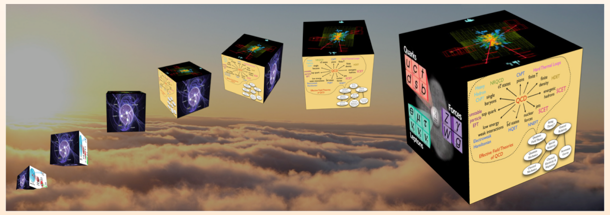


Interaction with the shock-wave

R. Venugopalan, A.T., to be published



New Windows into the Strong Interaction



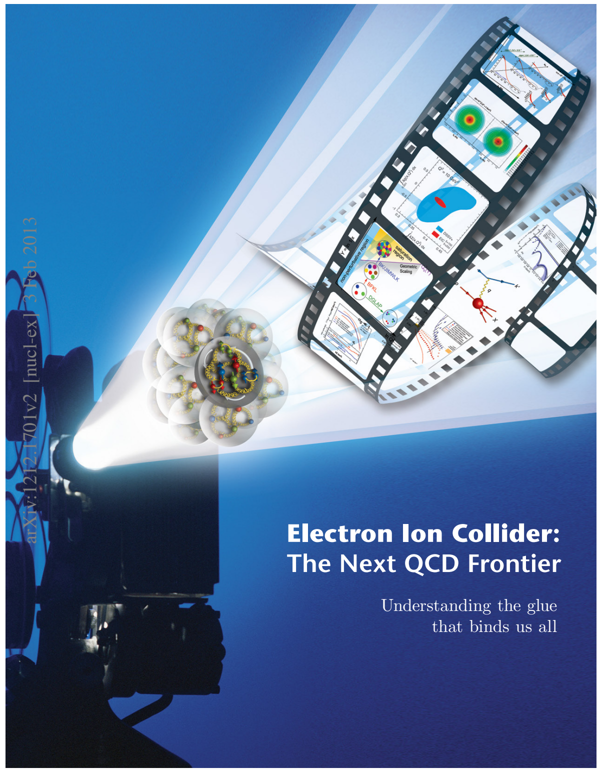
Iain Stewart
MIT

University of California, Los Angeles
Physics Colloquium
Nov. 2017

(Advances in QCD workshop at the Mani L. Bhaumik Institute)

Outline

- The Strong Force and Particle Colliders
- 10 Open Problems for the Strong Interaction
- Two Examples
    - i) Measuring the strong coupling with Jets
    - ii) Jet Substructure to answer: "What is the mass of the heaviest known elementary particle?"



arXiv:1212.1701v2 [nucl-ex] 3 Feb 2013

Electron Ion Collider:
The Next QCD Frontier

Understanding the glue
that binds us all

---

3. When do the dynamics of a high energy collision factorize?

Can we find a simple classification for the space of observables?

Factorize

small
factorization
violation

Don't
Factorize

TMD factorization

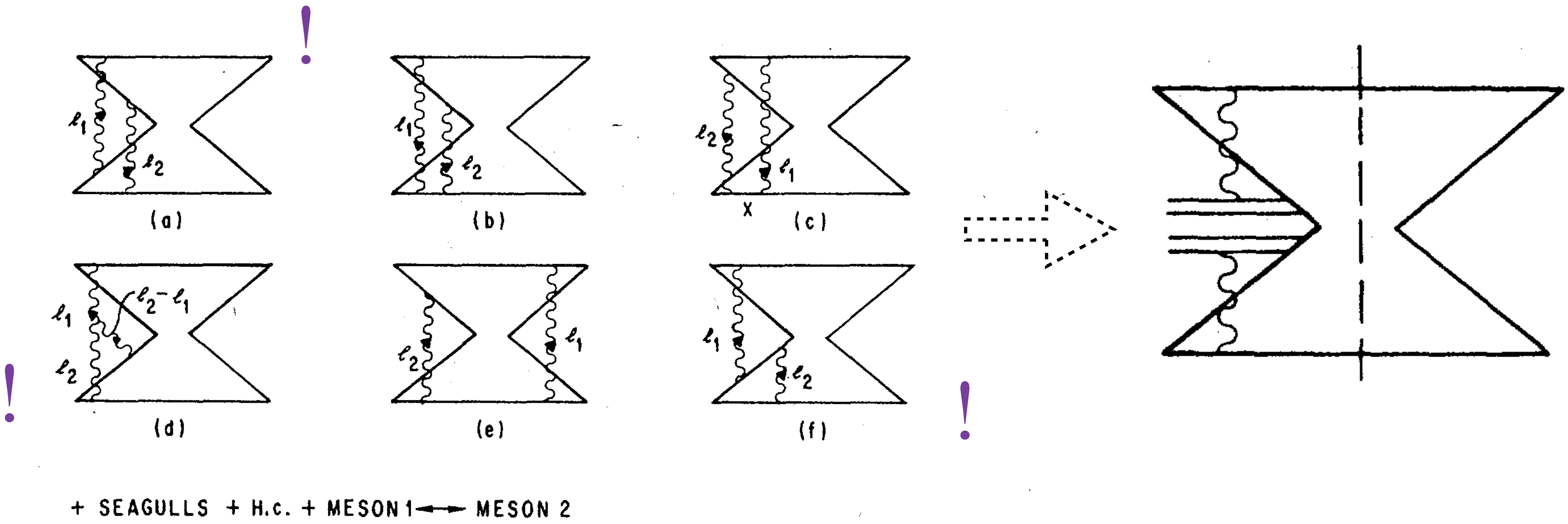


FIG. 33. Virtual double-spectator contributions to the Drell-Yan cross section.

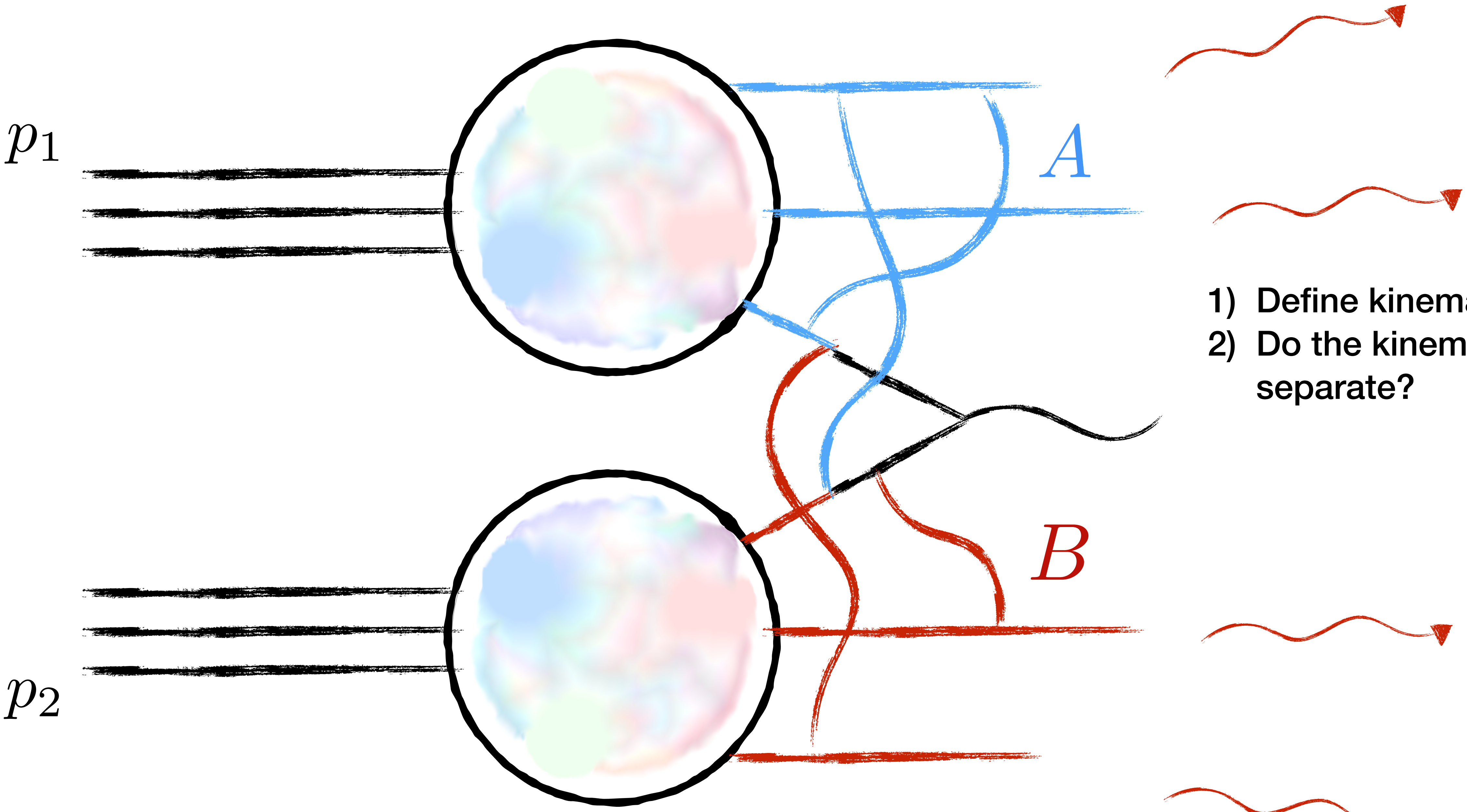
To obtain factorization we have to neglect any transverse momenta in the diagram

The problem becomes two dimensional

J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. B 109 (1982) 388;
J.C. Collins, D.E. Soper and G.F. Sterman, Nucl. Phys. B 250 (1985) 199;
G.T. Bodwin, Phys. Rev. D 31 (1985) 10;
X.-d. Ji, J.-p. Ma and F. Yuan, Phys. Rev. D 71 (2005) 034005;
M.G. Echevarria, A. Idilbi and I. Scimemi, JHEP 07 (2012) 002

Kinematic modes

$$\frac{d\sigma}{d\eta d^2 q_\perp} = \sum_f \int d^2 b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(f f \to H)$$



1) Define kinematical modes
2) Do the kinematics modes separate?

Cut-off parameters

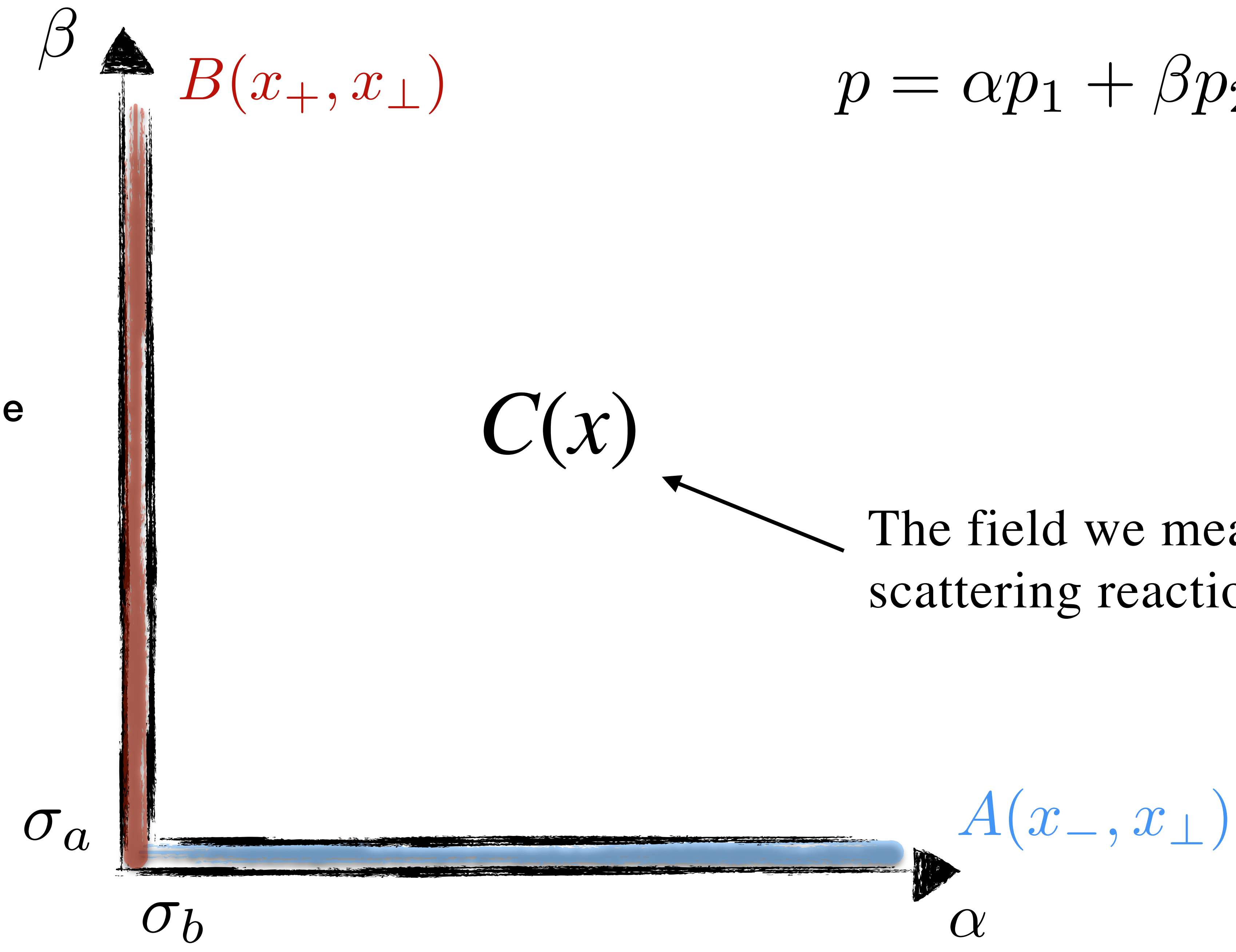
We work at the tree level. There is no large logarithms

$$\ln \frac{\sigma_a \sigma_b s}{M_Z^2}$$

We neglect dependence on cut-off parameters

$$\sigma_a, \sigma_b \to 0$$

$$p = \alpha p_1 + \beta p_2 + p_\perp$$



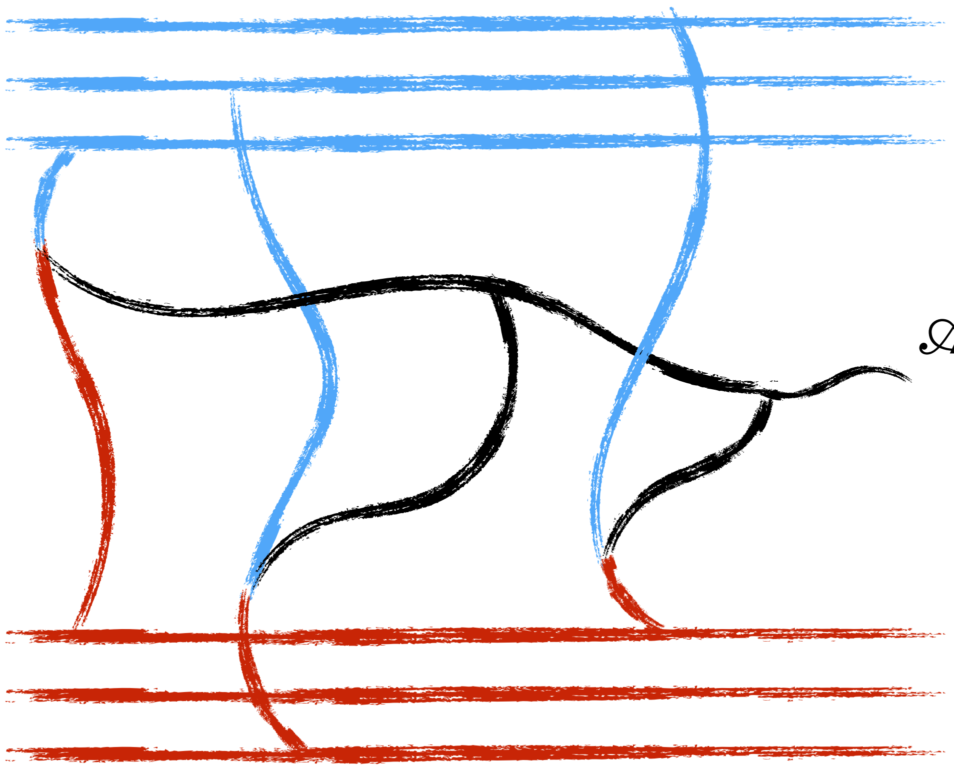
The field we measure in a scattering reaction

Large component of the background field

$$\Omega = \frac{1}{2}[x_+, -\infty][x_-, -\infty] + \frac{1}{2}[x_-, -\infty][x_+, -\infty] + \ldots$$

First few orders of the gauge matrix

$$A_+(x_-) \sim \sqrt{s}$$



$$\mathscr{A}_\mu = \Omega i \partial_\mu \Omega^\dagger$$

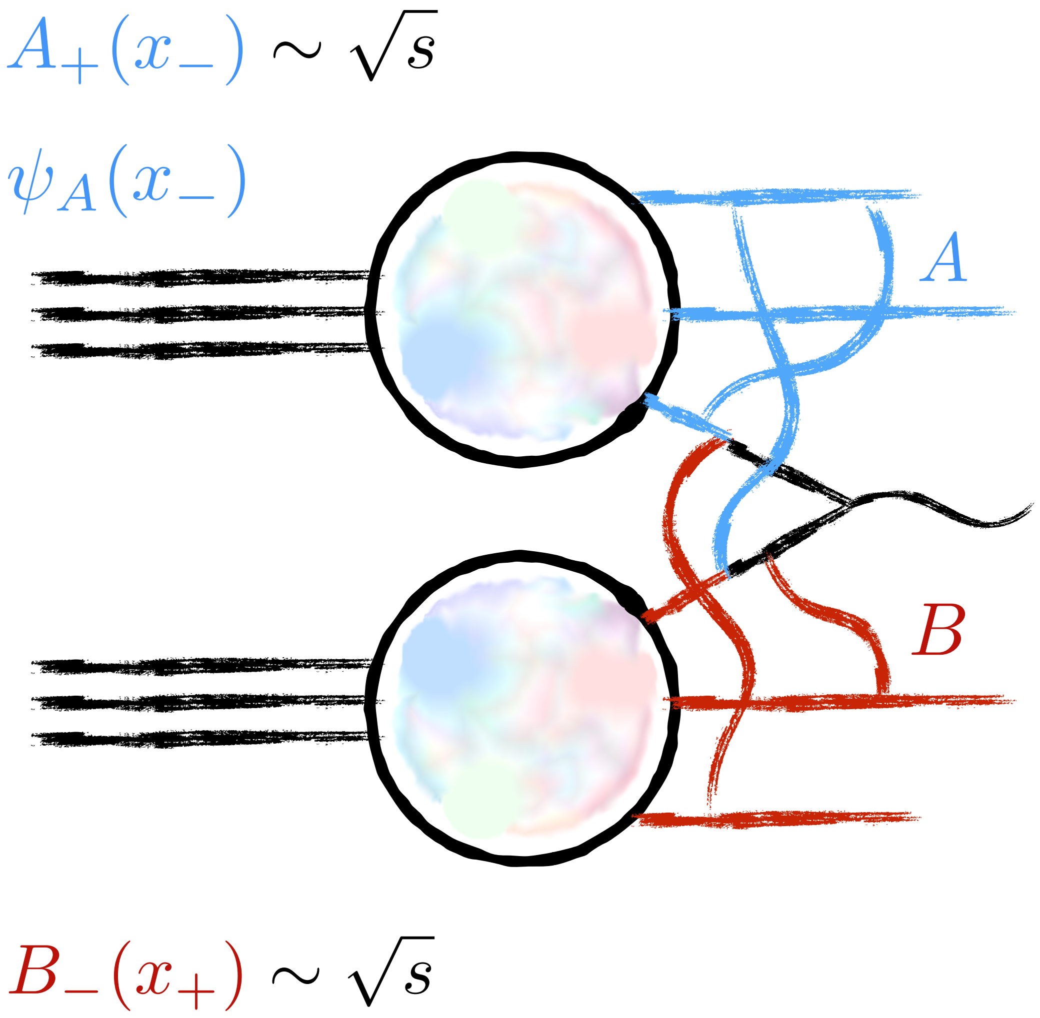
$$B_-(x_+) \sim \sqrt{s}$$

The gauge matrix satisfies boundary conditions:

$$\Omega(x) \overset{x_+ \to -\infty}{=} [x_-, -\infty]^{A_+}$$

$$\Omega(x) \overset{x_- \to -\infty}{=} [x_+, -\infty]^{B_-}$$

Large component of the background field



The gauge matrix satisfies boundary conditions:

$$\Omega(x) \overset{x_+ \to -\infty}{=} [x_-, -\infty]^{A_+}$$

$$\Omega(x) \overset{x_- \to -\infty}{=} [x_+, -\infty]^{B_-}$$

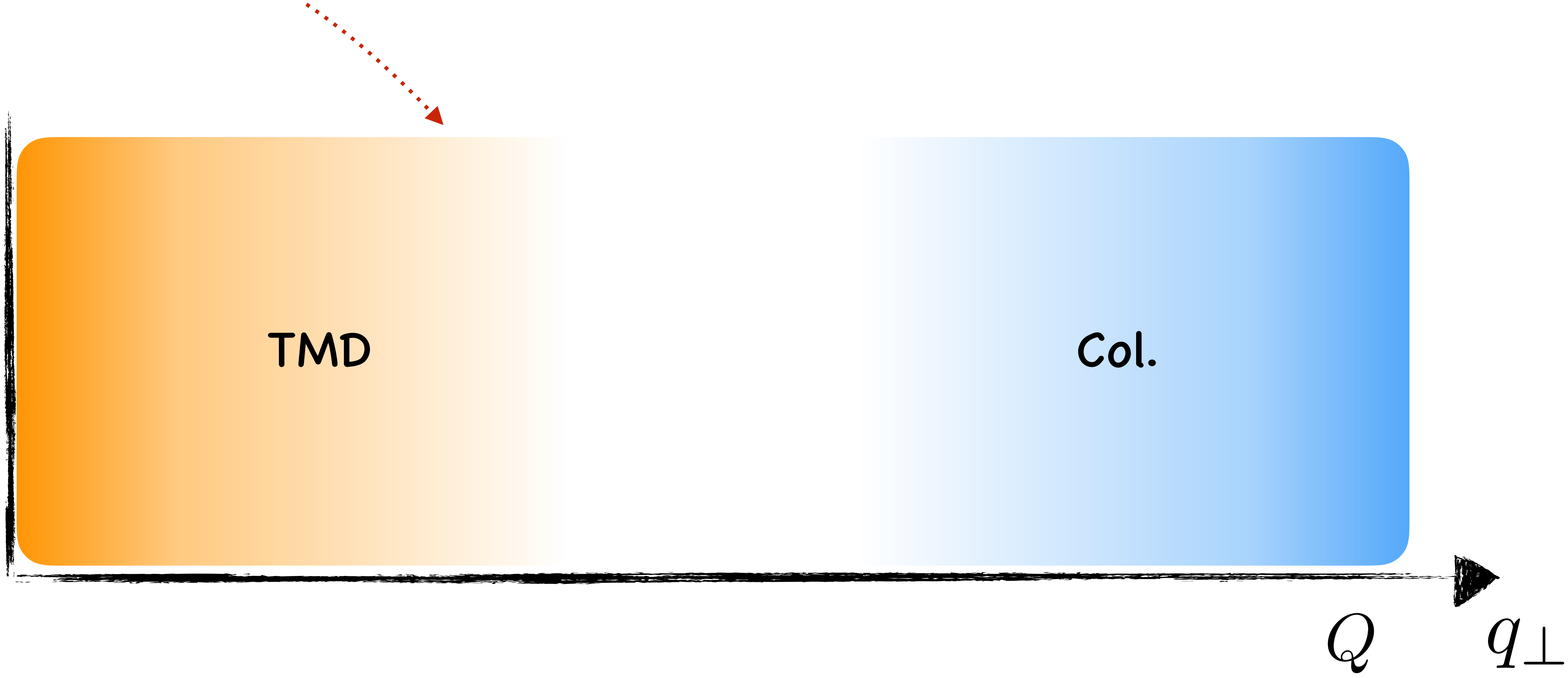
$$\psi_{\bar{A}}(x_-) = \Omega^\dagger \psi_A(x_-)$$

$$\psi_{\bar{A}}(x_-) = [-\infty, x_-]^{A_+} \psi_A(x_-)$$

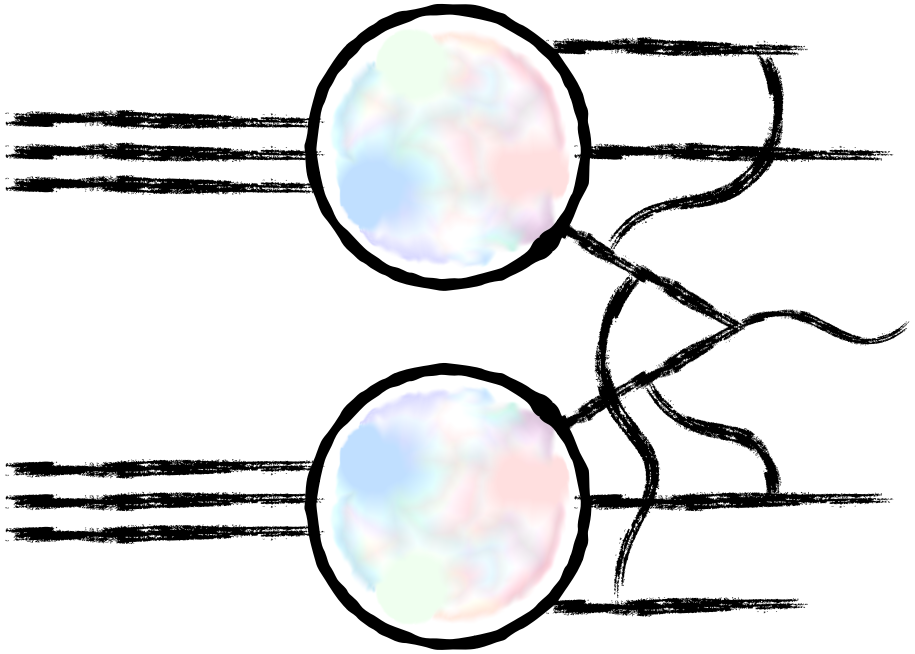
TMD vs. collinear factorization

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta)\mathcal{D}_{f/B}(x_B, b_\perp, \eta)\sigma(ff \to H)$$

Power corrections to TMD factorization



Z boson production



$$\mathcal{L}_Z = \int dx \; J_\mu Z^\mu(x)$$

$$J_\mu = -\frac{e}{2 s_W c_W} \sum_f \bar{\psi}_f \gamma_\mu (g_f^V - g_f^A \gamma_5) \psi_f$$

$$d\sigma = \frac{\pi}{2s} \frac{d^3 q}{E_q} [-W(p_A, p_B, q)]$$

$$W(p_A, p_B, q) \overset{\text{def}}{=} \frac{1}{(2\pi)^4} \int d^4 x \; e^{-iqx} \langle p_A, p_B | J_\mu(x) J^\mu(0) | p_A, p_B \rangle$$

Calculate the hadronic tensor and present the result in a factored form

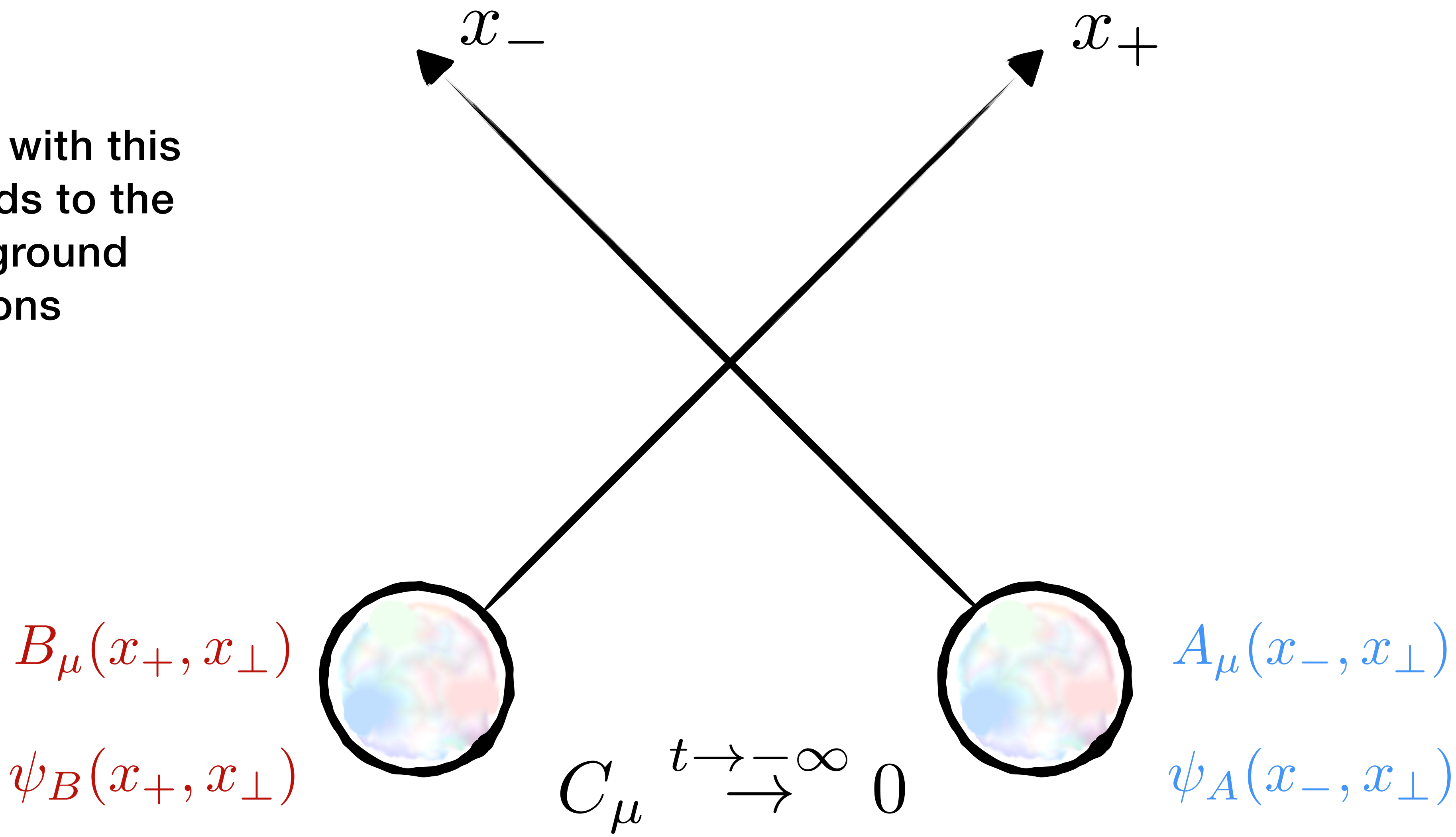
Boundary conditions

$$\mathcal{A}_\mu(x) \overset{x_+\to-\infty}{=} A_\mu(x_-,x_\perp), \qquad \Psi(x) \overset{x_+\to-\infty}{=} \psi_A(x_-,x_\perp)$$

$$\mathcal{A}_\mu(x) \overset{x_-\to-\infty}{=} B_\mu(x_+,x_\perp), \qquad \Psi(x) \overset{x_-\to-\infty}{=} \psi_B(x_+,x_\perp)$$

Solution of equations of motion with this boundary conditions corresponds to the sum of set of diagrams in background field with retarded Green functions

J.P. Blaizot, F. Gelis, and R. Venugopalan,
Nucl. Phys. A743, 13 (2004);
F. Gelis, T. Lappi, and R. Venugopalan,
Phys. Rev. D 78, 054019 (2008)

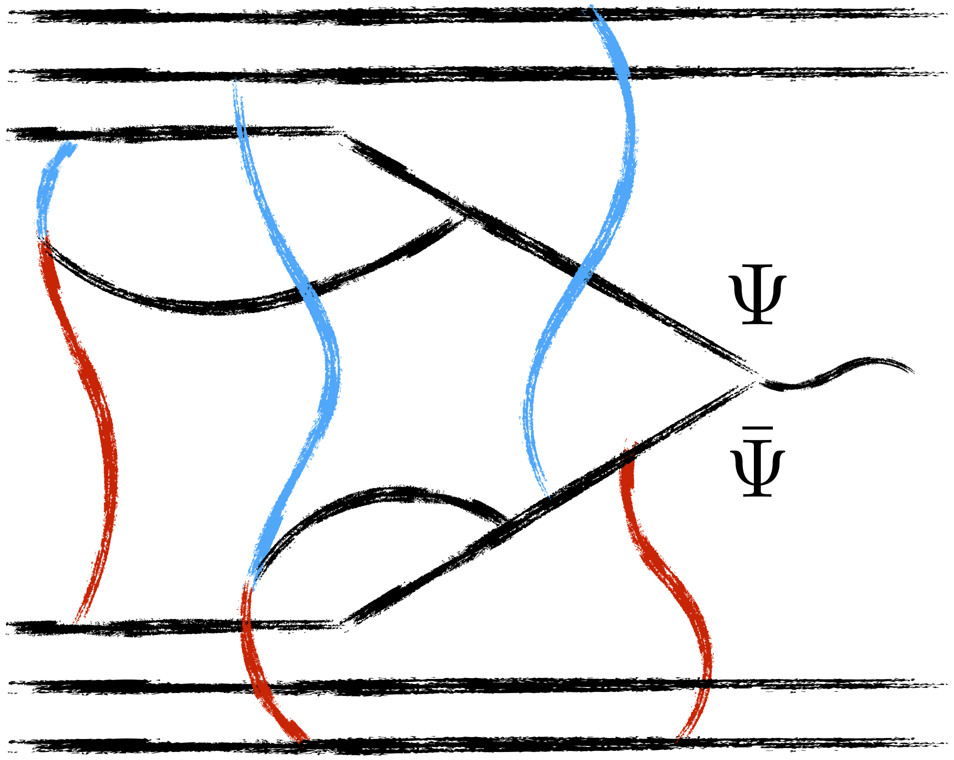


Equations of motion

To calculate $\mathcal{O}(q,x;A,\psi_a;B,\psi_b)$ we find solution of equations of motion

$$(i\,\partial\!\!\!/ + g A\!\!\!/ + g B\!\!\!/ + g\,C\!\!\!/)(\psi^f_A + \psi^f_B + \psi^f_C) \;=\; 0$$

$$D^\nu F^a_{\mu\nu}(A+B+C) \;=\; g\sum_f(\bar\psi^f_A + \bar\psi^f_B + \bar\psi^f_C)\gamma_\mu t^a(\psi^f_A + \psi^f_B + \psi^f_C)$$



Background fields satisfy equations of motion:

$$i\,D\!\!\!/_A\psi_A \;=\; 0, \quad D^\nu_A A^a_{\mu\nu} \;=\; g\sum_f\bar\psi^f_A\gamma_\mu t^a\psi^f_A$$

$$i\,D\!\!\!/_B\psi_B \;=\; 0, \quad D^\nu_B B^a_{\mu\nu} \;=\; g\sum_f\bar\psi^f_B\gamma_\mu t^a\psi^f_B$$

We will use solution of the equations of motion

$$\mathcal{A} = A + B + C \qquad \Psi = \psi_A + \psi_B + \psi_C$$

to calculate currents in the hadronic tensor

Perturbative solution. First order

We can construct perturbation
solution of the equation of motion

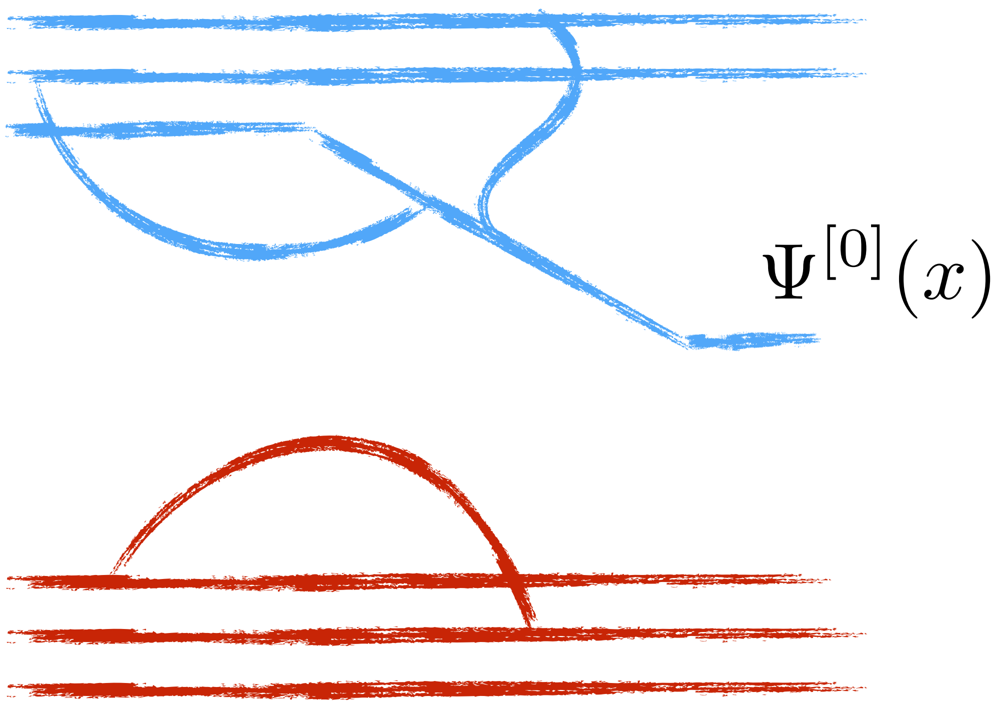
$$\mathcal{A}_\mu(x) = \mathcal{A}^{[0]}_\mu(x) + \mathcal{A}^{[1]}_\mu(x) + \mathcal{A}^{[2]}_\mu(x) + \dots$$

$$\Psi(x) = \Psi^{[0]}(x) + \Psi^{[1]}(x) + \Psi^{[2]}(x) + \dots$$

The leading order is trivial:

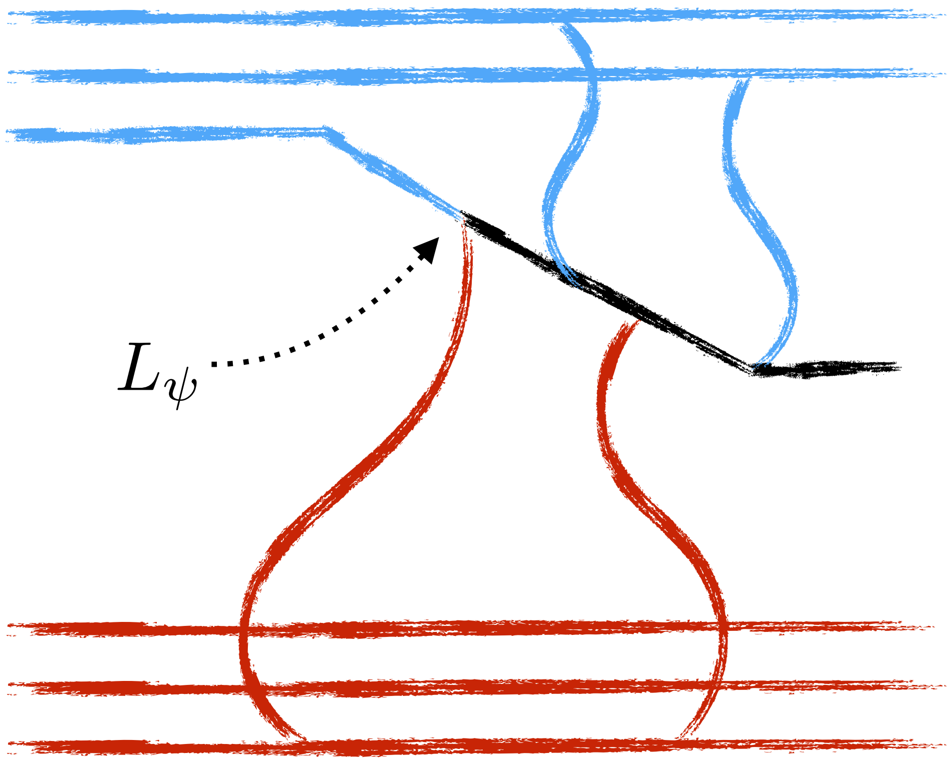
$$\mathcal{A}^{[0]}_\mu(x) \;=\; A_\mu(x_-, x_\perp) + B_\mu(x_+, x_\perp)$$

$$\Psi^{[0]}(x) \;=\; \psi_A(x_-, x_\perp) + \psi_B(x_+, x_\perp)$$



Background fields satisfy classical
equations of motions

Perturbative solution. Second order



$$\Psi^{[1]} = -\frac{1}{\mathcal{P}} L_\psi$$

quark propagates in two background fields

Covariant derivative for two background fields:

$$\mathcal{P}_\mu \equiv i\partial_\mu + gA_\mu + gB_\mu$$

Creation of the central sector fields from the background is described by the linear term

$$L_\psi \equiv \mathcal{P}\Psi^{[0]}$$

The gluon field in the second order of the perturbative expansion has a similar structure

$$\mathcal{A}_\mu^{[1]} = \frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu}} L^\nu$$

$$L_\mu^a \equiv \mathcal{D}^\xi \mathcal{F}_{\xi\mu}^{[0]a} + g\bar{\Psi}^{[0]}\gamma_\mu t^a \Psi^{[0]}$$

We should prove TMD factorization in all orders of perturbation theory. Need a new expansion parameter

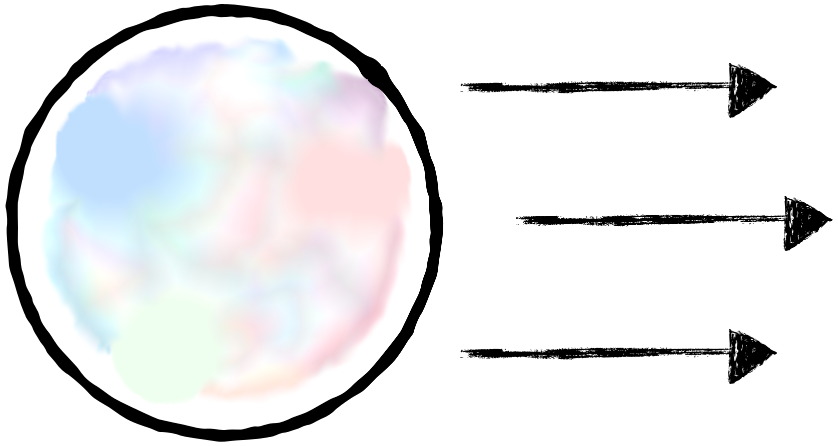
Parametrization of external fields. Lorentz boost

Hadron in the rest frame:

$A_+ \sim m, \quad A_- \sim m, \quad A_i \sim m$

Boost the system

Look at the limit: $s \to \infty$



Expansion parameter:

$m^2/s \sim p_\perp^2/s$

$A_+ \sim m^2/\sqrt{s}, \quad A_- \sim \sqrt{s}, \quad A_i \sim m$

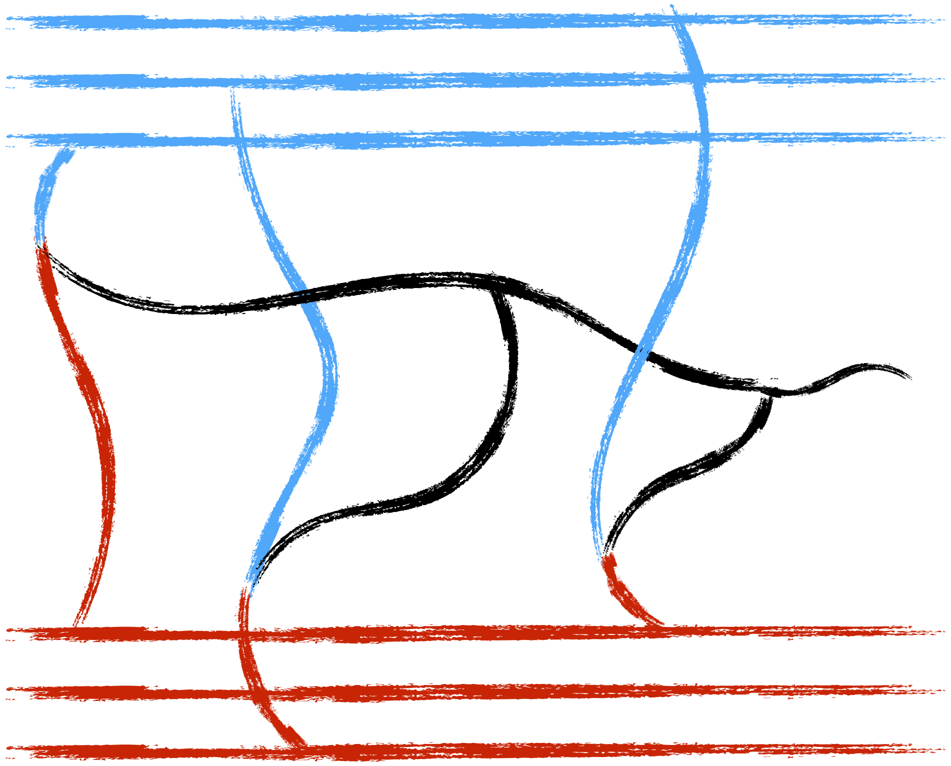
Use this parametrization to separate different contributions

Gauge rotation of the background fields

New background fields

$$\bar{A}_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + A_\mu(x)\Big)\Omega(x)$$

$$\psi_{\bar{A}}(x_-,x_\perp) = \Omega^\dagger\psi_A(x_-,x_\perp)$$



New initial conditions:

$$\mathcal{A}_\mu(x) \overset{x_+\to-\infty}{\to} \bar{A}_\mu(x_-,x_\perp), \quad \psi(x) \overset{x_+\to-\infty}{\to} \psi_{\bar{A}}(x_-,x_\perp)$$

$$\mathcal{A}_\mu(x) \overset{x_-\to-\infty}{\to} \bar{B}_\mu(x_+,x_\perp), \quad \psi(x) \overset{x_-\to-\infty}{\to} \psi_{\bar{B}}(x_+,x_\perp)$$

New background fields

$$\bar{B}_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + B_\mu(x)\Big)\Omega(x)$$

$$\psi_{\bar{B}}(x_-,x_\perp) = \Omega^\dagger\psi_B(x_+,x_\perp)$$

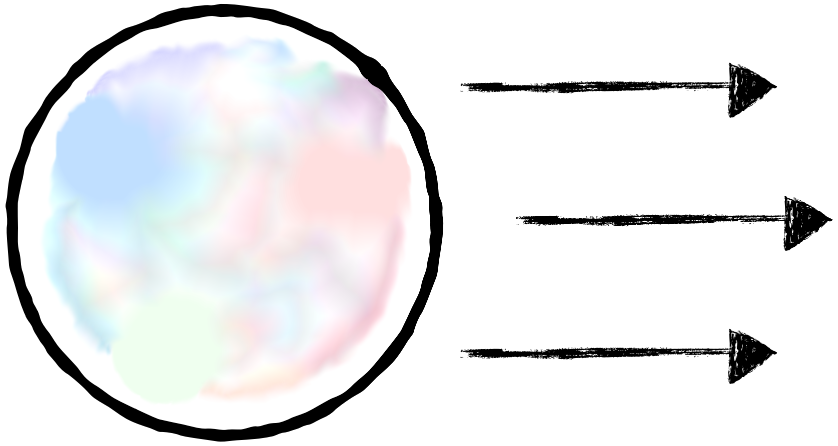
Parametrization of fields after gauge rotation

$$\bar{A}_-(x_-,x_\perp)\sim m^2/\sqrt{s},\quad \bar{A}_+(x_-,x_\perp)=0,\quad \bar{A}_i(x_-,x_\perp)\sim m$$

$$\not{p}_1\psi_{\bar{A}}(x_-,x_\perp)\ \sim\ m^{5/2}\qquad \gamma_i\psi_{\bar{A}}(x_-,x_\perp)\ \sim\ m^{3/2}$$

$$\not{p}_2\psi_{\bar{A}}(x_-,x_\perp)\ \sim\ s\sqrt{m}$$

Consider YM equation in these background fields



$$\bar{B}_-(x_+,x_\perp)=0;\quad \bar{B}_+(x_+,x_\perp)\sim m^2/\sqrt{s};\quad \bar{B}_i(x_*,x_\perp)\sim m$$

$$\not{p}_1\psi_{\bar{B}}(x_+,x_\perp)\ \sim\ s\sqrt{m}\qquad \gamma_i\psi_{\bar{B}}(x_+,x_\perp)\ \sim\ m^{3/2}$$

$$\not{p}_2\psi_{\bar{B}}(x_+,x_\perp)\ \sim\ m^{5/2}$$

Parametrization of perturbation solution

$$\mathcal{A}^{[0]}_\mu(x) \;=\; \bar{A}_\mu(x_-, x_\perp) + \bar{B}_\mu(x_+, x_\perp)$$

$$\mathcal{A}^{[1]}_\mu \;=\; \frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu}} L^\nu$$

We regroup terms of the perturbative solution using new expansion parameter: m^2/s

gluon propagator in two background fields

operator constructed from background fields

Perturbative expansion of the gluon propagator in two background fields

$$(x|\frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu} + i\epsilon p_0}|y) \;\equiv\; (x|\frac{1}{p^2 + i\epsilon p_0}|y) - g(x|\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}_{\mu\nu}\frac{1}{p^2 + i\epsilon p_0}|y)$$
$$+\; g^2(x|\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}_{\mu\xi}\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}^{\xi}{}_{\nu}\frac{1}{p^2 + i\epsilon p_0}|y) + \ldots$$

Separation of terms with the new parameter

$$\sim 1 + \frac{m^2}{s} + \left(\frac{m^2}{s}\right)^2 + \left(\frac{m^2}{s}\right)^3 + \ldots$$

$$(x|\frac{1}{p_{\|}^2 + i\epsilon p_0}|y)$$

Do we need these terms?

Parametrization of the linear term

Use explicit form of the linear term

$$L_\psi \;\equiv\; \not{P}\Psi^{[0]} \;=\; L_\psi^{(0)} + L_\psi^{(1)}$$

Expansion in terms of the new parameter

$$L_\psi^{(0)} \;=\; g\gamma^i \bar{A}_i \psi_{\bar{B}} + g\gamma^i \bar{B}_i \psi_{\bar{A}}, \qquad L_\psi^{(1)} \;=\; g\sqrt{\frac{2}{s}}\;\not{p}_2 \bar{A}_- \psi_{\bar{B}} + g\sqrt{\frac{2}{s}}\;\not{p}_1 \bar{B}_+ \psi_{\bar{A}}$$

Power counting: $L_\psi^{(0)} \;\sim\; m^{5/2}, \qquad L_\psi^{(1)} \;\sim\; \dfrac{m^{9/2}}{s}$

Linear term for gluons:

$$L_\mu^a \;\equiv\; \mathcal{D}^\xi \mathcal{F}_{\xi\mu}^{[0]a} + g\bar{\Psi}^{[0]}\gamma_\mu t^a \Psi^{[0]} \;=\; L_\mu^{(-1)a} + L_\mu^{(0)a} + L_\mu^{(1)a}$$

$$\sim s^{1/2}m^2 + \frac{m^4}{s^{1/2}} + \frac{m^6}{s^{3/2}}$$

Solution of the equations of motion

Equation of motion:

$$(i \not{\partial} + g \not{\bar{A}} + g \not{\bar{B}} + g \not{C})\Psi \ = \ 0$$

parametrization of the perturbative solution in terms of the new parameter

Perturbative solution:

$$\Psi(x) \ = \ \Psi^{[0]}(x) + \Psi^{[1]}(x) + \Psi^{[2]}(x) + \ldots \ = \ \Psi_A^{(0)} + \Psi_B^{(0)} + \Psi_A^{(1)} + \Psi_B^{(1)} + \ldots$$

Leading order solution:

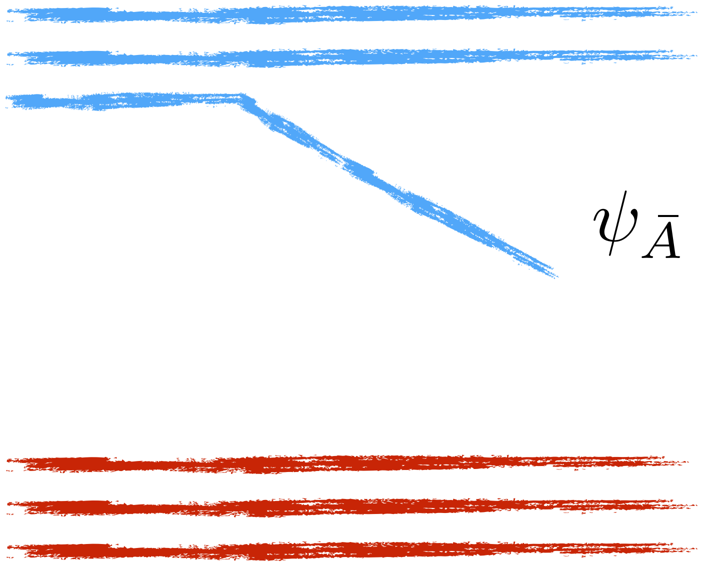
$$\Psi_A^{(0)} \ = \ \psi_{\bar{A}} + \Xi_{2A} \sim m^{3/2}, \qquad \Xi_{2A} \ = \ -\frac{g \not{p}_2}{s}\gamma^i \bar{B}_i \frac{1}{\alpha + i\epsilon}\psi_{\bar{A}}$$

$$\Psi_B^{(0)} \ = \ \psi_{\bar{B}} + \Xi_{1B} \sim m^{3/2}, \qquad \Xi_{1B} \ = \ -\frac{g \not{p}_1}{s}\gamma^i \bar{A}_i \frac{1}{\beta + i\epsilon}\psi_{\bar{B}}$$

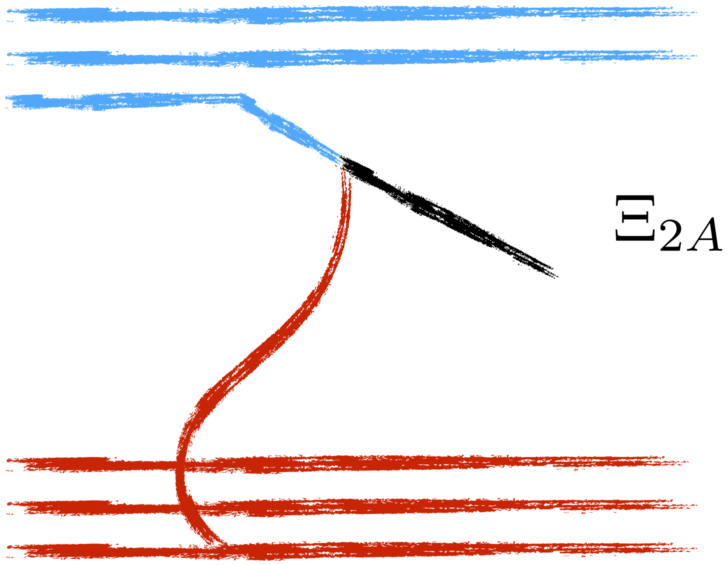
We use this solution to calculate hadronic tensor

The structure of the leading order solution

$$\Psi_A^{(0)} = \psi_{\bar{A}} + \Xi_{2A}$$

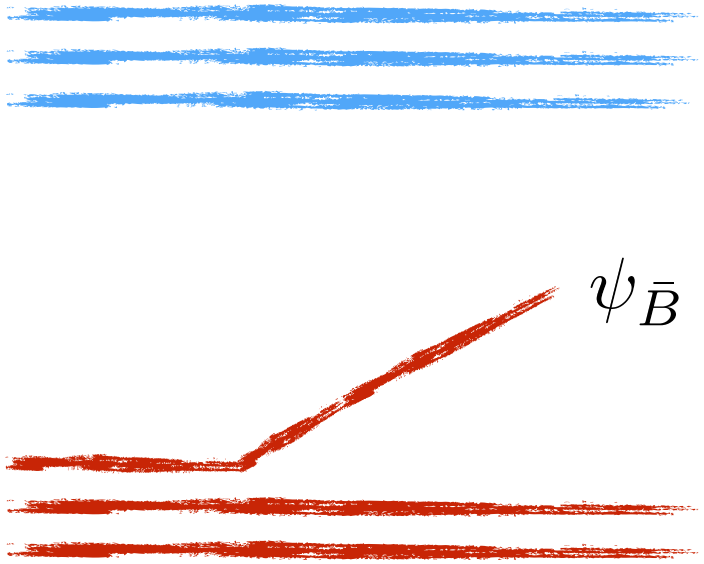


$\psi_{\bar{A}} \sim m^{3/2}$

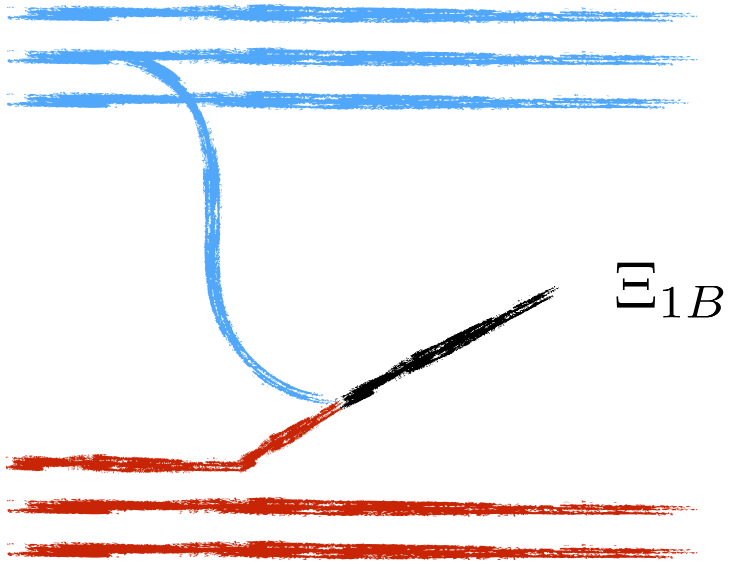


$\Xi_{2A} \sim m^{3/2}$

$$\Psi_B^{(0)} = \psi_{\bar{B}} + \Xi_{1B}$$



$\psi_{\bar{B}} \sim m^{3/2}$



$\Xi_{1B} \sim m^{3/2}$

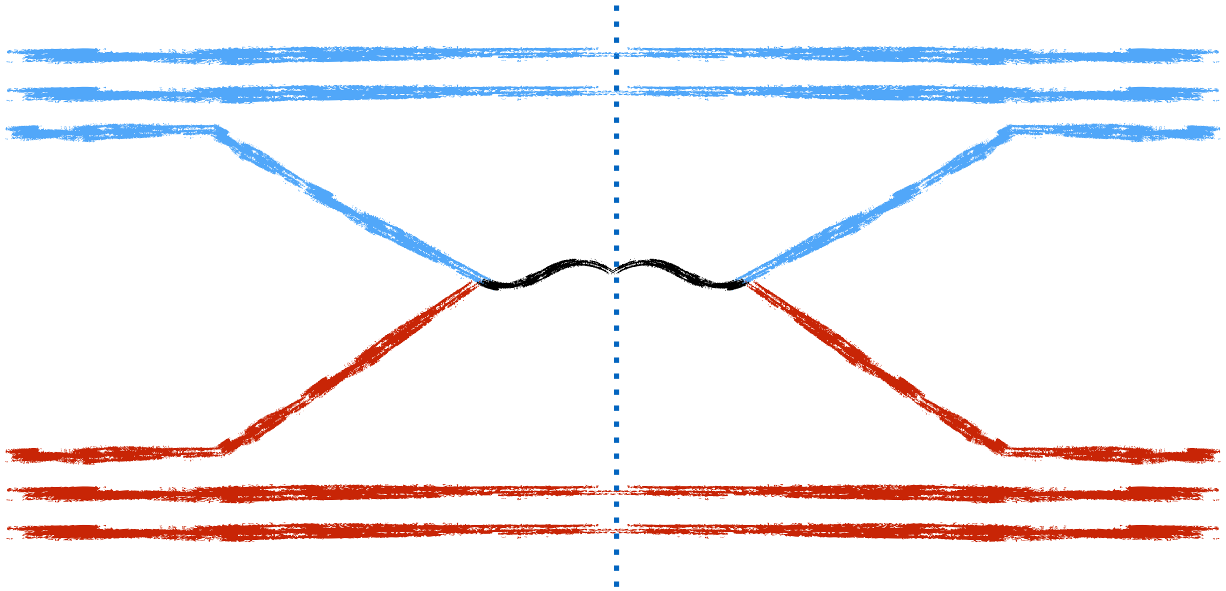
Hadronic tensor in the leading order

$$W(\alpha_z,\beta_z,x_\perp) \equiv \frac{1}{(2\pi)^4}\int dx_+ dx_-\; e^{-i\sqrt{\frac{s}{2}}\alpha_z x_- - i\sqrt{\frac{s}{2}}\beta_z x_+}\langle p_A,p_B|J_\mu(x_+,x_-,x_\perp)J^\mu(0)|p_A,p_B\rangle$$

In the leading order we use

$$\Psi_A^{(0)} = \psi_{\bar{A}}$$

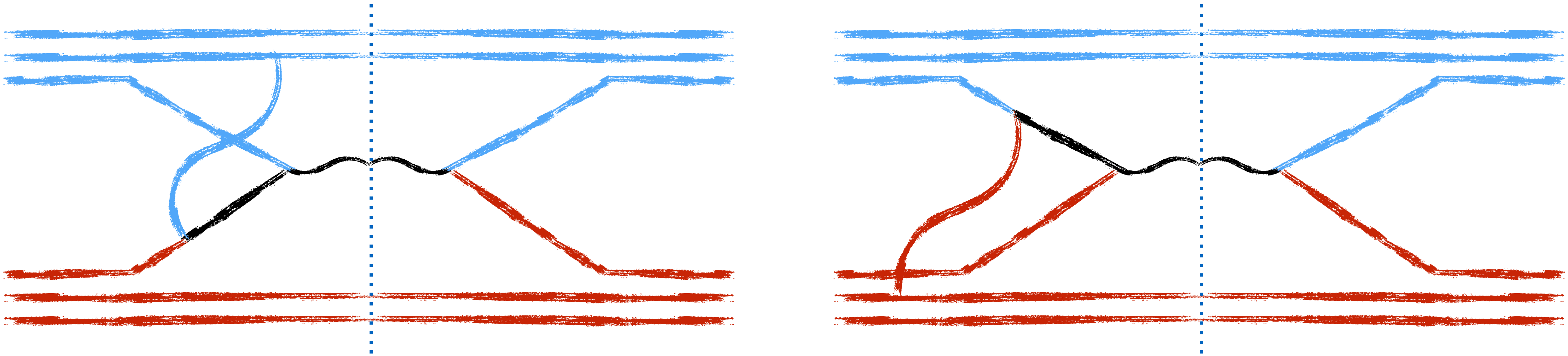
$$\Psi_B^{(0)} = \psi_{\bar{B}}$$



$$W^{\rm lt}(\alpha_z,\beta_z,q_\perp) = -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big(\big\{(1+a_u^2)\big[f_1^u(\alpha_z,k_\perp)\bar{f}_1^u(\beta_z,q_\perp-k_\perp)$$
$$+\bar{f}_1^u(\alpha_z,k_\perp)f_1^u(\beta_z,q_\perp-k_\perp)\big]\big\}+\{u\leftrightarrow c\}+\{u\leftrightarrow d\}+\{u\leftrightarrow s\}\Big)$$

$$a_{u,c} = (1-\frac{8}{3}s_W^2) \qquad a_{d,s} = (1-\frac{4}{3}s_W^2)$$

Suppressed contributions I



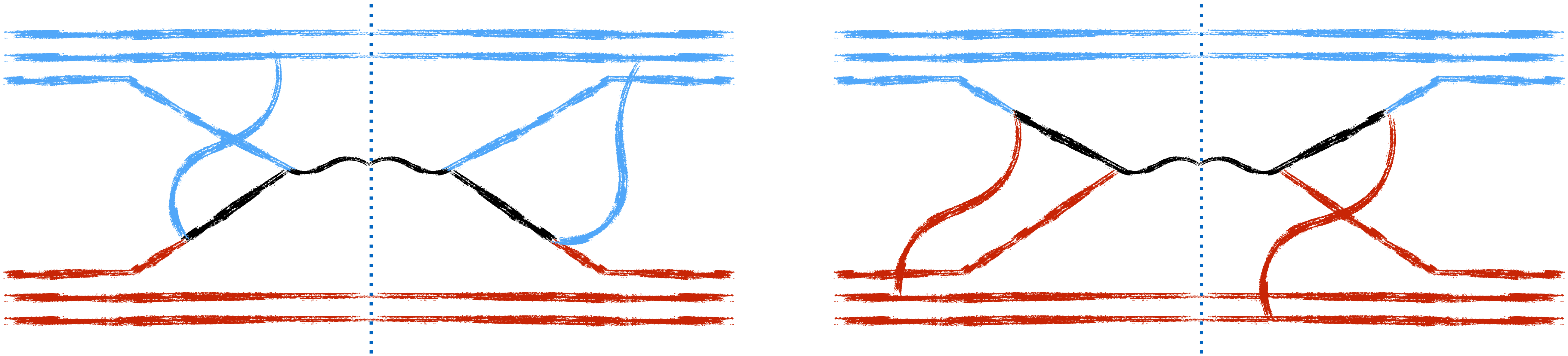
$\Psi_A^{(0)} = \psi_{\bar{A}}$ $\Psi_A^{(0)} = \Xi_{2A}$ $\Psi_B^{(0)} = \Xi_{1B}$ $\Psi_B^{(0)} = \psi_{\bar{B}}$

This contribution $\sim \dfrac{q_\perp^2}{\alpha_z s} W^{\mathrm{lt}} \ll \dfrac{q_\perp^2}{Q^2} W^{\mathrm{lt}}$

Looks like a leading power correction, but suppressed in the kinematic limit

$$s \gg Q^2 \gg Q_\perp^2$$

Suppressed contributions II



$\Psi_A^{(0)} = \psi_{\bar{A}}$

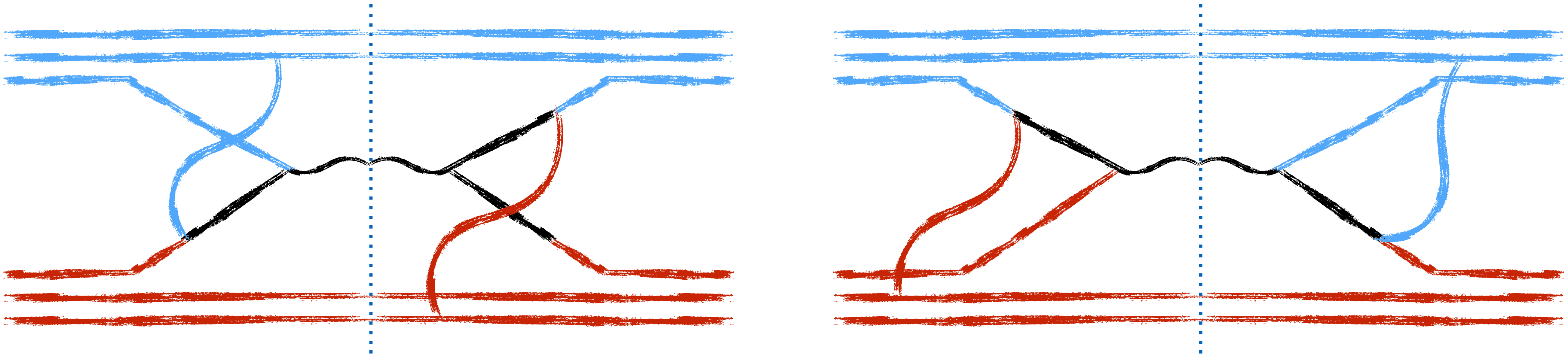
$\Psi_A^{(0)} = \Xi_{2A}$

$\Psi_B^{(0)} = \Xi_{1B}$

$\Psi_B^{(0)} = \psi_{\bar{B}}$

This contribution is suppressed as $\sim \dfrac{m^4}{s^2} W^{\mathrm{lt}}$

Leading in 1/N$_c$ contribution



$$\Psi_A^{(0)} = \Xi_{2A} \qquad \Psi_B^{(0)} = \Xi_{1B}$$

$$\begin{aligned} W(\alpha_z,\beta_z,q_\perp) = {} & \frac{e^2}{4s_W^2c_W^2N_cQ^2}\int d^2k_\perp\Big[\Big\{(1+a_u^2)(k,q-k)_\perp f_1^u(\alpha_z,k_\perp)\bar f_1^u(\beta_z,q_\perp-k_\perp) \\ & + \frac{1}{m^2}(1-a_u^2)k_\perp^2(q-k)_\perp^2 h_{1u}^\perp(\alpha_z,k_\perp)\bar h_{1u}^\perp(\beta_z,q_\perp-k_\perp) + (\alpha_z\leftrightarrow\beta_z)\Big\} \\ & + \Big\{u\leftrightarrow c\Big\} + \Big\{u\leftrightarrow d\Big\} + \Big\{u\leftrightarrow s\Big\}\Big] \end{aligned}$$

Leading power correction to TMD factorization

$$\sim \frac{q_\perp^2}{Q^2}W^{\rm lt}$$

Leading in 1/N$_c$ contribution

$$W(\alpha_z,\beta_z,q_\perp)=\frac{e^2}{4s_W^2c_W^2N_cQ^2}\int d^2k_\perp\Big[\Big\{(1+a_u^2)(k,q-k)_\perp f_1^u(\alpha_z,k_\perp)\bar{f}_1^u(\beta_z,q_\perp-k_\perp)$$
$$+\ \frac{1}{m^2}(1-a_u^2)k_\perp^2(q-k)_\perp^2 h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp)+\ (\alpha_z\leftrightarrow\beta_z)\Big\}$$
$$+\ \Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]$$

1) It is a leading power correction from the point of view of parametrization

2) Leading contribution in kinematic limit

$$s\gg Q^2\gg Q_\perp^2$$

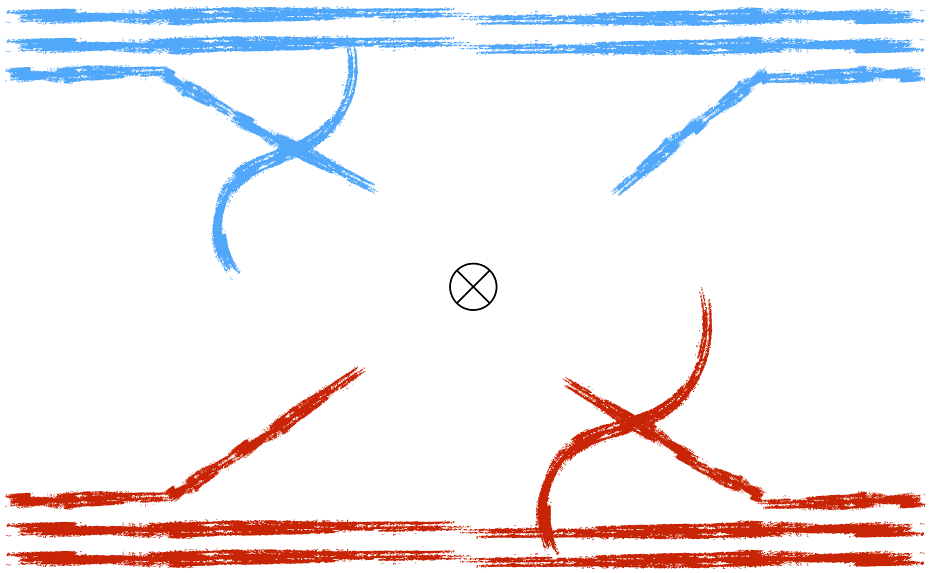
(the contribution is $\sim\frac{q_\perp^2}{Q^2}W^{\rm lt}$)

3) The contribution is proportional to $\frac{1}{N_c}$

(all other important terms $\sim\frac{1}{N_c^2},\ \frac{1}{N_c^3}$)

4) The leading power correction to TMD factorization can be expressed in terms of leading twist distribution functions. This is the most important observation.

Leading twist distribution functions



$f_1(\alpha_z, k_\perp)$ $h_1^\perp(\alpha_z, k_\perp)$

$f_1(\beta_z, k_\perp)$ $h_1^\perp(\beta_z, k_\perp)$

The structure of the TMD operator

$$\frac{g}{8\pi^3 s}\sqrt{\frac{s}{2}}\int dx_- dx_\perp e^{-i\alpha\sqrt{\frac{s}{2}}x_- + ik_\perp x_\perp}\langle A|\bar{\psi}_{\bar{A}}(x_-, x_\perp)\ \not{p}_2\left[\bar{A}_i(0) + i\gamma_5\tilde{\bar{A}}_i(0)\right]\psi_{\bar{A}}(0)|A\rangle = -k_i f_1(\alpha, k_T^2)$$

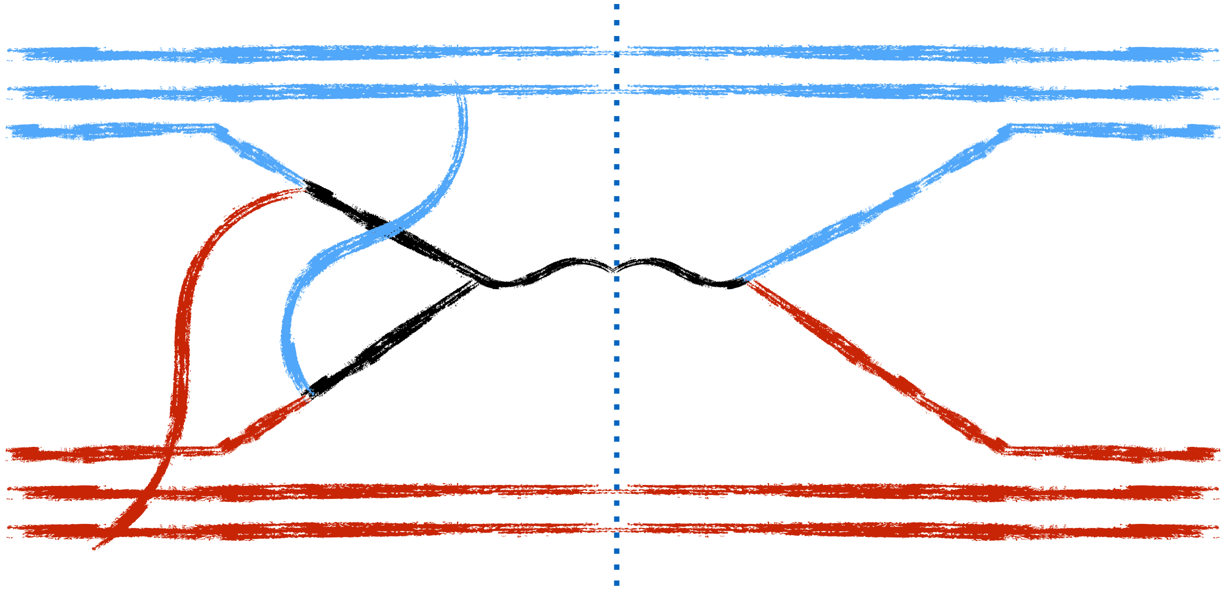
Leading twist TMD operator is constructed from quark fields only

$$f_1(\alpha, k_T^2) = \frac{1}{2}\int\frac{dx_- d^2x_\perp}{(2\pi)^3}e^{-i\alpha\sqrt{\frac{s}{2}}x_- + ik_\perp x_\perp}\langle A|\bar{\psi}_{\bar{A}}(x)\gamma_+\psi_{\bar{A}}(0)|A\rangle$$

R. D. Tangerman and P. J. Mulders,
Phys. Rev. D51 (1995)

The structure of Wilson lines can be restored by gauge rotation

Contribution suppressed by color

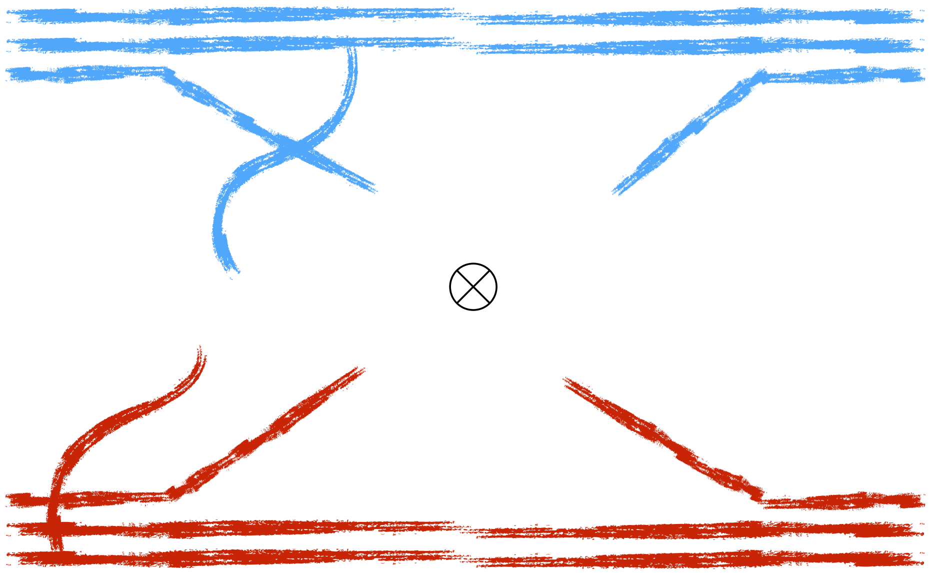


1) Introduce higher twist distribution functions

2) The contribution is suppressed as $\frac{1}{N_c^3}$

$$
\begin{aligned}
W(\alpha_z,\beta_z,q_\perp) \;&=\; -\frac{e^2}{4s_W^2c_W^2N_c(N_c^2-1)Q^2}\int d^2k_\perp\; k_\perp^2(q-k)_\perp^2 \\
\times\; &\Big\{\frac{1}{m^2}(a_u^2-1)[h_u^{\rm tw3}(\alpha_z,k_\perp)\bar{h}_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde{h}_u^{\rm tw3}(\alpha_z,k_\perp)\tilde{\bar{h}}_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)] \\
+\;&(\alpha_z\leftrightarrow\beta_z)\Big\} \;+\; \Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]
\end{aligned}
$$

New class of TMD distribution functions



$h_f^{tw3}(\alpha, k_\perp^2) \qquad \tilde{h}_f^{tw3}(\alpha, k_\perp^2)$

$h_f^{tw3}(\beta, k_\perp^2) \qquad \tilde{h}_f^{tw3}(\beta, k_\perp^2)$

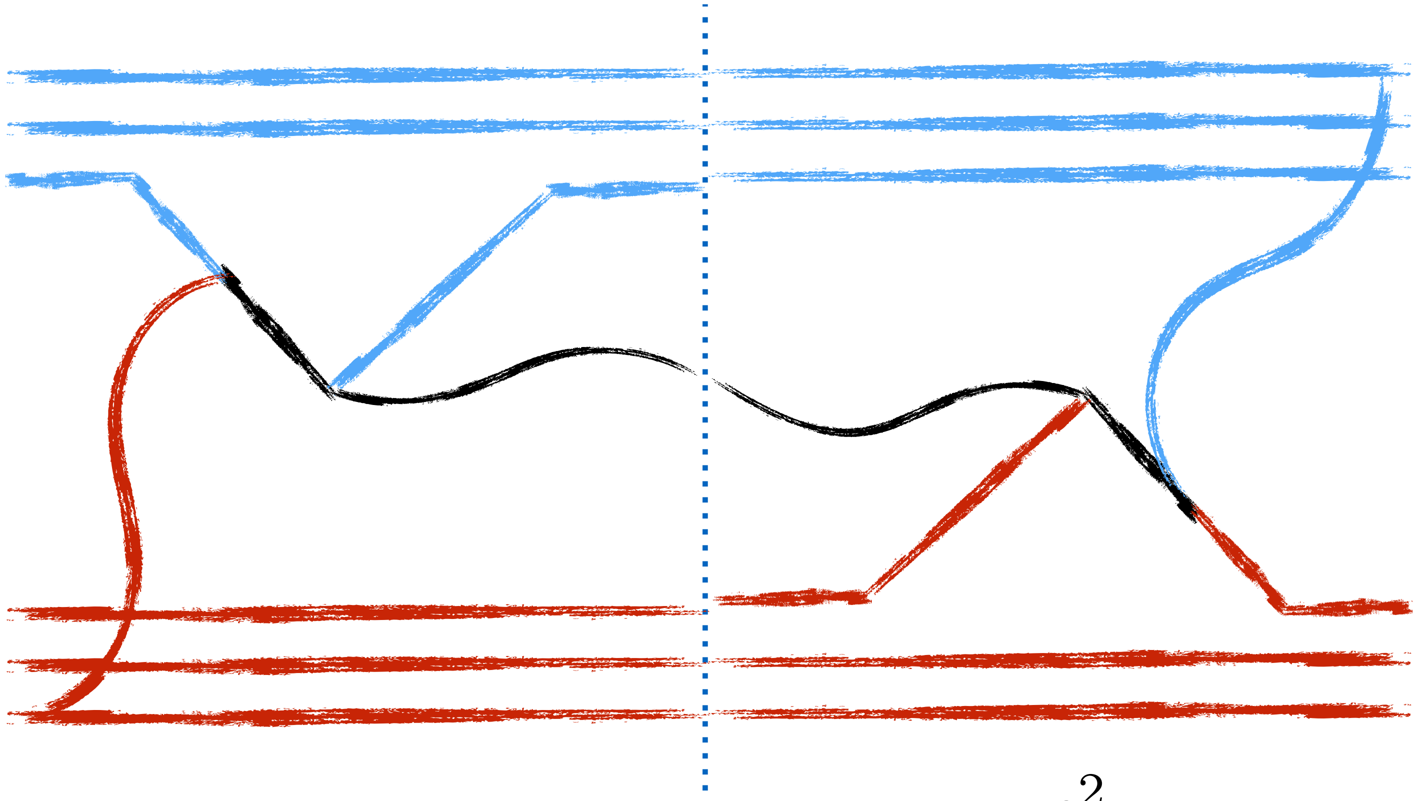
Higher twist distribution functions

$$\frac{g}{8\pi^3 s}\sqrt{\frac{s}{2}}\int dx_\perp dx_- \; e^{-i\alpha\sqrt{\frac{s}{2}}x_- + i(k,x)_\perp}\langle A|\bar{\psi}_{\bar{A}}(x_-, x_\perp)\; \not{p}_2\gamma^i\Big\{\bar{A}_i(0)\psi_{\bar{A}}(0) + \; \bar{F}_{+i}(0)\int_{-\infty}^{0} dx'_- \; \psi_{\bar{A}}(x'_-, 0_\perp)\Big\}|A\rangle$$

$$= \; i\frac{k_\perp^2}{m}\left[h_f^{\mathrm{tw}3}(\alpha, k_\perp^2) + i\tilde{h}_f^{\mathrm{tw}3}(\alpha, k_\perp^2)\right]$$

The structure of Wilson lines can be restored by gauge rotation

Contribution with gluon exchanges



The contribution is suppressed as $\frac{1}{N_c^2}$

$$
\begin{aligned}
W(\alpha_z,\beta_z,q_\perp) \;=\; & \frac{e^2}{8s_W^2c_W^2(N_c^2-1)Q^2}\int d^2k_\perp\;(k,q-k)_\perp\Big[\Big\{2(1+a_u^2)\\
\times\; & \big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)-\;\tilde{j}_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\big]\\
+\; & (1-a_u^2)\big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde{j}_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\\
+\; & j_{2u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde{j}_{2u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\big]\;+\;\alpha_z\leftrightarrow\beta_z\Big\}\\
+\; & \Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]
\end{aligned}
$$

The TMD operator is
constructed from quark
and gluon fields

Power correction to TMD factorization

leading twist contribution

leading power correction

$$
\begin{aligned}
W(\alpha_z,\beta_z,q_\perp) \;=\; & -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big[\Big\{(1+a_u^2)\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big]\\
&\times f_{1u}(\alpha_z,k_\perp)\bar f_{1u}(\beta_z,q_\perp-k_\perp)+2(a_u^2-1)\frac{k_\perp^2(q-k)_\perp^2}{m^2Q^2}h_{1u}^\perp(\alpha_z,k_\perp)\bar h_{1u}^\perp(\beta_z,q_\perp-k_\perp)\\
&+\frac{2k_\perp^2(q-k)_\perp^2}{(N_c^2-1)Q^2m^2}(a_u^2-1)[h_u^{\rm tw3}(\alpha_z,k_\perp)\bar h_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde h_u^{\rm tw3}(\alpha_z,k_\perp)\bar{\tilde h}_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)]\\
&-\frac{N_c}{N_c^2-1}\frac{(k,q-k)_\perp}{Q^2}\Big(2(1+a_u^2)\big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)-\tilde j_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)]\\
&+(1-a_u^2)\big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+j_{2u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\\
&+\tilde j_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde j_{2u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)]\Big)\\
&+(\alpha_z\leftrightarrow\beta_z)\Big\}\;+\;\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]\;+\;O\Big(\frac{m^8}{s}\Big)
\end{aligned}
$$

power correction (gluon contribution) suppressed by $1/N_c$

power correction suppressed by $1/N_c^2$

I. Balitsky, A.T. (2018)

Estimations

perturbative tails of TMDs:

$$f_1(\alpha_z,k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^\perp(\alpha_z,k_\perp^2) \simeq \frac{m^2 h(\alpha_z)}{k_\perp^4}, \qquad \bar{f}_1 \simeq \frac{\bar{f}(\alpha_z)}{k_\perp^2}, \quad \bar{h}_1^\perp \simeq \frac{m^2 \bar{h}(\alpha_z)}{k_\perp^4}$$

$$f_1(\beta_z,k_\perp^2) \simeq \frac{f(\beta_z)}{k_\perp^2}, \quad h_1^\perp(\beta_z,k_\perp^2) \simeq \frac{m^2 h(\beta_z)}{k_\perp^4}, \qquad \bar{f}_1 \simeq \frac{\bar{f}(\beta_z)}{k_\perp^2}, \quad \bar{h}_1^\perp \simeq \frac{m^2 \bar{h}(\beta_z)}{k_\perp^4}$$

J. Zhou, F. Yuan and Z.-T. Liang, Phys. Rev. D78 (2008)

Leading order power correction:

relative weight of the power correction

$$W(\alpha_z,\beta_z,q_\perp) \simeq -\frac{e^2}{8 s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k_\perp^2 (q-k)_\perp^2} \left[1 - 2\frac{(k,q-k)_\perp}{Q^2}\right]$$
$$\times \left[\left\{(1+a_u^2)[f_u(\alpha_z)\bar{f}_u(\beta_z)+\bar{f}_u(\alpha_z) f_u(\beta_z)]\right\} + \left\{u \leftrightarrow c\right\} + \left\{u \leftrightarrow d\right\} + \left\{u \leftrightarrow s\right\}\right]$$

Power corrections are important in the region where transverse momentum is not too small

Conclusion

$$\begin{aligned} W(\alpha_z,\beta_z,q_\perp) \;=\; & -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big[\Big\{(1+a_u^2)\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big] \\ & \times\;[f_{1u}(\alpha_z,k_\perp)\bar{f}_{1u}(\beta_z,q_\perp-k_\perp)+\bar{f}_{1u}(\alpha_z,k_\perp)f_{1u}(\beta_z,q_\perp-k_\perp)] \\ & +2(a_u^2-1)\frac{k_\perp^2(q-k)_\perp^2}{m^2Q^2}[h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp)+\bar{h}_{1u}^\perp(\alpha_z,k_\perp)h_{1u}^\perp(\beta_z,q_\perp-k_\perp)]\Big\} \\ & +\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big] \;+\; O\Big(\frac{m^8}{s}\Big) \end{aligned}$$

I. Balitsky and A.T., JHEP 07 (2017) 095; 05 (2018) 150

1) We calculated leading power correction to TMD factorization in Z boson production

2) We calculated hadronic tensor at the tree level using solution of the equations of motion

3) We constructed solution of the equations of motion by expansion in parameter m^2/s

4) We found leading contribution in kinematic limit $s \gg Q^2 \gg Q_\perp^2$

5) We found that the leading term can be expressed in terms of leading twist distribution functions