INT Program INT-18-3 Week 1: GPDs, 1–5 October 2018, Seattle, WA
"Probing Nucleons and Nuclei in High Energy Collisions"

Form factors of the energy-momentum tensor

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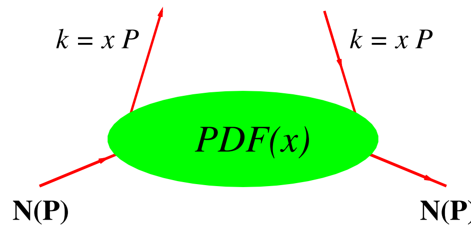
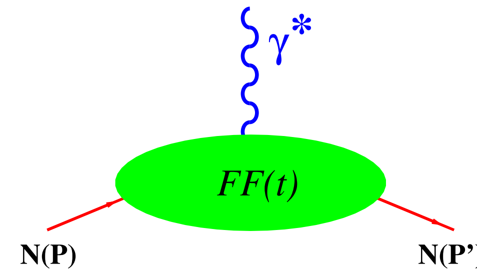
Outline

- **Introduction**
  What can we study at EIC?
  GPDs, Ji sum rule, tomography, and more!
- **Energy-momentum tensor**
  EMT form factors & D-term
  last unknown global property(!)
- **D-term**
  What is it? What do we know?
  theory & experiment
- **Physical interpretation**
  3D densities: limitations & uses
  stress tensor and stability
- **Applications**
  large-N_c baryons
  hadrocharmonia
- **Outlook**

based on: PS, Boffi, Radici, PRD**66** (2002)
Goeke et al, PRD**75**, 094021; PRC**75**, 055207
Cebulla et al, Nucl. Phys. A**794**, 87 (2007)
Mai, PS, PRD**86**, 076001 & **86**, 096002 (2012)
Cantara, Mai, PS, Nucl. Phys. A**953**, 1 (2016)
Bergabo, Cantara, PS, in preparation (2018)
Perevalova, Polyakov, PS, PRD**94**, 054024
Hudson, PS, PRD96 (2017) 114013
Hudson, PS, PRD97 (2018) 056003
Neubelt, Sampino, et al in progress
Polyakov, PS 1801.05858, 1802.09029
Polyakov, PS PRD98 (2018) 034030
review: Polyakov, PS IJMPA33 (2018) 1830025
supported by: NSF, DOE, DFG, Schuler Stiftung

Introduction

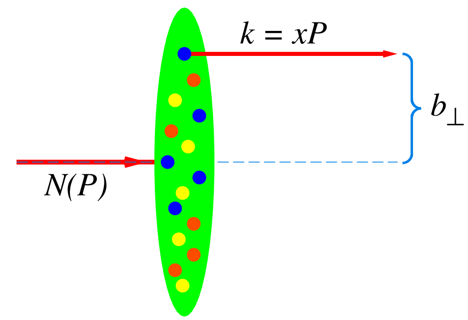




- {form factors, PDFs} $\in$ GPDs

  $\int \mathrm{d}x\; H^q(x,\xi,t) = F_1^q(t)$

  $\lim\limits_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$



- do tomography (M. Burkardt)

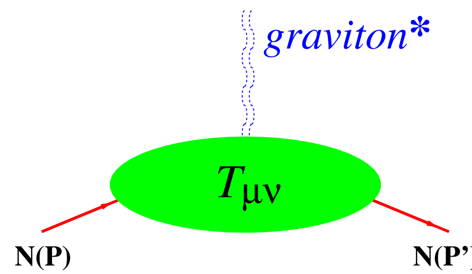
$$H^q(x,b_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\lim_{\xi\to 0} H^q(x,\xi,t)\right] e^{i\boldsymbol{\Delta}_T \boldsymbol{b}_T}$$

- **gravitational form factors** (polynomiality)

  $\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$

  $\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$

  Ji sum $A^q(t) + B^q(t) = 2J^q(t) \xrightarrow{t\to 0} 2J^q(0)$



- $\boldsymbol{T_{\mu\nu}}$ $\Rightarrow$ generators of Poincaré group

matrix elements of $\boldsymbol{T_{\mu\nu}}$: $\underbrace{\textbf{mass,}}_{T_{00}}\;\underbrace{\textbf{spin,}}_{\varepsilon^{ijk}x_j T_{0k}}\;\underbrace{\boldsymbol{D}\textbf{-term}}_{T_{ij}}$

$\left.\begin{matrix}\boldsymbol{M}\\ \boldsymbol{J}\end{matrix}\right\}$ external properties

$\boldsymbol{D}$ "internal" property

nucleon EMT form factors (Kobzarev & Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\biggl[\, A^a(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} \pm \bar{c}^a(t)g_{\mu\nu}\biggr]u(p) \quad (a = q, g)$$

- $\hat{T}^q_{\mu\nu}$, $\hat{T}^g_{\mu\nu}$ both gauge-invariant (not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$ (and $\sum_a \bar{c}^a(t) = 0$, $a = q, g$)
- constraints: **mass** $\Leftrightarrow$ $A^q(0) + A^g(0) = 1$ (quarks + gluons carry 100 % of nucleon momentum)

  spin $\Leftrightarrow$ $B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)*
- property: **D-term** $\Leftrightarrow$ $D^q(0) + D^g(0) \equiv D$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* also expressed as: vanishing of total gravitomagnetic moment

last global unknown: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\vert J^\mu_{\mathbf{em}}\vert N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \dots$
weak:	PCAC	$\langle N'\vert J^\mu_{\mathbf{weak}}\vert N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \dots$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\mathbf{grav}}\vert N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \dots$

global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40)\times 10^{-19}\,\text{C} \\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N \\
g_A &= 1.2694(28) \\
g_p &= 8.06(0.55) \\
M &= 938.272013(23)\,\text{MeV} \\
J &= \tfrac{1}{2} \\
\boldsymbol{D} &= \quad \mathbf{??}
\end{aligned}
$$

and more:

t-dependence ... ...

parton structure, etc ...

$\hookrightarrow$ $\boldsymbol{D}$ = *"last" global unknown*

which value does it have?

what does it mean?

Theoretical results for D

free spin 0 field

- free Klein-Gordon field $D = -1$
  (Pagels 1966; Hudson, PS 2017)

Goldstone bosons (decays of $\psi' \to J/\psi\,\pi\pi$, light Higgs $\to \pi\pi$)

- Goldstone bosons of chiral symmetry breaking $D = -1$ in soft pion limit
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

- chiral perturbation theory for Goldstone bosons
  Donoghue, Leutwyler (1991); Kubis, Meissner (2000); Diehl, Manashov, Schäfer (2005)

$$
\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

where
$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\,(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi \approx -0.97 \pm 0.01$
$D_K \approx -0.77 \pm 0.15$
$D_\eta \approx -0.69 \pm 0.19$ (estim. uncertainty, Hudson,PS 2017)

nuclei

- nuclei in liquid drop model $\boldsymbol{D = -0.2 \times A^{7/3}}$ $\rightarrow$ potential for DVCS with nuclei!
  Maxim Polyakov (2002) (see below)

- nuclei in Walecka model
  Guzey, Siddikov (2006)

$$
\begin{array}{rcl}
{}^{12}\text{C}: & D = & -6.2 \\
{}^{16}\text{O}: & D = & -115 \\
{}^{40}\text{Ca}: & D = & -1220 \\
{}^{90}\text{Zr}: & D = & -6600 \\
{}^{208}\text{Pb}: & D = & -39000
\end{array}
$$

Q-balls (toy model laboratory)

- Q-balls, non-topological solitons in strongly interacting theory: $90 \leq -D =\leq \infty$
  Mai, PS PRD**86**, 076001 (2012)

- N^{th} excited Q-ball state (decay into ground states): $D = -\,\text{const}\, N^8$
  Mai, PS PRD**86**, 096002 (2012)

- Q-cloud limit, most extreme instability we could find: $D = -\,\text{const}/\varepsilon^2$ in the limit $\varepsilon \to 0$
  Cantara, Mai, PS NPA**953**, 1 (2016)

- Q-cloud excitations, even more extreme instability: $D < 0$ even more negative
  Bergabo, Cantara, PS, in preparation (2018)

free spin $\frac{1}{2}$ fermion

- $D = 0$ Dirac equation predicts $g = 2$ anomalous magnetic moment
  analoguously it predicts $D = 0$ for non-interacting fermion
  implicit: Donoghue, Holstein, Garbrecht, Konstandin, PLB**529**, 132 (2002)
  explicit in Hudson, PS Phys.Rev. D97 (2018) 056003

if $D_{\text{fermion}} \neq 0$ $\leftarrow$ interactions!!

interacting fermion systems

- case study I: introduce boundary condition (bag model)

"switch on interaction" $D = N_c^2 \underbrace{\left(\frac{-4\,\pi^2 + 15}{45}\right)}_{=-0.54\ldots\ <\ 0}$ in limit $mR \to \infty$

- case study II: chiral quark-soliton model

$D = -F_\pi^2 M_N \int d^3r\; r^2\, P_2(\cos\theta)\, \mathrm{tr}_F[\nabla^3 U][\nabla^3 U^\dagger] + \mathcal{O}\big((\nabla U)^3\big)$ PS, Radici, Boffi (2002)

"switch off chiral interaction" i.e. pion fields $U = \exp(i\tau^a\pi^a/F_\pi) \to 1$ $\Rightarrow$ $D \to 0$

Hudson, PS Phys.Rev. D97 (2018) 056003

D-term distinguishes free bosons and fermions

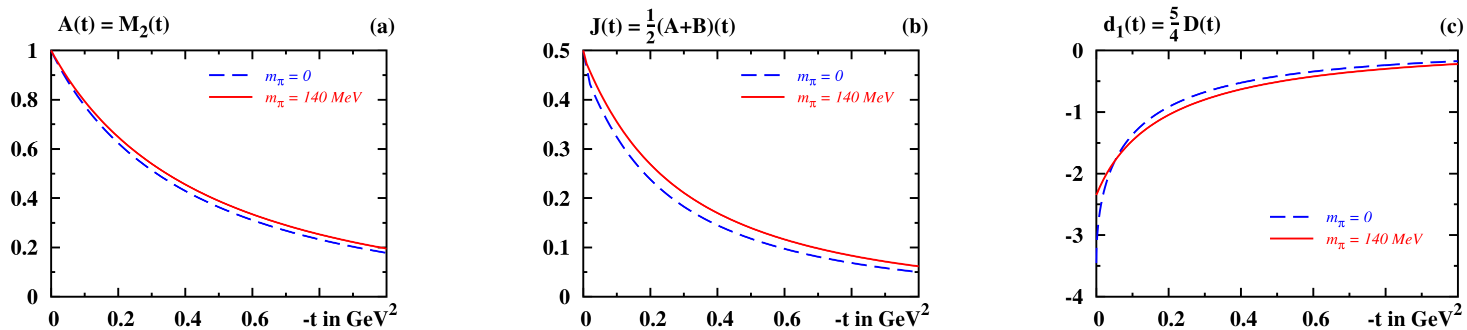
free spin-0 case $D = -1$ vs spin-$\frac{1}{2}$ case $D = 0$. What does this mean?

matter, visible universe made of fermions! All D-terms due to interaction!?

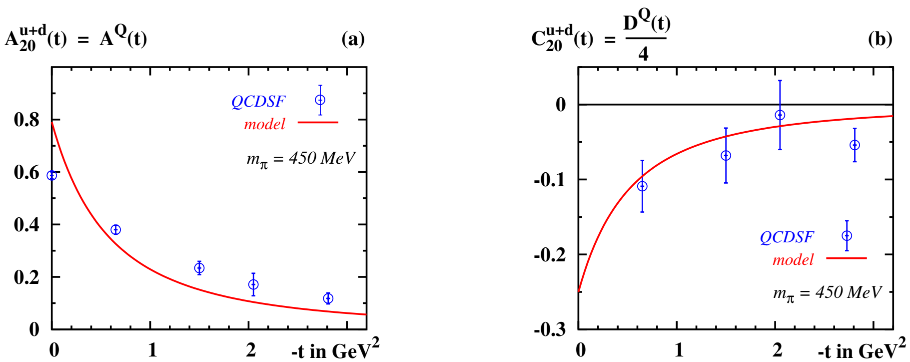
nucleon

- bag model (always good starting point!) $D = -1.145 < 0$ due to bag boundary!
  Ji, Melnitchouk, Song (1997); Neubelt, Sampino, et al (2018)

- chiral quark soliton model
  Petrov et al 1998, Goeke et al, PRD75 (2007) 094021



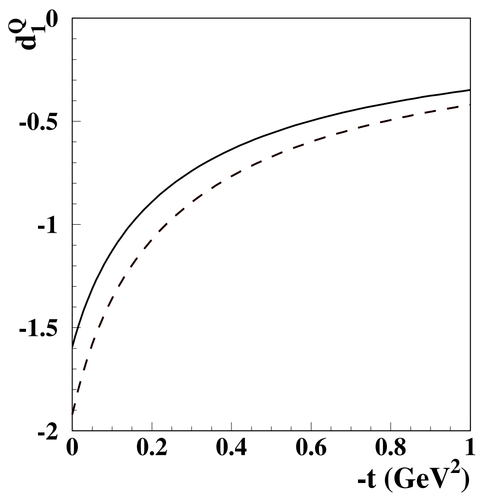
- lattice D^Q: QCDSF Collaboration, Göckeler et al, PRL92 (2004) 042002 & hep-ph/0312104



- χPT cannot predict D-term, but $d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5k\, g_A^2 M_N}{64\,\pi\, f_\pi^2}\, m_\pi + \dots, \quad \overset{\circ}{d}{}_1'(0) = -\frac{k\, g_A^2\, M_N}{32\,\pi f_\pi^2\, m_\pi} + \dots$
  $k = 1$ for finite N_c, and $k = 3$ for $N_c \to \infty$ Belitsky, Ji (2002), Diehl et al (2006), Goeke et al (2007)

nucleon dispersion relations

- unsubtracted t-channel dispersion relations (need pion PDFs) at $\mu^2 = 4\,\mathrm{GeV}^2$
  Barbara Pasquini, Maxim Polyakov, Marc Vanderhaeghen (2014)



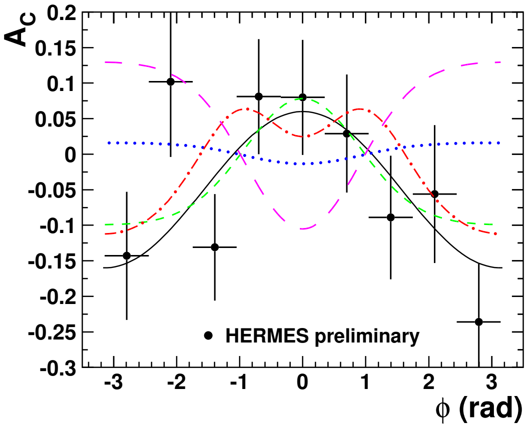
... predictions are made in models, lattice, dispersion relations.

(lattice can and will improve, "tomorrow" or in "3 years")

What do experiment and phenomenology say?

D-term of nucleon

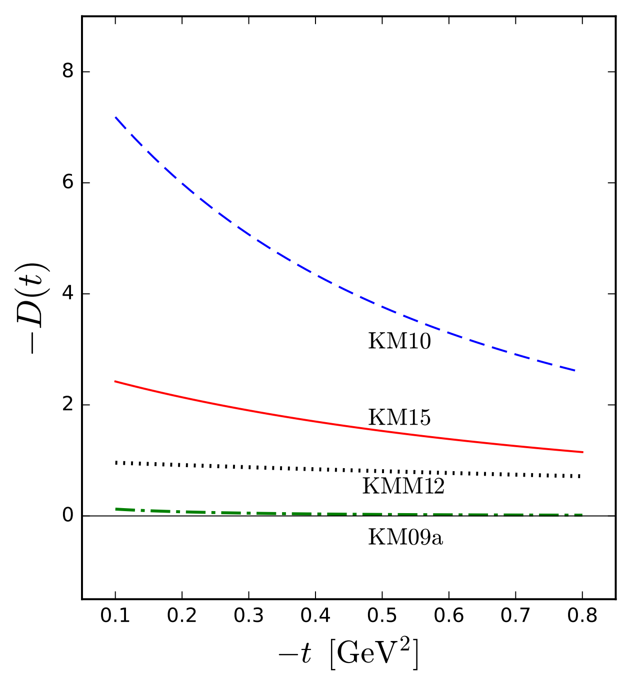
- HERMES proceeding NPA711, 171 (2002); Airapetian et al PRD 75, 011103 (2007)



beam charge asymmetry
dotted line: VGG model without D-term (ruled out)
dashed line: VGG model + positive D-term (ruled out)
dashed-dotted: VGG model + **negative** D-term (yeah!)
model-dependent Frank Ellinghaus, NPA711, 171 (2002)
Belitsky, Müller, Kirchner, NPB629 (2002) 323
COMPASS with $\mu^\pm$ beams!!? Nicole D'Hose, Primošten 17-22 Sep 2018

- fits by Kresimir Kumerički, Dieter Müller et al
  some tendency that $D < 0$ needed!

  DVCS parametrizations from:
  Kumerički, Müller, NPB 841 (2010) 1
  Kumerički, Müller, Murray, Phys. Part. Nucl. 45 (2004) 723
  Kumerički, Müller, EPJ Web Conf. 112 (2016) 01012
  Fig. 9 in ECT* workshop proceeding 1712.04198
  statistical uncertainty of D in KMM12: $\sim 50\,\%$,
  statistical uncertainty of D in KM15: $\sim 20\,\%$.
  unestimated systematic uncertainty
  Kresimir Kumerički private communication + talk yesterday

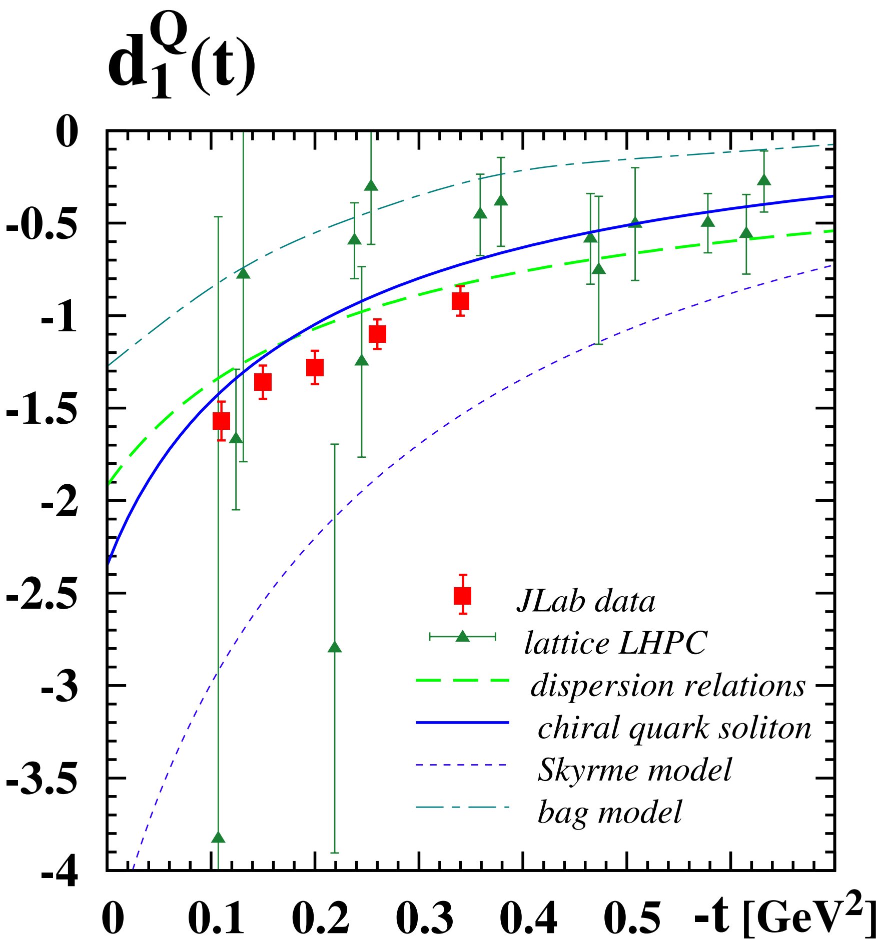


- **CLAS result**

Burkert, Elouadrhiri, Girod, **Nature 557, 396 (2018)** (talk: V. Burkert at SPIN 2016 Sep. 2016)

based on: Girod et al PRL 100 (2008) 162002, Jo et al PRL 115 (2015) 212003

beware: unestimated systematic bias (Kresimir yesterday). Not model independent, first pioneering step



D-term = subtraction term in fixed-t dispersion relations for $\mathcal{A}_{\text{DVCS}}$
Teryaev hep-ph/0510031
Anikin, Teryaev, PRD76, 056007 (2007)
Diehl and Ivanov, EPJC52, 919 (2007)
Radyushkin, PRD83, 076006 (2011)

subtraction term $\sim\ d_1 + d_3 + d_5 + \ldots$
the $d_i \to 0$ for $i > 1$ with $Q^2 \to \infty$

assumed d_3, d_5, ... small compared to d_1
working assumption (do better $\to$ future data)

chiral quark-soliton $d_3^q/d_1^q = 0.3$, $d_5^q/d_1^q = 0.1$
Kivel, Polyakov, Vanderhaeghen, PRD63 (2001)

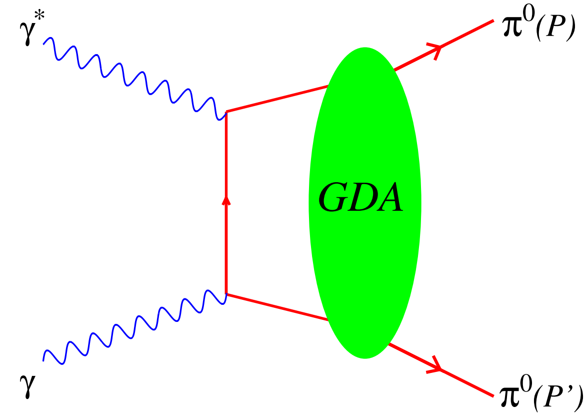
$D^q(t) = \frac{4}{5}\, d_1^q(t)$ + assumptions

$\Rightarrow$ CLAS, KM-fits, dispersion relations, models, lattice: **insight on $D(t)$!** What do we learn?

- ***D*-term of π^0**

access EMT form factors of unstable particles
through generalized distribution amplitudes
(analytic continuation of GPDs)
via $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^-

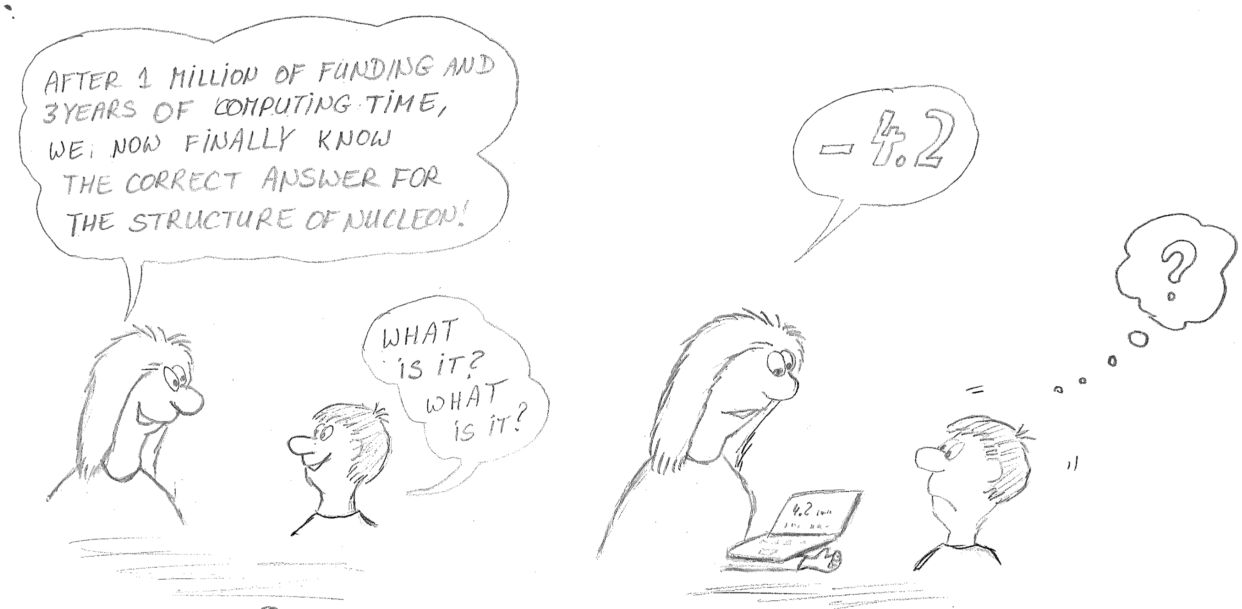
Masuda et al (Belle), PRD 93, 032003 (2016)



best fit to Belle data $\to$ $D^Q_{\pi^0} \approx -0.7$
at $\langle Q^2 \rangle = 16.6$ GeV^2
compatible with soft pion theorem $D_{\pi^0} \approx -1$
(assuming gluons contribute the rest which is reasonable)

Kumano, Song, Teryaev, PRD97, 014020 (2018)

(in principle also other hadrons, even $\bar{p}p$. But difficult to "extrapolate" from $t > 4m_p^2$ to $t = 0$. But for pion: very interesting, narrow region to "extrapolate over" + χPT can help!)



LATTICE QCD TELLS US THE CORRECT ANSWER FOR THE STRUCTURE OF HADRONS AND EVERYTHING!

BUT IT DOES NOT TELL US **WHY** WE GET THIS NUMBER...

MODELS DO NOT TELL US THE EXACT NUMBER...

BUT WE KNOW EXACTLY WHY WE GOT IT!

interpretation

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$
- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r}\, \rho_E(\vec{r})\, e^{i\vec{\Delta}\vec{r}} \to$ charge distribution
  Sachs, PR126 (1962) 2256
  $\hookrightarrow Q = \int d^3\vec{r}\, \rho_E(\vec{r})$
- static EMT $T_{\mu\nu}(\vec{r},\vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle \to$ mechanical properties of nucleon
  M.V.Polyakov, PLB 555 (2003) 57
  $\hookrightarrow M_N = \int d^3\vec{r}\, T_{00}(\vec{r})$, etc
- limitations: 2D densities exact partonic probability densities.
  3D densities not exact, reservations for $r \lesssim \lambda_{\text{Compt}} = \frac{\hbar}{mc} \sim 0.2\text{fm}$ for proton

known since earliest days (Sachs, 1962) comprehensive studies, e.g. by
- corrections ABSENT in large-N_c limit ($m \sim N_c$, $R \sim N_c^0$ and $1/(mR)^2 \sim 1/N_c^2$)
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
- G. Miller, PRC80 (2009) 045210 ("simple model" with FF from triangle diagram, dramatic effect)

mathematically well-defined, correct and consistent
relative correction for $\langle r_E^2\rangle = \int d^3r\, r^2 T_{00}(r)/m$ is $\delta_{\text{rel}} = 1/(2m^2R^2)$ Hudson, PS PRD (2007)
numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{{}^{4}\text{He}}_{5\times10^{-4}}$, $\underbrace{{}^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{{}^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{{}^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{{}^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{{}^{208}\text{Pb}}_{2\times10^{-8}}$

- important distinction:

  2D densities = partonic probability densities (unitarity)

  must be exact! $\rightarrow$ M. Burkardt (2000) is exact $\surd$

  apply to any particle including pion

  vs

  3D densities = mechanical response functions, *correlation functions*

  Subject to corrections (grain of salt, but okay since NOT probabilities!)

  can be studied for nucleon or heavier where corrections acceptably small $\surd$

- besides:

  no 2D interpretations exist for stress tensor T^{ij} (so far)

  $T^{ij} \neq$ diagonal in Fock space (ERBL region!) $\rightarrow$ Fock components interact!

  inherently 3D concepts, have to pay a prize (and pay attention to corrections)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{\mathrm{d}^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$

all formulae correct, interpretation in terms of 3D-densities has limitations (see above)

$$\int \mathrm{d}^3r\; T_{00}(\vec{r}) = M_N \qquad \text{known}$$

$$\int \mathrm{d}^3r\; \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r}, \vec{s}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5} M_N \int \mathrm{d}^3r \left(r^i r^j - \frac{r^2}{3}\delta^{ij}\right) T_{ij}(\vec{r}) \equiv \boldsymbol{D} \qquad \textbf{new!}$$

with: $$\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij}} \qquad \textbf{stress tensor}$$

$$\left.\begin{array}{l}\boldsymbol{s(r)} \text{ related to distribution of } \textit{shear forces} \\ \boldsymbol{p(r)} \text{ distribution of } \textit{pressure} \text{ inside hadron}\end{array}\right\} \longrightarrow \textbf{“mechanical properties”}$$

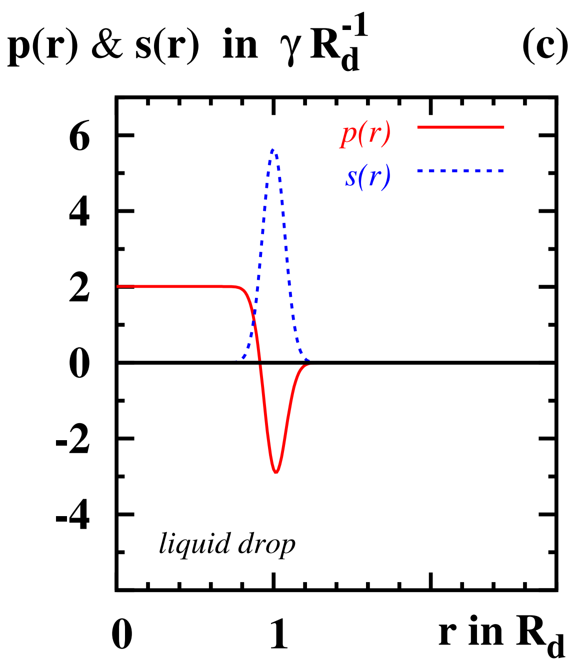
relation to stability: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15}\, m \int_0^\infty \mathrm{d}r\, r^4 s(r) = 4\pi m \int_0^\infty \mathrm{d}r\, \boldsymbol{r^4\, p(r)}$$ $\rightarrow$ shows how internal forces balance

let's gain intuition from models:

- **liquid drop model of nucleus**



radius $R_A = R_0\, A^{1/3}$, $m_A = m_0\, A$

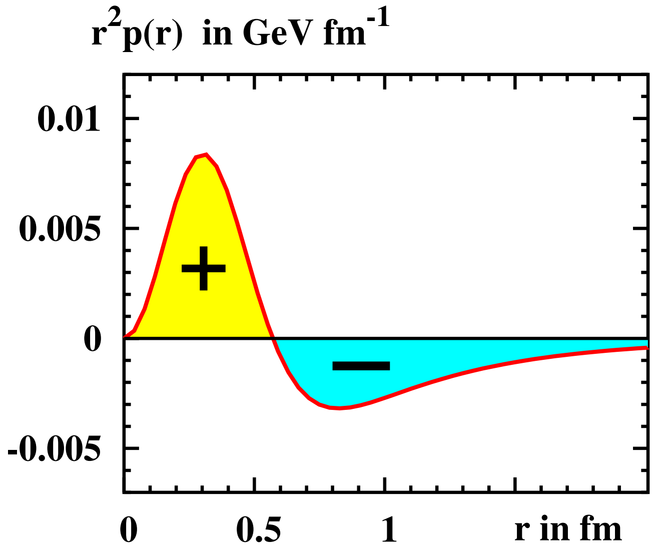
surface tension $\gamma = \frac{1}{2} p_0 R_A$, $s(r) = \gamma\, \delta(r - R_A)$

pressure $p(r) = p_0\, \Theta(R_A - r) - \frac{1}{3} p_0 R_A\, \delta(r - R_A)$

D-term $\boldsymbol{D = -\frac{4\pi}{3}\, m_A\, \gamma\, R_A^4 \approx -0.2\, A^{7/3}}$

M.V.Polyakov PLB555 (2003); confirmed in Walecka model by Guzey, Siddikov (2006) different result by Liuti, Taneja (2005) from a model of a non-rel. nuclear spectral function

- **chiral quark soliton model of nucleon**



- $p(0) = 0.23\,\mathrm{GeV/fm^3} \approx 4 \times 10^{34}\,\mathrm{N/m^2}$

  $\gtrsim$ 10-100×(pressure in center of neutron star)

- $p(r) = 0$ at $r = 0.57\,\mathrm{fm}$ change of sign in pressure

- $p(r) = \left(\dfrac{3g_A^2}{8\pi f_\pi}\right)^2 \dfrac{1}{r^6}$ at large r in chiral limit $m_\pi \to 0$

  Goeke et al, PRD75 (2007) 094021

- **How does it look like in nature?** Look in **Nature article** ☺

  see Burkert, Elouadrhiri, Girod Nature 557, 396 (2018)

  beware: additional assumptions!
  (early state of art, will be improved)

- **technical remark** on assumptions in Nature-article

JLab sensitive only to quarks! (also other experiments, see KM fits)

now one can define D-term $D^q = -\dfrac{16\pi}{15}\, m \displaystyle\int_0^\infty \mathrm{d}r\, r^4 s^q(r)$ for quarks, and analog D^g for gluons

"partial" (quark, gluon) contributions to shear forces <u>**can**</u> be defined

but pressure **only** defined for total (quark + gluon) system!

"partial" (quark, gluon) contributions to pressure <u>**cannot**</u> be obtained from $D^{q,g}(t)$

reason: $T_{ij}(\vec{r}) = s(r)\left(\dfrac{r_i r_j}{r^2} - \dfrac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}$ such that $\begin{cases} \text{shear forces} \propto \text{traceless part} \\ \text{pressure} \propto \text{trace of stress tensor} \end{cases}$

but remember: $\langle p'|\widehat{T}^q_{\mu\nu}|p\rangle = \bar{u}(p')\left[\cdots + \boldsymbol{D^q(t)}\,\dfrac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} \pm \bar{c}^q(t) g_{\mu\nu}\right]u(p)$

pressure requires $D^{q,g}(t)$ and $\bar{c}^{q,g}(t)$ M.Polyakov, H.-D. Son JHEP 1809 (2018) 156

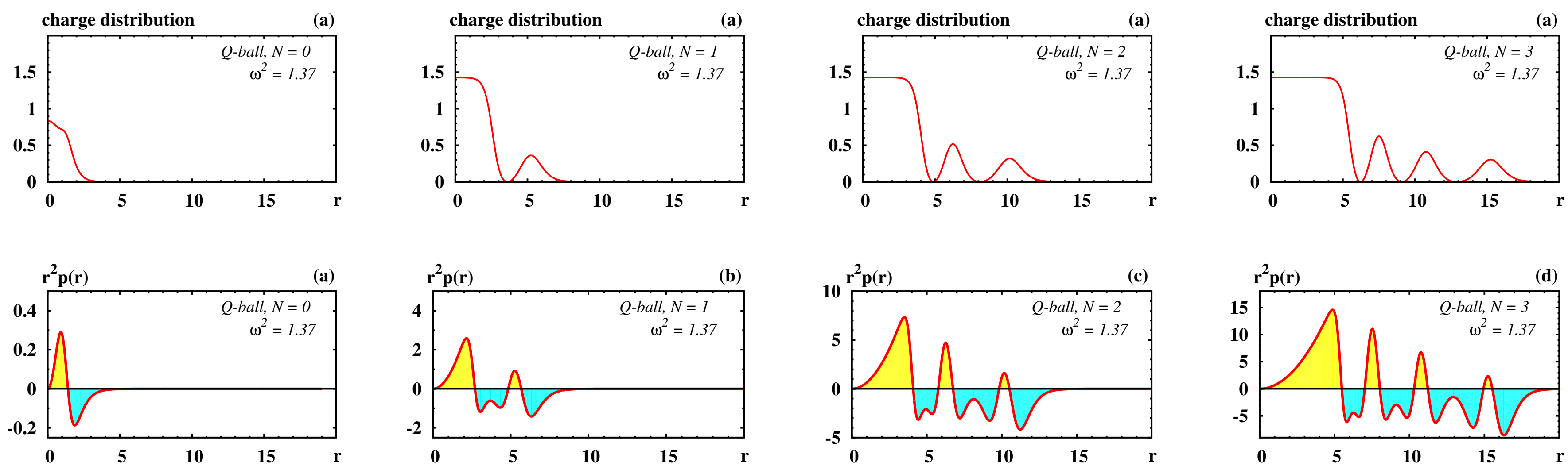
[for discussion of $\bar{c}^q(t)$ see M.Polyakov, H.-D.Son; Cédric Lorcé (2017), Keh-Fei Liu, Cédric Lorcé (2016)]

• more lessons from toy system: Q-ball

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$ with U(1) global symm., $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$, $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

$N = 0$ ground state, $N = 1$ first excited state, etc Volkov & Wohnert (2002), Mai, PS PRD86 (2012)

charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros



excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

so far **all D-terms** negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q-balls, Q-clouds

could perhaps the Roper resonance look like this? Or a "hallo nucleus"? (possible to measure??)

However e.g. Δ-resonance, similar to nucleon in model! Insights through transition form factors?

stress tensor and mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij} =$ symmetric 3×3 matrix
  $\rightarrow$ can be diagonalized with eigenvalues:
  $\frac{2}{3}\,s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)
  $-\frac{1}{3}\,s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$ degenerate for spin 0, $\frac{1}{2}$)

- mechanical stability $\Leftrightarrow$ normal force directed towards outside
  $\Leftrightarrow T^{ij} e_r^j\, dA = \underbrace{[\frac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\, dA \quad \Rightarrow \quad \boldsymbol{D < 0}$ crucial: positive definite density!
  recall $\langle r^2 \rangle_{el} < 0$ for neutron $\neq$ size of neutron!

- define: $\langle \boldsymbol{r^2} \rangle_{\mathbf{mech}} = \dfrac{\int d^3r\; r^2 [\frac{2}{3}\,s(r) + p(r)]}{\int d^3r\; [\frac{2}{3}\,s(r) + p(r)]} = \dfrac{6D(0)}{\int_{-\infty}^{0} \mathrm{d}t\, D(t)}$ vs $\langle r^2_{\text{ch}} \rangle = \dfrac{6\, G_E'(0)}{G_E(0)}$

  intuitive result for large nucleus $\frac{2}{3}\,s(r) + p(r) = p_0\, \Theta(R_A - r) \;\;\rightarrow\;\; \langle r^2 \rangle_{\text{mech}} = \frac{3}{5}\, R_A^2$
  M.Polyakov, PS arXiv:1801.05858 (Kumano, Song, Teryaev PRD (2018) used $D'(0)$ but inadequate)

- proton: $\langle r^2 \rangle_{\text{mech}} \approx 0.75\, \langle r^2_{\text{ch}} \rangle$ for $m_\pi = 140\,\text{MeV}$ (chiral quark soliton model)
  Notice: in chiral limit $\langle r^2 \rangle_{\text{mech}}$ finite vs $\langle r^2_{\text{ch}} \rangle$ which diverges

more on normal/tangential forces in future from Trawinski, Lorcé, Moutarde (talk Lightcone 2018)

- **Application I: investigating forces**

  prominent property of proton:
  life time $\tau_{\rm prot} > 2.1 \times 10^{29}$ years!

  question: how do strong forces
  balance to produce stability?

- answer in **model:** strong cancellation of
  repulsive forces due to quark core, and
  attractive forces from pion cloud

- answer in **QCD:** we do not know
  nice pictures, attractive insights
  underexplored propaganda(?)

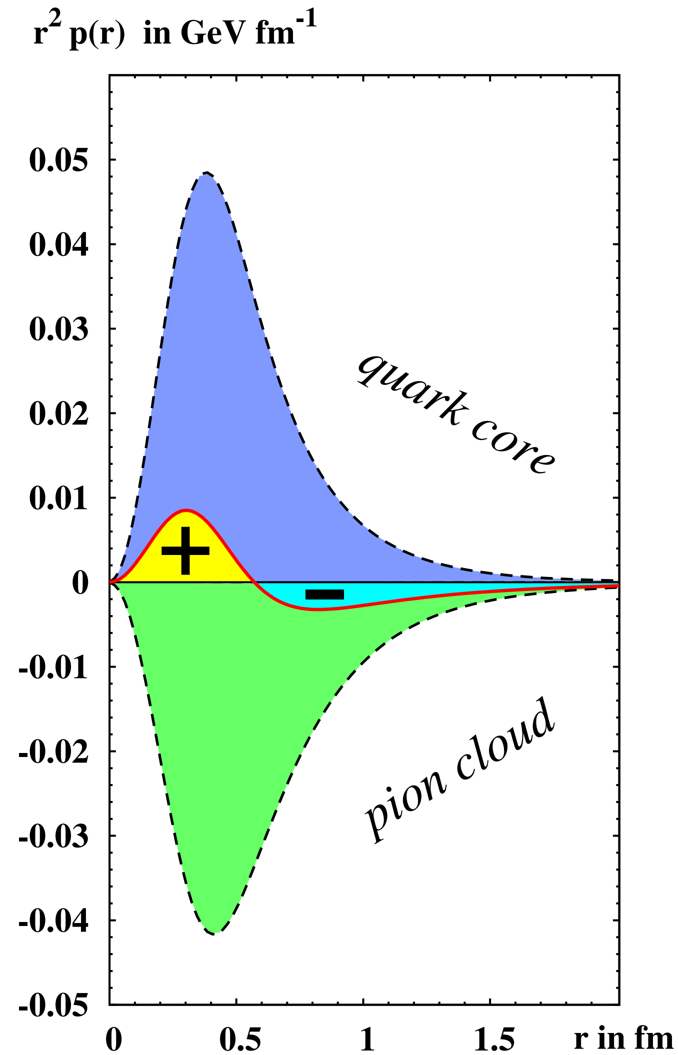
  be aware: same for neutron,
  $\tau_{\rm neut} = 14\,\text{min}\ 40\,\text{sec} \gg 10^{-23}\,\text{sec}$
  and even the same picture for Δ ...
  $\tau_\Delta \sim 10^{-23}\,\text{sec} \to$ necessary condition!

- as mental support for GPD program: okay

  ... but is there any practical use of that?

  answer before: *not really ...*

  answer today: *Yes!*



in chiral quark soliton model
chiral symmtry breaking ✓
realization of QCD in large-N_c ✓
built on instanton vacuum calculus ✓
not bad, but after all a model ...
Goeke et al, PRD75 (2007)

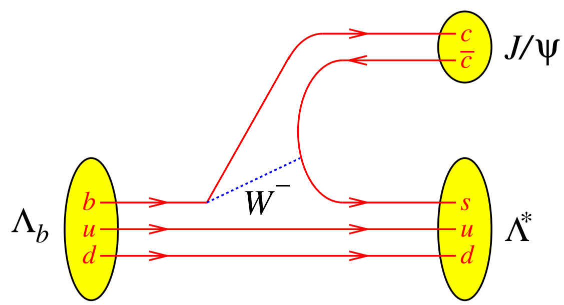
Application II: hidden-charm pentaquarks as hadrocharmonia

$\Lambda_b^0 \longrightarrow J/\Psi\, p\, K^-$ seen

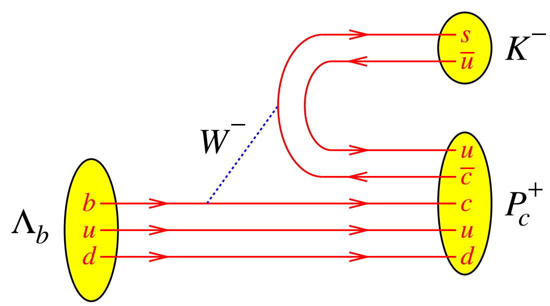
Aaij *et al.* PRL 115 (2015)

Λ_b^0 $m = 5.6$ GeV, $\tau = 1.5$ ps
J/Ψ $m = 3.1$ GeV, $\Gamma = 93$ keV, $\Gamma_{\mu^+\mu^-} = 6\,\%$
Λ^* $m = 1.4$ GeV or more, $\Lambda^* \to K^- p$ in 10^{-23}s

$\longrightarrow J/\Psi\, \Lambda^*$



$\longrightarrow K^-\, P_c^+$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\,\%$	$J/\psi\, p$	$\frac{3}{2}^{\mp}$ or $\frac{5}{2}^{+}$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\,\%$	$J/\psi\, p$	$\frac{5}{2}^{\pm}$ or $\frac{3}{2}^{-}$

appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- **theoretical approach**

  $R_{c\bar{c}} \ll R_N \;\Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
  baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

  $V_{\text{eff}} = -\frac{1}{2}\,\alpha\,\vec{\boldsymbol{E}}^2$ Voloshin, Yad. Fiz. **36**, 247 (1982)

- **chromoelectric polarizability**

$$\alpha(1S) \approx \quad 0.2\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S) \approx \quad 12\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S \to 1S) \approx \begin{cases} -0.6\,\text{GeV}^{-3}\ \text{(pert)}, \\ \pm 2\,\text{GeV}^{-3}\ \text{(pheno)}, \end{cases}$$

in heavy quark mass limit & large-N_c limit
$\rightsquigarrow$ "perturbative result" Peskin, NPB 156 (1979) 365

value for $2S \to 1S$ transition from
phenomenological analysis of $\psi' \to J/\psi\,\pi\,\pi$ data
Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- **chromoelectric field strength:**

$$\vec{\boldsymbol{E}}^2 = g^2 \left(\frac{8\pi^2}{bg^2}\, T^{\mu}{}_{\mu} + T_{00}^{G} \right)$$

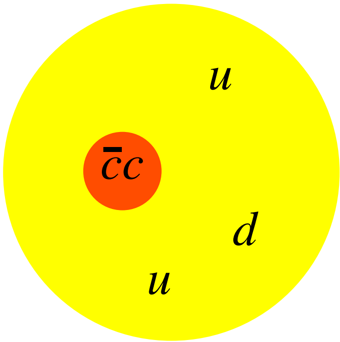
$b = \frac{11}{3}\,N_c - \frac{2}{3}\,N_F$ leading coeff. of β-function
$g =$ strong coupling at low (nucleon) scale $\lesssim 1$ GeV
$g_s =$ strong coupling at scale of heavy quark ($g_s \neq g$)
$T_{00}^{G} = \xi T_{00}$ with $\xi =$ fractional contributions of gluon to M_N
$T^{\mu}{}_{\mu} = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r}\frac{r^j}{r} - \frac{1}{3}\,\delta^{ij} \right) s(r) + \delta^{ij}\,p(r)$

- **universal effective potential**

$$\boldsymbol{V_{\text{eff}} = -\tfrac{1}{2}\,\alpha\,\frac{8\pi^2}{b}\,\frac{g^2}{g_s^2}\Big[\nu\,T_{00}(r) + 3p(r)\Big]\,, \quad \nu = 1 + \xi_s\,\frac{b\,g_s^2}{8\pi^2}}$$

$\boldsymbol{\nu \approx 1.5}$ estimate by Eides et al, op. cit.
Novikov & Shifman, Z.Phys.C8, 43 (1981);
X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)

- **compute quarkonium-nucleon bound state**



solve $\left(-\frac{\vec{\nabla}^2}{2\mu}+V_{\text{eff}}(r)\right)\psi = E_{\text{bind}}\,\psi$

$\mu =$ reduced quarkonium-baryon mass

V_{eff} from EMT of chiral quark soliton model (Eides et al, 2015); Skyrme (Perevalova et al 2016)

- **results:**

no J/ψ-nucleon bound state! Supported by lattice data on J/ψ-N potential! Sugiura et al, 1711.11219

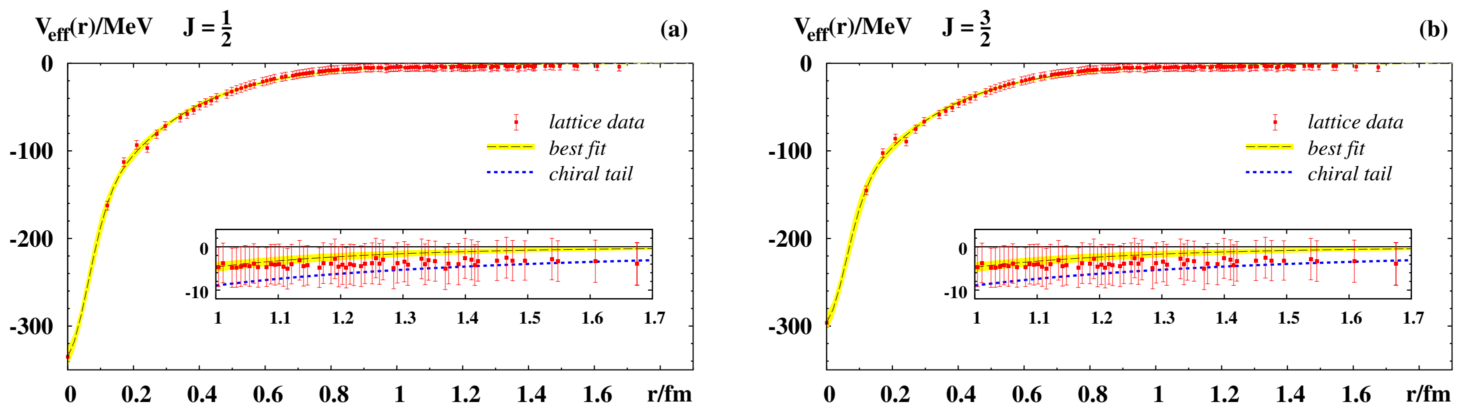
$\psi(2S)$-nucleon bound states if $\alpha(2S) \approx 17$ GeV^{-3}, $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ around 4450 MeV
with narrow width $\Gamma = |\alpha(2S \to 1S)|^2 \times \cdots =$ few tens MeV, mass-splitting $\mathcal{O}(20)$ MeV
supported by lattice data, Sugiura et al; Polyakov, PS, PRD (2018)

- **test approch:** predicted bound states of $\psi(2S)$ with Δ and hyperons!

Perevalova et al 2016, Eides et al, 2017 waiting for test at LHCb, JLab (Meziani et al, XXX), EIC (!!)

Application III: extract chromoelectric polarizabilities

- J/ψ-nucleon potential studied on lattice Sugiura et al, 1711.11219



- actually no model for EMT densities needed: $\int d^3r\, V_{\text{eff}}(r) = -\,\alpha(1S)\,\dfrac{4\pi^2}{b}\,\dfrac{g_s^2}{g_c^2}\,\nu\, M_N$

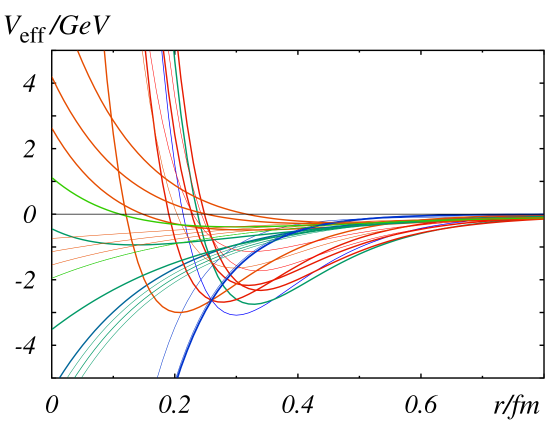
  $b = (\frac{11}{3}N_c - \frac{2}{3}N_f)$ leading coefficient of Gell-Mann–Low function
  $1 \lesssim g_s^2/g_c^2 \lesssim 1.7$ with g_c (g_s) is strong coupling at scale of charmonium (nucleon)
  $\nu = 1 + \xi_s \frac{b\,g_s^2}{8\pi^2} \approx 1.5 \pm 0.1$ with ξ_s fraction of nucleon momentum carried by gluons at μ_s

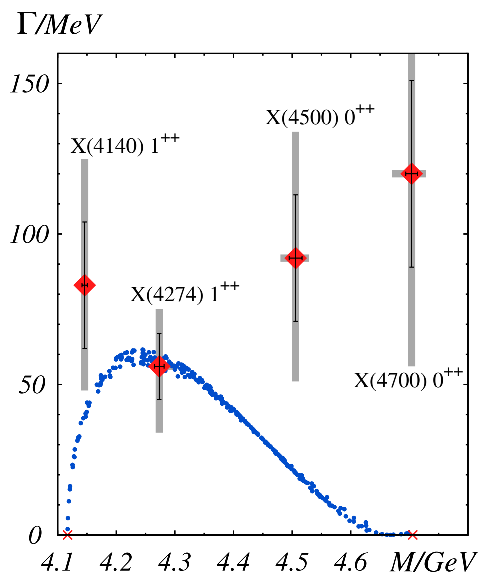
- non-perturbative method: $\boldsymbol{\alpha(1S) = (1.5 \pm 0.6)\mathrm{GeV}^{-3}}$ ("proof of principle")
  uncertainities (estimated + unestimated due to lattice systematic uncertainty)
  if $\alpha(2S)/\alpha(1) \approx 15$ lattice potential admits $\psi(2S)$-nucleon bound states, compatible!
  $1/m_Q$ corrections not large (small mass splitting of predicted $J^P = \frac{1}{2}^-,\ \frac{3}{2}^-$ pentaquarks)
  Polyakov, PS, PRD98 (2018) 034030

- applications: hadrocharmonia, hadronic decays of $\bar{c}c$, photo/hadro-production of $\bar{c}c$ and charmed hadrons on nuclear targets, diagnostics of quark gluon plasma in heavy-ion coll.

Application IV: hidden-charm hidden-strangeness tetraquarks

- decay in J/ψ and ϕ, $J^{PC} = 0^{++}$, 1^{++} Aaij et al PRD95 (2017) 012002
- $X(4140)$ possibly a rescattering effect Swanson, Int.J.Mod.Phys.E 25 (2016) 1642010
- $X(4500)$, $X(4700)$ hadronic molecules of D- or D^* mesons, bound states in diquark picture
  Karliner, Rosner (2016), Ding (2009), Branz et al (2009), Drenska et al (2009), Anisovich et al (2015)
- $X(4274)$ cannot be a molecular state, but is candidate for ϕ-$\psi(2S)$ hadrocharmonium!
- what do we know about EMT of ϕ-meson??? Nothing!!! Wide assumptions e.g.
  $A(t) = 1/(1 - t/M_1^2)^2$, $D(t) = D/(1 - t/M_2^2)^3$, $r_E^2 = 12/M_1^2$, $r^2_{\text{mech}} = 12/M_2^2$
  $0.05\ \text{fm}^2 < \langle r^2\rangle_{E,\text{mech}} < 1\ \text{fm}^2$
  and $-15 < D < 0$

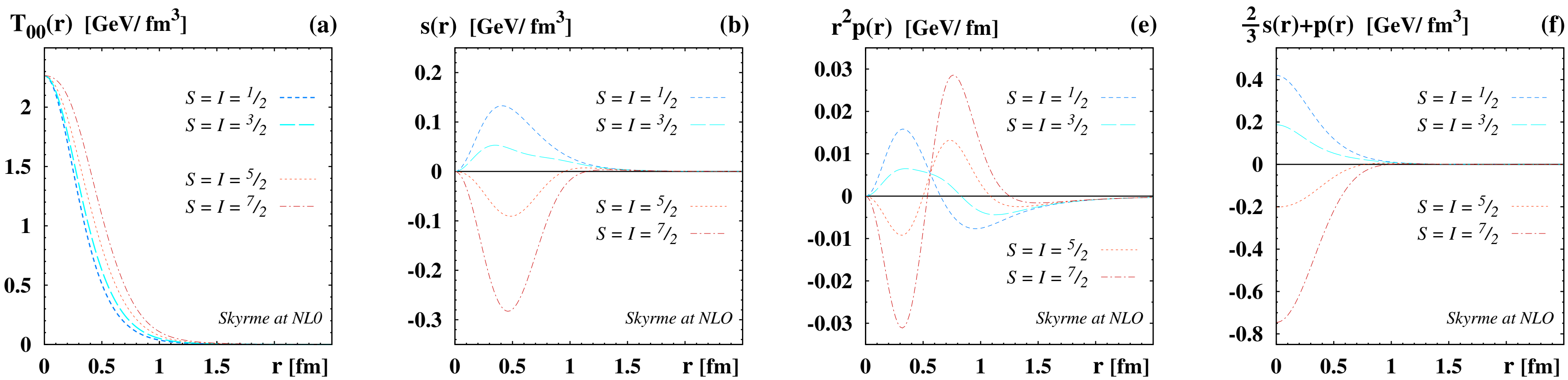




conclusion:
$X(4274)$ may be ϕ-$\psi(2S)$ bound state!
If this is was the case: we have insight on EMT of ϕ: $\langle r^2\rangle_E \in [0.1, 0.6]\,\text{fm}^2$, $\langle r^2\rangle_{\text{mech}} \in [0.1, 0.5]\,\text{fm}^2$, $D \in [-5, 0]$
(smaller radii $\leftrightarrow$ larger $|D|$)

Application V: nucleon, Δ, large-N_c artifacts   Witten 1979

in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2},}_{\text{observed}} \underbrace{\frac{5}{2}, \dots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

stability needs more:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \ \frac{2}{3}s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ **have positive D-term!!**
That's why they do not exist!
EMT: dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Summary & Outlook

- **GPDs, GDAs** $\rightarrow$ form factors of **energy momentum tensor**
  mass decomposition, spin decomposition, and D-term!

- **D-term**: last unknown global property, related to forces
  attractive and physically appealling $\rightarrow$ "motivation"

- **first results**(!) from experiment/phenomenology for proton, π^0
  compatible with results from theory and models (see review arXiv:1805.06596)

- define **pressure & mechanical radius** $\rightarrow$ complementary information!

- development: apply to **hadrocharmonia** pentaquarks & tetraquarks
  rich potential, new predictions, ongoing work

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Thank you!