Probing Nucleons and Nuclei in High Energy Collisions
INT - October 8th, 2018

Measurements of transverse momentum distributions in semi-inclusive DIS

- from a mainly European perspective -

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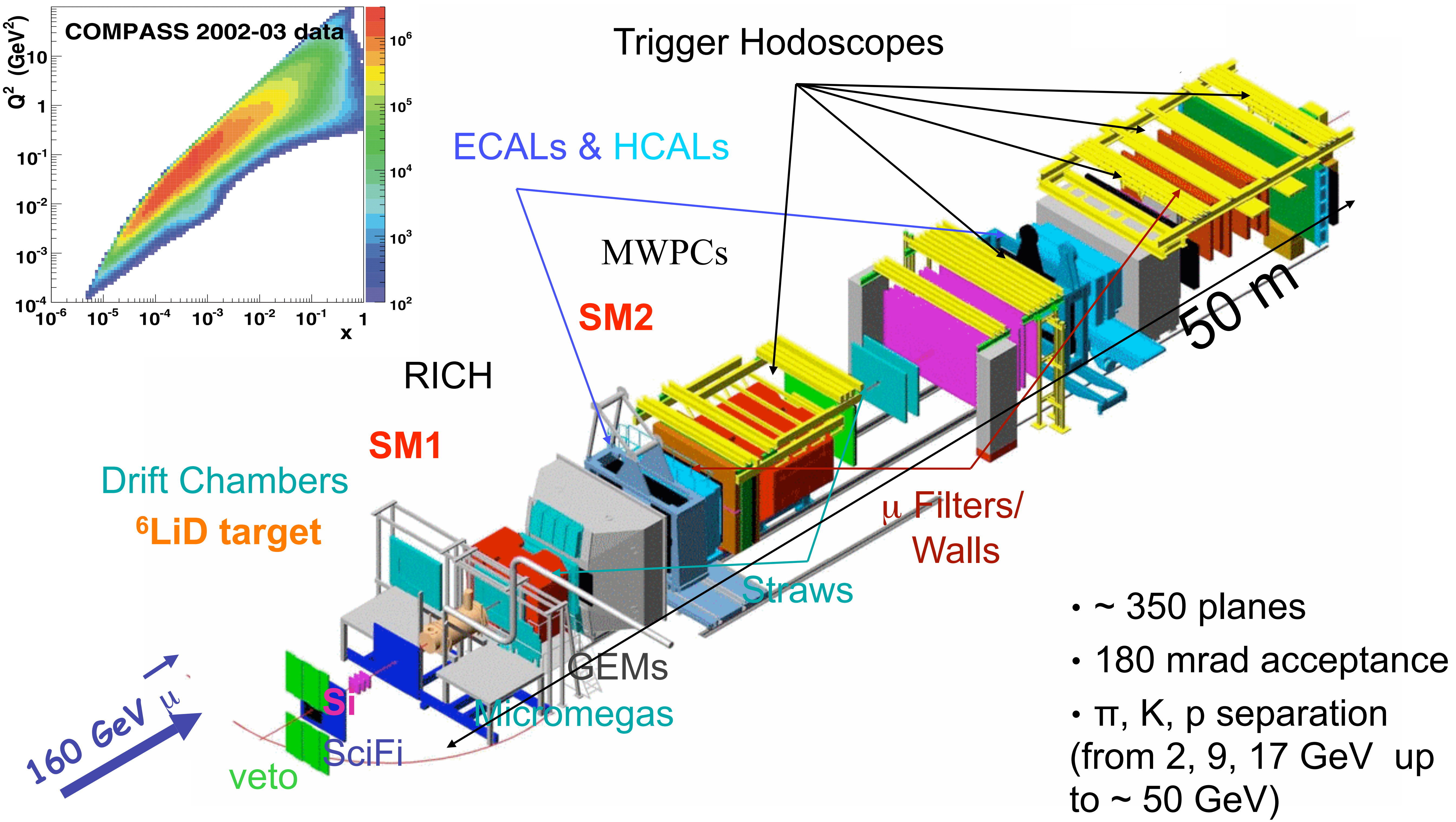
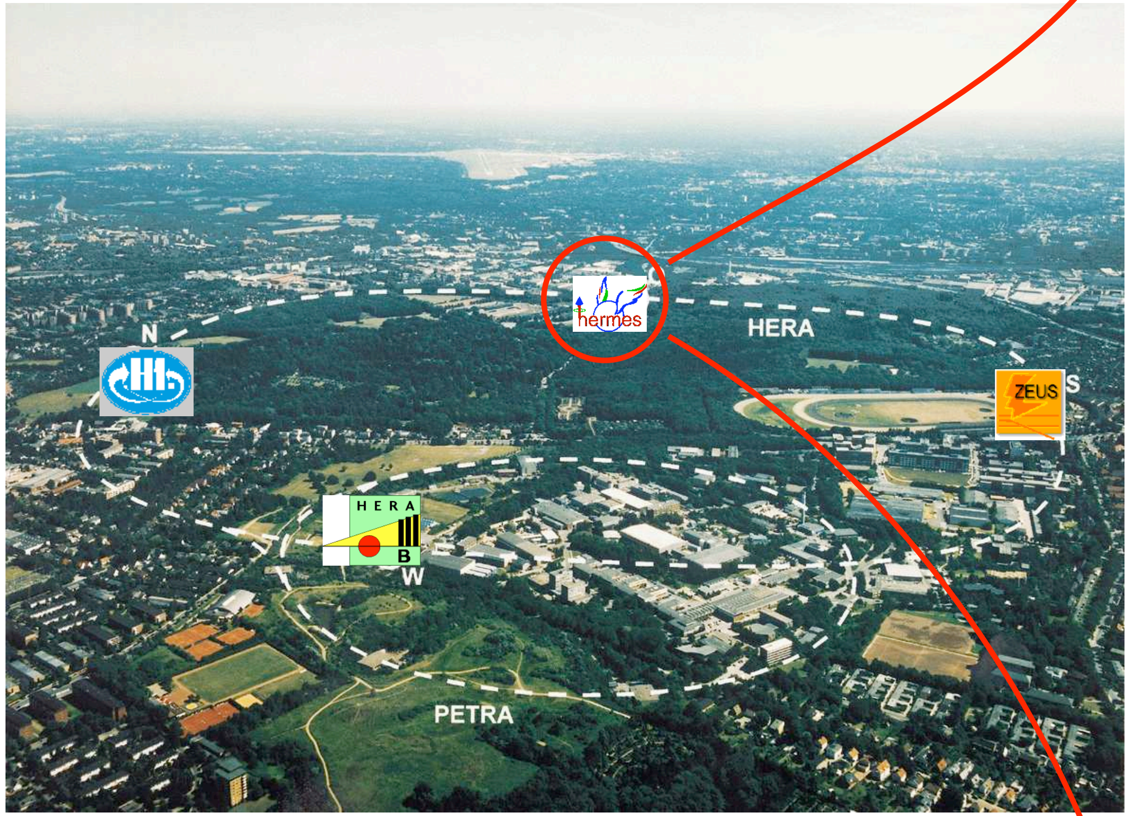
The COMPASS experiment @ CERN

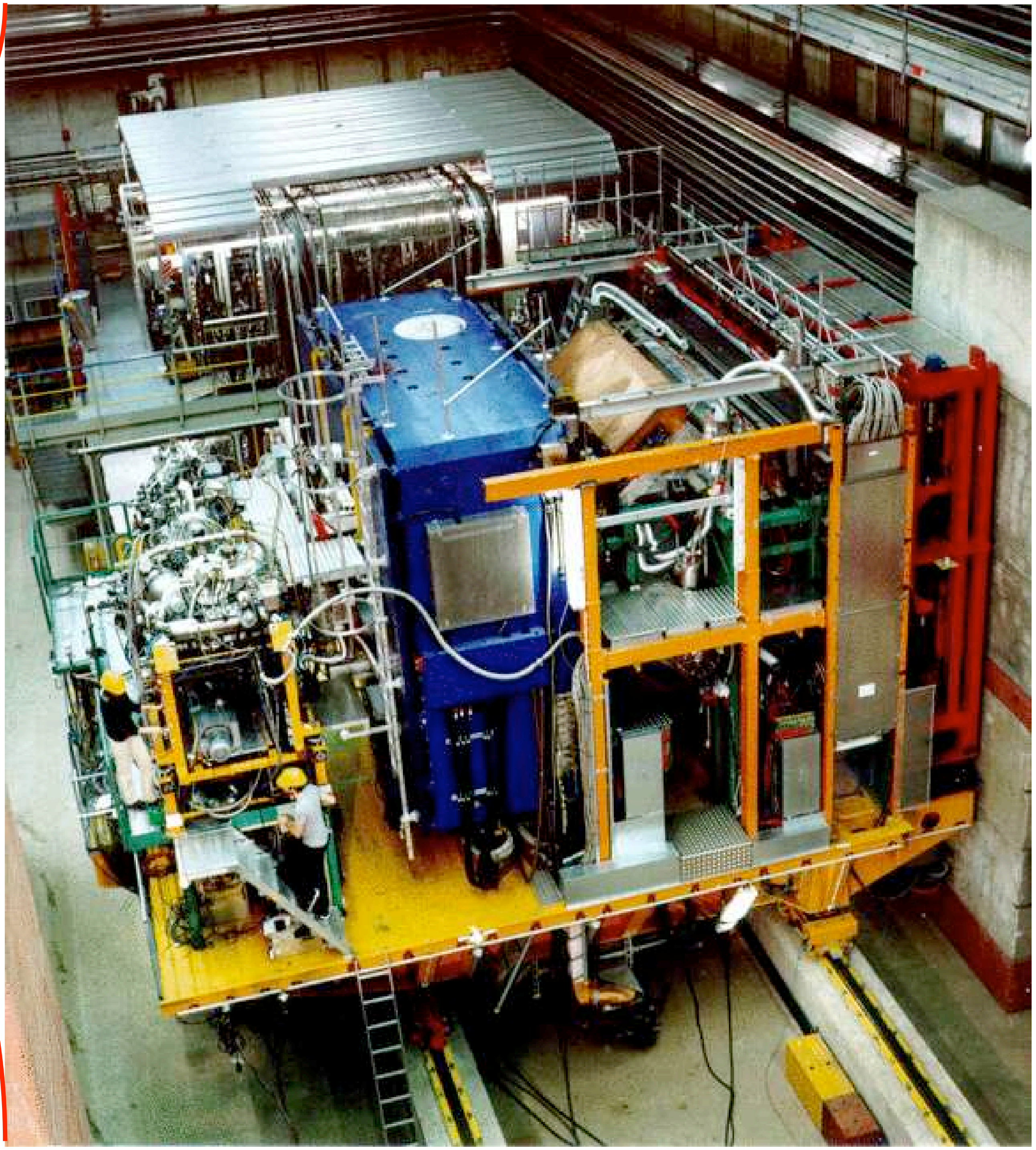


- ~ 350 planes
- 180 mrad acceptance
- π, K, p separation (from 2, 9, 17 GeV up to ~ 50 GeV)

HERMES Experiment (†2007) @ DESY

27.6 GeV polarized e$^+$/e$^-$ beam scattered off ...





- unpolarized (H, D, He,..., Xe)
- as well as **transversely (H)** and longitudinally (H, D, He) polarized (pure) gas targets

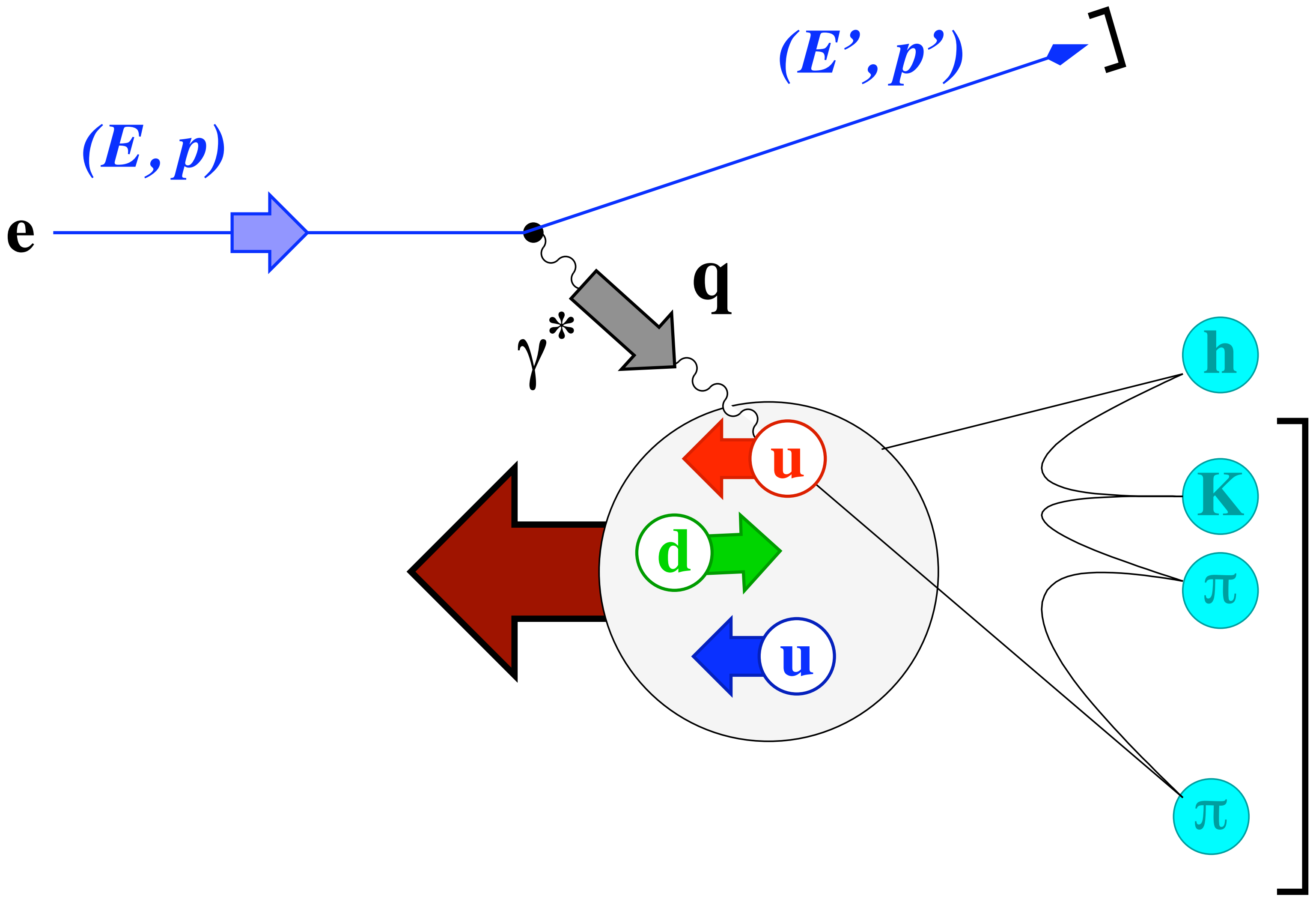
getting polarized nucleons

- common polarized targets
  - gas targets -> pure, but lower density
  - solid (e.g. NH_3) targets -> high density, but large dilution

getting polarized nucleons

- common polarized targets
  - gas targets -> pure, but lower density
  - solid (e.g. NH_3) targets -> high density, but large dilution
- statistical precision: $\sim \frac{1}{f P_B P_T} \frac{1}{\sqrt{N}}$ (f… dilution factor)
  - solid targets f≈0.2 -> directly scales uncertainties (as do P_B & P_T)
  - dilution also kinematics dependent (partially unknown systematics)

Semi-inclusive DIS



Spin-momentum structure of the nucleon

$$\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ + \lambda\gamma^+\gamma_5)\,\Phi\Big] = \frac{1}{2}\left[f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^\perp + \lambda\Lambda\,g_1 + \lambda\,S^ik^i\,\frac{1}{m}\,g_{1T}\right]$$

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ - s^j i\sigma^{+j}\gamma_5)\,\Phi\Big] = \frac{1}{2}\Bigg[& f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^\perp + s^i\epsilon^{ij}k^j\,\frac{1}{m}\,h_1^\perp + s^iS^i h_1 \\ & + s^i(2k^ik^j - \boldsymbol{k}^2\delta^{ij})S^j\,\frac{1}{2m^2}\,h_{1T}^\perp + \Lambda\,s^ik^i\,\frac{1}{m}\,h_{1L}^\perp\Bigg]\end{aligned}$$

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

- each TMD describes a particular spin-momentum correlation
- functions in black survive integration over transverse momentum
- functions in green box are chirally odd
- functions in red are naive T-odd

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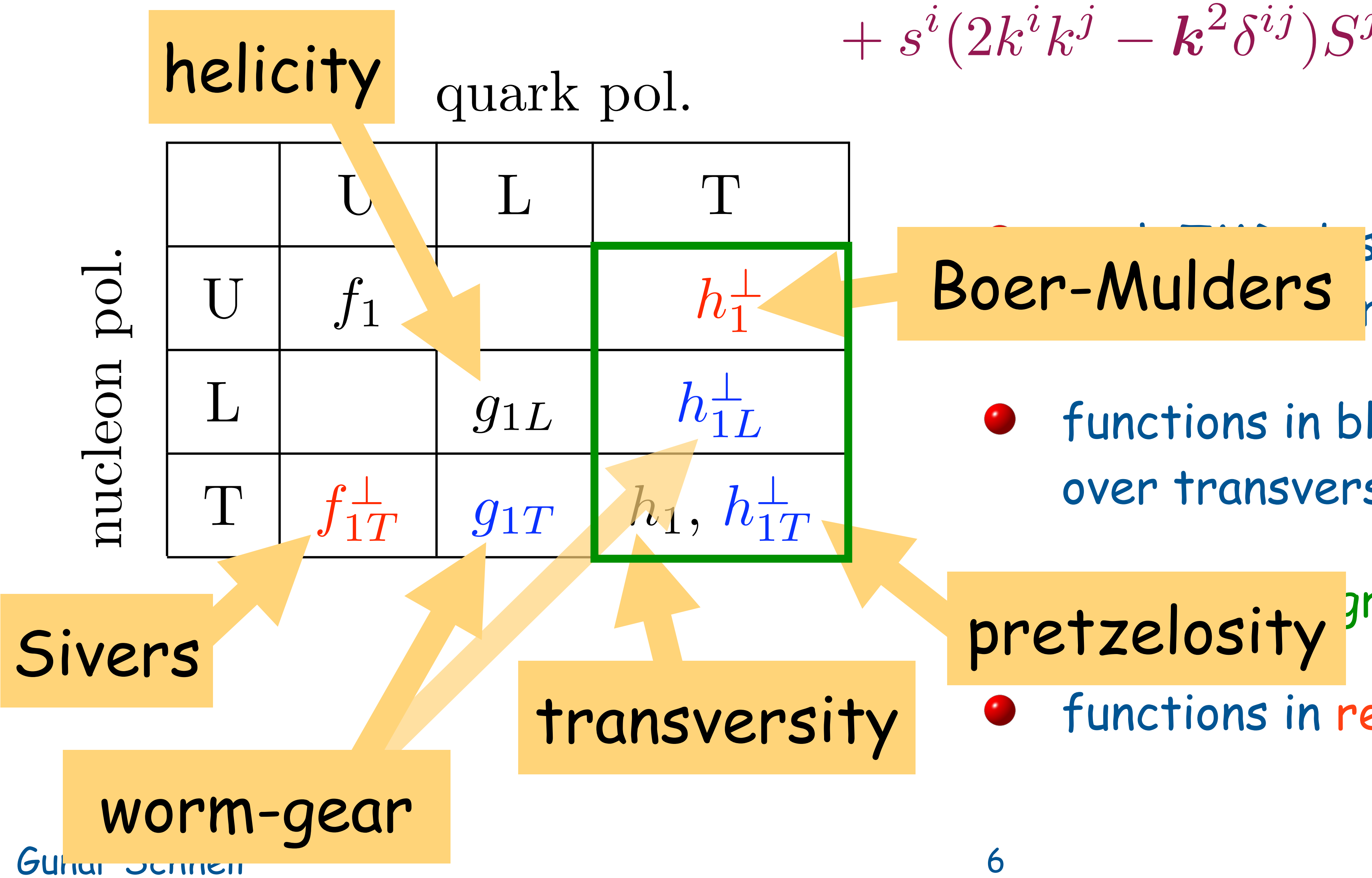

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TMDs in hadronization

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hadron pol.

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☛ R. Seidl, A. Vossen

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☛ relevant for unpolarized final state

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TMDs in hadronization

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☛ relevant for unpolarized final state

Collins FF: $H_1^{\perp,q\to h}$

ordinary FF: $D_1^{q\to h}$

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TMDs in hadronization

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hadron pol.

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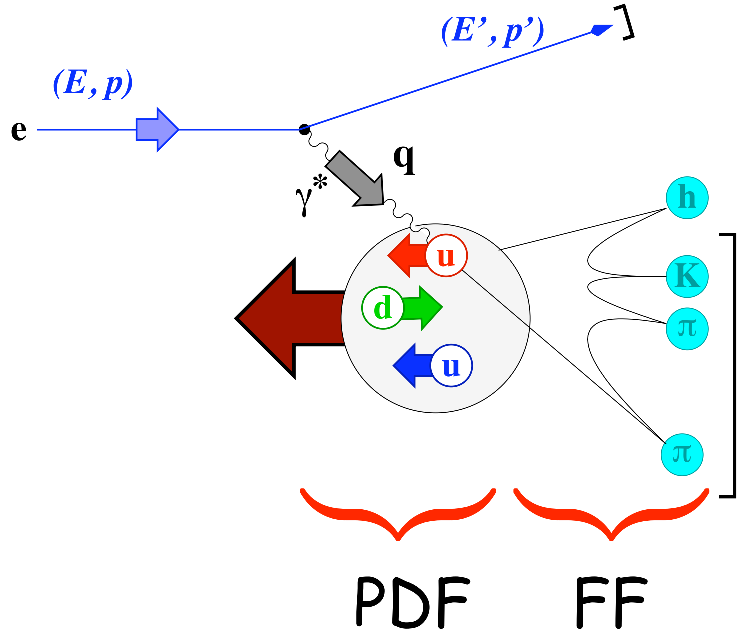
☛ relevant for unpolarized final state

polarized final-state hadrons

☛ R. Seidl, A. Vossen

Probing TMDs in semi-inclusive DIS



quark pol.

nucleon pol.	U	L	T
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T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

in SIDIS*) couple PDFs to:

Collins FF: $H_1^{\perp,q\to h}$

ordinary FF: $D_1^{q\to h}$

➟ give rise to characteristic azimuthal dependences

*) semi-inclusive DIS with unpolarized final state

one-hadron production (ep→ehX)

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi \, d\sigma^1_{UU} + \frac{1}{Q}\cos\phi \, d\sigma^2_{UU} + \lambda_e \frac{1}{Q}\sin\phi \, d\sigma^3_{LU}$$

$$+S_L \left\{ \sin 2\phi \, d\sigma^4_{UL} + \frac{1}{Q}\sin\phi \, d\sigma^5_{UL} + \lambda_e \left[d\sigma^6_{LL} + \frac{1}{Q}\cos\phi \, d\sigma^7_{LL} \right] \right\}$$

$$+S_T \left\{ \sin(\phi - \phi_S) \, d\sigma^8_{UT} + \sin(\phi + \phi_S) \, d\sigma^9_{UT} + \sin(3\phi - \phi_S) \, d\sigma^{10}_{UT} \frac{1}{Q} \right.$$

$$+\frac{1}{Q}\left(\sin(2\phi - \phi_S) \, d\sigma^{11}_{UT} + \sin\phi_S \, d\sigma^{12}_{UT} \right)$$

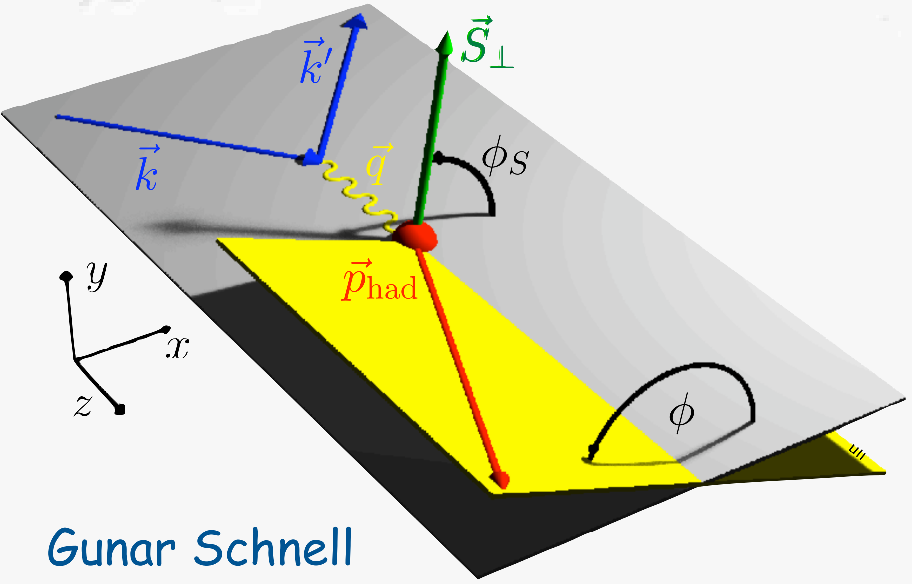
$$\left. +\lambda_e \left[\cos(\phi - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q}\left(\cos\phi_S \, d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \, d\sigma^{15}_{LT} \right) \right] \right\}$$

σ_{XY}

$\swarrow \quad \searrow$

Beam Target

Polarization



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

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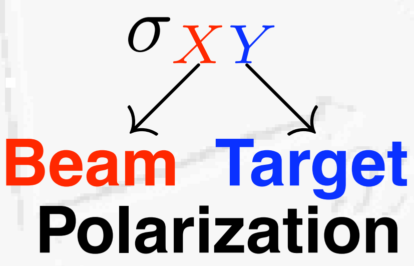

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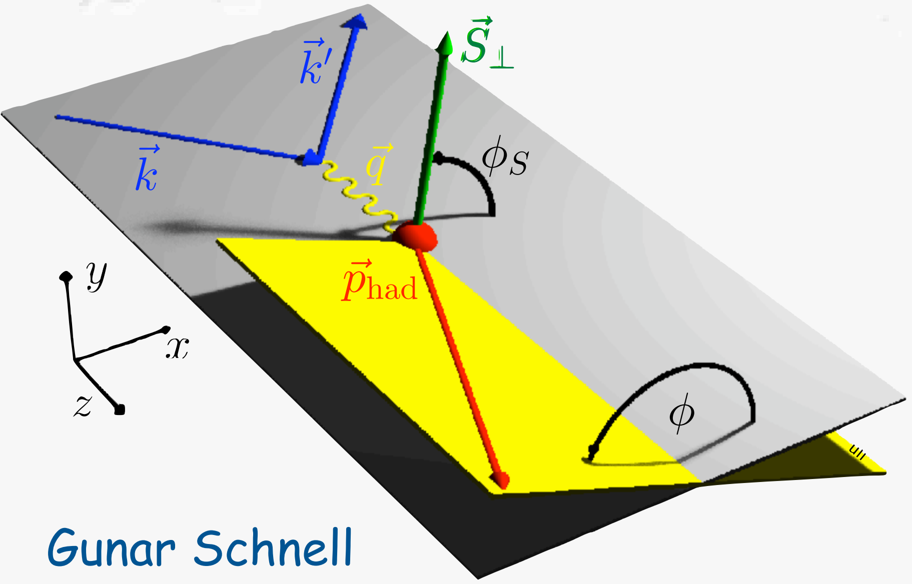
Beam **Target**

Polarization



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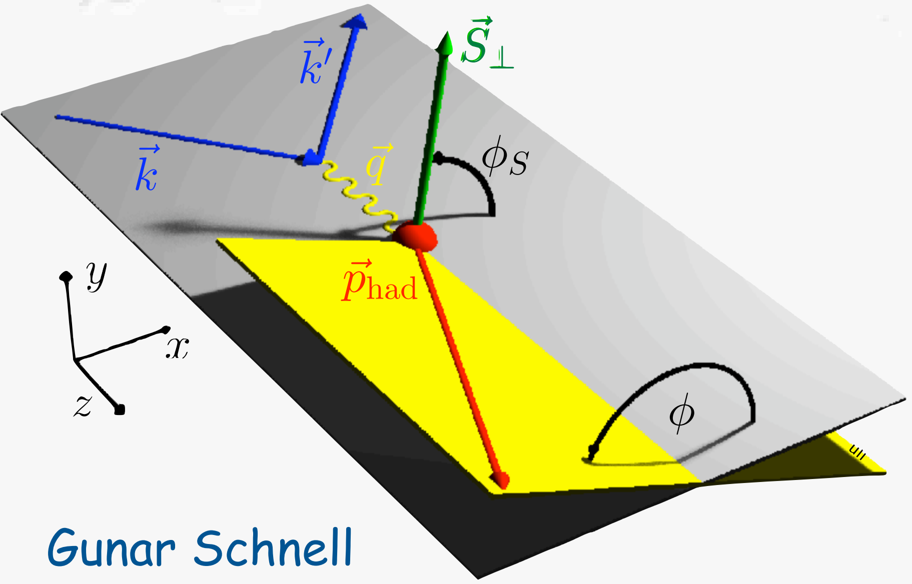

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… possible measurements

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\{F_{UU,T}+\epsilon F_{UU,L}$$

$$+\sqrt{2\epsilon(1-\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\}$$

… possible measurements

hadron multiplicity:

normalize to inclusive DIS cross section

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moments:
normalize to azimuth-independent cross-section

… possible measurements

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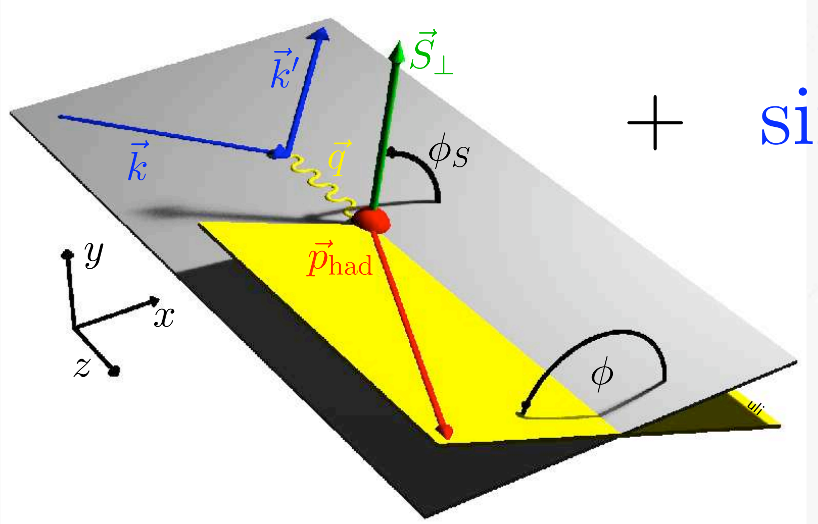
$$\approx \epsilon\frac{\sum_q e_q^2\ h_1^{\perp,q}(x,p_T^2)\otimes_{\text{BM}} H_1^{\perp,q\to h}(z,K_T^2)}{\sum_q e_q^2\ f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}$$

… azimuthal spin asymmetries

$$A_{UT}(\phi,\phi_S) = \frac{1}{\langle|S_\perp|\rangle}\frac{N_h^\uparrow(\phi,\phi_S)-N_h^\downarrow(\phi,\phi_S)}{N_h^\uparrow(\phi,\phi_S)+N_h^\downarrow(\phi,\phi_S)}$$

$$\sim \sin(\phi+\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{k_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M_h}h_1^q(x,p_T^2)H_1^{\perp,q}(z,k_T^2)\right]$$



$$+ \sin(\phi-\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{p_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M}f_{1T}^{\perp,q}(x,p_T^2)D_1^q(z,k_T^2)\right]$$

$$+ \quad \cdots$$

$\mathcal{I}[\ldots]$: convolution integral over initial (p_T) and final (k_T) quark transverse momenta

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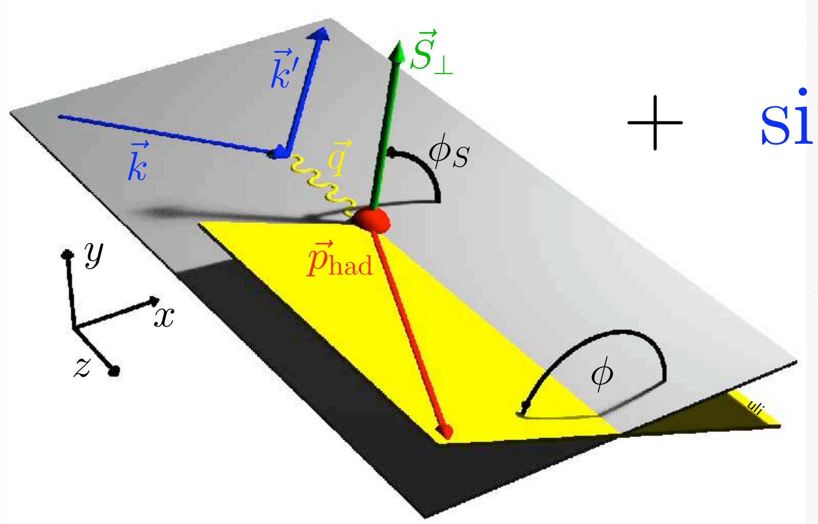

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$$+ \quad \cdots$$

$\mathcal{I}[\ldots]$: **convolution integral over initial (p_T) and final (k_T) quark transverse momenta**

fit azimuthal modulations, e.g., using maximum-likelihood method

$$\boxed{PDF(2\langle\sin(\phi\pm\phi_S)\rangle_{UT},\ldots,\phi,\phi_S) = \tfrac{1}{2}\{1+P_T(2\langle\sin(\phi\pm\phi_S)\rangle_{UT}\sin(\phi\pm\phi_s)+\ldots)\}}$$

“Qual der Wahl”

- SIDIS structure functions come with various kinematic prefactors
  - include in definition of asymmetries (“cross-section asym.”)

    $$\text{M.L. pdf } \propto [1 + \mathcal{A}^{\sin(\phi+\phi_s)}(x, y, z, P_{h\perp}) + \ldots]$$

  - factor out from asymmetries (“structure-fct. asym.”)

    $$\text{M.L. pdf } \propto [1 + D(y)A^{\sin(\phi+\phi_s)}(x, y, z, P_{h\perp}) + \ldots]$$

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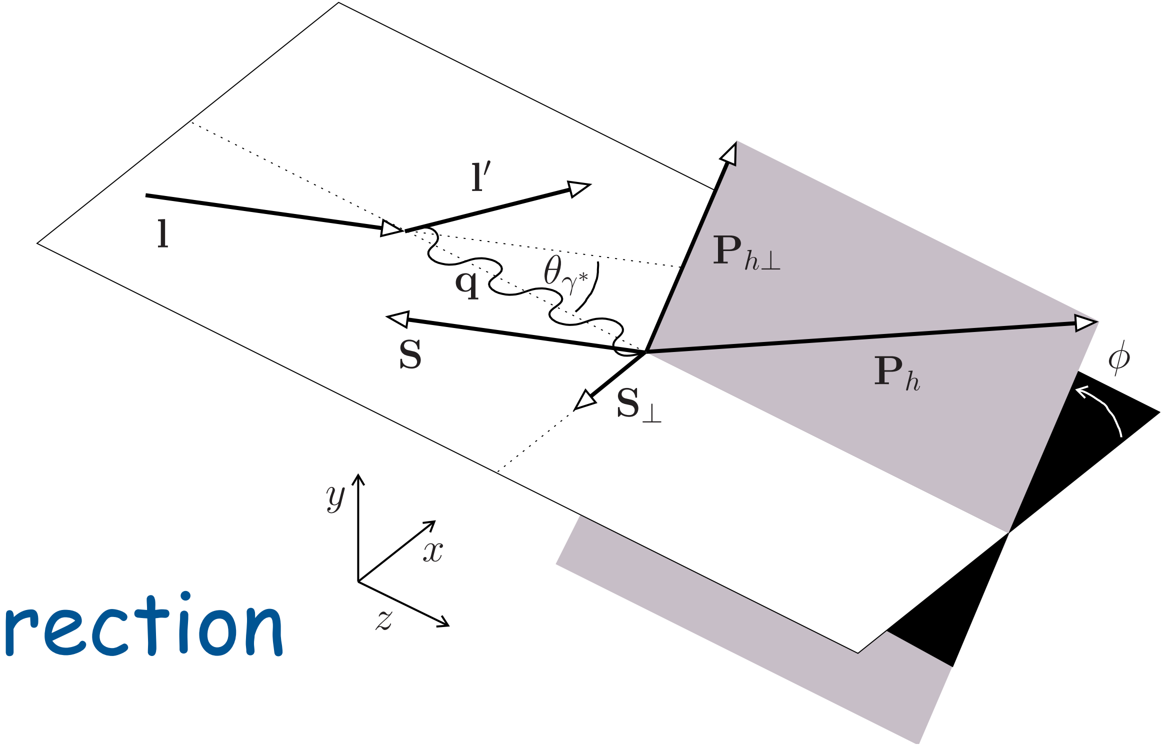
    $$\text{M.L. pdf} \;\propto [1 + D(y)A^{\sin(\phi+\phi_s)}(x, y, z, P_{h\perp}) + \ldots]$$

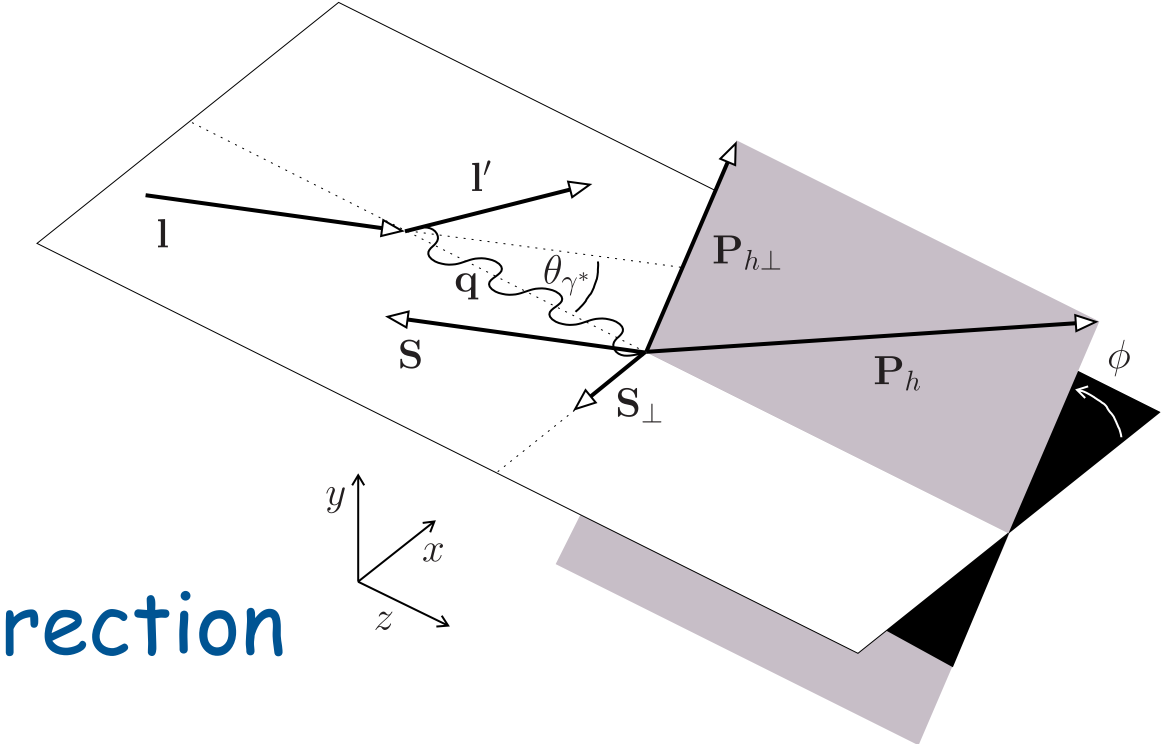
- latter facilitates comparisons between experiments and simplifies kinematic dependences by removing known dependences
  - but what about twist suppression, also factor out?
  - and what about other kinematically suppressed contributions?

… other complications



- theory done w.r.t. virtual-photon direction
- experiments use targets polarized w.r.t. lepton-beam direction

… other complications



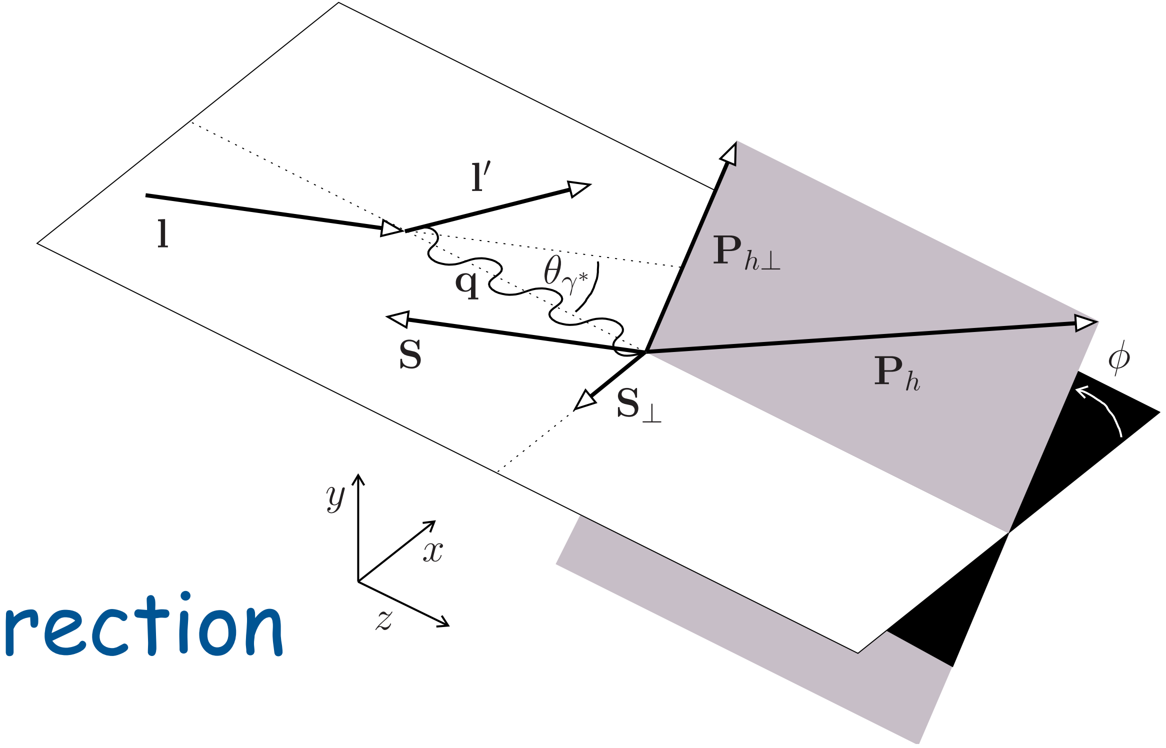
- theory done w.r.t. virtual-photon direction
- experiments use targets polarized w.r.t. lepton-beam direction

➡ mixing of longitudinal and transverse polarization effects [Diehl & Sapeta, EPJ C 41 (2005) 515], e.g.,

$$\begin{pmatrix} \left\langle \sin\phi \right\rangle_{UL}^{l} \\ \left\langle \sin(\phi-\phi_S) \right\rangle_{UT}^{l} \\ \left\langle \sin(\phi+\phi_S) \right\rangle_{UT}^{l} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\gamma^*} & -\sin\theta_{\gamma^*} & -\sin\theta_{\gamma^*} \\ \frac{1}{2}\sin\theta_{\gamma^*} & \cos\theta_{\gamma^*} & 0 \\ \frac{1}{2}\sin\theta_{\gamma^*} & 0 & \cos\theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \left\langle \sin\phi \right\rangle_{UL}^{q} \\ \left\langle \sin(\phi-\phi_S) \right\rangle_{UT} \\ \left\langle \sin(\phi+\phi_S) \right\rangle_{UT} \end{pmatrix}$$

($\cos\theta_{\gamma^*} \simeq 1$, $\sin\theta_{\gamma^*}$ up to 15% at HERMES energies)

… other complications



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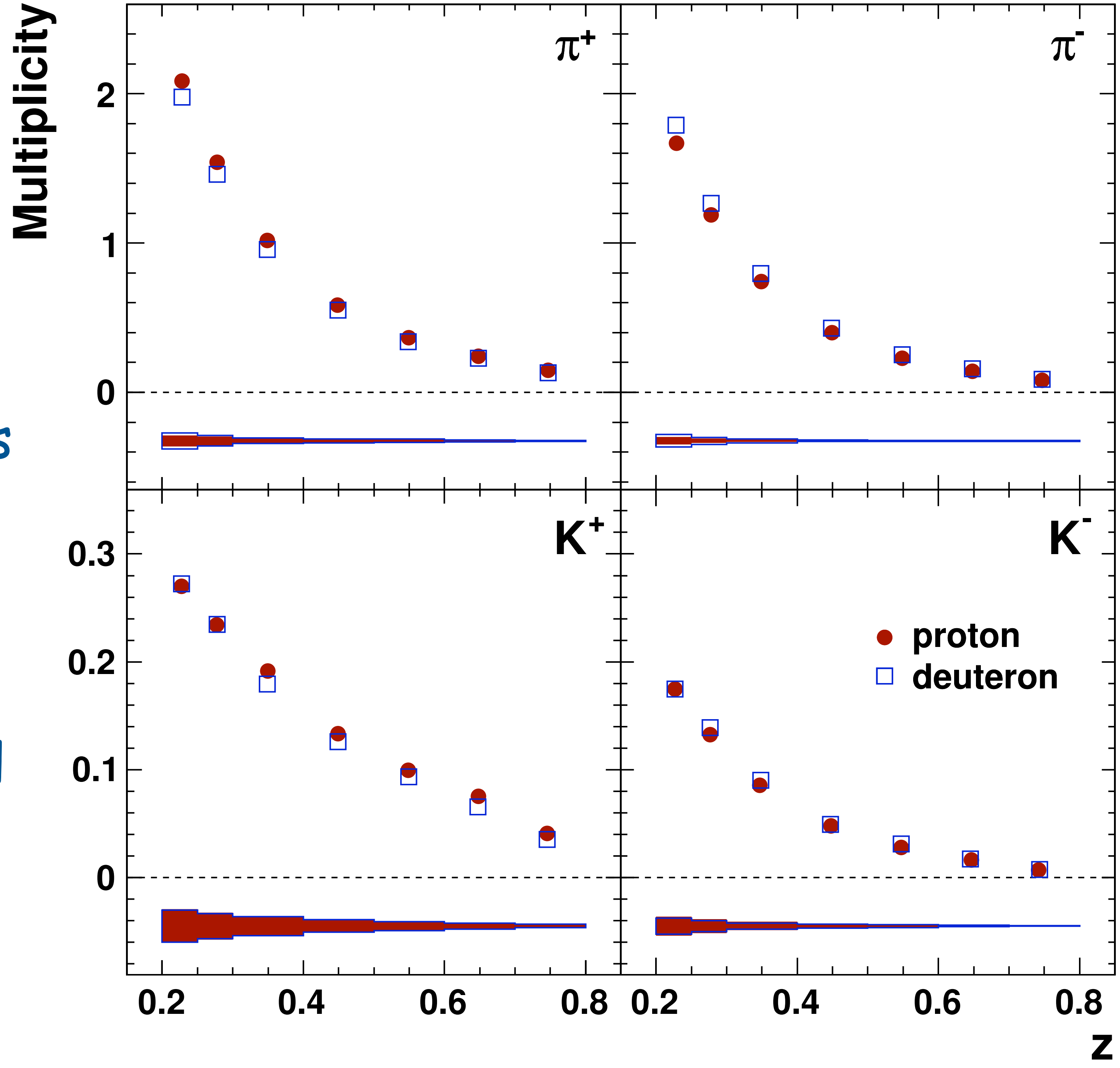
➡ need data on same target for both polarization orientations!

… results …

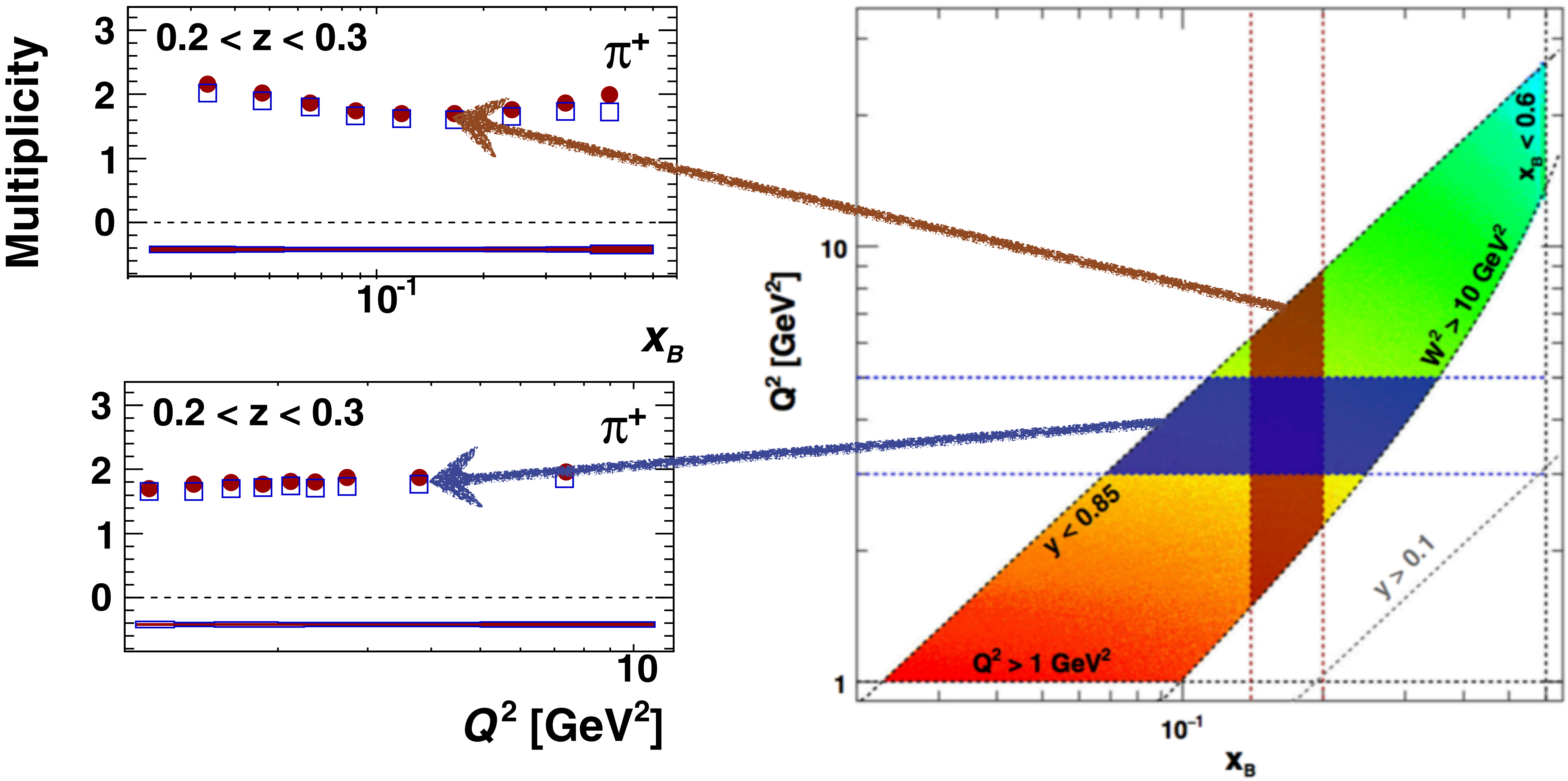
multiplicities @ HERMES

- extensive data set on pure proton and deuteron targets for identified charged mesons
  - access to flavor dependence of fragmentation through different mesons and targets
- input to fragmentation function analyses
- extracted in a multi-dimensional unfolding procedure:
  - $(x, z, P_{h\perp})$
  - $(Q^2, z, P_{h\perp})$

[Airapetian et al., PRD 87 (2013) 074029]



$\langle \mathcal{M}(Q^2) \rangle_{Q^2} \neq \mathcal{M}(\langle Q^2 \rangle)$



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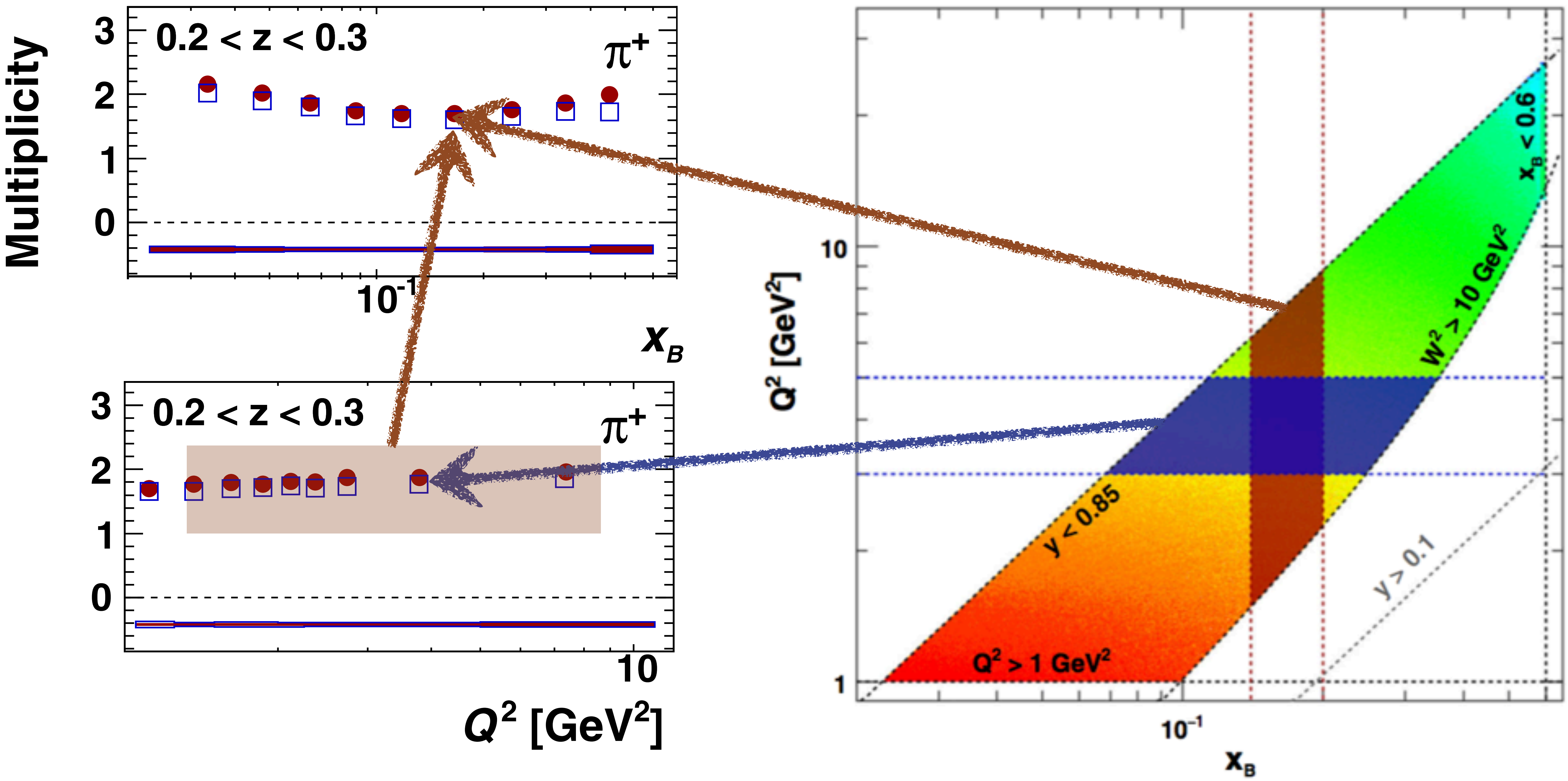

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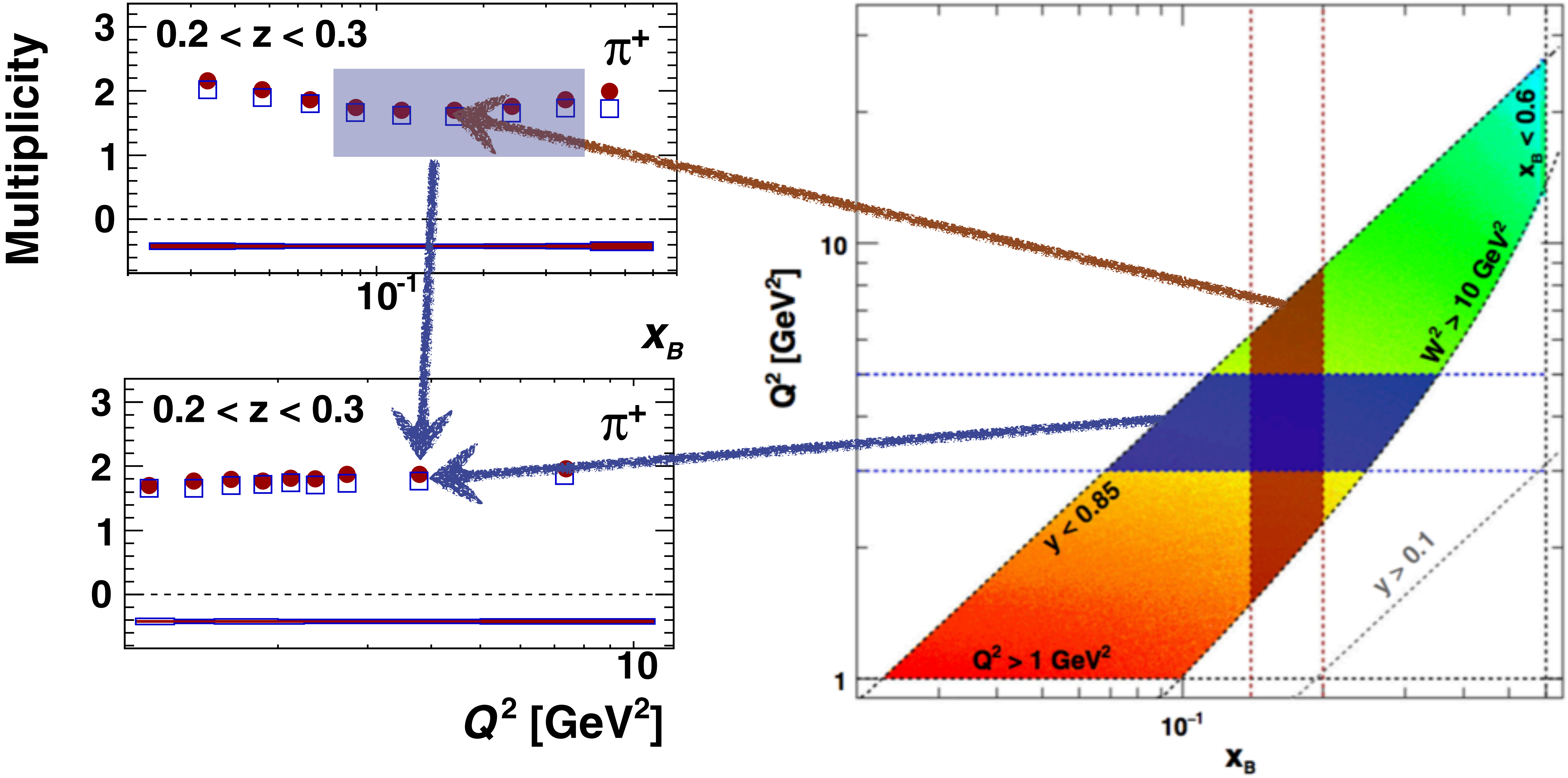

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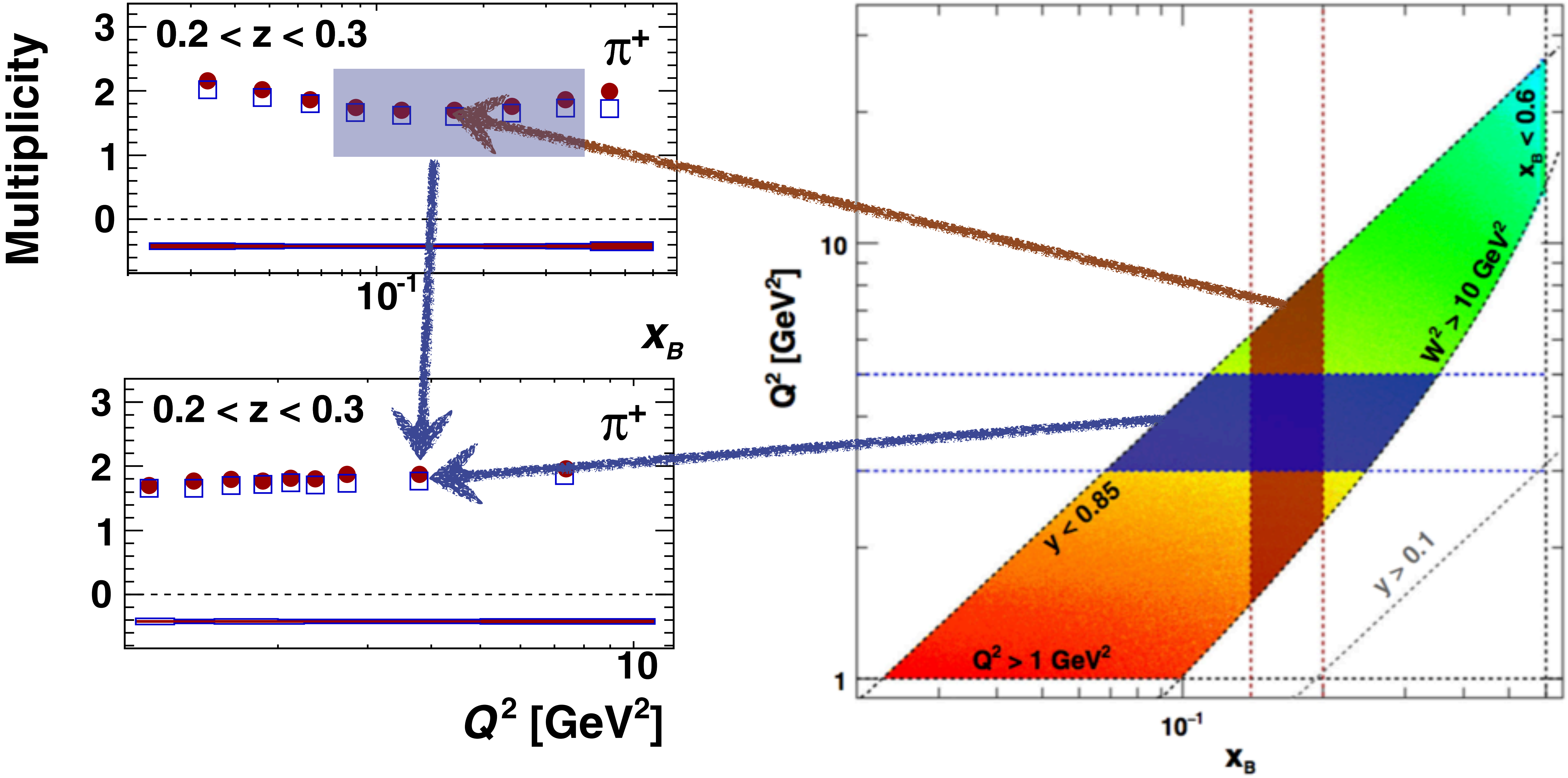
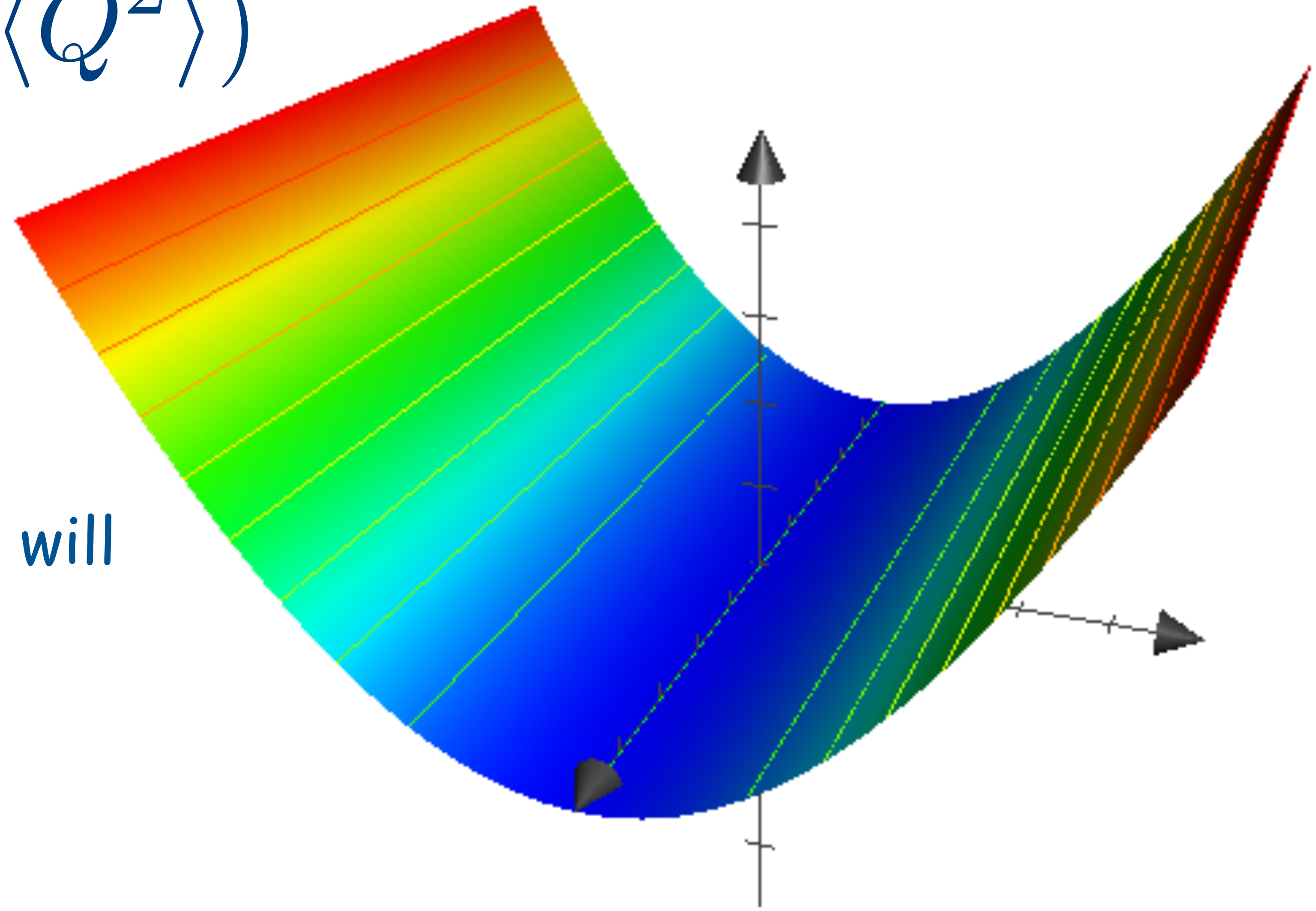


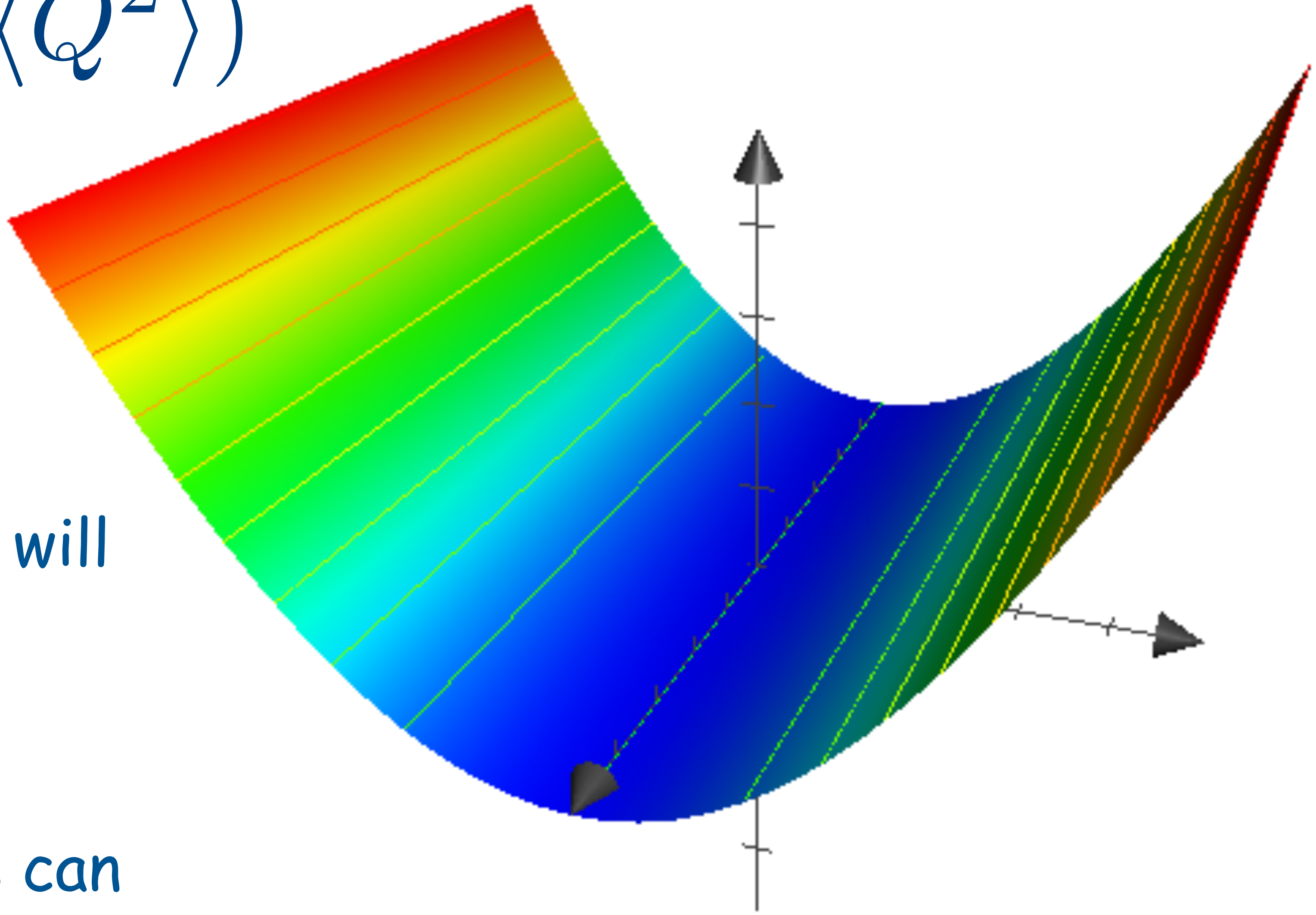
- even though having similar average kinematics, multiplicities in the two projections are different

$$\langle \mathcal{M}(Q^2) \rangle_{Q^2} \neq \mathcal{M}(\langle Q^2 \rangle)$$



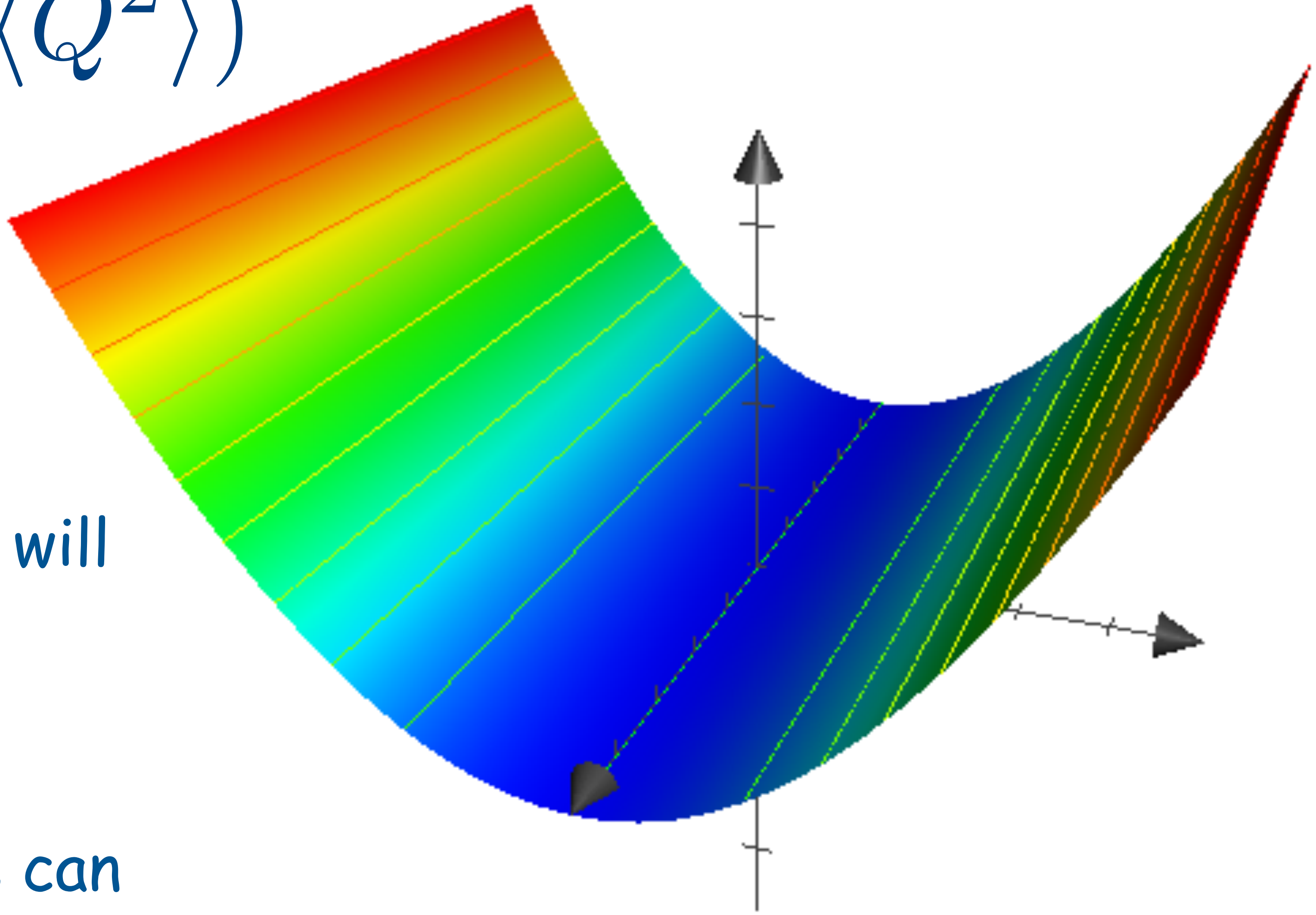
- the average along the valley will be smaller than the average along the gradient

$\langle \mathcal{M}(Q^2) \rangle_{Q^2} \neq \mathcal{M}(\langle Q^2 \rangle)$



- the average along the valley will be smaller than the average along the gradient
- still the **average kinematics** can be the same

$\langle \mathcal{M}(Q^2) \rangle_{Q^2} \neq \mathcal{M}(\langle Q^2 \rangle)$



- the average along the valley will be smaller than the average along the gradient
- still the **average kinematics** can be the same

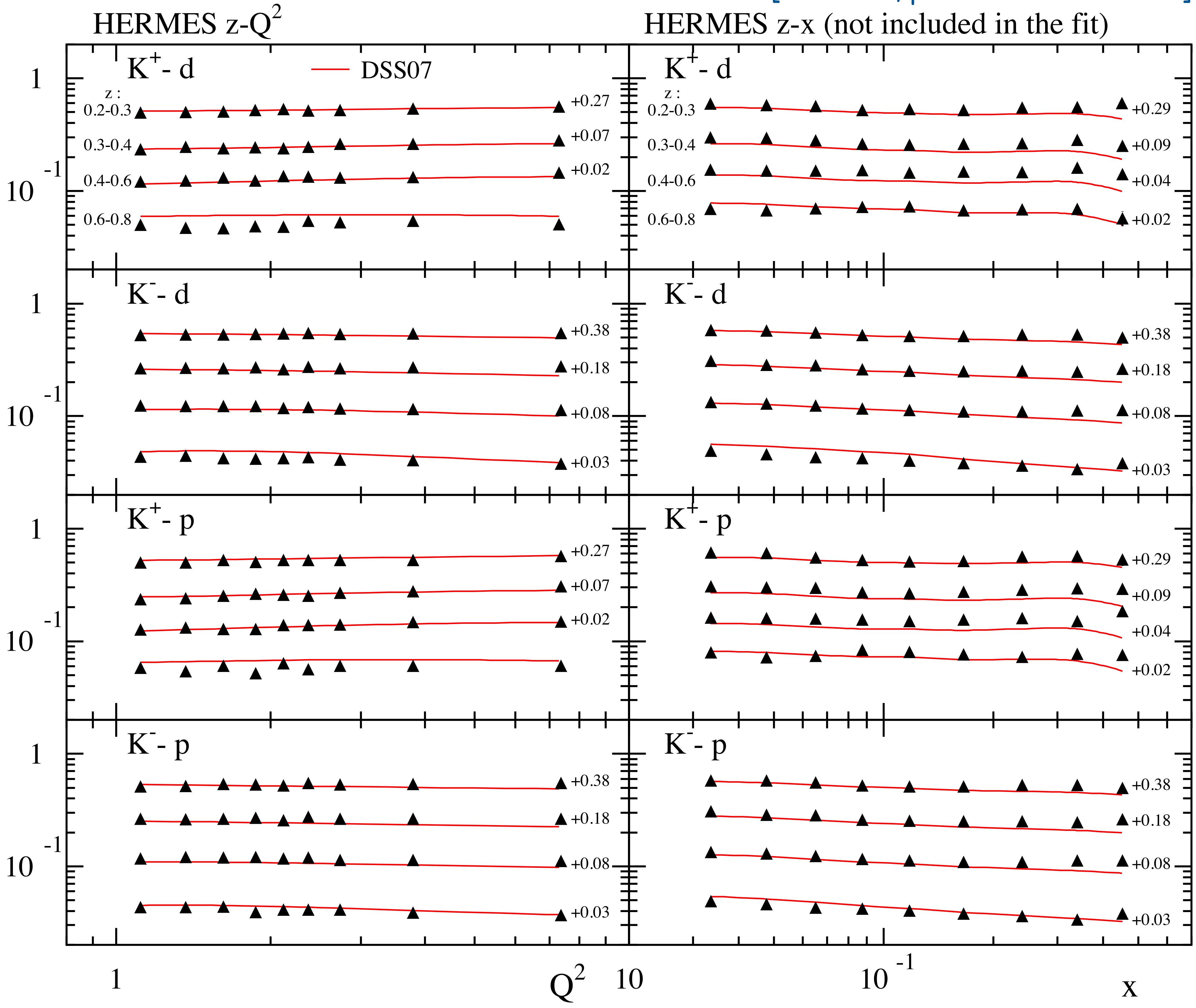
take-away messages: (when told so) integrate your cross section over the kinematic ranges dictated by the experiment (e.g., do not simply evaluate it at the average kinematics)

To experiments: fully differential analyses!

integrating vs. using average kinematics

- (by now old) **DSS07** FF fit to z-Q^2 projection

[R. Sassot, private communication]



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- z-x "prediction" reasonable well when using **integration** over phase-space limits (red lines)

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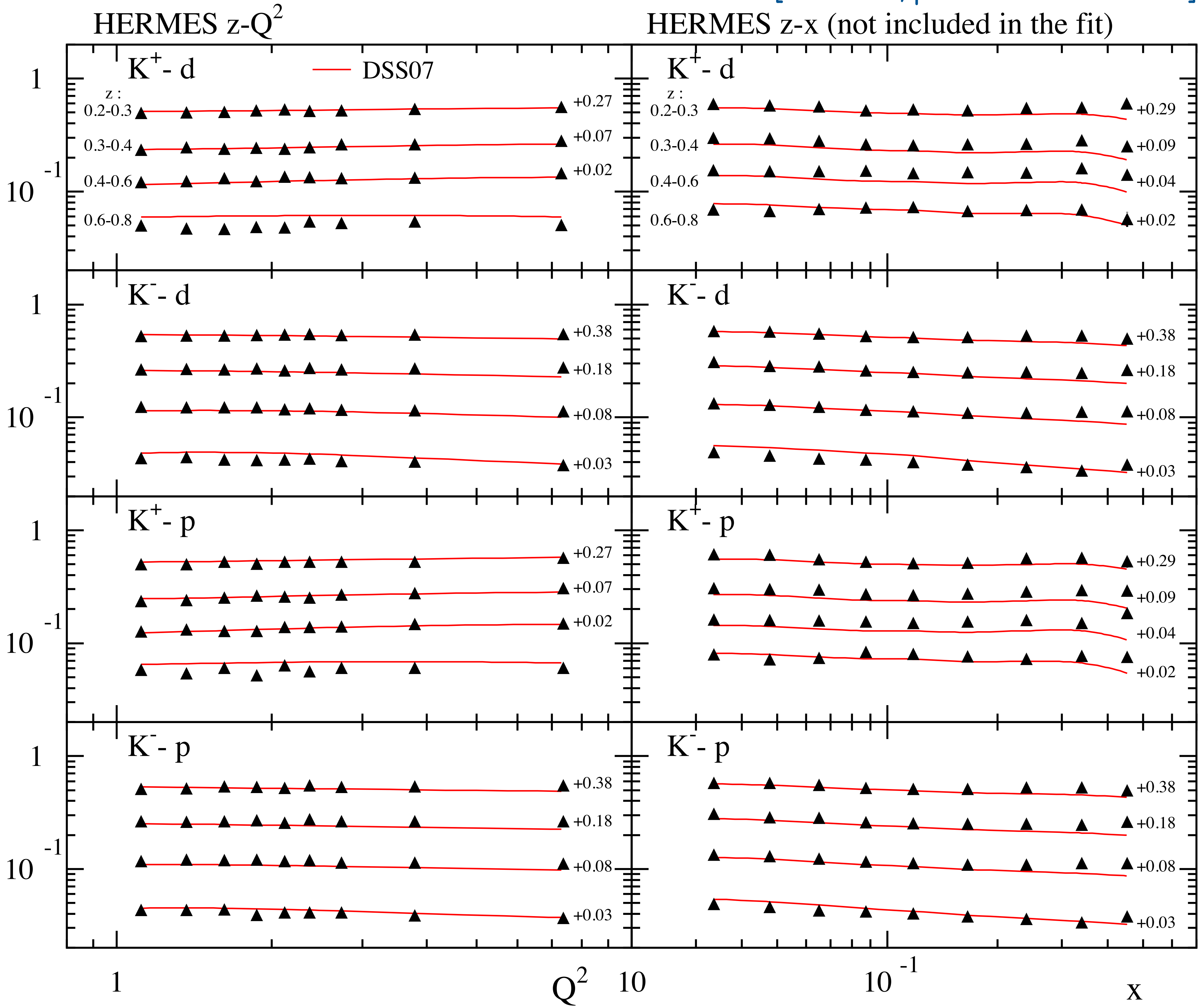
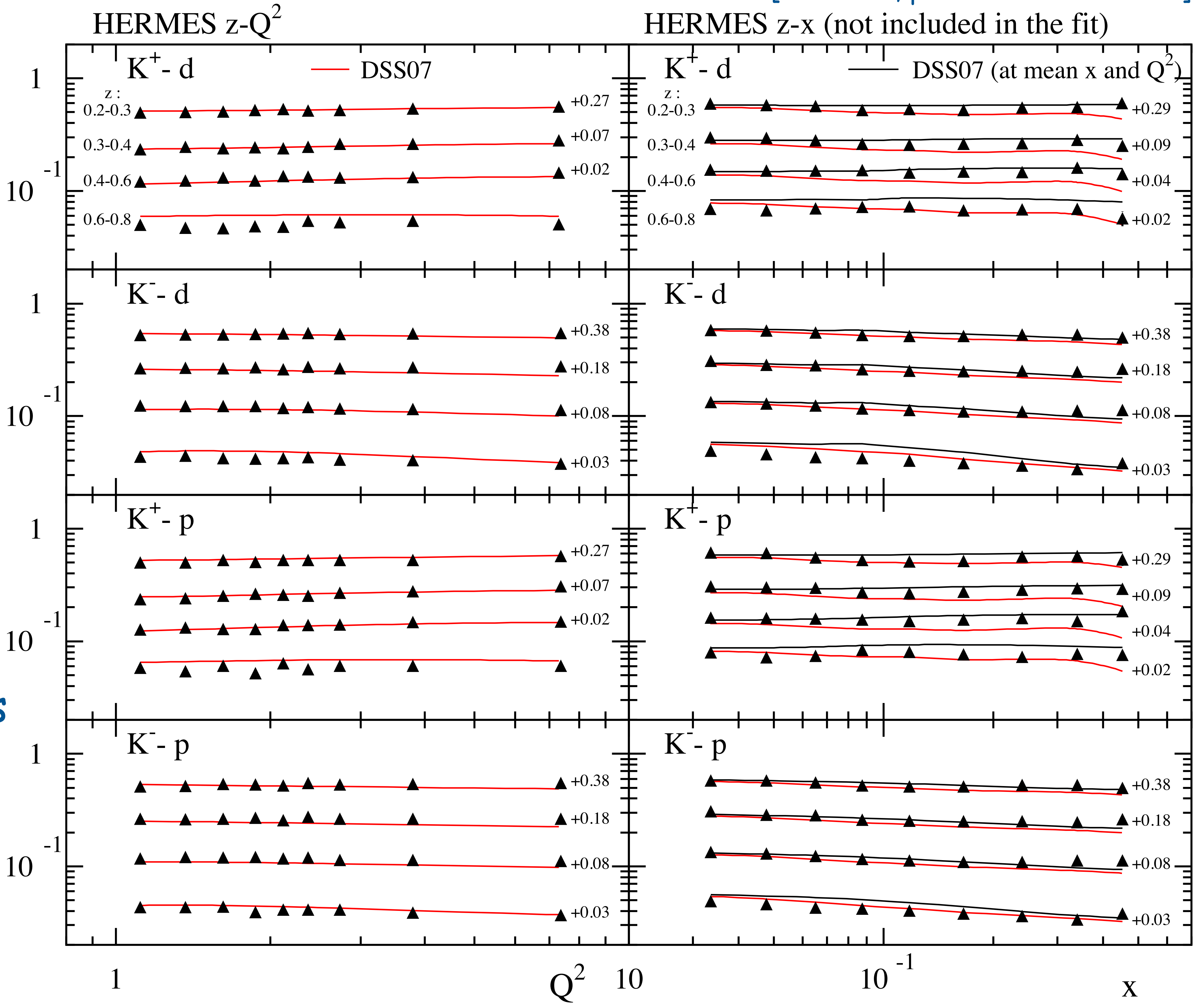


integrating vs. using average kinematics

- (by now old) **DSS07** FF fit to z-Q^2 projection
- z-x "prediction" reasonable well when using **integration** over phase-space limits (red lines)
- significant changes when using **average kinematics**



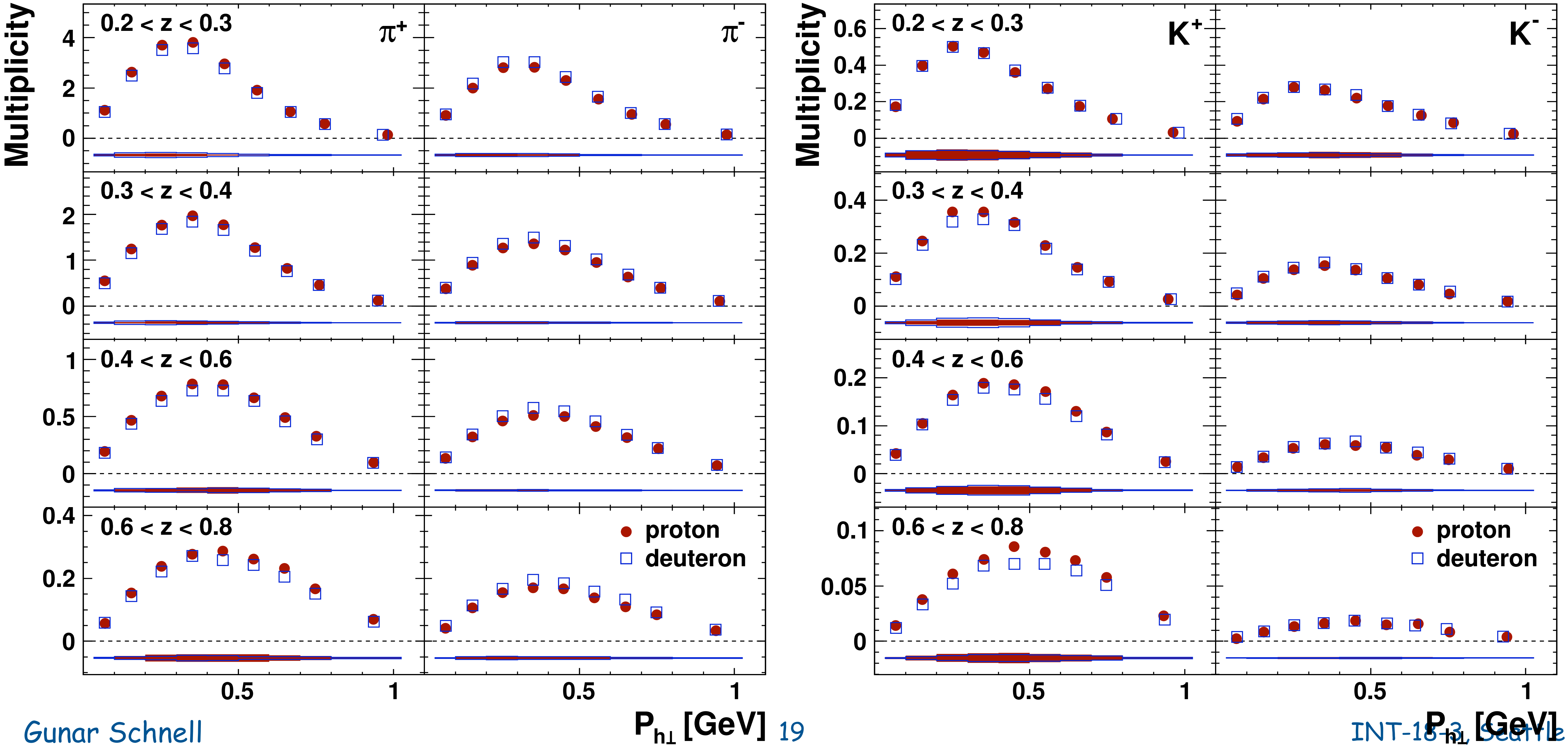
[R. Sassot, private communication]

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

$P_{h\perp}$ dependence

- multi-dimensional analysis allows going beyond collinear factorization
- flavor information on transverse momenta via target variation and hadron ID e.g. [A. Signori et al., JHEP 11(2013)194]

[Airapetian et al., PRD 87 (2013) 074029]



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T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

$P_{h\perp}$-multiplicity landscape

	EMC [11]	HERMES [15]	JLAB [31]	COMPASS [16]	COMPASS (This paper)
Target	p/d	p/d	d	d	d
Beam energy (GeV)	100–280	27.6	5.479	160	160
Hadron type	$h^\pm$	$\pi^\pm$, $K^\pm$	$\pi^\pm$	$h^\pm$	$h^\pm$
Observable	$M^{h^+ + h^-}$	M^h	σ^h	M^h	M^h
Q^2_{min} $(\text{GeV}/c)^2$	2/3/4/5	1	2	1	1
W^2_{min} $(\text{GeV}/c^2)^2$	-	10	4	25	25
y range	[0.2,0.8]	[0.1,0.85]	[0.1,0.9]	[0.1,0.9]	[0.1,0.9]
x range	[0.01,1]	[0.023,0.6]	[0.2,0.6]	[0.004,0.12]	[0.003,0.4]
P^2_{hT} range $(\text{GeV}/c)^2$	[0.081, 15.8]	[0.0047,0.9]	[0.004,0.196]	[0.02,0.72]	[0.02,3]

[11] J. Ashman et al. (EMC), Z. Phys.C 52, 361 (1991).

[15] A. Airapetian et al. (HERMES), Phys. Rev. D87, 074029 (2013).

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[31] R. Asaturyan et al., Phys. Rev. C 85, 015202 (2012).

["This paper"] M. Aghasyan et al. (COMPASS), Phys. Rev. D 97, 032006 (2018).

… as well as more limited measurements by H1 and Zeus

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U	f_1		$h_1^\perp$
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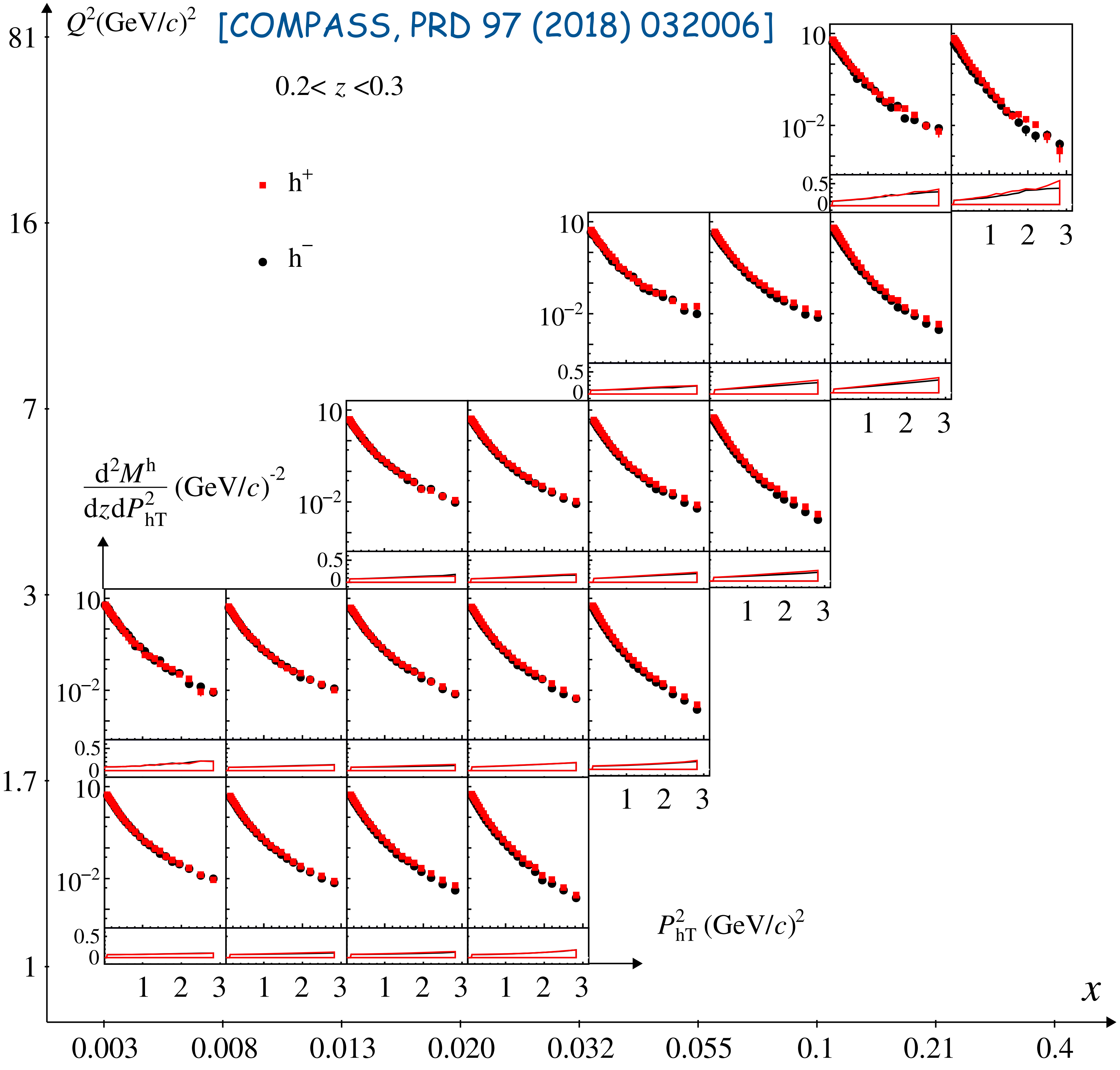
$P_{h\perp}$ dependence



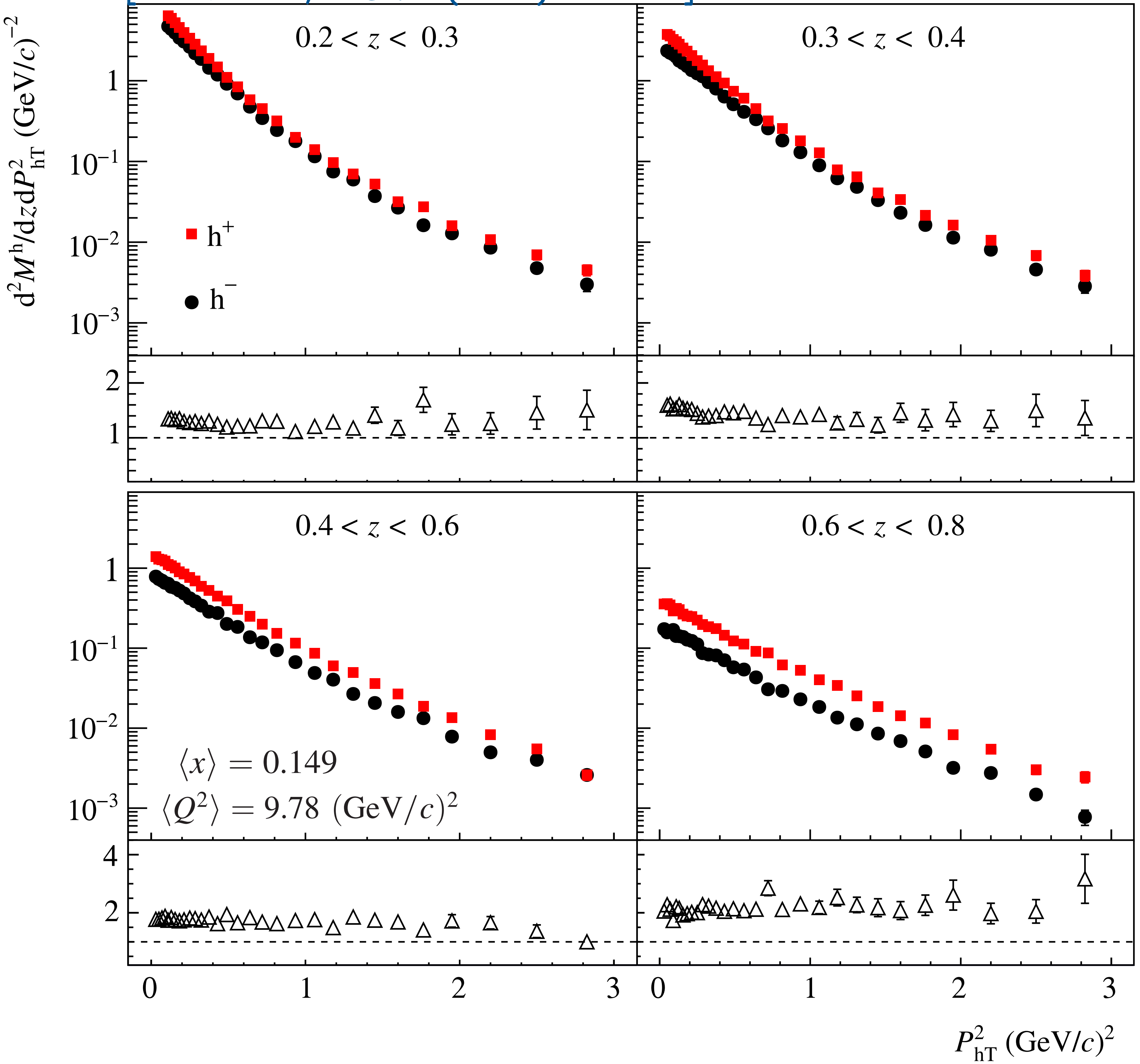
[COMPASS, PRD 97 (2018) 032006]

- data on LiD target
- differential in x, z, Q^2, $P_{h\perp}{}^2$
- one example (lowest z bin)
- high statistical precision allows detailed studies

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

$P_{h\perp}$ dependence

[COMPASS, PRD 97 (2018) 032006]

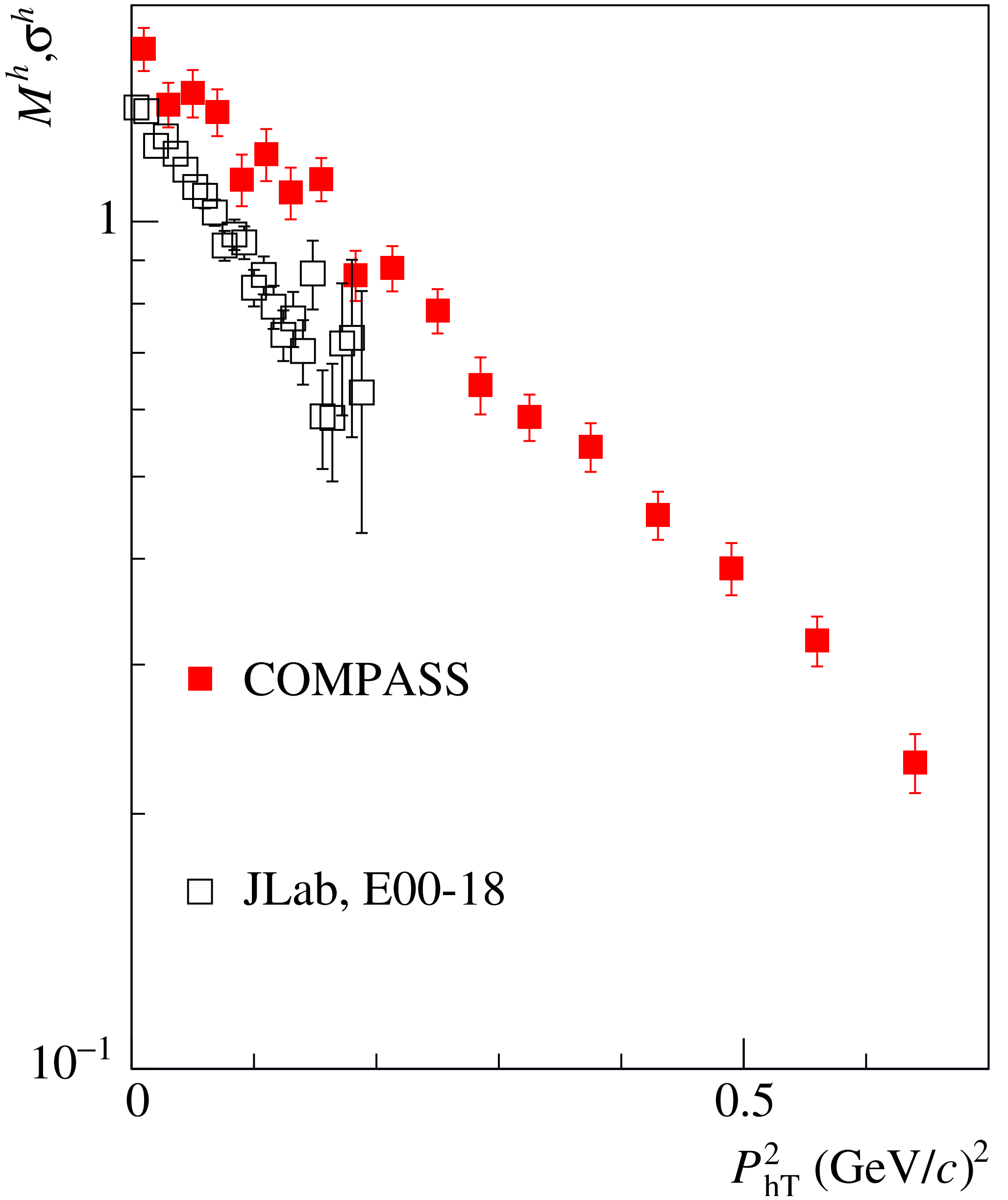


- differences between h^+ and h^- increase with z

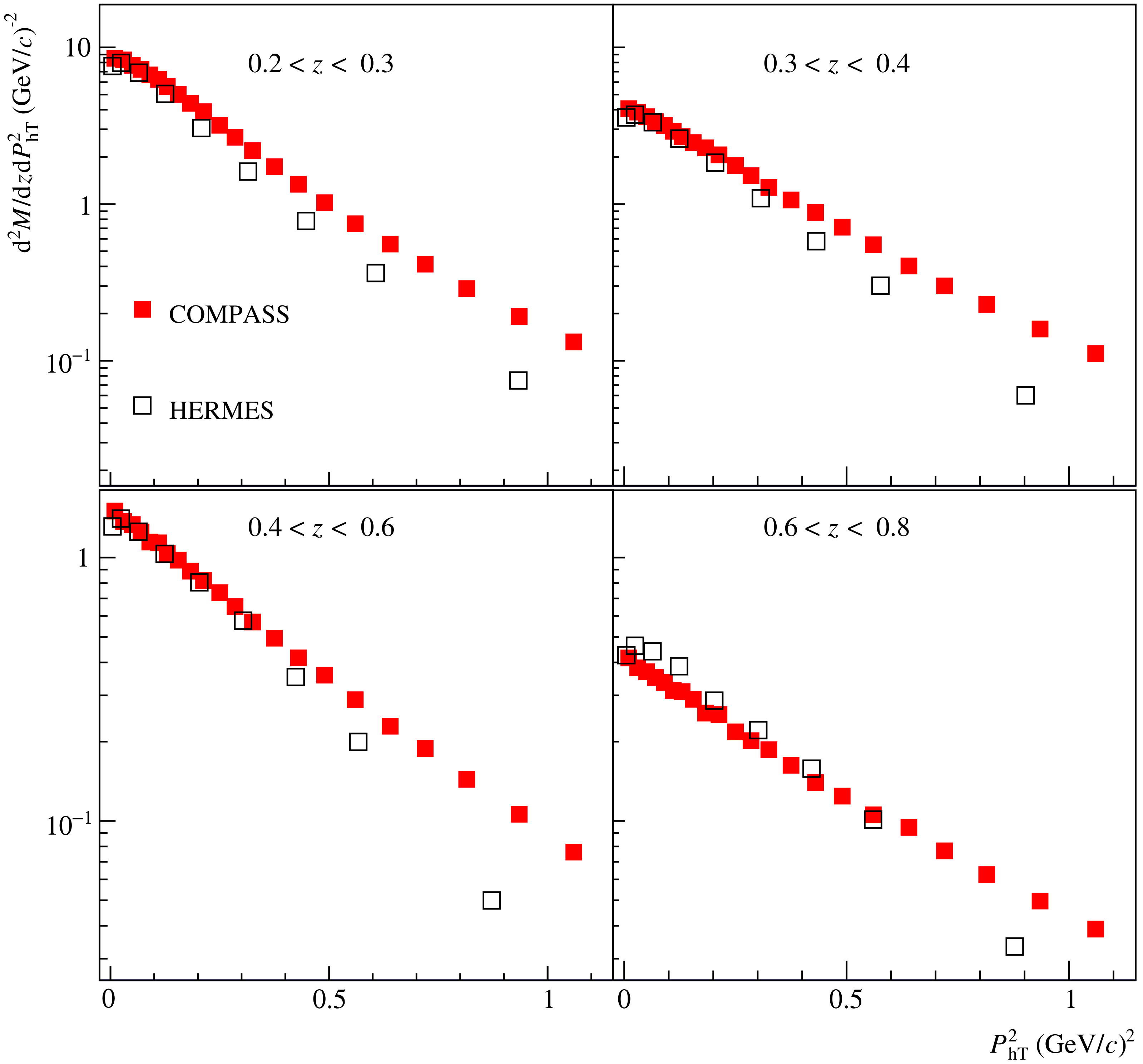
COMPASS vs. JLab & HERMES

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

[COMPASS, PRD 97 (2018) 032006]



[COMPASS, PRD 97 (2018) 032006]



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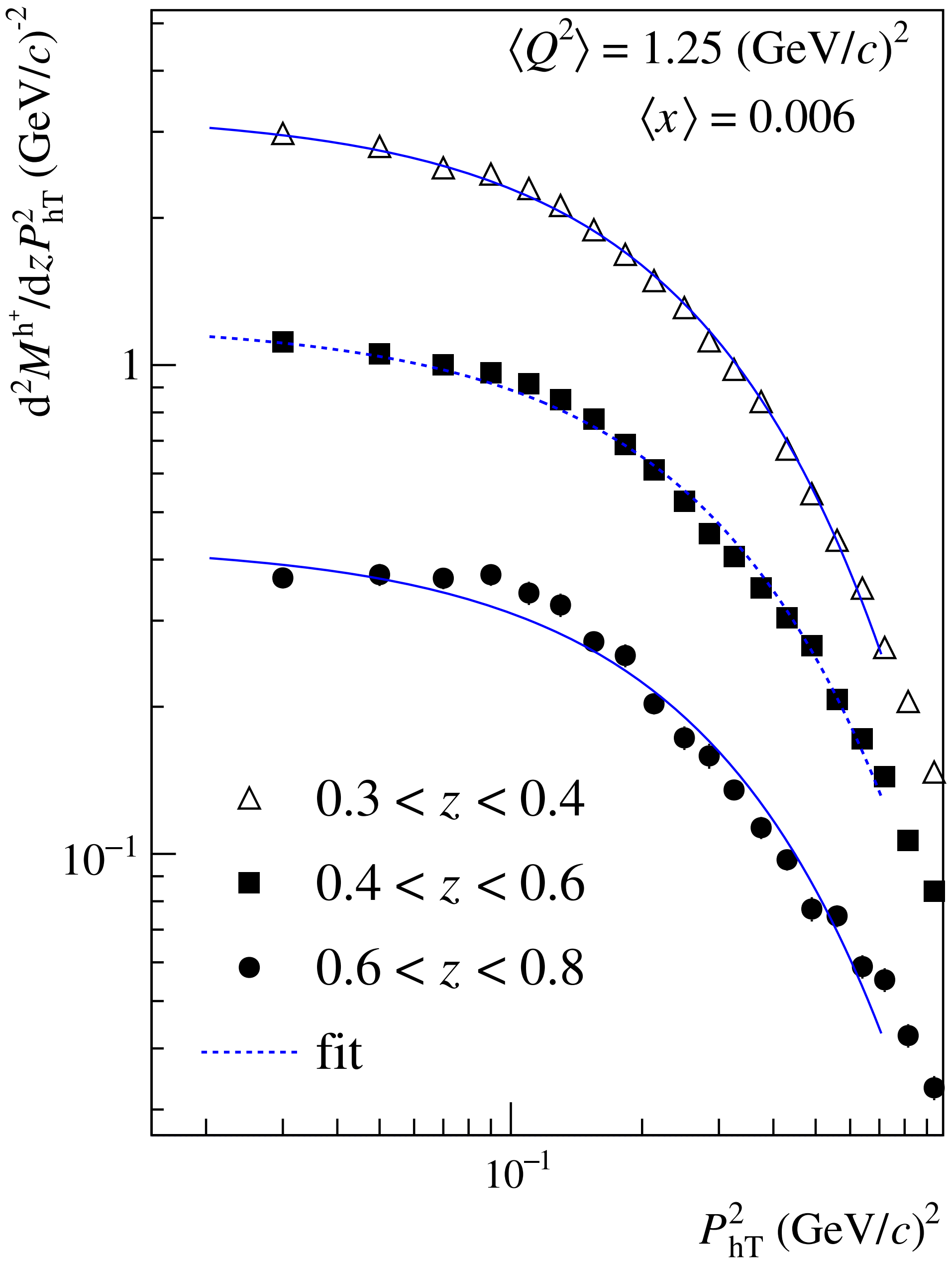
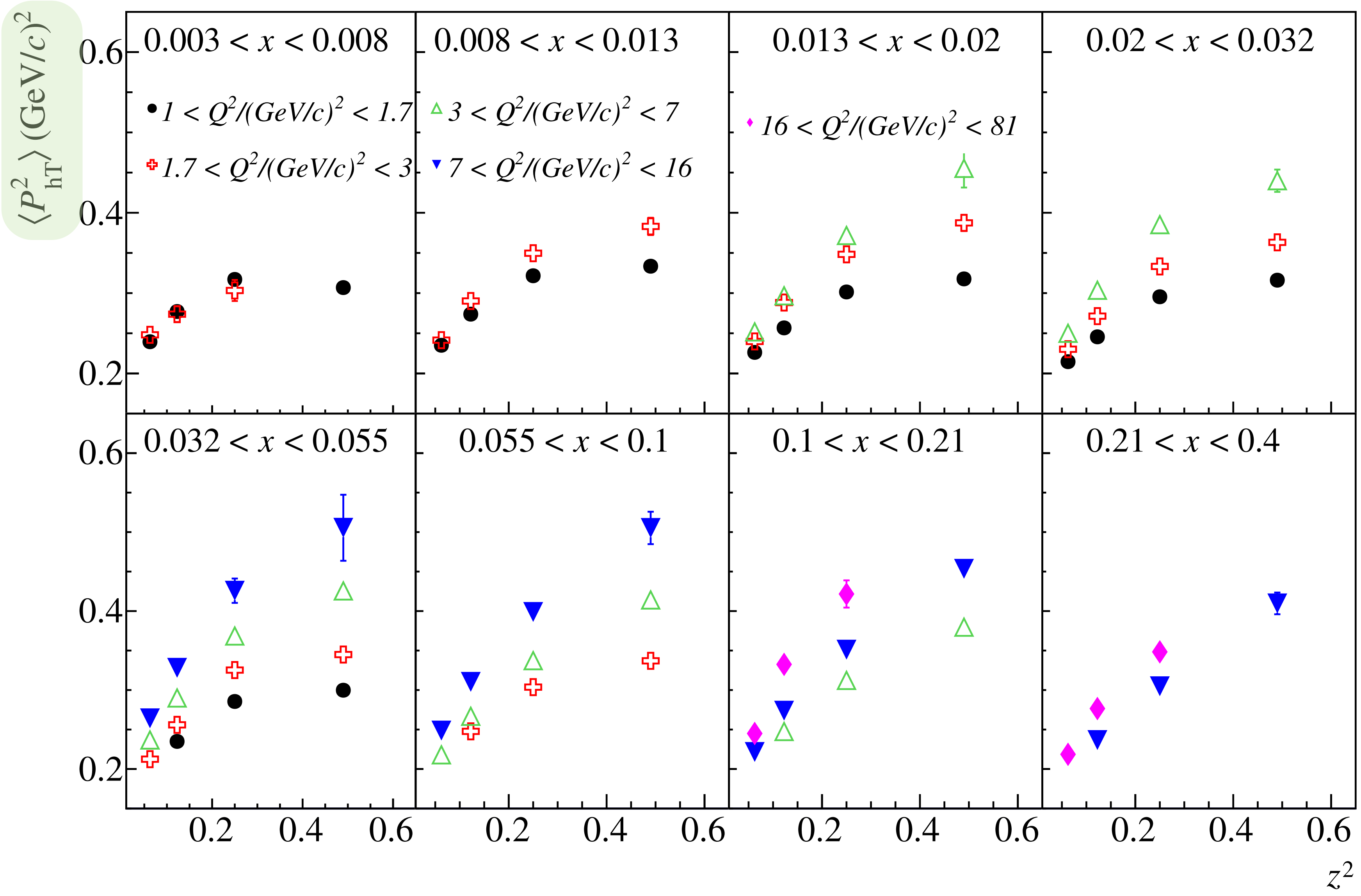
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

fitting the $P_{h\perp}$ dependence

$$\frac{\mathrm{d}^2 M^{\mathrm{h}}(x, Q^2; z)}{\mathrm{d}z\,\mathrm{d}P_{\mathrm{hT}}^2} = \frac{N}{\langle P_{\mathrm{hT}}^2 \rangle} \exp\left(-\frac{P_{\mathrm{hT}}^2}{\langle P_{\mathrm{hT}}^2 \rangle}\right)$$

[COMPASS, PRD 97 (2018) 032006]



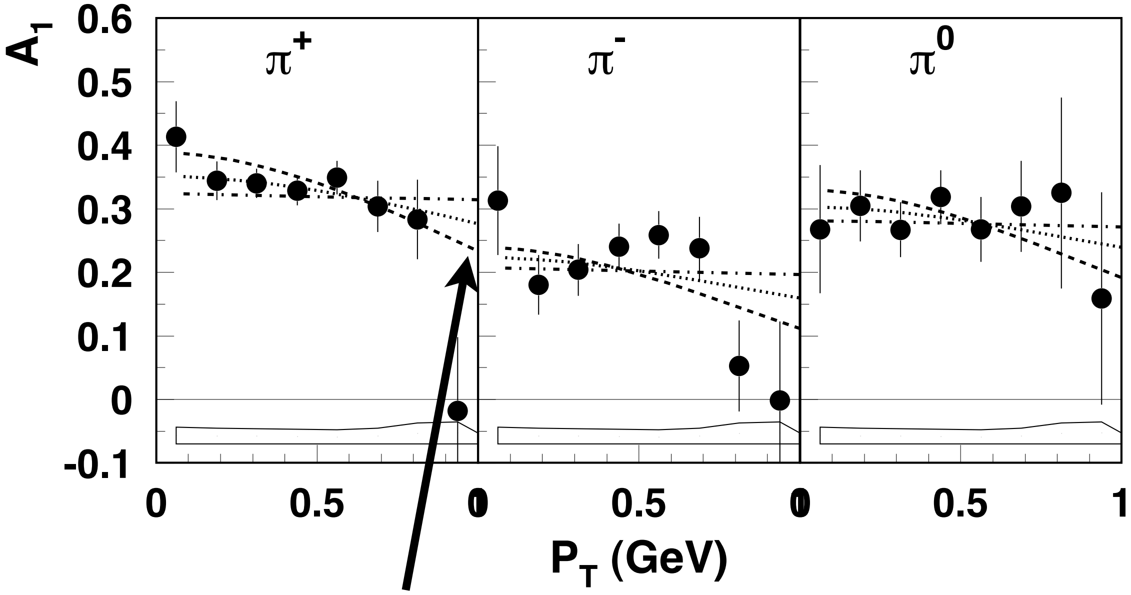


$\langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle$ does not work!

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Helicity density

[Avakian et al. [CLAS], PRL 105, 262002 (2010)]



CLAS data hints at width μ_2 of g_1 that is less than the width μ_0 of f_1

$$f_1^q(x,k_T) = f_1(x)\frac{1}{\pi\mu_0^2}\exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x,k_T) = g_1(x)\frac{1}{\pi\mu_2^2}\exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

… also suggested by lattice QCD

Helicity density

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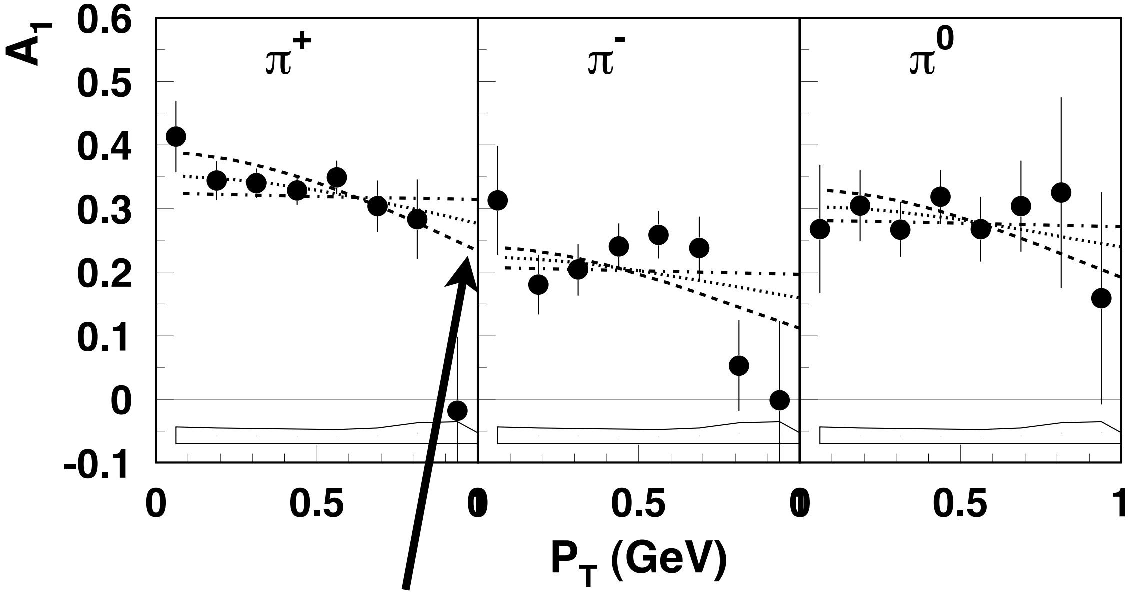

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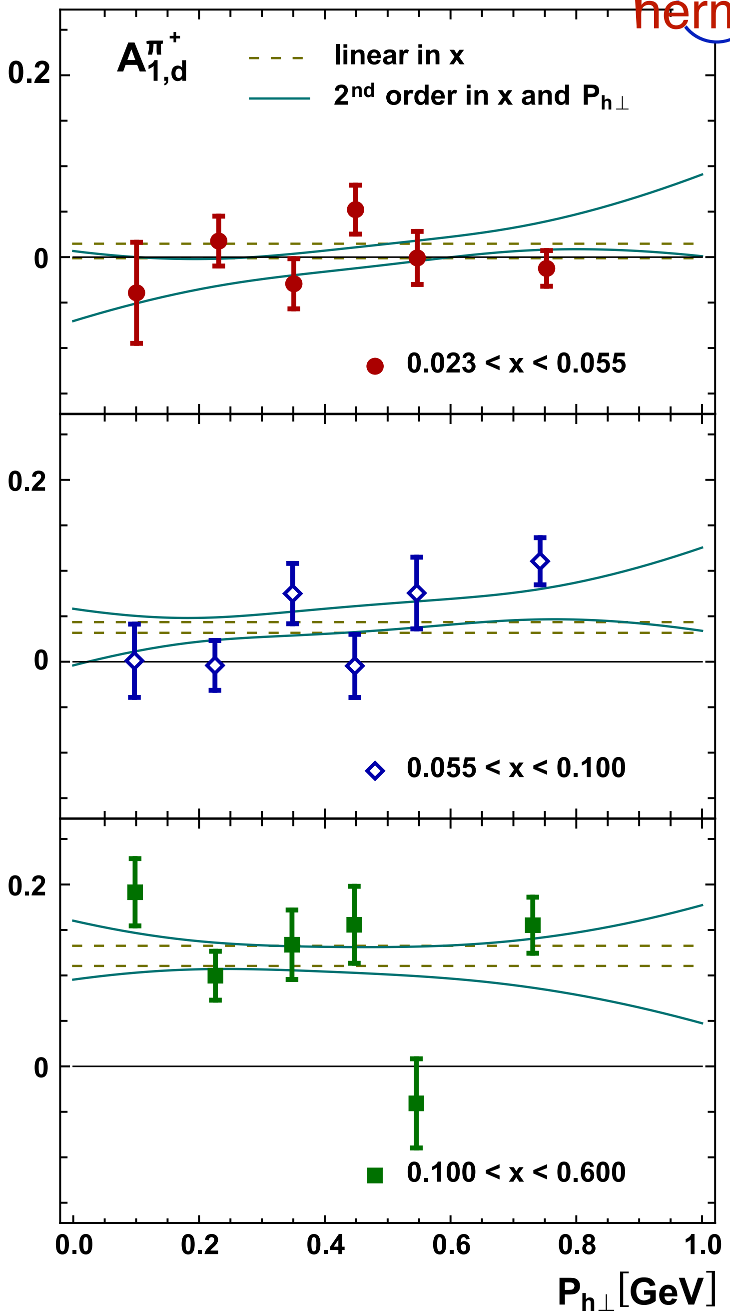
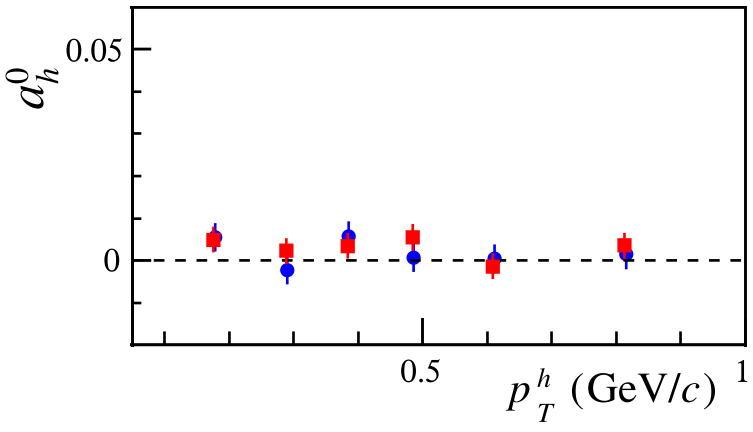


[COMPASS, arXiv:1609.06062]



no significant $P_{h\perp}$ dependences seen on D at HERMES and COMPASS

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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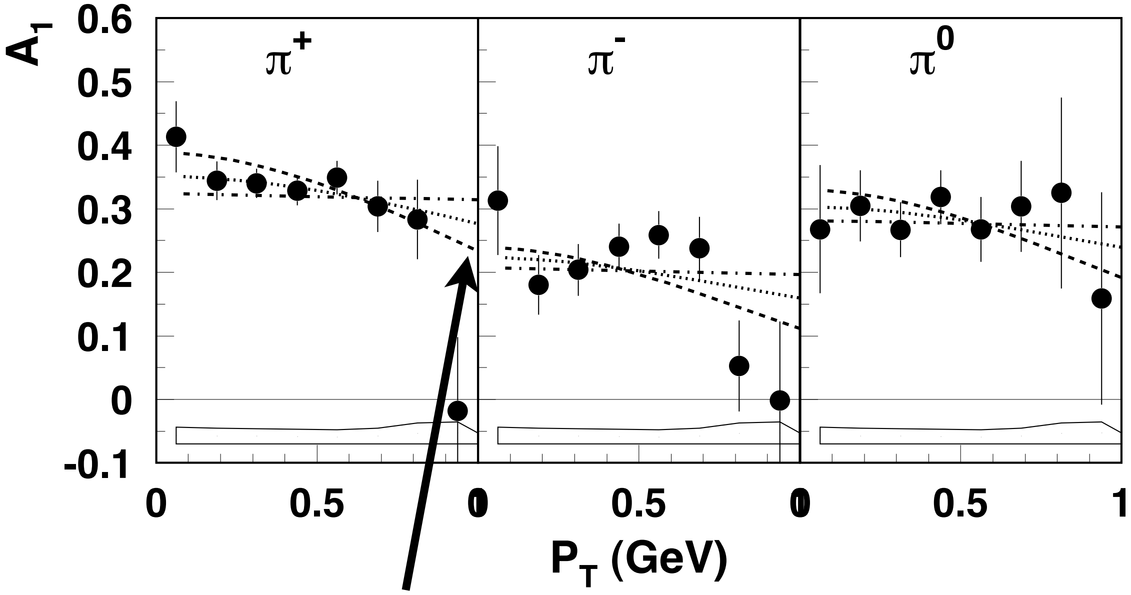

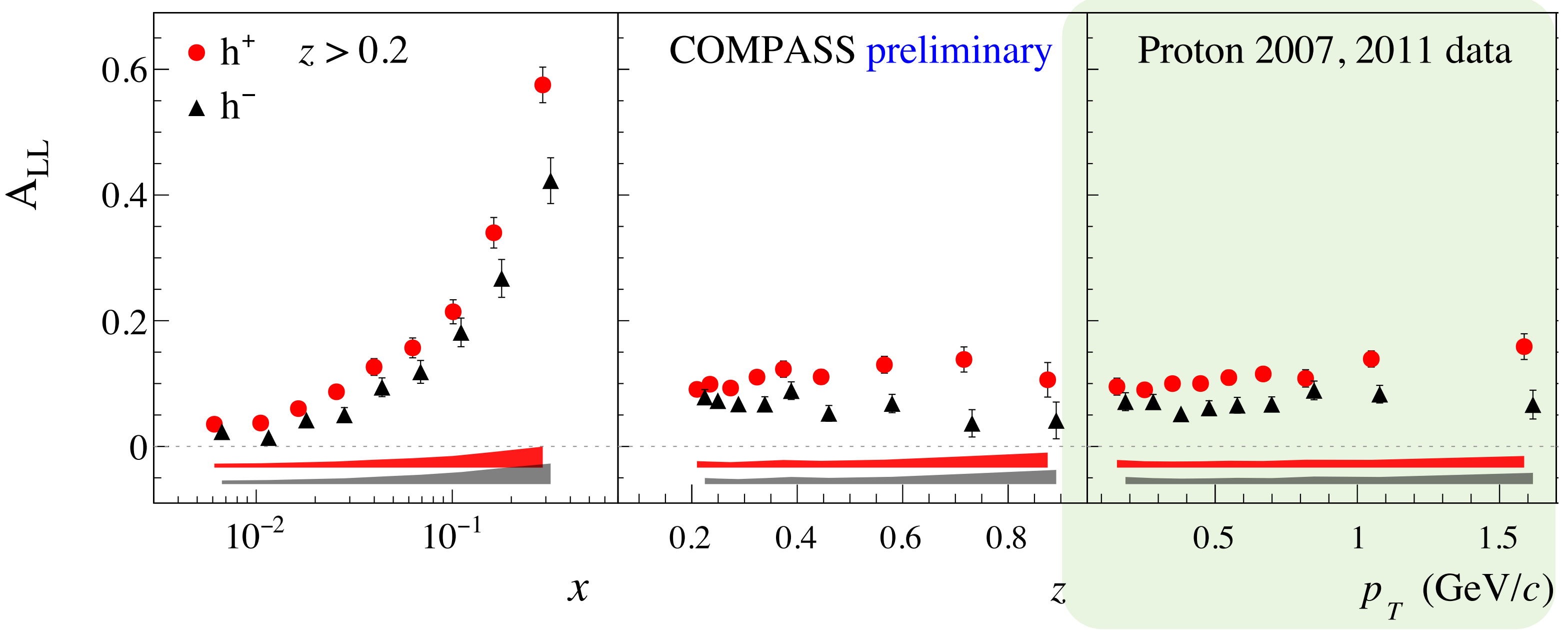

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… also suggested by lattice QCD

perhaps a hint on protons at COMPASS?
(but opposite trend than at CLAS)

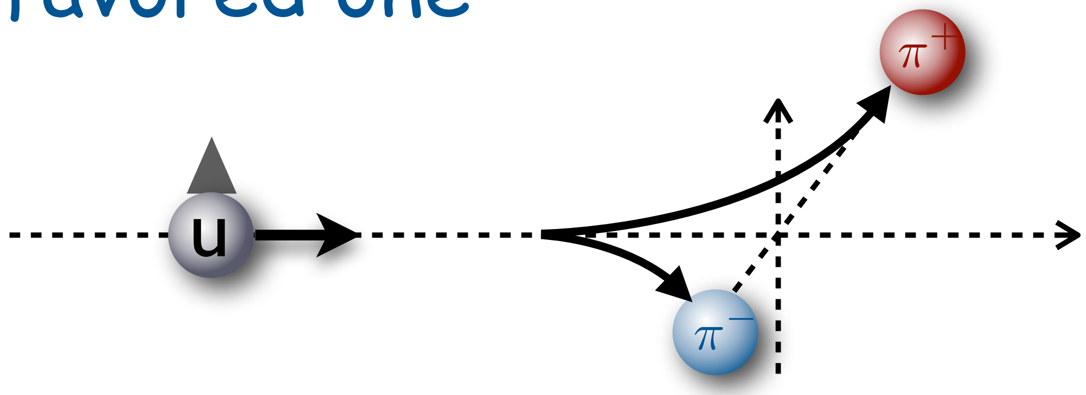
no significant $P_{h\perp}$ dependences seen on D at HERMES and COMPASS

The quest for transversity

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

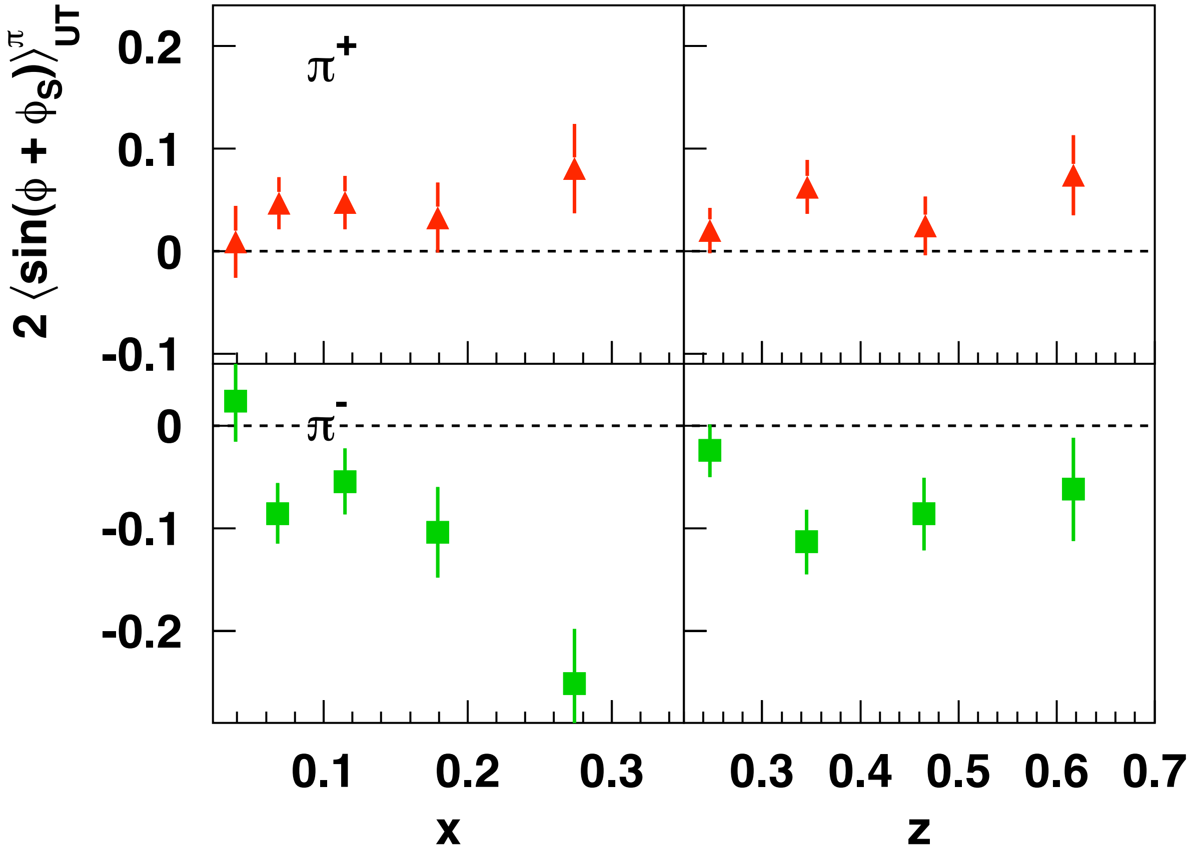
Transversity (Collins fragmentation)

- significant in size and opposite in sign for charged pions
- **disfavored** Collins FF **large** and **opposite in sign** to favored one



- leads to various cancellations in SSA observables

[A. Airapetian et al, PRL 94 (2005) 012002]



2005: First evidence from HERMES SIDIS on proton

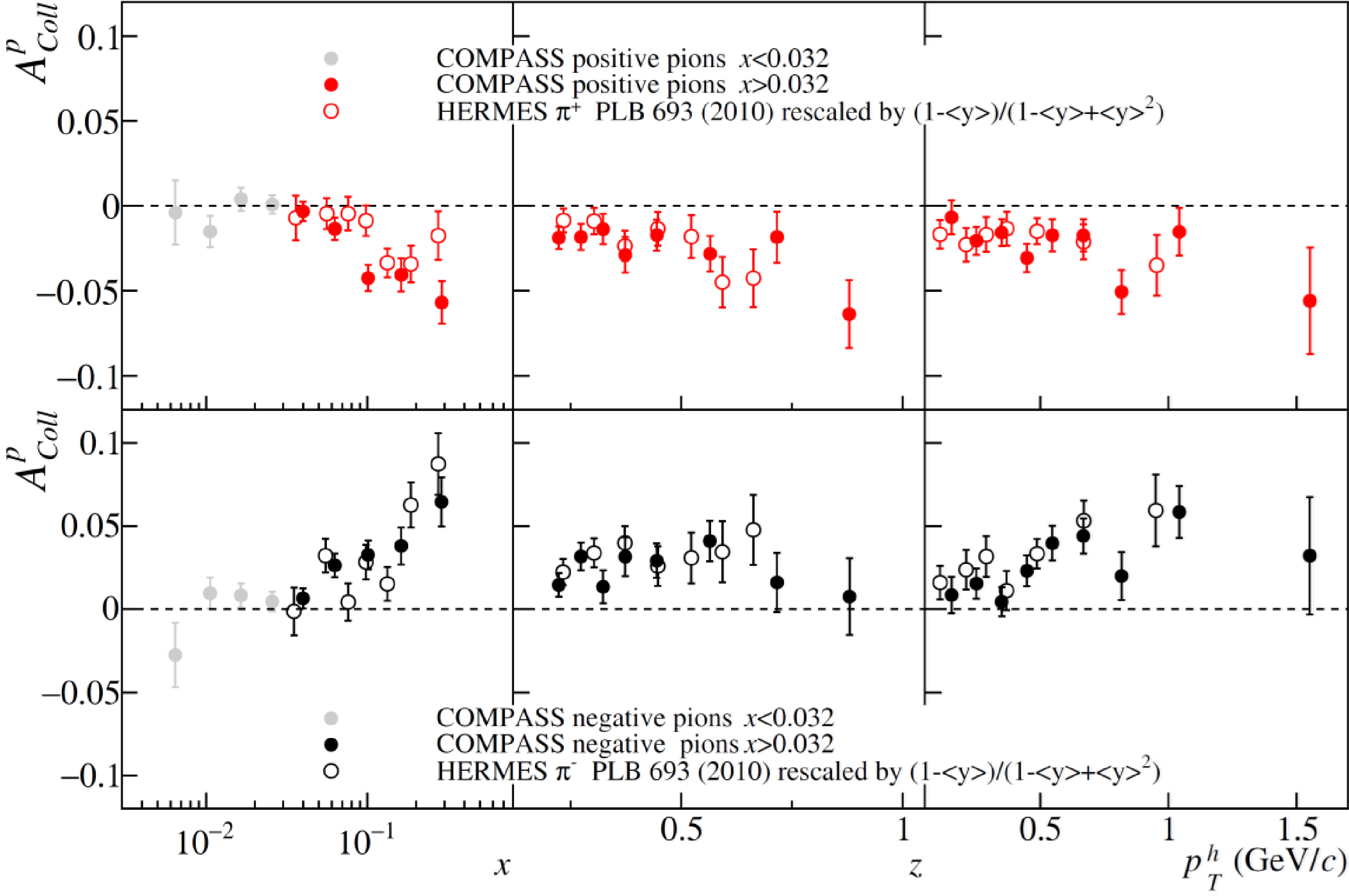
Non-zero transversity
Non-zero Collins function

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Collins amplitudes

- since those early days, a wealth of new results:
  - COMPASS
    [PLB 692 (2010) 240,
    PLB 717 (2012) 376, PLB 744 (2015) 250]
  - HERMES
    [PLB 693 (2010) 11]
  - Jefferson Lab
    [PRL 107 (2011) 072003]

[PLB 744 (2015) 250]



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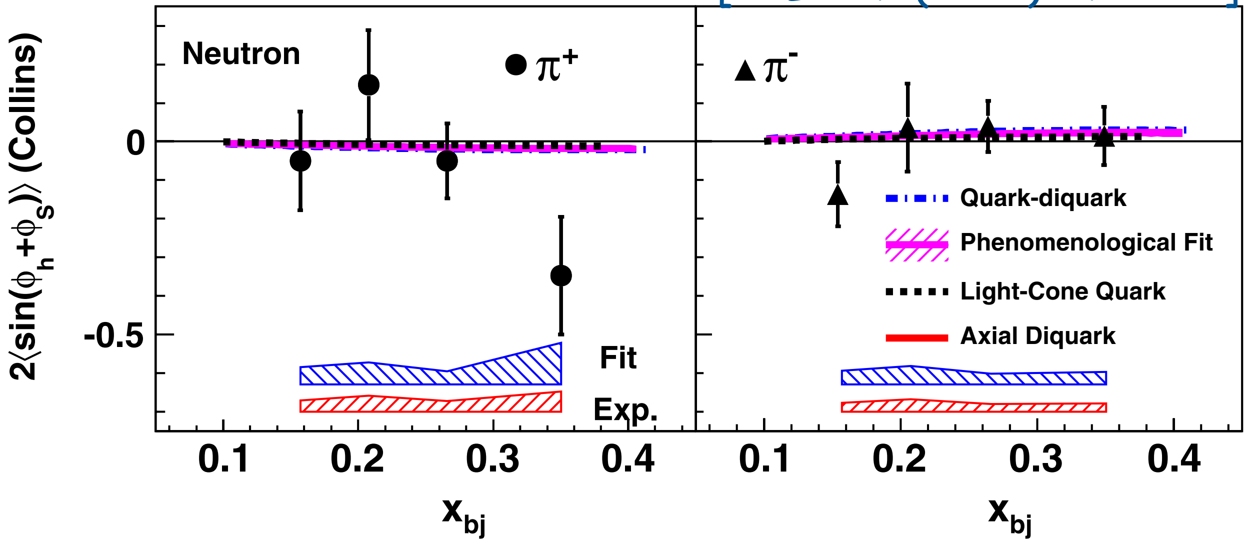

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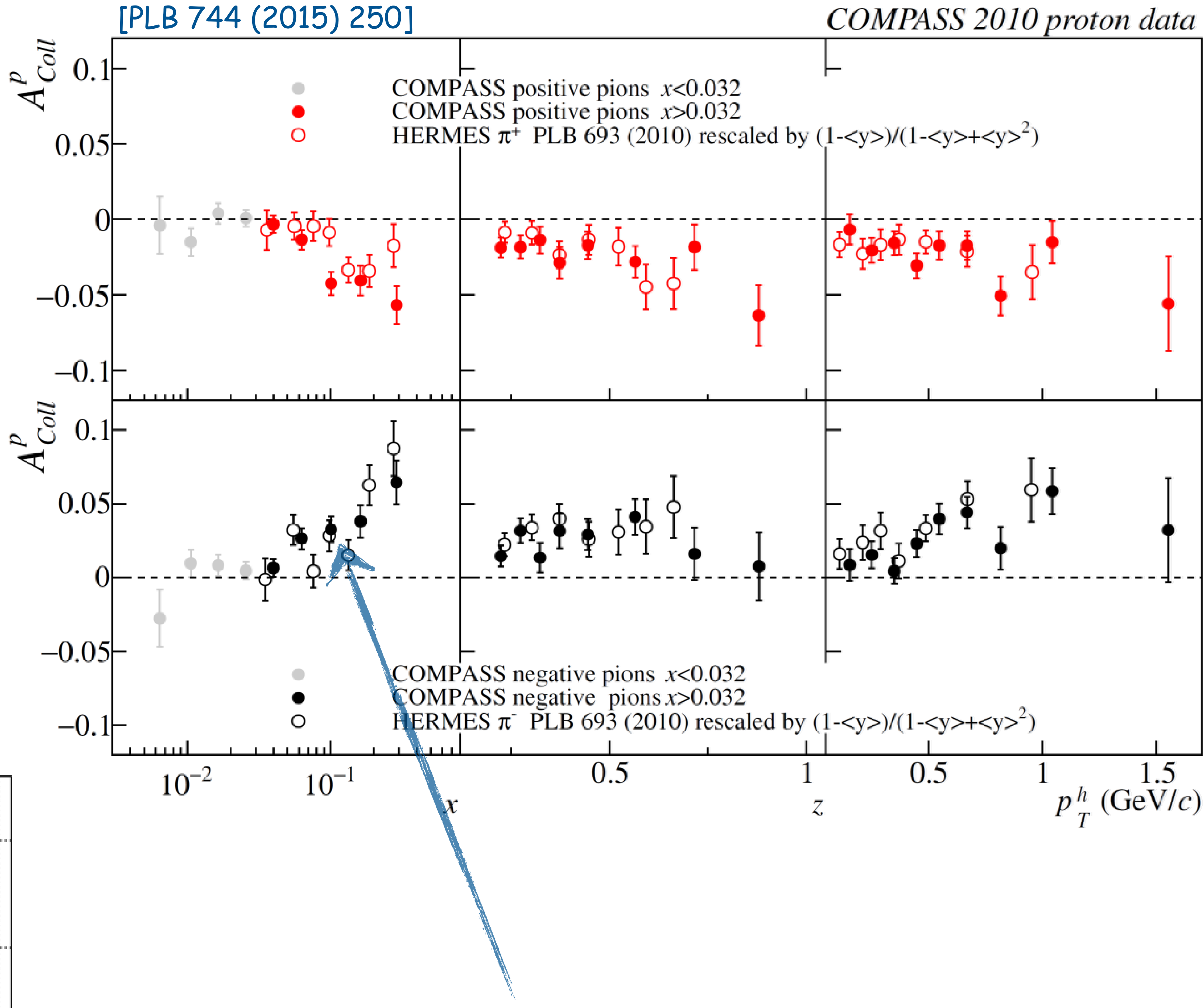
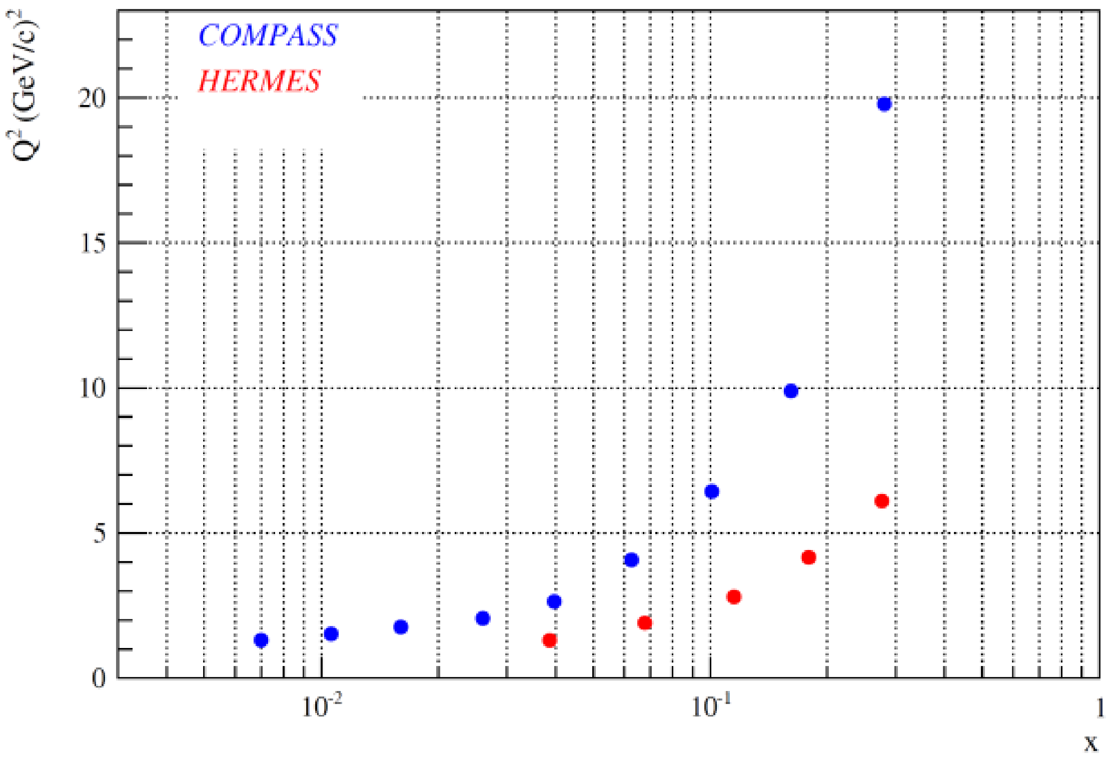


- excellent agreement of various proton data, also with neutron results
- no indication of strong evolution effects

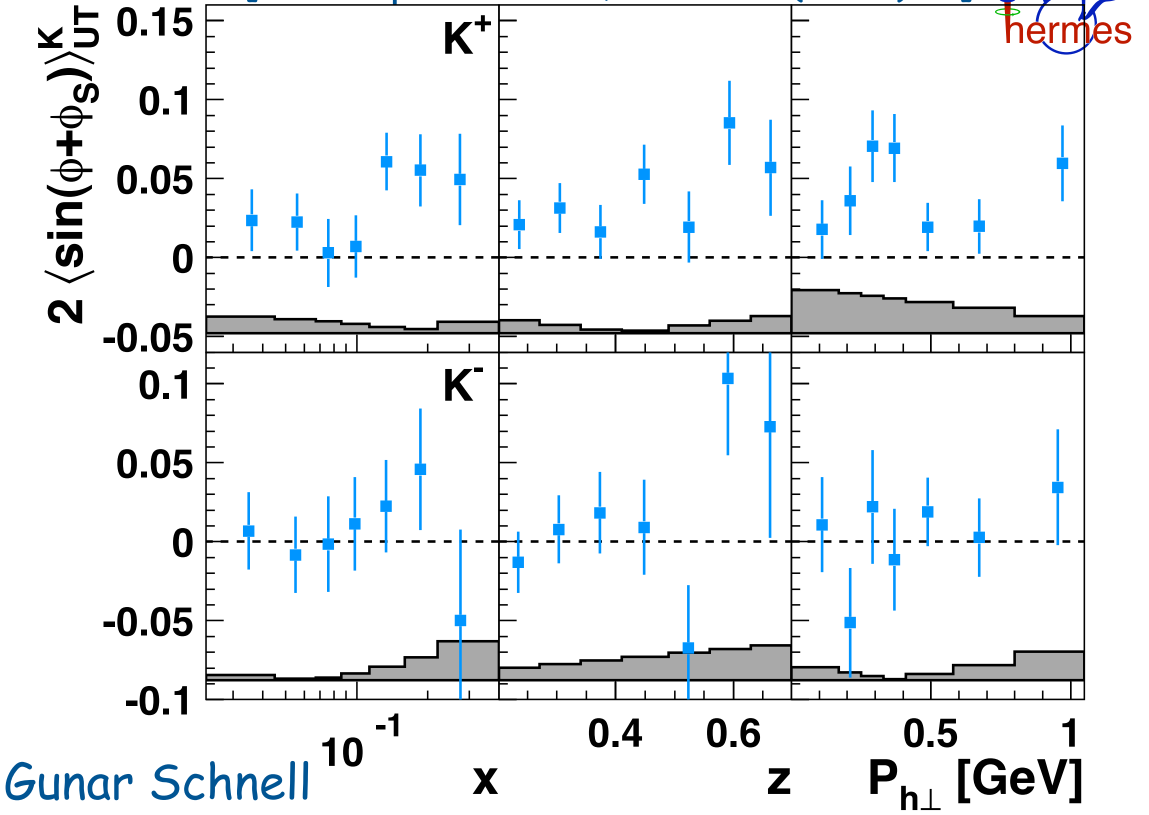


	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

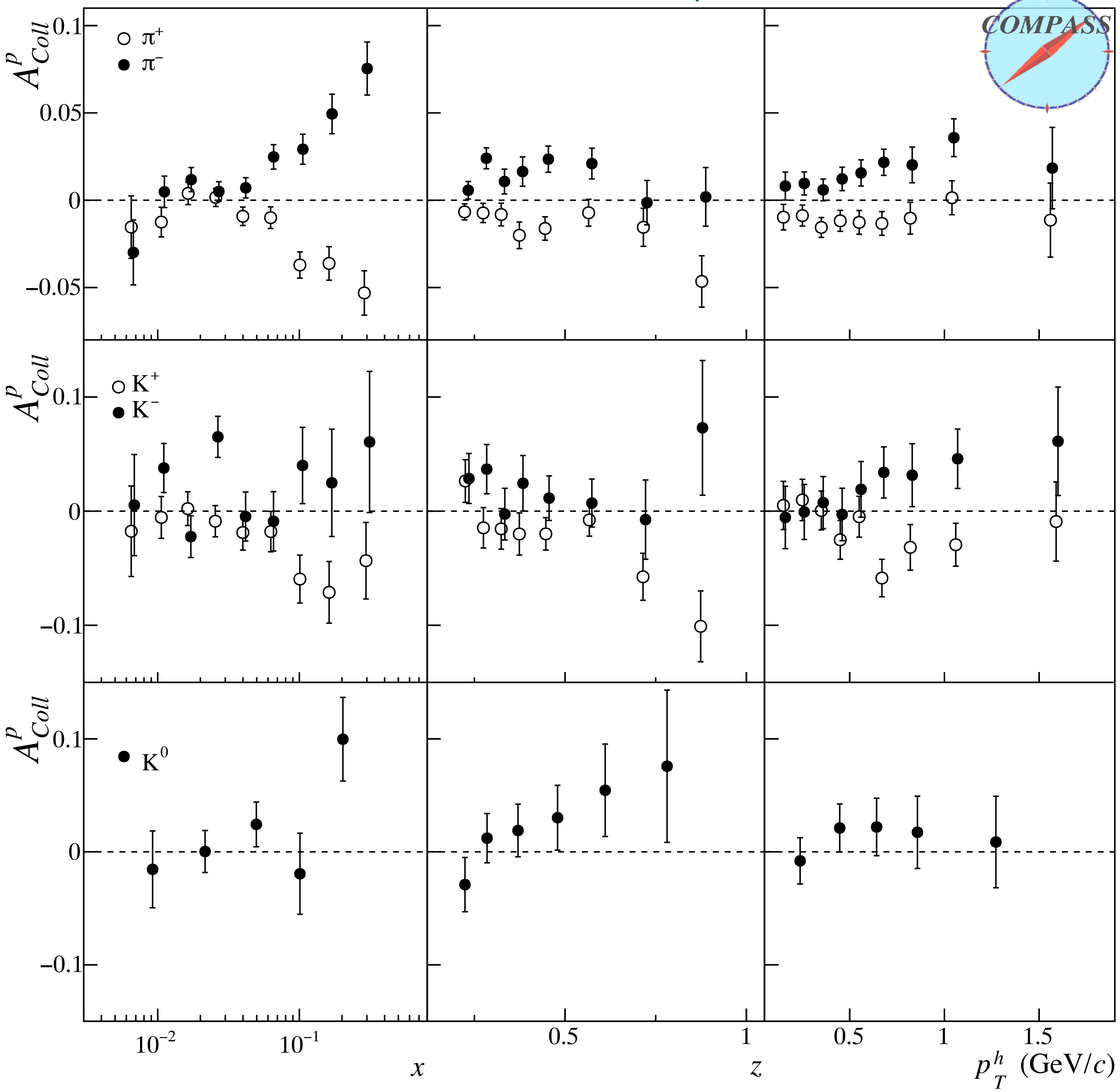
Collins amplitudes

- since those early days, a wealth of new results:
  - **COMPASS**
    [PLB 692 (2010) 240,
    PLB 717 (2012) 376, PLB 744 (2015) 250]
  - **HERMES**
    [PLB 693 (2010) 11]
  - **Jefferson Lab**
    [PRL 107 (2011) 072003]

[A. Airapetian et al, PLB 693 (2010) 11]



[C. Adolph, PLB 744 (2015) 250]



	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Collins amplitudes

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    [PLB 693 (2010) 11]
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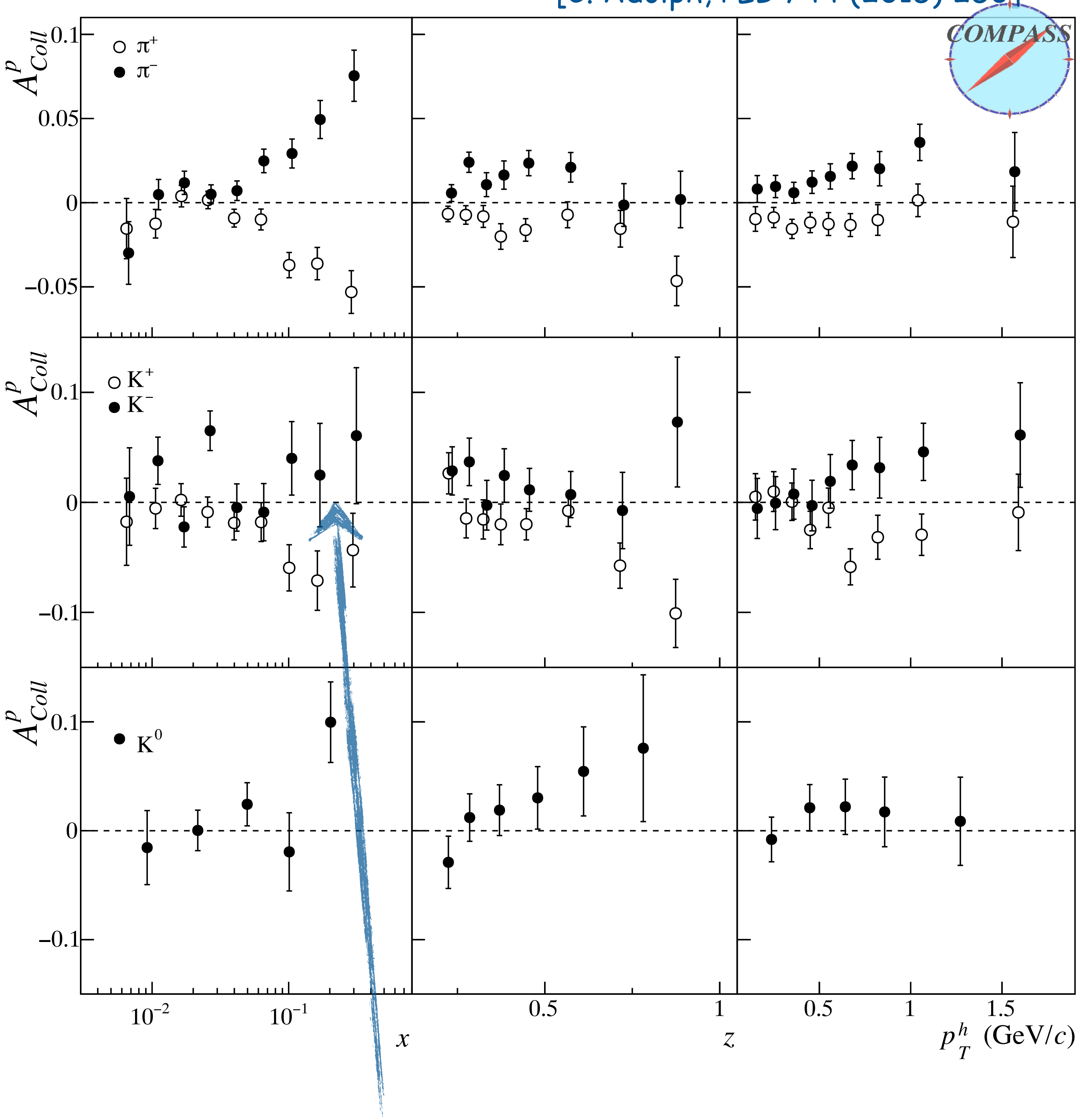
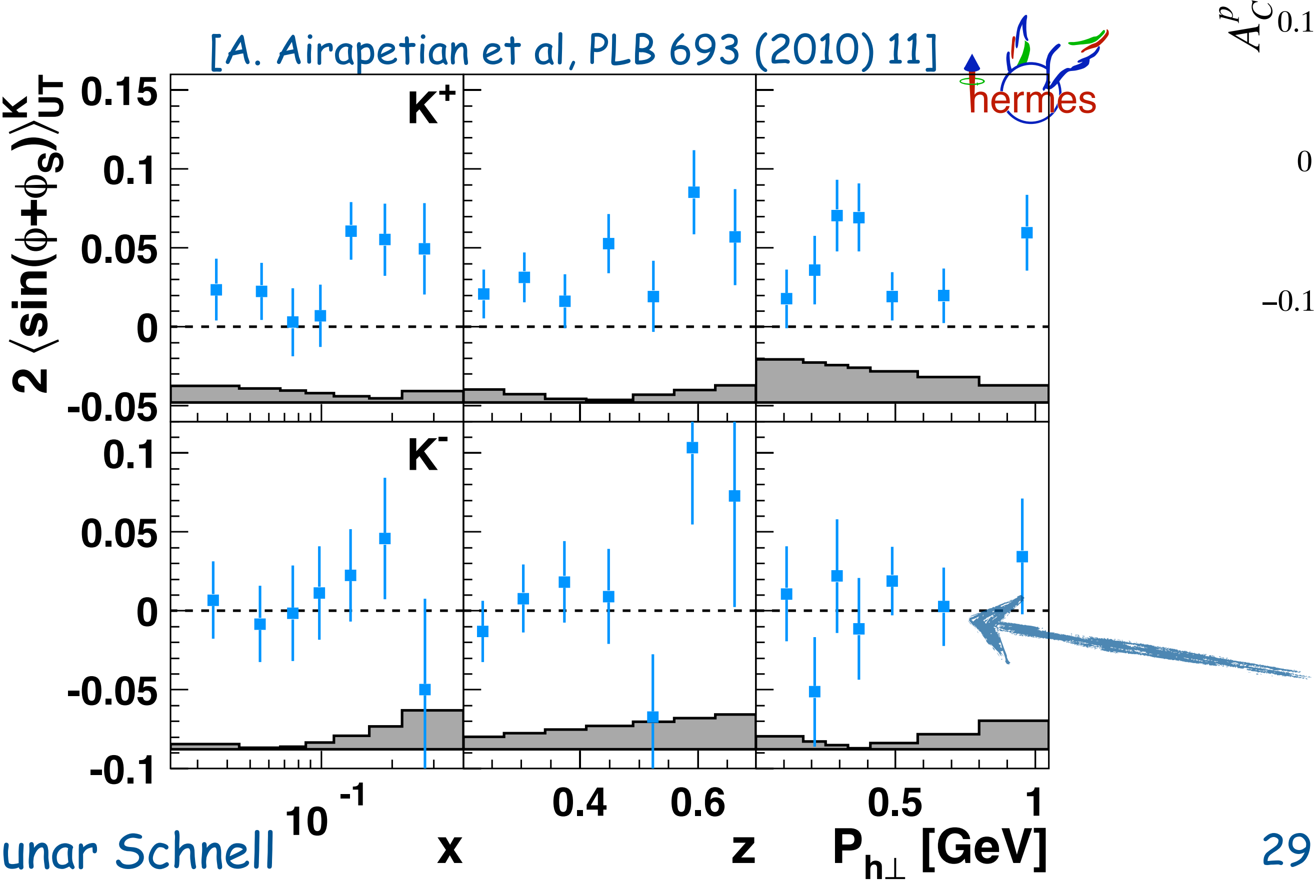
    [PRL 107 (2011) 072003]

[C. Adolph, PLB 744 (2015) 250]



[A. Airapetian et al, PLB 693 (2010) 11]



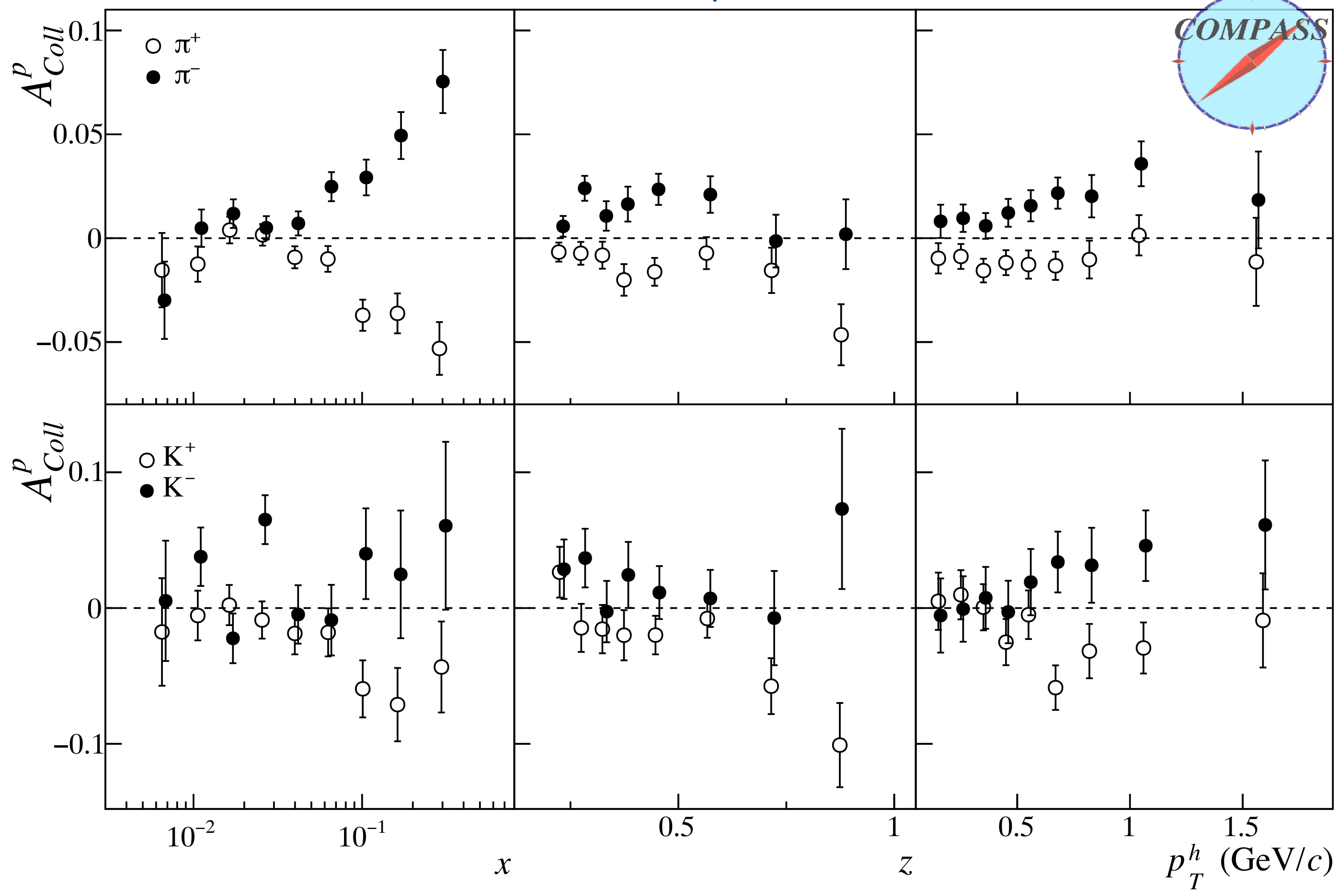
cancelation of (unfavored) u and d fragmentation (opposite signs of up and down transversity)?

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

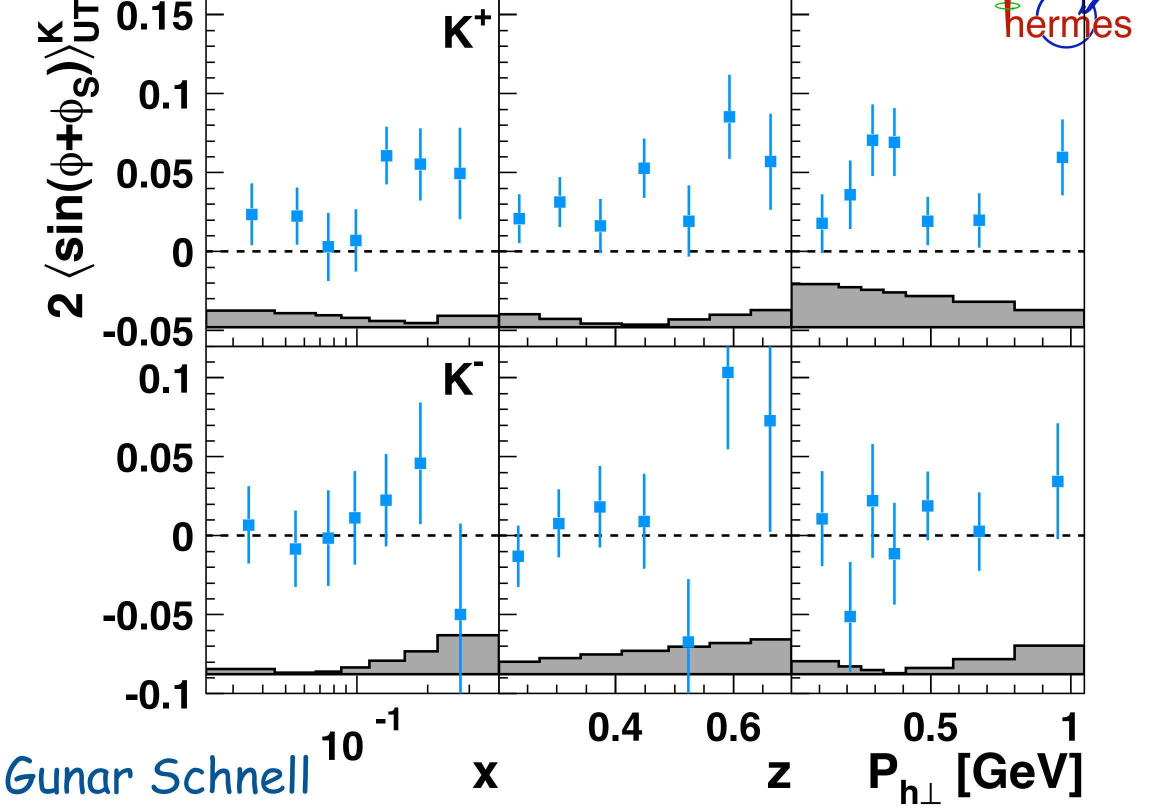
Collins amplitudes

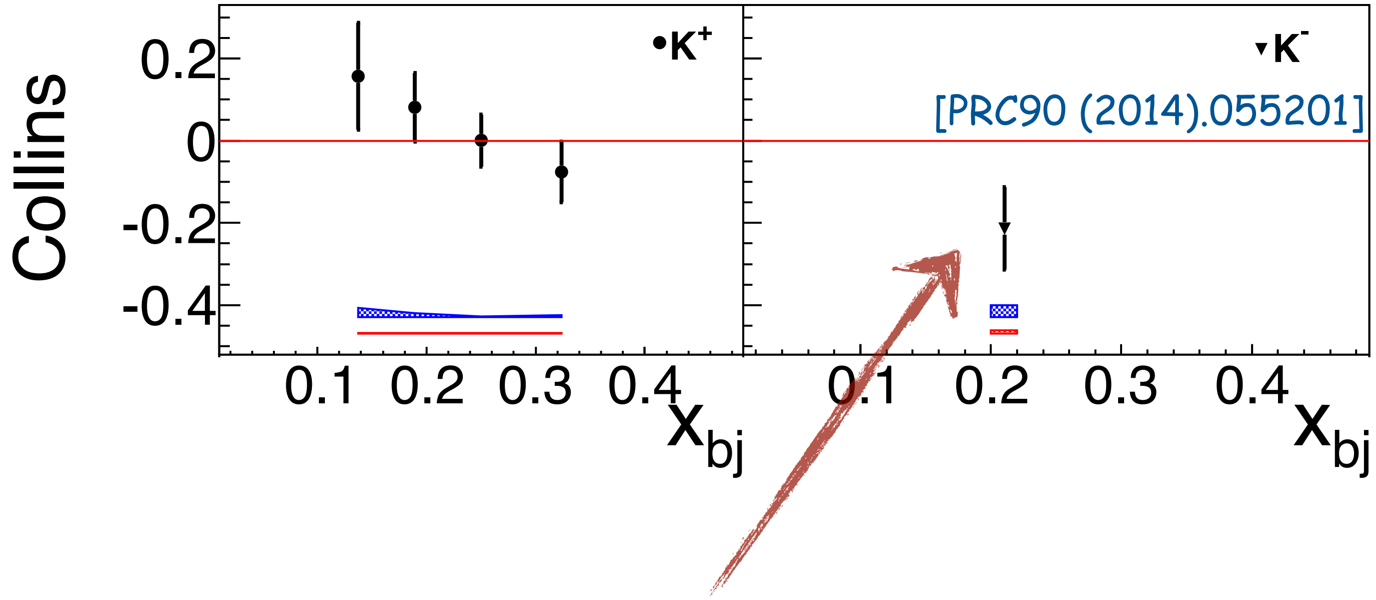
- since those early days, a wealth of new results:
  - COMPASS
    [PLB 692 (2010) 240,
    PLB 717 (2012) 376, PLB 744 (2015) 250]
  - HERMES
    [PLB 693 (2010) 11]
  - Jefferson Lab
    [PRL 107 (2011) 072003, PRC90 (2014).055201]

[Adolph et al., PLB 744 (2015) 250]



[A. Airapetian et al, PLB 693 (2010) 11]





but relatively large K^- asymmetry on ^{3}He?

the “Collins trap”

$$H_{1,\mathrm{fav}}^{\perp} \simeq -H_{1,\mathrm{dis}}^{\perp}$$

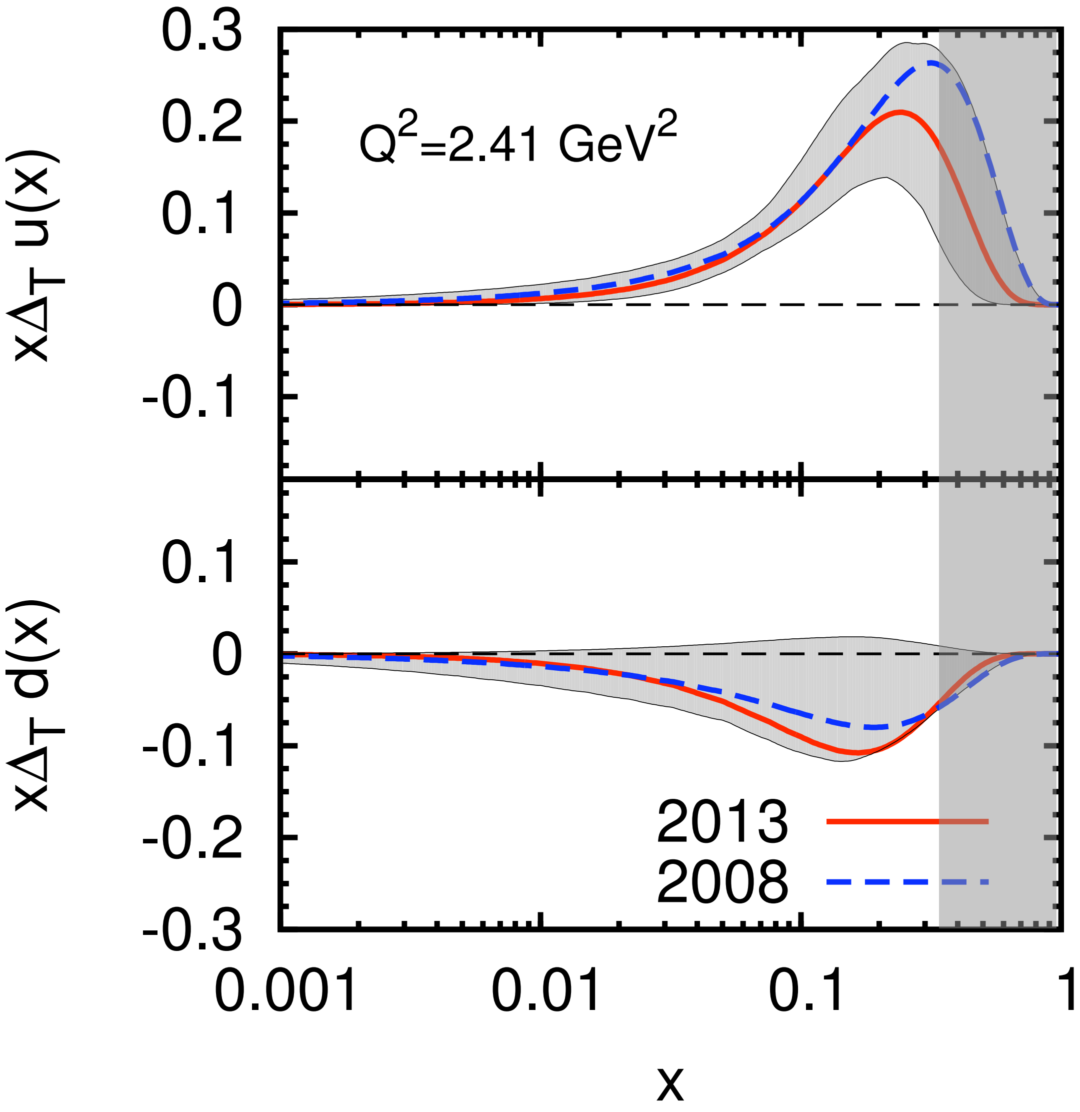
thus

$$\langle \sin(\phi+\phi_S)\rangle_{UT}^{\pi^+} \sim \left(4h_1^u - h_1^d\right) H_{1,\mathrm{fav}}^{\perp}$$

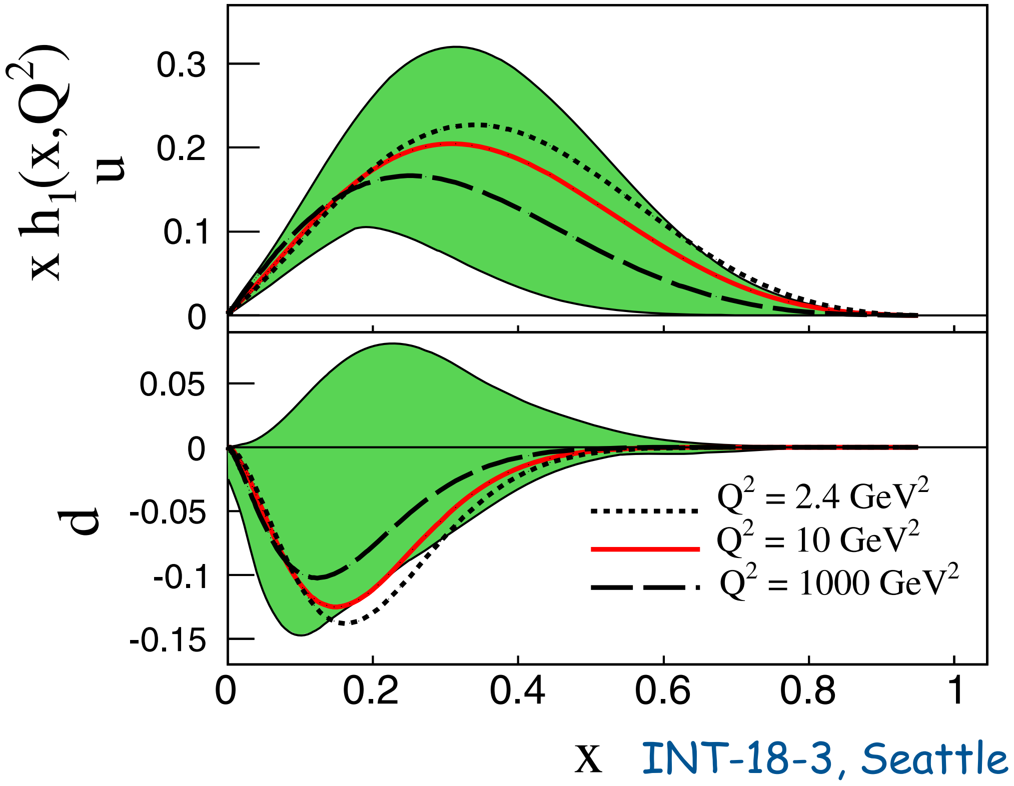
$$\langle \sin(\phi+\phi_S)\rangle_{UT}^{\pi^-} \sim -\left(4h_1^u - h_1^d\right) H_{1,\mathrm{fav}}^{\perp}$$

“impossible” to disentangle u/d transversity -> current limits driven mainly by Soffer bound?

[M. Anselmino et al., PRD 87 (2013) 094019]



[Z.B. Kang et al. PRD93 (2016) 014009]



the “Collins trap“

$$H_{1,\mathrm{fav}}^{\perp} \simeq -H_{1,\mathrm{dis}}^{\perp}$$

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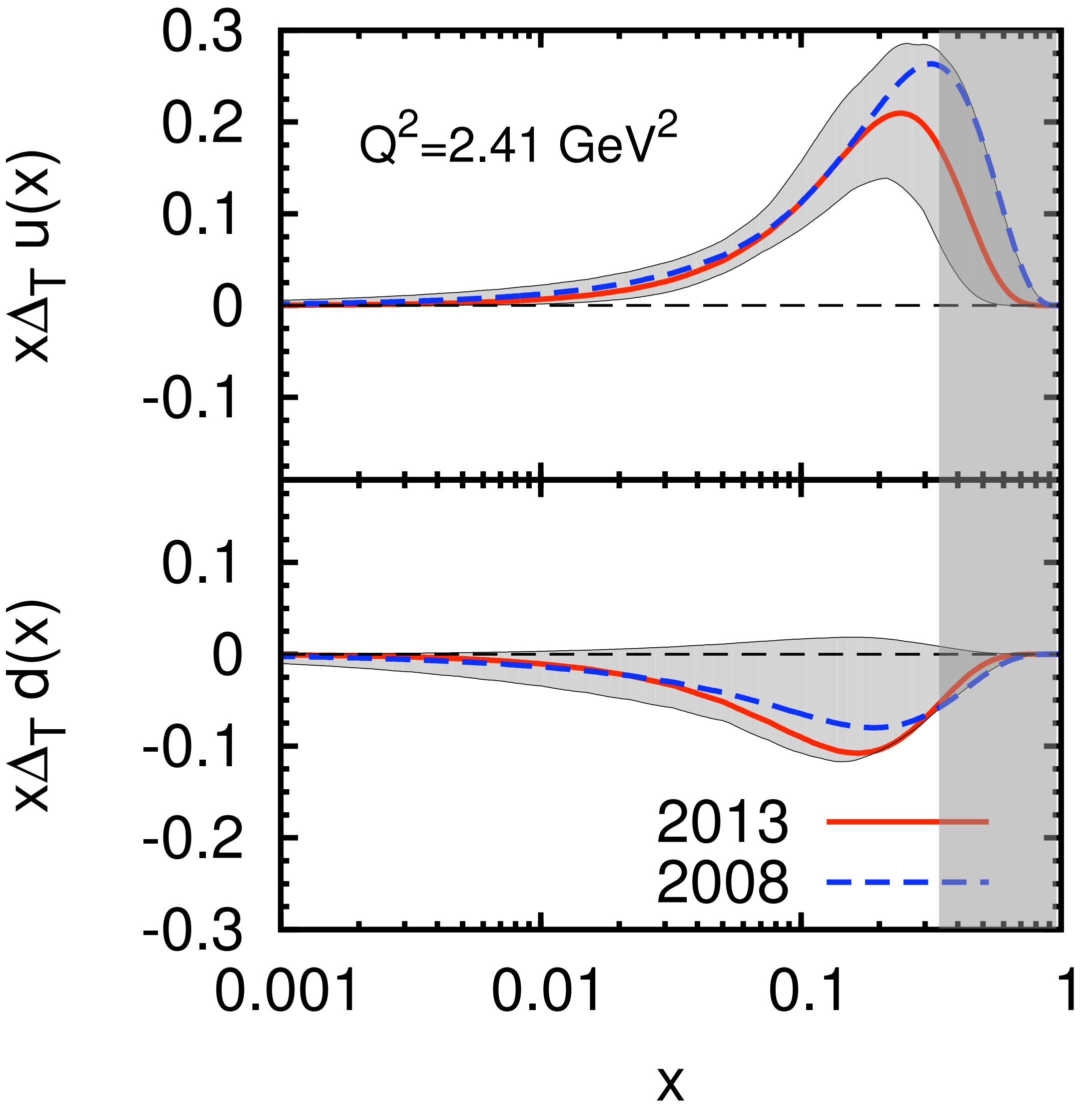
$$\langle \sin(\phi+\phi_S)\rangle_{UT}^{\pi^-} \sim -\left(4h_1^u - h_1^d\right) H_{1,\mathrm{fav}}^{\perp}$$

“impossible“ to disentangle u/d transversity -> current limits driven mainly by Soffer bound?

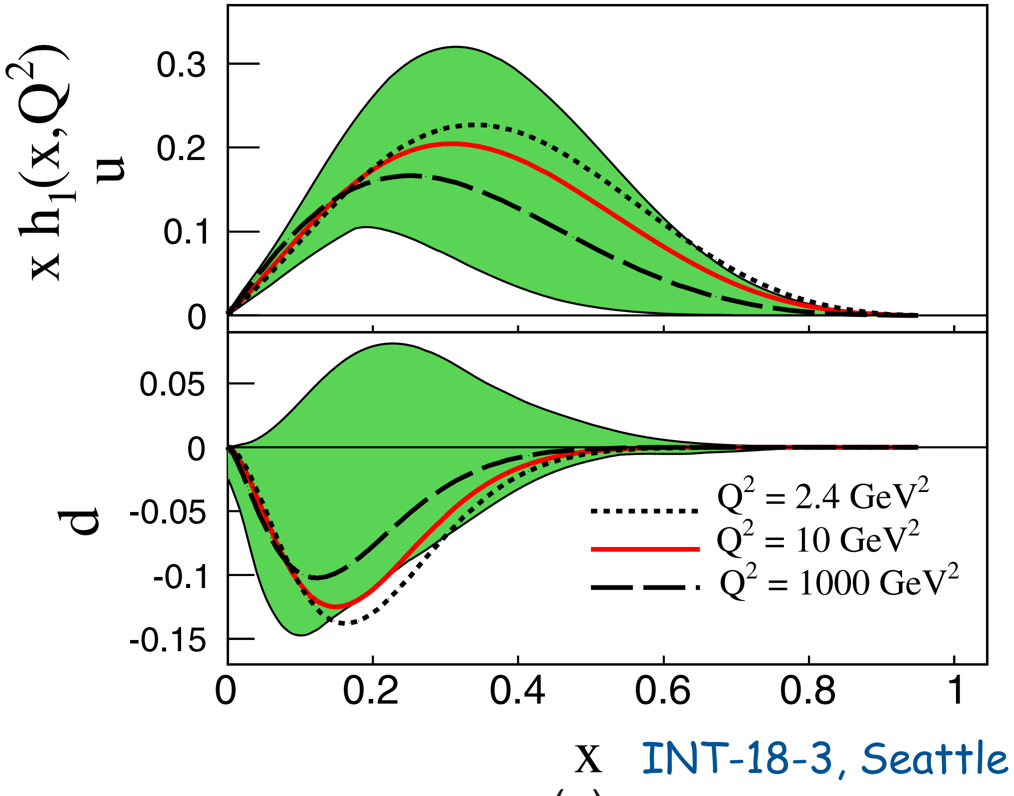
clearly need precise data from “neutron“ target(s), e.g., COMPASS d, and later JLab12 & EIC

(valid for all chiral-odd TMDs)

[M. Anselmino et al., PRD 87 (2013) 094019]

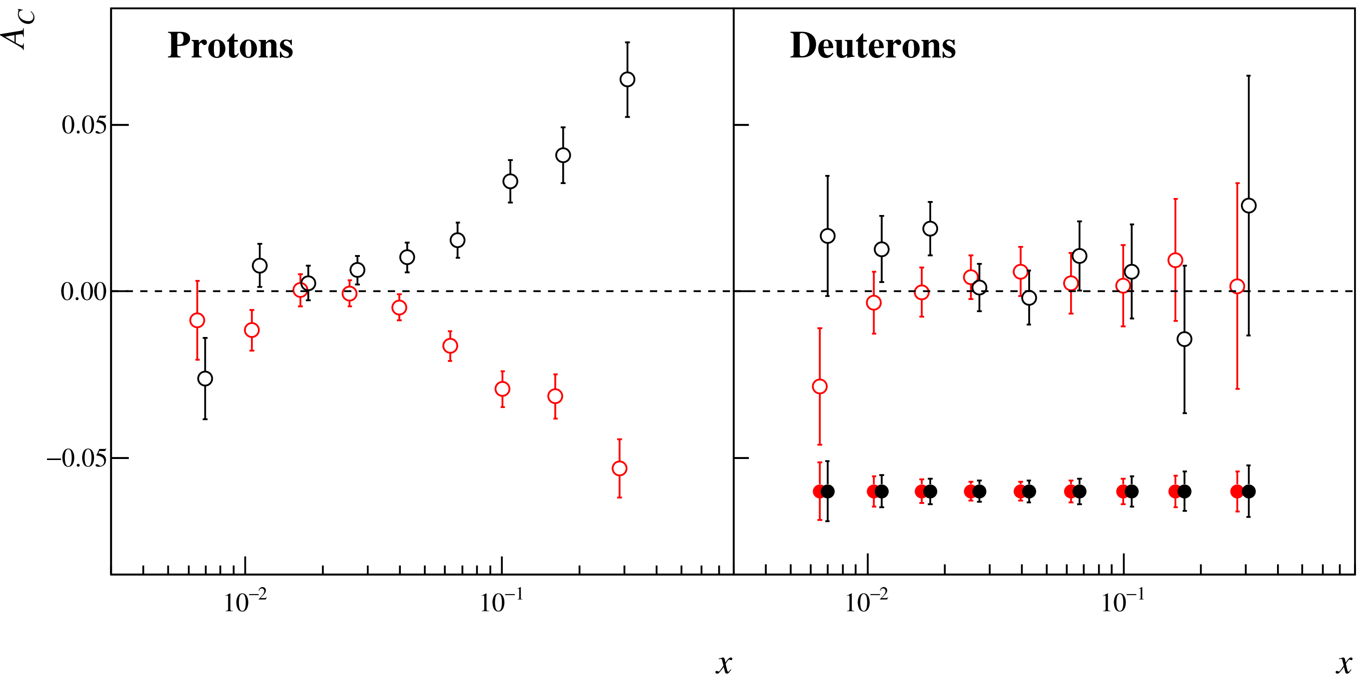


[Z.B. Kang et al. PRD93 (2016) 014009]

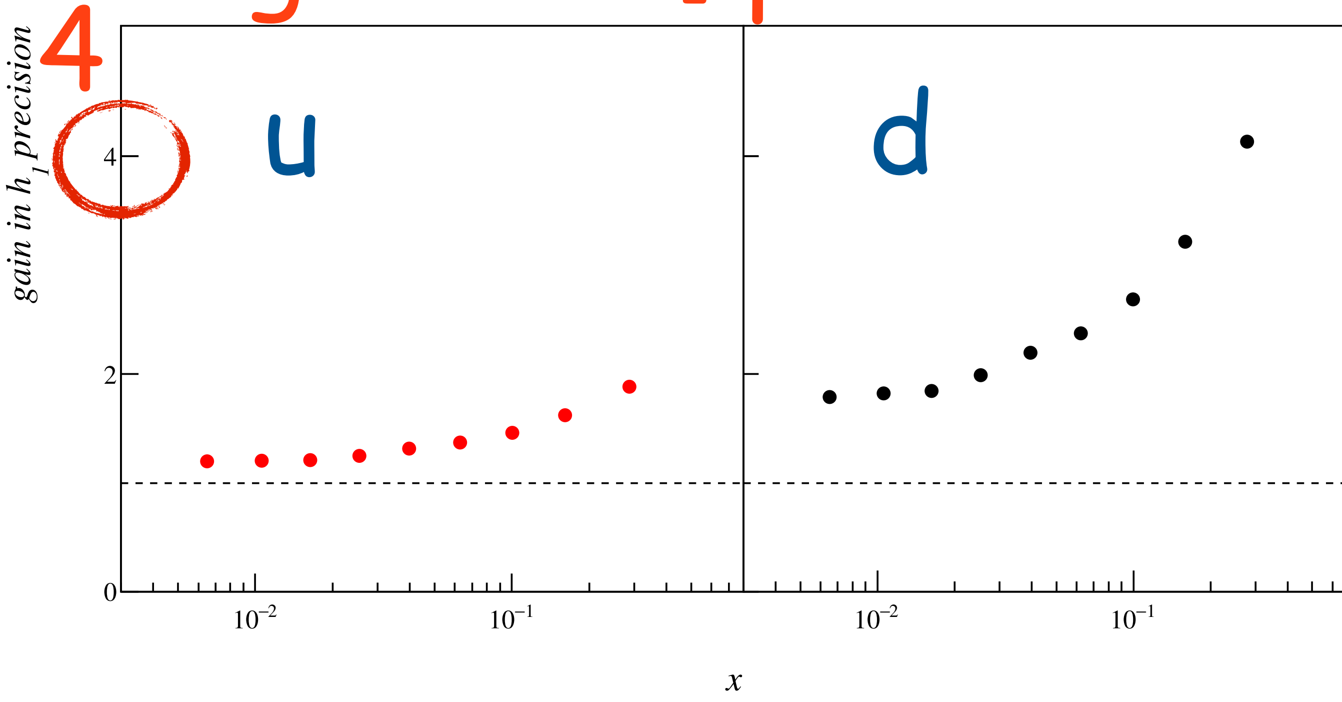


d-transversity running at COMPASS

- currently much more p than d data available
- add another year of d running after CERN LS2 (2021)
  - large impact on d-transversity
  - reduced correlations between u and d transversity (note, correlations important in tensor-charge calculation)



gain in h_1 precision

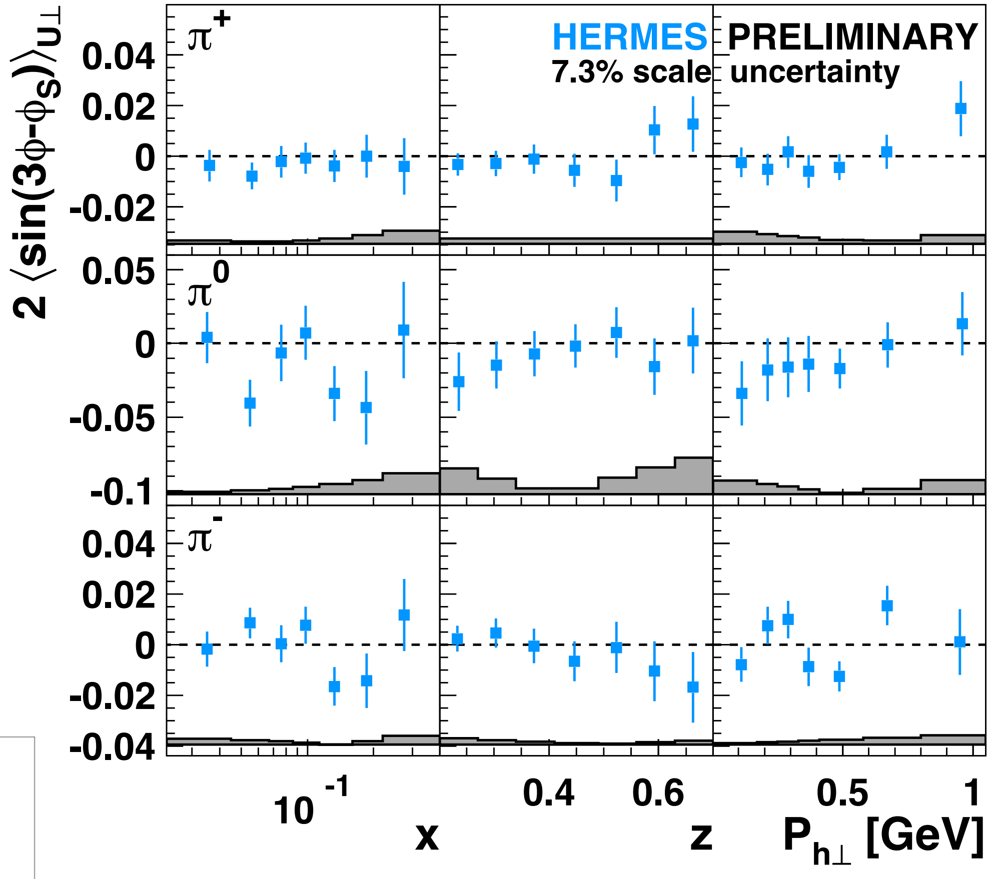


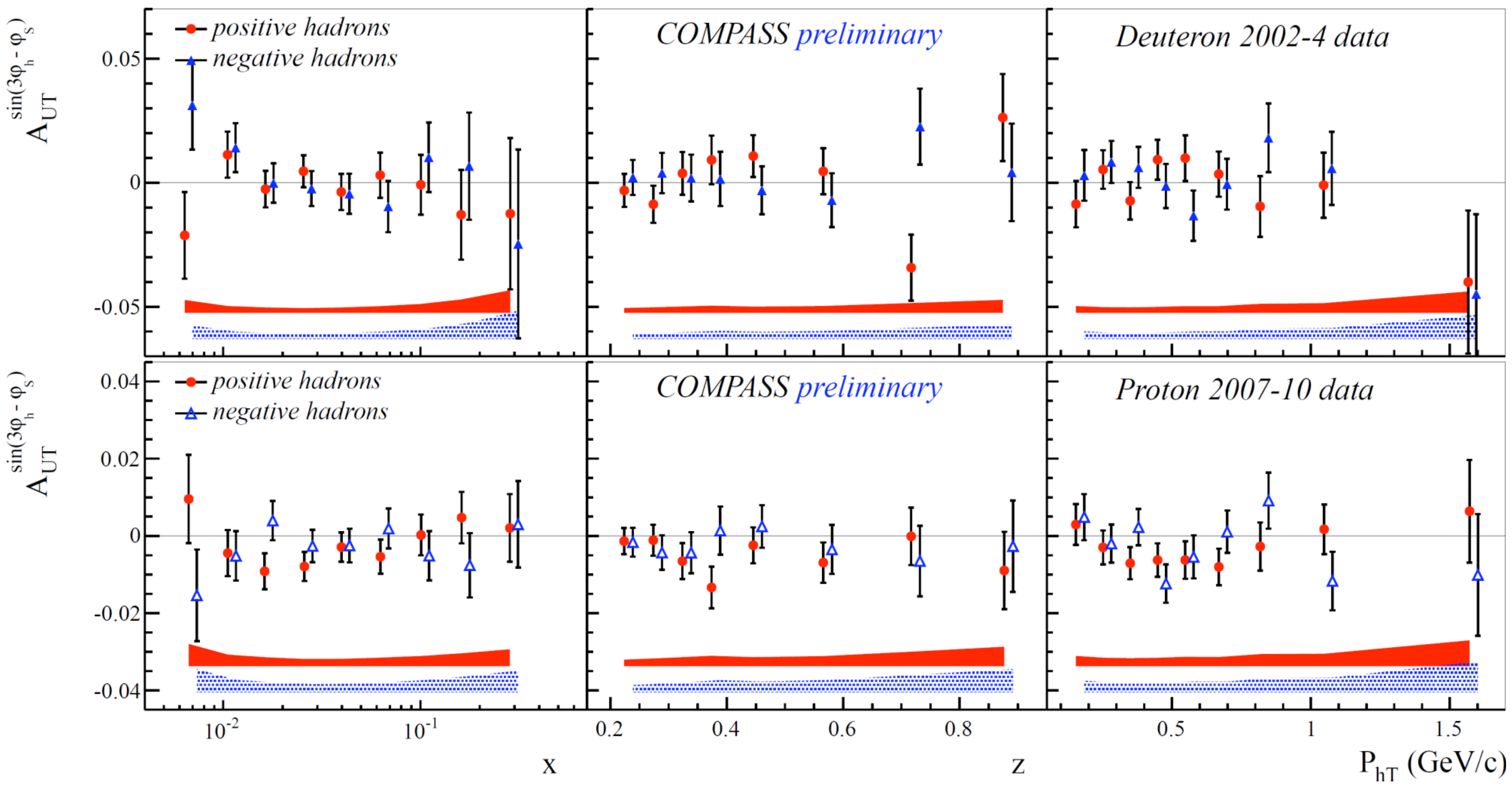
Transversity's friends

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

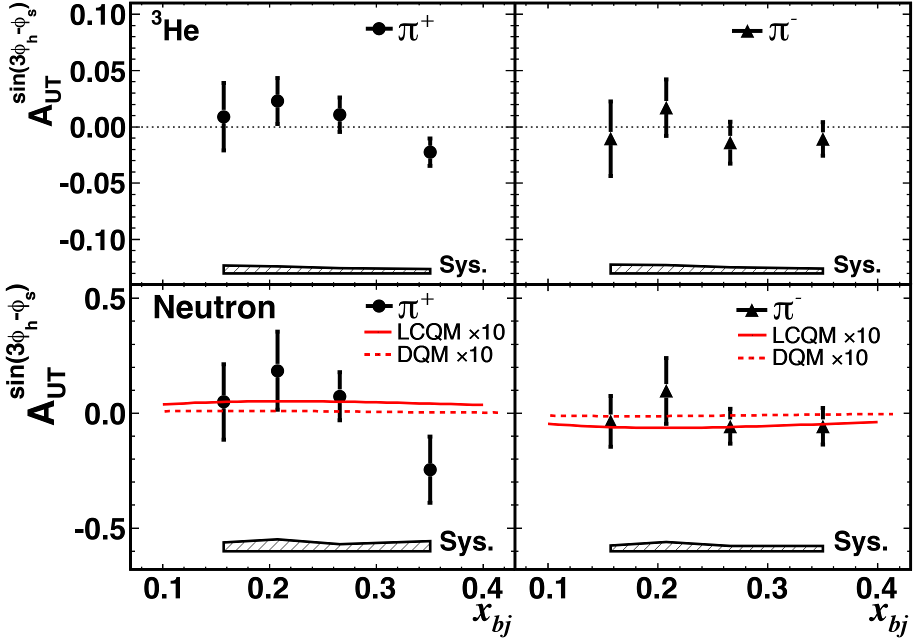
Pretzelosity

- chiral-odd ➥ needs Collins FF (or similar)
- ^{1}H, ^{2}H & ^{3}He data consistently small
- cancelations? pretzelosity=zero?
  or just the additional suppression by two powers of $P_{h\perp}$



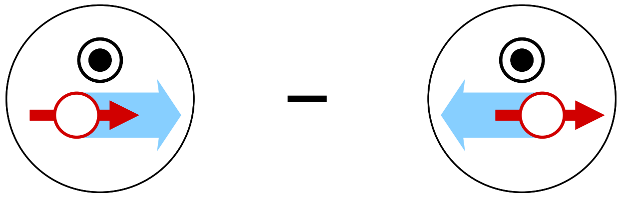


[JLab Hall A, PRC 90 (2014) 055209]



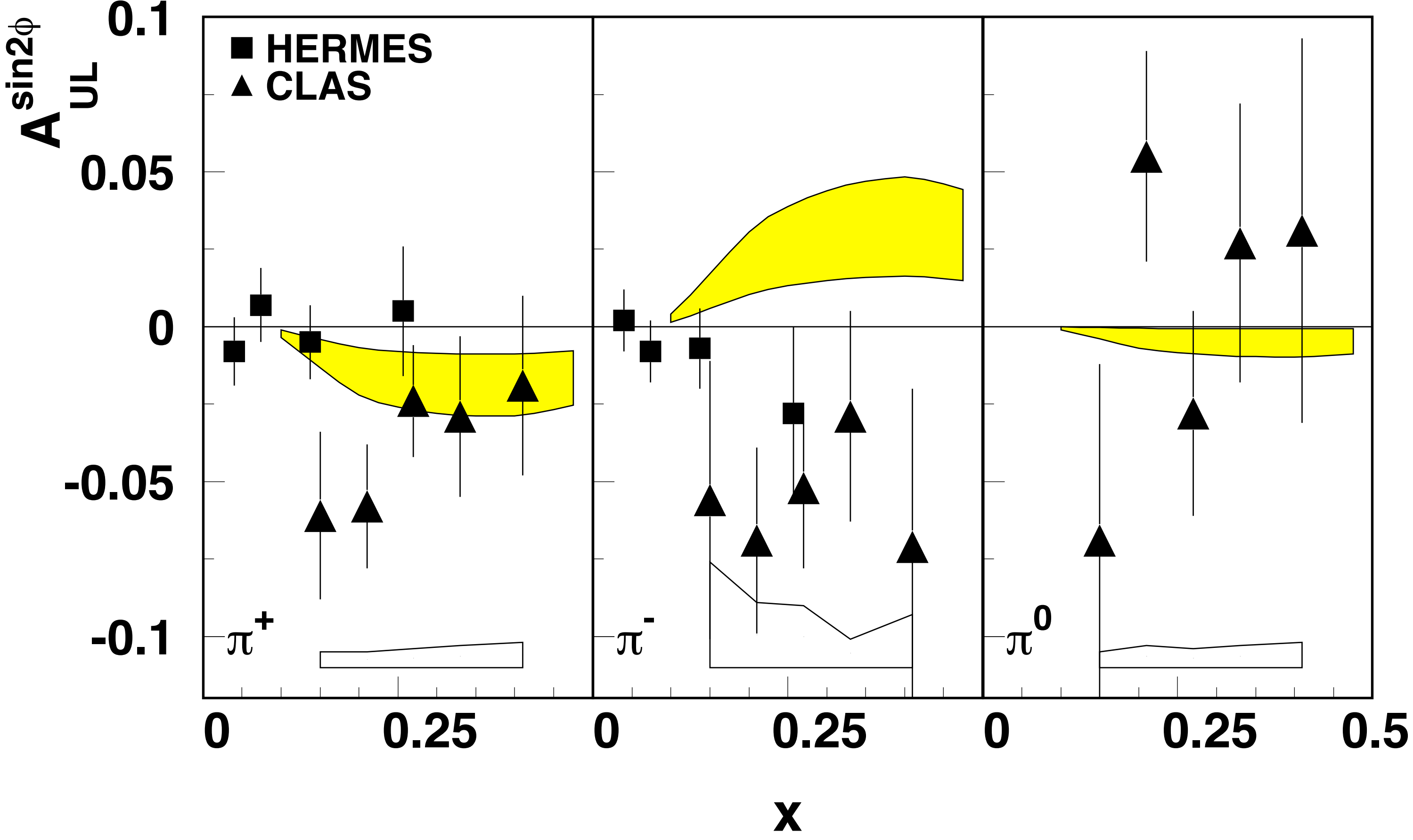
Worm-Gear I

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

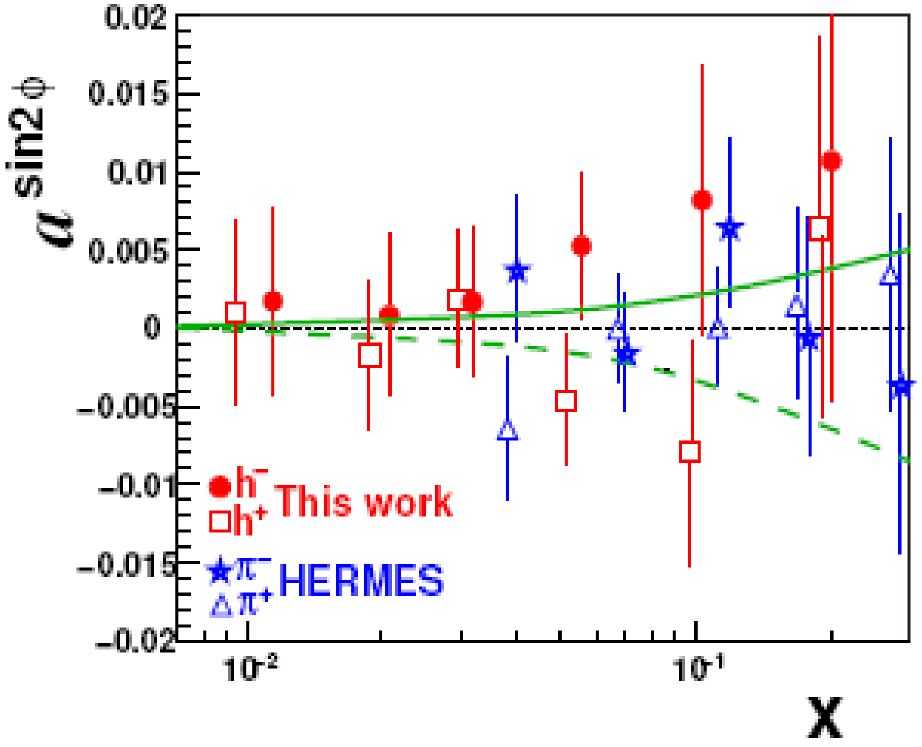


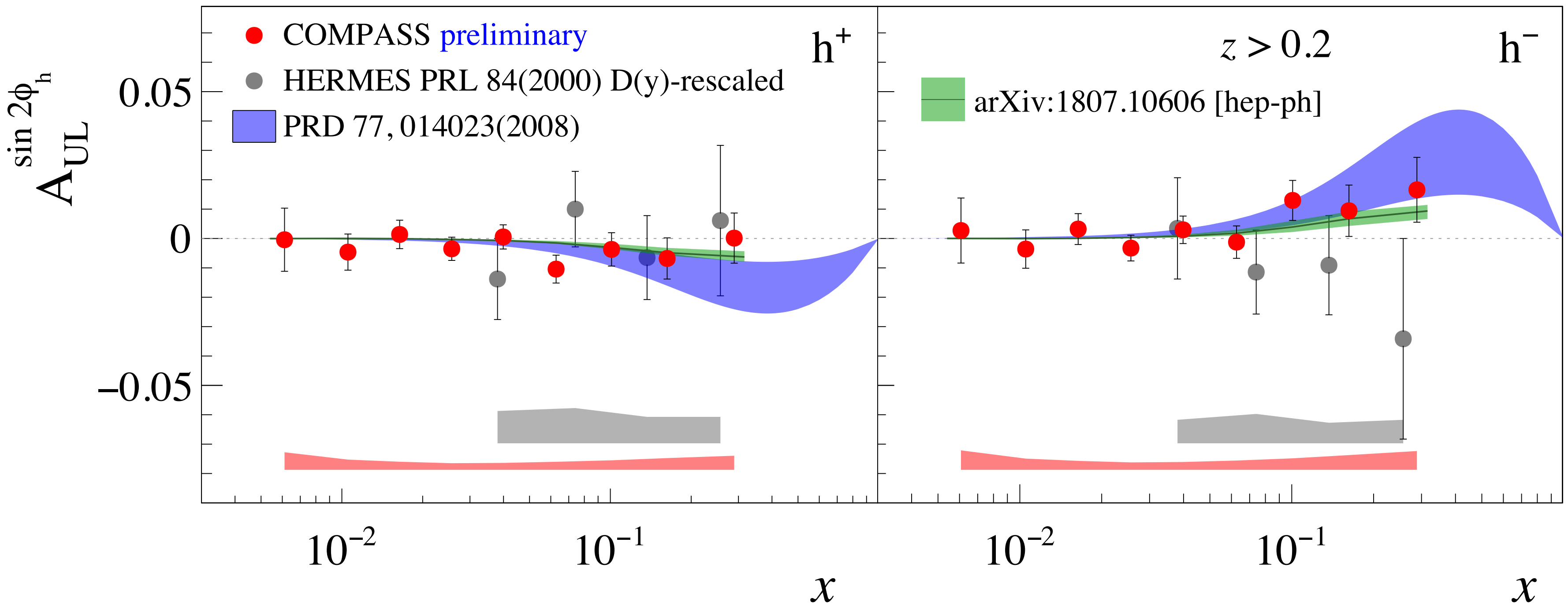
- again: chiral-odd
- evidence from CLAS?
- consistent with zero at COMPASS and HERMES

[CLAS, PRL 105 (2010) 262002]



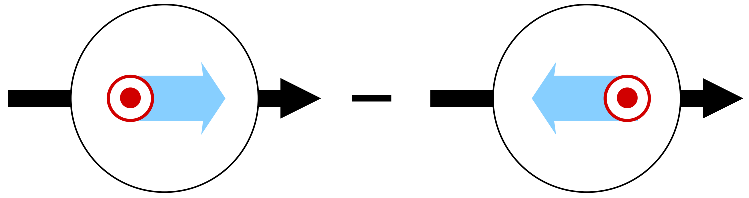
[COMPASS, EPJ C 70 (2010) 39]

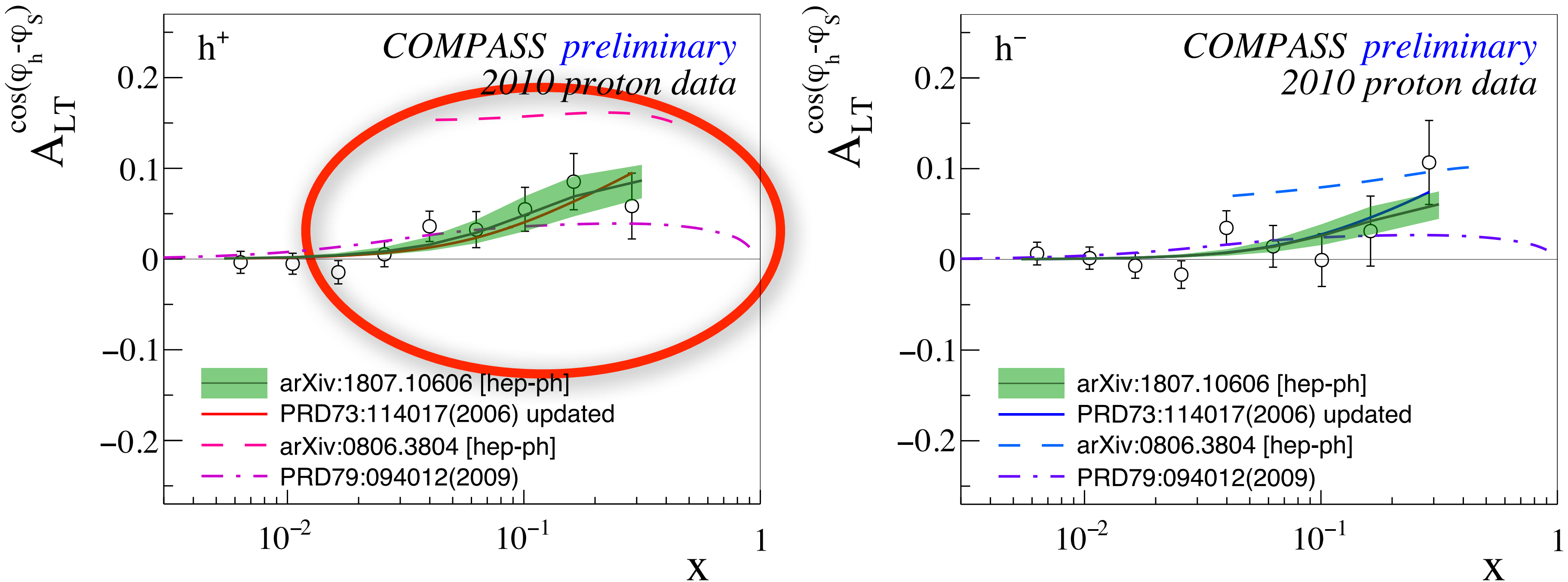




Worm-Gear II

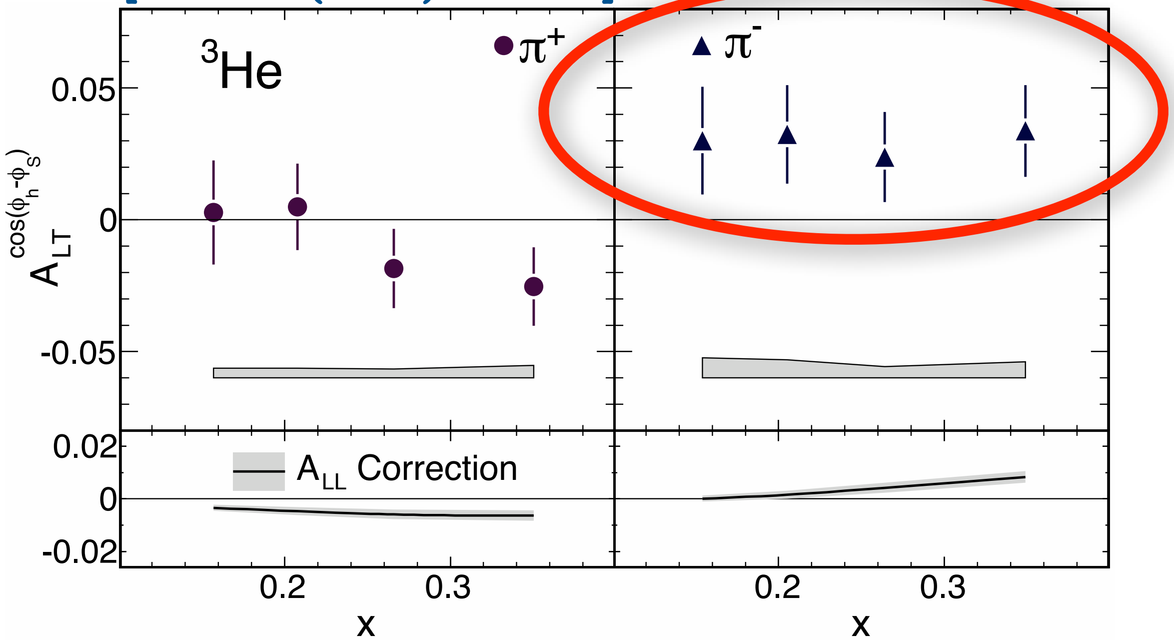
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

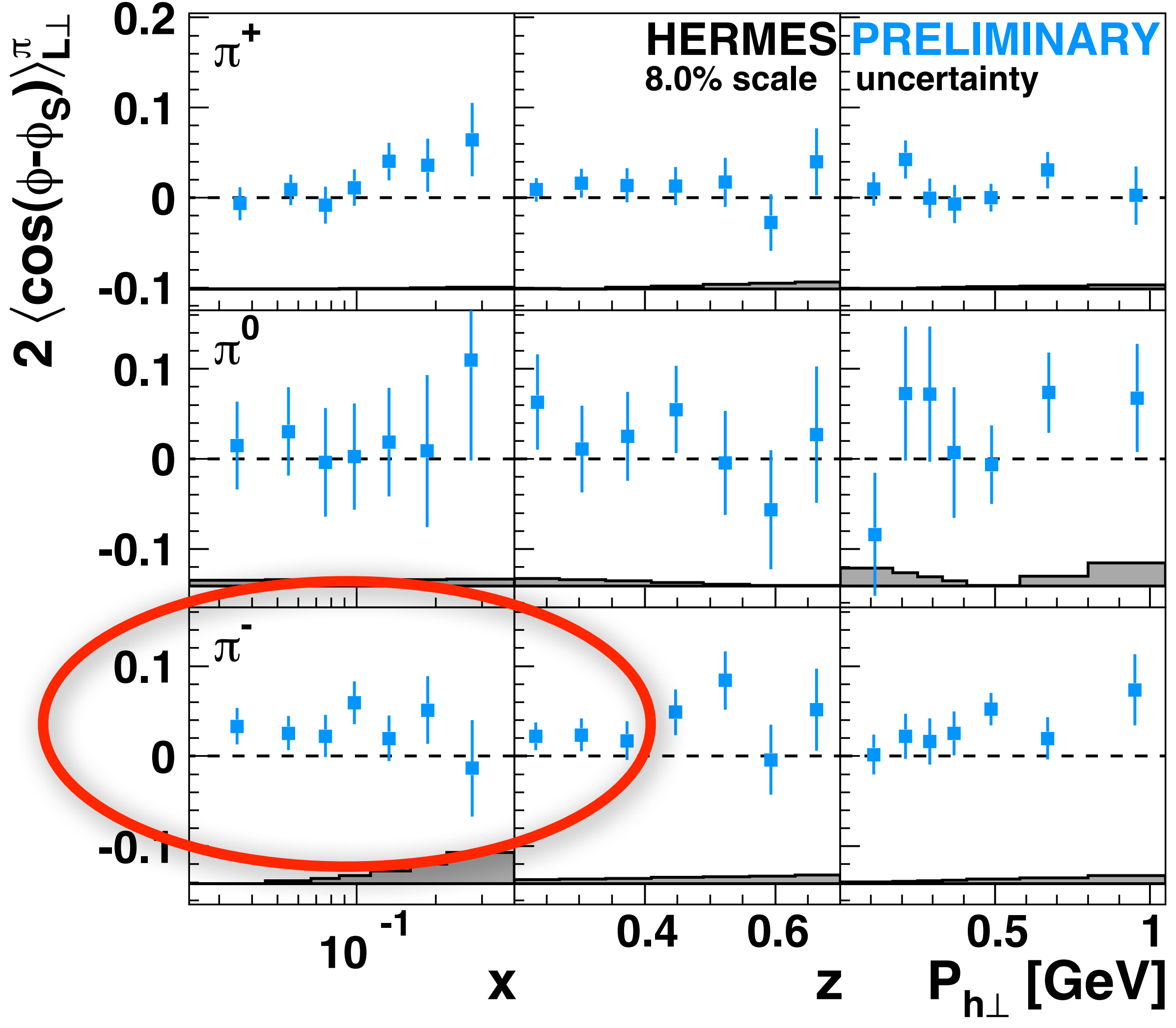




- first evidences:
  - ^{3}He target at JLab
  - H target at COMPASS & HERMES

[PRL 108 (2012) 052001]

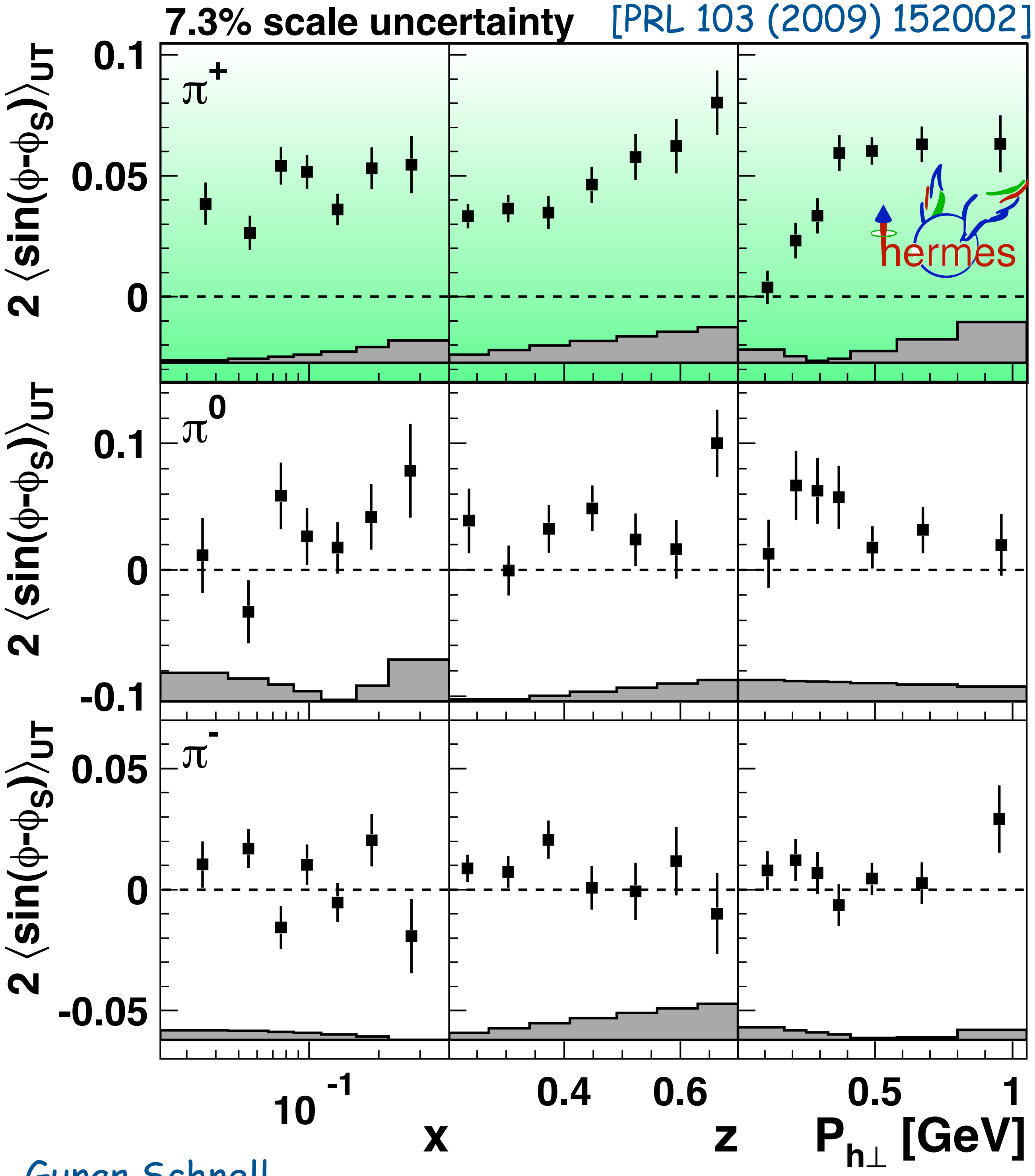




	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes for pions

$$2\langle\sin(\phi-\phi_S)\rangle_{\mathrm{UT}} = -\frac{\sum_q e_q^2 f_{1\mathrm{T}}^{\perp,q}(x,p_T^2) \otimes_{\mathcal{W}} D_1^q(z,k_T^2)}{\sum_q e_q^2 f_1^q(x,p_T^2) \otimes D_1^q(z,k_T^2)}$$



[PRL 103 (2009) 152002]

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes for pions

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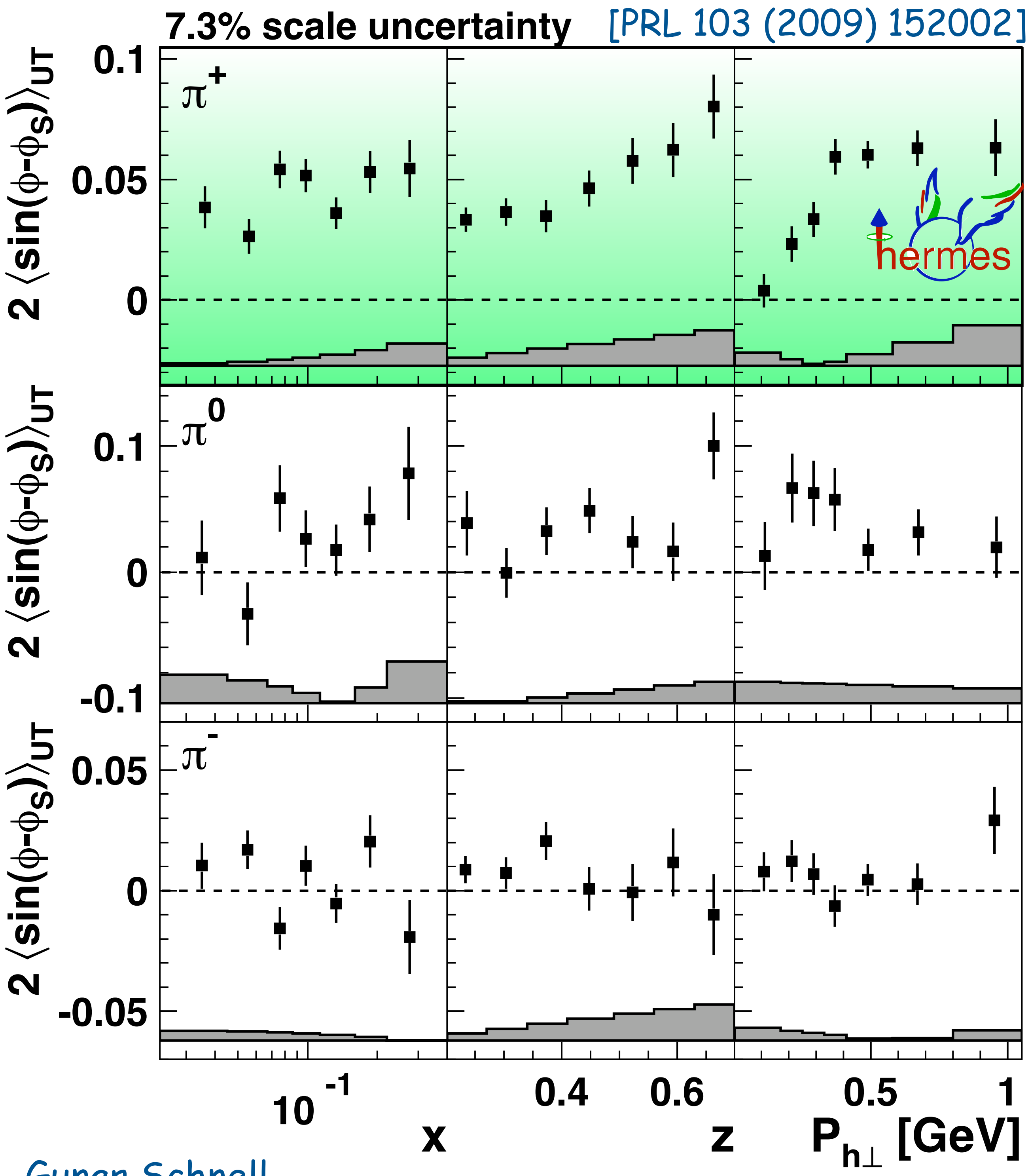


[PRL 103 (2009) 152002]

π^+ dominated by u-quark scattering:

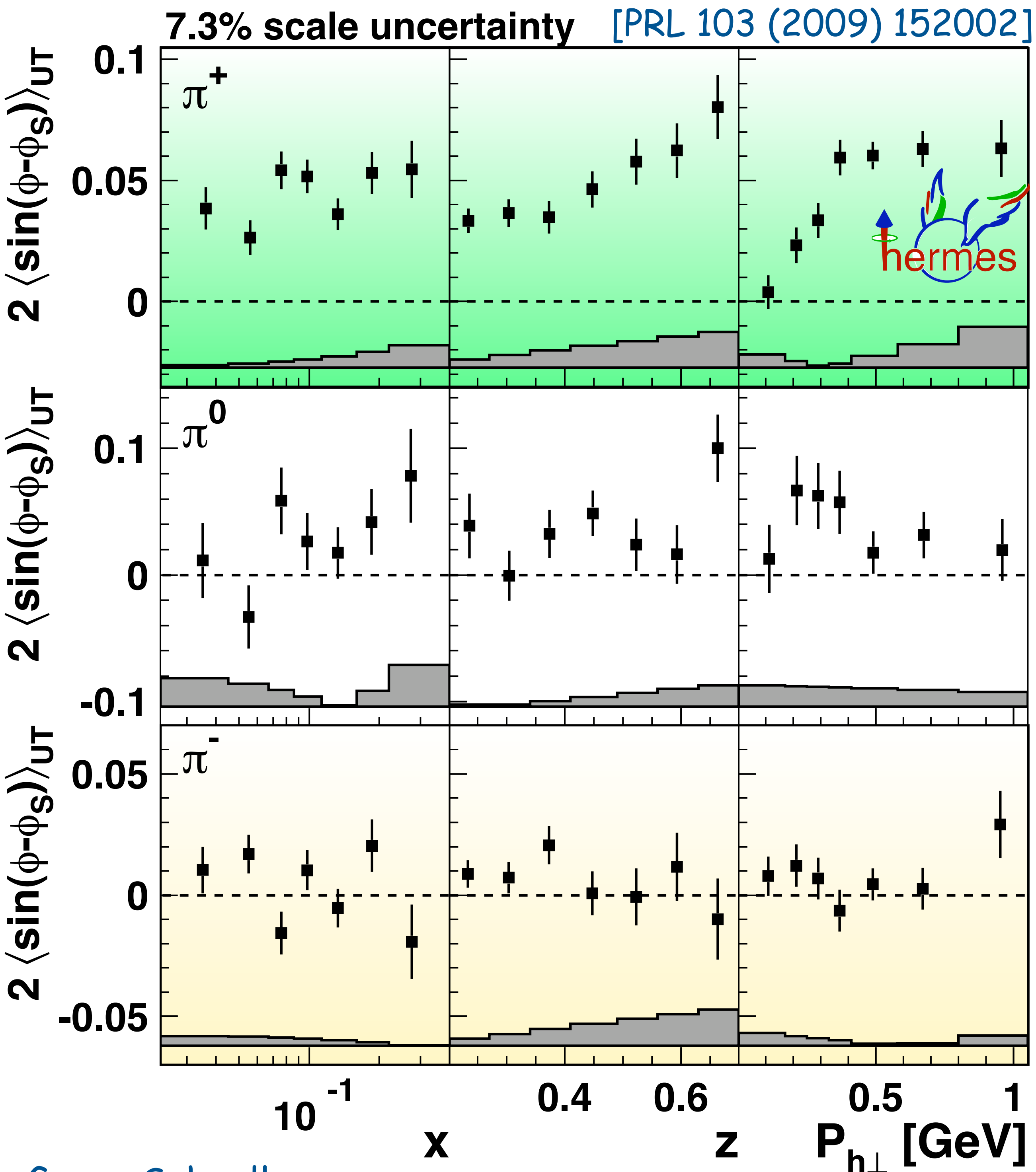
$$\simeq -\frac{f_{1\mathrm{T}}^{\perp,u}(x,p_T^2) \otimes_{\mathcal{W}} D_1^{u\to\pi^+}(z,k_T^2)}{f_1^u(x,p_T^2) \otimes D_1^{u\to\pi^+}(z,k_T^2)}$$

☛ u-quark Sivers DF < 0

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes for pions

$$2\langle\sin(\phi-\phi_S)\rangle_{\rm UT} = -\frac{\sum_q e_q^2 f_{1\rm T}^{\perp,q}(x,p_T^2) \otimes_{\mathcal{W}} D_1^q(z,k_T^2)}{\sum_q e_q^2 f_1^q(x,p_T^2) \otimes D_1^q(z,k_T^2)}$$



7.3% scale uncertainty [PRL 103 (2009) 152002]

π^+ dominated by u-quark scattering:

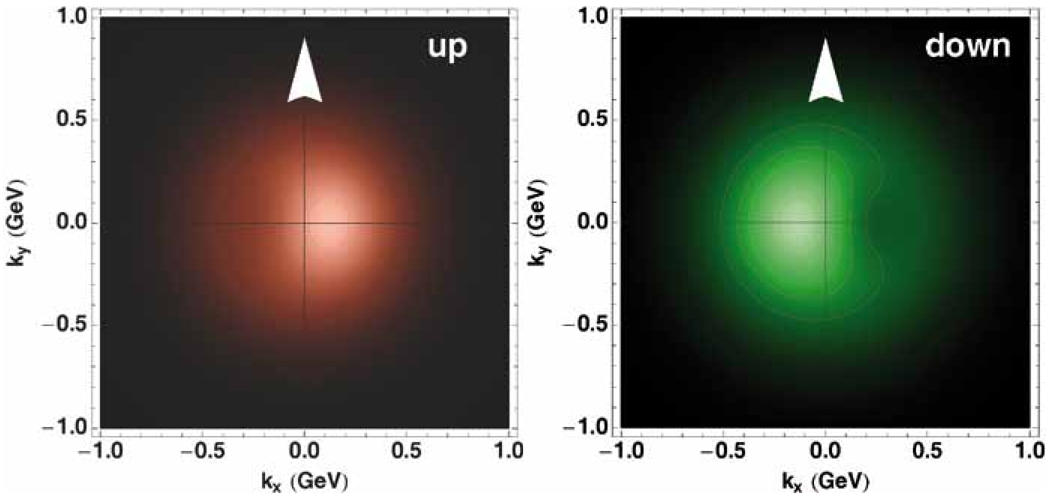
$$\simeq -\frac{f_{1\rm T}^{\perp,u}(x,p_T^2) \otimes_{\mathcal{W}} D_1^{u\to\pi^+}(z,k_T^2)}{f_1^u(x,p_T^2) \otimes D_1^{u\to\pi^+}(z,k_T^2)}$$

☛ u-quark Sivers DF < 0

☛ d-quark Sivers DF > 0 (cancelation for π^-)

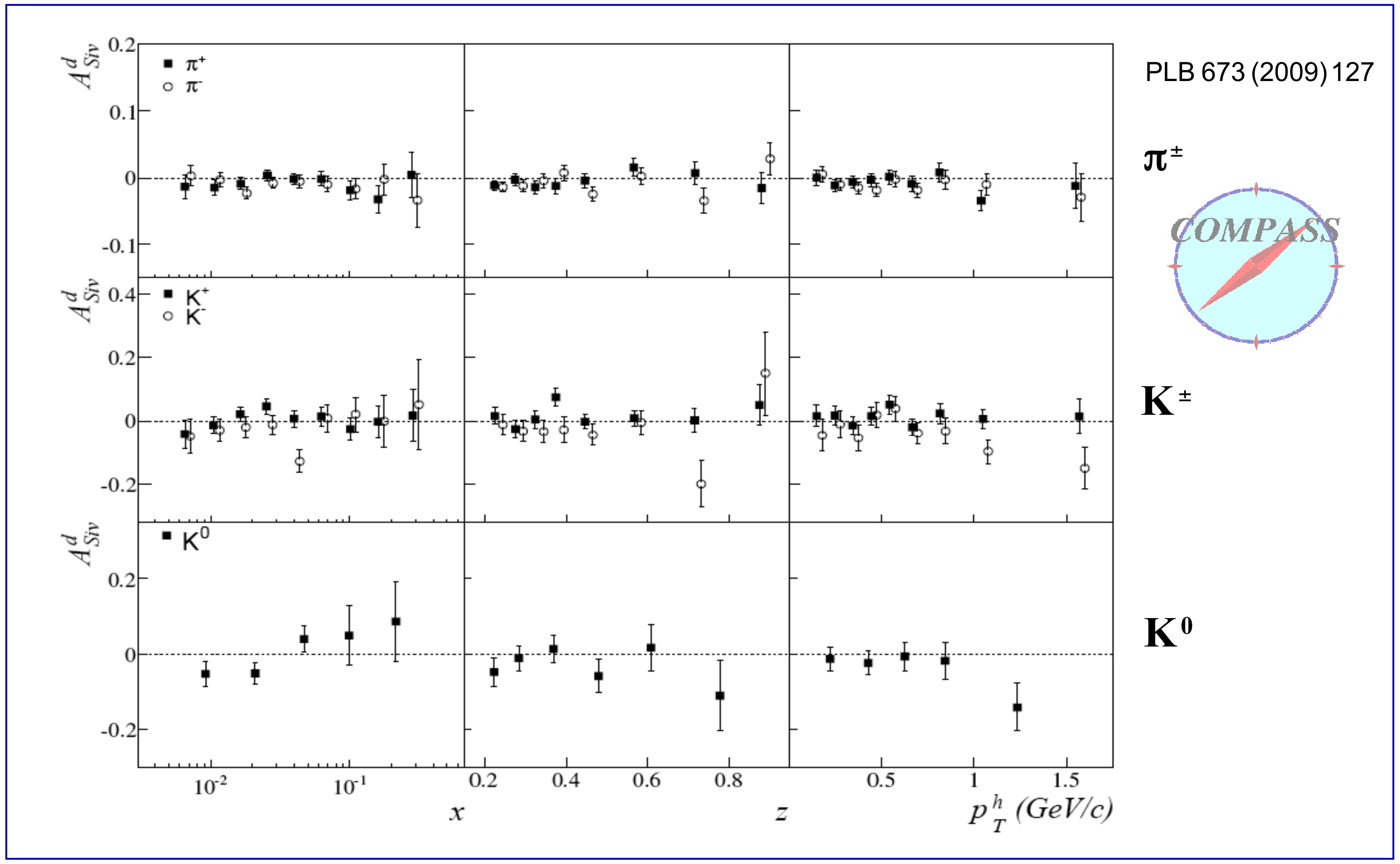
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes



[A. Bacchetta et al.]

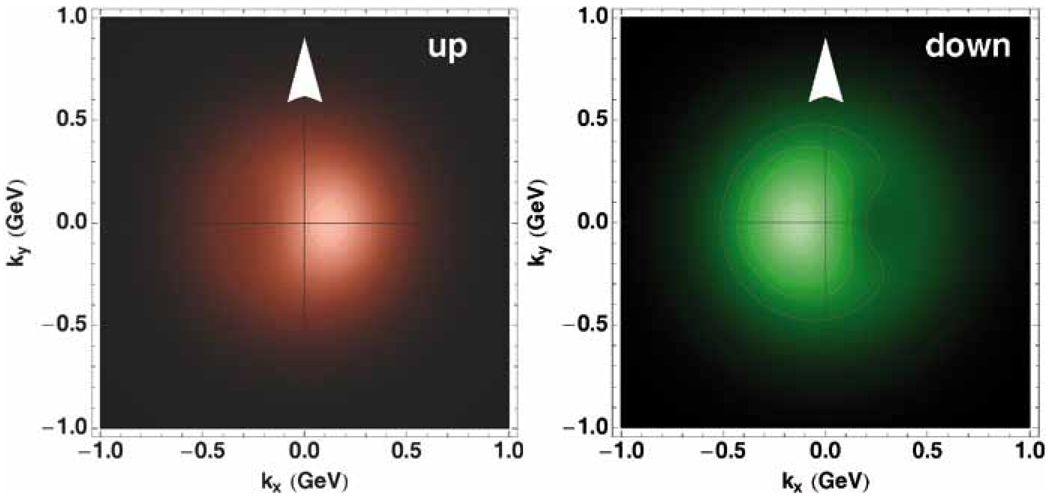
- cancelation for D target supports opposite signs of up and down Sivers



PLB 673 (2009) 127

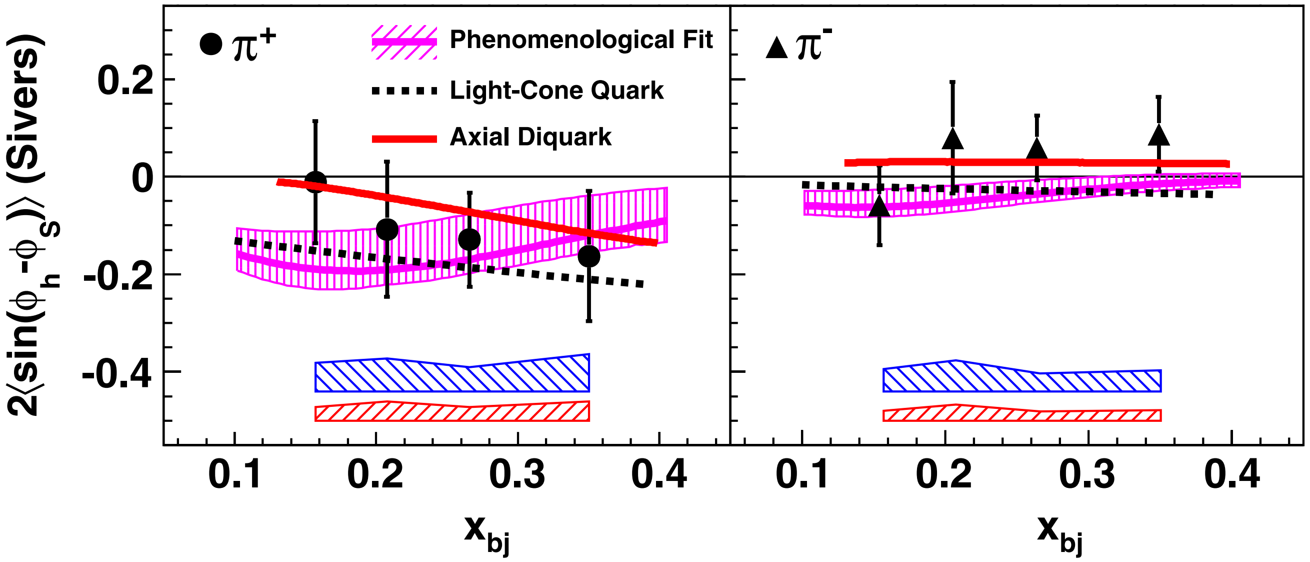
Sivers amplitudes

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

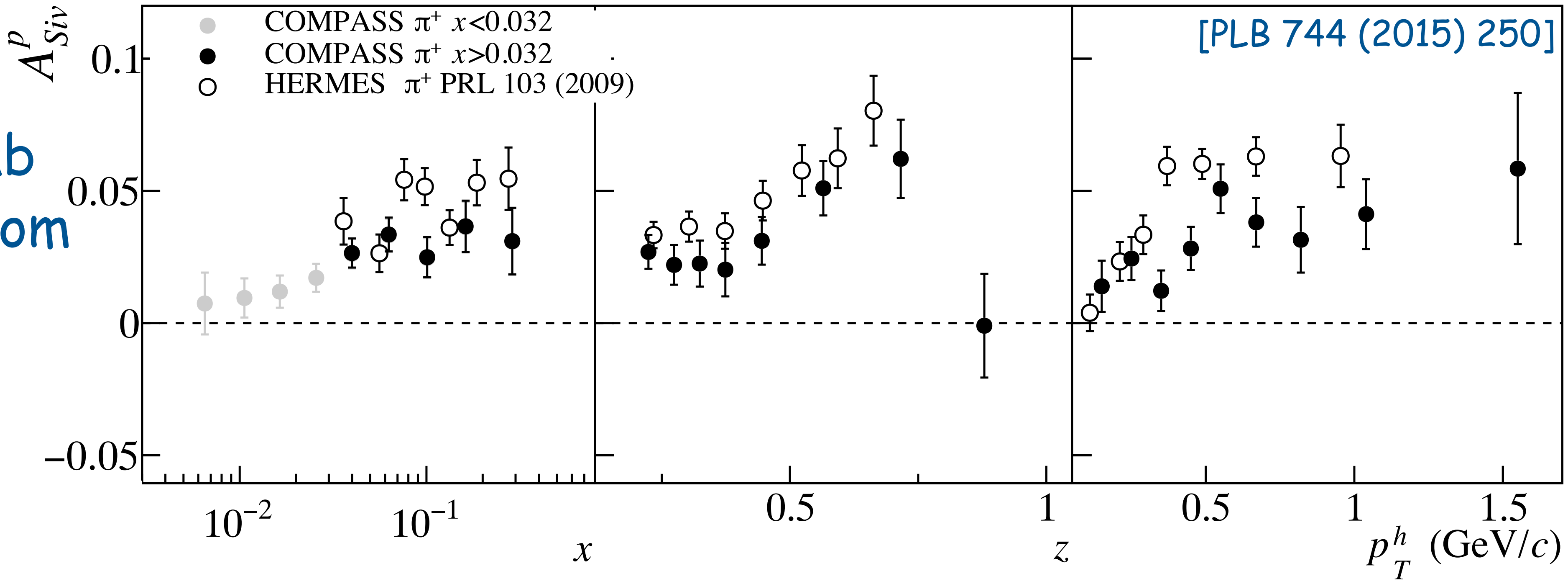


[A. Bacchetta et al.]

[PRL 107 (2011) 072003]



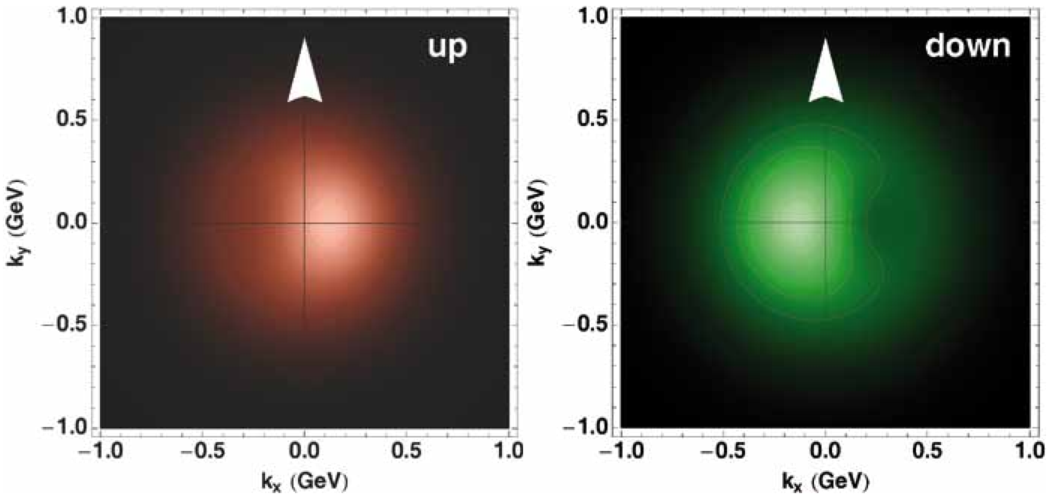
- cancelation for D target supports opposite signs of up and down Sivers
- newer results from JLab using ^{3}He target and from COMPASS for proton target (also multi-d)



[PLB 744 (2015) 250]

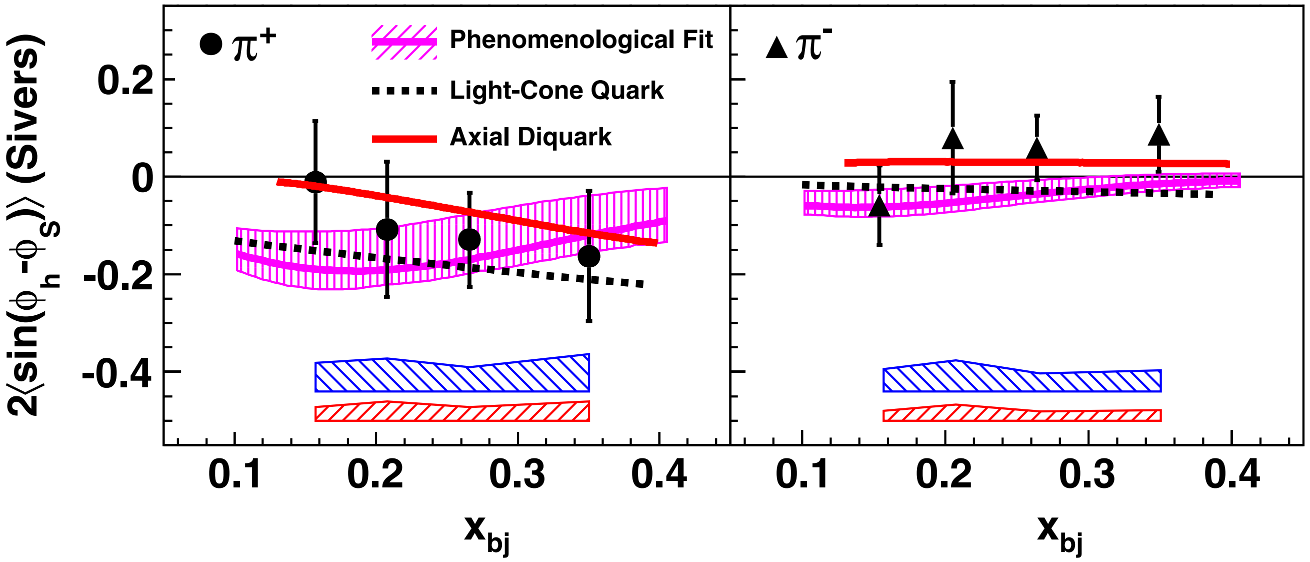
Sivers amplitudes

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

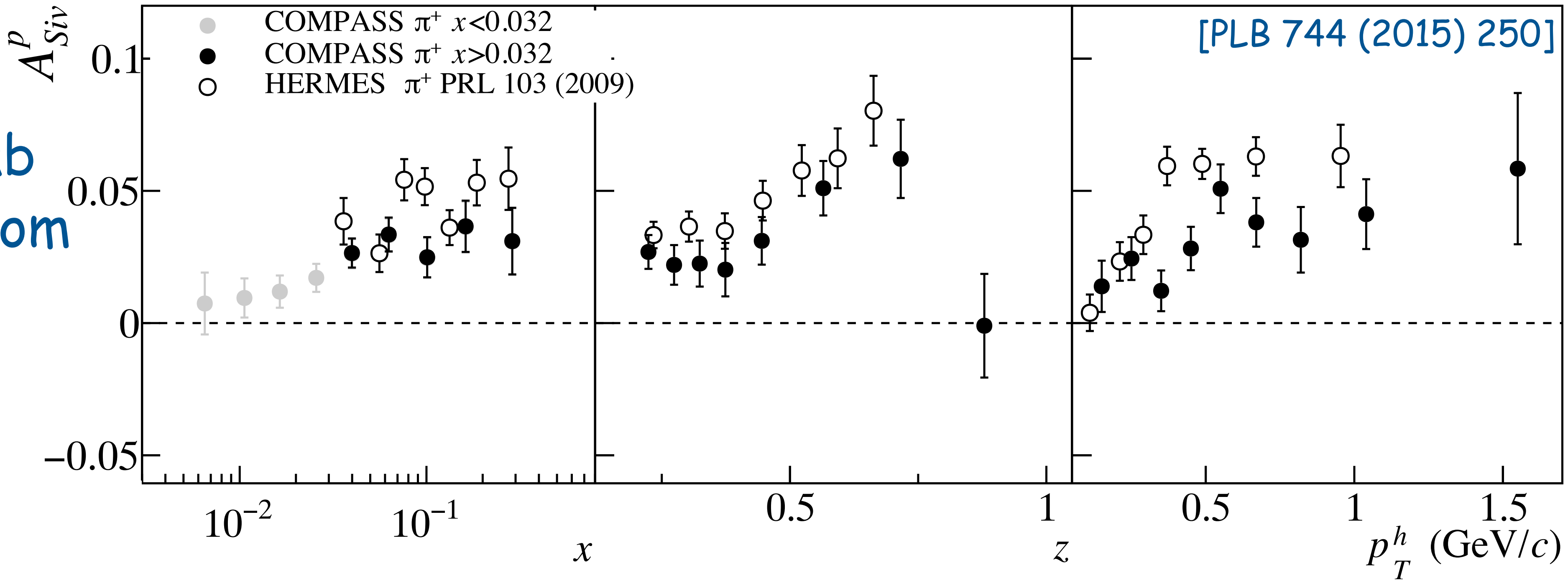


[A. Bacchetta et al.]

[PRL 107 (2011) 072003]



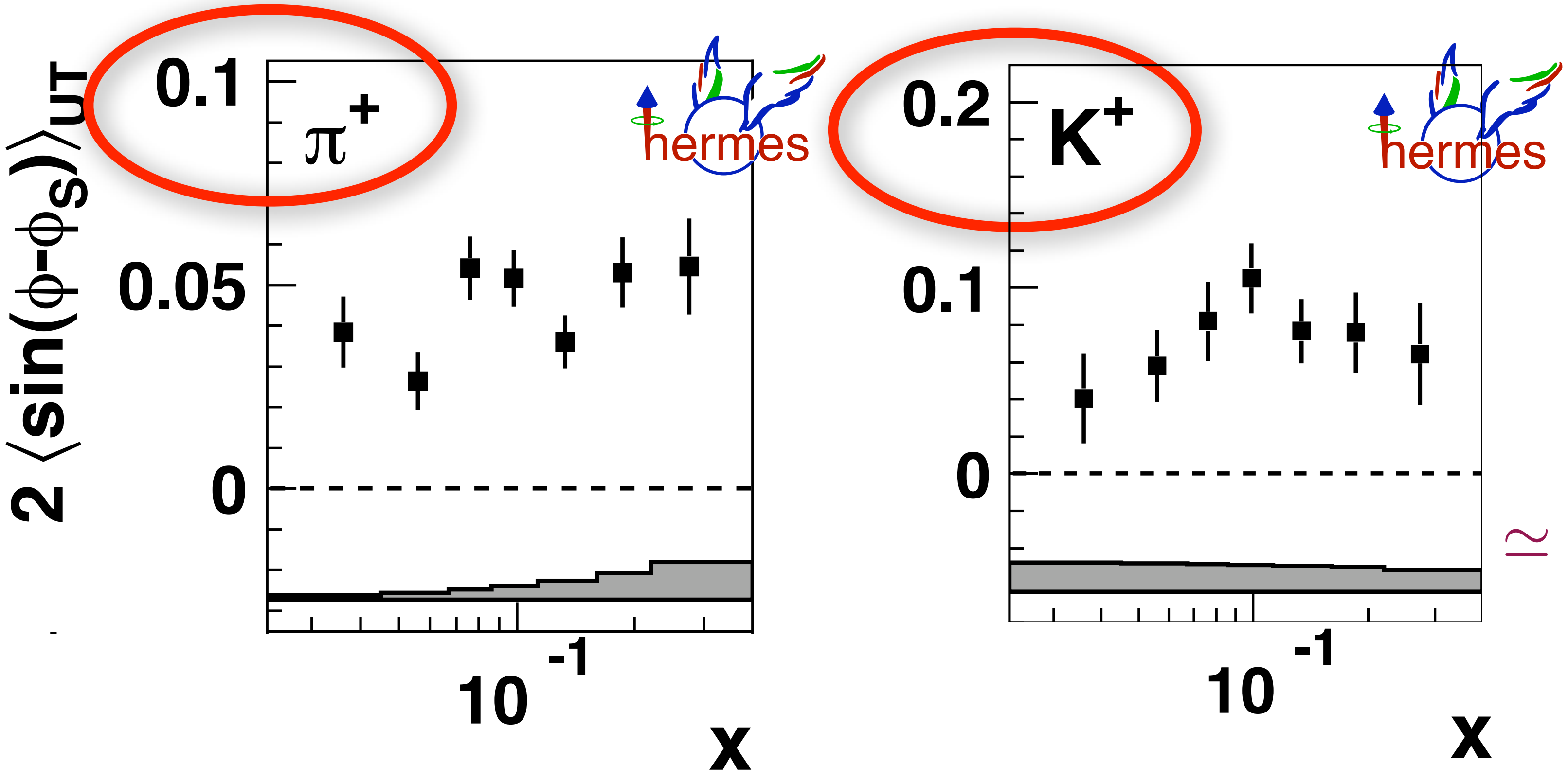
- cancelation for D target supports opposite signs of up and down Sivers
- newer results from JLab using ^{3}He target and from COMPASS for proton target (also multi-d)
- hint of Q^2 dependence from COMPASS vs. HERMES



[PLB 744 (2015) 250]

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Sivers amplitudes pions vs. kaons

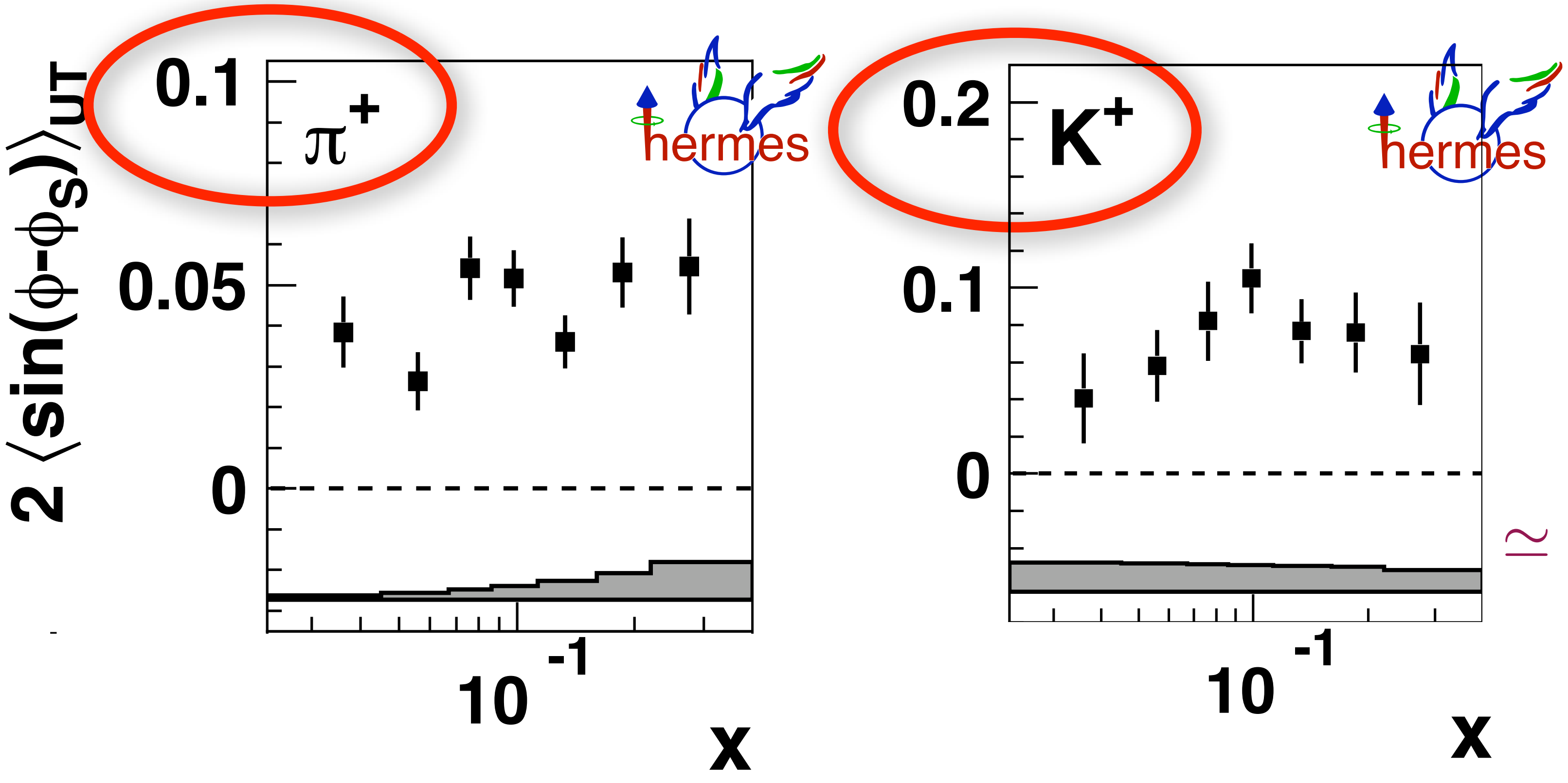


somewhat unexpected if dominated by scattering off u-quarks:

$$\simeq -\frac{\mathbf{f_{1T}^{\perp,u}(x,p_T^2)} \otimes_{\mathcal{W}} \mathbf{D_1^{u\to\pi^+/K^+}(z,k_T^2)}}{\mathbf{f_1^u(x,p_T^2)} \ \otimes \mathbf{D_1^{u\to\pi^+/K^+}(z,k_T^2))}}$$

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

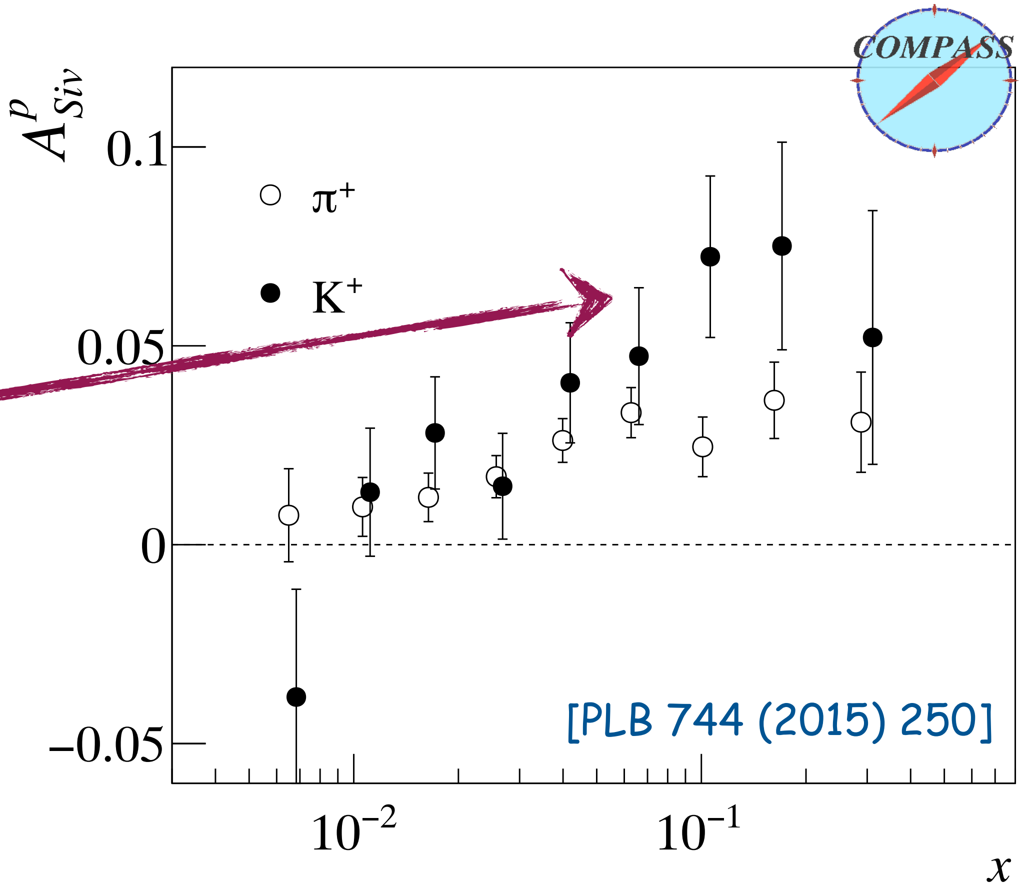
Sivers amplitudes pions vs. kaons



somewhat unexpected if dominated by scattering off u-quarks:

$$\simeq -\frac{f_{1T}^{\perp,u}(x,p_T^2)\otimes_{\mathcal{W}} D_1^{u\to\pi^+/K^+}(z,k_T^2)}{f_1^u(x,p_T^2)\ \otimes D_1^{u\to\pi^+/K^+}(z,k_T^2))}$$

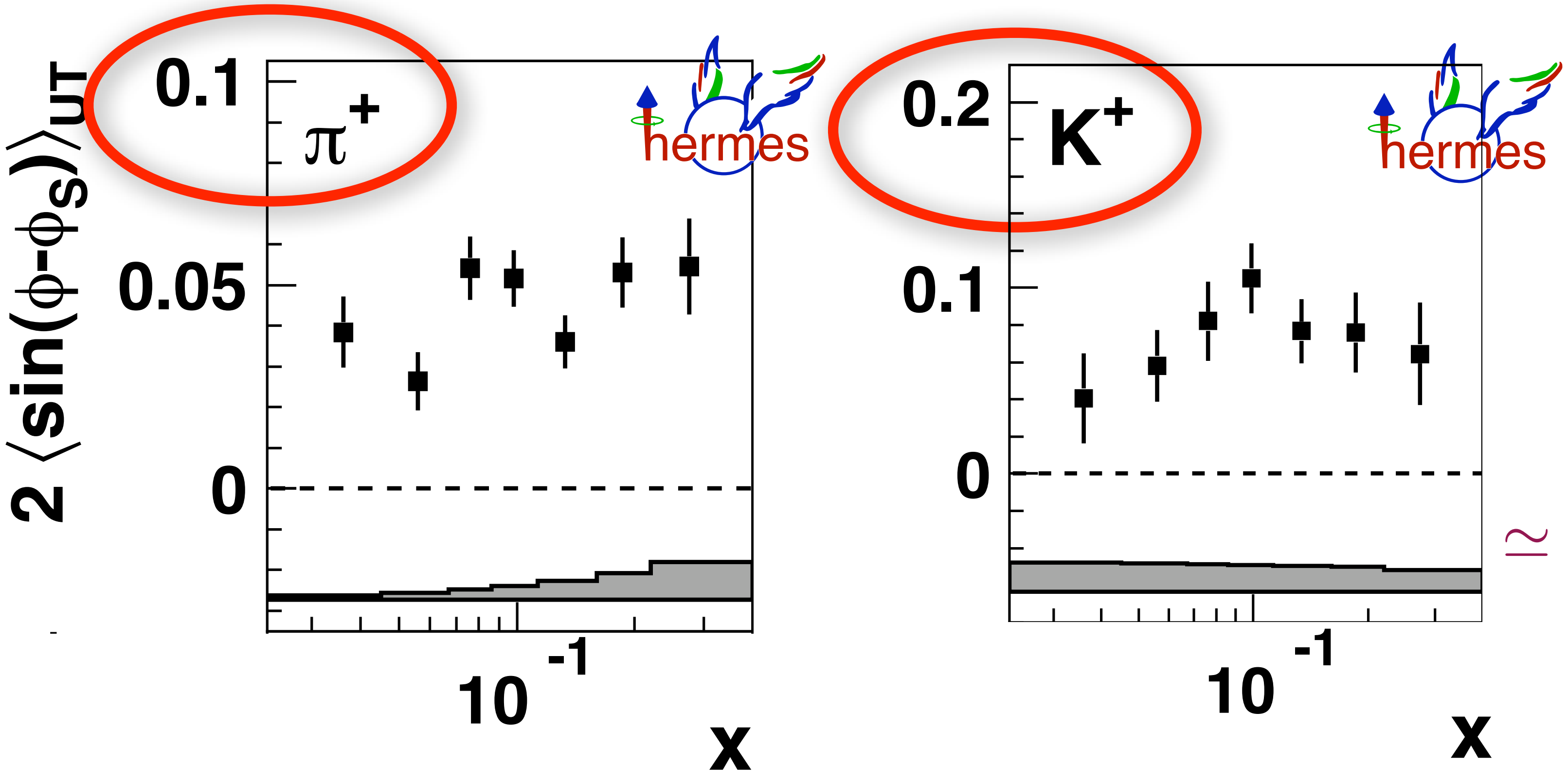
larger amplitudes seen also by COMPASS



[PLB 744 (2015) 250]

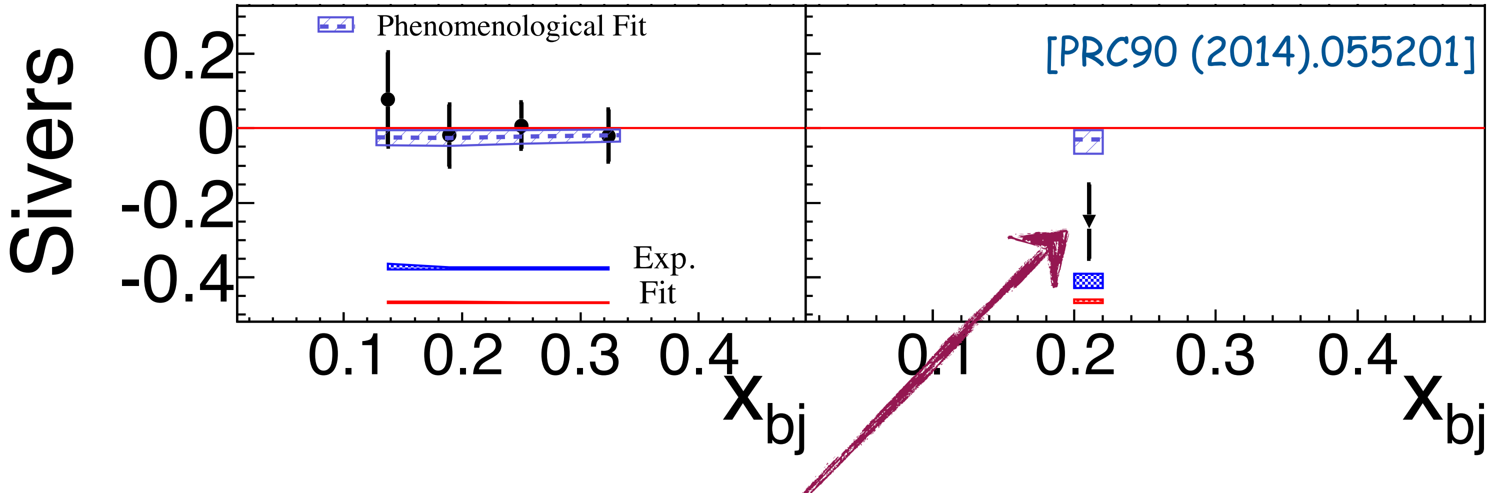
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes pions vs. kaons



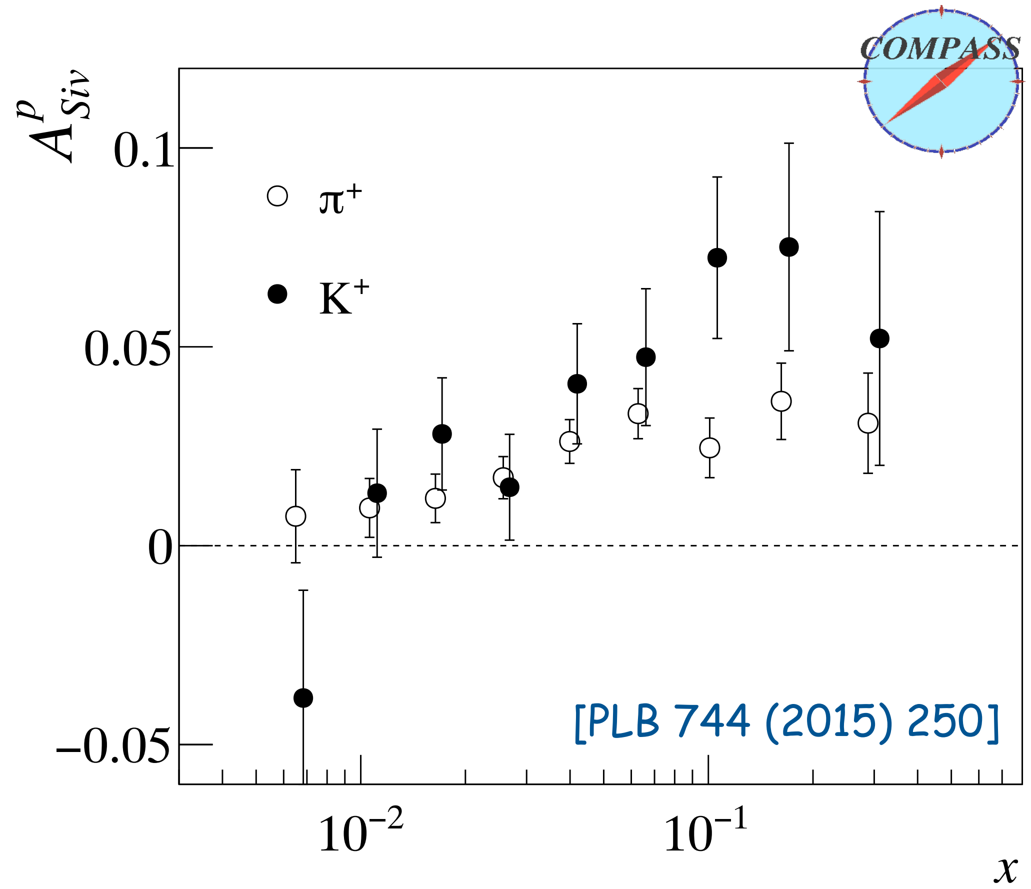
somewhat unexpected if dominated by scattering off u-quarks:

$$\simeq -\frac{f_{1T}^{\perp,u}(x,p_T^2) \otimes_{\mathcal{W}} D_1^{u\to\pi^+/K^+}(z,k_T^2)}{f_1^u(x,p_T^2)\ \otimes D_1^{u\to\pi^+/K^+}(z,k_T^2))}$$



[PRC90 (2014).055201]

surprisingly large K^- asymmetry for ^{3}He target (but zero for K^+?!)



[PLB 744 (2015) 250]

interlude: dealing with multi-d dependences

multi-d dependences

- TMD cross sections differential in at least 5 variables
  - some easily parametrized (e.g., azimuthal dependences)
  - others mostly unknown

multi-d dependences

- TMD cross sections differential in at least 5 variables
  - some easily parametrized (e.g., azimuthal dependences)
  - others mostly unknown
- one-dimensional binning provide only glimpse of true physics
  - even different kinematic bins can't disentangle underlying physics dependences
  - e.g., binning in x involves [incomplete] integration(s) over $P_{h\perp}$

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- one-dimensional binning provide only glimpse of true physics
  - even different kinematic bins can't disentangle underlying physics dependences
  - e.g., binning in x involves [incomplete] integration(s) over $P_{h\perp}$
- further complication: physics (cross sections) folded with acceptance
  - NO experiment has flat acceptance in full multi-d kinematic space

multi-d dependences

$$\frac{N^+(x) - N^-(x)}{N^+(x) - N^-(x)} = \frac{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \Delta\sigma(x,\omega)}{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \sigma(x,\omega)}$$

- measured cross sections / asymmetries often contain “remnants” of experimental acceptance ϵ

multi-d dependences

$$\frac{N^+(x) - N^-(x)}{N^+(x) - N^-(x)} = \frac{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \Delta\sigma(x,\omega)}{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \sigma(x,\omega)} \neq \frac{\int \mathrm{d}\omega \;\; \Delta\sigma(x,\omega)}{\int \mathrm{d}\omega \;\; \sigma(x,\omega)}$$

- measured cross sections / asymmetries often contain “remnants” of experimental acceptance ϵ

multi-d dependences

$$\frac{N^+(x) - N^-(x)}{N^+(x) - N^-(x)} = \frac{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \Delta\sigma(x,\omega)}{\int \mathrm{d}\omega \, \epsilon(x,\omega) \, \sigma(x,\omega)} \neq A(x, \langle\omega\rangle)$$

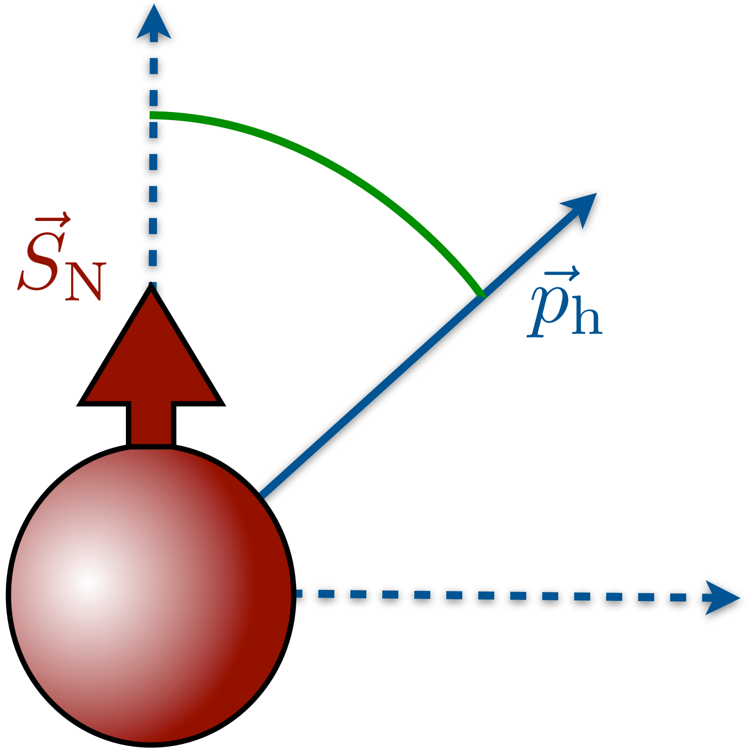
- measured cross sections / asymmetries often contain “remnants” of experimental acceptance ϵ
- difficult to evaluate precisely in absence of good physics model
  - general challenge to statistically precise data sets
  - avoid 1d binning/presentation of data
  - theorist: watch out for precise definition (if given!) of experimental results reported … and try not to treat data points of different projections as independent

inclusive hadrons: A_{UT} sinψ amplitude

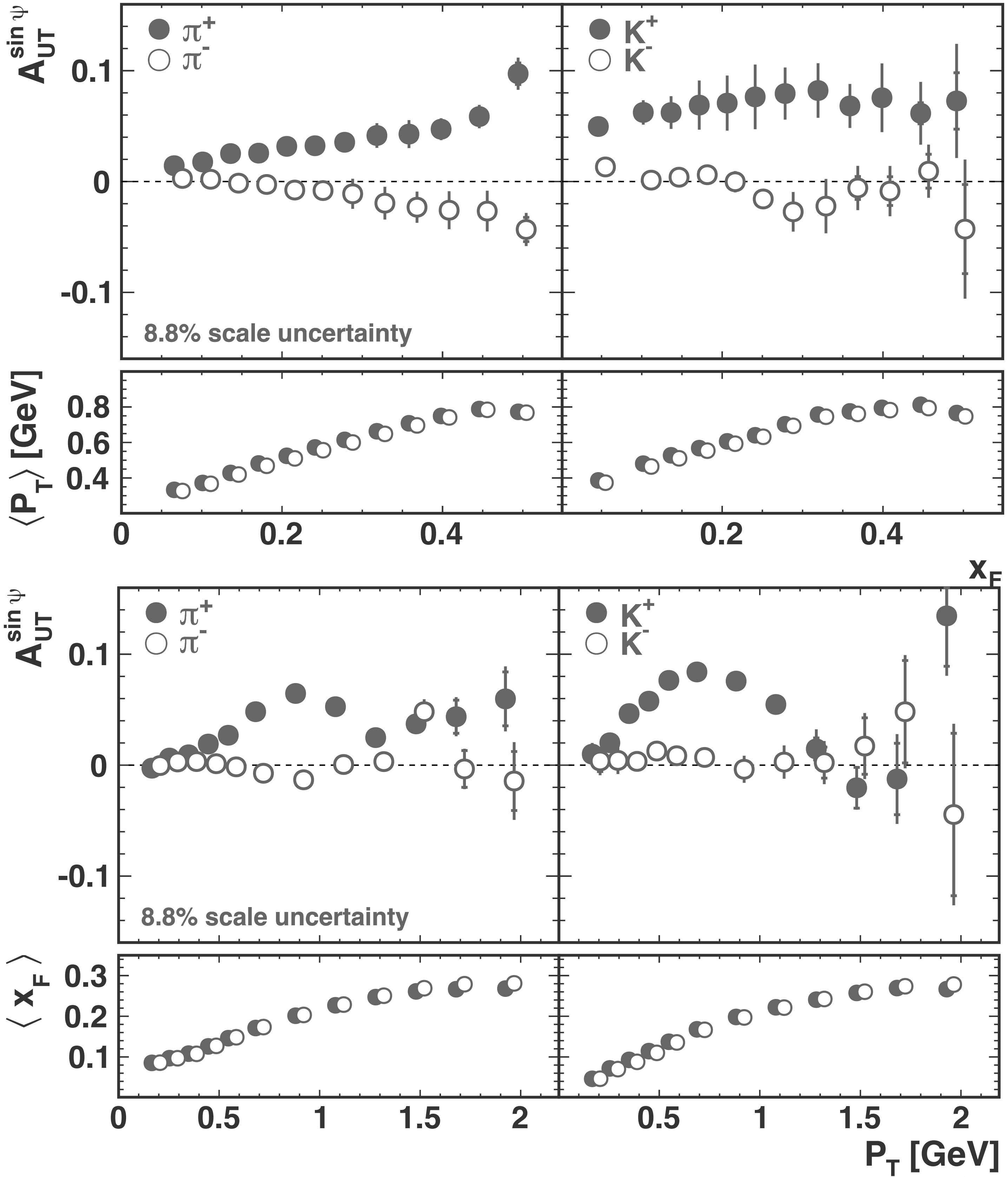
[Airapetian et al., Phys. Lett. B 728, 183-190 (2014)]

- clear left-right asymmetries for pions and positive kaons

$$ep^{\uparrow} \rightarrow hX$$

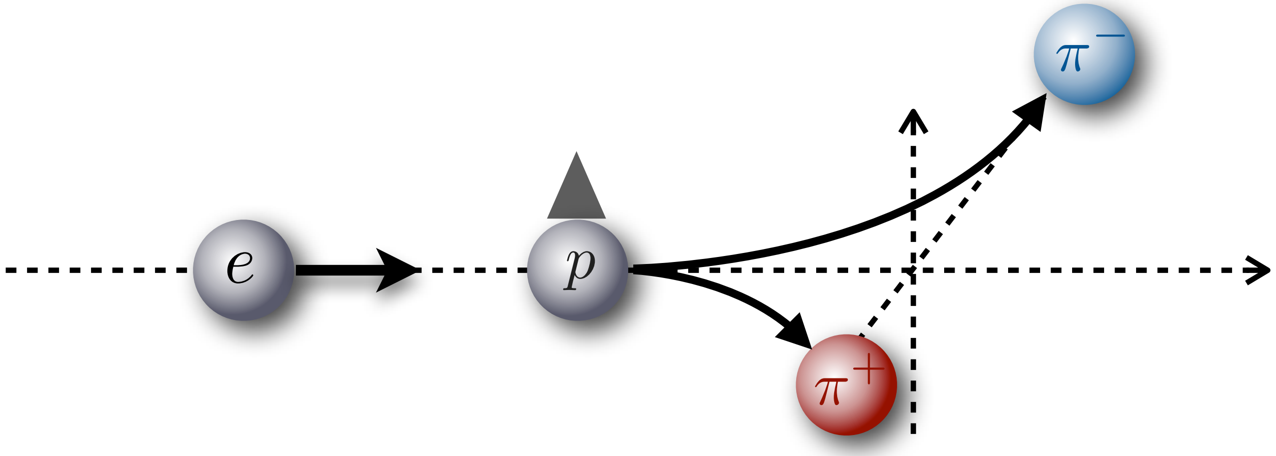


lepton going into the plane

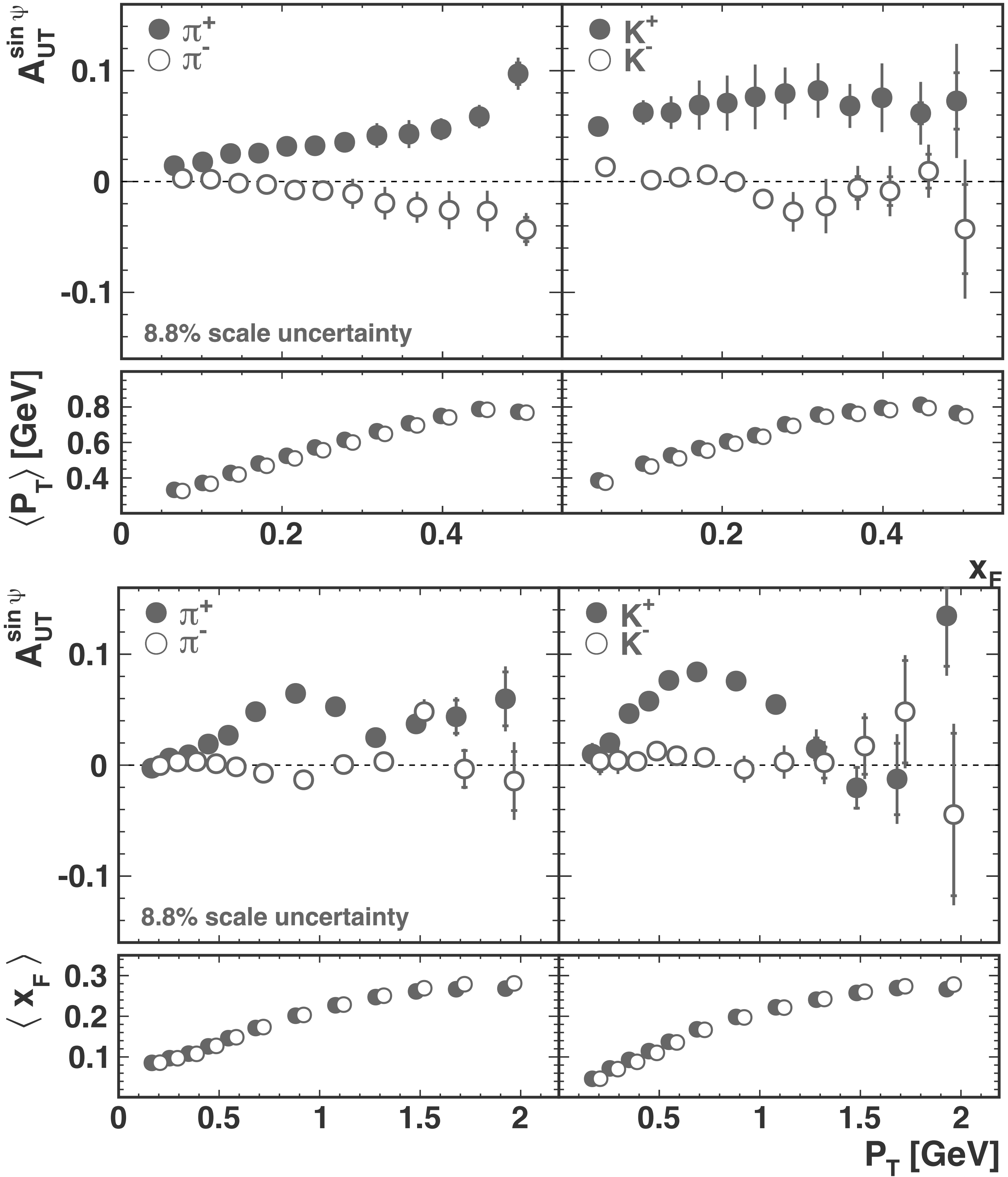


inclusive hadrons: A_{UT} sinψ amplitude

- clear left-right asymmetries for pions and positive kaons
- increasing with x_F (as in pp)



[Airapetian et al., Phys. Lett. B 728, 183-190 (2014)]



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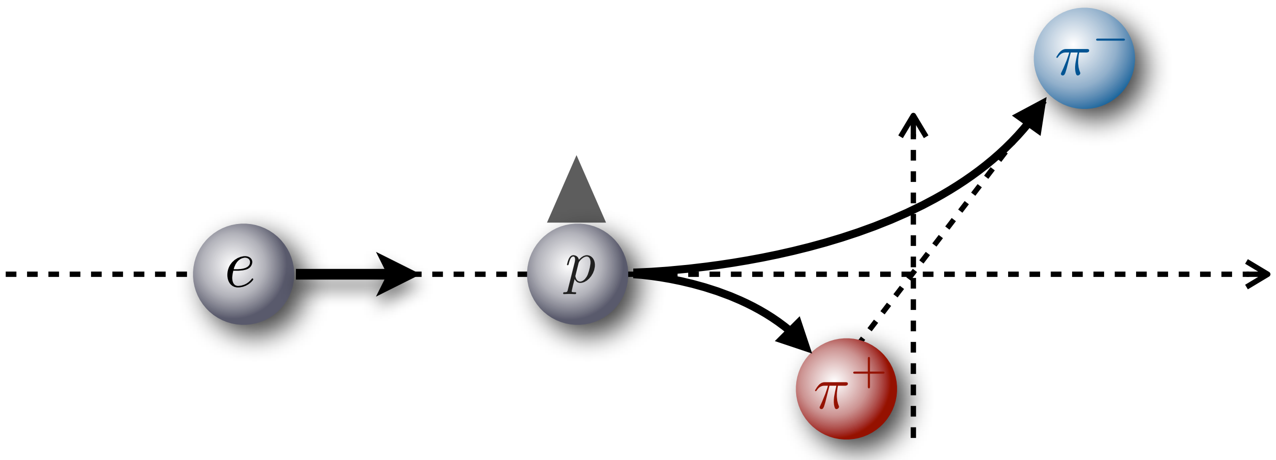
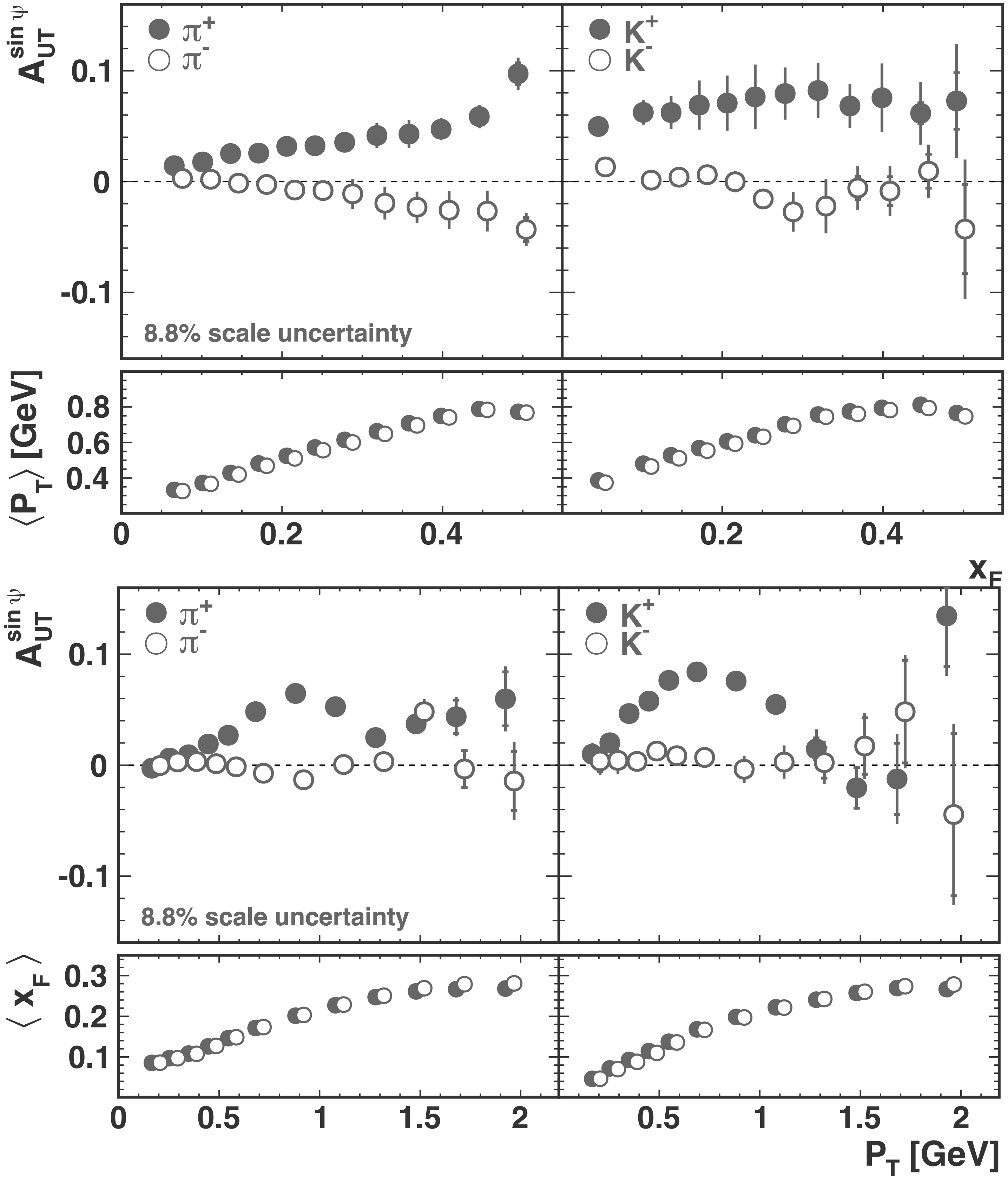


- initially increasing with P_T with a fall-off at larger P_T

[Airapetian et al., Phys. Lett. B 728, 183-190 (2014)]



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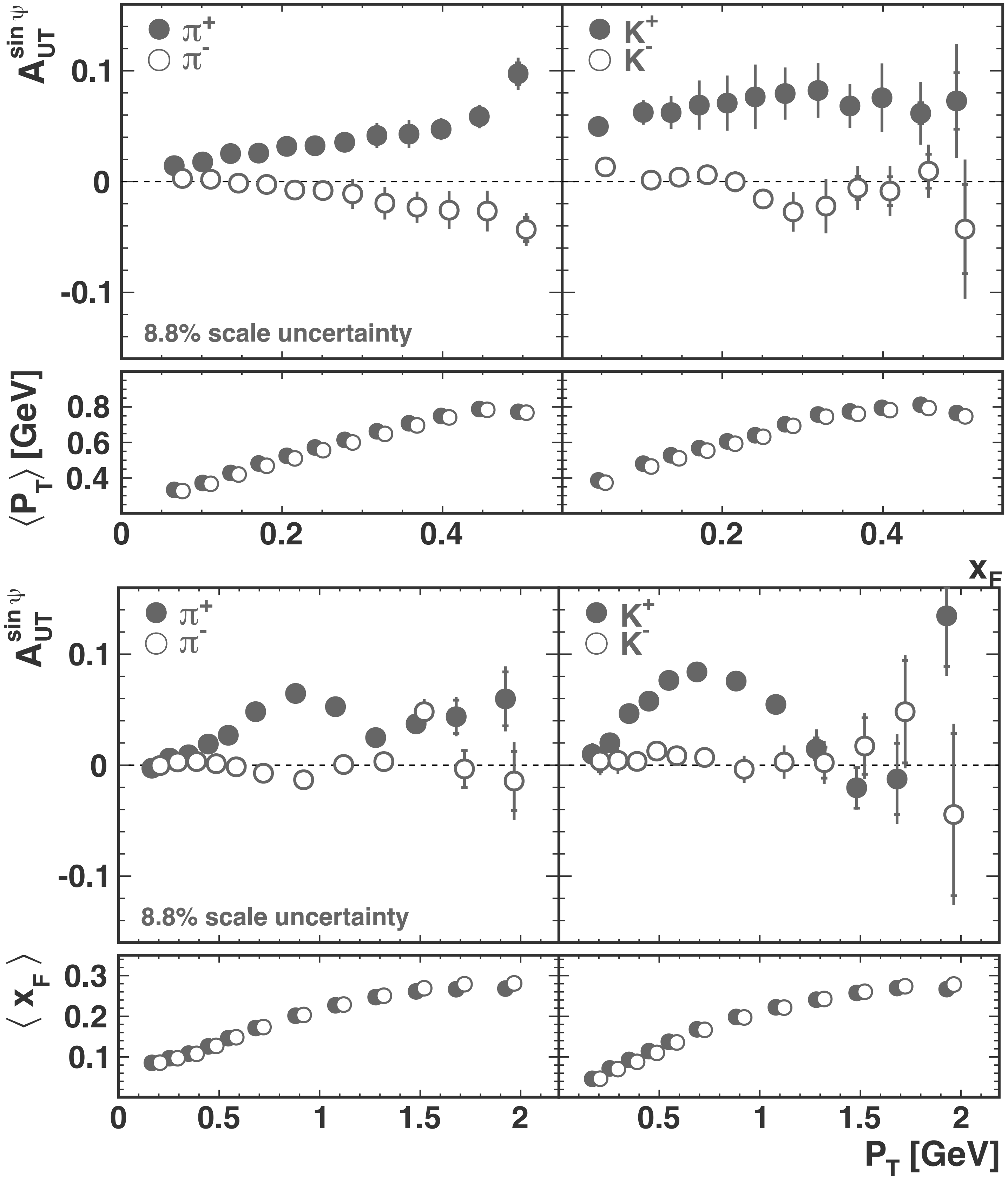
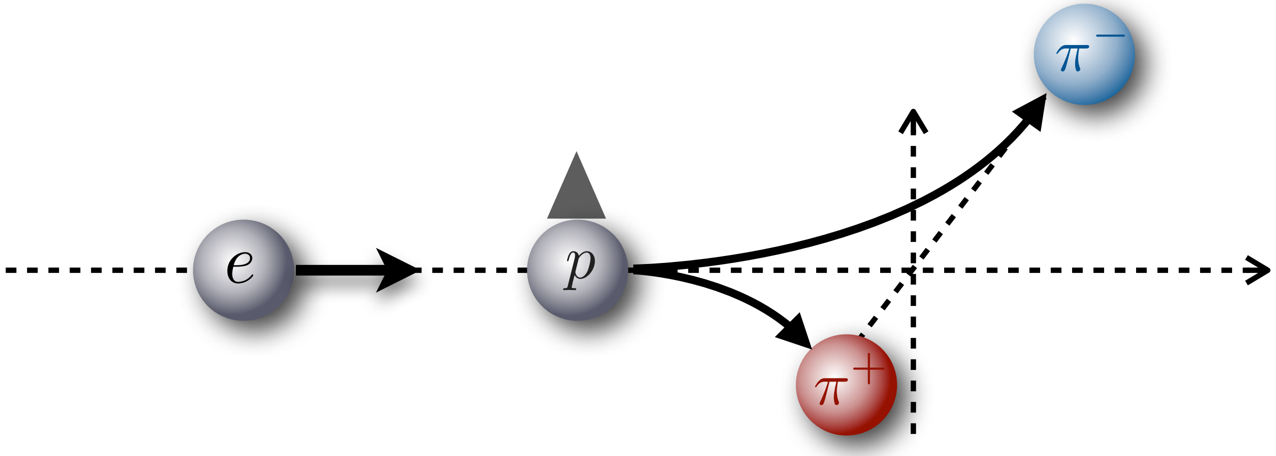


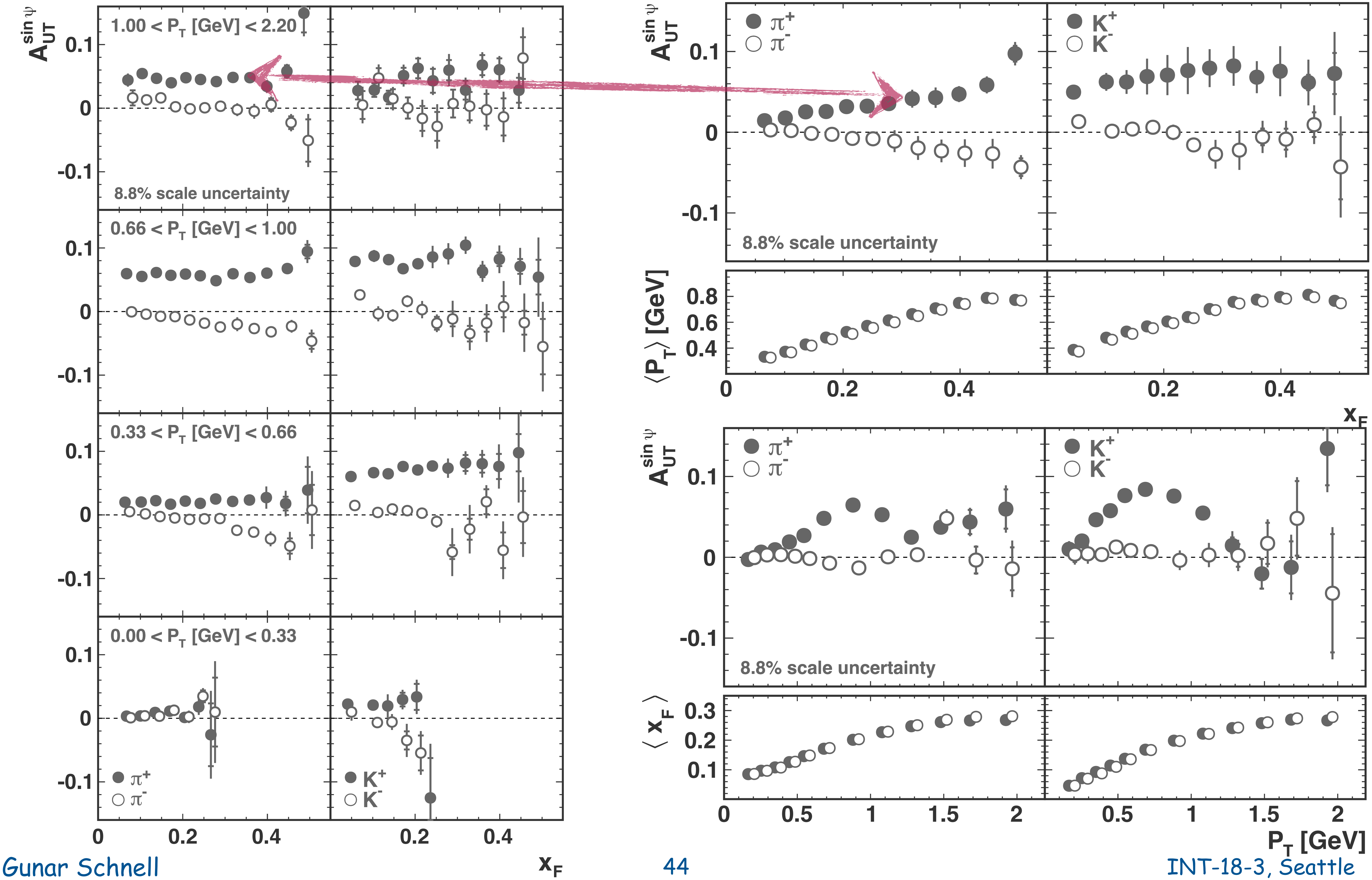
- clear left-right asymmetries for pions and positive kaons
- increasing with x_F (as in pp)



- initially increasing with P_T with a fall-off at larger P_T
- x_F and P_T correlated
  ➡ look at 2D dependences

inclusive hadrons: A_{UT} $\sin\psi$ amplitude

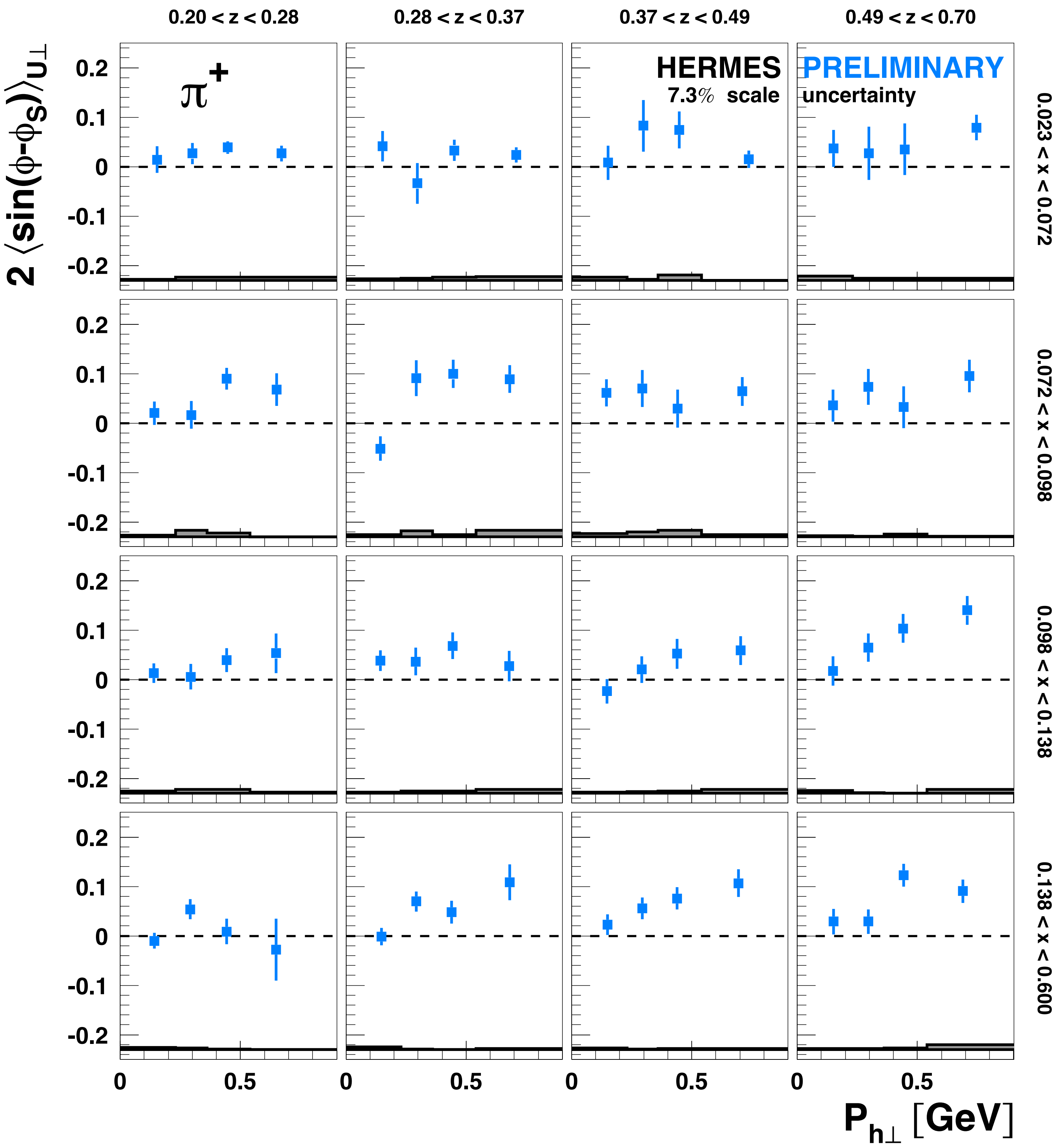
[Airapetian et al., Phys. Lett. B 728, 183-190 (2014)]



back to SIDIS

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes - 3d binning



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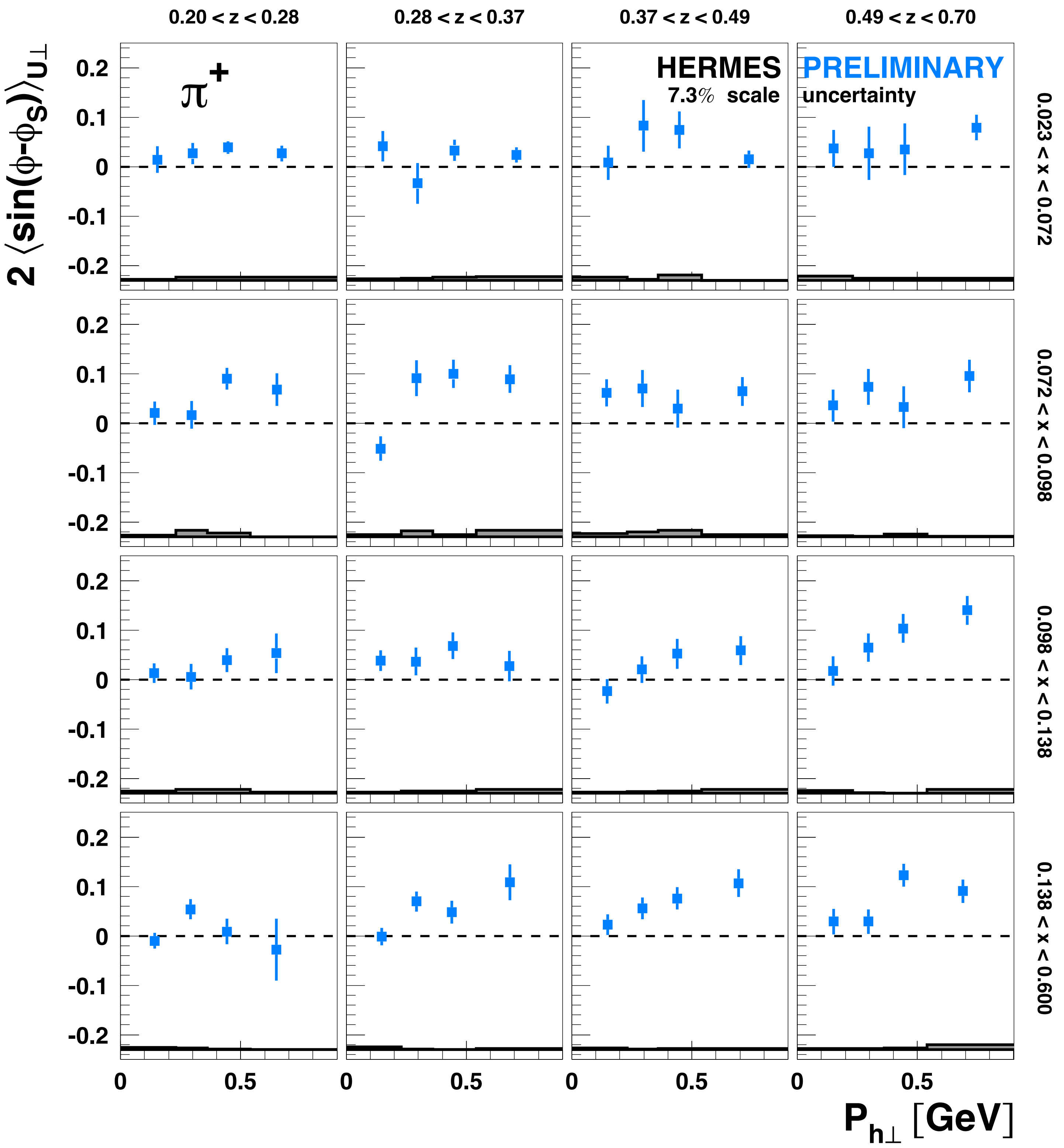

- 3d analysis: 4x4x4 bins in (x,z, $P_{h\perp}$)

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
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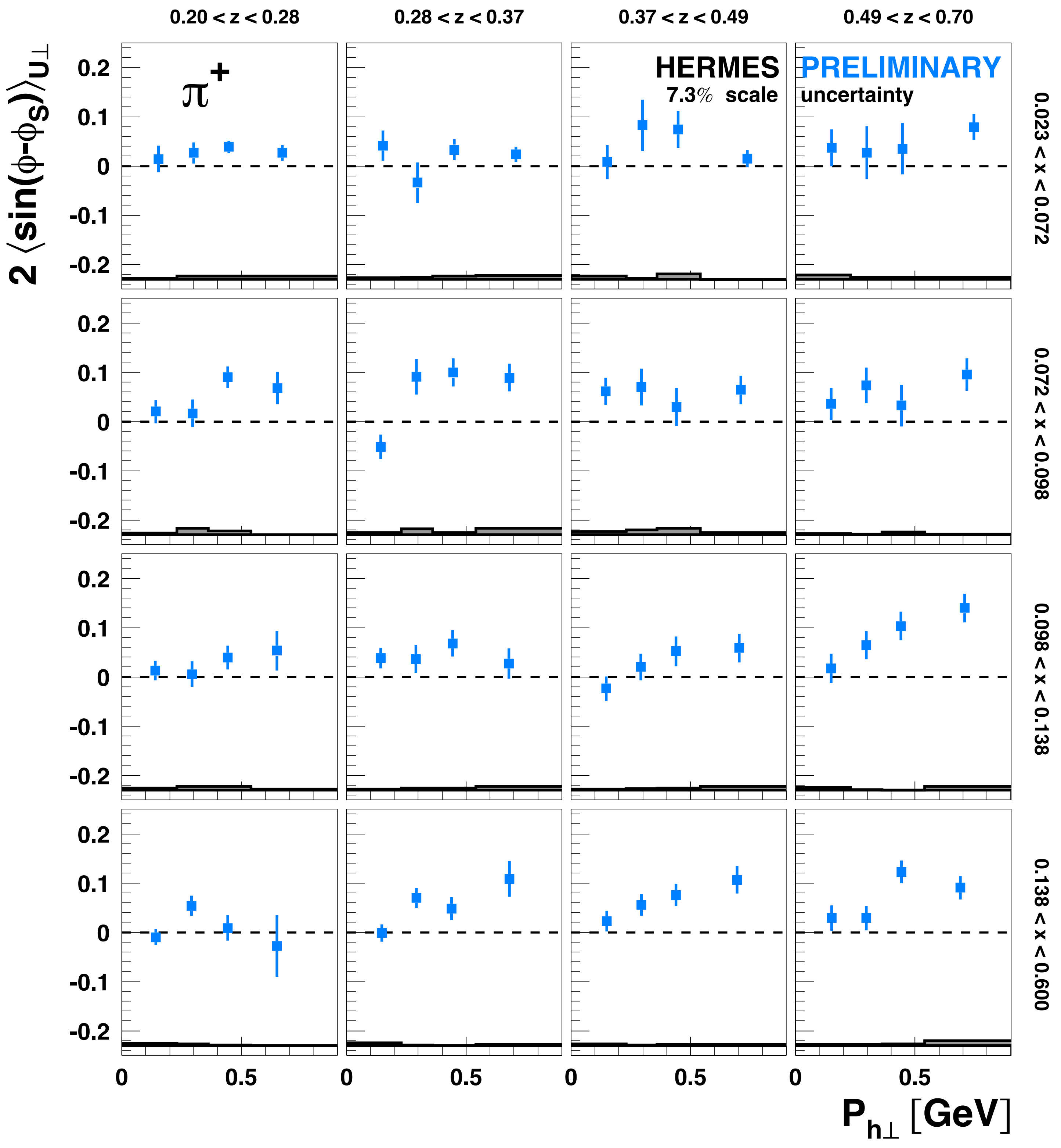

- 3d analysis: 4x4x4 bins in $(x,z, P_{h\perp})$
- reduced systematics
- disentangle correlations
- isolate phase-space region with large signal strength

	U	L	T
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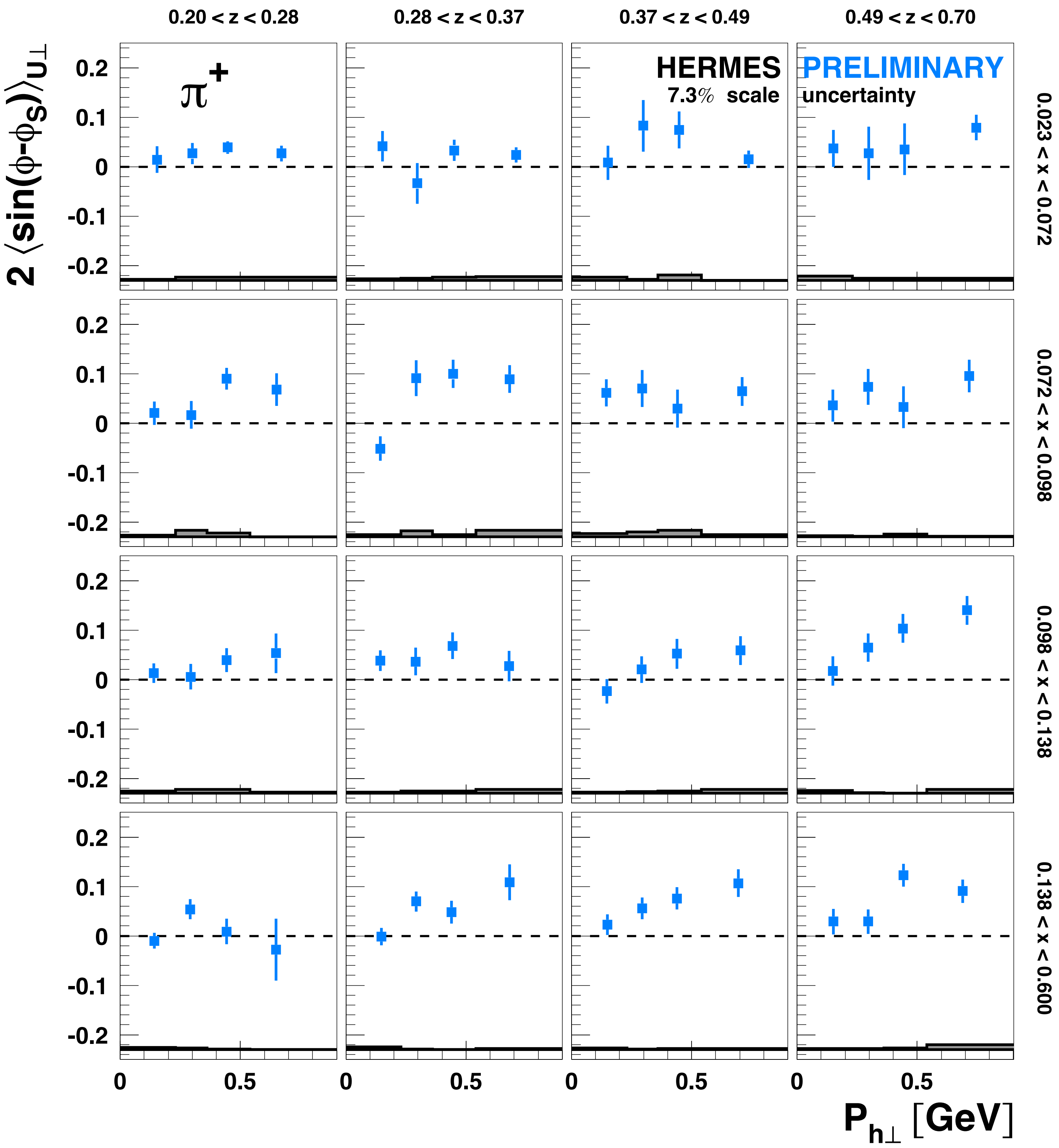
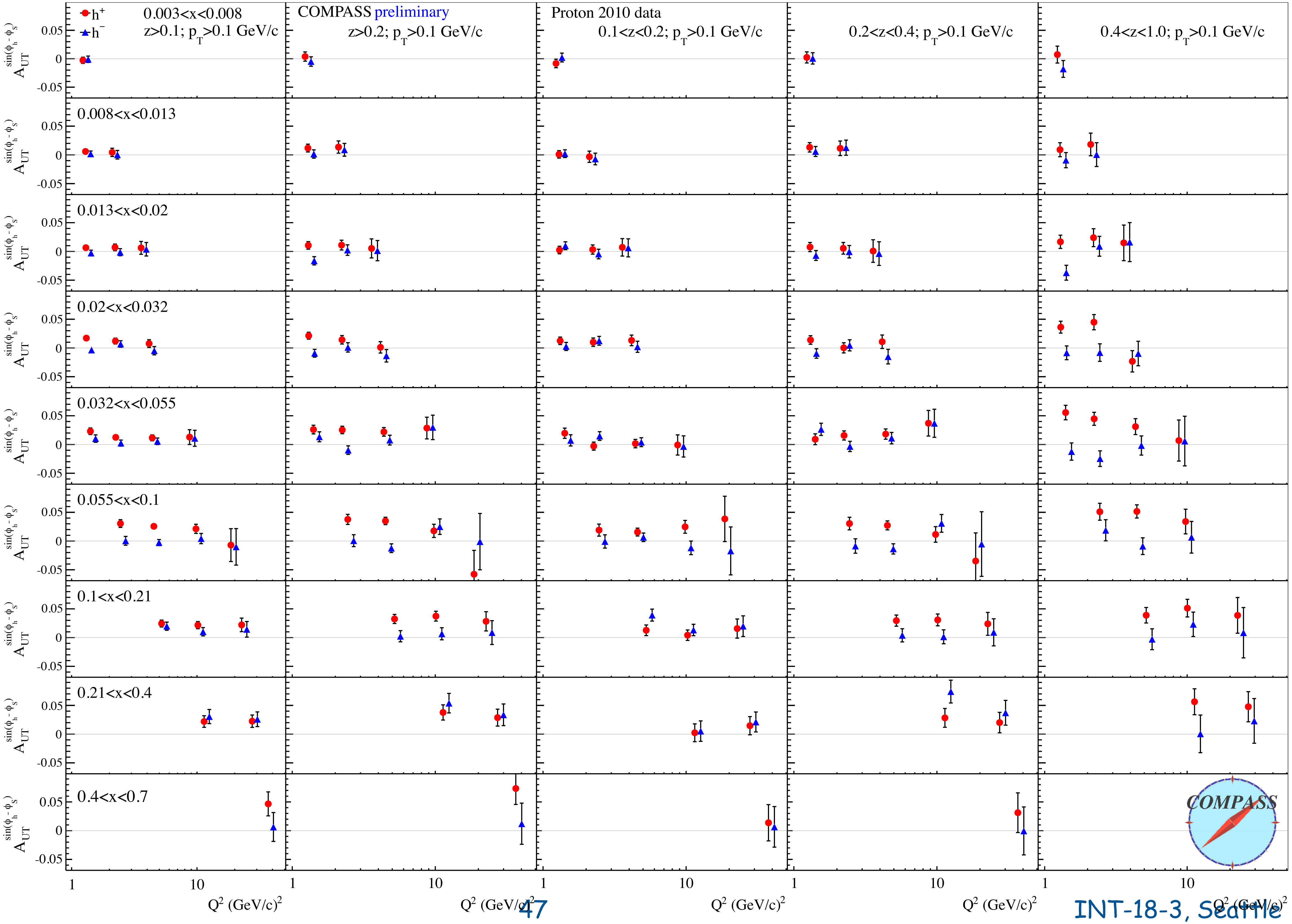


- 3d analysis: 4x4x4 bins in (x,z, $P_{h\perp}$)
- reduced systematics
- disentangle correlations
- isolate phase-space region with large signal strength
- allows more detailed comparison with calculations

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes - 3d binning



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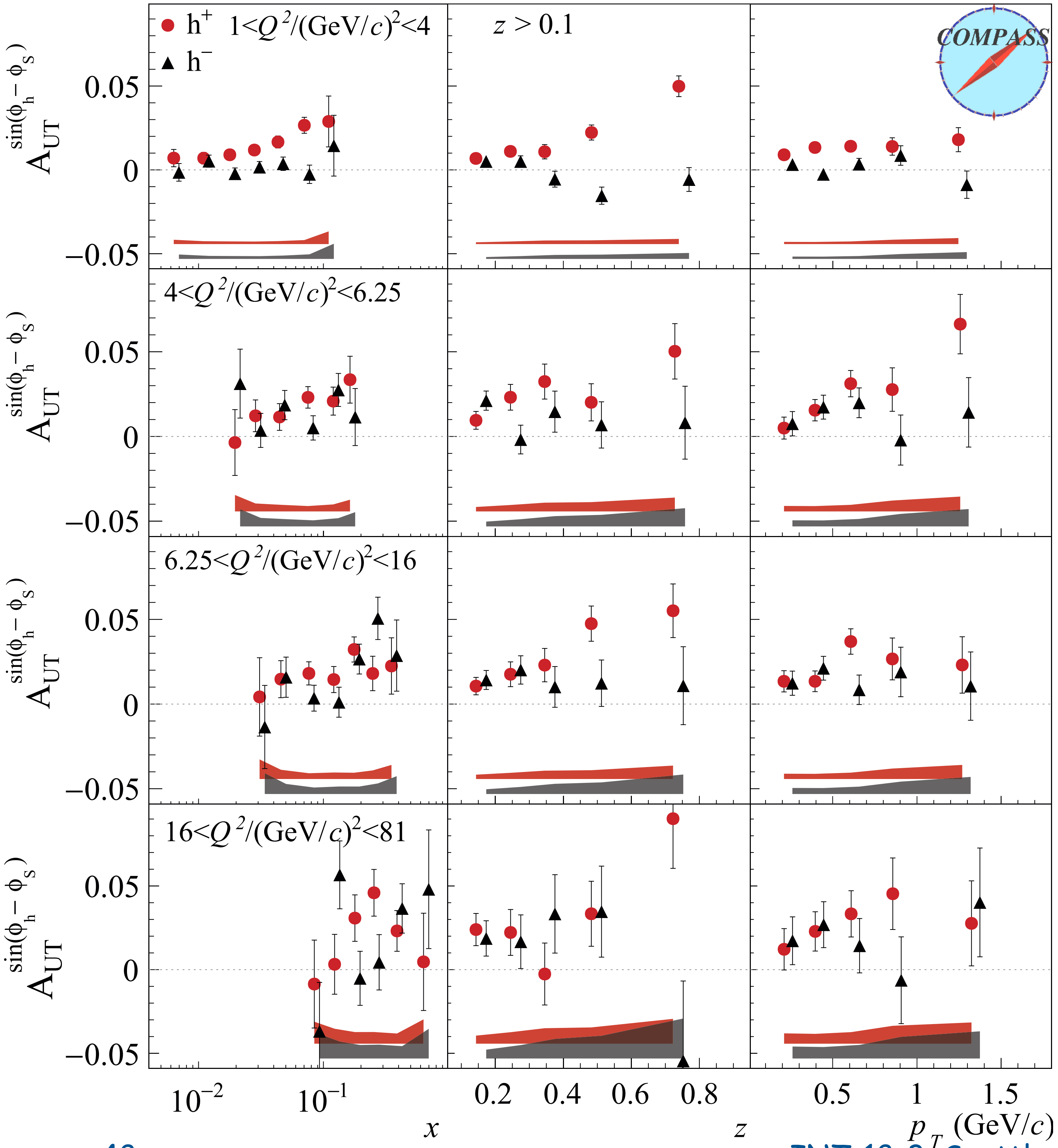
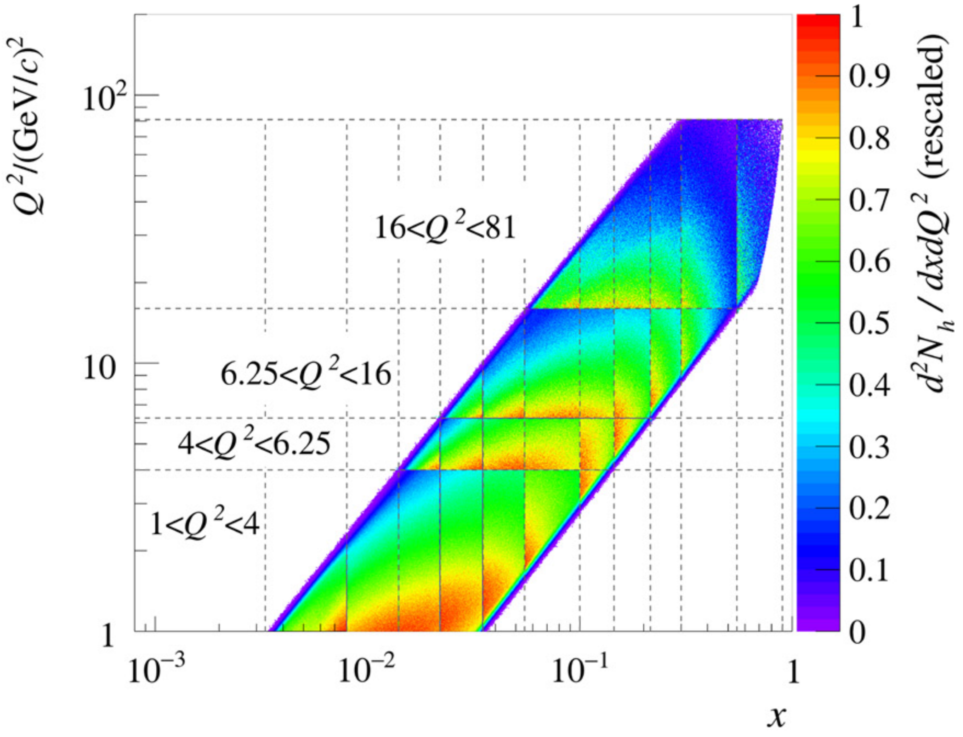
Sivers amplitudes - 2d binning

[Adolph et al., Phys. Lett. B 770, 138-145 (2017)]

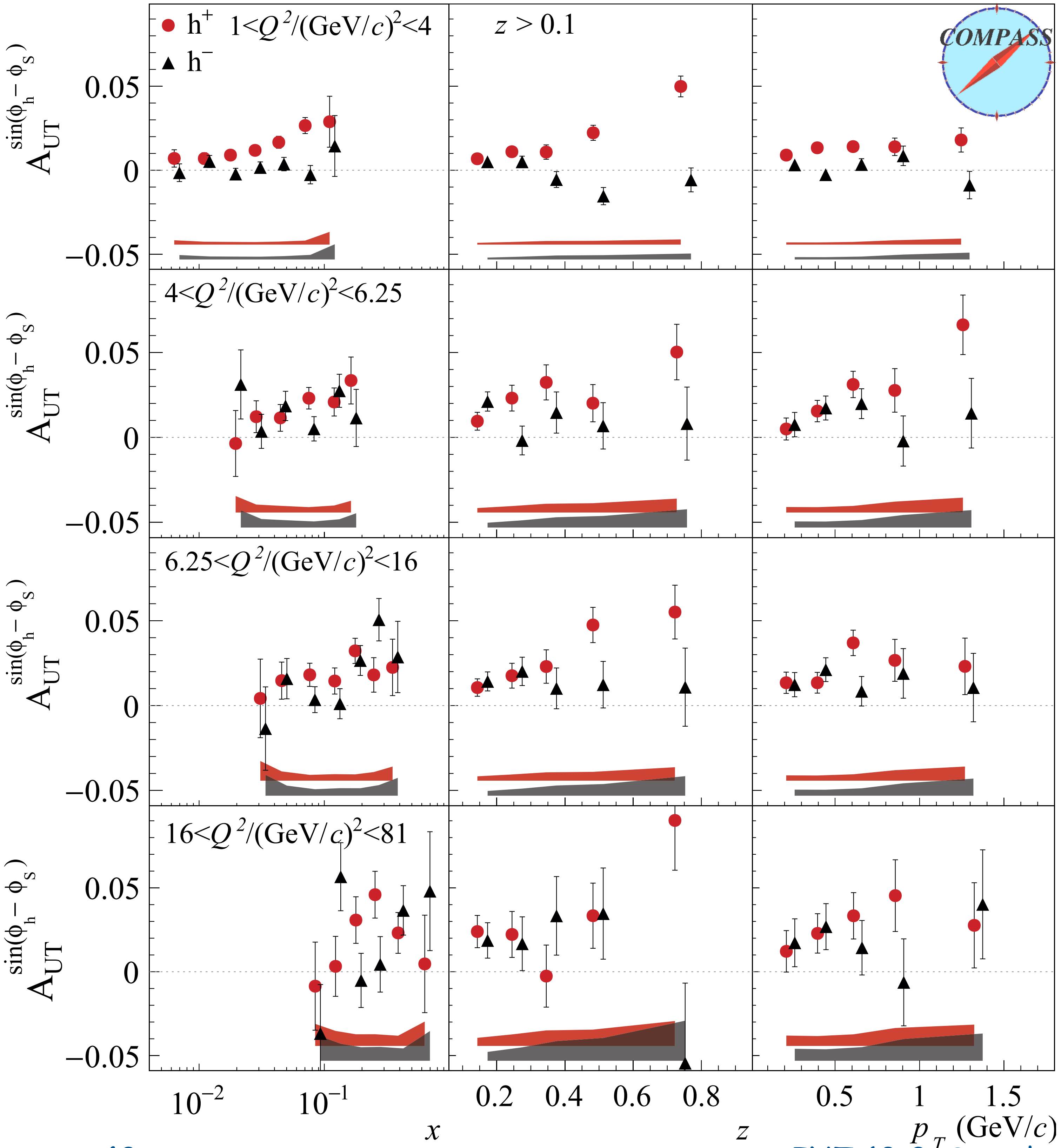




	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes - 2d binning

- 2d analysis to match Q^2 range probed in Drell-Yan



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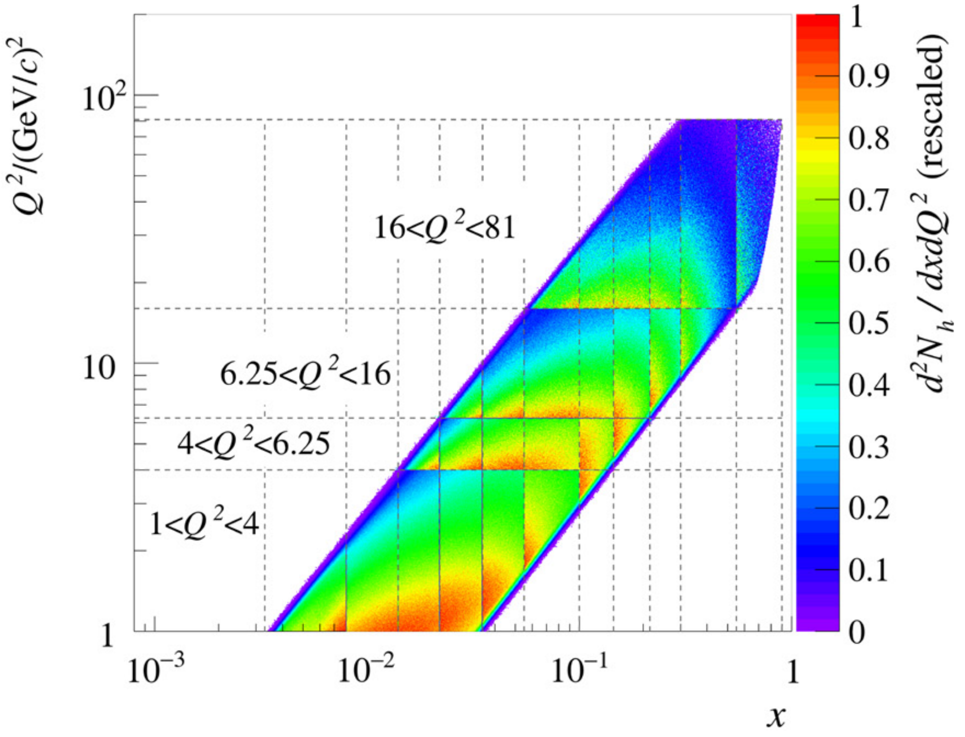

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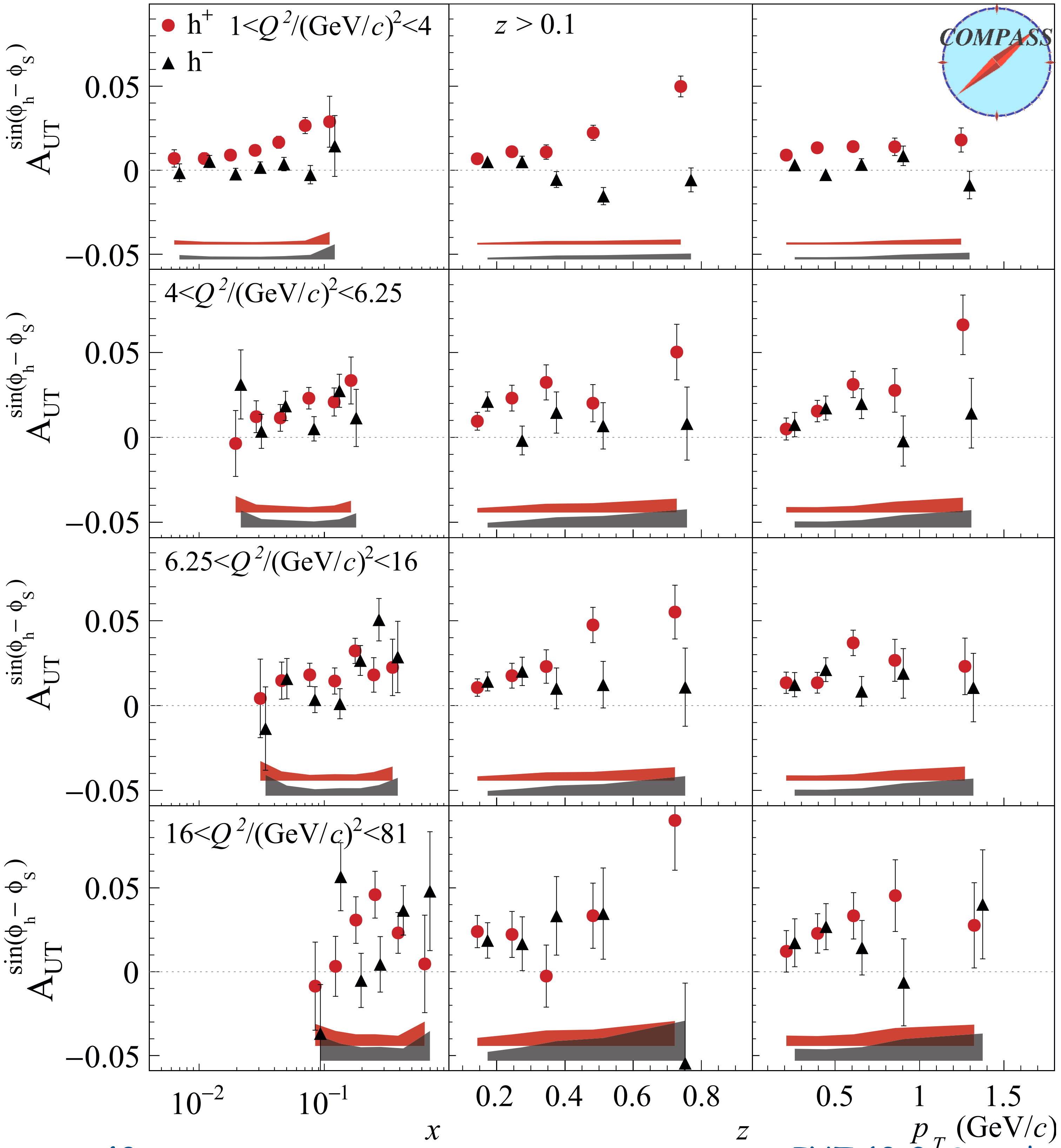
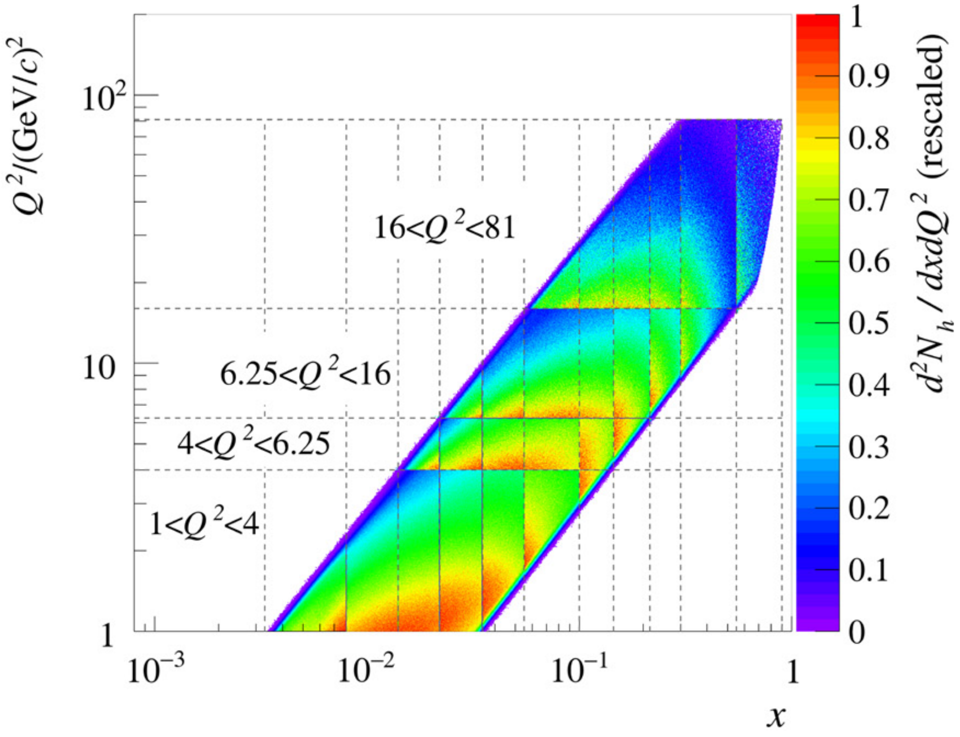
Sivers amplitudes - 2d binning

[Adolph et al., Phys. Lett. B 770, 138-145 (2017)]

- 2d analysis to match Q^2 range probed in Drell-Yan
- allows also more detailed evolution studies

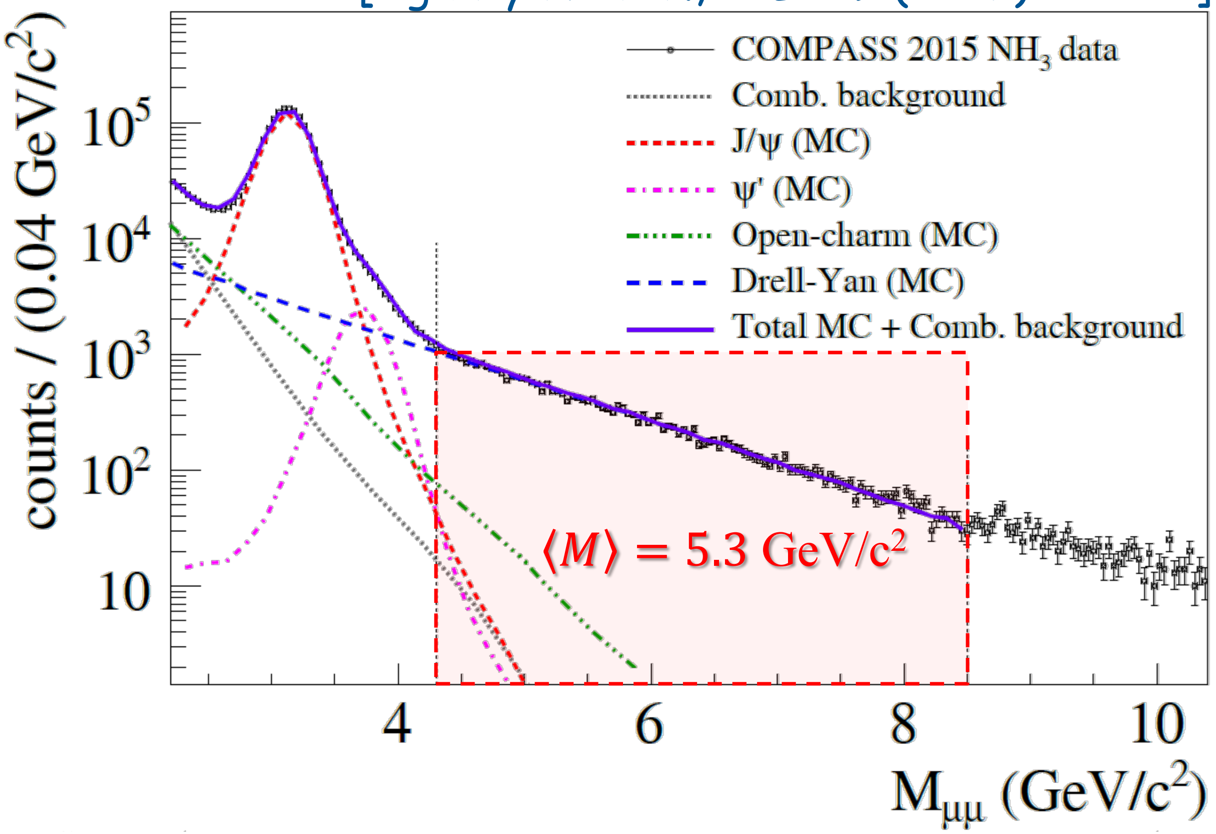


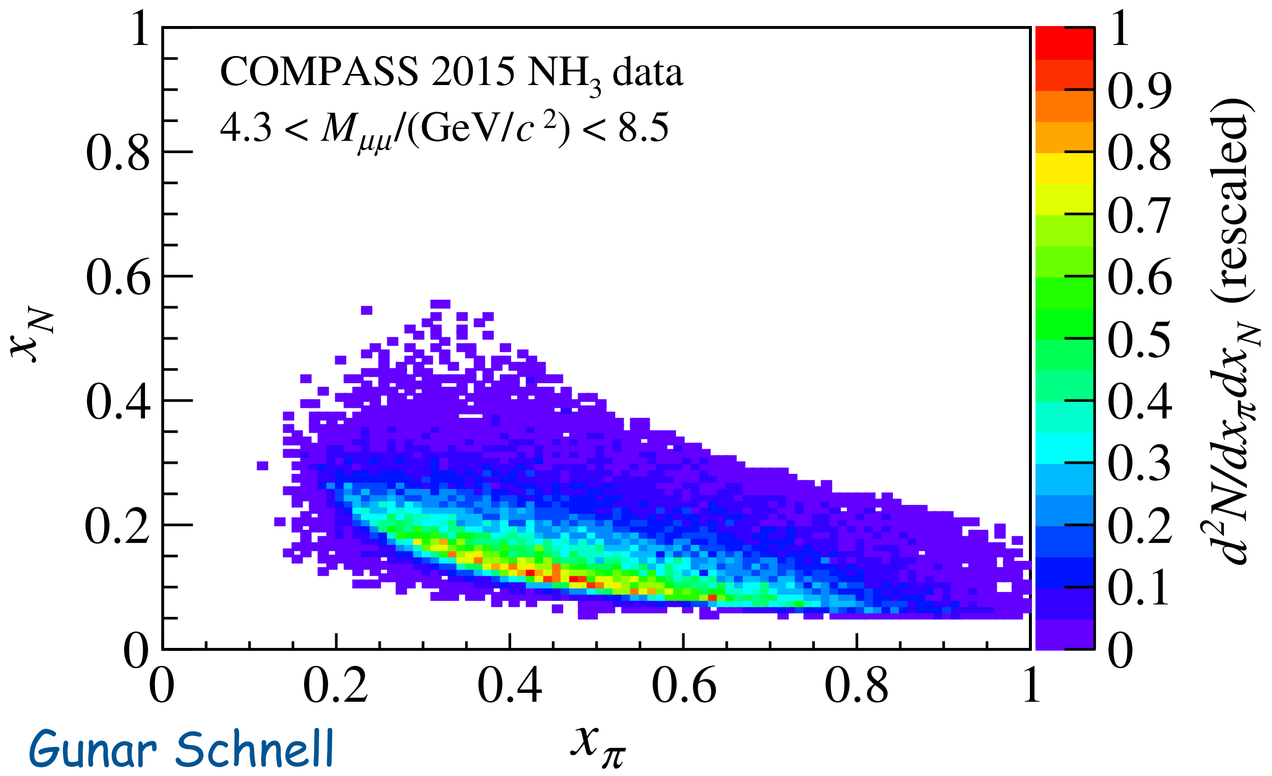


Sivers amplitudes - Drell-Yan

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

[Aghasyan et al., PRL 119 (2017) 112002]





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Sivers amplitudes - Drell-Yan

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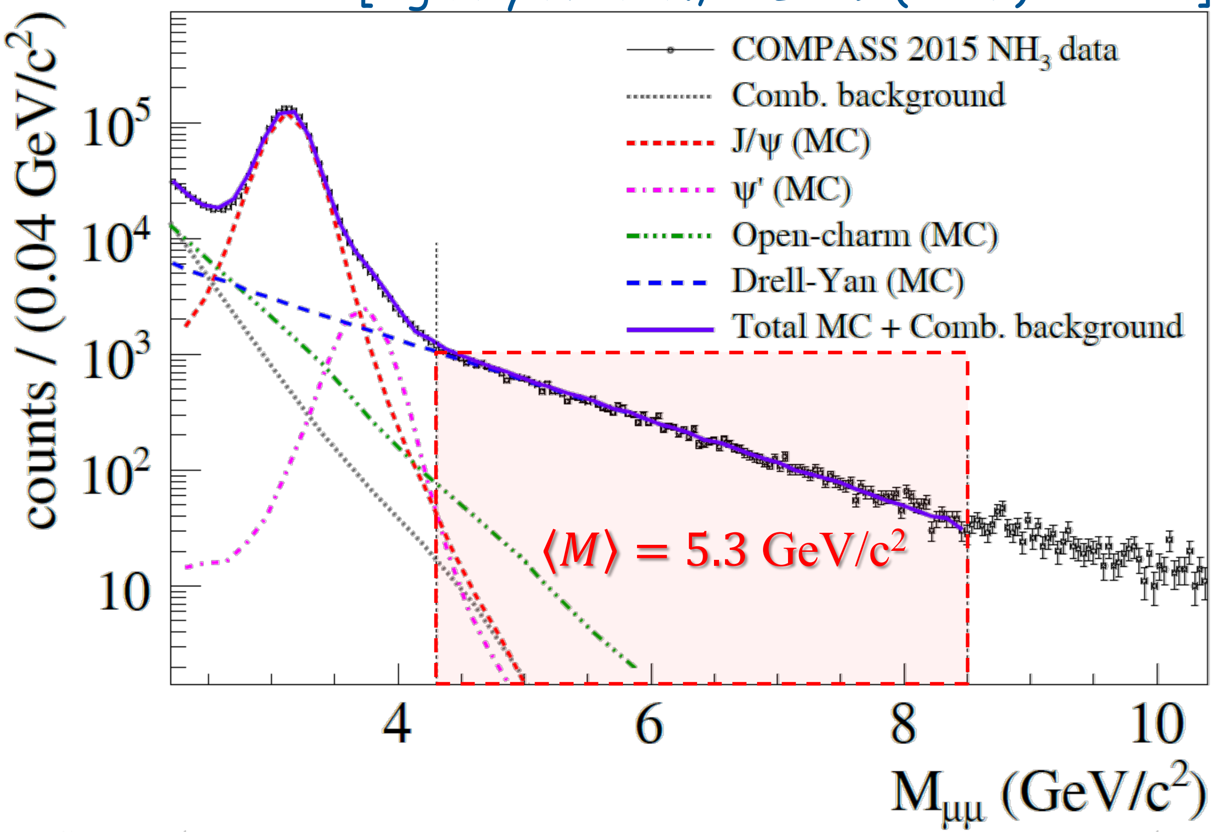
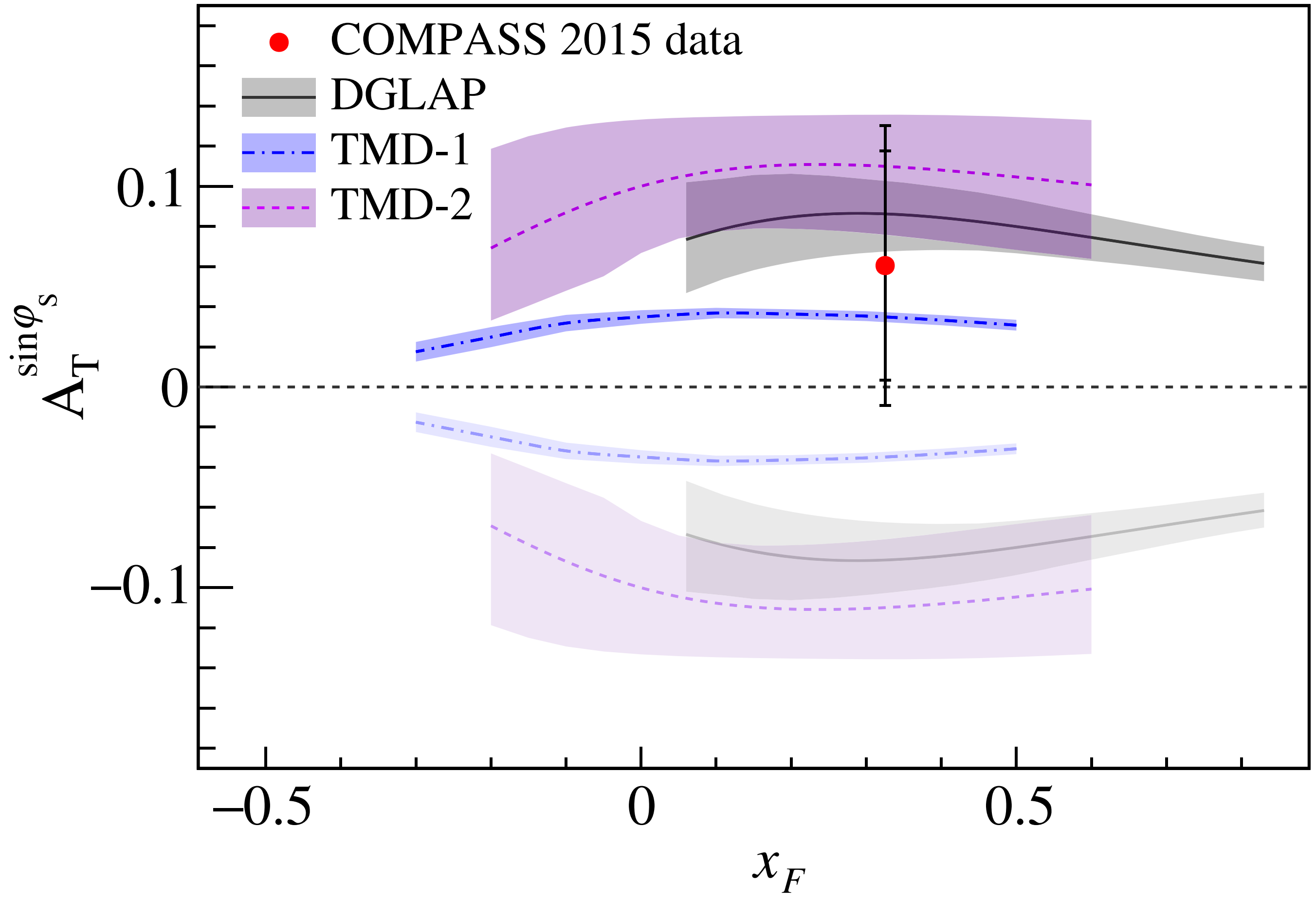
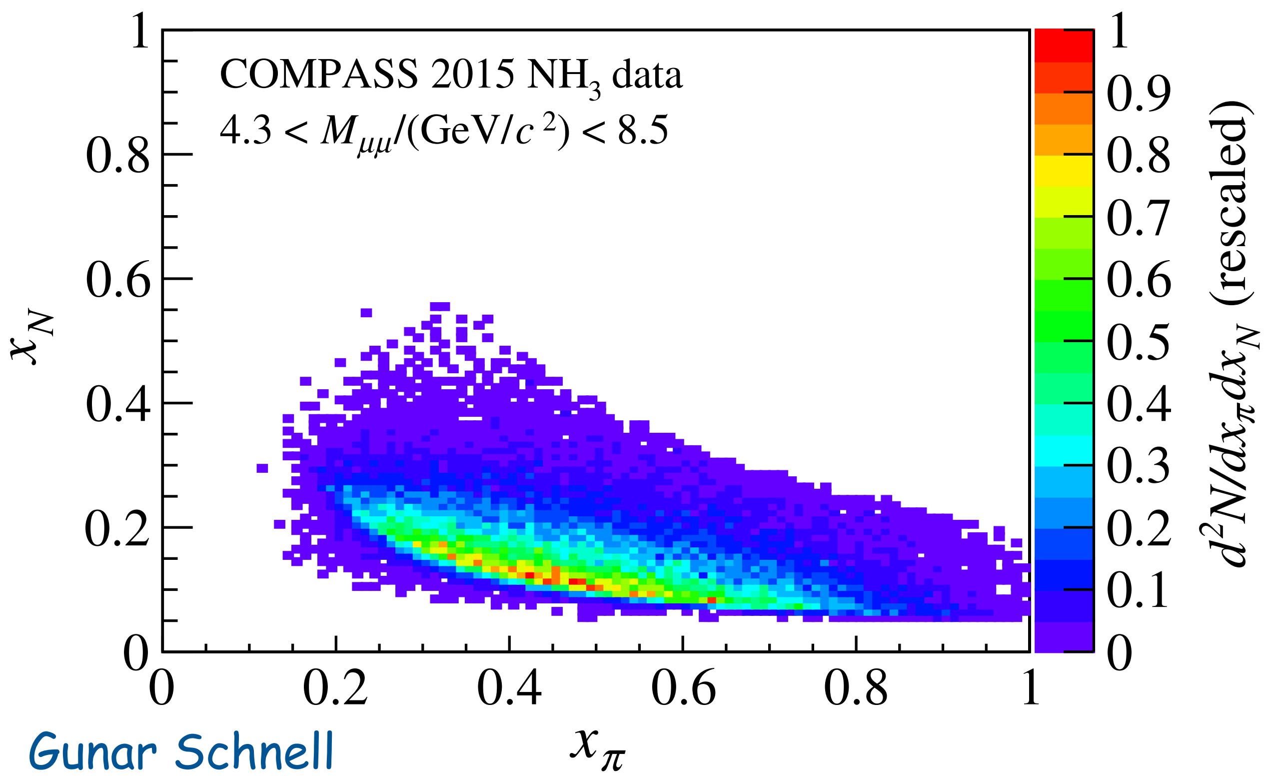




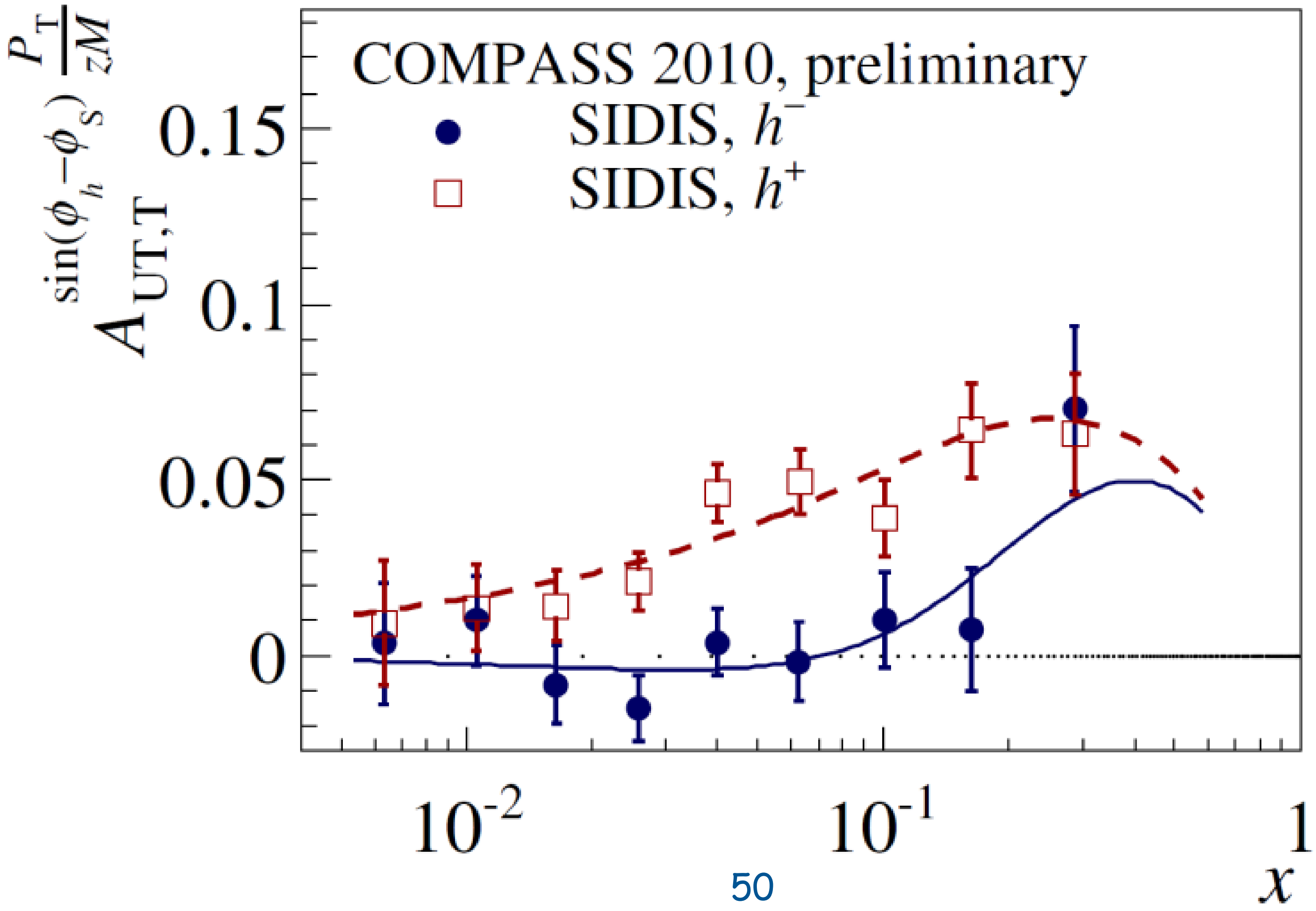


- (slight) preference for sign change
- some model curves move around when properly adjusted to exp.'s kinematics
- more data currently taken

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

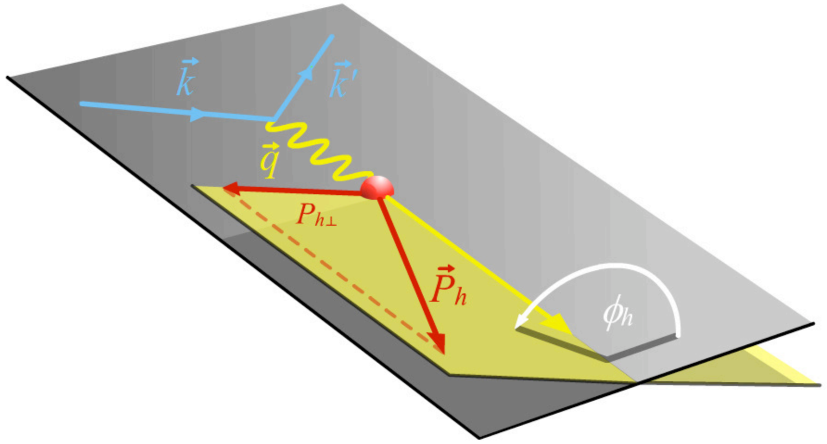
Sivers amplitudes - weighted

- $P_{h\perp}$ weighting, in principle, resolves convolutions [A. Kotzinian and P. Mulders, PLB 406 (1997) 373)]
- requires excellent control of detector efficiencies
- often no full integral (low- and high-$P_{h\perp}$ missing)



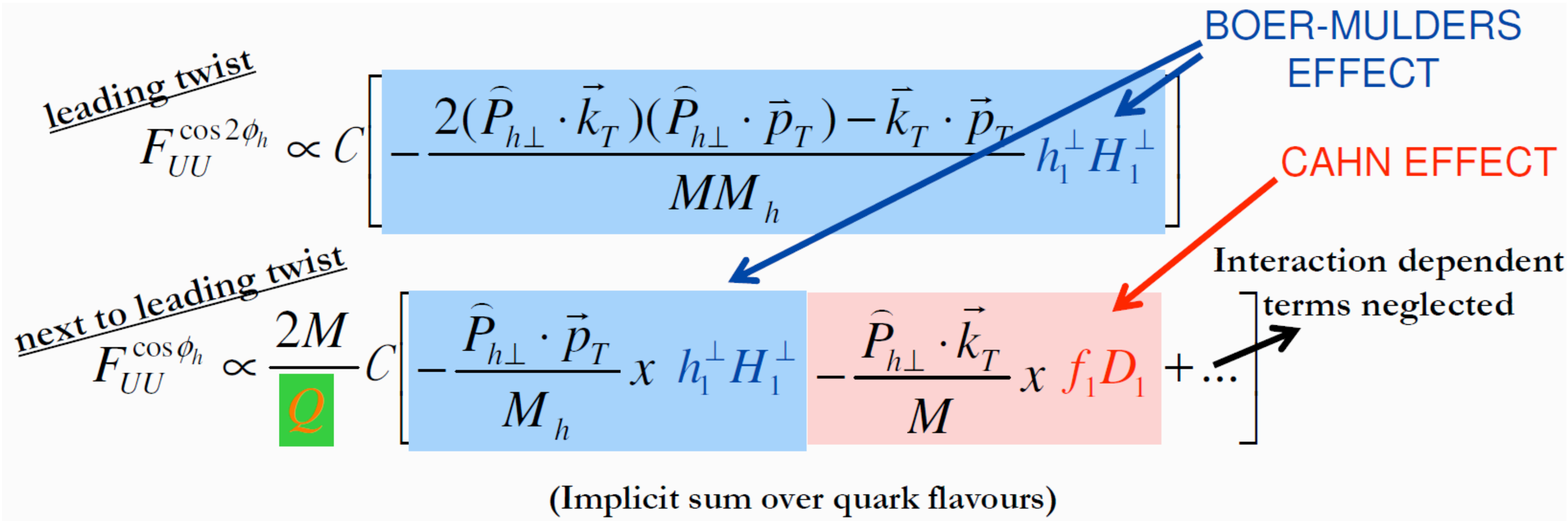
F. Bradamante (COMPASS at SPIN-2016) arXiv:1702.00621 [hep-ex]

modulations in spin-independent SIDIS cross section



$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_h\,\mathrm{d}P_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)\,F_{\mathrm{UU,T}} + B(y)F_{\mathrm{UU,L}}$$

$$+C(y)\cos\phi_h F_{\mathrm{UU}}^{\cos\phi_h} + B(y)\cos 2\phi_h F_{\mathrm{UU}}^{\cos 2\phi_h}\}$$



leading twist

$$F_{UU}^{\cos 2\phi_h} \propto C\left[-\frac{2(\hat{P}_{h\perp}\cdot\vec{k}_T)(\hat{P}_{h\perp}\cdot\vec{p}_T)-\vec{k}_T\cdot\vec{p}_T}{MM_h}\,h_1^\perp H_1^\perp\right]$$

BOER-MULDERS EFFECT

CAHN EFFECT

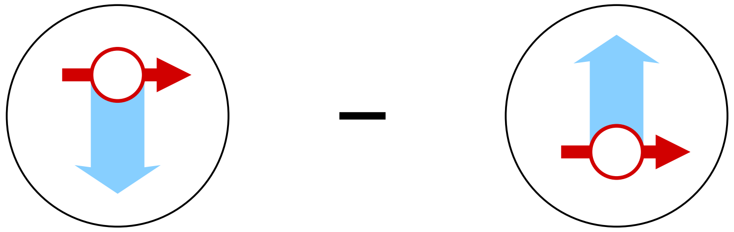
next to leading twist

$$F_{UU}^{\cos\phi_h} \propto \frac{2M}{Q}C\left[-\frac{\hat{P}_{h\perp}\cdot\vec{p}_T}{M_h}x\;h_1^\perp H_1^\perp - \frac{\hat{P}_{h\perp}\cdot\vec{k}_T}{M}x\;f_1 D_1 + \dots\right]$$

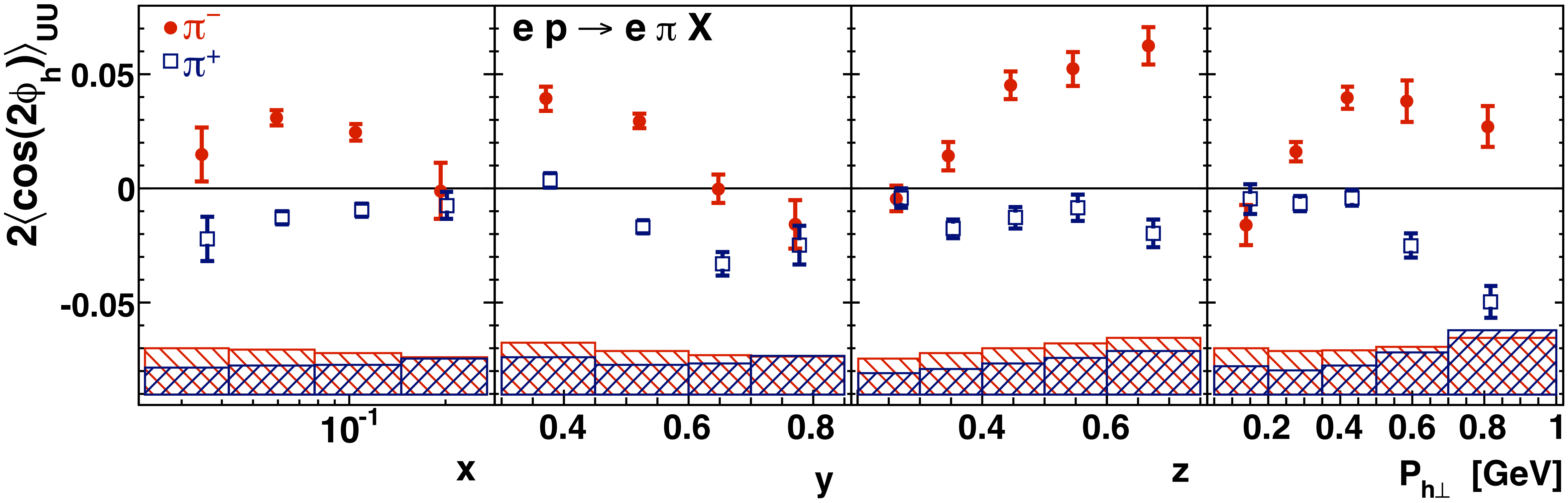
Interaction dependent terms neglected

(Implicit sum over quark flavours)

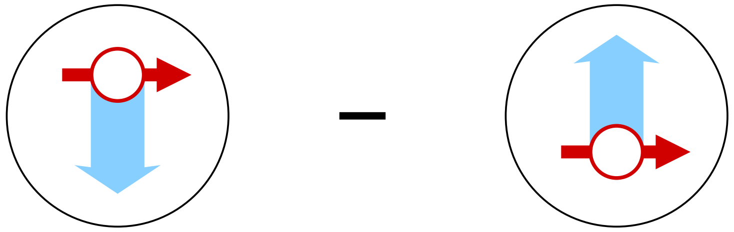
signs of Boer-Mulders



[Airapetian et al., PRD 87 (2013) 012010]



- not zero!



signs of Boer-Mulders

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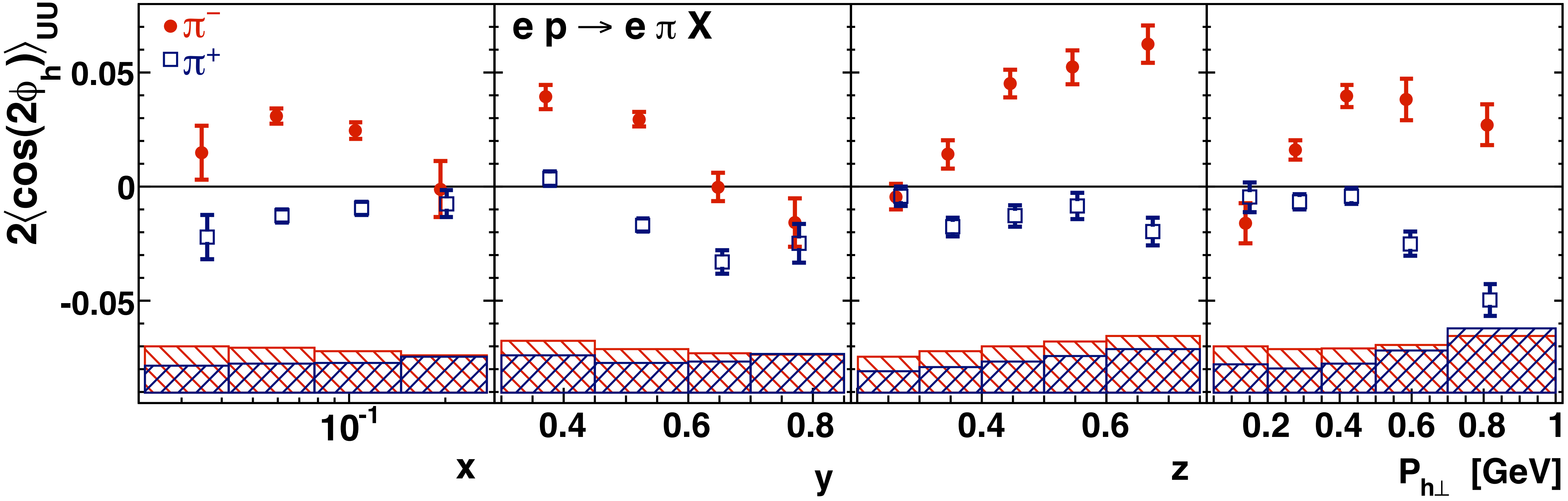
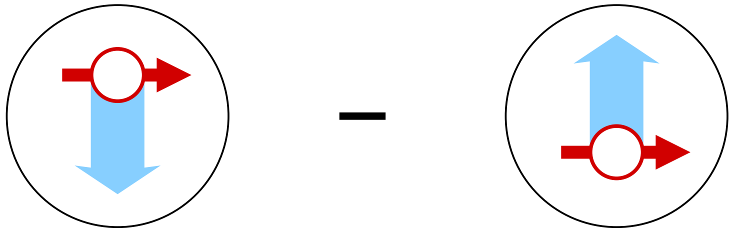
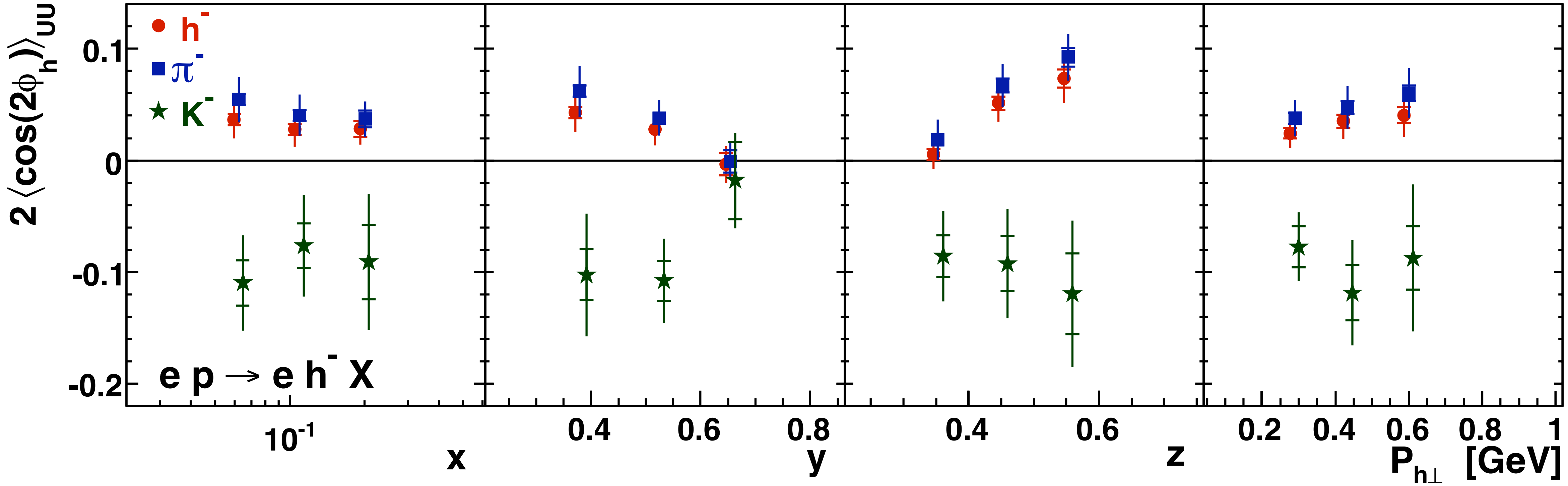


- not zero!
- opposite sign for charged pions with larger magnitude for π^-

  -> same-sign BM-function for valence quarks?

signs of Boer-Mulders





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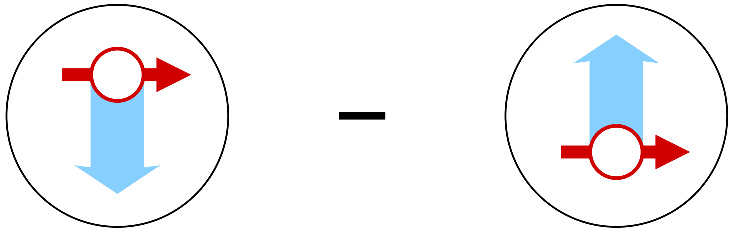
signs of Boer-Mulders



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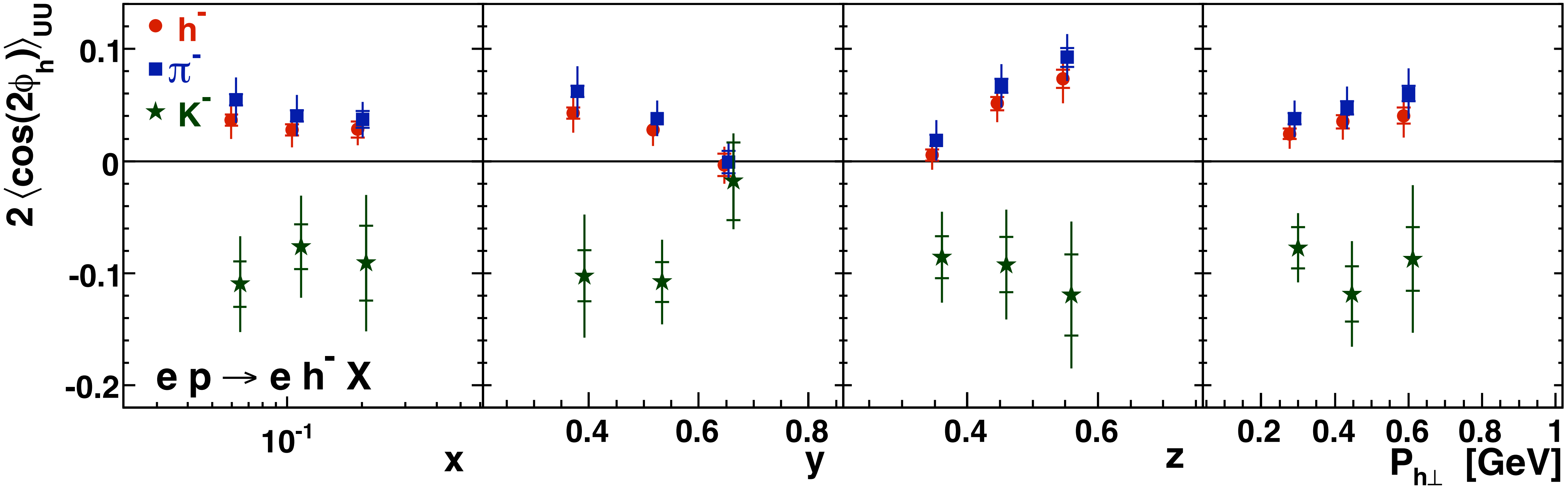
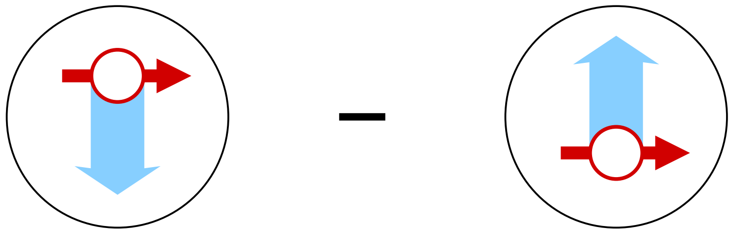
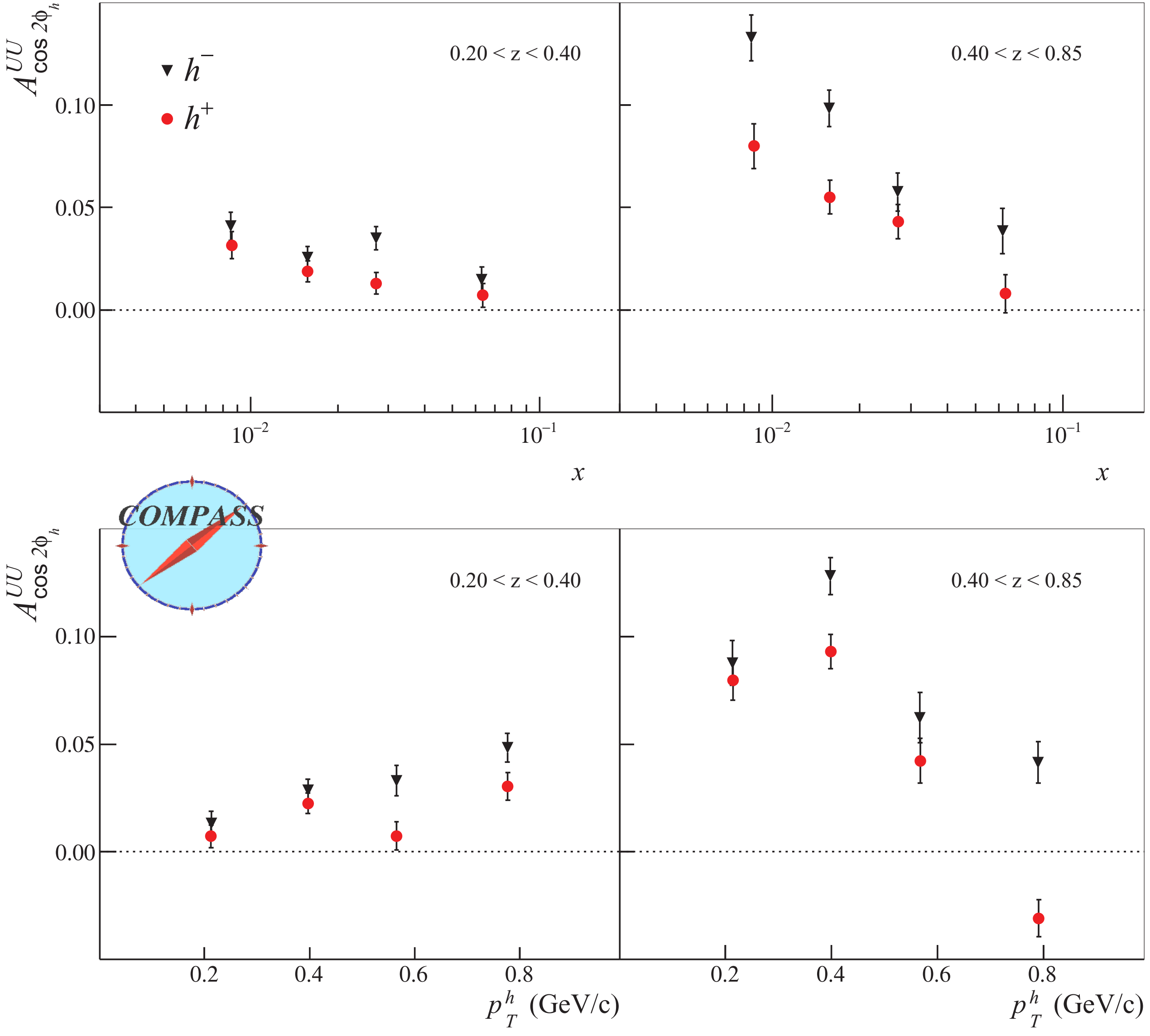


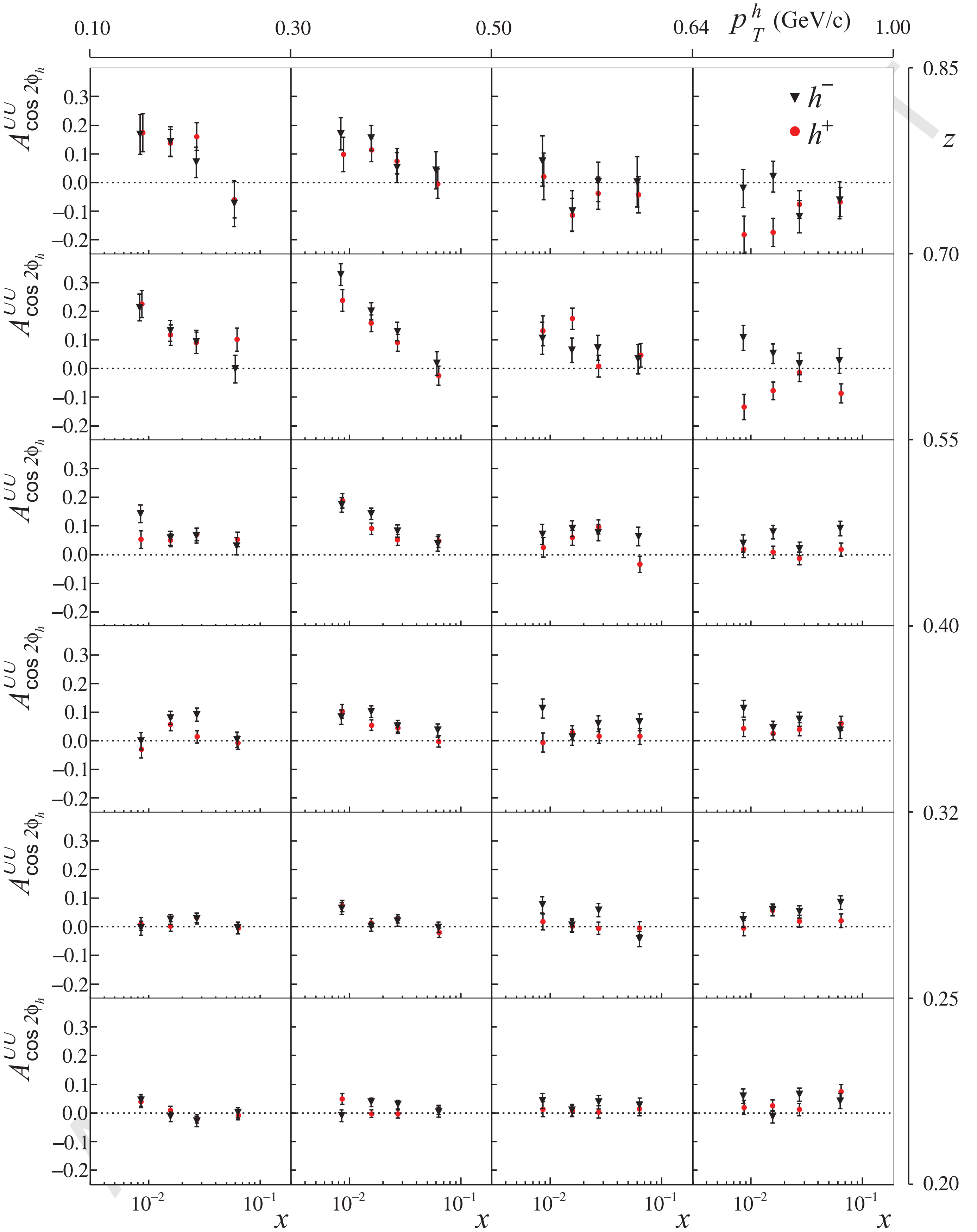
- not zero!
- opposite sign for charged pions with larger magnitude for π^-
  -> same-sign BM-function for valence quarks?
- intriguing behavior for kaons
- available in multidimensional binning both from HERMES and from COMPASS

signs of Boer-Mulders

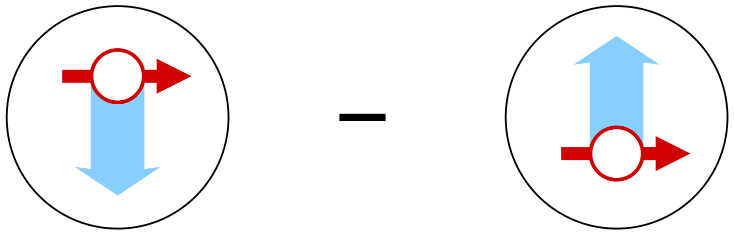


[10.1016/j.nuclphysb.2014.07.019]





signs of Boer-Mulders



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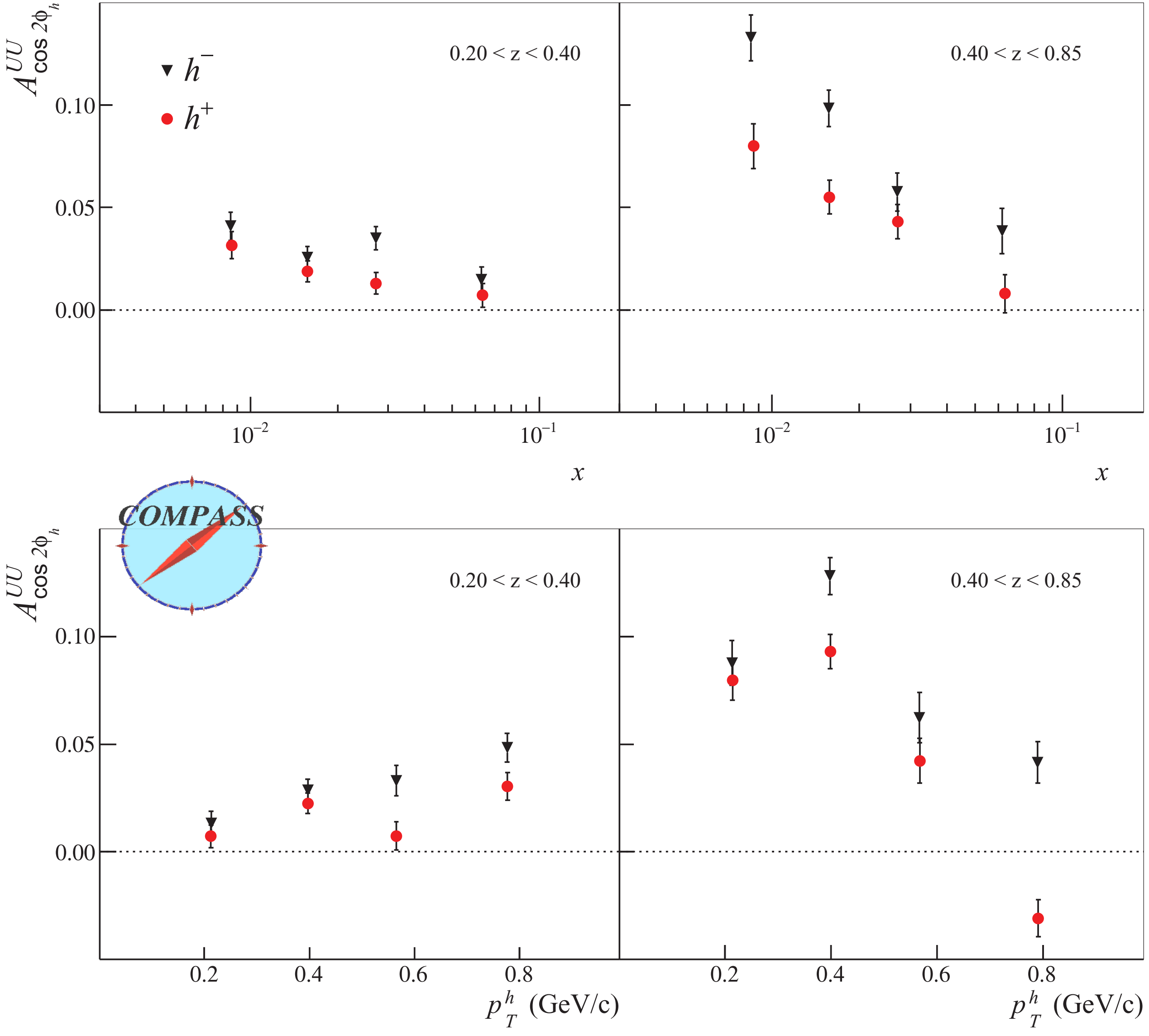
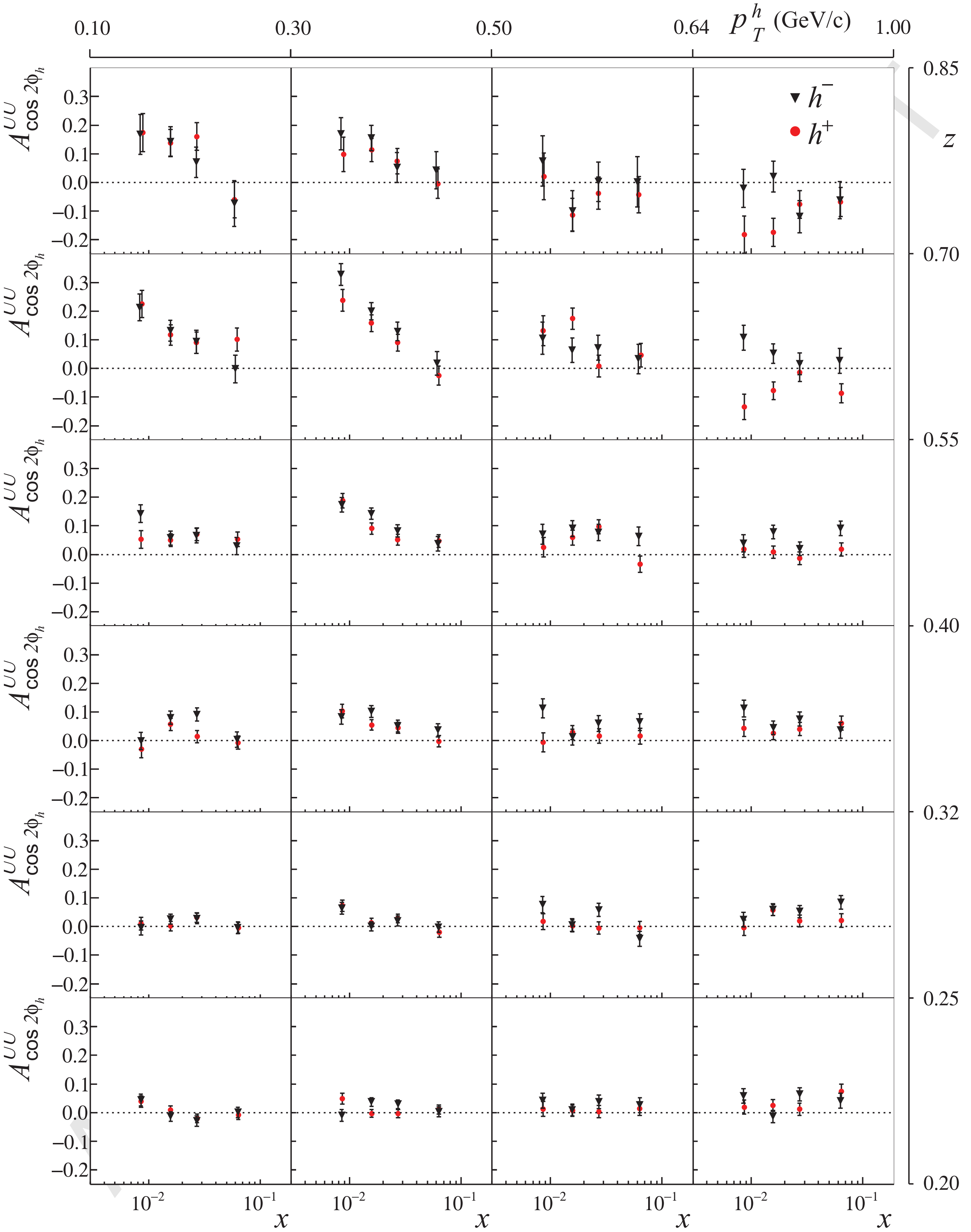
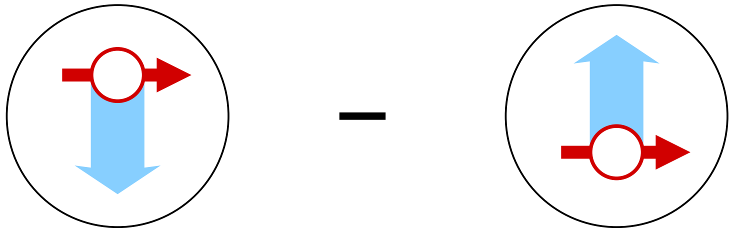


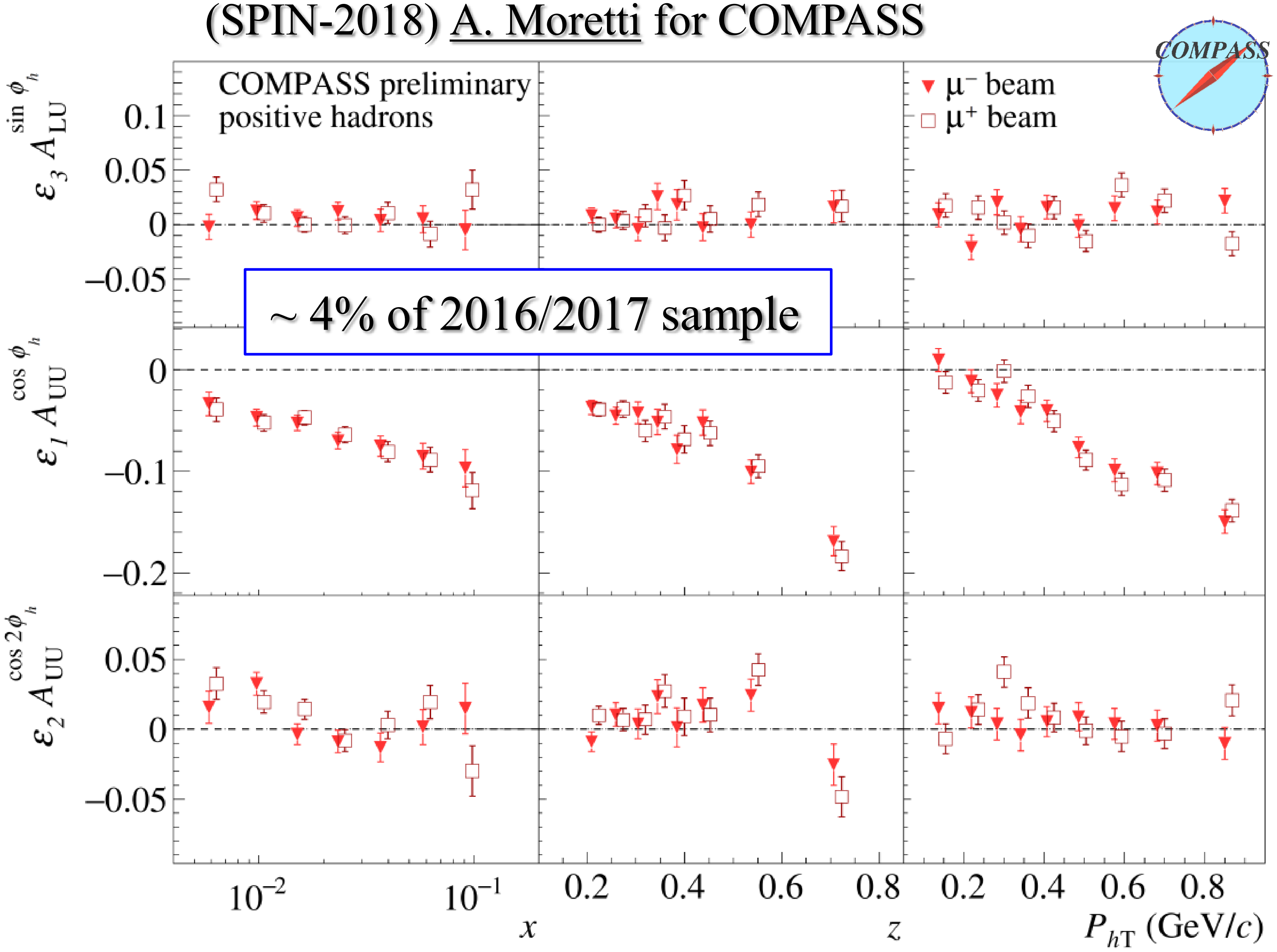


- unlike HERMES same sign for h^+ and h^-, though still different from each other

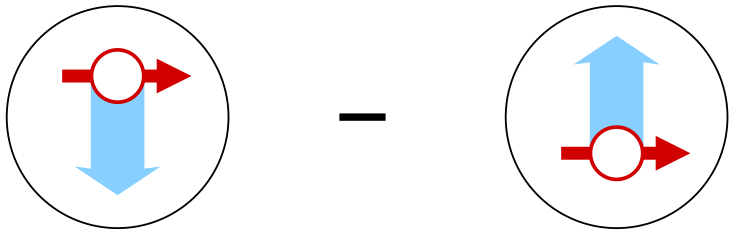
signs of Boer-Mulders



(SPIN-2018) A. Moretti for COMPASS



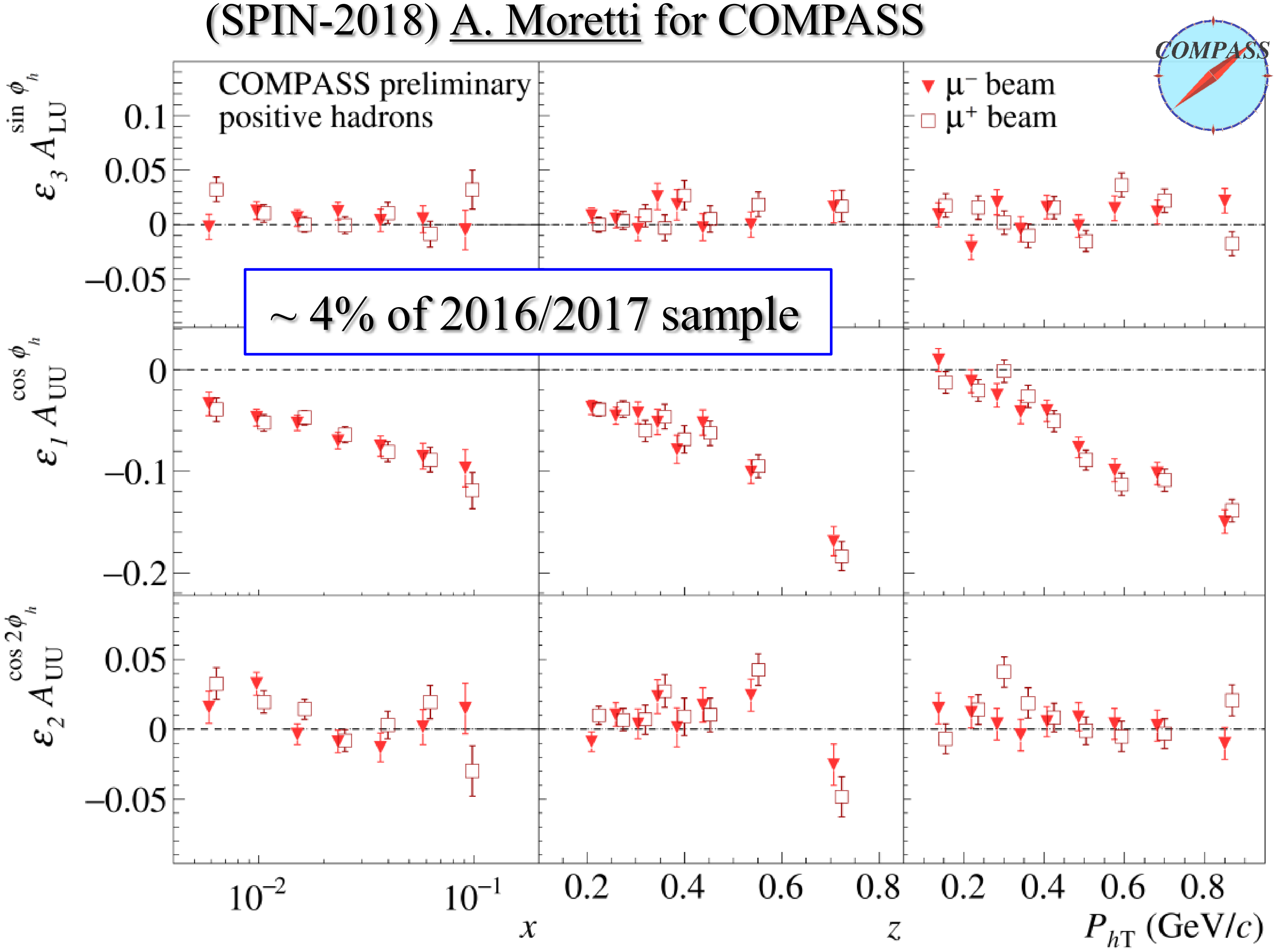
~ 4% of 2016/2017 sample



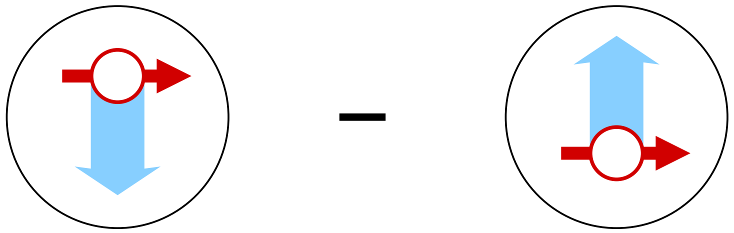
signs of Boer-Mulders

- in 2016/17 extensive data set collected on liquid-H target (DVCS program)

(SPIN-2018) A. Moretti for COMPASS



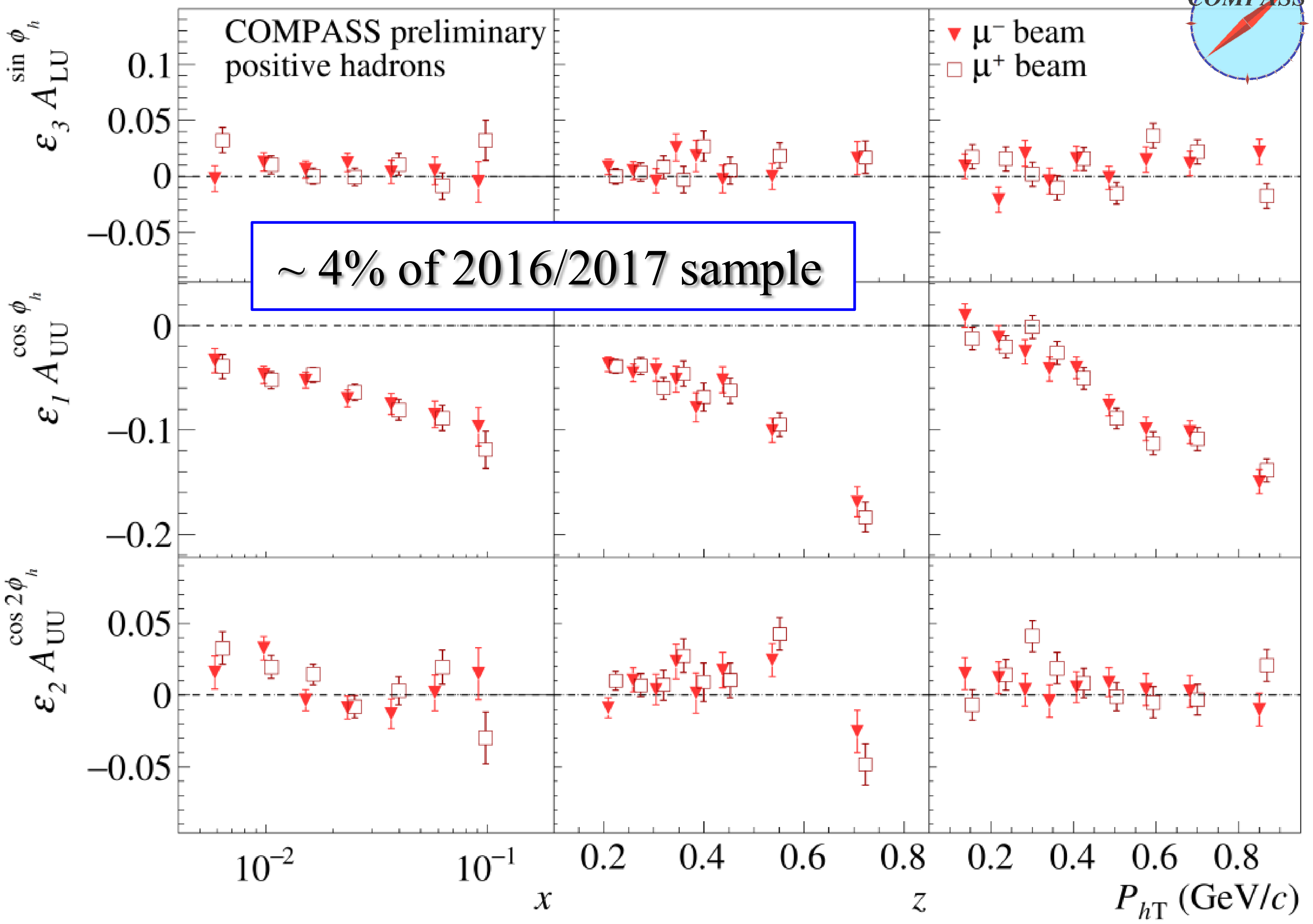
~ 4% of 2016/2017 sample



signs of Boer-Mulders

- in 2016/17 extensive data set collected on liquid-H target (DVCS program)
- will allow precision studies of multiplicities and A_{UU} & A_{LU} modulations

(SPIN-2018) A. Moretti for COMPASS



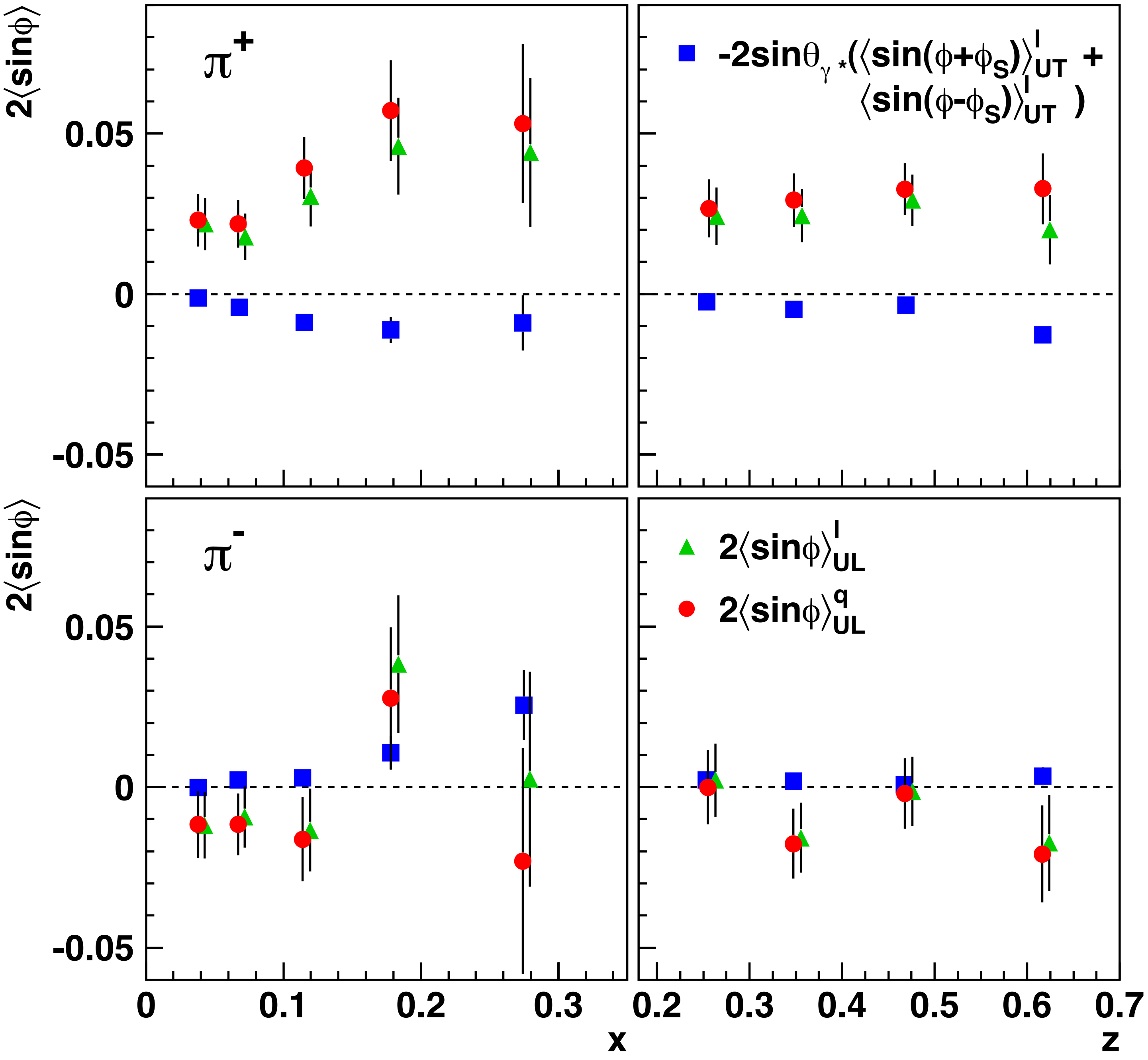
~ 4% of 2016/2017 sample

non-vanishing twist-3

subleading twist I - <sin(ϕ)>$_{UL}$

$$\left\langle \sin\phi \right\rangle^{q}_{UL} = \left\langle \sin\phi \right\rangle^{l}_{UL} + \sin\theta_{\gamma^*} \left(\left\langle \sin(\phi+\phi_S) \right\rangle^{l}_{UT} + \left\langle \sin(\phi-\phi_S) \right\rangle^{l}_{UT} \right)$$

[Airapetian et al., PLB 622 (2005) 14]



- experimental A_{UL} dominated by twist-3 contribution
  - correction for A_{UT} contribution increases purely longitudinal asymmetry for positive pions
  - consistent with zero for π^-

subleading twist I - <sin(ϕ)>$_{UL}$

$$\left\langle \sin\phi \right\rangle^{q}_{UL} = \left\langle \sin\phi \right\rangle^{l}_{UL} + \sin\theta_{\gamma^*}\left(\left\langle \sin(\phi+\phi_S) \right\rangle^{l}_{UT} + \left\langle \sin(\phi-\phi_S) \right\rangle^{l}_{UT} \right)$$

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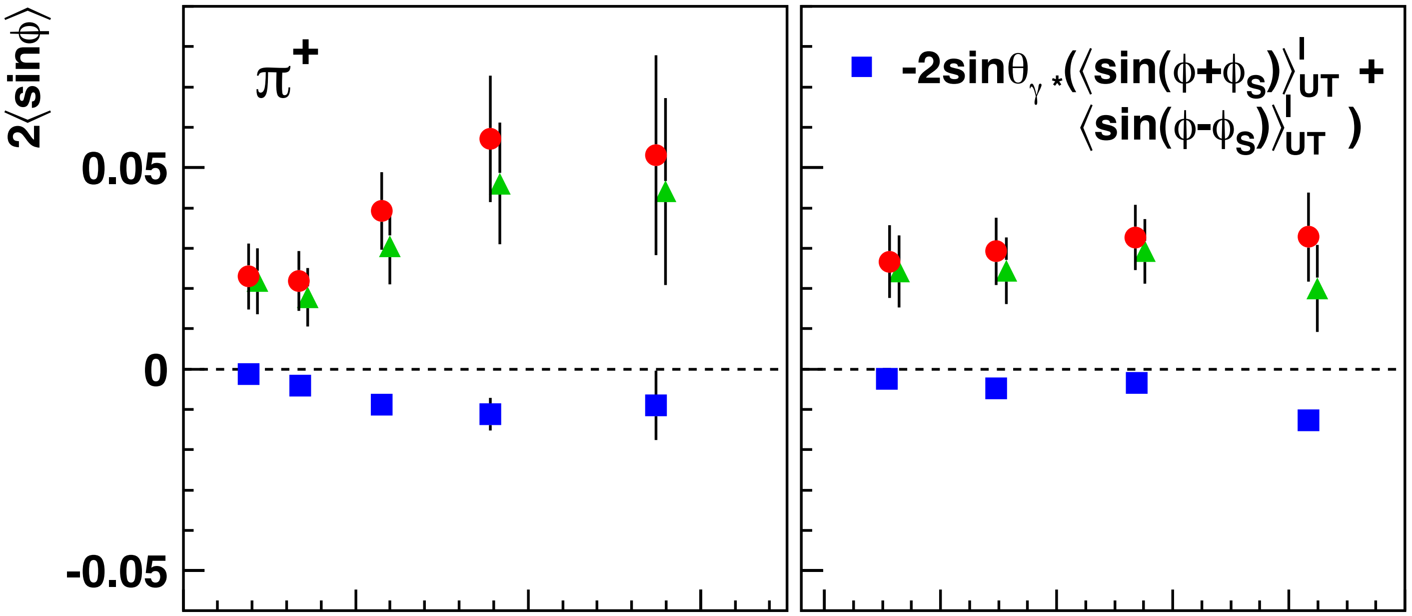
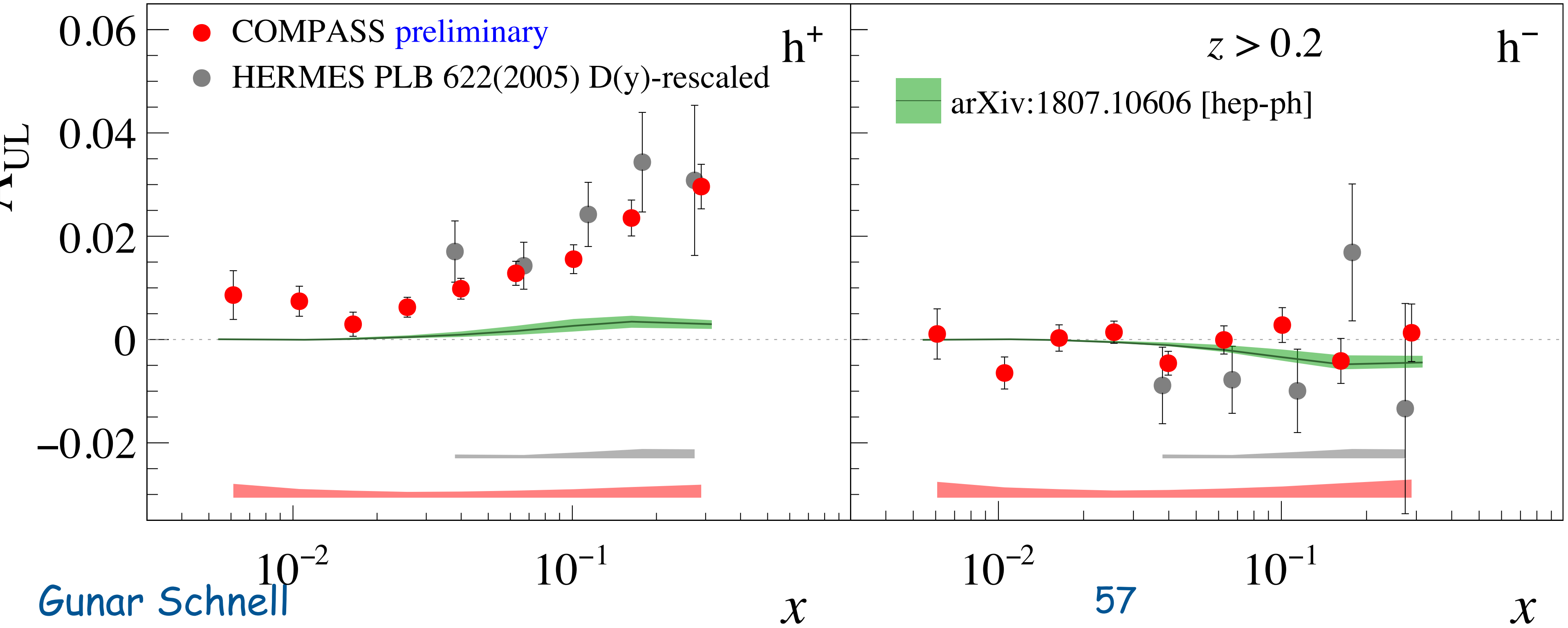
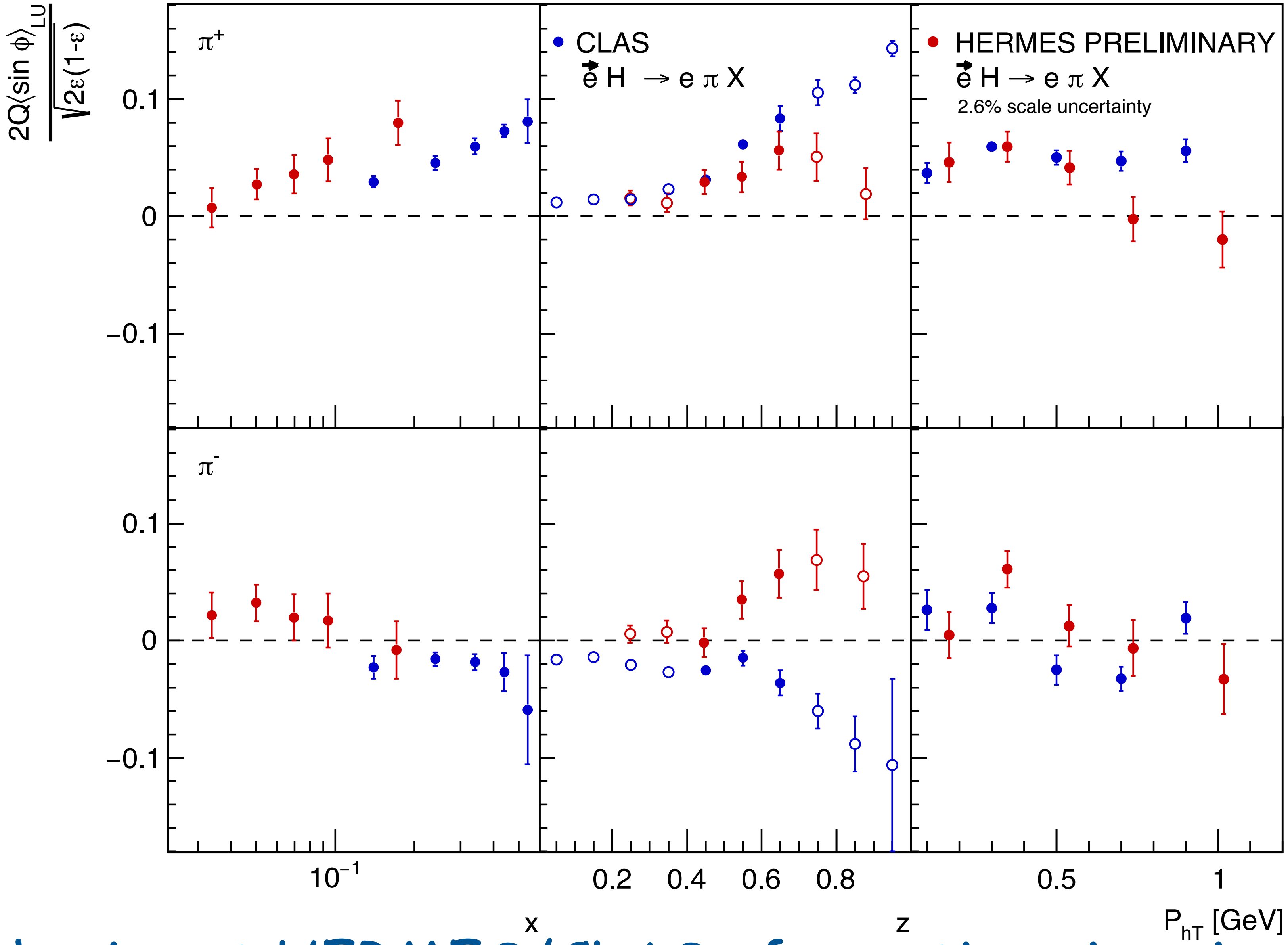




- experimental A_{UL} dominated by twist-3 contribution
- in contrast to WW-type approximation [1807.10606]

subleading twist II - <sin(ϕ)>$_{LU}$

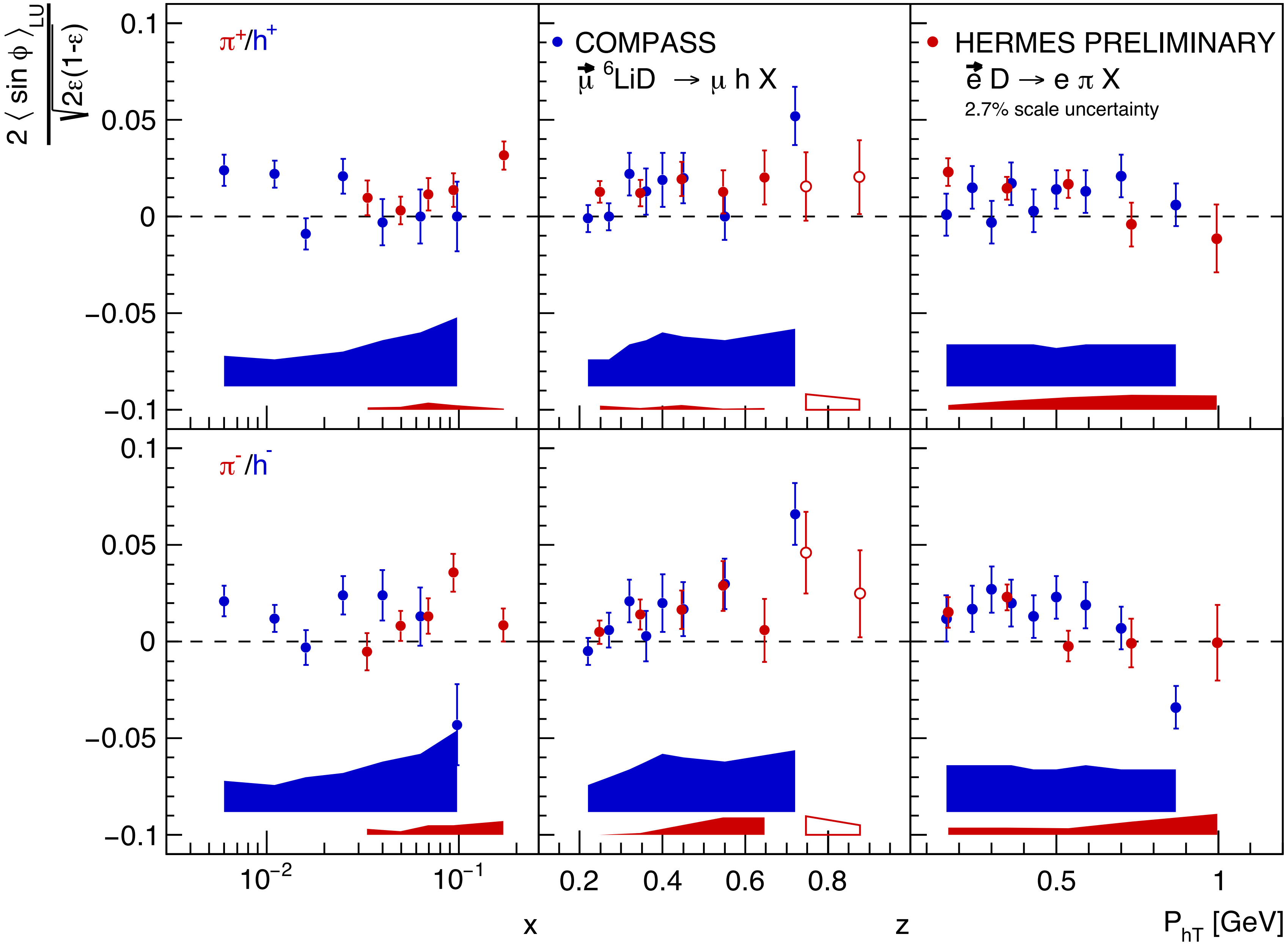
$$\frac{M_h}{Mz}h_1^\perp E \oplus xg^\perp D_1 \oplus \frac{M_h}{Mz}f_1 G^\perp \oplus xeH_1^\perp$$



- opposite behavior at HERMES/CLAS of negative pions in z projection due to different x-range probed
- CLAS more sensitive to e(x)Collins term due to higher x probed?

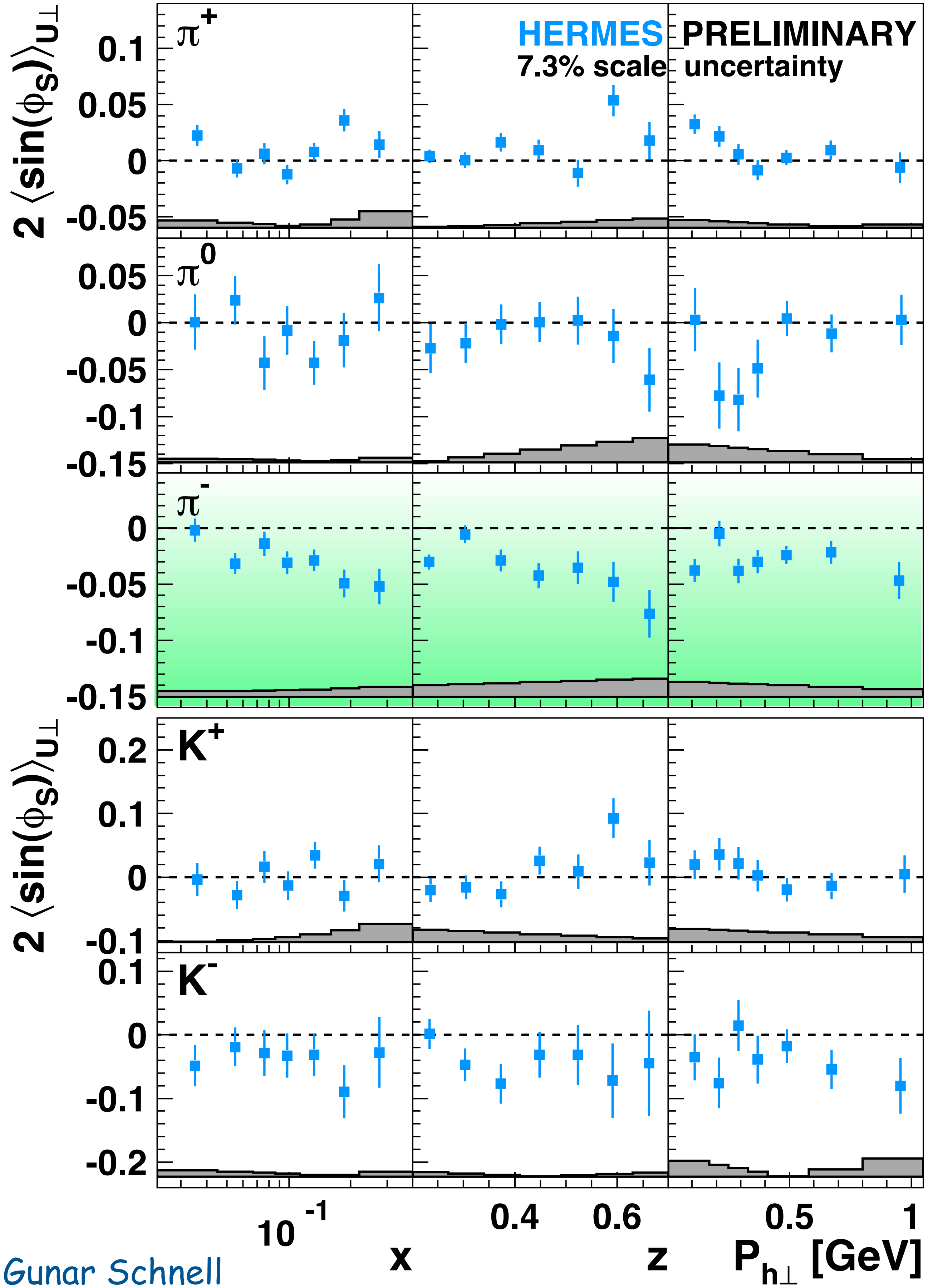
subleading twist II - <sin(ϕ)>$_{LU}$

$$\frac{M_h}{Mz}h_1^\perp E \oplus xg^\perp D_1 \oplus \frac{M_h}{Mz}f_1G^\perp \oplus xeH_1^\perp$$



- consistent behavior for charged pions / hadrons at HERMES / COMPASS for isoscalar targets

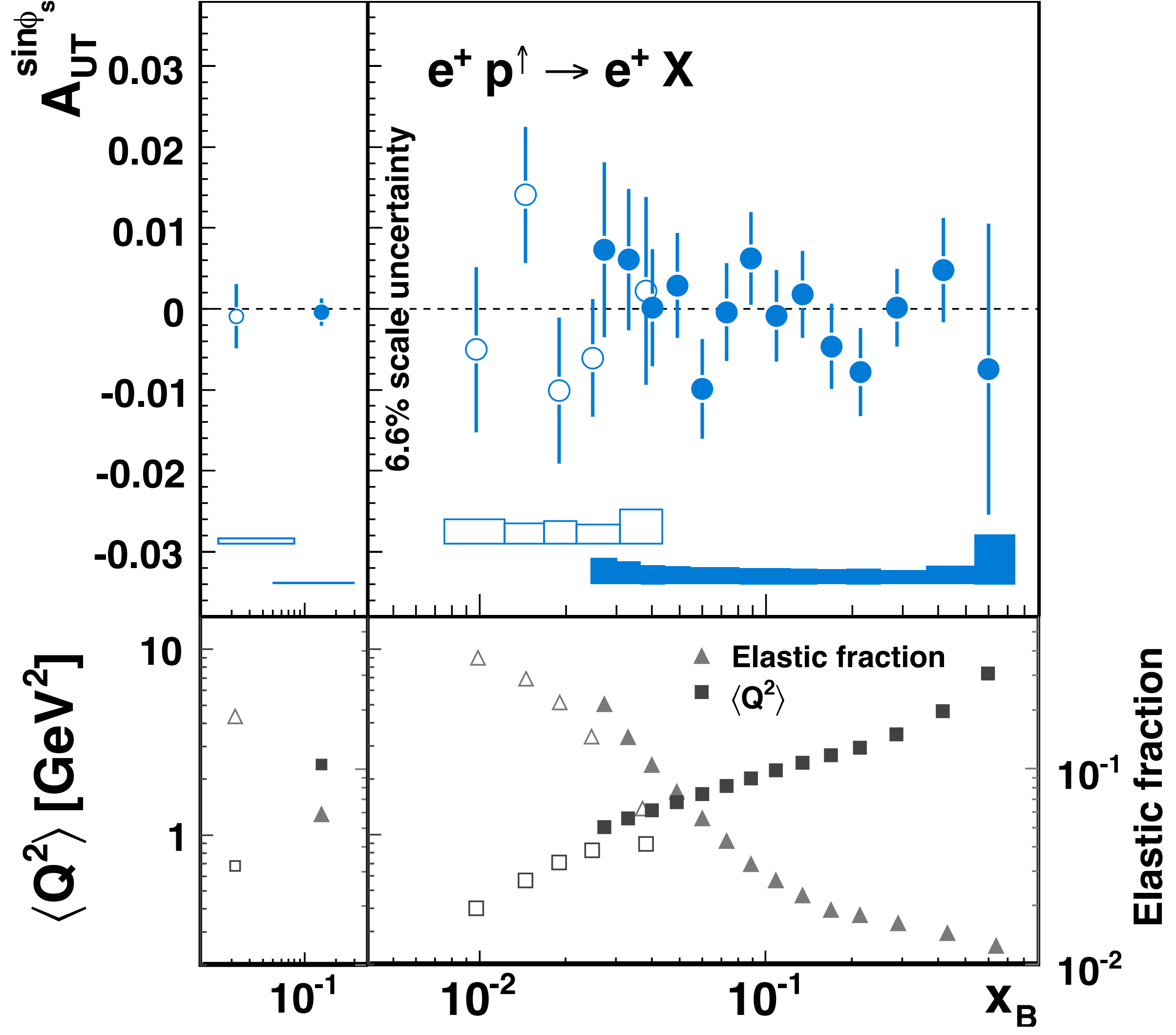
subleading twist III - $\langle \sin(\phi_s) \rangle_{UT}$



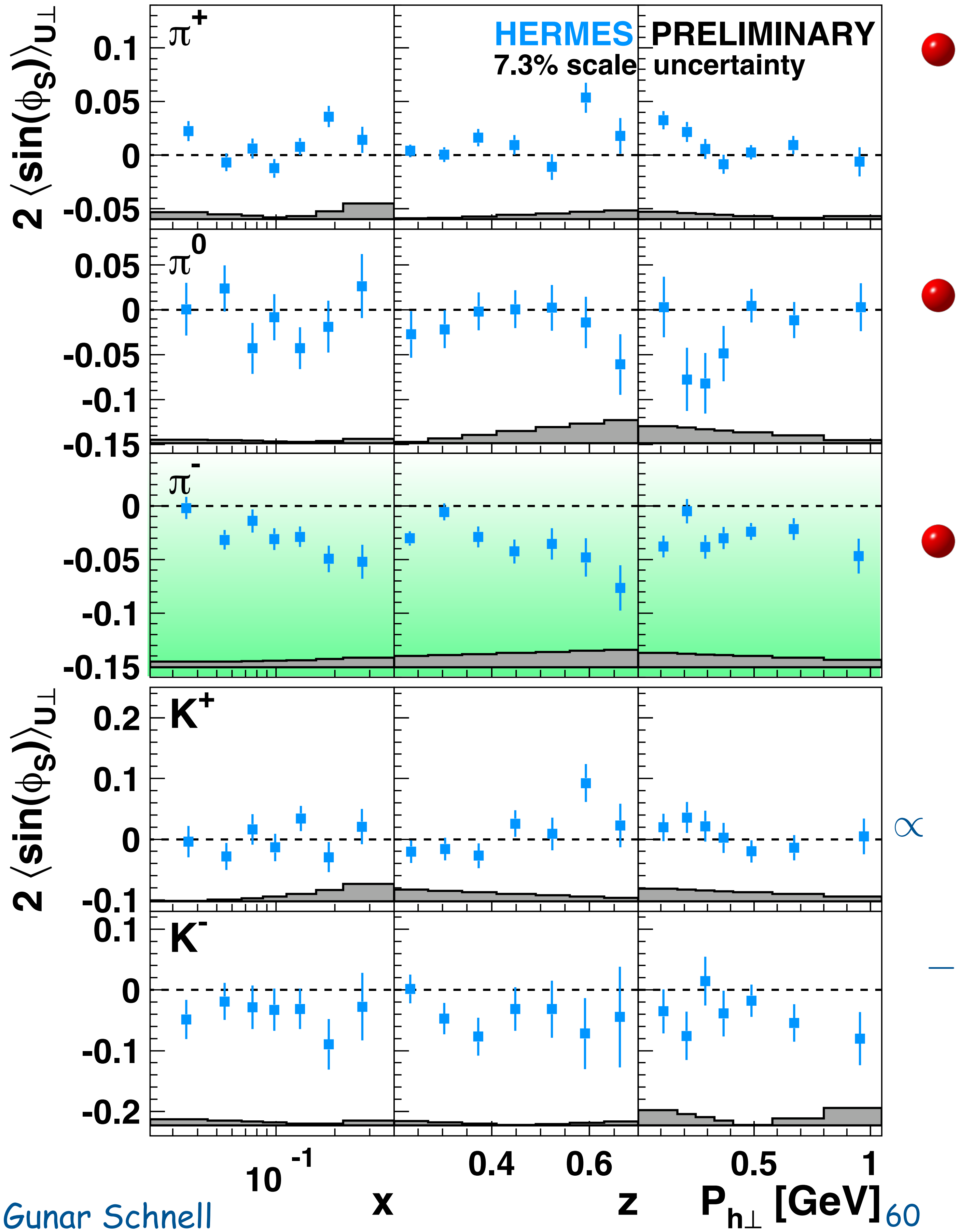
- significant non-zero signal observed for negatively charged mesons

- vanishes in inclusive limit, e.g. after integration over $P_{h\perp}$ and z, and summation over all hadrons

[Phys. Lett. B682 (2010) 351]



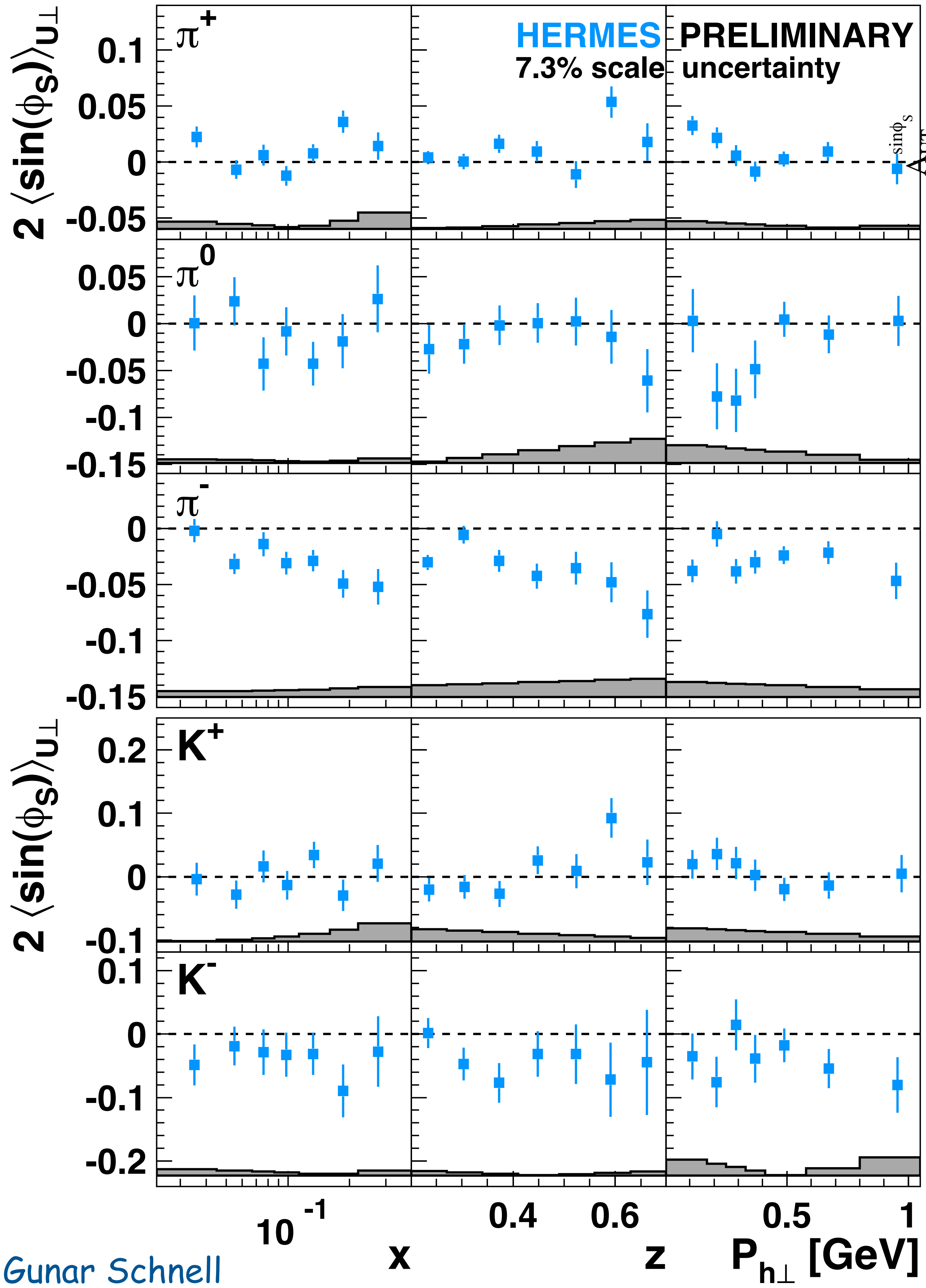
subleading twist III - $\langle\sin(\phi_s)\rangle_{UT}$

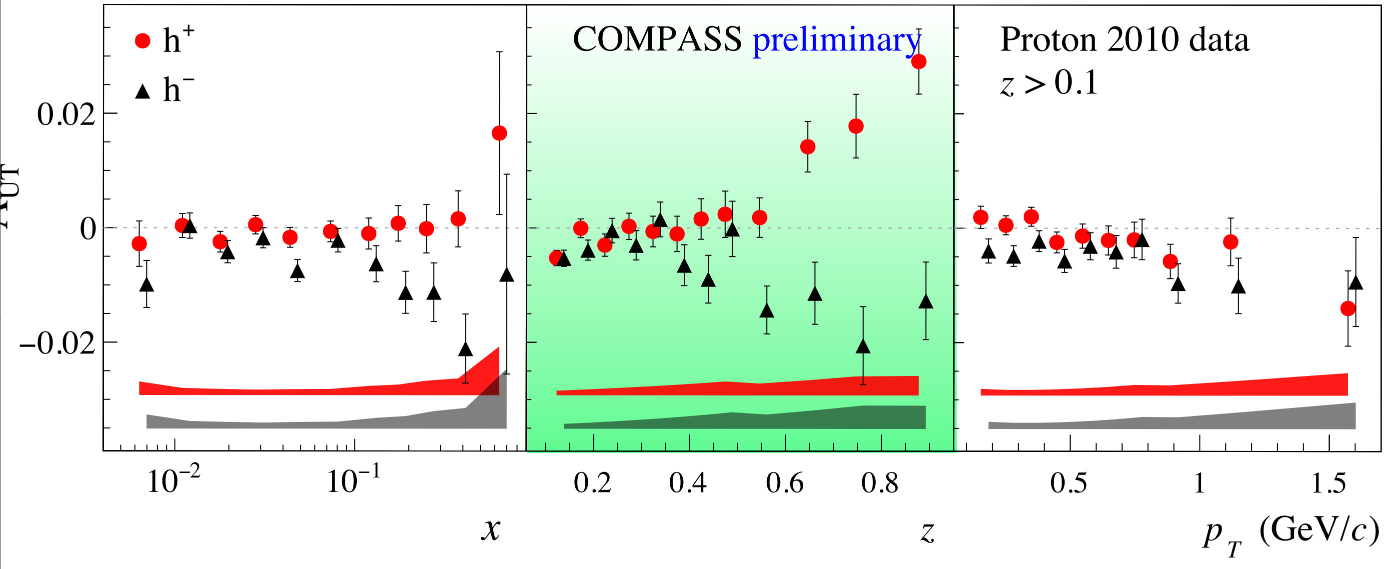


- significant non-zero signal observed for negatively charged mesons
- vanishes in inclusive limit, e.g. after integration over $P_{h\perp}$ and z, and summation over all hadrons
- various terms related to transversity, worm-gear, Sivers etc.:

$$\propto \left(xf_T^\perp D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z}\right) - \mathcal{W}(p_T, k_T, P_{h\perp}) \left[\left(xh_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z}\right) - \left(xh_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z}\right)\right]$$

subleading twist III - <sin(ϕ_s)>$_{UT}$



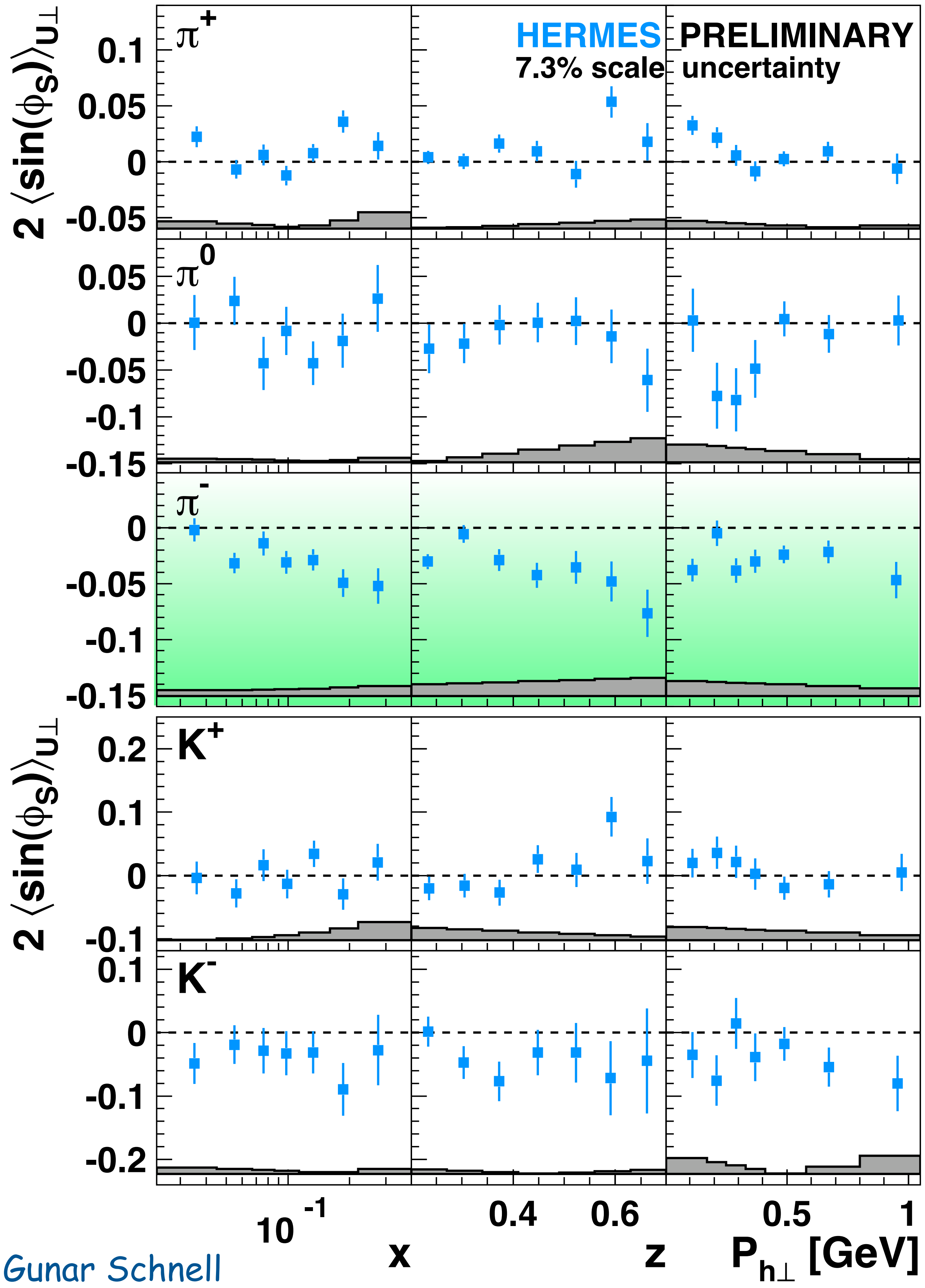


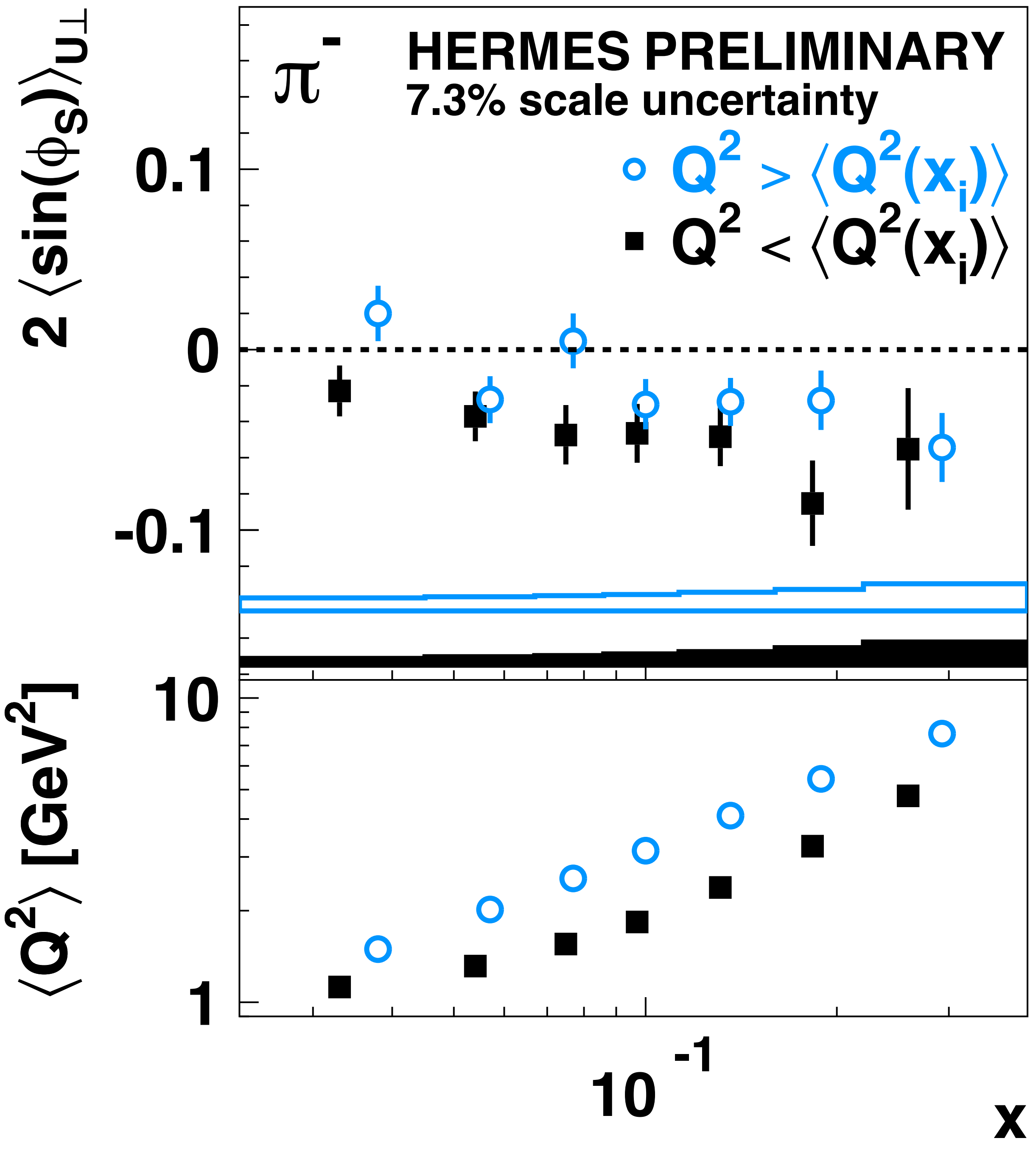
- opposite signs at large z
  -> Collins-like behavior

- indeed $\tilde{H}$ related to Collins fct.

$$\propto \left(x f_T^\perp D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) - \mathcal{W}(p_T, k_T, P_{h\perp}) \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right]$$

subleading twist III - $\langle \sin(\phi_S) \rangle_{UT}$





- hint of Q^2 dependence seen in signal for negative pions

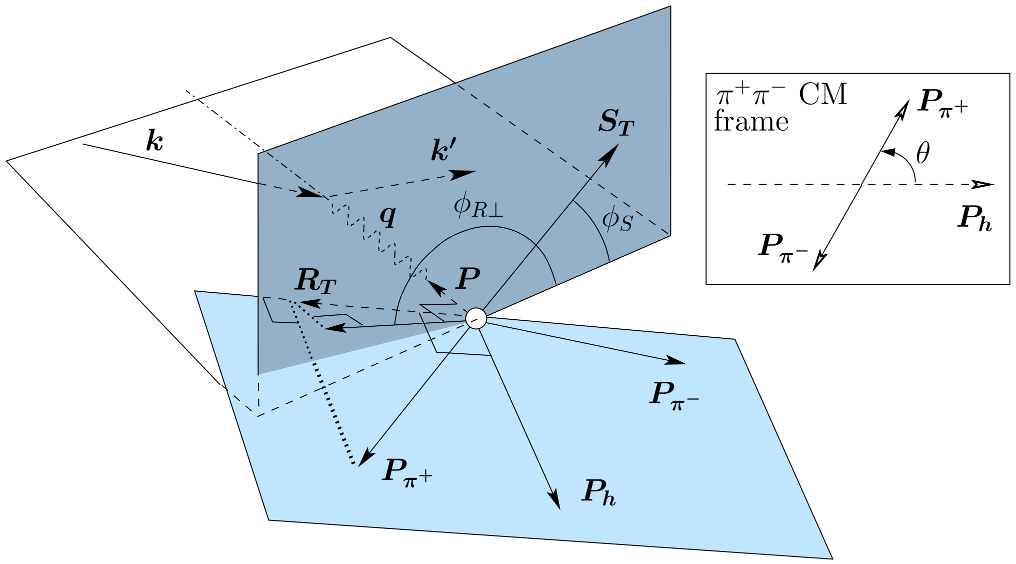
conclusions

- 1st round of SIDIS measurements coming to an end
- various indications of flavor-& spin-dependent transverse momentum
- transversity is non-zero and quite sizable
  - d-quark transversity difficult to access with only proton targets
- Sivers and chiral-even worm-gear function also clearly non-zero
- various sizable twist-3 effects
- highlights still to come
  - HERMES transverse-target, A_{LU} & A_{LL} asymmetries
  - COMPASS transverse d; high-statistics data set on unpol. pure H; multi-d asymmetries
- precision measurements needed to fully map TMD landscape (fully differential!)
- need also program with polarized D and ^{3}He

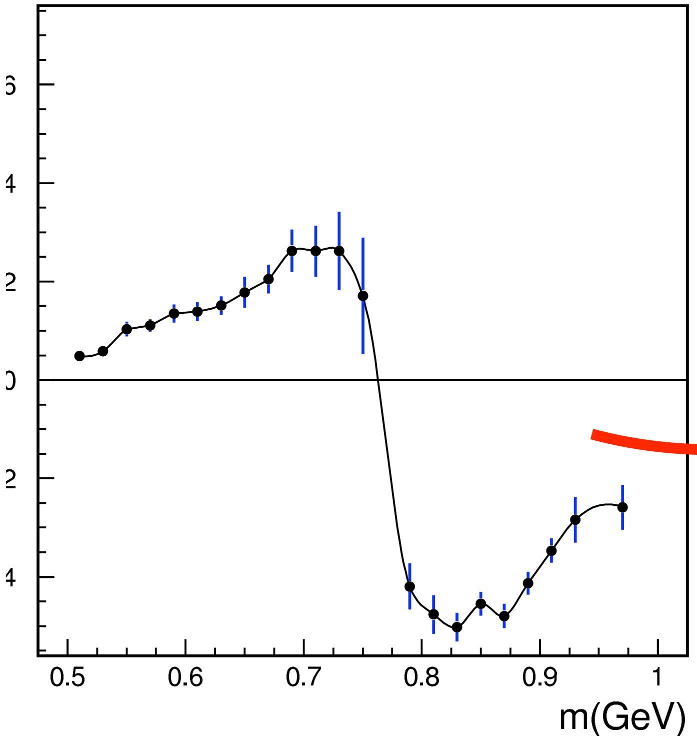
backup

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Transversity (2-hadron fragmentation)



$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S)\sin\theta h_1 H_1^{\sphericalangle}$$



Jaffe et al. [**hep-ph/9709322**]:

$$H_1^{\sphericalangle,sp}(z, M_{\pi\pi}^2) = \underline{\sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)} H_1^{\sphericalangle,sp'}(z)$$

δ_0 (δ_1) → S(P)-wave phase shifts

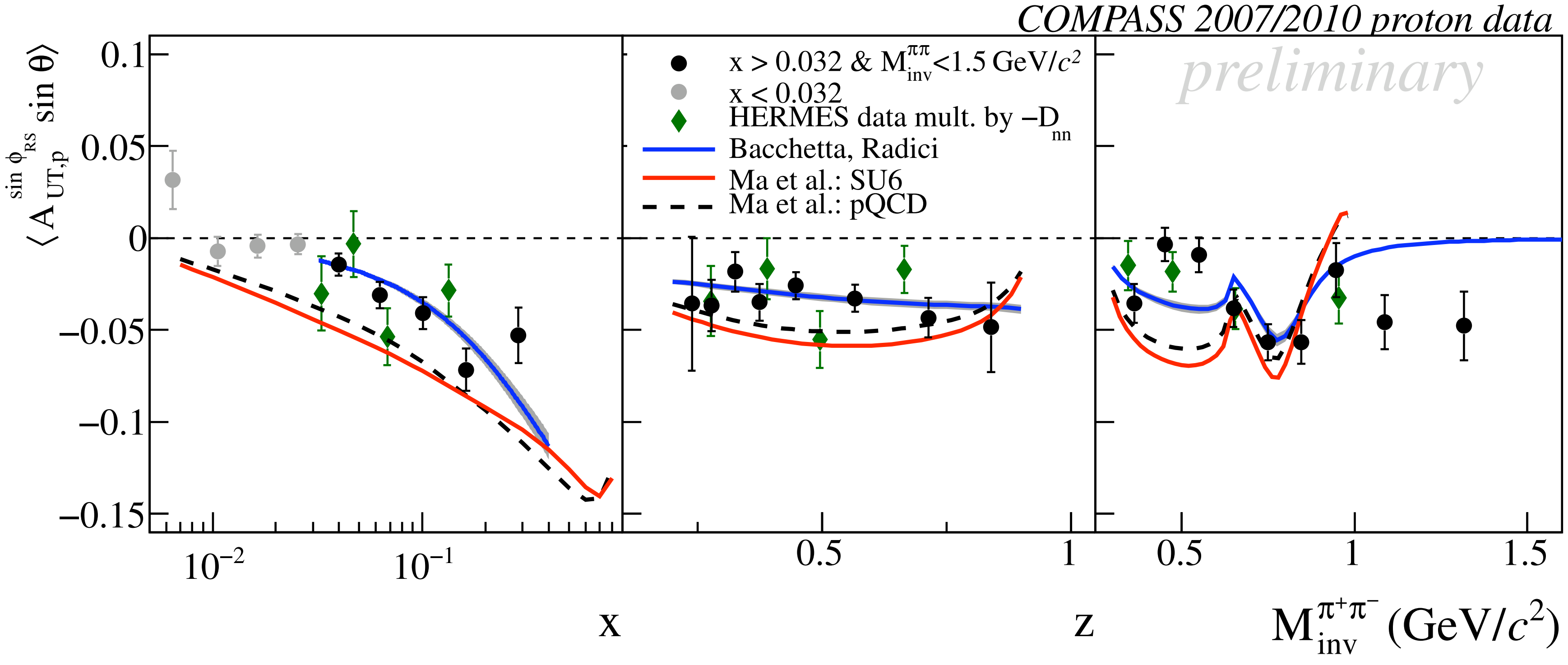
$$= \mathcal{P}(M_{\pi\pi}^2) H_1^{\sphericalangle,sp'}(z)$$

$\Rightarrow$ A_{UT} might depend strongly on $M_{\pi\pi}$

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Transversity (2-hadron fragmentation)

[A. Airapetian et al., JHEP 06 (2008) 017]
COMPASS 2007: [C. Adolph et al., Phys. Lett. B713 (2012) 10]
COMPASS 2010: [C. Braun et al., Nuovo Cimento C 035 (2012) 02]



	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Transversity (2-hadron fragmentation)

- HERMES, COMPASS: for comparison scaled HERMES data by depolarization factor and changed sign

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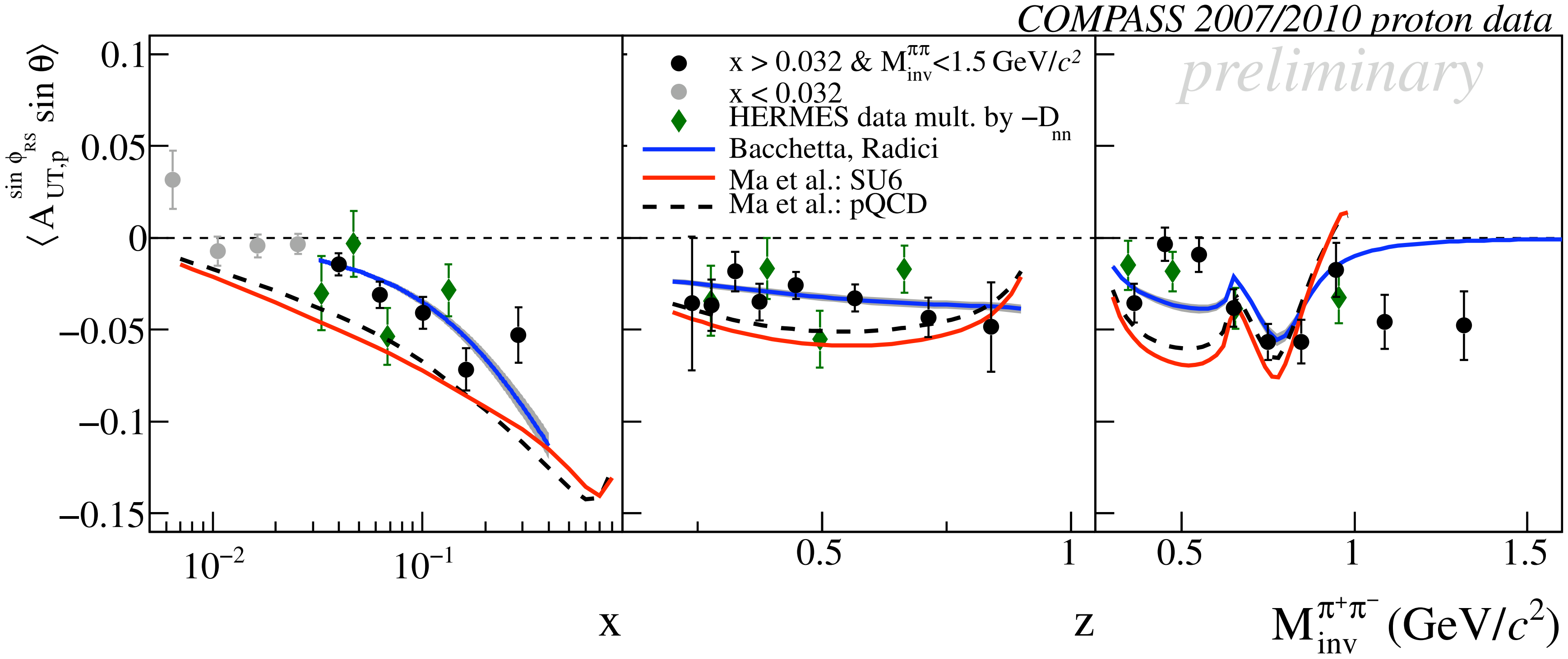

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Transversity (2-hadron fragmentation)

- HERMES, COMPASS: for comparison scaled HERMES data by depolarization factor and changed sign
- ^{2}H results consistent with zero

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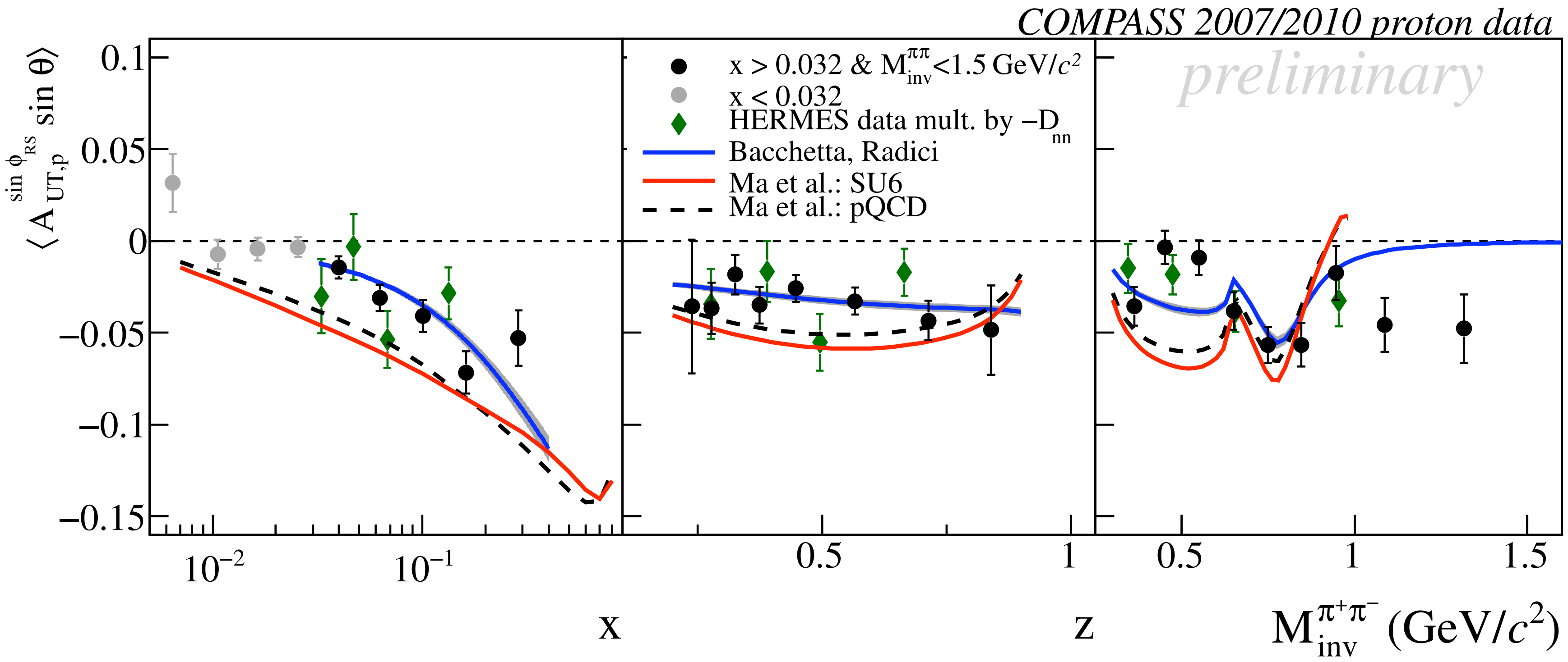

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

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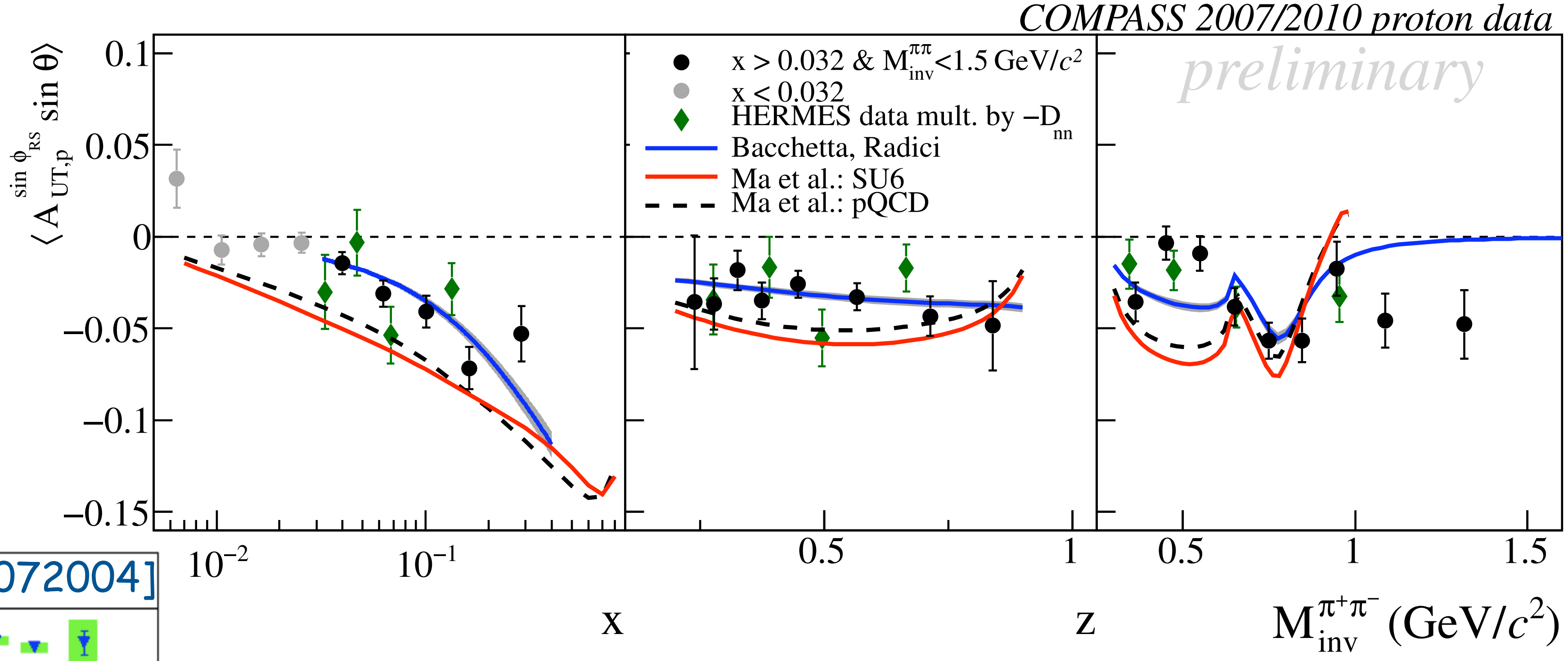
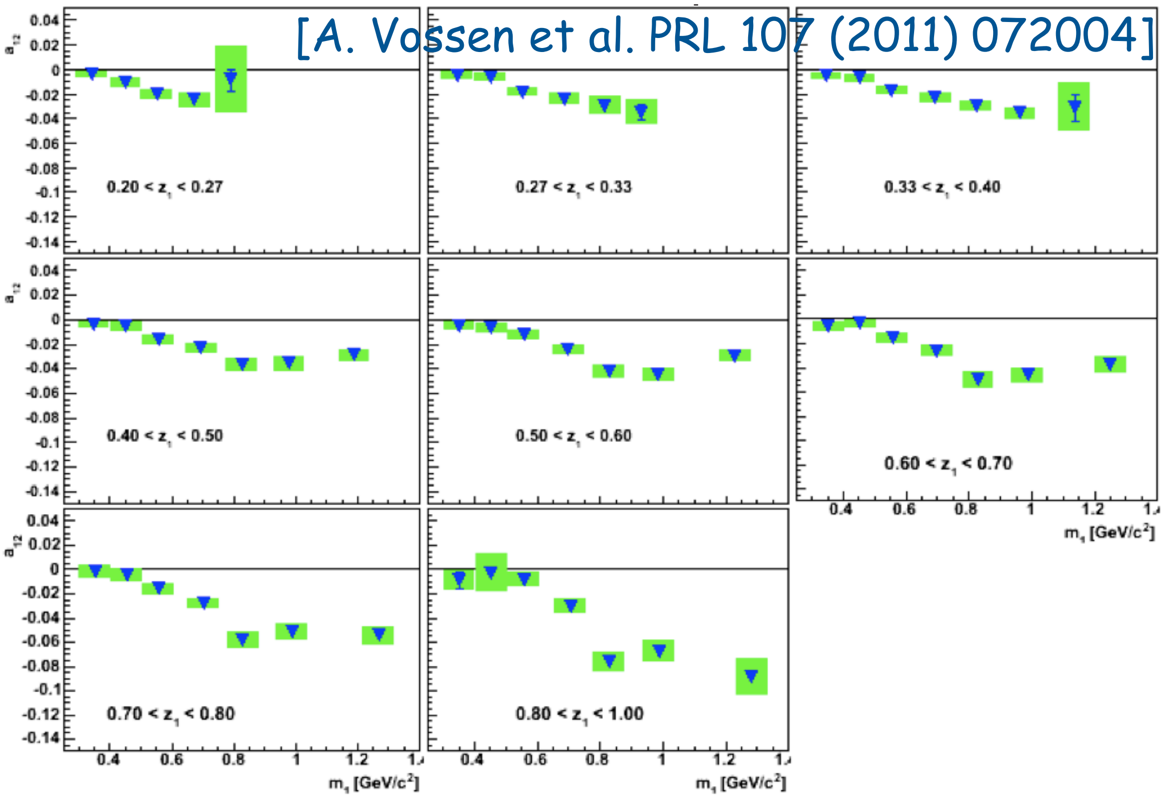
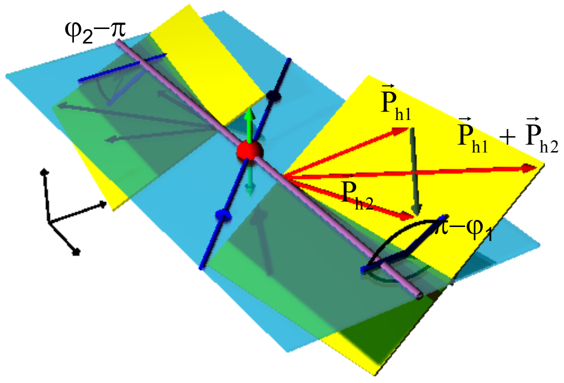


- ^{2}H results consistent with zero



[A. Vossen et al. PRL 107 (2011) 072004]

- data from e$^+$e$^-$ by BELLE



	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

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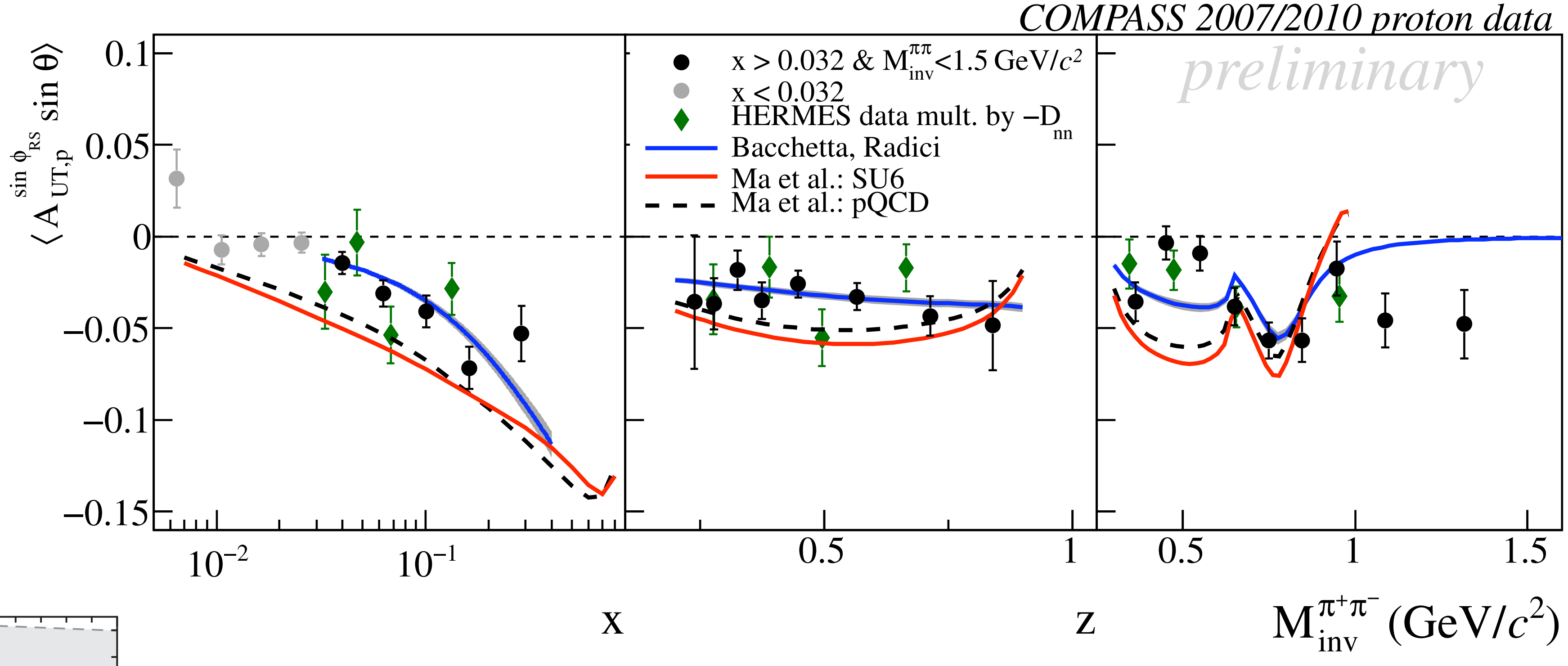
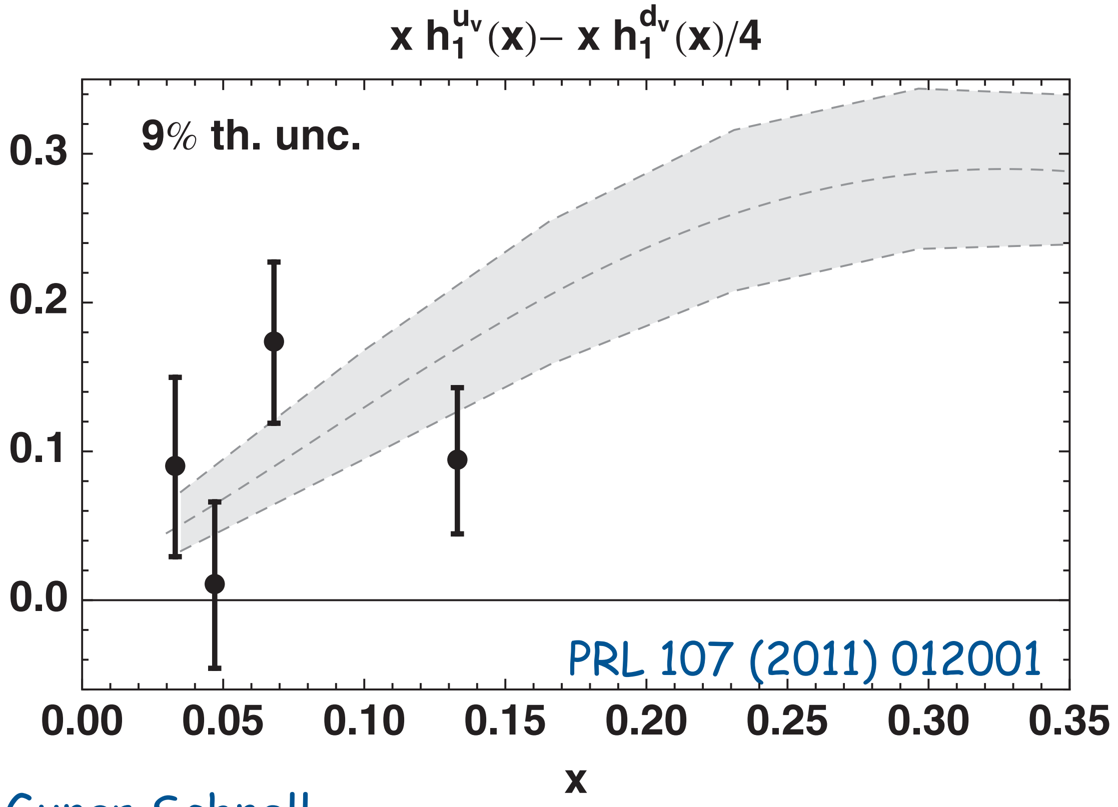
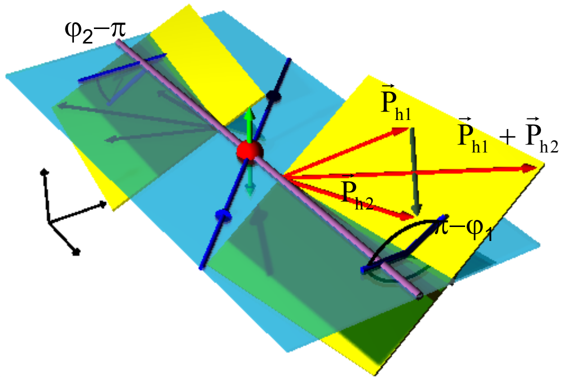




PRL 107 (2011) 012001

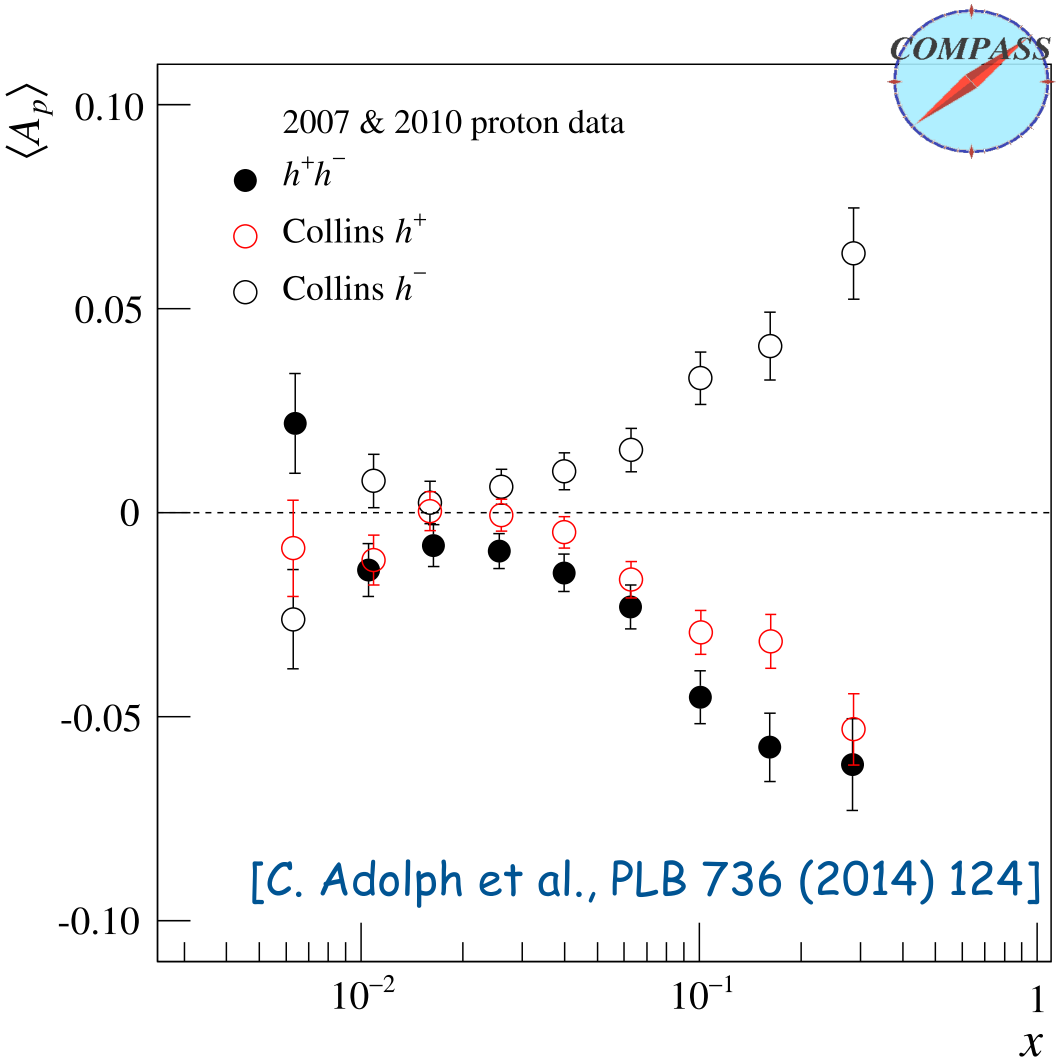
- data from e^+e^- by BELLE allow **first (collinear) extraction of transversity** (compared to Anselmino et al.)



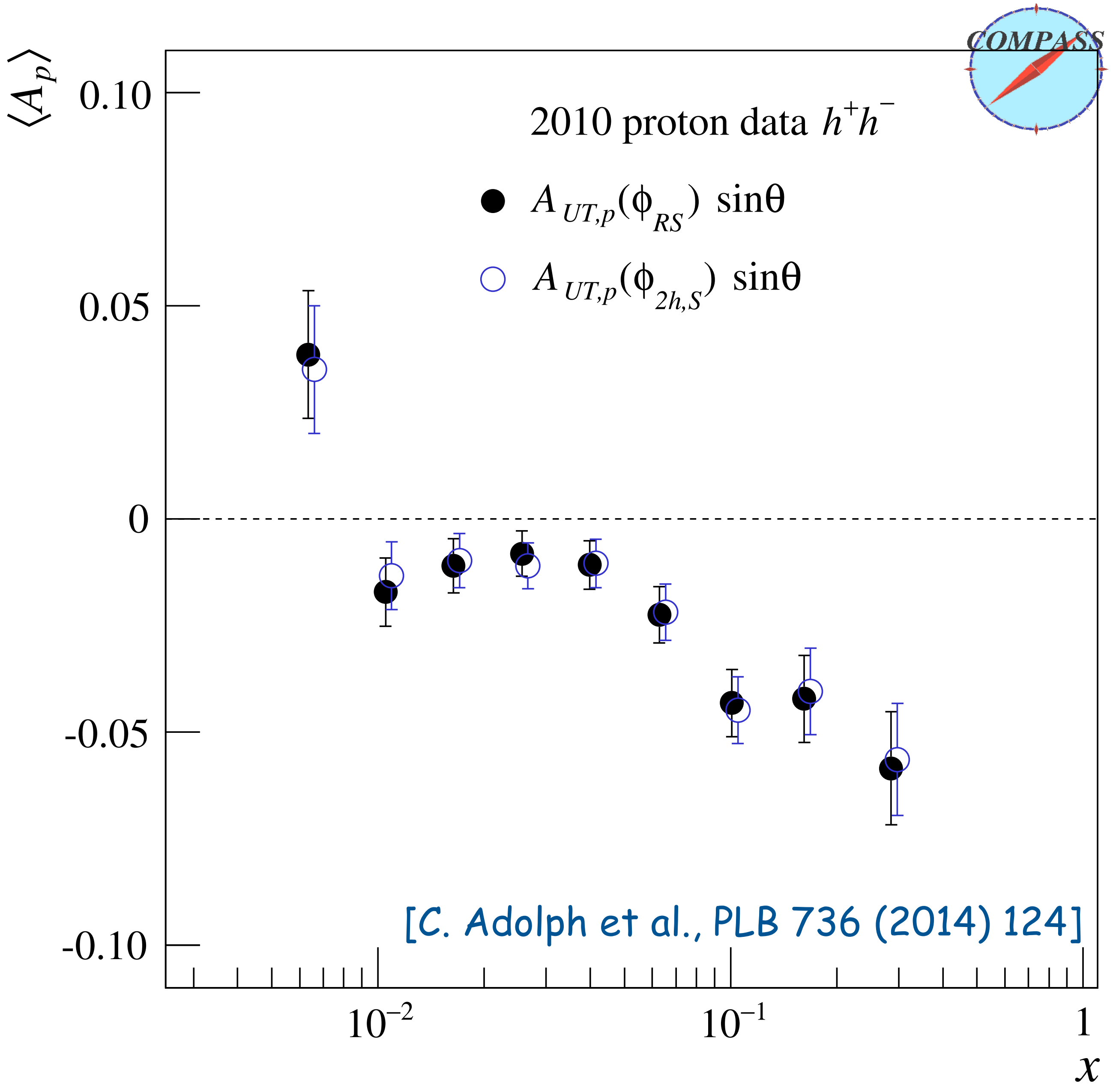
- updated analysis available (incl. COMPASS)

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

Di-hadron vs. Collins fragmentation



[C. Adolph et al., PLB 736 (2014) 124]

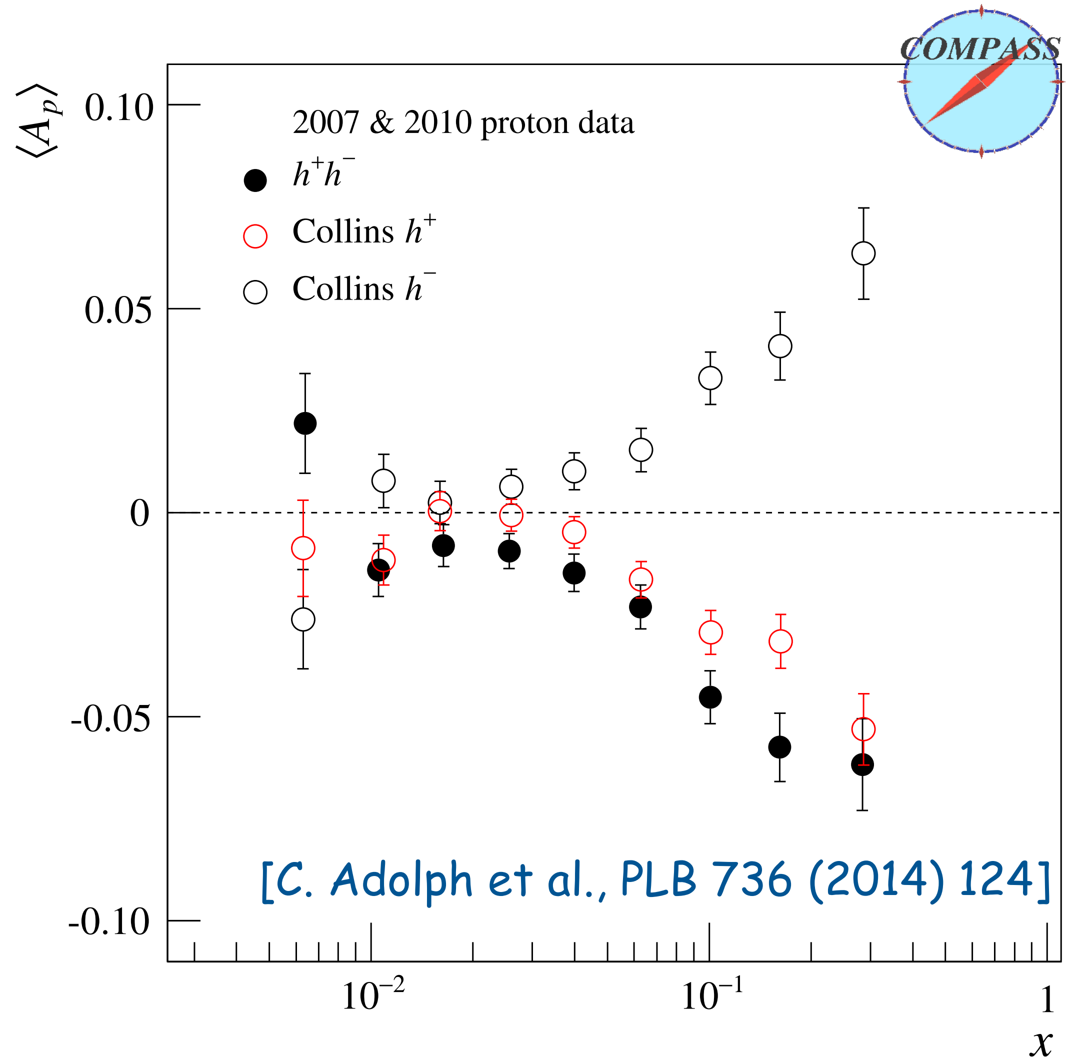


[C. Adolph et al., PLB 736 (2014) 124]

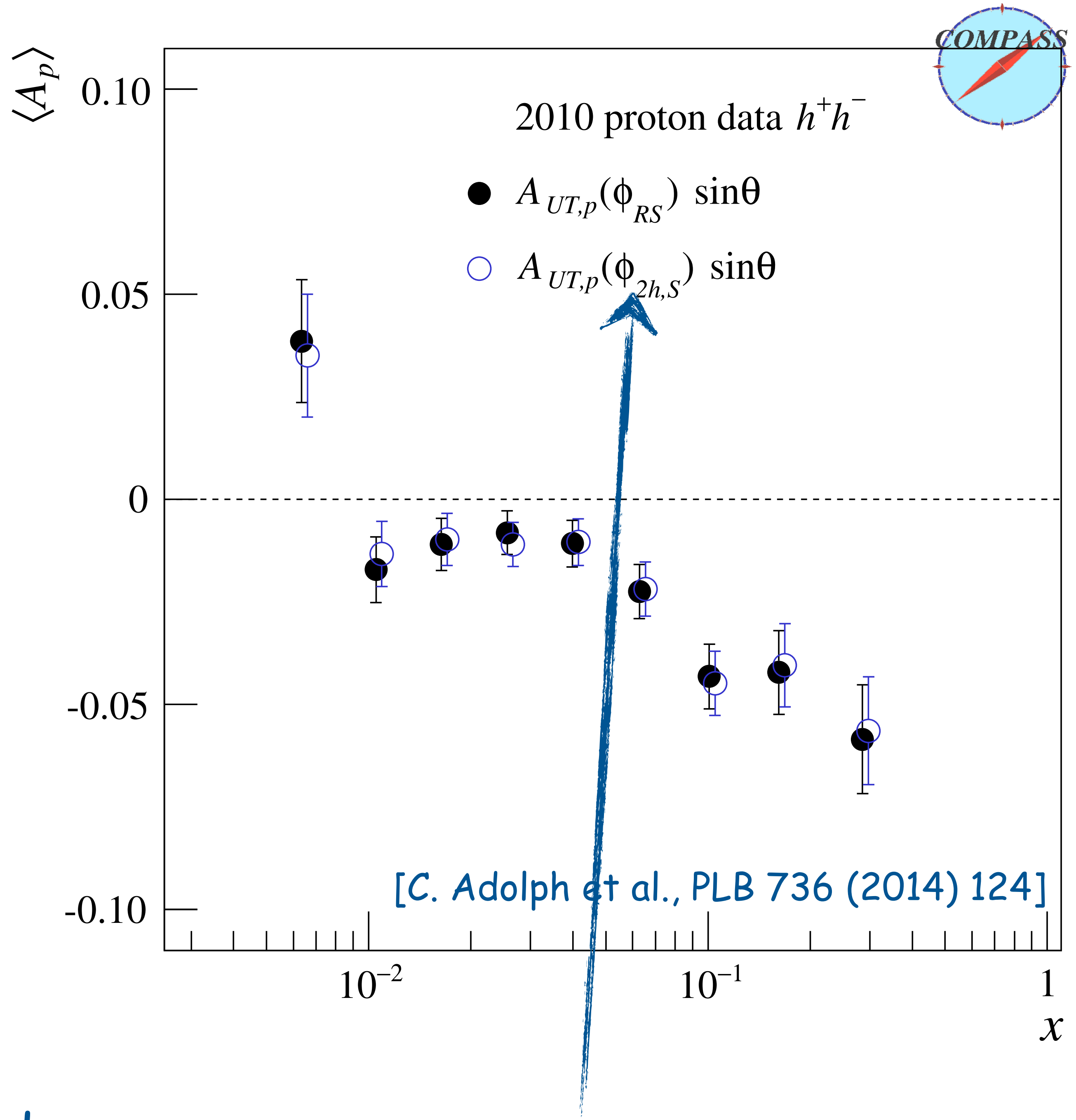
- apparent similarity of Collins and di-hadron asymmetries
- suggested common origin of Collins and di-hadron FF in PLB 736 (2014) 124

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Di-hadron vs. Collins fragmentation



[C. Adolph et al., PLB 736 (2014) 124]



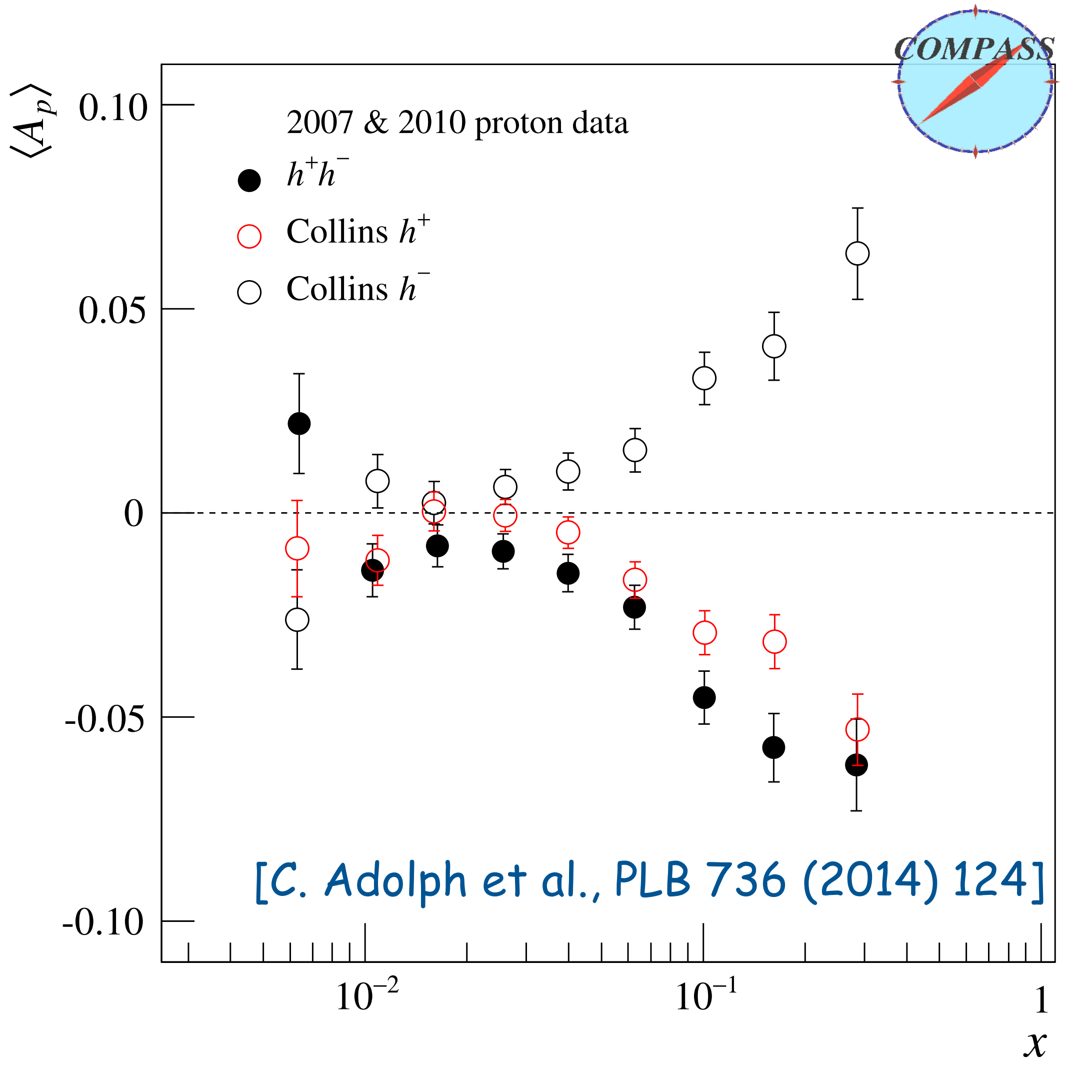
[C. Adolph et al., PLB 736 (2014) 124]

- apparent similarity of Collins and di-hadron asymmetries
- suggested common origin of Collins and di-hadron FF in PLB 736 (2014) 124

“Collins angle” of $\boldsymbol{R}_N = \hat{\boldsymbol{p}}_{T,h^+} - \hat{\boldsymbol{p}}_{T,h^-}$

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Di-hadron vs. Collins fragmentation



[C. Adolph et al., PLB 736 (2014) 124]

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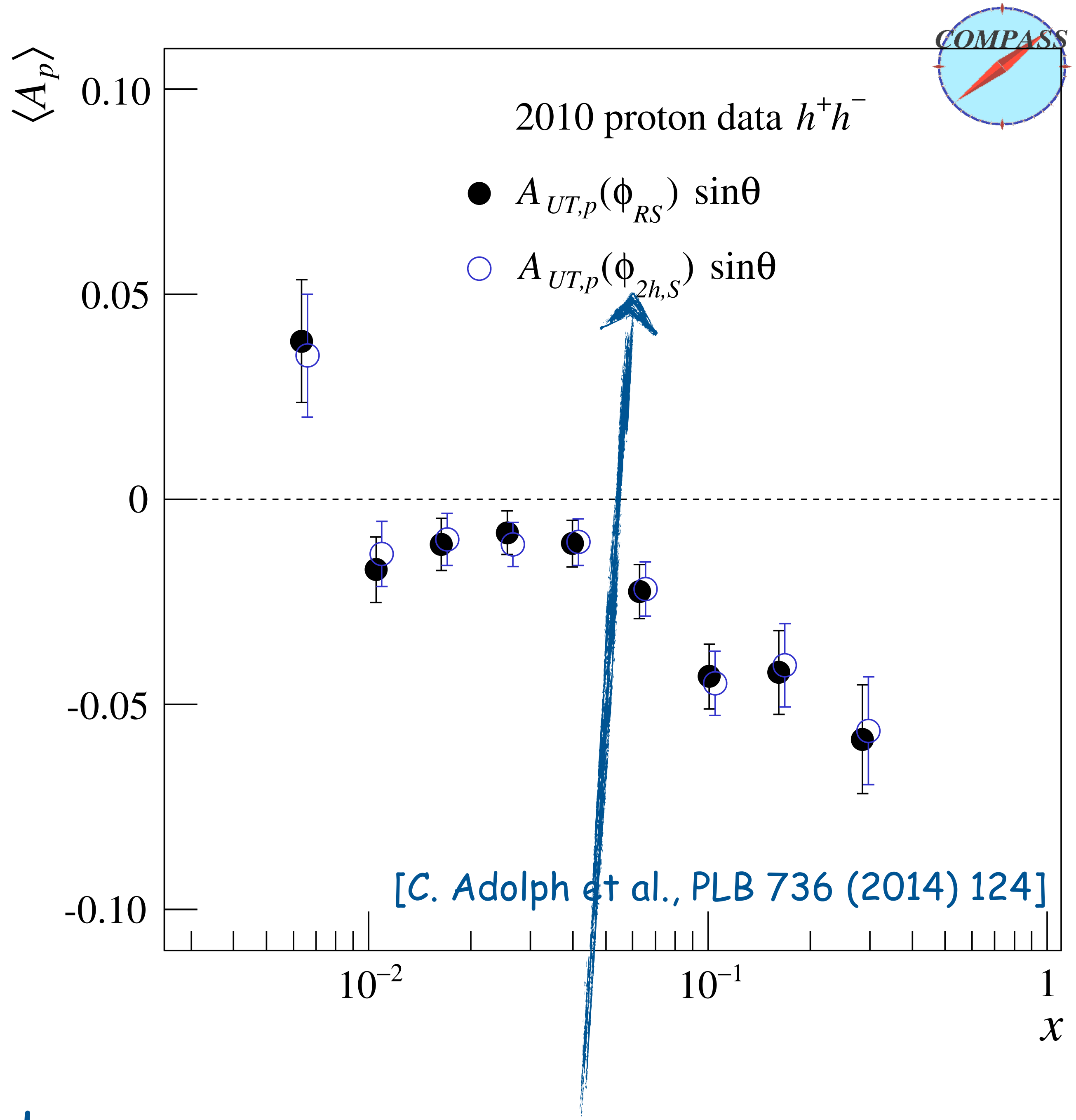

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	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

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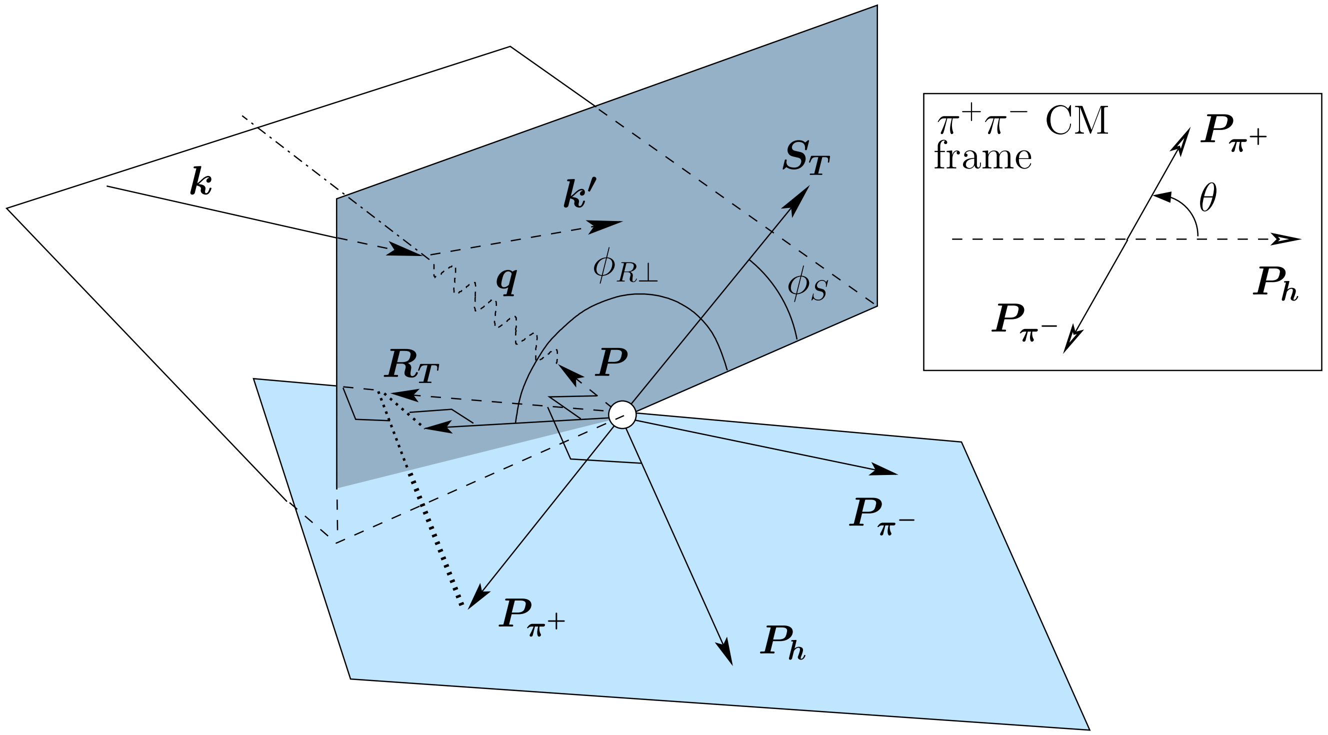
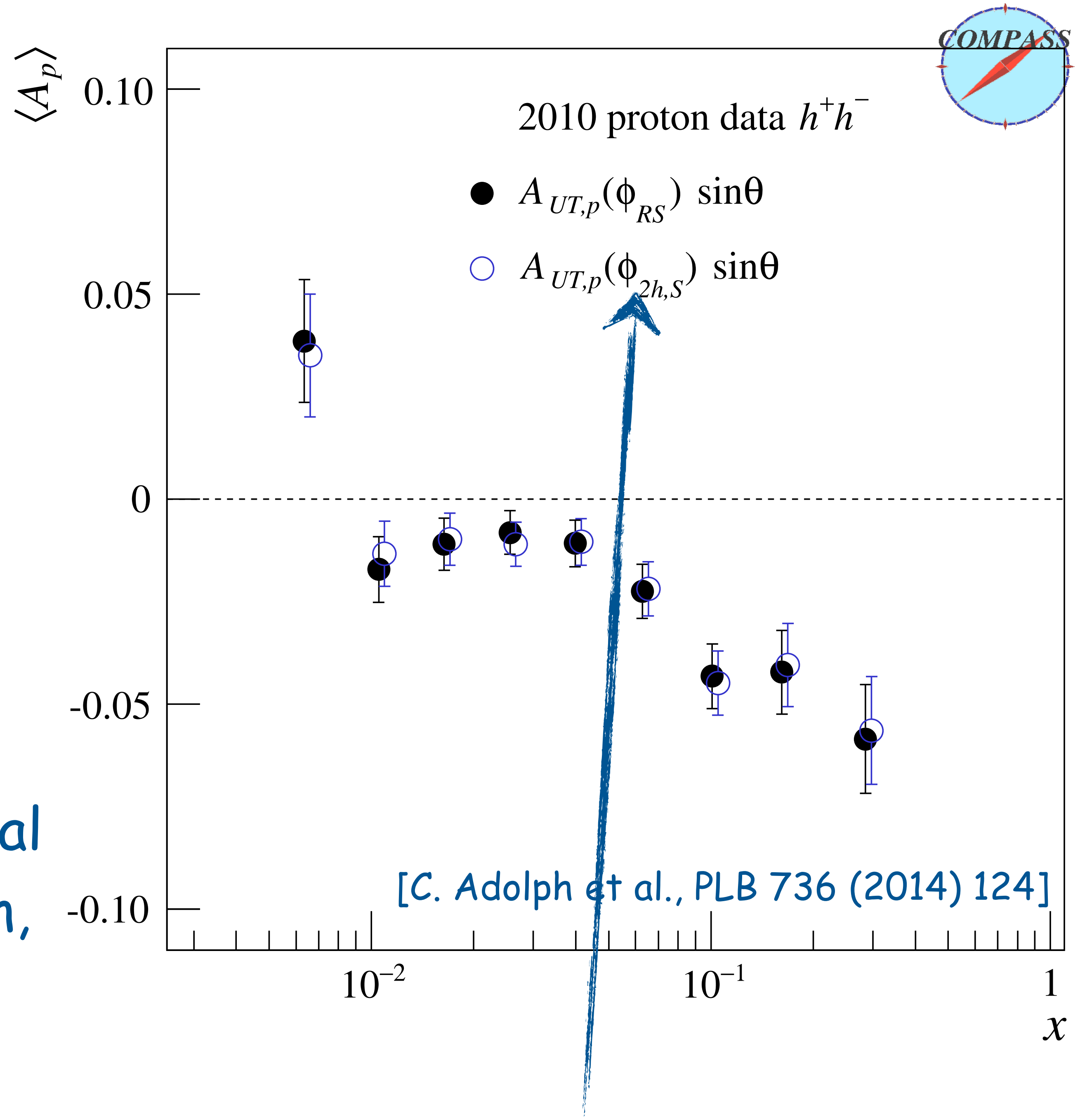




[C. Adolph et al., PLB 736 (2014) 124]

"Collins angle" of $\boldsymbol{R}_N = \hat{\boldsymbol{p}}_{T,h^+} - \hat{\boldsymbol{p}}_{T,h^-}$

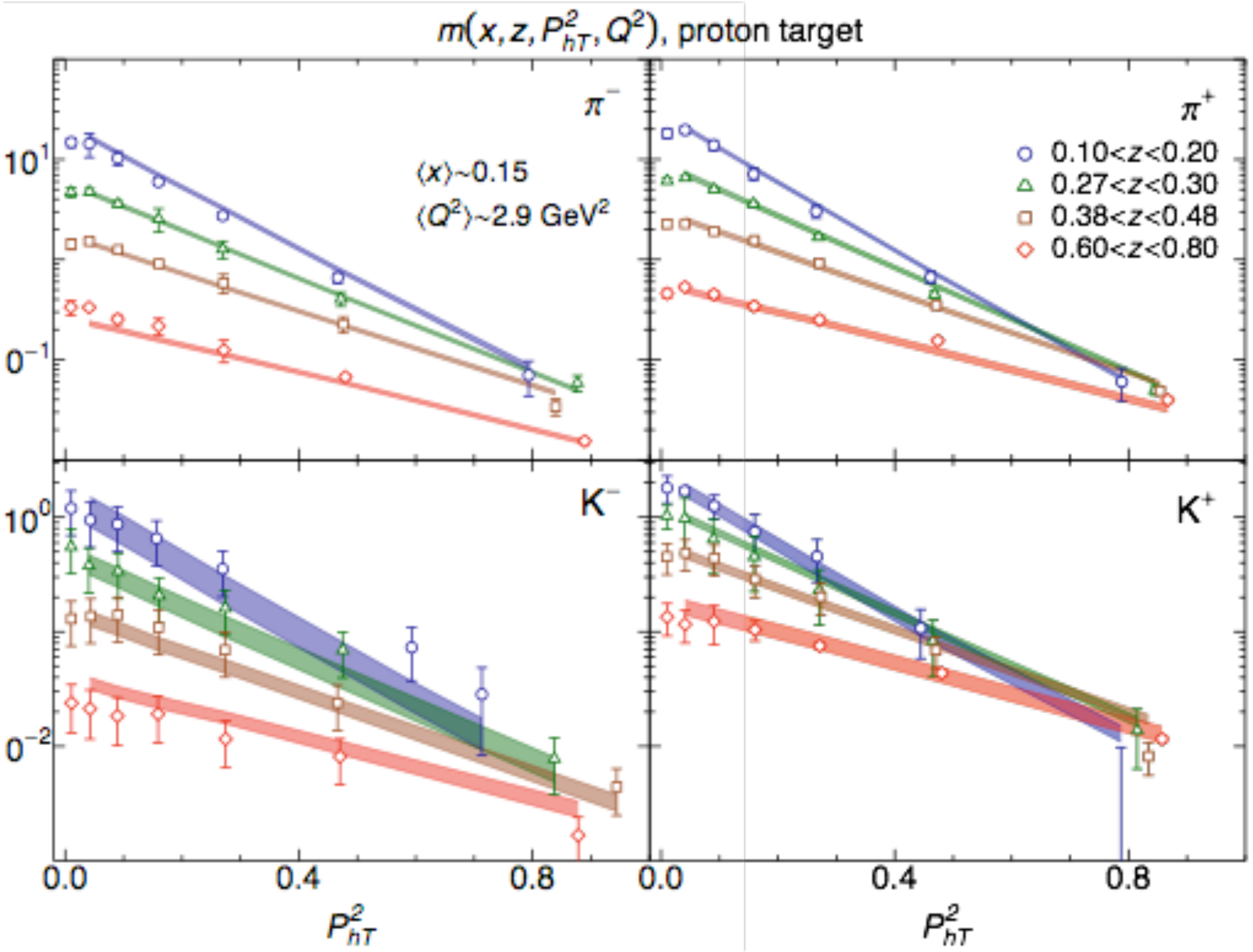
- in the limit of collinear P_h (w.r.t. virtual photon), e.g., in collinear factorization, $\phi_{2h,S}$ reduces just to ϕ_{RS}

➡ no big surprise that those two asymmetries are very similar?

FF TMD flavor dependence

- fit to HERMES multiplicity data:

$$m_N^h(x,z,\boldsymbol{P}_{hT}^2;Q^2) = \frac{\pi}{\sum_q e_q^2\, f_1^q(x;Q^2)} \sum_q e_q^2\, f_1^q(x;Q^2)\, D_1^{q\to h}(z;Q^2)\, \frac{e^{-\boldsymbol{P}_{hT}^2/\langle \boldsymbol{P}_{hT,q}^2\rangle}}{\pi\,\langle \boldsymbol{P}_{hT,q}^2\rangle}$$



$$f_1^q(x,\boldsymbol{k}_\perp^2;Q^2) = f_1^q(x;Q^2)\, \frac{e^{-\boldsymbol{k}_\perp^2/\langle \boldsymbol{k}_{\perp,q}^2\rangle}}{\pi\langle \boldsymbol{k}_{\perp,q}^2\rangle}$$

$$D_1^{q\to h}(z,\boldsymbol{P}_\perp^2;Q^2) = D_1^{q\to h}(z;Q^2)\, \frac{e^{-\boldsymbol{P}_\perp^2/\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}}{\pi\langle \boldsymbol{P}_{\perp,q\to h}^2\rangle}$$

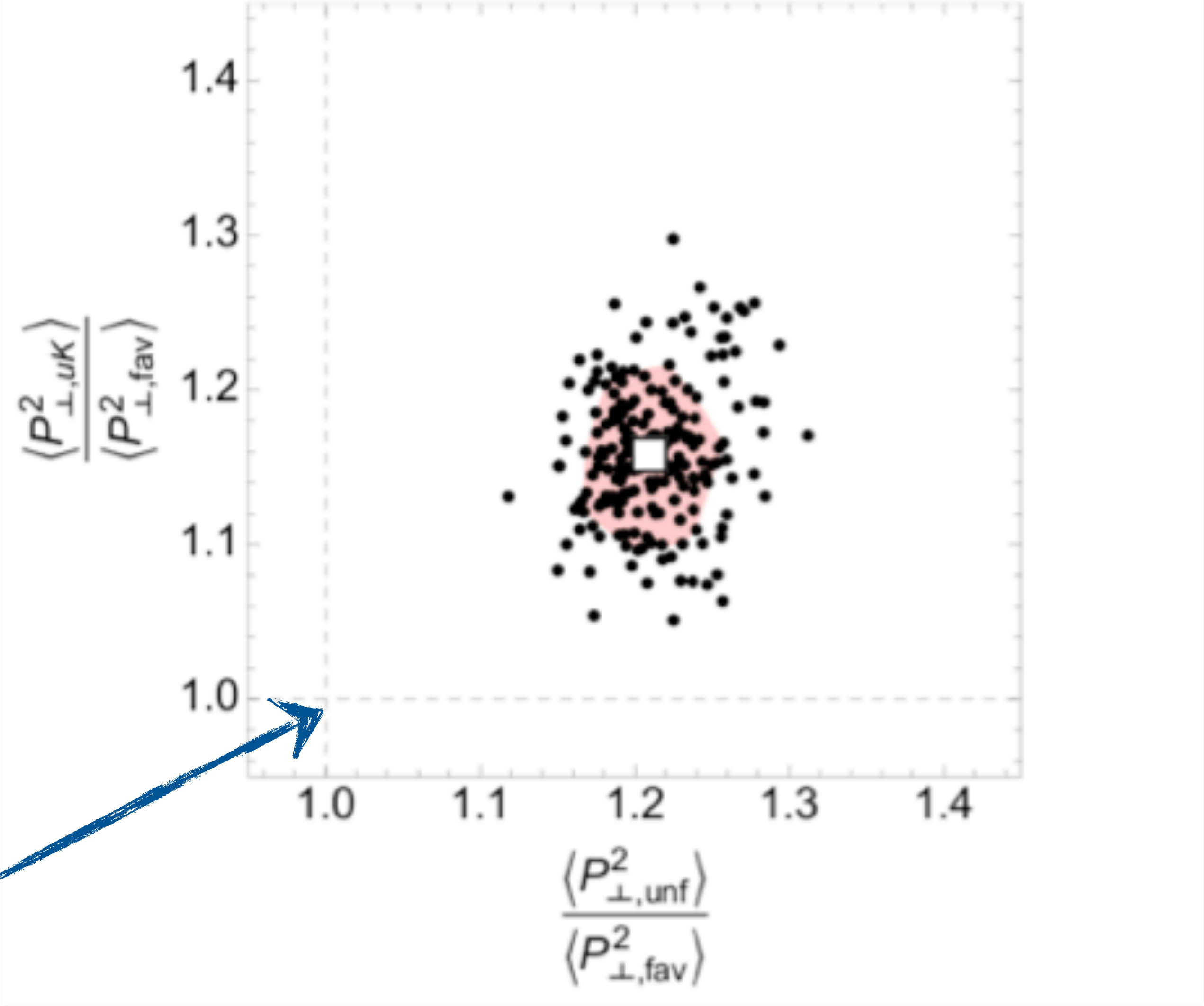
$$\langle \boldsymbol{P}_{hT,q}^2\rangle = z^2\langle \boldsymbol{k}_{\perp,q}^2\rangle + \langle \boldsymbol{P}_{\perp,q\to h}^2\rangle$$

[A. Signori, A. Bacchetta, M. Radici and GS, JHEP 11(2013)194]

FF TMD flavor dependence

- fit to HERMES multiplicity data:

[A. Signori, A. Bacchetta, M. Radici and GS, JHEP 11(2013)194]



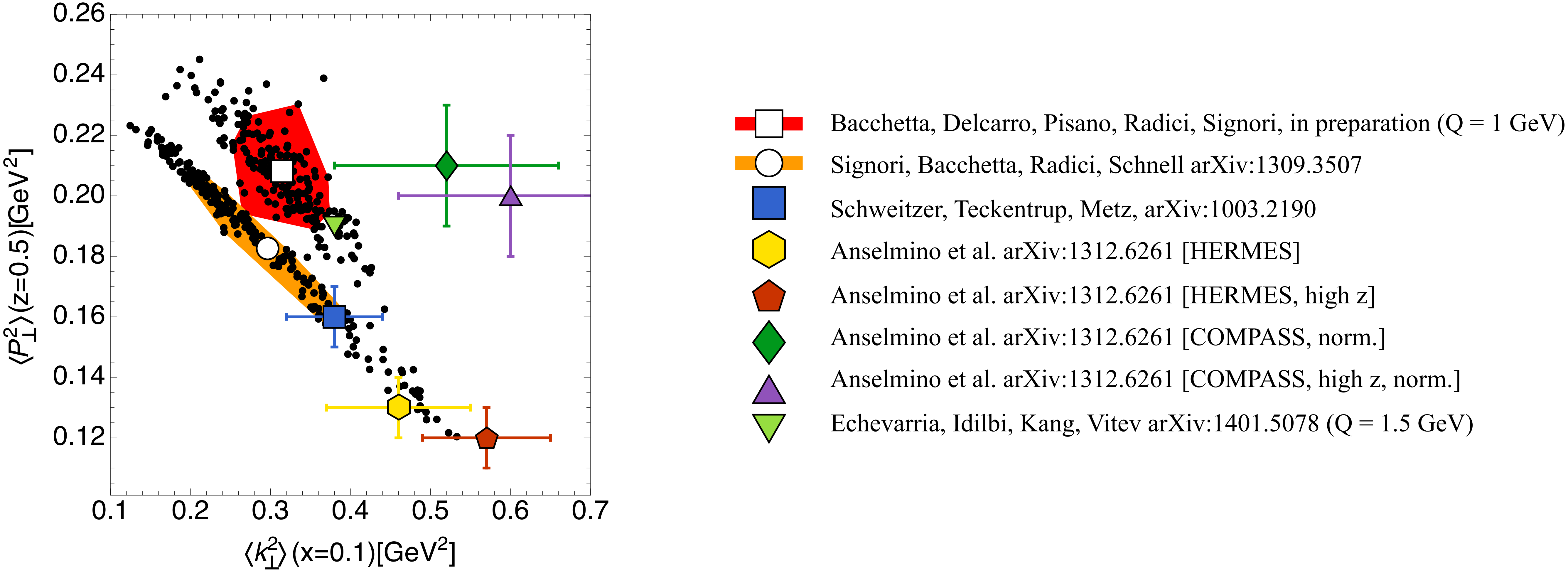
point of
no flavor dep.

q→π favored width
<
q→K favored width

q→π favored width < unfavored

FF TMD flavor dependence

- fit to SIDIS, DY & Z boson production: JHEP 06 (2017) 081



- fit to e^+e^- data: PLB 772 (2017) 78–86

- new data: COMPASS arXiv:1709.07374