

Unpolarized TMDs: extractions and predictive power

Andrea Signori

INT 18-3

**Probing nucleons and nuclei
in high-energy collisions**

Transverse spin and TMDs

Oct. 8th, 2018

Outline of the talk

- 1) TMDs and their evolution
- 2) first attempt to a global fit
- 3) predictive power vs
relevance of nonperturbative corrections
- 4) outlook



TMDs & their evolution

References (intro and reviews) :

- “The 3D structure of the nucleon” **EPJ A (2016) 52**
- J.C. Collins “**Foundations of perturbative QCD**”
- material from the TMD collaboration **summer school**
- ...



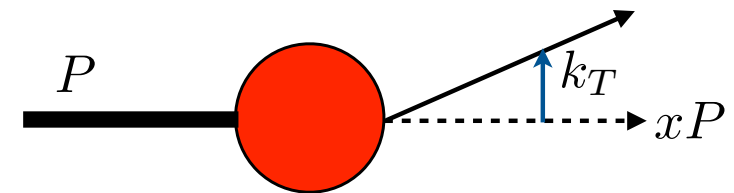
Quark TMD PDFs

$$\Phi_{ij}(k, P; S, T) \sim \text{F.T.} \langle PS | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PS \rangle_{LF}$$

	U	L	T
Quarks	γ^+	$\gamma^+ \gamma^5$	$i\sigma^{i+} \gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Sivers TMD PDF

unpolarized TMD PDF



extraction of a **quark**
not collinear with the proton

encode all the possible
spin-spin and **spin-momentum**
correlations
between the proton
and its constituents

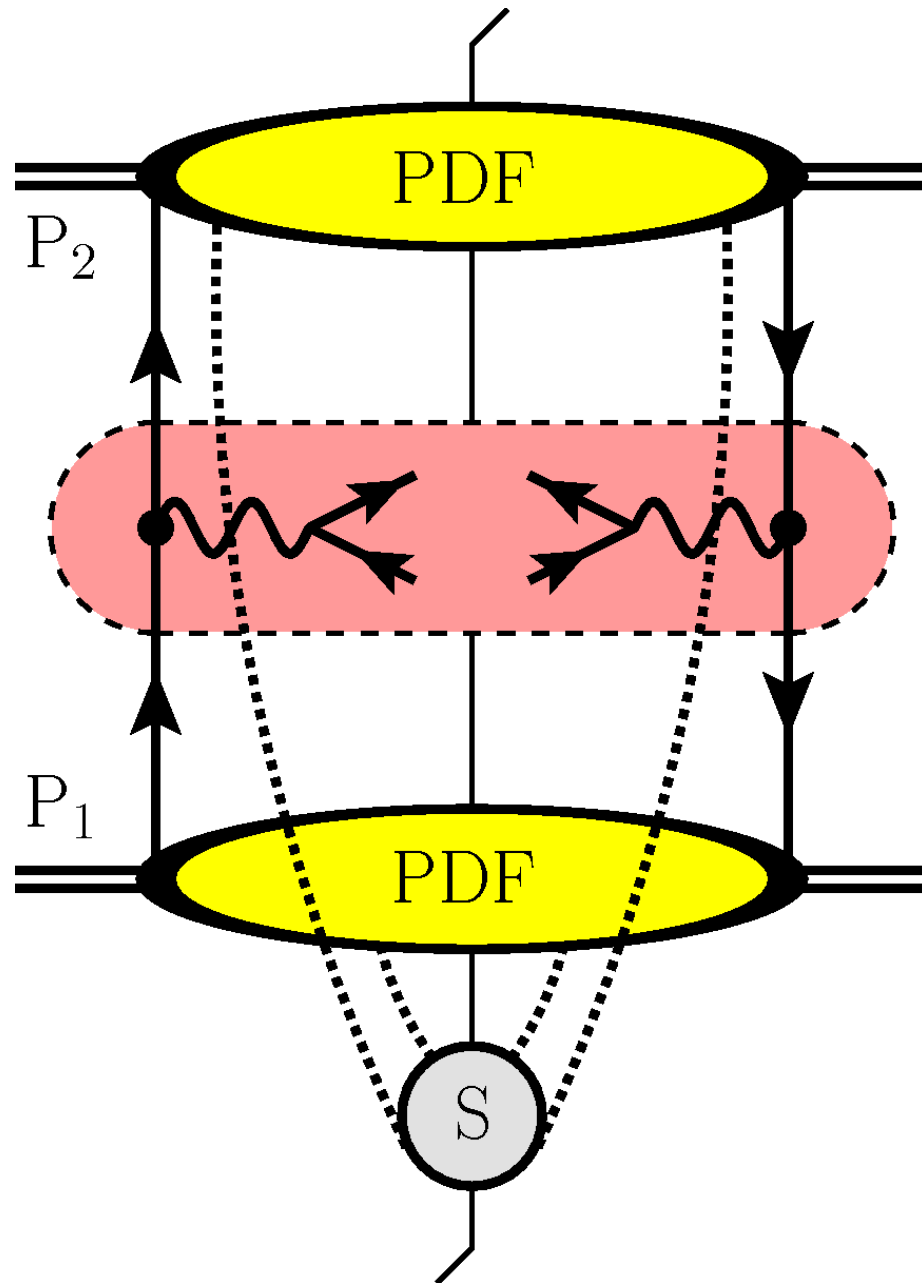
similar table for **gluons** and for **fragmentation**

bold : also collinear

red : time-reversal odd (universality properties)

Factorization and evolution

$$p p \rightarrow \ell \bar{\ell} X$$



In certain processes
the cross section can be **factorized**
in contributions characterized by a specific
scaling of the momenta

$$d\sigma \sim \mathcal{H} f_1^{bare} f_1^{bare} S$$

$$\sim \mathcal{H} f_1 f_1$$

renormalized TMD PDF :

IR div. : long-distance physics
UV div. and **rapidity div.** cancelled
by UV-renormalization and soft factor S

$$f_1(x, k_T^2; \mu, \zeta)$$

Evolution with respect to two scales

Evolution of TMDs

$$f_1^a(x, b_T^2, \mu_f, \zeta_f) = f_1^a(x, b_T^2, \mu_i, \zeta_i)$$

b_T , Fourier conjugate of k_T

two "evolution scales"

$$\times \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\}$$

evolution in mu
 $\mu_i \rightarrow \mu_f$

$$\times \left(\frac{\zeta_f}{\zeta_i} \right)^{-K(b_T, \mu_i)}$$

evolution in zeta
 $\zeta_i \rightarrow \zeta_f$

Input TMD distribution can be **expanded at low b_T** onto a basis of collinear distributions

$$f_1^a(x, b_T^2, \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i)$$

A sensible choice is to set the initial and final scale as:

$$\zeta_i = \mu_i^2 = 4e^{-2\gamma_E} / b_T^2 \equiv \mu_b^2$$

$$\zeta_f = \mu_f^2 = Q^2$$



Evolution of TMDs

$$f_1^a(x, b_T^2, \mu_f, \zeta_f) = f_1^a(x, b_T^2, \mu_i, \zeta_i)$$

b_T , Fourier conjugate of k_T

two "evolution scales"

$$\times \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\}$$

evolution in mu
 $\mu_i \rightarrow \mu_f$

$$\times \left(\frac{\zeta_f}{\zeta_i} \right)^{-K(b_T, \mu_i) - g_K(b_T, \{\lambda\})}$$

evolution in zeta
 $\zeta_i \rightarrow \zeta_f$

need corrections
at large b_T

Input TMD distribution can be **expanded at low b_T** onto a basis of collinear distributions

$$f_1^a(x, b_T^2, \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) F_{NP}^a(x, b_T; \{\lambda\})$$

A sensible choice is to set the
initial and final scale as:

$$\zeta_i = \mu_i^2 = 4e^{-2\gamma_E} / b_T^2 \equiv \mu_b^2$$

$$\zeta_f = \mu_f^2 = Q^2$$



"Global" analysis

References :

- Bacchetta, Delcarro, Pisano, Radici, **AS**: [JHEP 1706 \(2017\) 081](#)



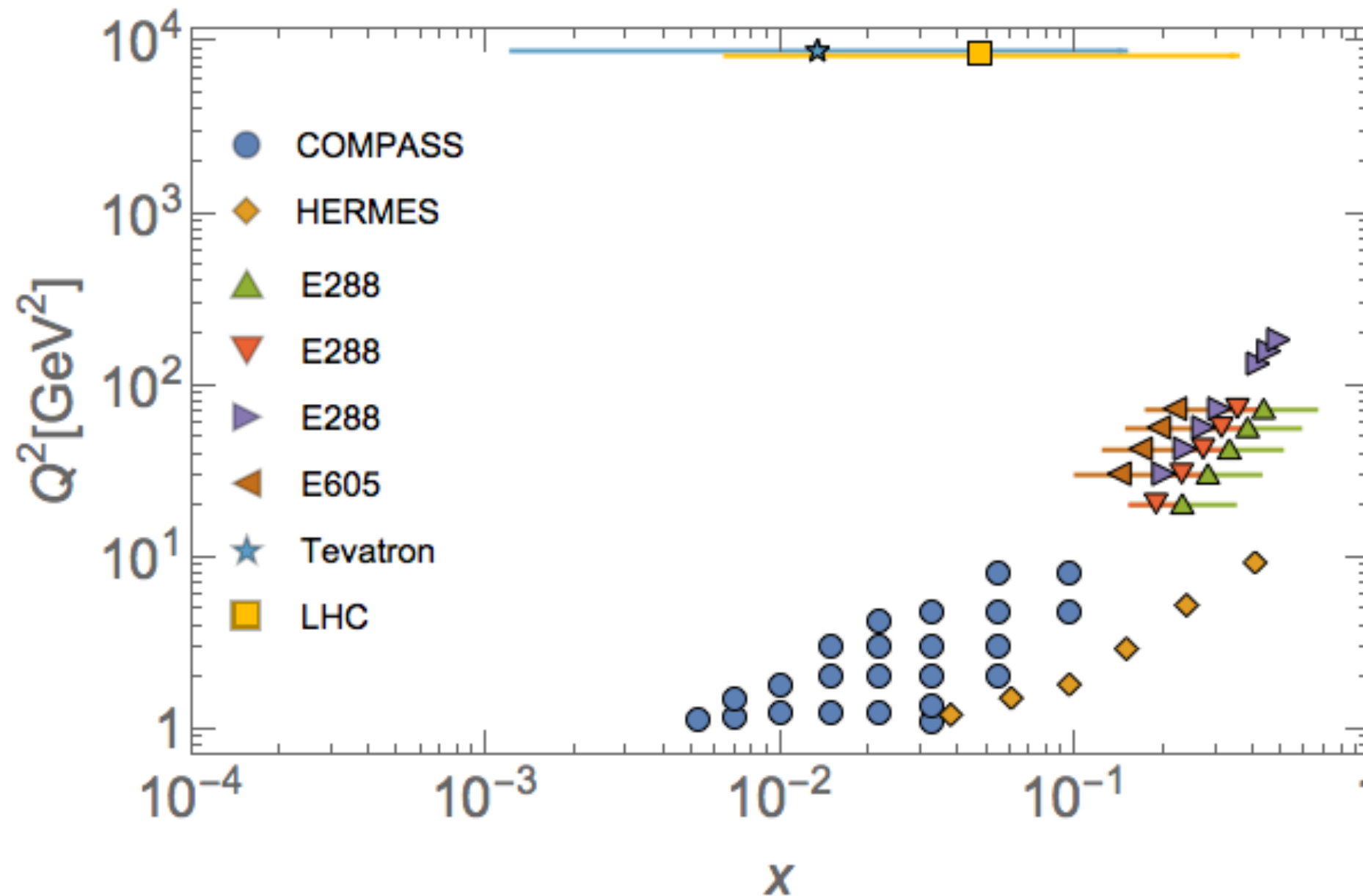
What do we know ?

(only a selection of results!)

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✗	✗	✓	✓	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✓	✗	✗	✗	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q ²) bin	1 (x,Q ²) bin	✓	✓	500 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	✗	✗	✓	✓	309

[courtesy A. Bacchetta]

Data sets and kinematic coverage



Electron-positron annihilation data are still missing
(only some azimuthal asymmetries are available)

crucial for analyses
of TMD FFs



Features

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 arXiv:1703.10157	LO-NLL	✓	✓	✓	✓	8059

PROs

almost a **global fit** of quark unpolarized TMDs

includes **TMD evolution**

replica (bootstrap) fitting methodology

kinematic dependence in intrinsic part of TMDs

intrinsic momentum: **beyond the Gaussian** assumption

CONs

no “pure” info on TMD FFs

accuracy of TMD evolution : not the state of the art

only “low” transverse momentum (no fixed order and Y-term)

flavor separation in the transverse plane : problematic



Intrinsic transverse momentum

$$f_{1NP}^a(x, k_{\perp}^2) = \frac{1}{\pi} \frac{(1 + \lambda k_{\perp}^2)}{g_{1a} + \lambda g_{1a}^2} e^{-\frac{k_{\perp}^2}{g_{1a}}}$$

$$\hat{x} = 0.1$$

$$g_1(x) = N_1 \frac{(1-x)^{\alpha} x^{\sigma}}{(1-\hat{x})^{\alpha} \hat{x}^{\sigma}}$$

weighted sum of two Gaussian distributions:
 same widths for TMD PDFs
 different widths for TMD FFs

There are **11 free parameters**
 in a flavor independent scenario
 (one for evolution)

$$D_{1NP}^{a \rightarrow h}(z, P_{\perp}^2) = \frac{1}{\pi} \frac{1}{g_{3a \rightarrow h} + (\lambda_F / z^2) g_{4a \rightarrow h}^2} \left(e^{-\frac{P_{\perp}^2}{g_{3a \rightarrow h}}} + \lambda_F \frac{P_{\perp}^2}{z^2} e^{-\frac{P_{\perp}^2}{g_{4a \rightarrow h}}} \right)$$

$$\hat{z} = 0.5$$

$$g_{3,4}(z) = N_{3,4} \frac{(z^{\beta} + \delta)(1-z)^{\gamma}}{(\hat{z}^{\beta} + \delta)(1-\hat{z})^{\gamma}}$$

Inspired by **model calculations**:

Matevosyan et al.

Phys. Rev. D85, 014021 (2012), 1111.1740

Bacchetta et al.

Phys. Lett. B659, 234 (2008), 0707.3372

Bacchetta et al.

Phys. Rev. D65, 094021 (2002),

hep-ph/0201091



Models - evolution and b_T regions

$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

Large b_T correction to evolution
[other functional forms to be explored]

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)$$

$\nearrow b_{\max}, \quad b_T \rightarrow +\infty$

avoid Landau pole

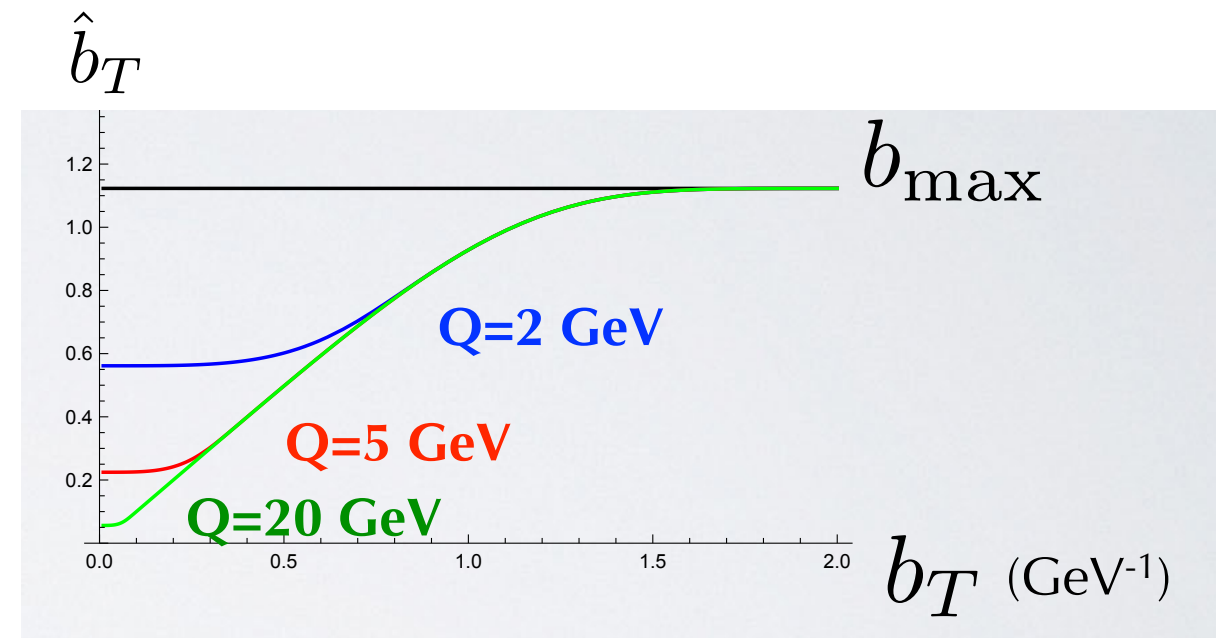
$\searrow b_{\min}, \quad b_T \rightarrow 0$

recover collinear fact.

$$b_{\min} \sim 1/Q, \quad \mu_{\hat{b}} < Q$$

Regularization needed to **recover the cross section integrated over q_T** in collinear factorization.

Crucial from the **theory** point of view and for the **phenomenology of SIDIS (low Q)**



Data sets and selections – SIDIS

	HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Reference	[61]			
Cuts	$Q^2 > 1.4 \text{ GeV}^2$ $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$			
Points	190	190	189	187
Max. Q^2	9.2 GeV ²			
x range	$0.06 < x < 0.4$			

TMD factorization
($P_{hT}^2/z^2 \ll Q^2$)

avoid target fragmentation [?]
[low z]
and exclusive contributions [?]
[high z]

Problem with normalization
in the previous release

	HERMES $D \rightarrow \pi^+$	HERMES $D \rightarrow \pi^-$	HERMES $D \rightarrow K^+$	HERMES $D \rightarrow K^-$	COMPASS $D \rightarrow h^+$	COMPASS $D \rightarrow h^-$
Reference	[74]				[75]	
Cuts	$Q^2 > 1.4 \text{ GeV}^2$ $0.20 < z < 0.74$ $P_{hT} < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$					
Points	190	190	189	189	3125	3127
Max. Q^2	9.2 GeV ²				10 GeV ²	
x range	$0.04 < x < 0.4$				$0.005 < x < 0.12$	
Notes					Observable: $m_{\text{norm}}(x, z, P_{hT}^2, Q^2)$, eq. (3.1)	



Agreement data-theory

Flavor independent scenario

Flavor independent configuration | 11 parameters

Points	Parameters	χ^2	$\chi^2/\text{d.o.f.}$
8059	11	12629 ± 363	1.55 ± 0.05

	HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Points	190	190	189	187
χ^2/points	4.83	2.47	0.91	0.82

Hermes P/D into π^+ :
problems at low z

	HERMES $D \rightarrow \pi^+$	HERMES $D \rightarrow \pi^-$	HERMES $D \rightarrow K^+$	HERMES $D \rightarrow K^-$	COMPASS $D \rightarrow h^+$	COMPASS $D \rightarrow h^-$
Points	190	190	189	189	3125	3127
χ^2/points	3.46	2.00	1.31	2.54	1.11	1.61

	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
χ^2/points	0.99	0.84	0.32	1.12

Hermes kaons better than pions:
larger uncertainties from FFs

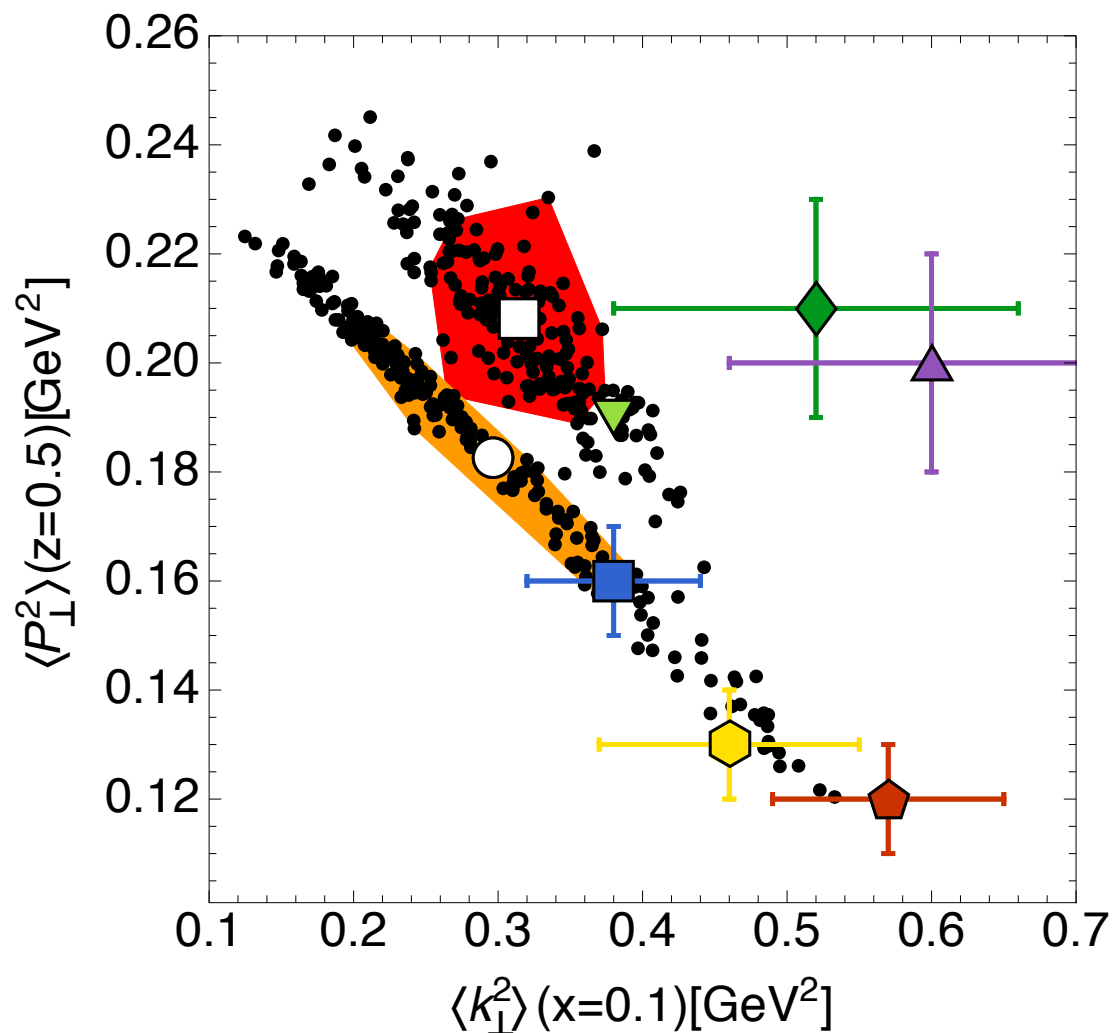
Compass : better agreement due to
#points and normalization

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36	1.11	2.00	1.73

Let's see what
happens with the new data

Average transverse momenta

Flavor ind. scenario



- 1 Bacchetta, Delcarro, Pisano, Radici, Signori (JHEP 2017)
- Signori, Bacchetta, Radici, Schnell arXiv:1309.3507
- Schweitzer, Teckentrup, Metz, arXiv:1003.2190
- ⬡ Anselmino et al. arXiv:1312.6261 [HERMES]
- ⬠ Anselmino et al. arXiv:1312.6261 [HERMES, high z]
- ◆ Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]
- ▲ Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]
- ▼ Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 ($Q = 1.5 \text{ GeV}$)

Red/orange regions : **68% CL** from replica method

Inclusion of **Compass** increases the $\langle P_{\perp}^2 \rangle$
and reduces its spread

Inclusion of **DY/Z** diminishes the **correlation**

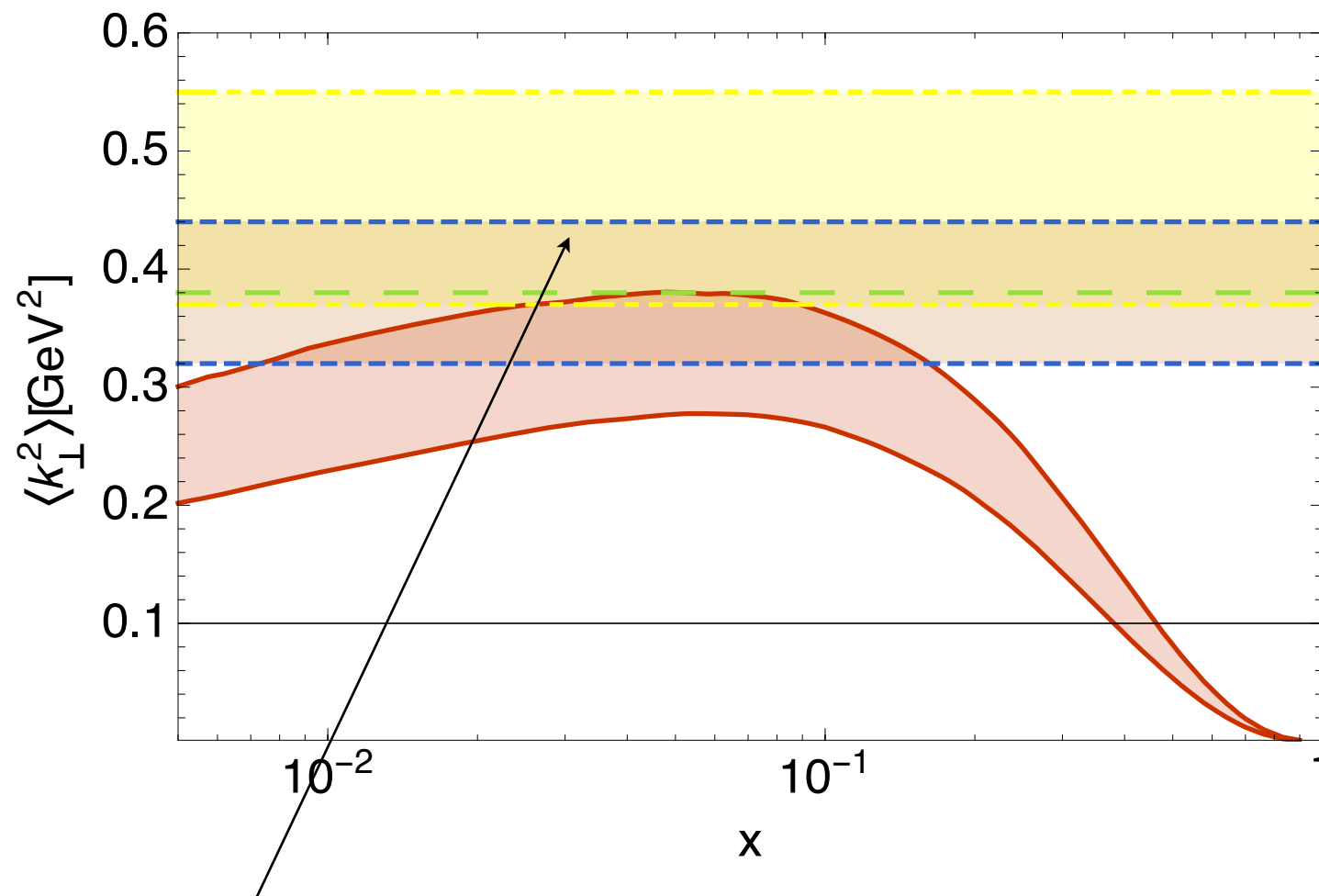
e+e- data would further reduce the correlation



Kinematic dependence

$$\langle k_{\perp}^2 \rangle(x) = \frac{\int d^2 k_{\perp} k_{\perp}^2 f_1^a(x, k_{\perp}^2, Q = 1 \text{ GeV})}{\int d^2 k_{\perp} f_1^a(x, k_{\perp}^2, Q = 1 \text{ GeV})}$$

Average square
transverse momentum
in TMD PDF



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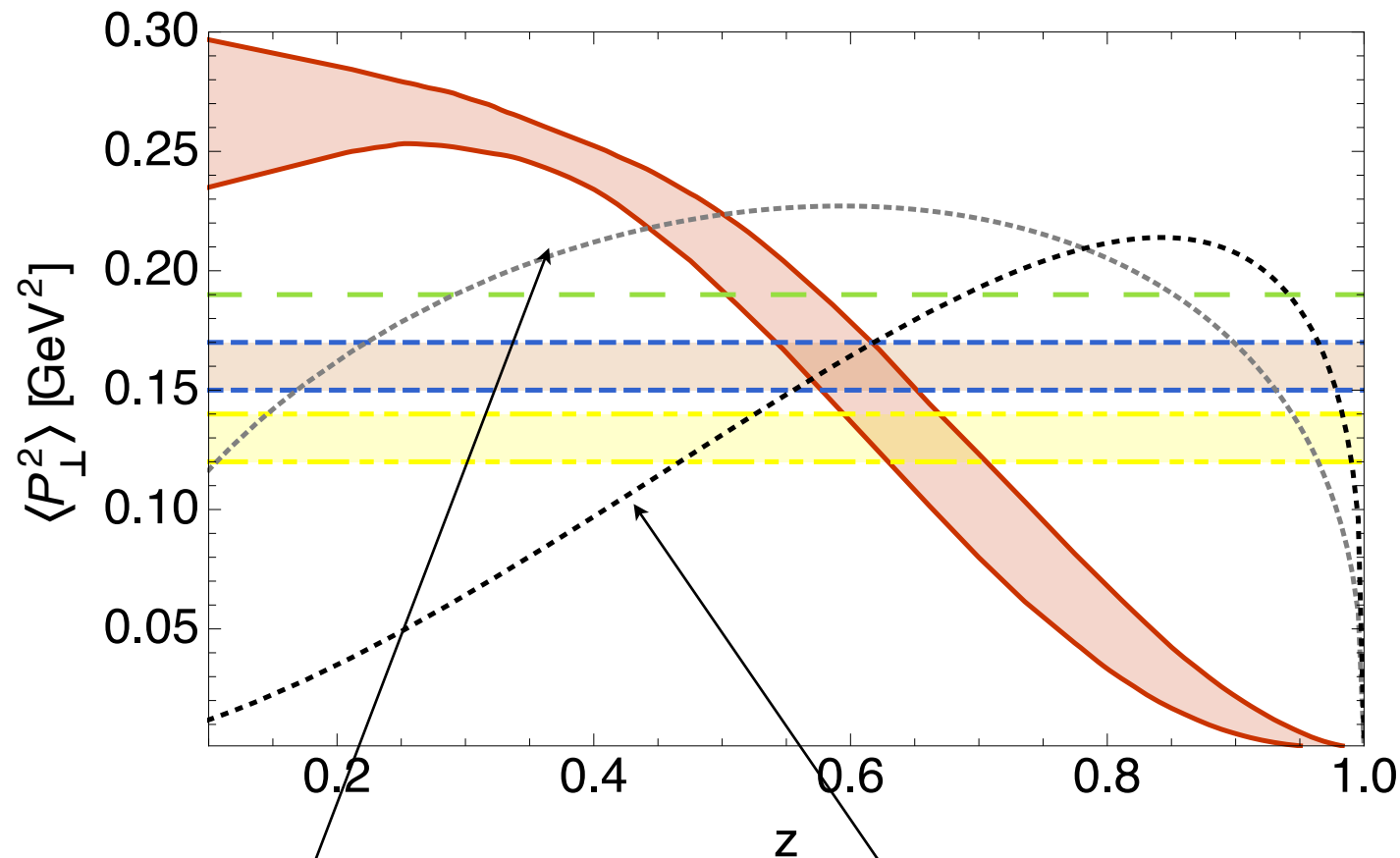
Flavor-independent scenario:
no differences in quark/hadron
flavor

x-independent extractions

Kinematic dependence

$$\langle P_{\perp}^2 \rangle(z) = \frac{\int d^2 P_{\perp} P_{\perp}^2 D_1^{a \rightarrow h}(z, P_{\perp}^2, Q = 1 \text{ GeV})}{\int d^2 P_{\perp} D_1^{a \rightarrow h}(z, P_{\perp}^2, Q = 1 \text{ GeV})}$$

Average square
transverse momentum in TMD FF



Color code : same as previous slide

Flavor-independent scenario:
no differences in quark/hadron
flavor

z-dependence :
important to fit the data

GMC trans

Anselmino et al.
hep-ph/9901442

What's next

Urgent things we need:

- **SIDIS**: distinction of different fragmentation mechanism
- **SIDIS**: NLO description of low q_T and high q_T data (separately)
- all processes: **matching** low and high q_T
(actually we could also start from high q_T to describe data)
- independent extractions of **TMD FFs**: formalism but no data
- **gluon TMDs**: we have the (effective) formalism and the data
- a faithful **Monte Carlo** implementation of TMD sensitive processes



What about TMD FFs ?

Forthcoming **unpolarized data** with transverse momentum dependence:

- 1) Belle-2 : e^+e^- to $h X$ (TM dependence with respect to thrust axis)
collinear factorization ? TMDs ?
- 2) Belle-2 : e^+e^- to $h_1 h_2 X$ - Definitely TMD factorization
- 3) BES-3 ... ?



From the **theory viewpoint** we are working on the formalism, to prepare the ground for forthcoming extractions:

- **high- q_T** limit in **collinear** factorization
- **low- q_T** limit in **TMD** factorization

E. Moffat, T. Rogers, AS

Comparison with Pythia pseudo data for the moment,



Gluon TMDs

gluon-gluon correlator

$$\Gamma^{\alpha\beta}(x, \mathbf{k}_T) \sim \text{F.T.} \langle P | F^{+\alpha}(0) U_{[0,\xi]} F^{+\beta}(\xi) U'_{[\xi,0]} | P \rangle_{|\xi^+=0}$$

Wilson loop correlator

$$\Gamma_0(x, \mathbf{k}_T) \sim \delta(x) \text{F.T.} \langle P | U_{[0,\xi]}^{[+]} U_{[\xi,0]}^{[-]} | P \rangle_{|\xi^+=0}$$

	Ref. [30]	Ref. [7]	Rank	T	C	Limit $x \rightarrow 0$
xf_1	xf_1	xG	0	even	even	$e^{(1)}$
xh_1^\perp	xh_1^\perp	xH^\perp	2	even	even	e
xg_1	xg_{1L}	$-x\Delta G_L$	0	even	odd	0
xh_{1L}^\perp	xh_{1L}^\perp	$-x\Delta H_L^\perp$	2	odd	even	0
xf_{1T}^\perp	xf_{1T}^\perp	$-xG_T$	1	odd	odd	$e_T^{(1)}$
xg_{1T}	xg_{1T}	$-x\Delta G_T$	1	even	even	0
xh_1	$xh_{1T} + xh_{1T}^{\perp(1)}$	$-x\Delta H_T$	1	odd	odd	$e_T^{(1)}$
xh_{1T}^\perp	xh_{1T}^\perp	$-x\Delta H_T^\perp$	3	odd	odd	$-e_T$
xf_{1LL}			0	even	even	$e_{LL}^{(1)}$
xh_{1LL}^\perp			2	even	even	e_{LL}
xf_{1LT}			1	even	odd	$e_{LT}^{(1)}/2$
xg_{1LT}			1	odd	even	0
xh_{1LT}			1	even	odd	$e_{LT}^{(1)}/2$
xh_{1LT}^\perp			3	even	odd	$-e_{LT}$
xf_{1TT}			2	even	even	$e_{TT}^{(1)}/3$
xg_{1TT}			2	odd	odd	0
xh_{1TT}			0	even	even	$e_{TT}^{(2)}$
xh_{1TT}^\perp			2	even	even	$-2e_{TT}^{(1)}/3$
$xh_{1TT}^{\perp\perp}$			4	even	even	e_{TT}

Boer, Cotogno, van Daal, Mulders, AS, Zhou
JHEP 1610 (2016) 013

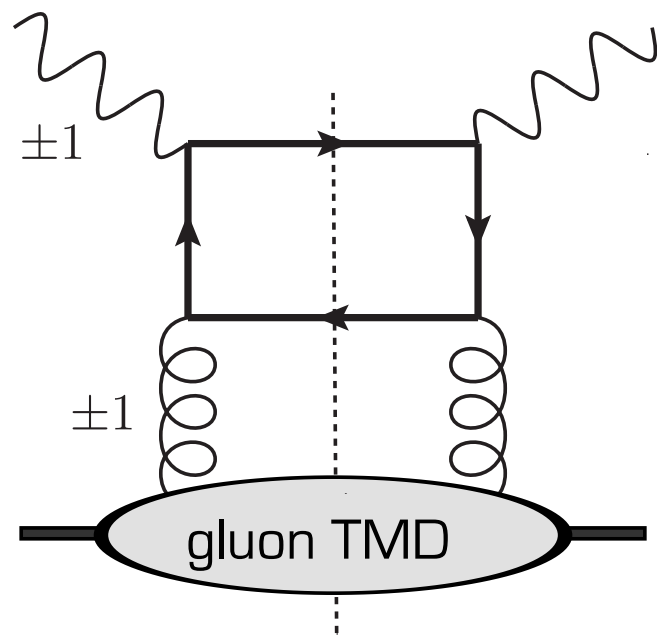
Table 1. An overview of the leading twist gluon TMDs for unpolarized, vector polarized, and tensor polarized hadrons. In the second and third column, the names of the functions in this paper are compared to the ones in refs. [7, 30]. In the fourth column we list the rank of the function. Furthermore, we list the properties (even/odd) under time reversal (T) and charge conjugation (C), see appendix A. In the last column it is indicated to which e -type function the TMD reduces in the limit $x \rightarrow 0$. As a shorthand, we use the moment notation $f_{\dots}^{(n)}(x, \mathbf{k}_T^2) \equiv [\mathbf{k}_T^2/(2M^2)]^n f_{\dots}(x, \mathbf{k}_T^2)$.



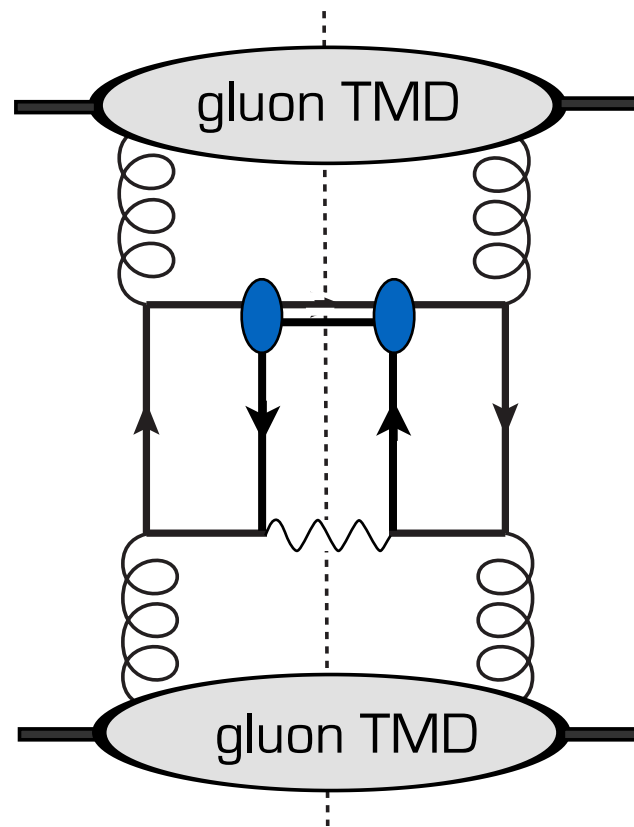
Gluon TMDs

$$e p \rightarrow e \text{ jet jet } X$$

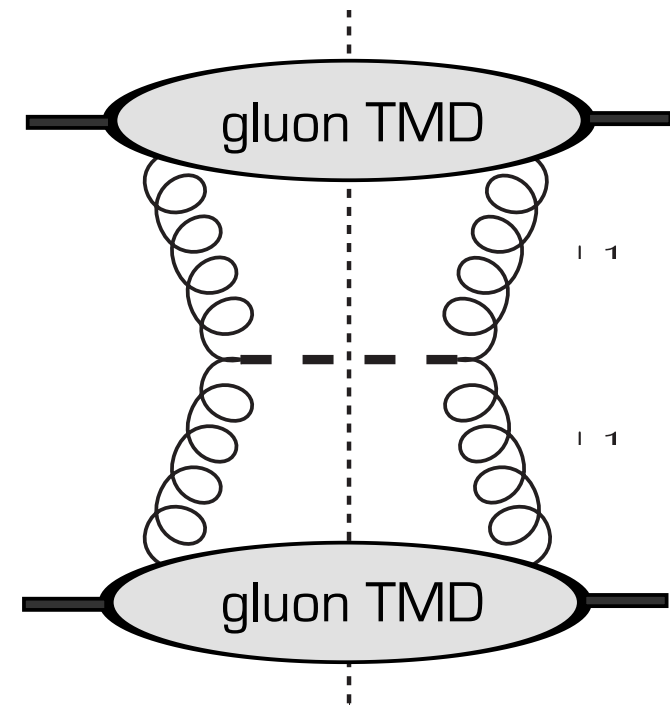
EIC !



$$p p \rightarrow J/\psi \gamma X$$



$$p p \rightarrow \eta_c X$$



- factorization properties in effective theories
- no extractions beyond parton model yet

See dedicated talks during the workshop

Predictive power

References :

- Parisi, Petronzio: Nucl. Phys. B154, 427 (1979)
- Collins, Soper, Sterman: Nucl. Phys. B250, 199 (1985)
- Qiu, Berger: Phys. Rev. Lett. 91, 222003 (2003)
- Grewal, Kang, Qiu, **AS**: in preparation

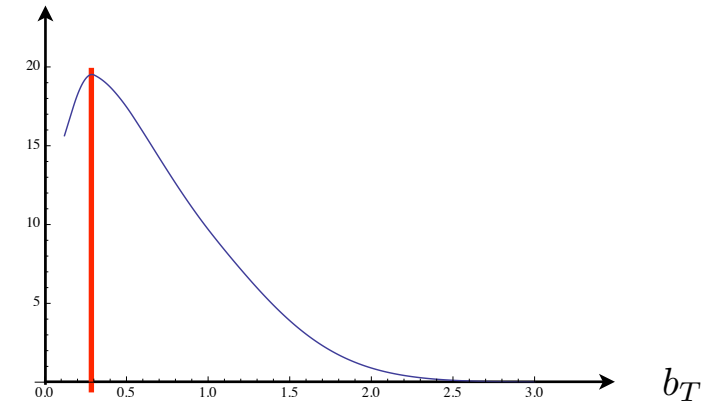


Saddle point approximation

Given a generic function $f \in C^2(a, b)$ and a positive constant A

Given x_0 , maximum in $[a, b]$ for f :

$$I(x_0, A) = \int_a^b dx e^{Af(x)} = e^{Af(x_0)} \sqrt{\frac{2\pi}{A(-f''(x_0))}} \left(1 + \mathcal{O}\left(\frac{1}{A}\right)\right)$$

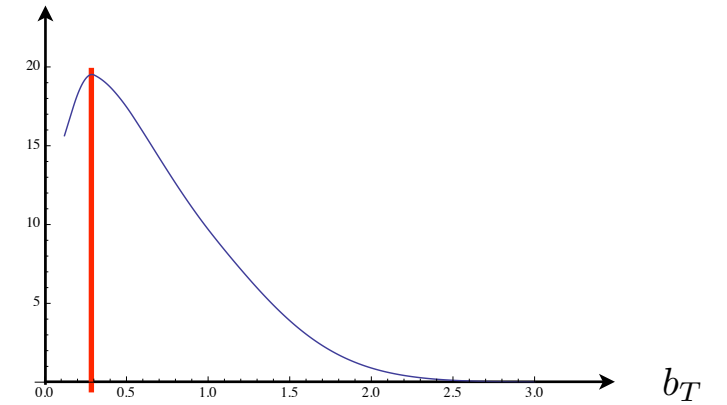


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Let's apply this to a TMD PDF evaluated at $k_T = 0$:

$$f_1^a(x, k_T; \mu_f, \zeta_f) = \text{F.T.} [f_1^a(x, b_T; \mu_f, \zeta_f)]$$

$$f_1^a(x, k_T = 0; \mu_f, \zeta_f) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} d(\ln b_T^2) \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[\sum_b C_{a/b} \otimes f_b \right] \right\}$$

Saddle point of the TMD PDF :
stationary point of the exponent

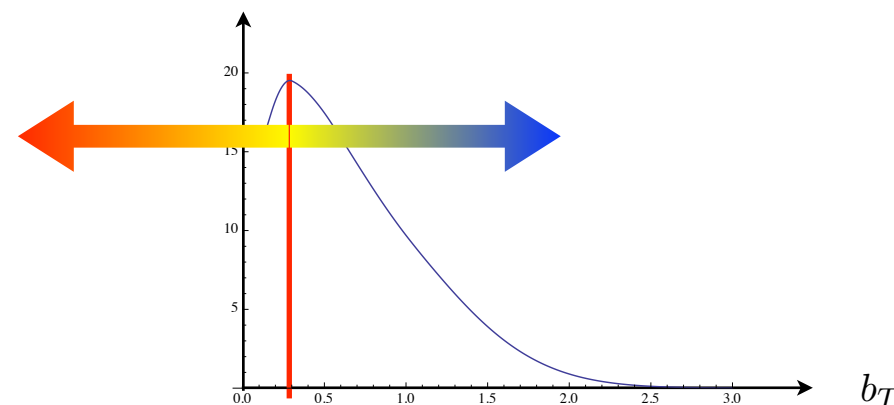


Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[\sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T = b_T^{sp}} = 0$$

← Generate the **scale-dependence** of the saddle point
 ← Generates the **x-dependence** of the saddle point

The **lower** the **saddle point**, the more the **TMD PDF is perturbatively dominated** : strong predictive power



The **higher** the **saddle point** [$b_T > b_{max}$], the more the **nonperturbative corrections are important**

Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[\sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T = b_T^{sp}} = 0$$

← Generate the **scale-dependence** of the saddle point

Parisi and Petronzio (1979) and Collins, Soper, Sterman (1982) :
 the same analysis at the level of the cross section, **neglecting the x-dependent part:**

Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[\sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

Generate the **scale-dependence** of the saddle point

Parisi and Petronzio (1979) and Collins, Soper, Sterman (1982) :
the same analysis at the level of the cross section, **neglecting the x-dependent part**:

Conclusion : the large b_T corrections are more relevant at low Q

Working at LL the solution is :

$$\zeta = \mu^2 = Q^2$$

$$b_T^{sp} \approx \frac{c}{\Lambda} \left(\frac{Q}{\Lambda} \right)^{-\Gamma_1^{cusp} / (\Gamma_1^{cusp} + 8\pi b_0)}$$



Determination of the saddle point

$$\begin{aligned}
 & \frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right. \\
 & \quad - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} \\
 & \quad + \ln b_T^2 \\
 & \quad \left. + \ln \left[\sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0
 \end{aligned}$$

← Generate the **scale-dependence** of the saddle point
 ← Generates the **x-dependence** of the saddle point

Qiu, Zhang (2001) introduced the x-dependent term in the analysis at the level of the cross section.

We repeat the same at the level of the TMD PDF, using the language of TMD evolution



Determination of the saddle point

$$\frac{d}{db_T} \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] - K(b_T, \mu_i) \ln \frac{\zeta_f}{\zeta_i} + \ln b_T^2 + \ln \left[\sum_b C_{a/b}(x, b_T^2, \mu_i, \zeta_i) \otimes f_b(x, \mu_i) \right] \right\}_{b_T=b_T^{sp}} = 0$$

Generate the **scale-dependence** of the saddle point

Generates the **x-dependence** of the saddle point

Working at LL the solution is :

$$b_T^{sp} = \frac{c}{\Lambda} \left(\frac{Q}{\Lambda} \right)^{-\Gamma_1^{\text{cusp}} / [\Gamma_1^{\text{cusp}} + 8\pi b_0 (1 - \mathcal{X}(x, \mu_b^*))]}$$

$$\mathcal{X}(x, \mu) = \frac{d}{d \ln \mu^2} \ln f_a(x, \mu) \quad \zeta = \mu^2 = Q^2$$

$$\mu_b^* = 2e^{-\gamma_E} / b_T^{sp} \quad \text{Requires iterative solution}$$

Conclusion : the predictive power is governed by both Q and x

The sign of the derivative of the collinear PDF determines the behavior



Large b_T corrections

$$f_1^a(x, b_T^2; Q) = \begin{cases} f_1^a(x, b_T^2; Q) \\ f_1^a(x, b_{max}^2; Q) F^{NIP}(x, b_T, Q; b_{max}) \end{cases}$$

Calculable in pQCD
(modulo PDFs)

To be modeled and
fit to data!

Which one is the most
relevant part and in
which **kinematic**
region?

$$b_T \leq b_{max}$$

$$b_T > b_{max}$$

Large b_T corrections

$$f_1^a(x, b_T^2; Q) = \begin{cases} f_1^a(x, b_T^2; Q) & b_T \leq b_{max} \\ f_1^a(x, b_{max}^2; Q) F^{NP}(x, b_T, Q; b_{max}) & b_T > b_{max} \end{cases}$$

Calculable in pQCD
(modulo PDFs)

To be modeled and
fit to data!

Which one is the most
relevant part and in
which **kinematic**
region?

$$F^{NP}(x, b_T, Q; b_{max}) = \exp \left\{ \begin{aligned} & - \ln \left(\frac{Q^2 b_{max}^2}{c^2} \right) \{ g_1 [(b^2)^\alpha - (b_{max}^2)^\alpha] \} \\ & - \ln \left(\frac{Q^2 b_{max}^2}{c^2} \right) \{ g_2 (b^2 - b_{max}^2) \} \\ & - \bar{g}_2 (b^2 - b_{max}^2) \} \end{aligned} \right.$$

“**extrapolation term**”
(see also Qiu-Zhang
PRD63 114011)

**low b_T behavior
extrapolated to large b_T
region**

g_1, α
fixed as a function of the other
parameters, requiring continuity of the
first and second derivatives



Large b_T corrections

$$f_1^a(x, b_T^2; Q) = \begin{cases} f_1^a(x, b_T^2; Q) & b_T \leq b_{max} \\ f_1^a(x, b_{max}^2; Q) F^{NP}(x, b_T, Q; b_{max}) & b_T > b_{max} \end{cases}$$

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“**extrapolation term**”
(see also Qiu-Zhang
PRD63 114011)

Correction to evolution

g_1, α
fixed as a function of the other
parameters, requiring continuity of the
first and second derivatives



Large b_T corrections

$$f_1^a(x, b_T^2; Q) = \begin{cases} f_1^a(x, b_T^2; Q) & b_T \leq b_{max} \\ f_1^a(x, b_{max}^2; Q) F^{NP}(x, b_T, Q; b_{max}) & b_T > b_{max} \end{cases}$$

Calculable in pQCD
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“**extrapolation term**”
(see also Qiu-Zhang
PRD63 114011)

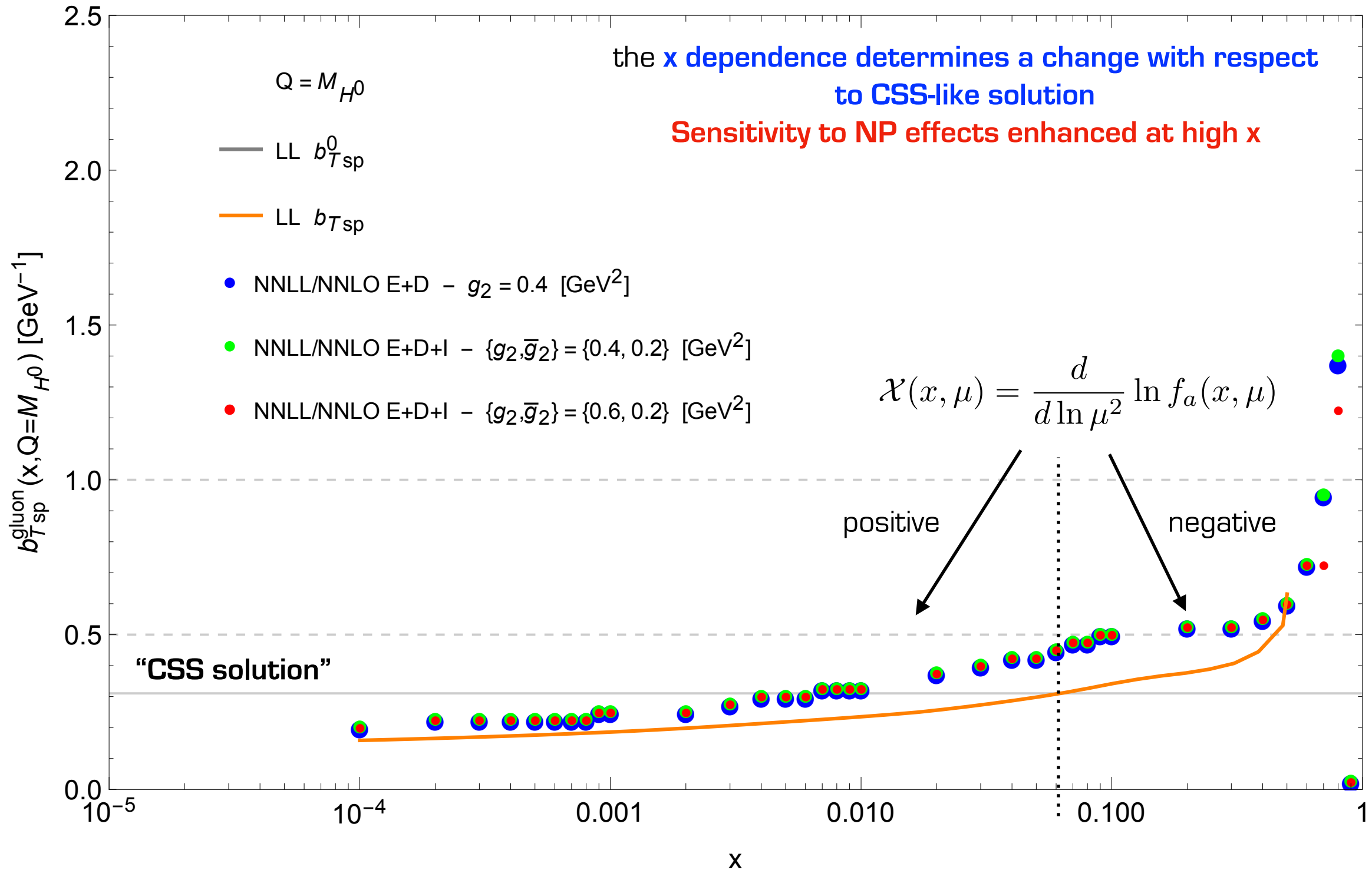
Correction to evolution

Correction to OPE at small b_T
(intrinsic transverse momentum)

g_1, α
fixed as a function of the other
parameters, requiring continuity of the
first and second derivatives

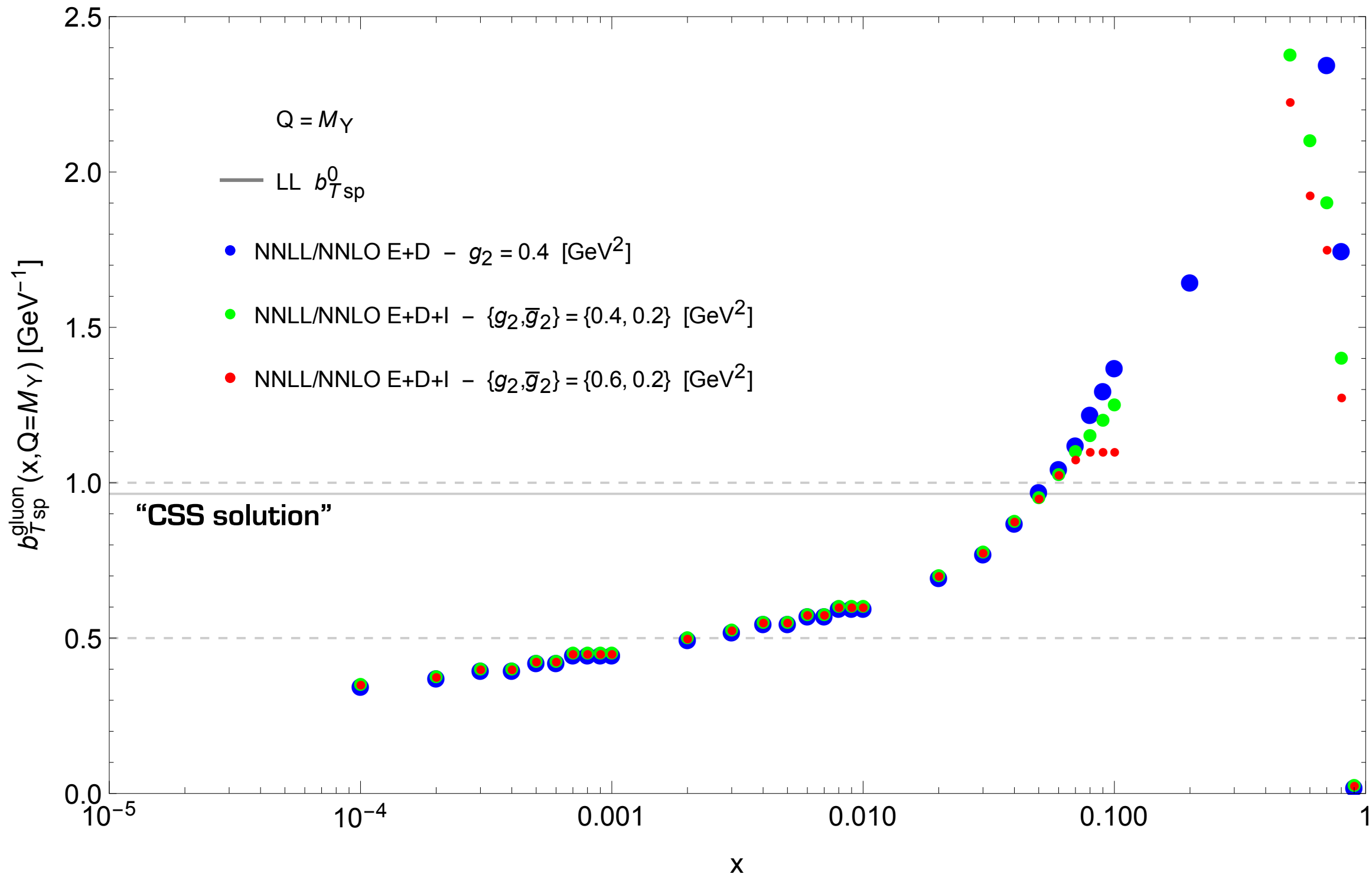
gluon - M_H (~ 125 GeV)

preliminary



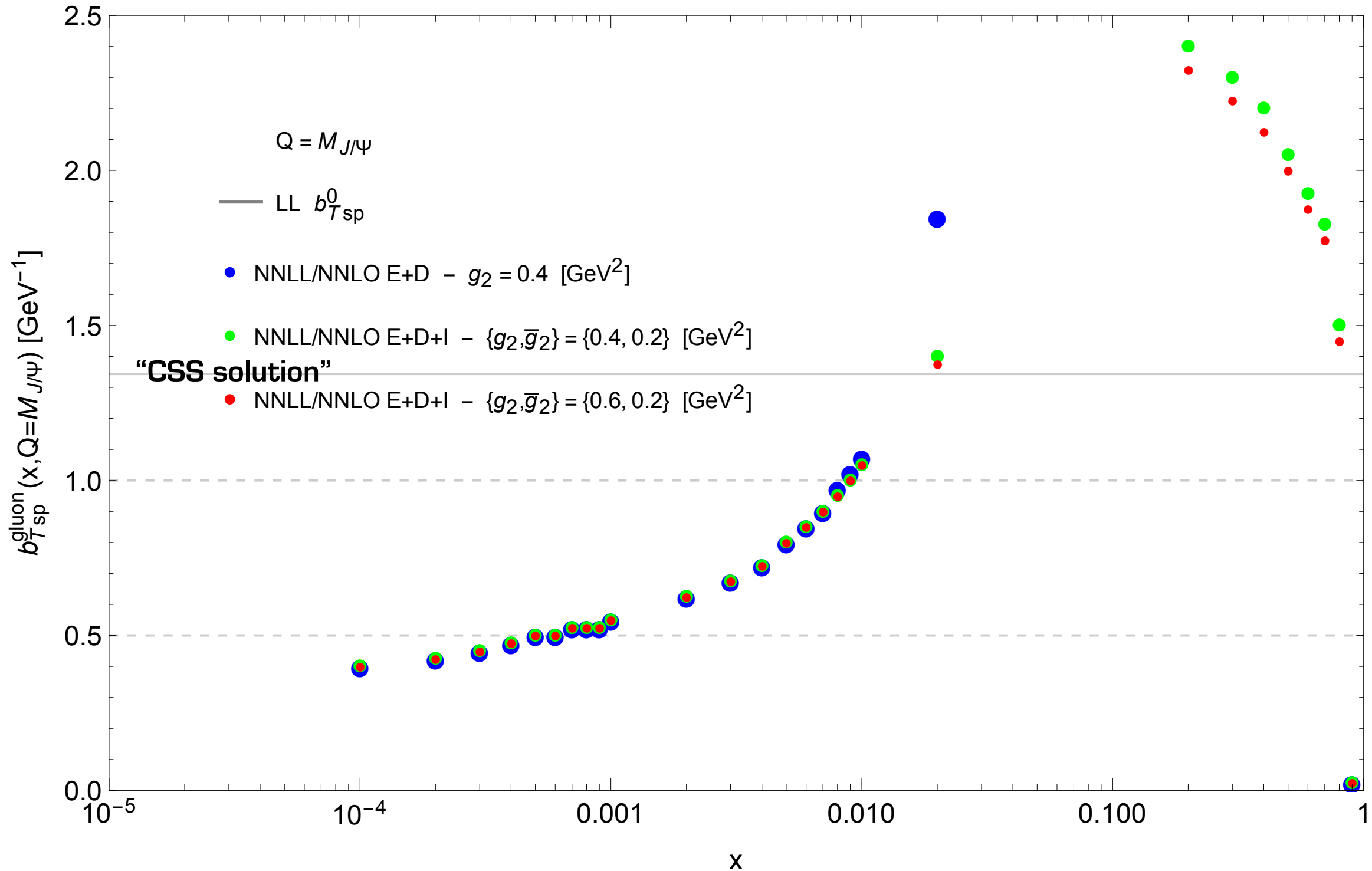
gluon - M_Y (~ 9 GeV)

preliminary



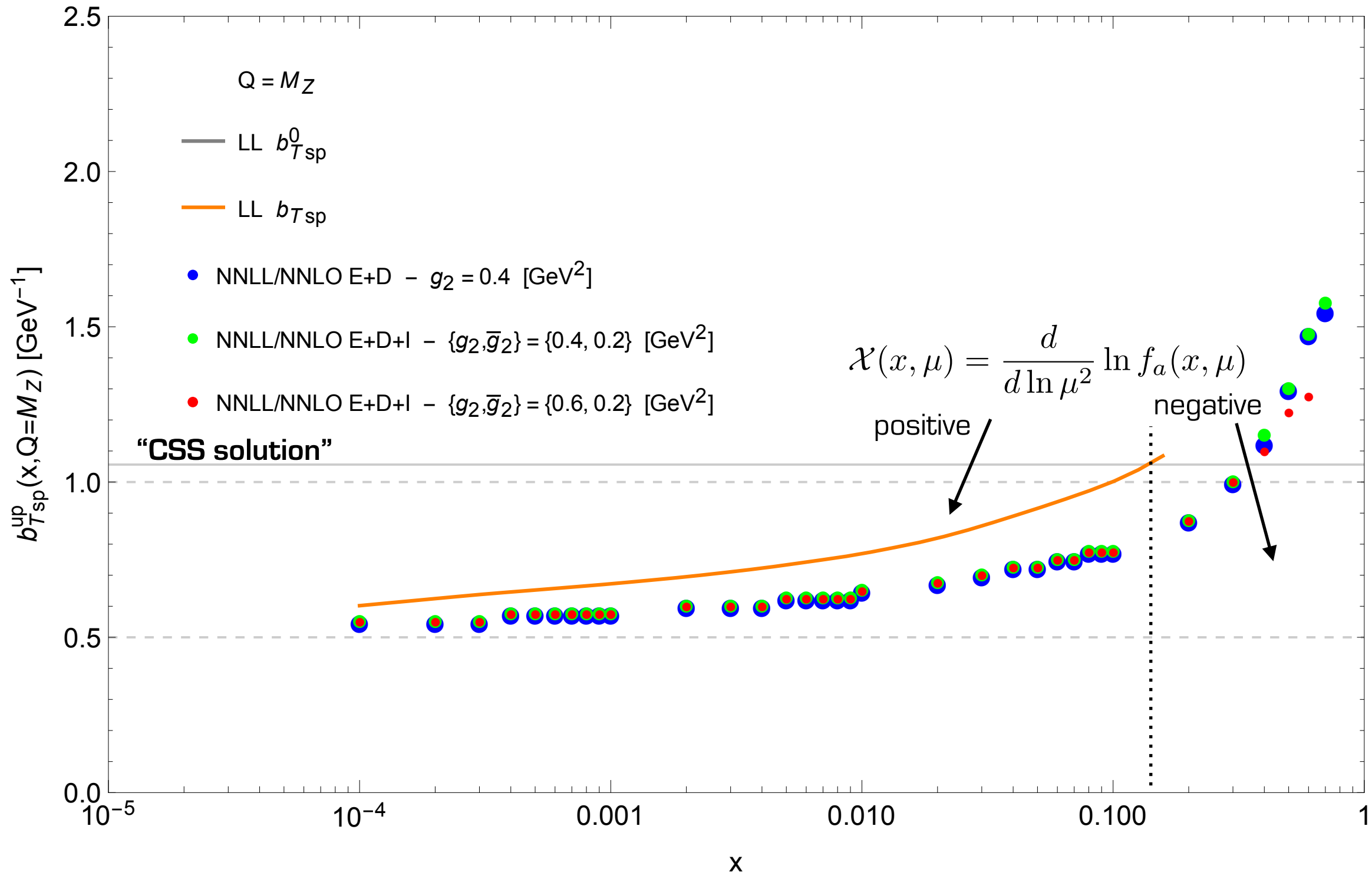
gluon - $M_{J/\psi}$ (~ 3 GeV)

preliminary



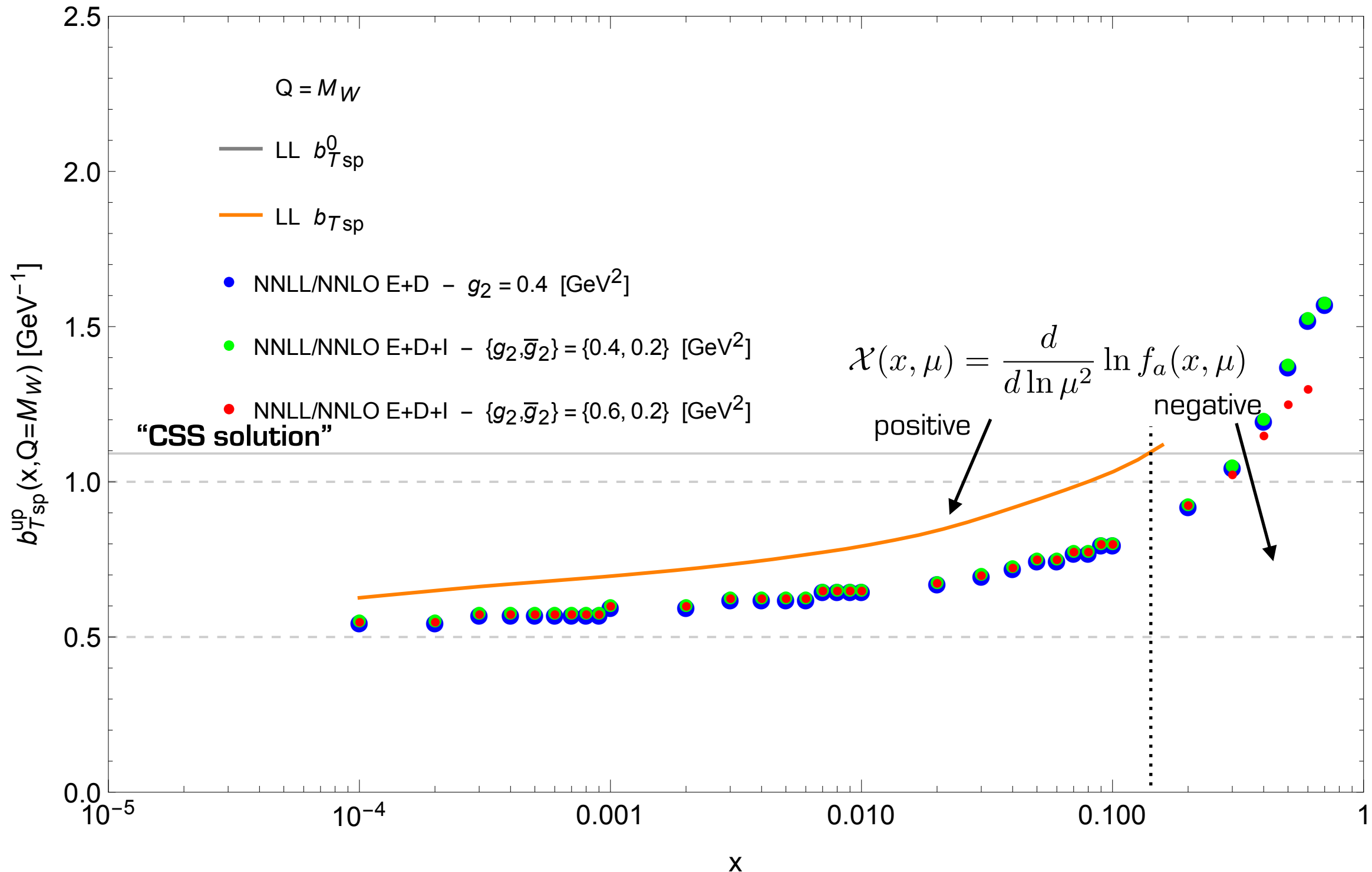
up quark - M_Z (~ 91 GeV)

preliminary



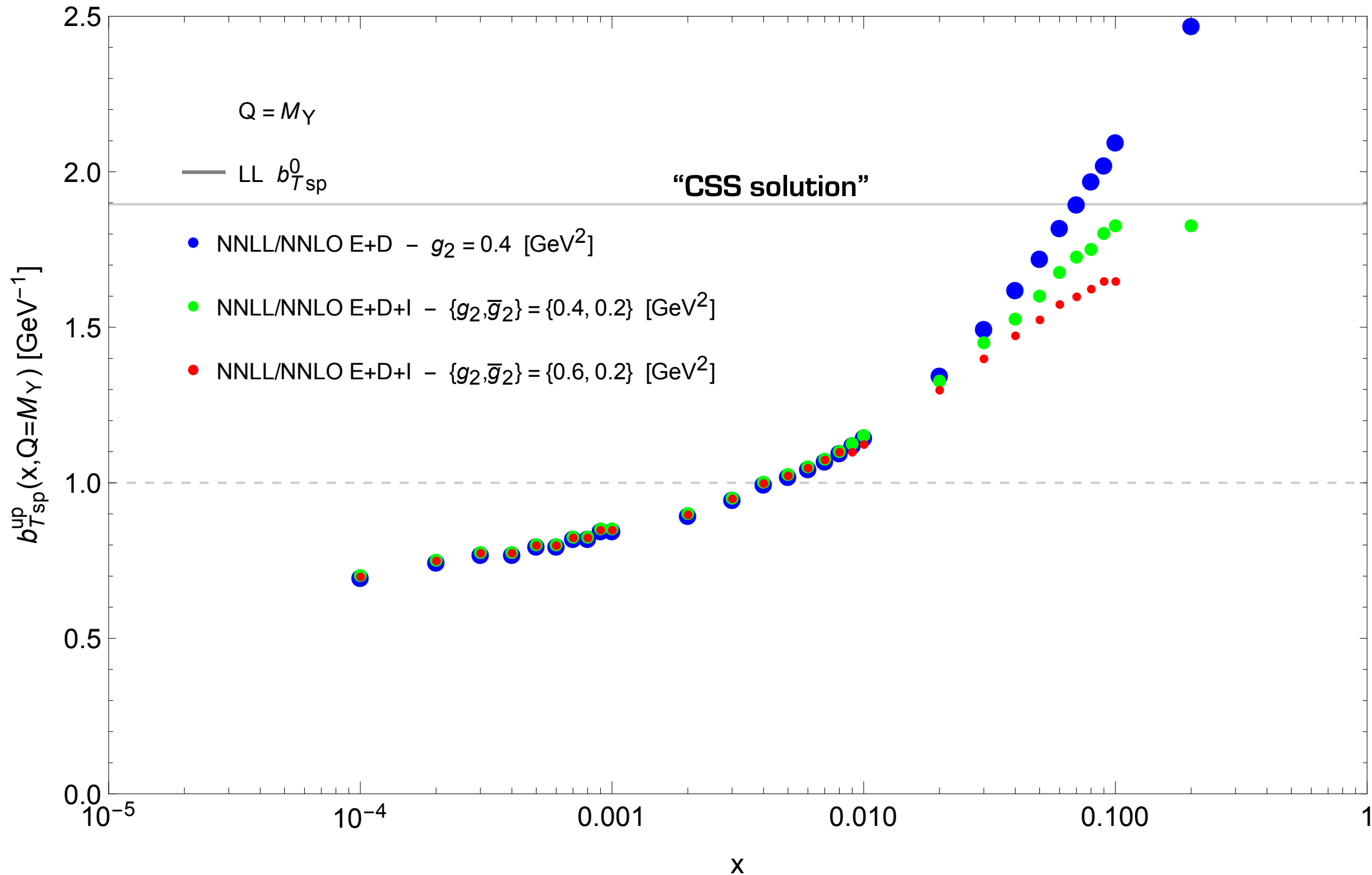
up quark - M_W (~ 80 GeV)

preliminary



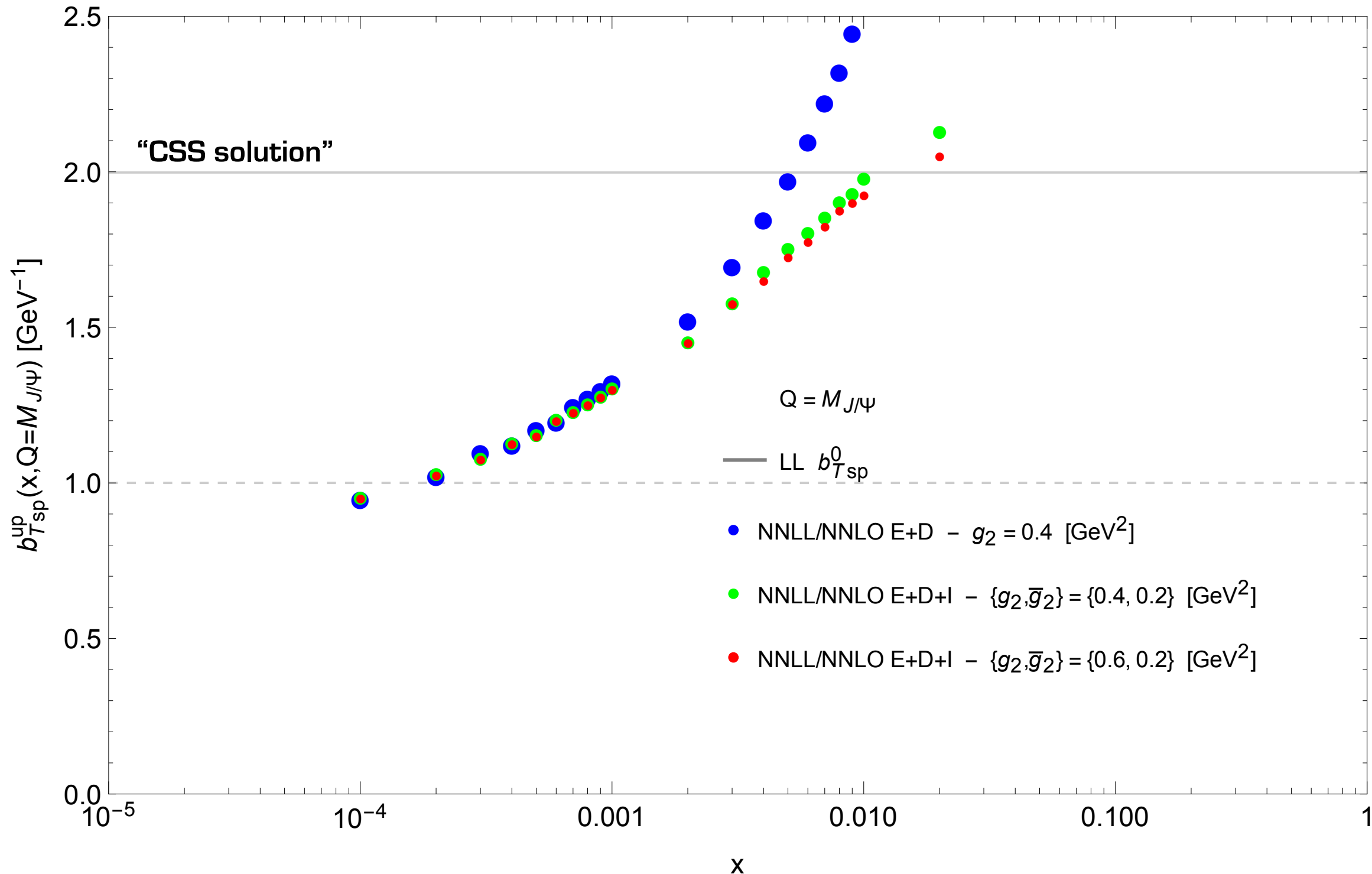
up quark - M_Y (~ 9 GeV)

preliminary

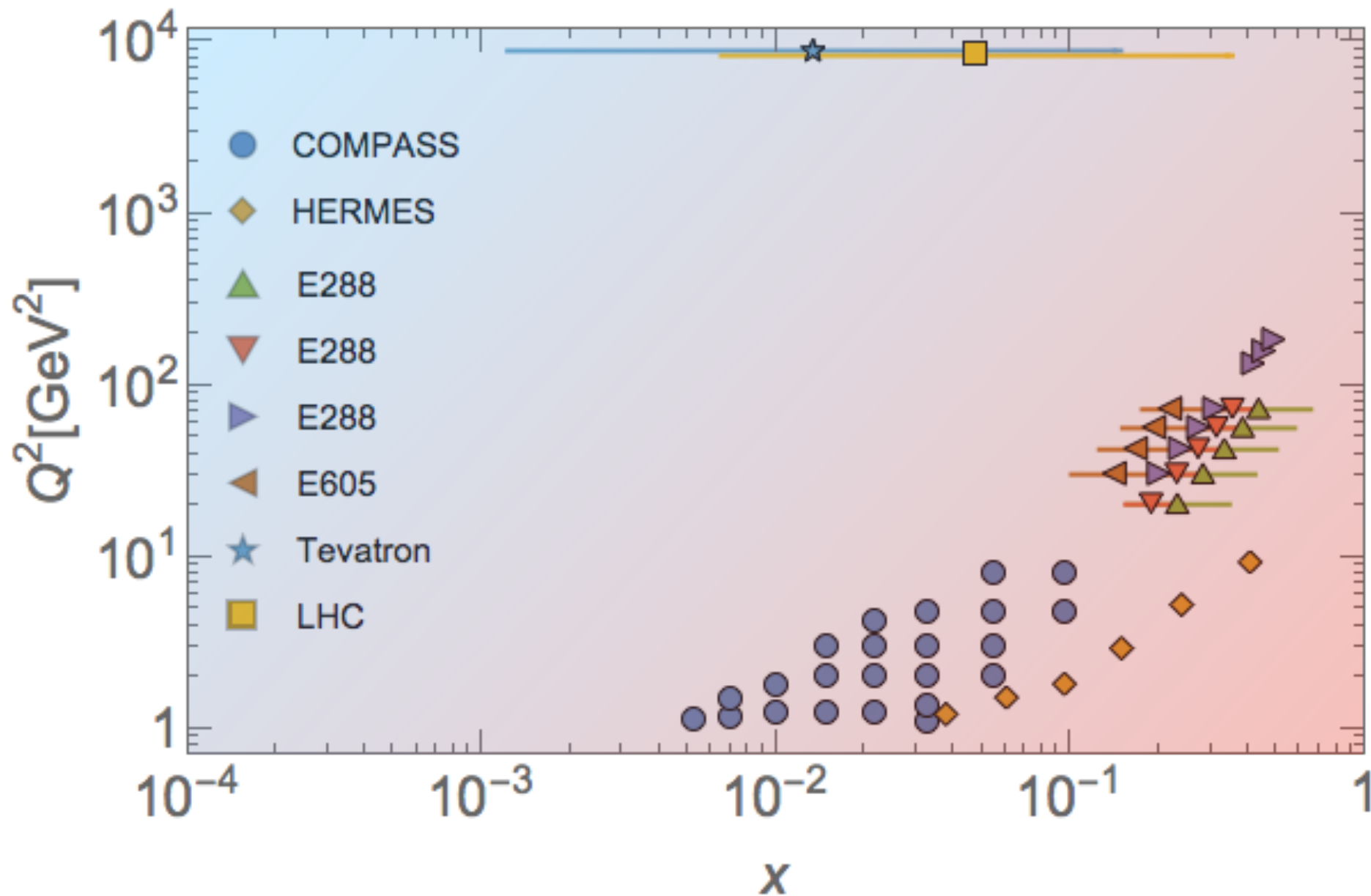


up quark - $M_{J/\psi}$ (~ 3 GeV)

preliminary



Predictive power in x - Q plane



high predictive power
weak influence of NP

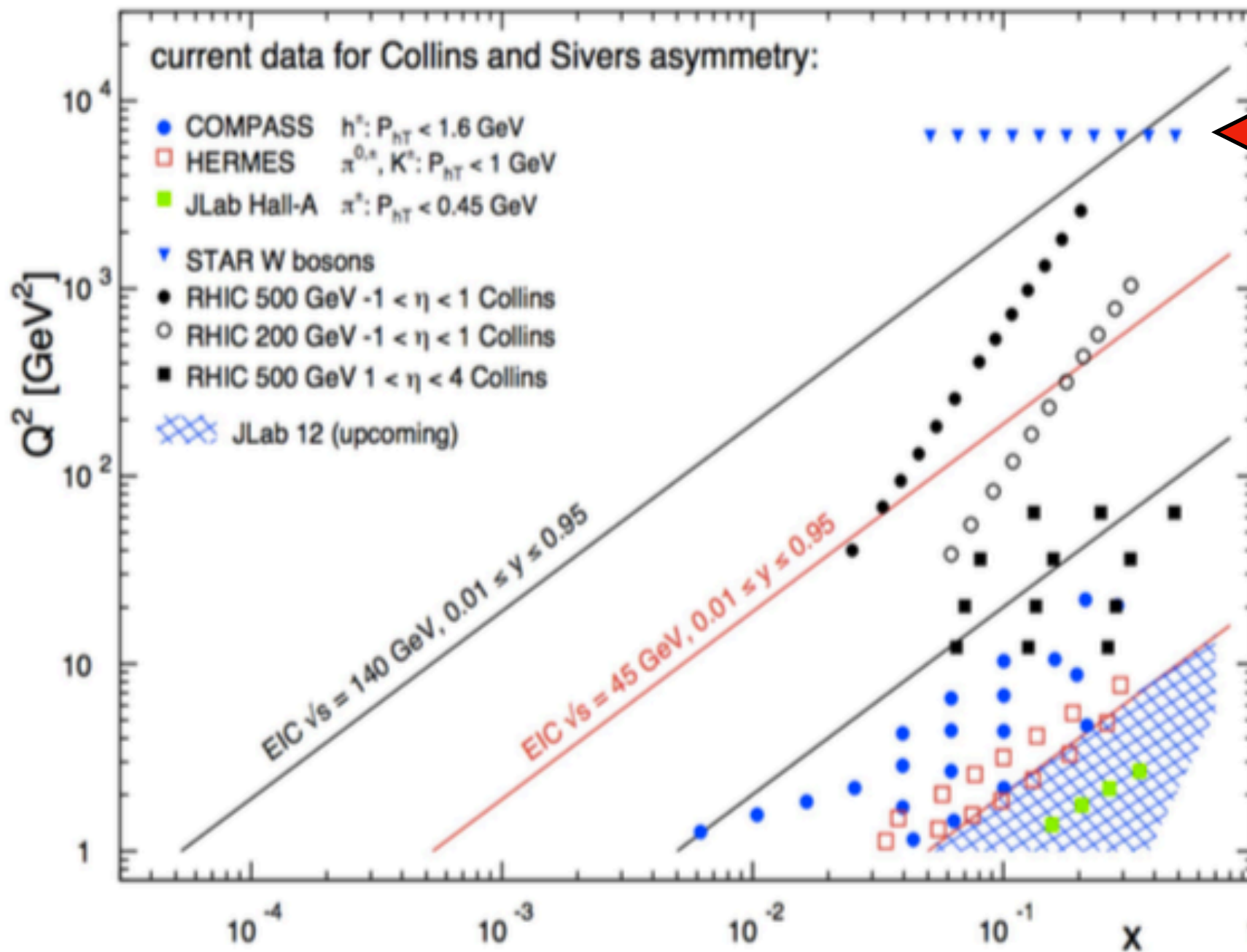
low predictive power
strong influence of NP

Small- x , high- Q :
strong predictive power

Rapidity dependence too



Kinematic coverage



W-boson production at
RHIC probes TMDs in
the high Q - high x
region

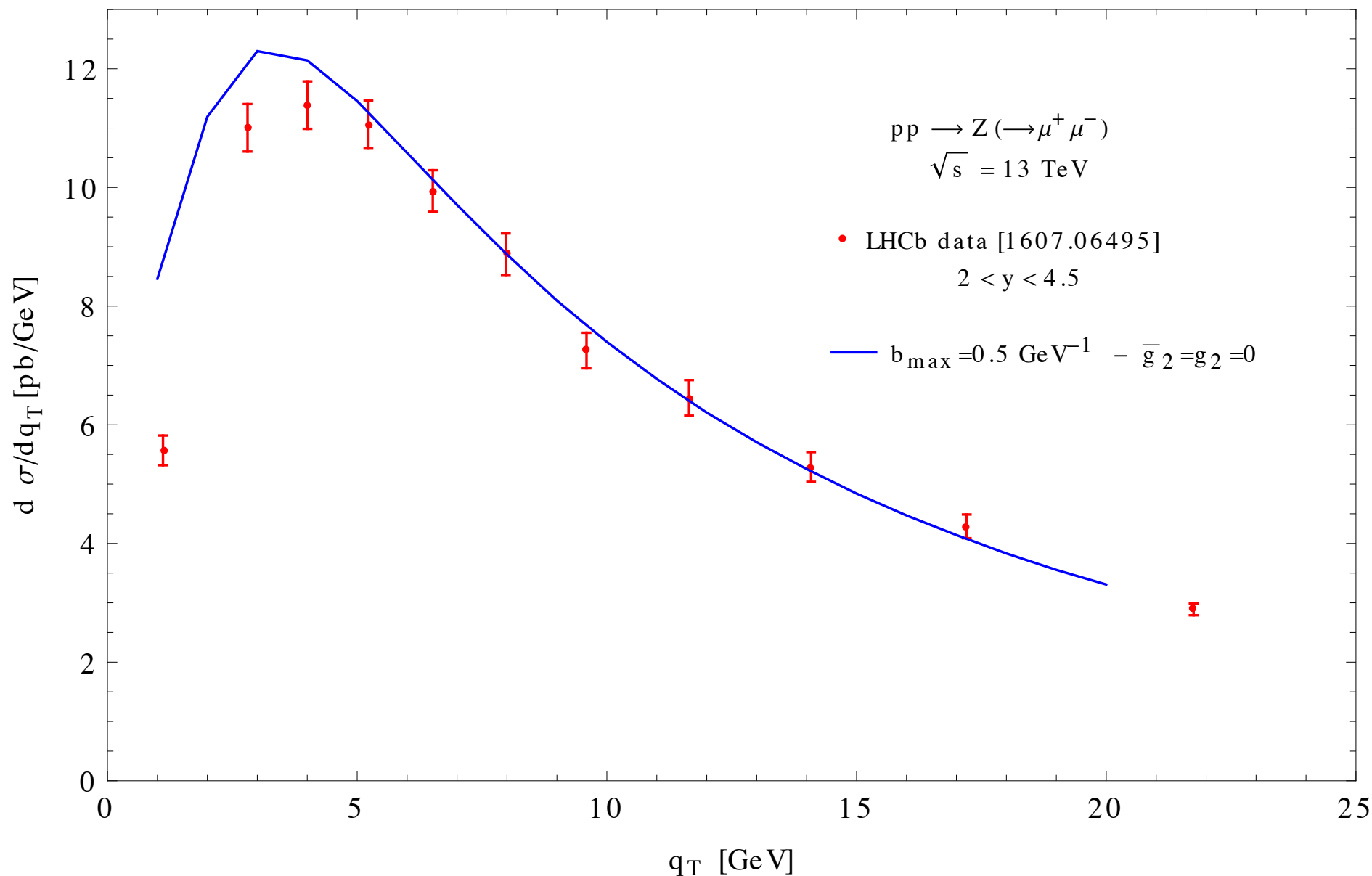
High Q : TMD
factorization under
control

High x : enhanced
sensitivity to
nonperturbative effects

Interesting combination

Z production @LHC 13 TeV

preliminary



Extrapolation to large bT
without NP corrections

Forward rapidity probes
significantly large x values
and we need NP
corrections to describe the
very low q_T bins

The **more precise the observable** is, the more **relevant** the **NP corrections** can be: see e.g. W mass and flavor dependence of intrinsic transverse momentum ([arXiv:1807.02101](https://arxiv.org/abs/1807.02101))



Conclusions and outlook

A first attempt to a **global fit of TMD PDFs and FFs** has been completed

We need independent information about TMD FFs to break the anti correlation with TMD PDFs

TMDs have predictive power in the small- x / high- Q limit.

Z production @LHC 13 TeV in different rapidity ranges can be used to prove the point.

Forward rapidity probes large x region, which implies sensitivity to the form of the NP part.

Conversely observables are more sensitive to NP corrections in the low- Q / large- x limit (**JLab**).

An interesting region to **extract the nonperturbative contributions** to TMD PDFs could be the region at **high Q** (to better control the corrections to factorization) and **high x** (to enhance the sensitivity to the large b_T region)

RHIC can provide data in this region, e.g. **W-boson production** to study both **polarized and unpolarized TMDs**, providing a complementary view on the results available from JLab

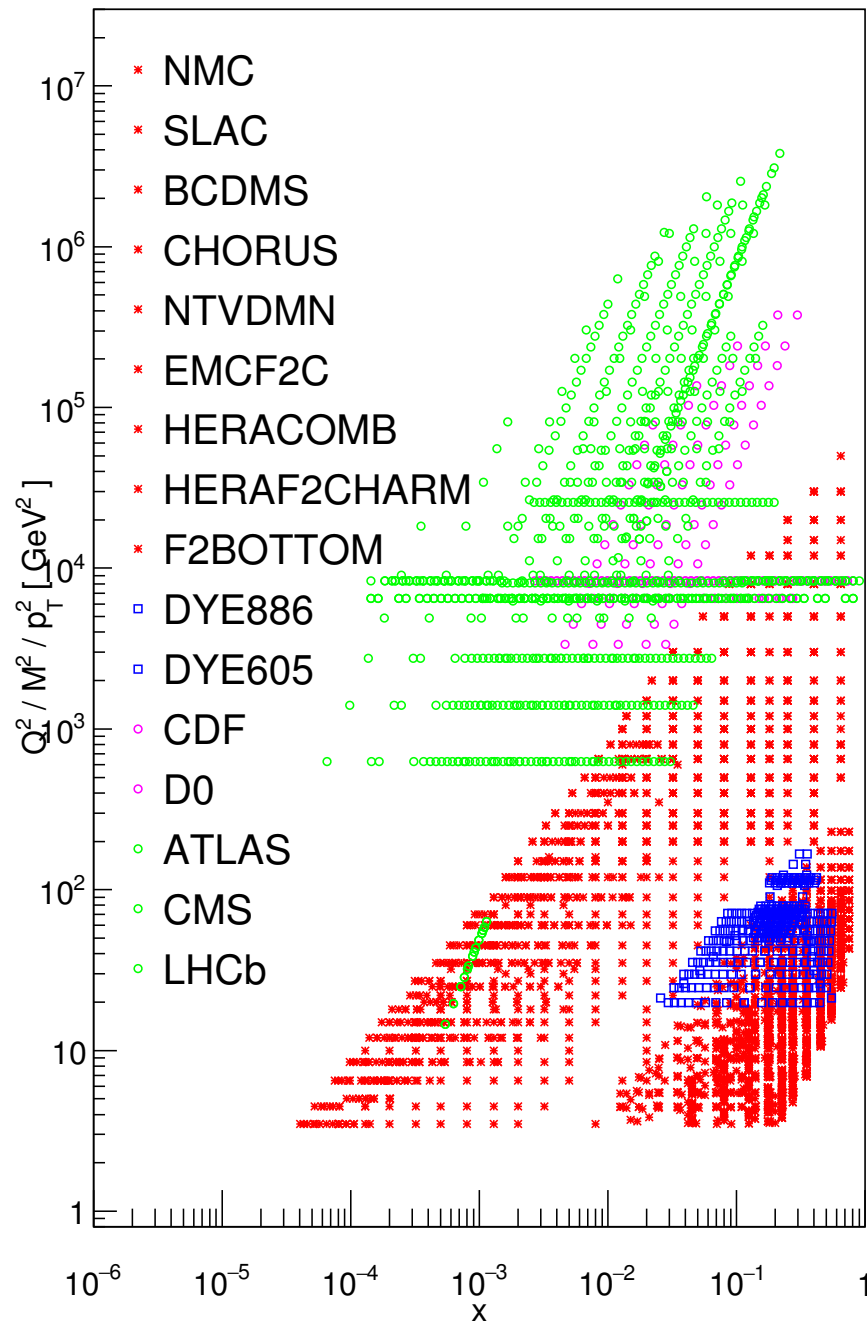


Backup



Collinear vs TMD PDFs

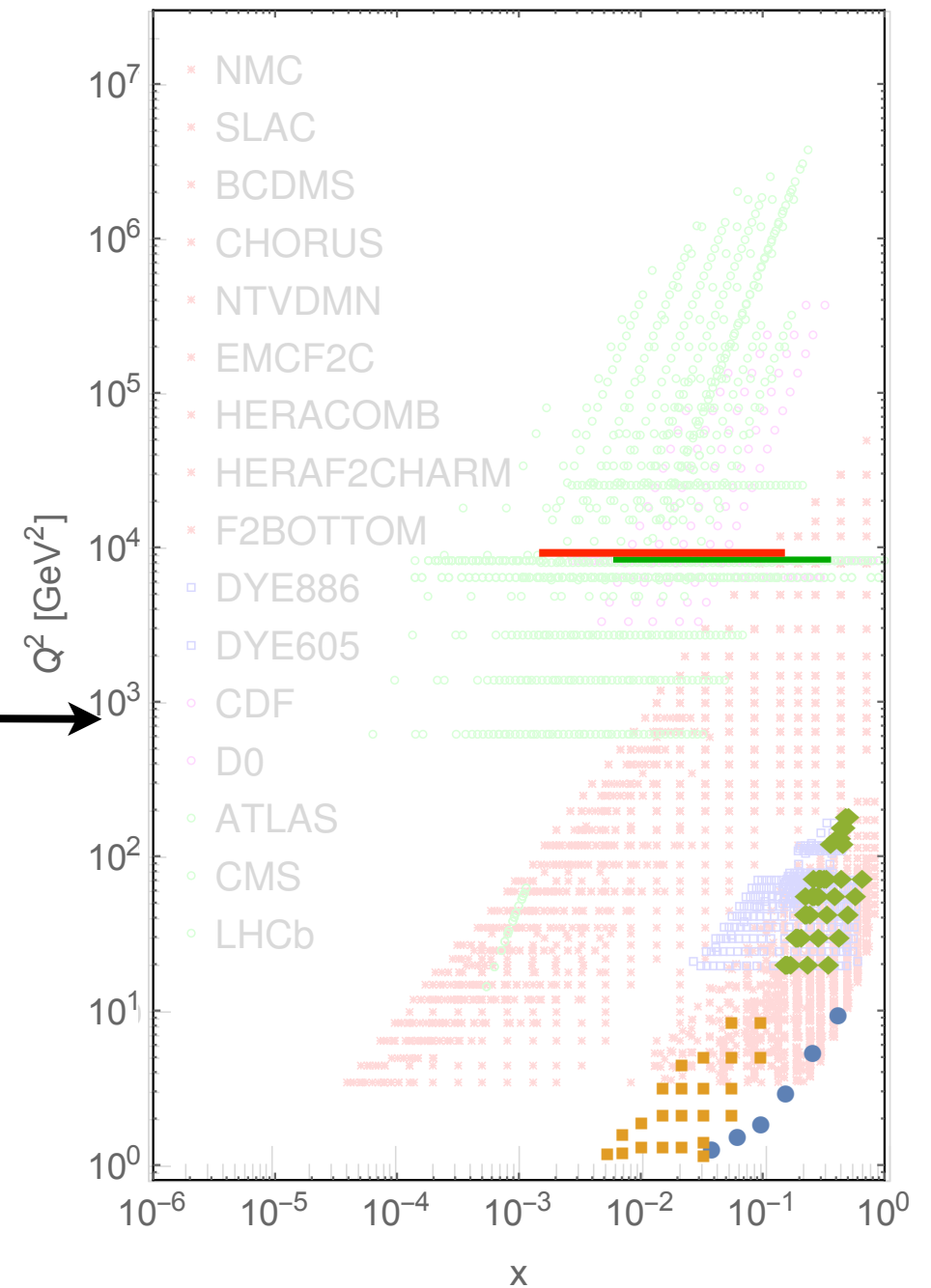
see E. Nocera - POETIC2016



data driven science

data sets available:

← collinear PDFs
vs
TMD PDFs →



x : momentum

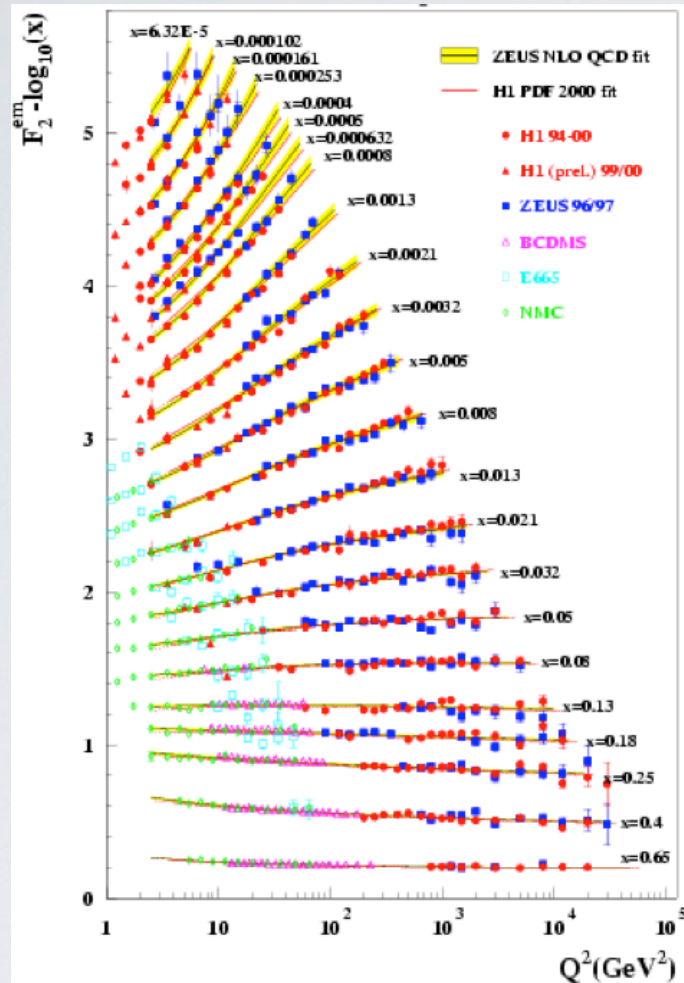
fraction carried by the parton

Q : resolution of the probe



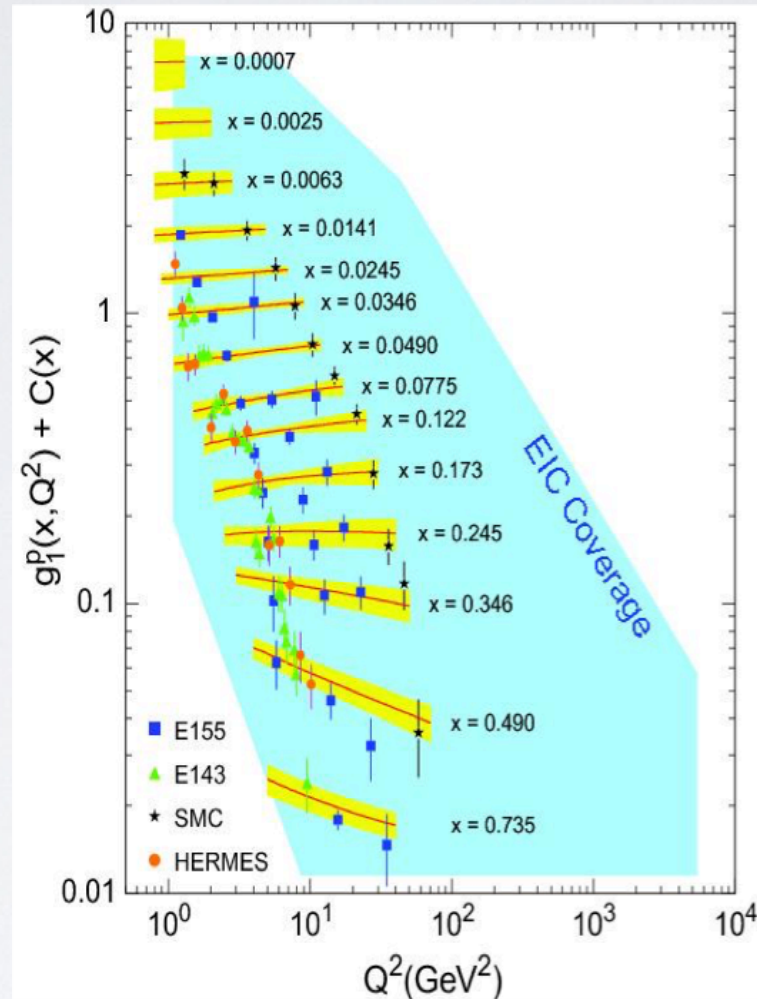
(Un)polarized collinear PDFs

World data for F_2^p



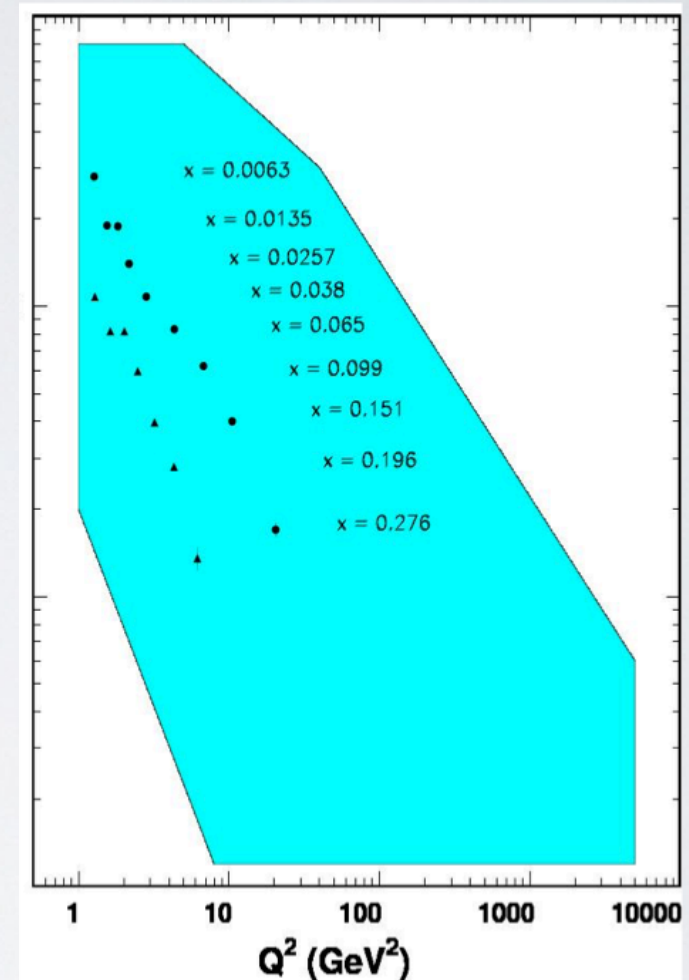
f_1 from fits of
thousands data

World data for g_1^p



g_1 from fits of
hundreds data

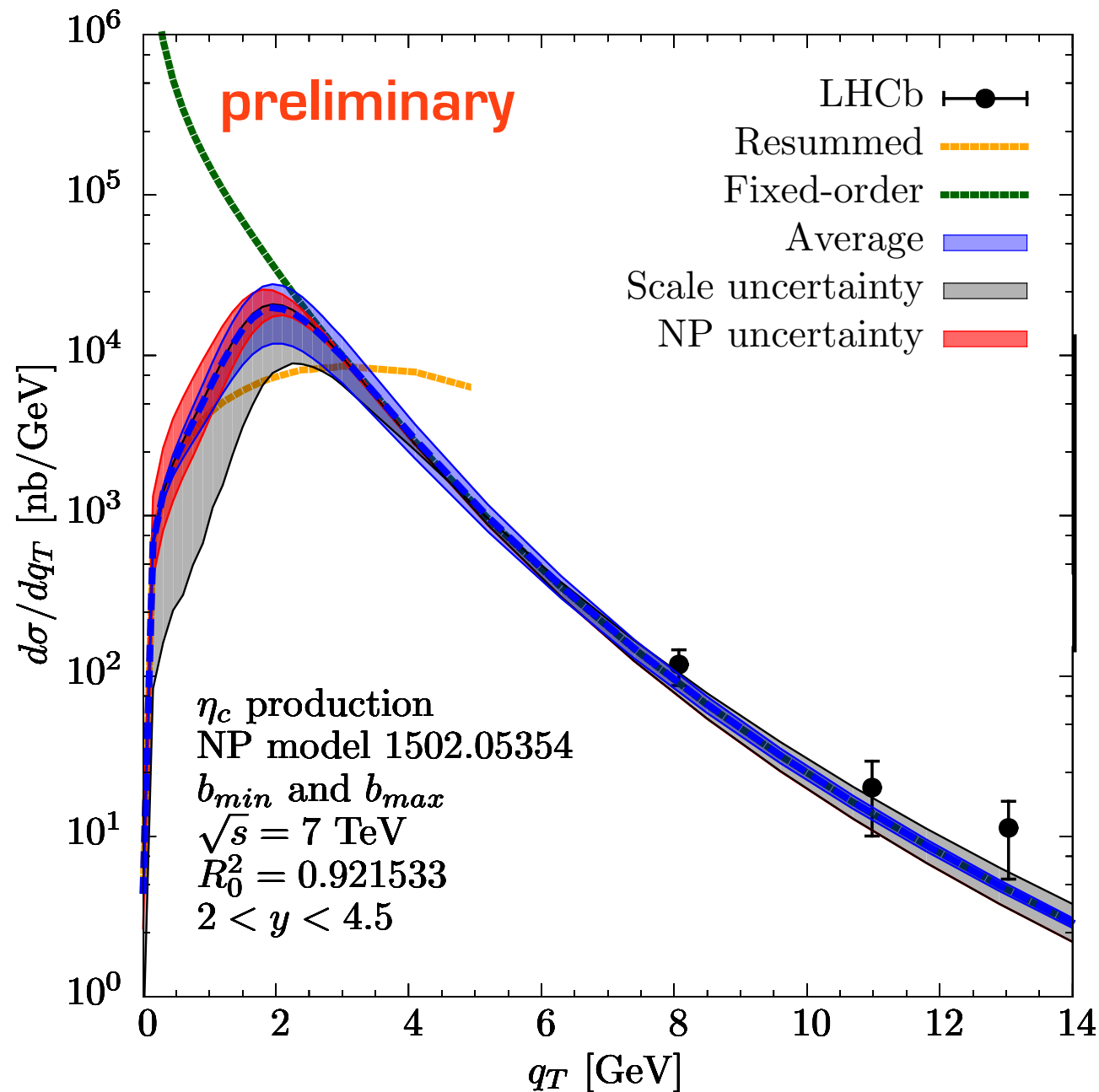
World data for h_1



h_1 from fits of
tens data

η_c production at LHC

full transverse momentum spectrum: inverse-error weighting :



Echevarria, Kasemets, Lansberg, AS, Pisano
 Phys.Lett. B781 (2018) 161-168

blue band: uncertainty from matching

grey band: scale uncertainty

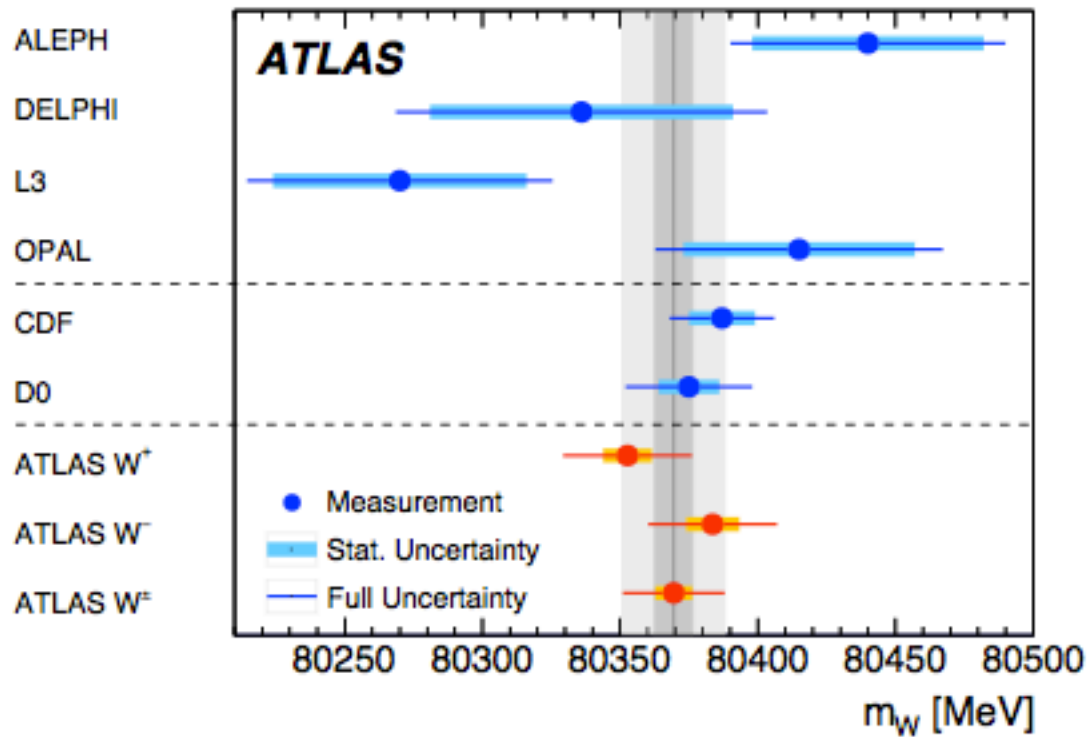
red band: uncertainty associated to the nonperturbative evolution and intrinsic transverse momenta

the formalism is in good shape
 we need the data at low q_T



W mass

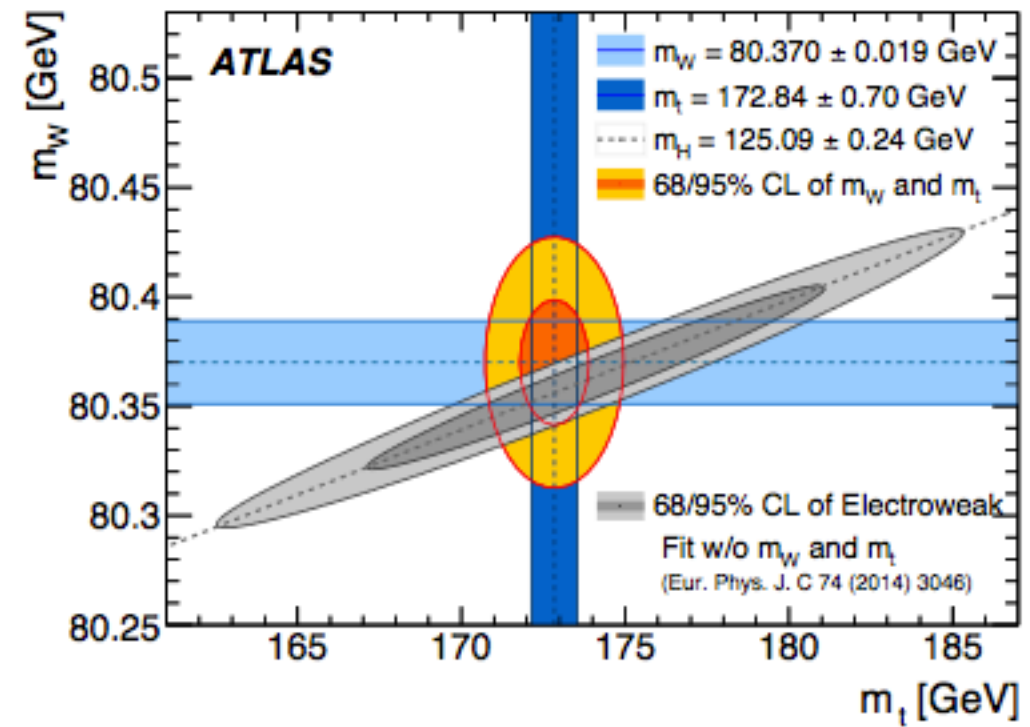
ATLAS, arxiv:1701.07240



Experimental measurements

$$m_W = 80370 \pm 19 \text{ MeV}$$

(7 stat, 11 exp, 14 th)



Global EW fit

$$m_W = 80356 \pm 8 \text{ MeV}$$

Need to **better control the uncertainties** associated to **direct** determinations of m_W

Is it possible to reduce the uncertainty to less than 10 MeV ?

Are we estimating all the **uncertainties of hadronic nature** in the best way possible?