Odd azimuthal anisotropy & gluon correlations in CGC

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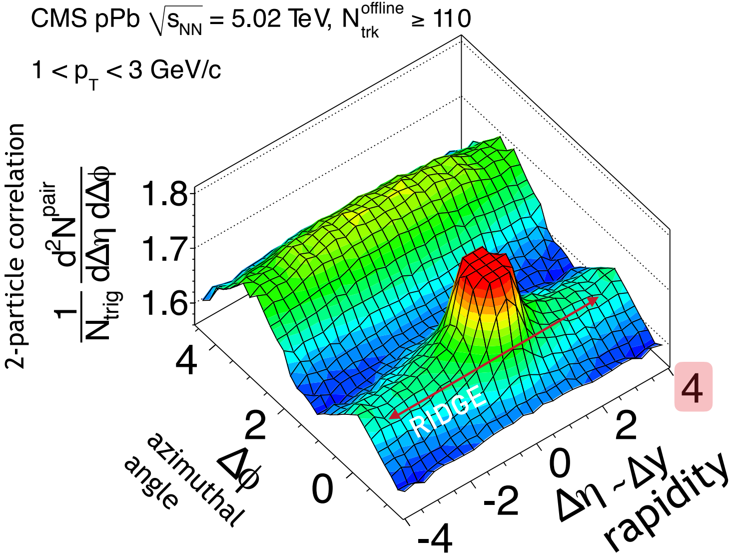
Outline

- Odd azimuthal anisotropy in saturation/CGC framework
- High multiplicity p(d)-A collisions & “glittering” glasma
- Effect of projectile geometry on azimuthal anisotropy in CGC



CMS, Phys. Lett. B 718 (2013) 795

Odd azimuthal anisotropy

- Forward region: odd anisotropy due to quark/anti-quark asymmetry

M. K. Davy, C. Marquet, Yu Shi, B.-W. Xiao, & C. Zhang, '18
M. Mace, K. Dusling & R. Venugoapalan, '16
T. Lappi, '14

- Central region:
  - No odd azimuthal anisotropy in strict dilute-dense approximation

A. Kovner & M. Lublinsky, '12
Y. V. Kovchegov & D. E. Wertepny, '12

  - ?

  - Non-zero odd azimuthal anisotropy in numerical dense-dense calculation

T. Lappi, S. Srednyak, R. Venugopalan, '09

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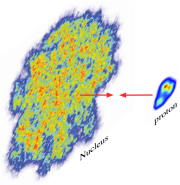
What do we know analytically in classical approximation?

Asymmetric collisions, when Q_s of projectile $\neq Q_s$ of target, is the easiest case.



Single inclusive production

- In general

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} f\left(\frac{Q_{sp}^2}{k_\perp^2}, \frac{Q_{sA}^2}{k_\perp^2}\right)$$ is known only numerically;

A. Krasnitz, R. Venugopalan, arXiv:9809433

For large $k_\perp \gg Q_{sA}^2$: $\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2}\frac{Q_{sA}^2}{k_\perp^2} f^{(1,1)}$

E. Kuraev, L. Lipatov, V. Fadin, '77

- If $k_\perp > Q_{sp}$,

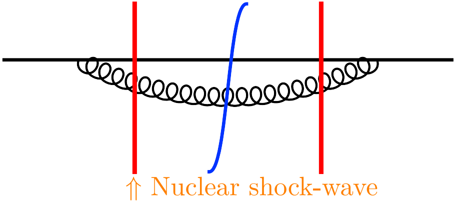
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

Functions $f^{(n)}$ are calculable!

Single inclusive production

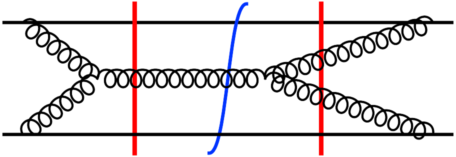
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

- $f^{(1)}$ is known since '98



Y. V. Kovchegov and A. H. Mueller, arXiv:hep-ph/9802440
A. Dumitru and L. D. McLerran, arXiv:hep-ph/0105268
J.-P. Blaizot, F. Gelis, R. Venugopalan, arXiv:0402256

- $f^{(2)}$: no complete result yet



I. Balitsky, arXiv:hep-ph/0409314
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

Double inclusive production

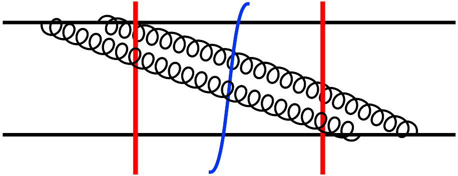
$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \; h^{(1)} \left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6 \; h^{(2)} \left(Q_{sA}\right) + \cdots$$

Momentum dependence is omitted to simplify notation

- Dilute-dilute "Glasma" graph: $\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 Q_{sA}^4 \; h^{(1,1)}$

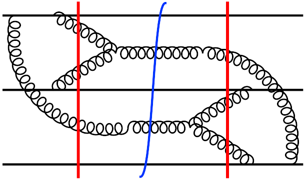
A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858

- $h^{(1)}$ is known since '12 ; invariant under $(k_\perp \to -k_\perp)$



A. Kovner and M. Lublinsky, arXiv:1211.1928
Y. V. Kovchegov and D. E. Wertepny, arXiv:1212.1195

- $h^{(2)}$: no complete result yet



L. McLerran and V. S., arXiv:1611.09870;
Yu. Kovchegov and V. S., arXiv:1802.08166

What does presence of odd harmonics mean?

- Double inclusive production

$$\frac{d^2N}{d^2k_1dy_1d^2k_2dy_2} = \frac{d^2N}{k_1dk_1dy_1k_2dk_2dy_2}\left(1 + 2v_2^2\{2\}\cos 2(\phi_1 - \phi_2) + 2v_3^2\{2\}\cos 3(\phi_1 - \phi_2) + \ldots\right)$$

- A non-vanishing $v_3^2\{2\}$

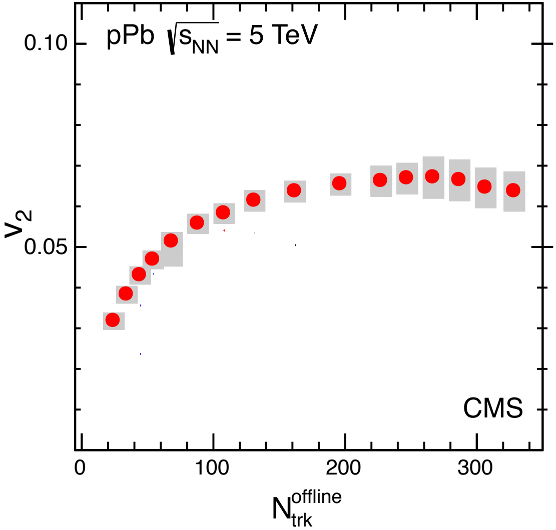
$$\int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1d^2k_2}\left(\Delta\phi\right) = \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1d^2k_2}\left(\Delta\phi\right) - \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1d^2k_2}\left(\Delta\phi + \pi\right)$$

$$= \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \left[\frac{d^2N}{d^2k_1d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) - \frac{d^2N}{d^2k_1d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)\right]$$

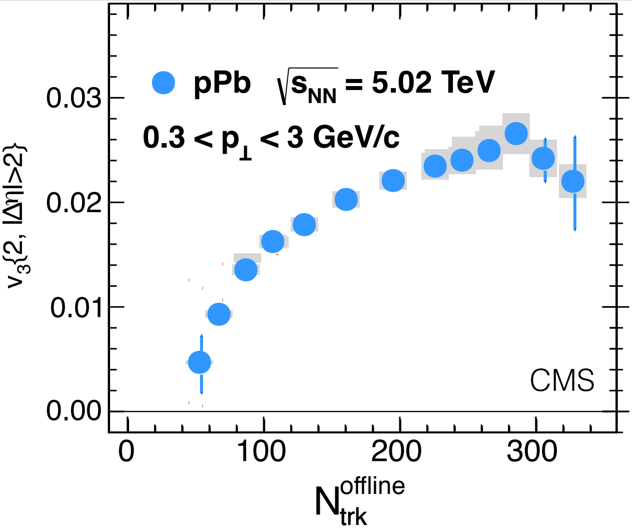
- Therefore, non-zero $v_3 \rightsquigarrow$

$$\frac{d^2N}{d^2k_1d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) \neq \frac{d^2N}{d^2k_1d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)$$

and is absent in “Glasma” graph and $h^{(1)}$


pPb $\sqrt{s_{NN}}$ = 5 TeV
CMS
v_2
0.10
0.05
0
100
200
300
$N_{trk}^{offline}$

Experimental data: $v_3\{2\}$



- Suppressed compared to v_2, but non-zero!

Can saturation dynamics account for observed long-range rapidity correlations with non-zero odd azimuthal harmonics?

Odd contribution is buried somewhere in multiple rescattering i.e. in high order $h^{(N\gg 1)}$ $\Downarrow$

$$\frac{d^2N}{d^3k\,d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4\; h^{(1)}\left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6\; h^{(2)}\left(Q_{sA}\right) + \cdots$$

Solving CYM on lattice: T. Lappi, S. Srednyak and R. Venugopalan, arXiv:0911.2068

- Theoretically this is unsatisfactory

- Phenomenologically this is problematic

  $v_3\{2\}$ is observed in p-A

  $v_3\{2\}$ is not much smaller than $v_2\{2\}$

Inspiration from Single Transverse Spin Asymmetry

- Consider single gluon production

$$\frac{d\sigma}{d^2k} \sim |M(\underline{k})|^2 = \int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, M(\underline{x})\, M^*(\underline{y})$$

- Amplitude may have two contributions

$$M(\underline{x}) = M_1(\underline{x}) + M_3(\underline{x}) + \ldots$$

- Asymmetry under $\underline{k} \to -\underline{k}$ would mean that

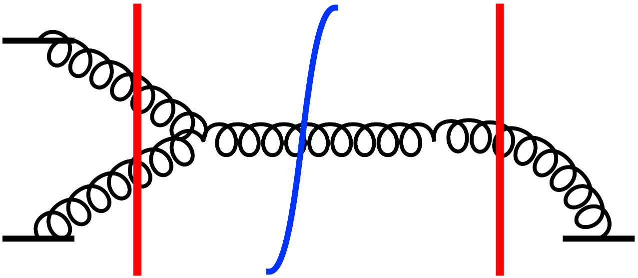
$$M_1(\underline{x})\, {M_3}^*(\underline{y}) + M_3(\underline{x})\, {M_1}^*(\underline{y}) = -M_1(\underline{y})\, {M_3}^*(\underline{x}) - M_3(\underline{y})\, {M_1}^*(\underline{x})$$

⇝ $M_1(\underline{x})\, {M_3}^*(\underline{y})$ is imaginary

⇝ Phase difference between M_1 and M_3 in coordinate space

In coordinate space, but not dissimilar from STSA
S. Brodsky, D. S. Hwang, Y. Kovchegov, I. Schmidt, M. Sievert, arXiv:1304.5237

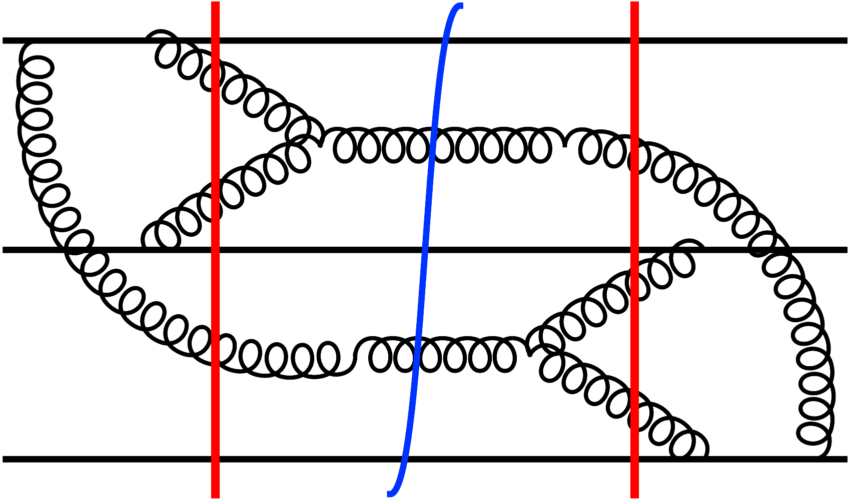
Natural candidate



M_3 M_1

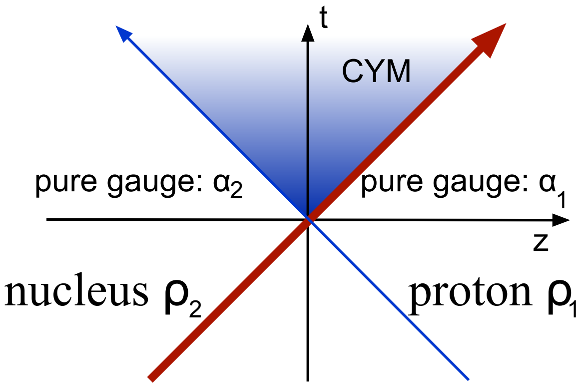
- Vanishes for single-inclusive production after performing average with respect to projectile configurations. . .

Double inclusive gluon production



- Non-zero!

Classical Yang-Mills



- Just after collision, $\tau \to 0+$, initial conditions are known (Fock-Schwinger gauge $A_\tau = 0$)
  A. Kovner, L. McLerran, H. Weigert, arXiv:9506320
- In forward light-cone $[D_\mu, F^{\mu\nu}] = 0$
- Solve equations perturbatively in ρ_1; use LSZ

Classical Yang-Mills

- Before collision, pure gauge soft fields created by "valence" currents

$$\partial_i \alpha^i_{1,2}(\mathbf{x}_\perp) = g\ \rho_{1,2}(\mathbf{x}_\perp)$$
$$\alpha^i_{1,2}(\mathbf{x}_\perp) = -\tfrac{1}{ig} U_{1,2}(\mathbf{x}_\perp)\partial^i U^\dagger_{1,2}(\mathbf{x}_\perp)$$

- Just after collision, $\tau \to 0+$, (Fock-Schwinger gauge $A_\tau = 0$)

A. Kovner, L. McLerran, H. Weigert, arXiv:9506320

$$\alpha^i(\tau \to 0, \mathbf{x}_\perp) = \alpha^i_1(\mathbf{x}_\perp) + \alpha^i_2(\mathbf{x}_\perp)$$

$$A_\eta(\tau \to 0, \mathbf{x}_\perp) = \tau^2 \alpha(\tau \to 0, \mathbf{x}_\perp); \quad \alpha(\tau \to 0, \mathbf{x}_\perp) = \frac{ig}{2}[\alpha^i_1(\mathbf{x}_\perp), \alpha^i_2(\mathbf{x}_\perp)]$$

- Expansion in $g\rho_1$ $(\Phi_1 = \frac{g}{\partial^2_\perp}\rho_1)$:

$$\alpha^i_1 = \partial^i \Phi_1 - \tfrac{ig}{2}\left(\delta_{ij} - \frac{\partial_i \partial_j}{\partial^2}\right)\left[\partial^j \Phi_1, \Phi_1\right] + \mathcal{O}(\Phi_1^3)$$

- In forward light-cone $\quad [D_\mu, F^{\mu\nu}] = 0$

Classical Yang-Mills: expansion in $g\rho_1$

- In order to perform expansion, it is convenient to rotate out **nucleus field** from initial conditions:

$$\alpha(\tau, \mathbf{x}_\perp) = U_2(\mathbf{x}_\perp)\beta(\tau, \mathbf{x}_\perp)U_2^\dagger(\mathbf{x}_\perp)$$

$$\alpha^i(\tau, \mathbf{x}_\perp) = U_2(\mathbf{x}_\perp)\left(\beta^i(\tau, \mathbf{x}_\perp) - \frac{1}{ig}\partial_i\right)U_2^\dagger(\mathbf{x}_\perp)$$

$$\beta(\tau \to 0, \mathbf{x}_\perp) = U_2^\dagger(\mathbf{x}_\perp)\alpha(\tau \to 0, \mathbf{x}_\perp)U_2^\dagger(\mathbf{x}_\perp)$$

$$\beta^i(\tau \to 0, \mathbf{x}_\perp) = U_2^\dagger(\mathbf{x}_\perp)\alpha_1^i(\mathbf{x}_\perp)U_2^\dagger(\mathbf{x}_\perp)$$

- Perform expansion in powers of ρ_1: $\beta_\gamma = \beta_\gamma^{(1)} + \beta_\gamma^{(2)} + \cdots$

First order, $\beta^{(1)}$

- At leading order, CYM equations are (in Milne coordinates)

$$\left[\partial_\tau^2 + \frac{3}{\tau}\partial_\tau - \partial_\perp^2\right]\beta^{(1)}(\tau, \mathbf{x}_\perp) = 0,$$
$$\partial_\tau \partial_i \beta_i^{(1)}(\tau, \mathbf{x}_\perp) = 0,$$
$$\left[\delta^{ij}\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \partial_\perp^2\right) + \partial_i\partial_j\right]\beta_j^{(1)}(\tau, \mathbf{x}_\perp) = 0$$

No non-linear terms $\rightsquigarrow$ solution can be found trivially. Analogous to $\Box\phi = 0$

- Solutions are in momentum space:

$$\beta^{(1)}(\tau, \mathbf{k}_\perp) = b_1(\mathbf{k}_\perp)\frac{J_1(k_\perp \tau)}{k_\perp \tau},$$
$$\beta_i^{(1)}(\tau, \mathbf{k}_\perp) = i\frac{\varepsilon^{ij}k_j}{k_\perp^2} b_2(\mathbf{k}_\perp) J_0(k_\perp \tau) + i k_i \Lambda(\mathbf{k}_\perp)$$

$$b_1(\mathbf{x}_\perp) = \delta^{ij}\Omega_{ij}(\mathbf{x}_\perp),$$
$$b_2(\mathbf{x}_\perp) = \epsilon^{ij}\Omega_{ij}(\mathbf{x}_\perp),$$
$$\Omega_{ij}(\mathbf{x}_\perp) = g\left[\frac{\partial_i}{\partial_\perp^2}\rho_1^a(\mathbf{x}_\perp)\right]\partial^j\left(U^\dagger(\mathbf{x}_\perp) t_a U(\mathbf{x}_\perp)\right)$$

Second order, $\beta^{(2)}$

- At next-to-leading order:

$$\left[\partial_\tau^2 + \frac{3}{\tau}\partial_\tau - \partial_\perp^2\right]\beta^{(2)}(\tau,\mathbf{x}_\perp) = -ig\left(\partial_i[\beta^{(1)}{}_i(\tau,\mathbf{x}_\perp),\beta^{(1)}(\tau,\mathbf{x}_\perp)] + [\beta^{(1)}{}_i(\tau,\mathbf{x}_\perp),\partial_i\beta^{(1)}(\tau,\mathbf{x}_\perp)]\right)$$

$$\left[\delta^{ij}\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \partial_\perp^2\right) + \partial_i\partial_j\right]\beta^{(2)}{}_j(\tau,\mathbf{x}_\perp) = -ig\left(\partial_j[\beta^{(1)}{}_j(\tau,\mathbf{x}_\perp),\beta^{(1)}{}_i(\tau,\mathbf{x}_\perp)]\right.$$
$$\left. + [\beta^{(1)}{}_j(\tau,\mathbf{x}_\perp),\partial_j\beta^{(1)}{}_i(\tau,\mathbf{x}_\perp) - \partial_i\beta^{(1)}{}_j(\tau,\mathbf{x}_\perp)] - \tau^2[\beta^{(1)}(\tau,\mathbf{x}_\perp),\partial_i\beta^{(1)}(\tau,\mathbf{x}_\perp)]\right)$$

First non-linear corrections!

- This looks very discouraging as (in momentum space) β^1 are Bessel functions
- Goal is to compute $g^6\rho_1^3$ correction to particle production.
  Do we have to solve these equations?

Particle production: Lehmann-Symanzik-Zimmermann I

- For simplicity – Minkowski space and semi-classical scalar field $\phi(x)$.
  The creation operator

$$a^+(\mathbf{k},t) = \frac{1}{i}\int d^3x \;\; \exp(-ik\cdot x) \;\; \phi(x) \qquad\qquad k\cdot x = k_\mu x^\mu$$

- The difference

$$a^+(\mathbf{k},t\to\infty) - a^+(\mathbf{k},t\to t_0) = \frac{1}{i}\int_{t_0}^{\infty} dt\; \partial_0 \left(\int d^3x \;\; \exp(-ik\cdot x) \;\; \phi(x)\right)$$

$$= \frac{1}{i}\int_{t_0}^{\infty} dt\; d^3x \;\; \exp(-ik\cdot x) \;\; \underbrace{\left(\Box + m^2\right) \;\; \phi\;(x)}_{\text{interaction}}$$

  Instead of a usual choice $t_0 \to -\infty$, in order to mimic initial conditions on light cone, $t_0 \to 0$.

- Thus for creation operator at out-state, there are two contributions

$$a^+(\mathbf{k},\infty) = \underbrace{\frac{1}{i}\int_{t=0} d^3x \exp(-ik\cdot x) \;\; \phi(x)}_{\text{initial flux through t=0 hypersurface}} + \underbrace{\frac{1}{i}\int_0^{\infty} dt \int d^3x \exp(-ik\cdot x) \;\; \left(\Box + m^2\right) \;\; \phi(x)}_{\text{interaction; evolution in the forward light cone}}$$

- Single-inclusive gluon production $E_k \frac{dN}{d^3k} = \frac{1}{2(2\pi)^3} a^+(\mathbf{k},\infty)\; a(\mathbf{k},\infty)$

Particle production: Lehmann-Symanzik-Zimmermann II

- Single-inclusive gluon production

$$E_k \frac{dN}{d^3k} = \frac{1}{2(2\pi)^3} \left| \overbrace{\underbrace{\frac{1}{i}\int_{t=0} d^3x \exp(-ik\cdot x) \;\; \phi(x)}_{\text{initial flux}}}^{\text{''surface''}} + \overbrace{\underbrace{\frac{1}{i}\int_0^\infty dt \int d^3x \exp(-ik\cdot x) \;\left(\Box + m^2\right)\; \phi(x)}_{\text{interaction; evolution in the forward light cone}}}^{\text{''bulk''}} \right|^2$$

- Leading order: no "bulk" contribution
- Both contributions schematically:

$$E_k \frac{dN}{d^3k} = \left(\underbrace{a^{(1)}(\mathbf{k}_\perp)}_{\text{surface only}} + a^{(2)}(\mathbf{k}_\perp) + \ldots \right) \left(a^{(1)}(\mathbf{k}_\perp) + \underbrace{a^{(2)}(\mathbf{k}_\perp)}_{\text{surface and bulk}} + \ldots \right)^*$$

$$\approx \underbrace{a^{(1)}(\mathbf{k}_\perp)a^{(1)}(-\mathbf{k}_\perp)}_{\text{symmetric}} + \underbrace{a^{(1)}(\mathbf{k}_\perp)\left(a^{(2)}(\mathbf{k}_\perp)\right)^* + a^{(1)}(-\mathbf{k}_\perp)a^{(2)}(\mathbf{k}_\perp)}_{\text{odd asymmetry is possible}}$$

Gluon production

- Leading order and saturation correction

$$\frac{dN^{\rm even}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}{\rm Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$

$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Here $\delta_{ij}\Omega_{ij}=\Omega_{xx}+\Omega_{yy}$ and $\epsilon_{ij}\Omega_{ij}=\Omega_{xy}-\Omega_{yx}$ and

$$\Omega^a_{ij}(\mathbf{x}_\perp) = \underbrace{g\left[\frac{\partial_i}{\partial^2}\overbrace{\rho^b(\mathbf{x}_\perp)}^{\text{val. sour.}}\right]\partial_j\overbrace{U^{ab}(\mathbf{x}_\perp)}^{\text{target W line}}}_{\text{valence sources rotated by the target}}$$

$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right]$ is suppressed by extra $\alpha_s\rho_p$

L. McLerran and V. S., arXiv:1611.09870

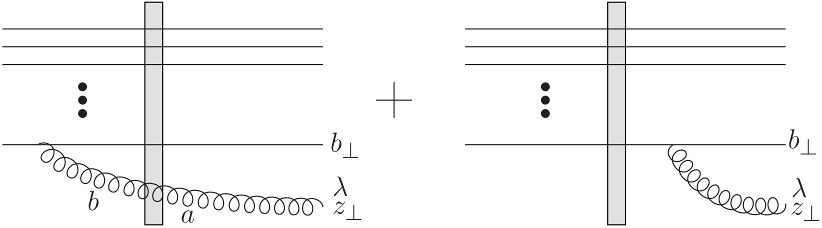
Alternative approach

- This was obtained in Fock-Schwinger gauge $A_\tau = 0$

- Motivation to compute in global gauge $A^+ = 0$

Yu. Kovchegov and V. S., arXiv:1802.08166

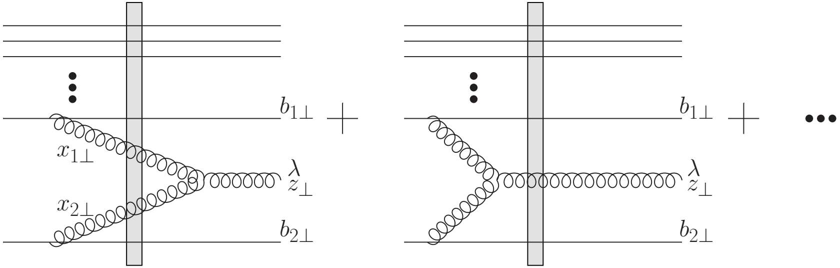
Leading order amplitude



$$\underline{\epsilon}^*_\lambda \cdot \underline{M}_1(\underline{z}, \underline{b}) = \frac{i\,g}{\pi} \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b})}{|\underline{z} - \underline{b}|^2} \left[U^{ab}_{\underline{z}} - U^{ab}_{\underline{b}} \right] (V_{\underline{b}}\, t^b)$$

- We have to track the phases $\Uparrow$ of the light-cone wave functions

First saturation correction



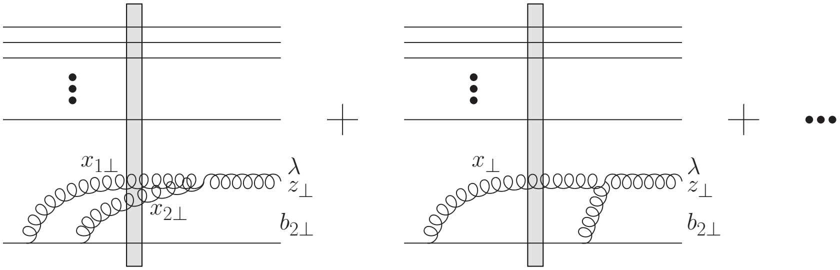
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

First saturation correction

$$
\begin{aligned}
\underline{\epsilon}^*_\lambda \cdot \underline{M}_3^{ABC} = &- \frac{g^3}{4\,\pi^4} \int d^2 x_1\, d^2 x_2\, \delta[(\underline{z} - \underline{x}_1) \times (\underline{z} - \underline{x}_2)] \Bigg[\frac{\underline{\epsilon}_\lambda{}^* \cdot (\underline{x}_2 - \underline{x}_1)}{|\underline{x}_2 - \underline{x}_1|^2} \frac{\underline{x}_1 - \underline{b}_1}{|\underline{x}_1 - \underline{b}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} - \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{x}_1 - \underline{b}_1)}{|\underline{x}_1 - \underline{b}_1|^2} \frac{\underline{z} - \underline{x}_1}{|\underline{z} - \underline{x}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}}{|\underline{x}_2 - \underline{b}} \\
&+ \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{x}_2 - \underline{b}_2)}{|\underline{x}_2 - \underline{b}_2|^2} \frac{\underline{x}_1 - \underline{b}_1}{|\underline{x}_1 - \underline{b}_1|^2} \cdot \frac{\underline{z} - \underline{x}_2}{|\underline{z} - \underline{x}_2|^2} \Bigg] f^{abc} \left[U^{bd}_{\underline{x}_1} - U^{bd}_{\underline{b}_1} \right] \left[U^{ce}_{\underline{x}_2} - U^{ce}_{\underline{b}_2} \right] \left(V_{\underline{b}_1} t^d \right)_1 \left(V_{\underline{b}_2} t^e \right)_2 + \frac{i\, g^3}{4\,\pi^3} f^{abc} \left(V_{\underline{b}_1} t^d \right)_1 \left(V_{\underline{b}_2} t^e \right)_2 \\
&\times \int d^2 x \Bigg[U^{bd}_{\underline{b}_1} \left(U^{ce}_{\underline{x}} - U^{ce}_{\underline{b}_2} \right) \Bigg(\frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{x})}{|\underline{z} - \underline{x}|^2} \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} - \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b}_1)}{|\underline{z} - \underline{b}_1|^2} \frac{\underline{z} - \underline{x}}{|\underline{z} - \underline{x}|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} - \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b}_1)}{|\underline{z} - \underline{b}_1|^2} \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} \cdot \frac{\underline{x} -}{|\underline{x} -} \\
&- \left(U^{bd}_{\underline{x}} - U^{bd}_{\underline{b}_1} \right) U^{ce}_{\underline{b}_2} \Bigg(\frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{x})}{|\underline{z} - \underline{x}|^2} \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} - \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \frac{\underline{z} - \underline{x}}{|\underline{z} - \underline{x}|^2} \cdot \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} - \frac{\underline{\epsilon}^{\lambda *} \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} \Bigg) \Bigg] \\
&- \frac{i\, g^3}{4\,\pi^2} f^{abc} \left(V_{\underline{b}_1} t^d \right)_1 \left(V_{\underline{b}_2} t^e \right)_2 \Bigg[(U^{bd}_{\underline{z}} - U^{bd}_{\underline{b}_1})\, U^{ce}_{\underline{b}_2} \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b}_1)}{|\underline{z} - \underline{b}_1|^2} \ln \frac{1}{|\underline{z} - \underline{b}_2|\,\Lambda} - U^{bd}_{\underline{b}_1} (U^{ce}_{\underline{z}} - U^{ce}_{\underline{b}_2}) \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \ln \frac{1}{|\underline{z} - \underline{b}_1|\,\Lambda} \Bigg] \\
&- \frac{i\, g^3}{4\,\pi^3} \int d^2 x \left[U^{ab}_{\underline{x}} - U^{ab}_{\underline{z}} \right] f^{bde} \left(V_{\underline{b}_1} t^d \right)_1 \left(V_{\underline{b}_2} t^e \right)_2 \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{x})}{|\underline{z} - \underline{x}|^2} \frac{\underline{x} - \underline{b}_1}{|\underline{x} - \underline{b}_1|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} \,\mathrm{Sign}(b_2^- - b_1^-)
\end{aligned}
$$

G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

First saturation correction



G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

First saturation correction

$$\underline{\epsilon}_\lambda^* \cdot \underline{M}_3^{DE} = -\frac{g^3}{8\,\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\underline{z}-\underline{x}_1) \times (\underline{z}-\underline{x}_2)] \left[\frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{x}_1)}{|\underline{x}_2 - \underline{x}_1|^2} \frac{\underline{x}_1 - \underline{b}_2}{|\underline{x}_1 - \underline{b}_2|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} \right.$$

$$\left. - \frac{\underline{\epsilon}^{\lambda *} \cdot (\underline{x}_1 - \underline{b}_2)}{|\underline{x}_1 - \underline{b}_2|^2} \frac{\underline{z} - \underline{x}_1}{|\underline{z} - \underline{x}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} + \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{b}_2)}{|\underline{x}_2 - \underline{b}_2|^2} \frac{\underline{x}_1 - \underline{b}_2}{|\underline{x}_1 - \underline{b}_2|^2} \cdot \frac{\underline{z} - \underline{x}_2}{|\underline{z} - \underline{x}_2|^2} \right]$$

$$\times\ f^{abc} \left[U_{\underline{x}_1}^{bd} - U_{\underline{b}_2}^{bd} \right] \left[U_{\underline{x}_2}^{ce} - U_{\underline{b}_2}^{ce} \right] \left(V_{\underline{b}_1} \right)_1 \left(V_{\underline{b}_2} t^e\, t^d \right)_2$$

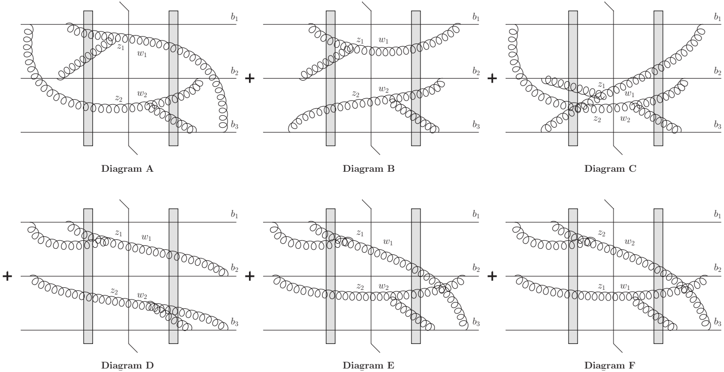
$$+ \frac{i\, g^3}{4\,\pi^3} \int d^2x\, f^{abc}\, U_{\underline{b}_2}^{bd} \left[U_{\underline{x}}^{ce} - U_{\underline{b}_2}^{ce} \right] \left(V_{\underline{b}_1} \right)_1 \left(V_{\underline{b}_2} t^e\, t^d \right)_2 \left(\frac{\underline{\epsilon}_\lambda^* \cdot (\underline{z} - \underline{x})}{|\underline{z} - \underline{x}|^2} \frac{1}{|\underline{x} - \underline{b}_2|^2} \right.$$

$$\left. - \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \frac{\underline{z} - \underline{x}}{|\underline{z} - \underline{x}|^2} \cdot \frac{\underline{x} - \underline{b}_2}{|\underline{x} - \underline{b}_2|^2} - \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \frac{1}{|\underline{x} - \underline{b}_2|^2} \right)$$

$$+ \frac{i\, g^3}{4\,\pi^2} f^{abc}\, U_{\underline{b}_2}^{bd} \left[U_{\underline{z}}^{ce} - U_{\underline{b}_2}^{ce} \right] \left(V_{\underline{b}_1} \right)_1 \left(V_{\underline{b}_2} t^e\, t^d \right)_2 \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{z} - \underline{b}_2)}{|\underline{z} - \underline{b}_2|^2} \ln \frac{1}{|\underline{z} - \underline{b}_2|\, \Lambda}$$

G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

Collecting all diagrams



- Reproduces result obtained in Fock-Schwinger gauge!
- Six adjoint Wilson lines multiplying a non-trivial function.

Approximations

- The sum of all contributions can be computed numerically; relatively low cost

- However, our goal is to obtain an analytical result

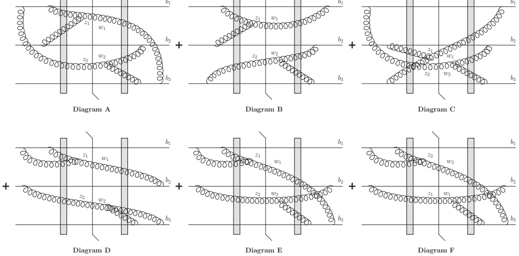
- Approximations:

  – Large N_c
  – Golec-Biernat–Wusthoff model

$$S = \exp\left(-\frac{1}{8}Q_s^2 r^2 \ln\frac{1}{r^2\Lambda^2}\right) \to \exp\left(-\frac{1}{8}Q_s^2 r^2\right)$$

  – Only lowest non-trivial order in interaction with the target

$$\frac{Q_s^2}{k^2} \ll 1$$



- Under these approximations, non-vanishing contributions from diagrams A, B and C

$$\frac{d\sigma_{odd}}{d^2k_1\,dy_1\,d^2k_2\,dy_2} = \frac{1}{[2(2\pi)^3]^2}\int d^2B\,d^2b\,[T_1(\underline{B}-\underline{b})]^3\; g^8\,Q_{s0}^6(b)\,\frac{1}{\underline{k}_1^6\,\underline{k}_2^6}$$

$$\times\left\{\underbrace{\left[\frac{(\underline{k}_1^2+\underline{k}_2^2+\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1+\underline{k}_2)^6} - \frac{(\underline{k}_1^2+\underline{k}_2^2-\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1-\underline{k}_2)^6}\right]}_{A} + \underbrace{\frac{10\,c^2}{(2\pi)^2}\,\frac{1}{\Lambda^2}\,\frac{\underline{k}_1\cdot\underline{k}_2}{k_1\,k_2}}_{B}\right.$$

$$\left. + \underbrace{\frac{1}{4\pi}\,\frac{k_1^4}{\Lambda^4}\left[\delta^2(\underline{k}_1-\underline{k}_2)-\delta^2(\underline{k}_1+\underline{k}_2)\right]}_{C}\right\}$$

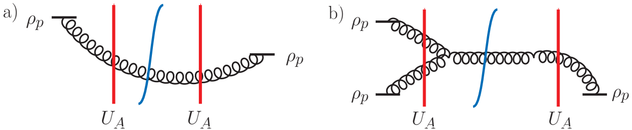
CGC perspective on v_3

- Leading order and the first saturation correction

$$\text{a)}\quad \frac{dN^{\text{even}}(\underline{k})}{d^2k\,dy}\Big[\rho_p,\rho_t\Big] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\,\Omega^a_{ij}(\underline{k})\,\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\text{b)}\quad \frac{dN^{\text{odd}}(\underline{k})}{d^2k\,dy}\Big[\rho_p,\rho_T\Big] = \frac{2}{(2\pi)^3}\text{Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{\text{Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\,\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$
$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Recall that $\Omega \propto \rho_{\text{proton}}$



- Odd azimuthal harmonics is a sign of emerging coherence in proton wave function: the first saturation correction!

Non-zero long-range odd harmonics in high energy p-A is evidence of saturation!

Multiplicity dependence: scaling argument

- Physical two-particle anisotropy coefficients can be simply expressed as

$$v_n^2\{2\}(N_{\rm ch}) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t \; W[\rho_p] \; W[\rho_t] \; |Q_n[\rho_p,\rho_t]|^2 \; \delta\left(\frac{dN}{dy}\left[\rho_p,\rho_t\right] - N_{\rm ch}\right)$$

with

$$Q_{2n}[\rho_p,\rho_t] = \frac{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i2n\phi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}, \quad Q_{2n+1}[\rho_p,\rho_t] = \frac{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i(2n+1)\phi} \frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}$$

- High multiplicity is driven by fluctuations in ρ_p
- To study multiplicity dependence, rescale $\rho_p \to c\,\rho_p$
- Under this rescaling:

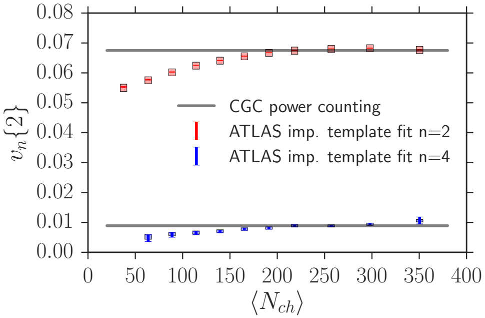
$$\frac{dN}{dy} \to c^2 \frac{dN}{dy}; \qquad v_{2n}^2\{2\} \to v_{2n}^2\{2\}; \qquad v_{2n+1}^2\{2\} \to c^2\; v_{2n+1}^2\{2\}$$

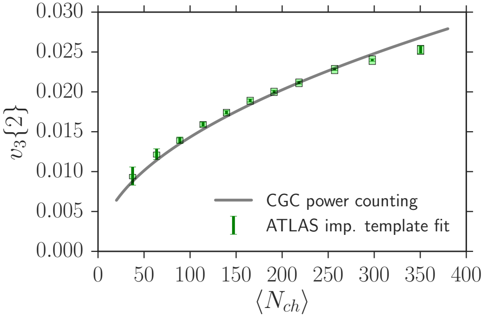
- Therefore in the first approximation: $v_{2n}\{2\}$ is independent of multiplicity

$$v_{2n+1}\{2\} \propto \sqrt{\frac{dN}{dy}}$$

Multiplicity dependence: scaling argument

M. Mace , V. S., P. Tribedy, & R. Venugopalan, arXiv:1807.00825





Conclusions I

- Odd azimuthal harmonics
  are an inherent property of particle production in the saturation framework

- Non-zero long range in y odd azimuthal harmonics $\Leftrightarrow$ evidence of saturation

- Phenomenological applications:
  – able to qualitatively describe multiplicity dependence in p-A at LHC
  – talk by Mark Mace next week:
  quantitative results for p-A at LHC and small systems at RHIC

Outline

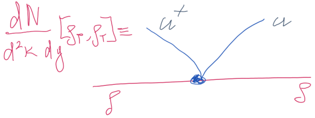
- Odd azimuthal anisotropy in saturation/CGC framework
- High multiplicity p(d)-A collisions & "glittering" glasma

  A. Kovner & V.S., '18
- Effect of projectile geometry on azimuthal anisotropy in CGC

  A. Kovner & V.S., '18

Gluon production: functional form

- Functional form for gluon production:



$$\left.\frac{dN}{d^2k\,dy}\right|_{\rho_{\rm p},\rho_{\rm t}} = \frac{2g^2}{(2\pi)^3}\int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\Gamma(\underline{k},\underline{q},\underline{q}')\rho^a_{\rm p}(-\underline{q}')\left[U^\dagger(\underline{k}-\underline{q}')U(\underline{k}-\underline{q})\right]_{ab}\rho^b_{\rm p}(\underline{q}),$$

where the square of Lipatov vertex is $\Gamma(\underline{k},\underline{q},\underline{q}') = \left(\frac{\underline{q}}{\underline{q}^2} - \frac{\underline{k}}{\underline{k}^2}\right)\cdot\left(\frac{\underline{q}'}{\underline{q}'^2} - \frac{\underline{k}}{\underline{k}^2}\right).$

- The single (double) inclusive production:

$$\frac{dN}{d^2k\,dy} = \left\langle\left\langle \left.\frac{dN}{d^2k\,dy}\right|_{\rho_{\rm p},\rho_{\rm t}}\right\rangle_{\rm p}\right\rangle_{\rm t}, \quad \frac{d^2N}{d^2k_1dy_1d^2k_2dy_2} = \left\langle\left\langle \left.\frac{dN}{d^2k_1dy_1}\right|_{\rho_{\rm p},\rho_{\rm t}} \left.\frac{dN}{d^2k_2dy_2}\right|_{\rho_{\rm p},\rho_{\rm t}}\right\rangle_{\rm p}\right\rangle_{\rm t}.$$

Averaging is performed over projectile and target color charge configurations:

$$\langle O(\rho_{\rm p,t})\rangle_{\rm p,t} = \frac{1}{Z_{\rm p,t}}\int \mathcal{D}\rho_{\rm p,t}\; W_{\rm p,t}(\rho_{\rm p,t})\; O(\rho_{\rm p,t})$$

Gluon production: functional form

- In general, one can compute $\frac{d^n N}{d^2k_1 dy_1 d^2k_2 dy_2 \cdots d^2k_n dy_n}$, cumulants and factorial cumulants

– Dilute-dilute approx. $\rightsquigarrow$ "Glittering glasmas" $\equiv$ color density fluctuations

GLITTER = GLuon Intensification Through Tenacious Emission of Radiation

F. Gelis, T. Lappi, & L. McLerran, Nucl. Phys. A ***828****, 149 (2009), arXiv:0905.3234*

- These fluctuations are approximately negative binomial:

derived for $k \gg Q_{st}$!

F. Gelis, T. Lappi, & L. McLerran, Nucl. Phys. A ***828****, 149 (2009), arXiv:0905.3234*

- Instead, consider generating function

$$G(t) = \left\langle \left\langle \exp\left[t \int_{k_{\min}} d^2k \, \frac{dN}{d^2k dy}\bigg|_{\rho_{\mathrm{p}},\rho_{\mathrm{t}}} \right] \right\rangle_{\mathrm{p}} \right\rangle_{\mathrm{t}}, \qquad \underbrace{k_{\min} \gg \Lambda_{\mathrm{QCD}}}_{\text{Detector cut for produced gluons}}$$

Moments $\int d^2k_1 \cdots d^2k_n \frac{d^n N}{d^2k_1 dy_1 d^2k_2 dy_2 \cdots d^2k_n dy_n} \equiv$ derivatives of $G(t)$ at $t = 0$.

Generating function:

- MV model:

$$\langle \rho_{\rm p}^a(\underline{p})\rho_{\rm p}^b(\underline{k})\rangle_{\rm p} = (2\pi)^2\mu_{\rm p}^2(p)\delta(\underline{p}+\underline{k})\delta^{ab} \Leftrightarrow W_{\rm p}(\rho_{\rm p}) = \exp\left(-\int \frac{d^2q}{(2\pi)^2}\rho_{\rm p}^a(-\underline{q})\frac{1}{2\mu_{\rm p}^2(q)}\rho_{\rm p}^a(\underline{q})\right)$$

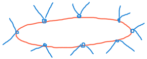
- Reminder: $\left.\frac{dN}{d^2kdy}\right|_{\rho_{\rm p},\rho_{\rm t}}$ is quadratic in ρ_p:

$$\left.\frac{dN}{d^2kdy}\right|_{\rho_{\rm p},\rho_{\rm t}} = \frac{2g^2}{(2\pi)^3}\int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\Gamma(\underline{k},\underline{q},\underline{q}')\rho_{\rm p}^a(-\underline{q}')\left[U^\dagger(\underline{k}-\underline{q}')U(\underline{k}-\underline{q})\right]_{ab}\rho_{\rm p}^b(\underline{q})$$

- Average w.r.t. ρ_p can be done analytically

$$G(t) = \left\langle\left\langle \exp\left[t\int_{k_{\rm min}} d^2k \left.\frac{dN}{d^2kdy}\right|_{\rho_{\rm p},\rho_{\rm t}}\right]\right\rangle_{\rm p}\right\rangle_{\rm t} = \frac{1}{Z_t}\int D\rho_{\rm t} W[\rho_{\rm t}]\exp\left[-\frac{1}{2}\mathrm{tr}\ln\left[1-tM\right]\right]$$

where M is defined by its matrix elements



$$M_{ab}(q',q) = \frac{4g^2}{(2\pi)^3}\mu^2(q)\int_{k_{\rm min}}\frac{d^2k}{(2\pi)^2}\Gamma(k,q,q')\left[U^\dagger(\underline{k}-\underline{q}')U(\underline{k}-\underline{q})\right]_{ab}$$

Target averaging

- Any combination of target Wilson lines into pairs with

$$\left\langle U_{ab}(\underline{p})U_{cd}(\underline{q})\right\rangle_{\mathrm{t}} = \frac{(2\pi)^2}{N_c^2-1}\delta_{ac}\delta_{bd}\delta(\underline{p}+\underline{q})D(\underline{p})$$

- Adjoint dipole

$$D(p) = \frac{1}{N_c^2-1}\int d^2x e^{ix\cdot p}\langle \mathrm{tr}\ \left[U^\dagger(x)U(0)\right]\rangle_{\mathrm{t}}$$

- The logic behind this approximation:

– dense regime for the target
– small size color singlets in the projectile:
any non-singlet states separated by distance $> 1/Q_{s\,t}$ have zero S-matrix
– leading in $S_\perp$ of projectile:
any singlet state containing more than 2 proj. gluons is suppressed by powers of $S_\perp$

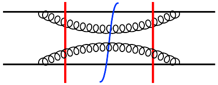
- This approximation is very restrictive and cannot be applied to many processes

- Approximating target averaging
  is not sufficient to move forward analytically

- In order to understand the structure
  of higher moments/generating function
  lets consider double inclusive production...

Double incl. production: dissecting connected terms

- Dipole contribution



$$\frac{2g^2}{(2\pi)^3}\int d^2k_1 \int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\Gamma(\underline{k}_1,\underline{q},\underline{q}') \quad \rho_p^a(-\underline{q}')\left[U^\dagger(\underline{k}_1-\underline{q}')U(\underline{k}_1-\underline{q})\right]_{ab}\rho_p^b(\underline{q})$$

$$\frac{2g^2}{(2\pi)^3}\int d^2k_2 \int \frac{d^2p}{(2\pi)^2}\frac{d^2p'}{(2\pi)^2}\Gamma(\underline{k}_2,\underline{p},\underline{p}') \quad \rho_p^c(-\underline{p}')\left[U^\dagger(\underline{k}_2-\underline{p}')U(\underline{k}_2-\underline{p})\right]_{cd}\rho_p^d(\underline{p})$$

$$\Rightarrow \frac{2g^2}{(2\pi)^3}\int d^2k_1 \int \frac{d^2q}{(2\pi)^2}\mu^2(q_1)\Gamma(\underline{k}_1,\underline{q},\underline{q}) \quad \underbrace{\mathrm{tr}\left[U^\dagger(\underline{k}_1-\underline{q})U(\underline{k}_1-\underline{q})\right]}_{\text{dipole}}$$

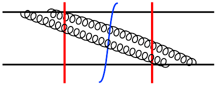
$$\frac{2g^2}{(2\pi)^3}\int d^2k_2 \int \frac{d^2p}{(2\pi)^2}\mu^2(q_2)\Gamma(\underline{k}_2,\underline{p},\underline{p}) \quad \underbrace{\mathrm{tr}\left[U^\dagger(\underline{k}_2-\underline{p})U(\underline{k}_2-\underline{p})\right]}_{\text{dipole}}$$

- For a connected contribution one will have to break both adjoint traces

$$\left\langle \mathrm{tr}\left[U^\dagger(\underline{k}_1-\underline{q})U(\underline{k}_1-\underline{q})\right]\mathrm{tr}\left[U^\dagger(\underline{k}_2-\underline{p})U(\underline{k}_2-\underline{p})\right]\right\rangle_t^{\text{conn.}}$$
$$= 2S_\perp \delta(\underline{k}_1+\underline{k}_2-\underline{q}-\underline{p})D^2(\underline{k}_1-\underline{q}) \sim \mathcal{O}(N_c^0)$$

Double incl. production: dissecting connected terms

- Quadrupole contribution



$$\frac{2g^2}{(2\pi)^3}\int d^2k_1 \int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\Gamma(\underline{k}_1,\underline{q},\underline{q}') \quad \rho_{\rm p}^a(-\underline{q}')\left[U^\dagger(\underline{k}_1-\underline{q}')U(\underline{k}_1-\underline{q})\right]_{ab}\rho_{\rm p}^b(\underline{q})$$

$$\frac{2g^2}{(2\pi)^3}\int d^2k_2 \int \frac{d^2p}{(2\pi)^2}\frac{d^2p'}{(2\pi)^2}\Gamma(\underline{k}_2,\underline{p},\underline{p}') \quad \rho_{\rm p}^c(-\underline{p}')\left[U^\dagger(\underline{k}_2-\underline{p}')U(\underline{k}_2-\underline{p})\right]_{cd}\rho_{\rm p}^d(\underline{p})$$

$$\Rightarrow \left(\frac{2g^2}{(2\pi)^3}\right)^2\int d^2k_1 d^2k_2 \int \frac{d^2q}{(2\pi)^2}\frac{d^2p}{(2\pi)^2}\mu^2(q)\mu^2(p)\Gamma(\underline{k}_1,\underline{q},\underline{p})\Gamma(\underline{k}_2,\underline{q},\underline{p})$$

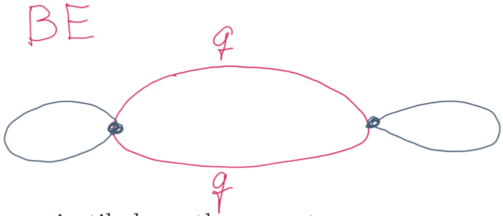
$$\times \underbrace{\mathrm{tr}[U^\dagger(\underline{k}_1-\underline{p})U(\underline{k}_1-\underline{q})U^\dagger(\underline{k}_2-\underline{q})U(\underline{k}_2-\underline{p})]}_{\text{quadrupole}}$$

- The remaining contraction leading to a quadrupole restores the infamous symmetry $\underline{k}_2 \to -\underline{k}_2$, which precluded odd azimuthal harmonics

Leading large N_c contractions in the quadrupole

- Wilson lines in the quadrupole can be contracted in multiple ways

$$\mu^2(q)\mu^2(p)\mathrm{tr}[U^\dagger(\underline{k}_1-\underline{p})U(\underline{k}_1-\underline{q})U^\dagger(\underline{k}_2-\underline{q})U(\underline{k}_2-\underline{p})]$$
$$\propto (N_c^2-1)\, S_\perp \underbrace{\mu^2(q)\mu^2(p)}_{\mu^4(q)} \delta(\underline{q}-\underline{p})D(\underline{k}_1-\underline{p})D(\underline{k}_2-\underline{p})$$



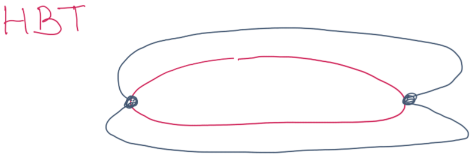
- Incoming gluons from projectile have the same transverse momentum $(=\underline{q}) \Rightarrow$ BE

A. Dumitru, F. Gelis, L. McLerran & R. Venugopalan arXiv:0804.3858
Y. Kovchegov and D. Wertepny arXiv:1212.1195
T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and M. Lublinsky, arXiv:1503.07126
Y. Kovchegov & V. S. arXiv:1802.08166

Leading large N_c contractions in the quadrupole

- Wilson lines in the quadrupole can be contracted in multiple ways

$$\mu^2(q)\mu^2(p)\ \mathrm{tr}[U^\dagger(\underline{k}_1-\underline{p})U(\underline{k}_1-\underline{q})U^\dagger(\underline{k}_2-\underline{q})U(\underline{k}_2-\underline{p})]$$
$$\propto (N_c^2-1)\ S_\perp\ \mu^2(q)\mu^2(p)\ \delta(\underline{k}_1-\underline{k}_2)\ D(\underline{k}_1-\underline{p})D(\underline{k}_2-\underline{q})$$



- Produced gluons have the same transverse momentum $(=\underline{k}_1) \Rightarrow$ HBT

- There is also an "anti"-HBT contribution with $\delta(\underline{k}_1+\underline{k}_2)$. I will refer to both $\delta(\underline{k}_1\pm\underline{k}_2)$ as to HBT contribution.

A. Dumitru, F. Gelis, L. McLerran & R. Venugopalan, arXiv:0804.3858

N_c suppressed contraction

$$\mu^2(q)\mu^2(p)\ \mathrm{tr}[U^\dagger(\underline{k}_1-\underline{p})U(\underline{k}_1-\underline{q})U^\dagger(\underline{k}_2-\underline{q})U(\underline{k}_2-\underline{p})]$$
$$\propto S_\perp\mu^2(q)\mu^2(p)\ \delta(\underline{k}_1+\underline{k}_2-\underline{p}-\underline{q})\ D(\underline{k}_1-\underline{p})D(\underline{k}_1-\underline{q})$$

- I will neglect N_c^2 suppressed contribution

Bose-Einstein contribution

- Returning to BE and collecting all together

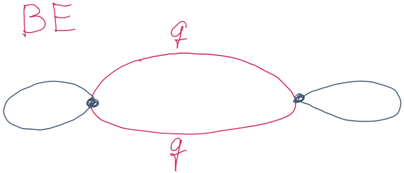
$$\left[\frac{d^2N}{dy_1dy_2}\right]_{\rm BE} = \left(\frac{2g^2}{(2\pi)^3}\right)^2 \int d^2k_1 d^2k_2 \int \frac{d^2q}{(2\pi)^2}\frac{d^2p}{(2\pi)^2}\mu^2(q)\mu^2(p)\Gamma(\underline{k}_1,\underline{q},\underline{p})\Gamma(\underline{k}_2,\underline{q},\underline{p})$$

$$\times \underbrace{{\rm tr}[U^\dagger(\underline{k}_1-\underline{p})U(\underline{k}_1-\underline{q})U^\dagger(\underline{k}_2-\underline{q})U(\underline{k}_2-\underline{p})]}_{(N_c^2-1)\;S_\perp\mu^2(q)\mu^2(p)\delta(\underline{q}-\underline{p})D(\underline{k}_1-\underline{p})D(\underline{k}_2-\underline{p})}$$

$$\left[\frac{d^2N}{dy_1dy_2}\right]_{\rm BE} = 2(N_c^2-1)S_\perp \int d^2q\;|\mu_{\rm p}^2(\underline{q})|^2\left|\frac{2g^2}{(2\pi)^3}\int_{k_{\rm min}} d^2k\,\Gamma(\underline{k},\underline{q},\underline{q})D(\underline{q}-\underline{k})\right|^2$$

- Important property of Lipatov vertex

$$\Gamma(\underline{k},\underline{q},\underline{q}) = \frac{(\underline{k}-\underline{q})^2}{q^2k^2} \to \frac{1}{q^2}$$



Bose-Einstein contribution

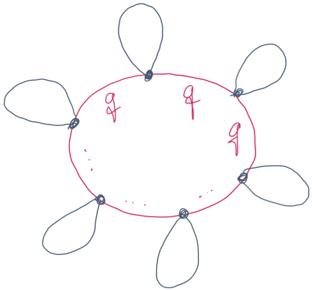
$$\left[\frac{d^2N}{dy_1 dy_2}\right]_{\mathrm{BE}} \approx 2(N_c^2-1)S_\perp \underbrace{\int d^2q \, \frac{|\mu_{\mathrm{p}}^2(\underline{q})|^2}{q^4}}_{\sim\, S_\perp \mu_{\mathrm{p}}^4} \underbrace{\left|\frac{2g^2}{(2\pi)^3}\int_{k_{\min}} d^2k D(\underline{k})\right|^2}_{\mathcal{D}^2}$$

- Effectively

$$\left[\frac{d^2N}{dy_1 dy_2}\right]_{\mathrm{BE}} \propto S_\perp^2$$

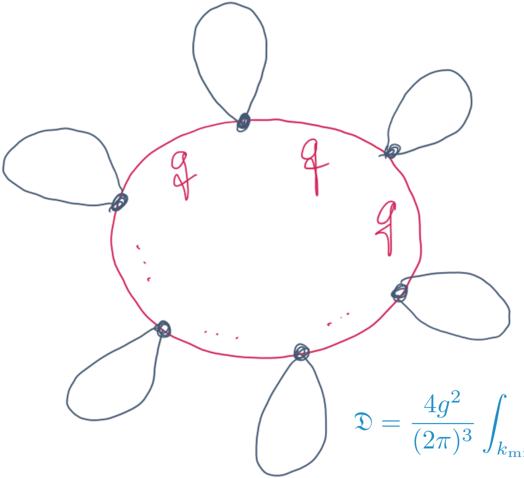
as for uncorrelated two-gluon production $(\mathrm{SIP})^2$.

- HBT contribution is suppressed by $S_\perp^{-1}$
- Lesson learned: Keep BE only!



Bose-Einstein contribution & Generating function

- Connected contribution $\ln G \equiv$ resummation of rainbow diagrams



$$\mathfrak{D} = \frac{4g^2}{(2\pi)^3} \int_{k_{\min}} d^2k \; D(\underline{k})$$

Cumulants & Phenomenological Conclusion

- Average number of gluons

$$\kappa_1 = \frac{N_c^2 - 1}{8\pi} \underbrace{S_\perp \mu_p^2} \mathfrak{D} \ln \frac{k_{\min}^2}{\Lambda^2}$$

- Higher order cumulants

$$\kappa_{n\geq 2} = \frac{\partial}{\partial t^n} \ln G_{\mathrm{LO}}(t)\Big|_{t=0} = (n-2)! \frac{(N_c^2 - 1)\ S_\perp \Lambda^2}{8\pi} \left(\frac{\mu_p^2 \mathfrak{D}}{\Lambda^2}\right)^n$$

- Properties ($\Lambda^2 \approx S_\perp^{\text{proton}}$)

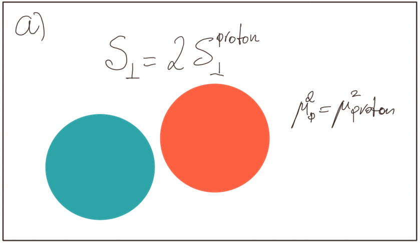
  - κ_1 is a function of $\underbrace{S_\perp \; \mu_{\text{p}}^2}$
  - Consider configurations a) and b):

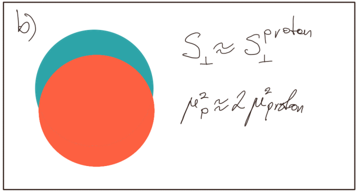
    $\kappa_1[a)] = \kappa_1[b)]$

    $\kappa_n[b)] \propto \mathbf{2} \; S_\perp^{\text{proton}} \left(\mu_p^{\text{proton}}\right)^{2n}$

    $\kappa_n[b)] \propto \mathbf{2^n} \; S_\perp^{\text{proton}} \left(\mu_p^{\text{proton}}\right)^{2n}$

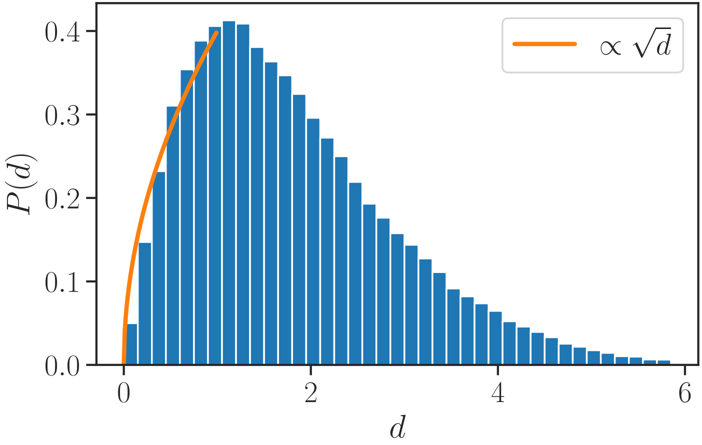
High multiplicity tail ≡
≡ configurations with
overlapping nucleons



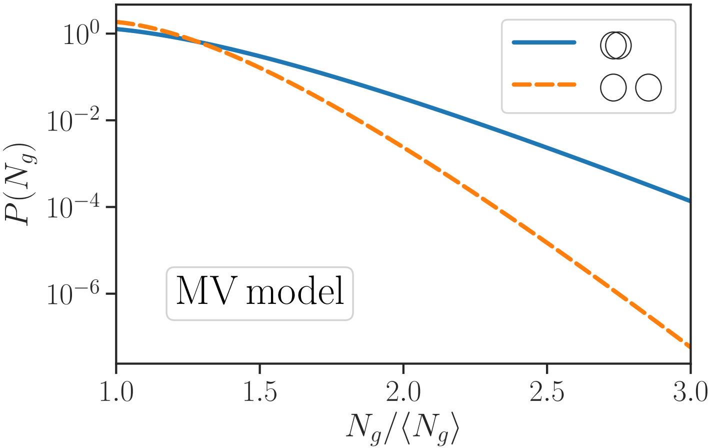


But...

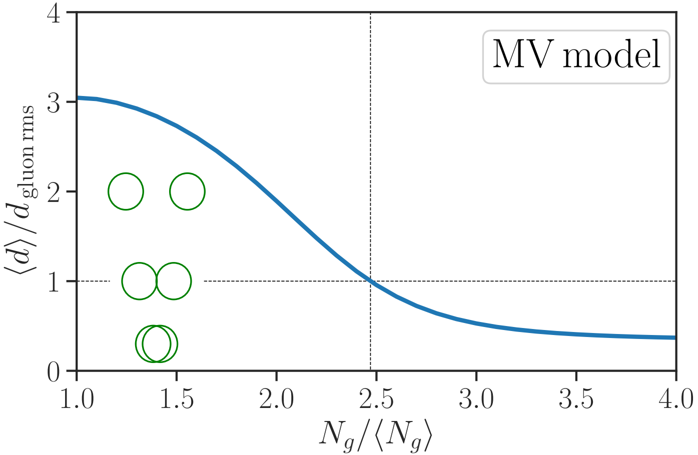
probability to have overlapping nucleus in an actual collision:



MV model numerical calculations

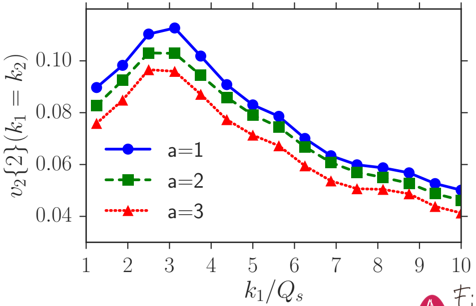


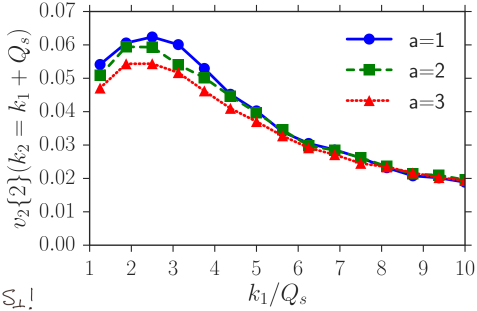
MV model plus “glauber”

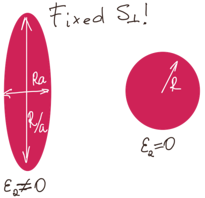


- High multiplicity bias $\rightsquigarrow$ overlapping configurations of deuteron

CGC response to projectile geometry







- $v_2\{2\}$ anticorrelates with ϵ_2

A. Kovner & V.S., 2018

Combining phenomenological conclusions...

- High multipl. $\equiv$ overlapping nucleons in projectile (d-A)
- Overlapping nucleons in projectile $\rightsquigarrow$ smaller ϵ_2
- Smaller $\epsilon_2 \rightsquigarrow$ larger $v_2\{2\}$

The latter effect is rather mild, but it is manifestly present!


BACKUP

More functionals

- BE is property of projectile
- Need for effective theory of gluons in projectile
- Constraint effective action for projectile gluon distribution

$$e^{-V_{\text{eff}}[\eta(\underline{q})]} = \frac{1}{Z_{\text{p}}} \int \mathcal{D}\rho_{\text{p}} \underbrace{W(\rho_{\text{p}})}_{\text{all possible fluct.}} \underbrace{\delta\left(\eta(\underline{q}) - \frac{g^2 \text{tr}|A^+(\underline{q})|^2}{\langle g^2 \text{tr}|A^+(\underline{q})|^2\rangle}\right)}_{\text{keeping only interesting stuff}}$$

$$A^+(\underline{q}) = g/q^2 \rho_{\text{p}}(\underline{q}), \qquad \langle g^2 \text{tr}|A^+(\underline{q})|^2\rangle = \tfrac{1}{2}(N_c^2 - 1) S_\perp \frac{g^4 \mu_{\text{p}}^2}{q^4}$$

- Exact expression for effective potential (modulo $S_\perp^{-1}$ corrections)

$$V_{\text{eff}}[\eta(\underline{q})] = \frac{1}{2}(N_c^2 - 1) S_\perp \int \frac{d^2 q}{(2\pi)^2} \left\{\eta(\underline{q}) - 1 - \ln \eta(\underline{q})\right\} \approx \frac{1}{2}(N_c^2 - 1) S_\perp \int \frac{d^2 q}{(2\pi)^2} \frac{1}{2} \ln^2 \eta(\underline{q})$$

A. Dumitru & V. S., arXiv:1704.05917

Liouville potential & high multiplicity tail

- Back to generating function

$$G_{\mathrm{LO}}(t) = \left\langle \exp\left[t \int_{\Lambda}^{k_{\min}} \frac{d^2q}{(2\pi)^2} \rho^a(-\underline{q}) \frac{\mathfrak{D}}{2q^2} \rho^a(\underline{q}) \right] \right\rangle_{\mathrm{p}}$$

- In terms of effective potential

$$G_{\mathrm{LO}}(t) = \int \mathcal{D}\eta \exp\left(-V_{\mathrm{eff}}[\eta(q)] + \underbrace{\frac{1}{2}(N_c^2 - 1) S_\perp \int_{\Lambda}^{k_{\min}} \frac{d^2q}{(2\pi)^2}\, t\, \frac{\mu_{\mathrm{p}}^2 \mathfrak{D}}{q^2}\, \eta(\underline{q})}_{\text{reweighting! derivatives in } t \text{ probe Liouville potential}} \right)$$

Liouville potential & high multiplicity tail

- For large $S_\perp$: saddle point approximation

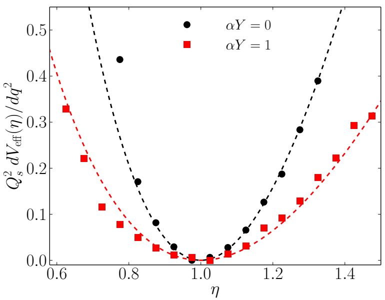
$$\eta_s(\underline{q}) = \begin{cases} \left(1 - t\frac{\mu_{\mathrm{p}}^2 \mathfrak{D}}{q^2}\right)^{-1}, \text{ if } \Lambda \leq q \leq k_{\min} \\ 1, \text{ otherwise} \end{cases}$$

to yield

$$\ln G_{\mathrm{LO}}(t) = \frac{1}{2}(N_c^2 - 1)S_\perp \int \frac{d^2q}{(2\pi)^2} \ln \eta_s(\underline{q}) = -\frac{1}{2}(N_c^2 - 1)S_\perp \int_\Lambda^{k_{\min}} \frac{d^2q}{(2\pi)^2} \ln \left(1 - t\frac{\mu_{\mathrm{p}}^2 \mathfrak{D}}{q^2}\right)$$

- We recovered previously derived result. Origin of ln $\equiv$ Liouville's ln!

Liouville potential and small x evolution



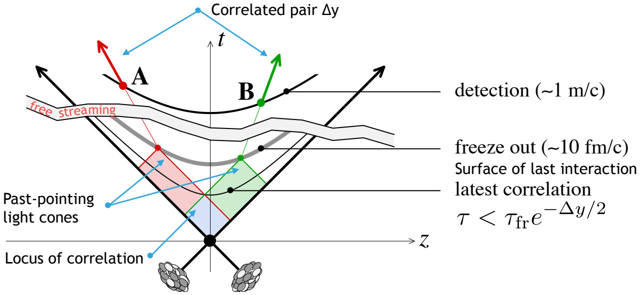
A. Dumitru & V. S., arXiv:1704.05917

- Form does not change $V_{\mathrm{eff}}[\eta(\underline{q})] \approx \frac{1}{2}(N_c^2 - 1)S_\perp \int \frac{d^2q}{(2\pi)^2} \frac{1}{2} \ln^2 \eta(\underline{q})$
- $S_\perp \to S_\perp^{\mathrm{eff}} \equiv \frac{S_\perp}{\sigma^2}$:

  partially responsible for phenomenological parameter σ
- C.f. $P[\rho] \propto \exp\left[-\frac{\rho^2}{2\sigma^2}\right]$ with $\rho \equiv \ln Q_s^2/\bar{Q}_s^2$

L. McLerran & P. Tribedy, arXiv:1508.03292

Long-range correlations



Suppress final state effects in order to probe initial state physics

- Regardless of nature of the ridge
  - long-range rapidity correlations either pre-exist in initial wave function or develop very early after collision
  - understanding initial/early stage is of paramount importance for phenomenology of p-A and p-p.

figure adapted from A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858