De-correlation of di-jets in p+A(r) collision

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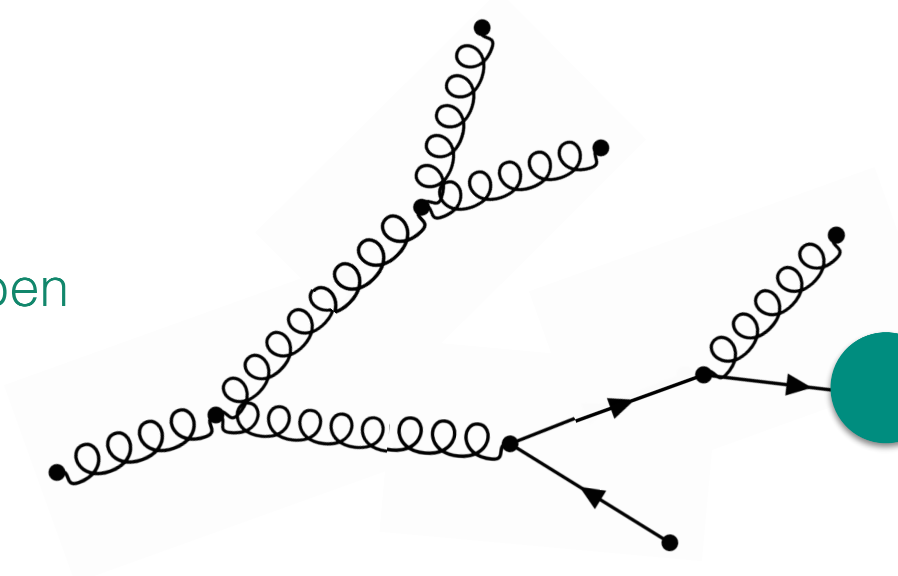
Based in part on work in progress with
C.Royon, F. Deganutti, M. Hentschinski, T. Raben



INT Program 18-3:
Probing Nucleons and Nuclei
in High Energy Collisions

Seattle, Nov 2018

CRC-TR 211
Strong-interaction matter
under extreme conditions

Universität Bielefeld

Experimental possibilities

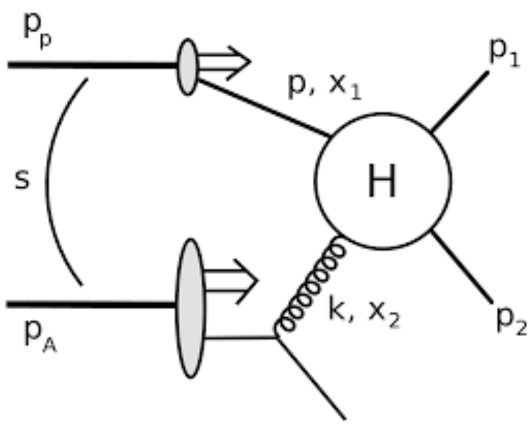
Search for gluon saturation at small x

Non-linear evolution effects?   Emergence of semi-hard scale Q_s?

High precision measurements in e+A with future EIC (>2020)

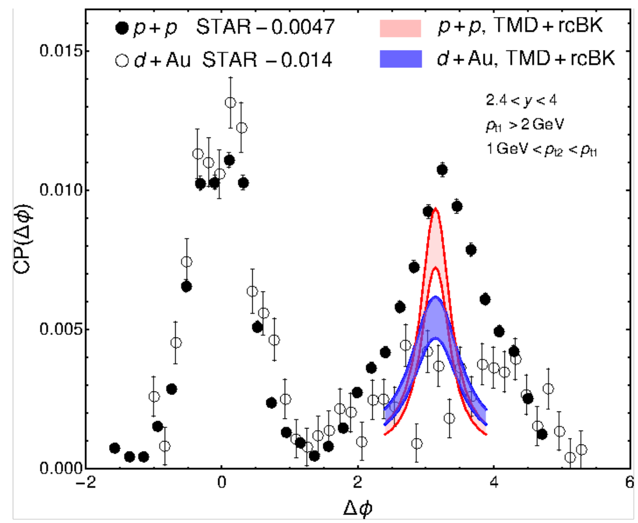
Hadronic collisions (p+p/A) at RHIC & LHC energies

forward di-hadrons/di-jets



$x_1 \sim 1$

$x_2 << 1$



Albacete, Giacalone, Marquet (2018)

RHIC: high-pT hadrons at forward rapidities

Note that presence of additional effects, e.g. soft gluon radiation can obstruct physics we are after

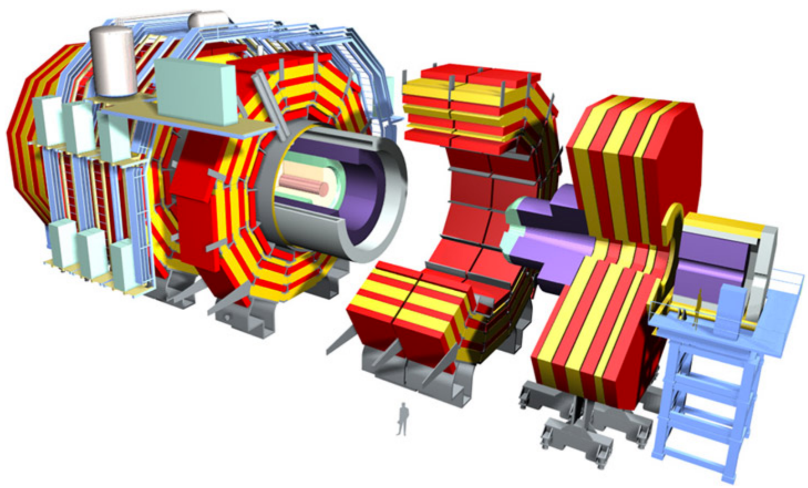
Experimental possibilities

Search for gluon saturation at small x

Non-linear evolution effects?   Emergence of semi-hard scale Q_s?

High precision measurements in e+A with future EIC (>2020)

Hadronic collisions (p+p/A) at RHIC & LHC energies



Eta Coverage
Hadronic Barrel: HB
$|\eta| \leq 1.4$ (barrel)
Hadronic Endcaps: HE
$1.3 \leq |\eta| \leq 3$ (endcap)
Hadronic Forward: HF
$3 \leq |\eta| \leq 5$ (forward)
Hadronic Outer: HO
$|\eta| \leq 1.26$ (outer)
CASTOR
$5.32 \leq \eta \leq 6.86$

c.f. CERN-CMS-DP-2014-022

LHC: reconstructed jets (pT>3 GeV) at forward rapidities

Note that presence of additional effects, e.g. soft gluon radiation can obstruct physics we are after

Outline

Di-jet production in dilute-dense CGC & TMD factorization

— Small-x TMDs at finite Nc & (very) preliminary results for cross-sections

Di-jet cross-section in CGC/JIMWLK

— Direct numerical evaluation of CGC di-jet cross-section

Calculation of di-jet production at unequal rapidities

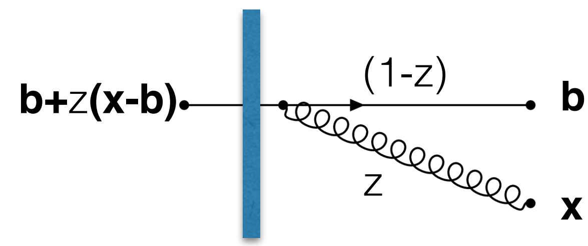
— First steps towards including non-linear evolution effects on BFKL-like emissions

Disclaimer: So far all results preliminary, but (hopefully) interesting nevertheless

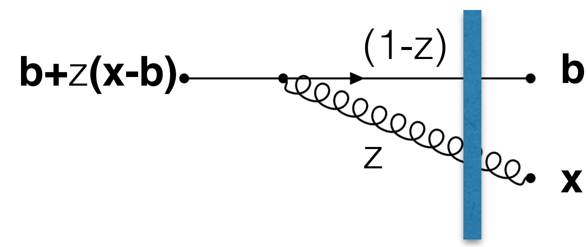
Di-jet production in dilute-dense CGC

Cross-section calculated at LO by C. Marquet (2007)

$$\frac{d\sigma(pA \to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \alpha_s C_F (1-z) p_1^+ x_1 f_{q/p}(x_1, \mu^2)\, |\mathcal{M}(p, p_1, p_2)|^2$$



$$|\mathcal{M}(p,p_1,p_2)|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')}$$
$$\times \sum_{\lambda\alpha\beta} \phi^{\lambda *}_{\alpha\beta}(p, p_1^+, \mathbf{x}'-\mathbf{b}') \phi^{\lambda}_{\alpha\beta}(p, p_1^+, \mathbf{x}-\mathbf{b})$$
$$\times \Big\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b}, \mathbf{x}, \mathbf{b}', \mathbf{x}'; x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b}, \mathbf{x}, \mathbf{b}' + z(\mathbf{x}'-\mathbf{b}'); x_2]$$
$$- S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}), \mathbf{x}', \mathbf{b}'; x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}), \mathbf{b}'+z(\mathbf{x}'-\mathbf{b}'); x_2] \Big\}$$



Even at LO so far no direct evaluation of the cross-section

-> Current phenomenology based on simplifications

Di-jet production in dilute-dense CGC

So far no direct evaluation of the cross-section

-> Current phenomenology based on different approximations

A) Gaussian / large Nc approximations of Wilson line correlators
(e.g. Marquet; Lappi, Mäntysaari)

$$S^{(4)}(\mathbf{b}_T, \mathbf{b}'_T, \mathbf{x}_T, \mathbf{x}'_T) \approx \frac{{N_c}^2}{{N_c}^2 - 1}\left[S(\mathbf{x}_T, \mathbf{x}'_T)Q(\mathbf{b}_T, \mathbf{b}'_T, \mathbf{x}_T, \mathbf{x}'_T) - \frac{1}{{N_c}^2}S(\mathbf{b}_T, \mathbf{b}'_T)\right]$$

B) Simplifications in certain kinematic limits (e.g. correlation limit $p_{T1}, p_{T2} >> \Delta p_T \sim Q_s$) (e.g. Kotko,Kutak,Marquet, Petreska,Sapeta, van Hameren, …)

Expansion of $S^{(4)}(b_T, b'_T, x_T, x'_T)$ around $b_T = x_T$ and $b'_T = x'_T$

c.f. Marquet, Petreska, Roiesnel (2016)

Di-jets in CGC / TMD factorization

Cross section in correlation limit $p_{T1}, p_{T2} >> \Delta p_T \sim Q_s$ equivalent to TMD factorization

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}} \,,$$

Calculation essentially reduces to evaluation of a set of small-x TMDs

- large N_c, Gaussian approximation, JIMWLK

c.f. Kotko,Kutak,Marquet, Petreska,Sapeta, van Hameren (2015)
Marquet, Petreska, Roiesnel (2016)

Small x TMDs

General expressions for small x TMDs

qg

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{qg}^{(1)}(\mathbf{k}) = \int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}^\dagger\right)\left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right)\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{qg}^{(2)}(\mathbf{k}) = \frac{-1}{N_c}\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{y}}^\dagger \left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{x}}^\dagger\Big]\mathrm{Tr}\Big[V_{\mathbf{y}} V_{\mathbf{x}}^\dagger\Big]\,,$$

gg

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(1)}(\mathbf{k}) = \frac{+1}{N_c}\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}^\dagger\right)\left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right)\Big]\mathrm{Tr}\Big[V_{\mathbf{x}} V_{\mathbf{y}}^\dagger\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(2)}(\mathbf{k}) = \frac{-1}{N_c}\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{y}}^\dagger\Big]\mathrm{Tr}\Big[\left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{x}}^\dagger\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(3)}(\mathbf{k}) = -\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{y}}^\dagger \left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{x}}^\dagger\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(4)}(\mathbf{k}) = -\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{x}}^\dagger \left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{y}}^\dagger\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(5)}(\mathbf{k}) = -\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{y}}^\dagger V_{\mathbf{x}} V_{\mathbf{y}}^\dagger \left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{x}}^\dagger V_{\mathbf{y}} V_{\mathbf{x}}^\dagger\Big]\,,$$

$$\frac{g^2(2\pi)^3}{4S_\perp} F_{gg}^{(6)}(\mathbf{k}) = \frac{-1}{N_c^2}\int d^2(\mathbf{x}-\mathbf{y})\, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, \mathrm{Tr}\Big[\left(\partial_i^{\mathbf{x}} V_{\mathbf{x}}\right) V_{\mathbf{y}}^\dagger \left(\partial_i^{\mathbf{y}} V_{\mathbf{y}}\right) V_{\mathbf{x}}^\dagger\Big]\mathrm{Tr}\Big[V_{\mathbf{x}} V_{\mathbf{y}}^\dagger\Big]\mathrm{Tr}\Big[V_{\mathbf{y}} V_{\mathbf{x}}^\dagger\Big]\,,$$

c.f. Marquet, Petreska, Roiesnel (2016)

Small x TMDs

Gaussian approximation (MV) at finite N_c (denote $G_{xy} = log(D_{xy})$)

qg

$$\frac{1}{N_c}\tilde{F}^{(2)}_{gq}(\mathbf{x},\mathbf{y}) = \frac{\left((N_c+2)(N_c-1)e^{\frac{3N_c-1}{N_c^2-1}G_{\mathbf{xy}}} + (N_c-2)(N_c+1)^2 e^{\frac{3N_c+1}{N_c^2-1}G_{\mathbf{xy}}} - 2N_c(N_c^2-3)e^{G_{\mathbf{xy}}}\right) G^{(i,i)}_{\mathbf{xy}}}{4N_c(N_c^2-1)G_{\mathbf{xy}}} + \frac{1}{N_c^2-1}\left(G^{(i,i)}_{\mathbf{xy}} - G^{(i,0)}_{\mathbf{xy}}G^{(i,0)}_{\mathbf{xy}}\right) e^{G_{\mathbf{xy}}}\,,$$

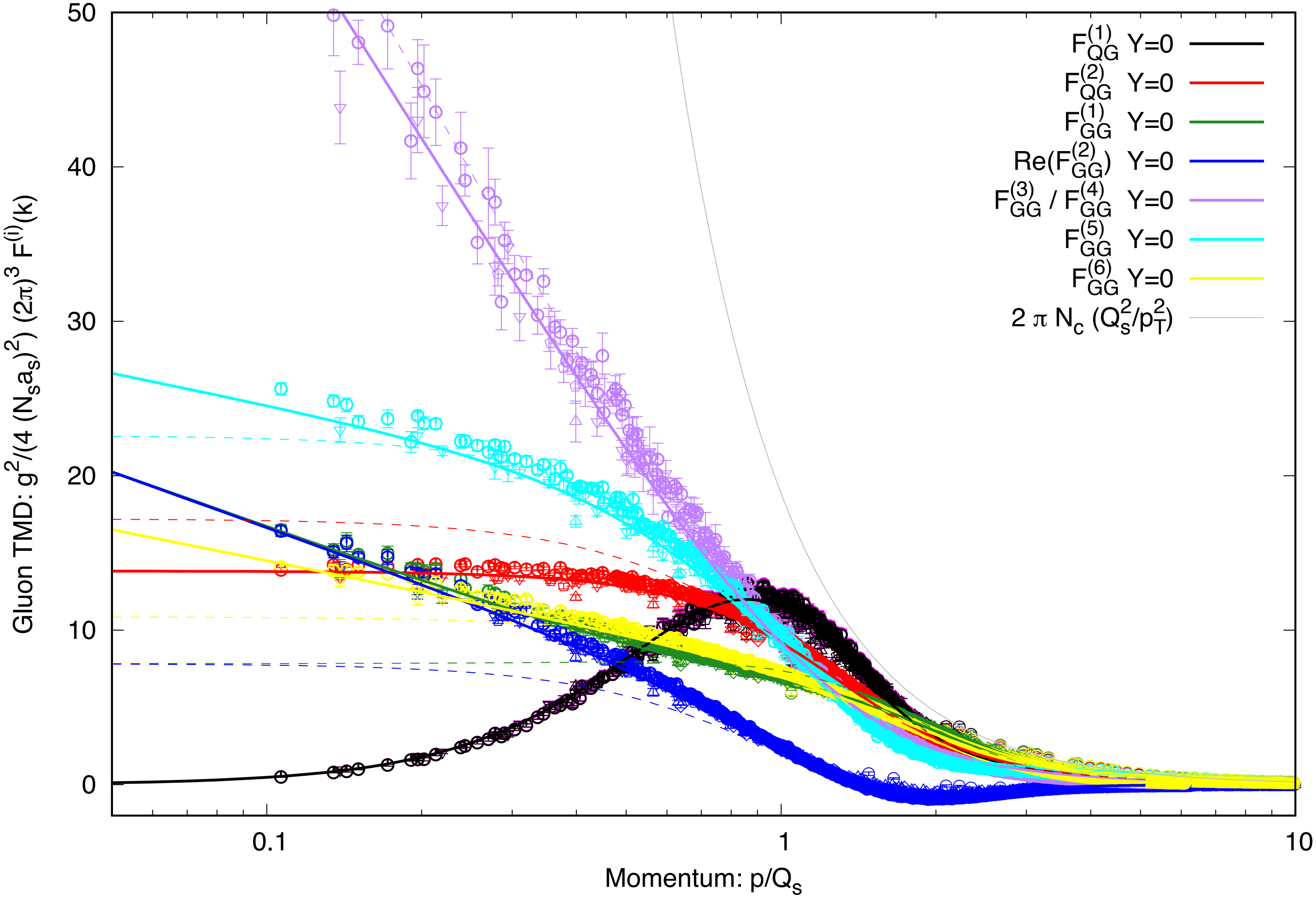
gg

$$\frac{1}{N_c}\tilde{F}^{(5)}_{gg}(\mathbf{x},\mathbf{y}) = \frac{G^{(i,i)}_{\mathbf{xy}}\left(N_c^3\left((N_c^3-7N_c-6)\,e^{\frac{4N_cG_{\mathbf{xy}}}{N_c+1}} + (N_c^3-7N_c+6)\,e^{\frac{4N_cG_{\mathbf{xy}}}{N_c-1}}\right) - 2\left(N_c^2-4\right)^2\left(N_c^2-1\right)e^{\frac{4N_c^2G_{\mathbf{xy}}}{N_c^2-1}}\right)}{8N_c^2\left(N_c^2-4\right)G_{\mathbf{xy}}} + \frac{G^{(i,i)}_{\mathbf{xy}}\left(-4\left(N_c^4-13N_c^2+12\right)e^{\frac{2N_c^2G_{\mathbf{xy}}}{N_c^2-1}} - 4\left(N_c^2-4\right)\right)}{8N_c^2\left(N_c^2-4\right)G_{\mathbf{xy}}} + \frac{2N_c^2G^{(i,0)}_{\mathbf{xy}}G^{(i,0)}_{\mathbf{xy}}e^{\frac{2N_c^2G_{\mathbf{xy}}}{N_c^2-1}}}{N_c^2-1}$$

$$\frac{1}{N_c}\tilde{F}^{(6)}_{gg}(\mathbf{x},\mathbf{y}) = \frac{G^{(i,i)}_{\mathbf{xy}}\left((N_c^3-7N_c-6)\,N_c^3e^{\frac{4N_cG_{\mathbf{xy}}}{N_c+1}} + (N_c^3-7N_c+6)\,N_c^3e^{\frac{4N_cG_{\mathbf{xy}}}{N_c-1}} + 16\left(N_c^2-4\right)N_c^2G_{\mathbf{xy}}e^{\frac{2N_c^2G_{\mathbf{xy}}}{N_c^2-1}}\right)}{8(N_c-2)N_c^4(N_c+2)G_{\mathbf{xy}}} + \frac{G^{(i,i)}_{\mathbf{xy}}\left(2\left(N_c^2-4\right)^2\left(N_c^2-1\right)e^{\frac{4N_c^2G_{\mathbf{xy}}}{N_c^2-1}} - 4\left(N_c^6-9N_c^4+16N_c^2-8\right)e^{\frac{2N_c^2G_{\mathbf{xy}}}{N_c^2-1}} - 4\left(N_c^2-4\right)N_c^2\right)}{8(N_c-2)N_c^4(N_c+2)G_{\mathbf{xy}}} - \frac{2G^{(i,0)}_{\mathbf{xy}}G^{(i,0)}_{\mathbf{xy}}e^{\frac{2N_c^2G_{\mathbf{xy}}}{N_c^2-1}}}{N_c^2-1}$$

could be used directly for phenomenology with rcBK

Small x TMDs in MV model

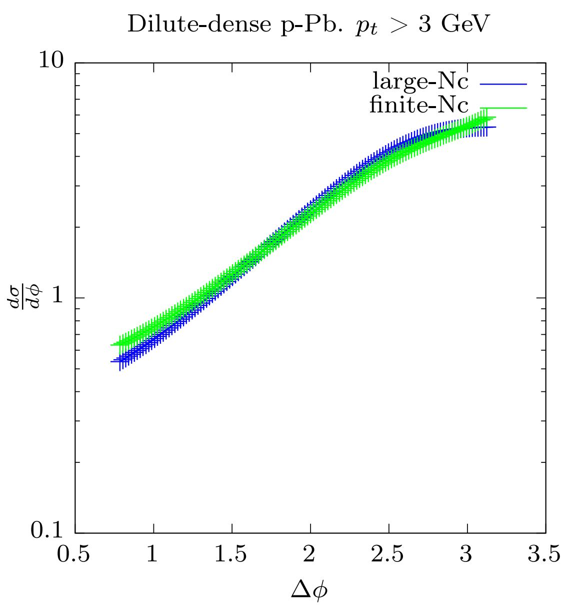


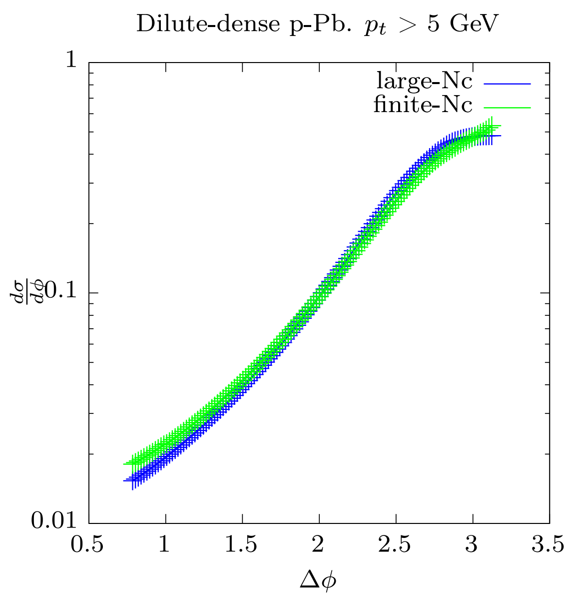
numerical lattice

finite Nc

large Nc

Very preliminary results for cross-section





credits to F. Deganutti

Improved TMD factorization formula

LHC (Castor) kinematics

5.2> $y_{1/2}$ < 6.6

Geometric scaling of TMDs

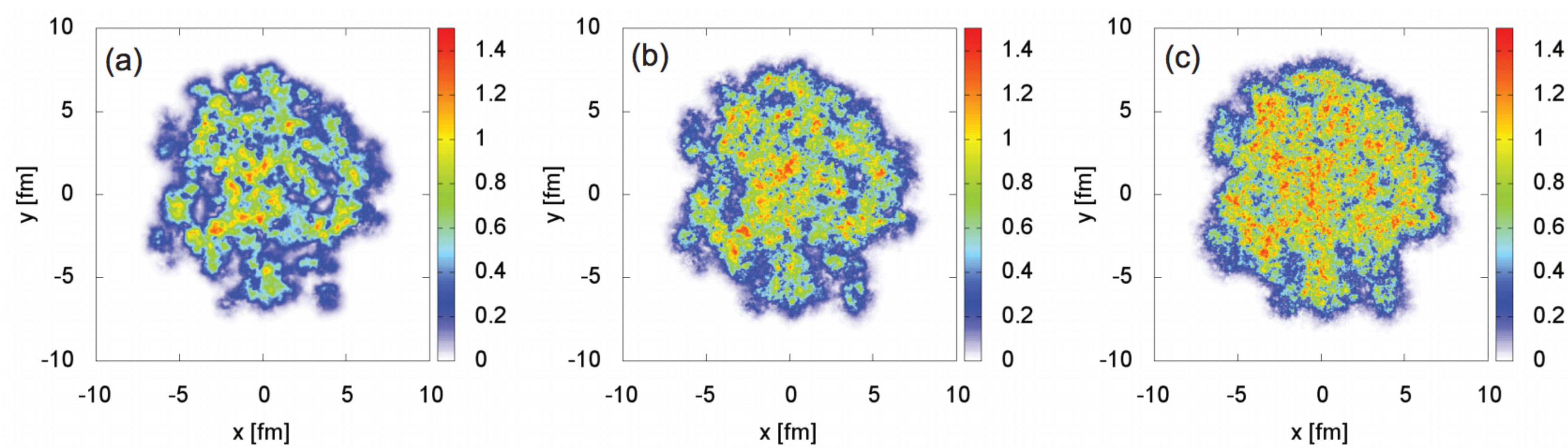
$$Q_s^2(x,A) = c(b)A^{1/3}\left(\frac{x}{x_0}\right)^{\lambda}$$

$$\lambda \approx 0.28\ ,\ \ x_0 \approx 3\times 10^{-4}\ ,\ \ c(b) = 0.6$$

Direct evaluation of CGC cross-section

Besides checking quality of various approximations, there are other good motivations to do so

Connect to saturation models (IP-Glasma) used in Heavy-Ion Phenomenology, explicitly based on statistical ensembles of Wilson lines



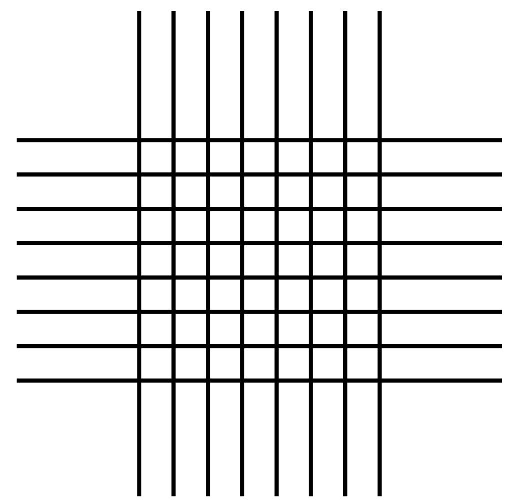
Eliminate uncertainties in $Q_{s,A}$ vs. $Q_{s,p}$ due to improved treatment of impact parameter dependence

Direct evaluation of CGC cross-section

<u>Example:</u> Single inclusive particle production at LO

$$\sigma \propto \int d^2(x-y) \left\langle \frac{1}{N_c} \mathrm{tr}\left[V_x V_y^\dagger \right] \right\rangle \; e^{-ik(x-y)}$$

Statistical ensemble of configurations $V_x(Y)$
discretized on a 2D transverse lattice



Calculate amplitude
on each configuration

$$M^{ij}(k) = \int d^2x \; e^{-ikx} \; V_x^{ij}$$

Calculate average of <|M|^2> to obtain cross-section

Direct evaluation of CGC cross-section

Di-jet production at LO

$$\frac{d\sigma^{qA\to qg+X}}{dy_k d^2\mathbf{k} dy_q d^2\mathbf{q}} = \frac{\alpha_S}{(2\pi)^8}\, p^+\, \delta(p^+ - k^+ - q^+)\, 8\pi^2 z(1-z)\hat{P}(z)\, \frac{1}{N_c} \sum_{i,j,\lambda,c} |M_{ij}^{\lambda,c}(\mathbf{k},\mathbf{q},z)|^2$$

Decompose into square amplitude

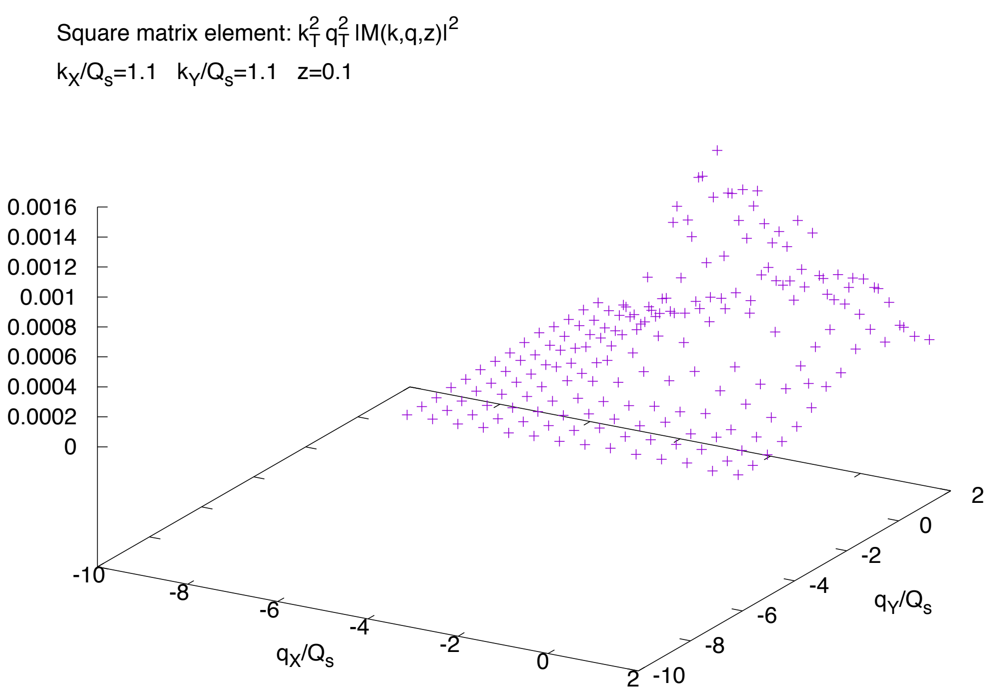
$$M_{ij}^{\lambda,c}(\mathbf{k},\mathbf{q},z) = \int d^2\mathbf{x} \int d^2\mathbf{b}\; K^{(\lambda)}(\mathbf{x}-\mathbf{b})\; \left[V_{\mathbf{b}+z(\mathbf{x}-\mathbf{b})} t^c - \left(V_{\mathbf{x}} t^c V_{\mathbf{x}}^\dagger\right) V_{\mathbf{b}}\right]_{ij}\; e^{-i\mathbf{k}\mathbf{x}}\; e^{-i\mathbf{q}\mathbf{b}}\;.$$

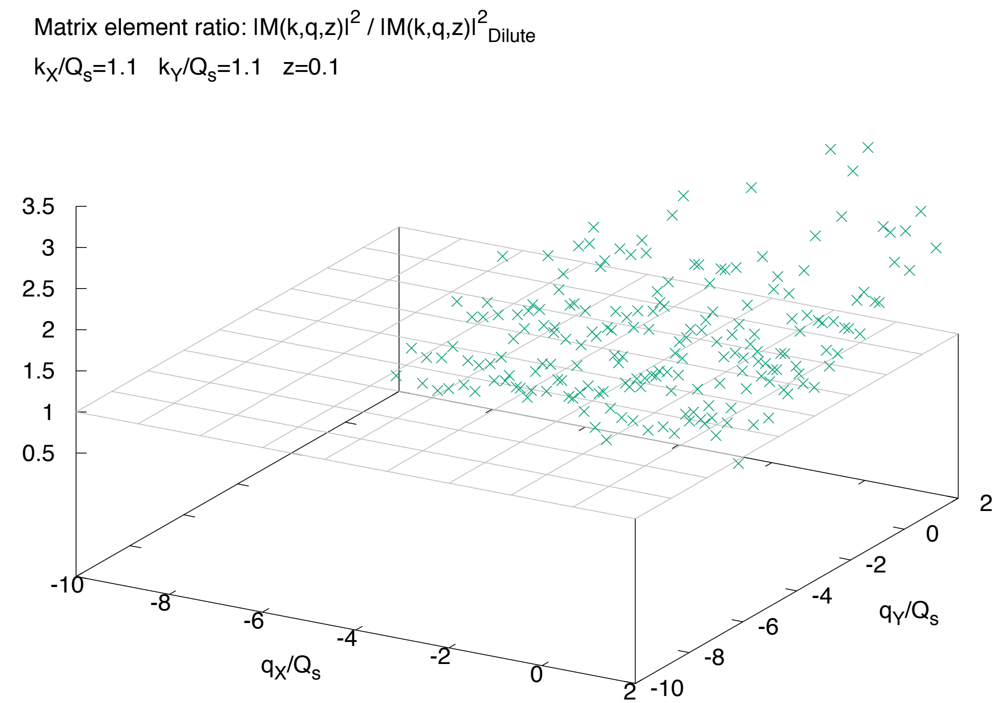
Evaluate amplitude as a single integral in momentum space

$$M_{ij}^{\lambda c}(k,q,z) = K^{(\lambda)}\left(k - z(k+q)\right) \left[V_{k+q} t_c\right]_{ij} - \int \frac{d^2 l}{(2\pi)^2} K^{(\lambda)}(l) \left[\left(V t^c V^\dagger\right)_{k-l} V_{q+l}\right]_{ij}$$

NB: beware of lattice related subtleties due to z(k+q)

First results for partonic cross section

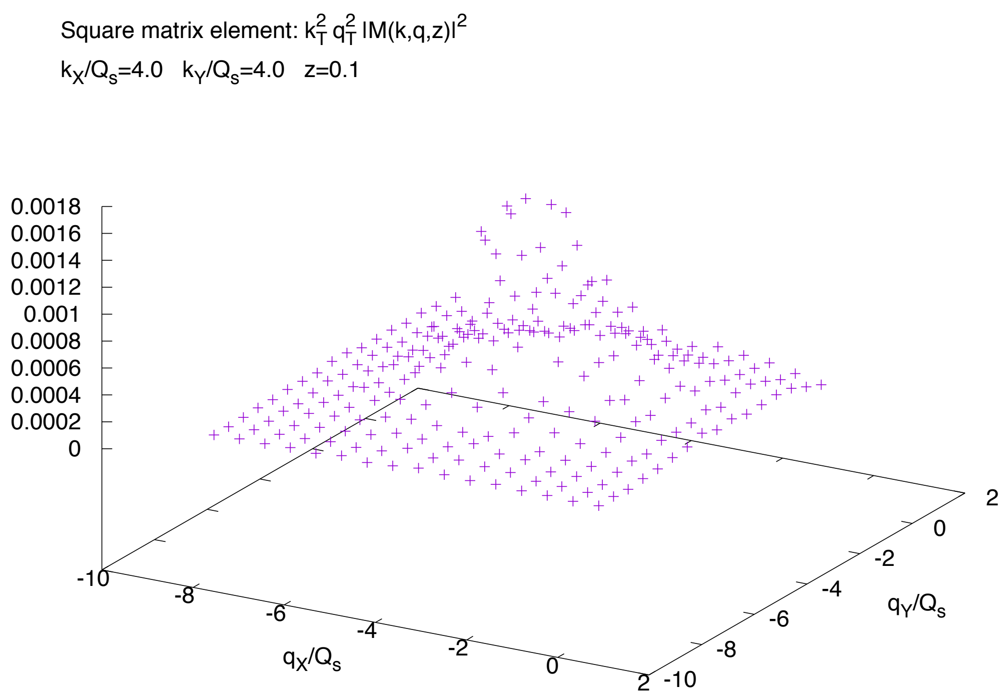


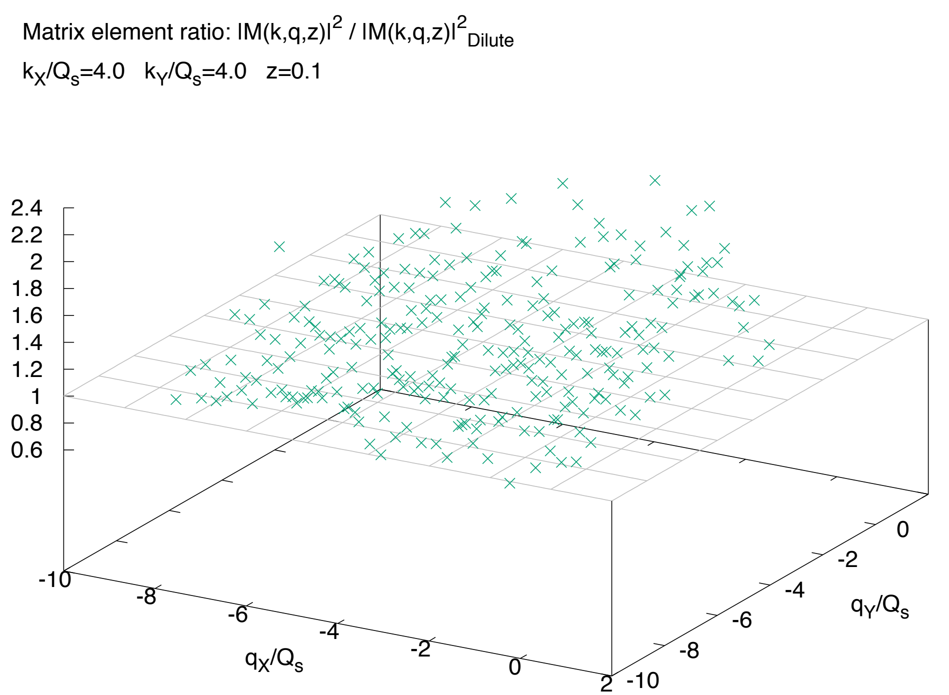


Comparison to dilute limit

$$\frac{1}{N_c}\sum_{i,j,\lambda,c}|M_{ij}^{\lambda,c}(\mathbf{k},\mathbf{q},z)|^2 \;=\; (2\pi)^2\,\frac{N_c|\mathbf{k}+\mathbf{q}|^2 S_{q\bar{q}}^{(2)}(\mathbf{k}+\mathbf{q})}{2\mathbf{k}^2\mathbf{q}^2}\left[1+\frac{(1-z)^2\mathbf{k}^2}{\mathbf{P}^2}-\frac{1}{N_c^2}\frac{z^2\mathbf{q}^2}{\mathbf{P}^2}\right]$$

First results for partonic cross section





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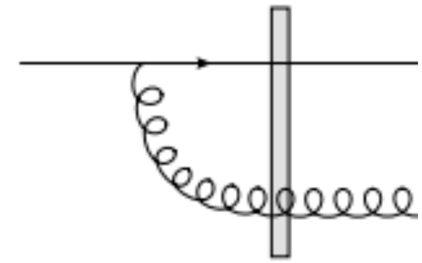
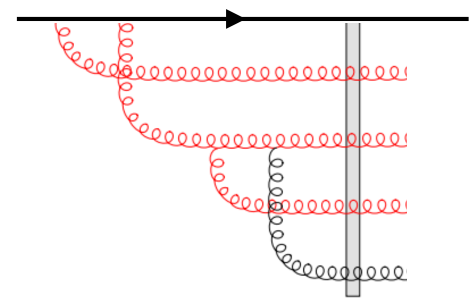
Di-jet production at unequal rapidities



So far rapidities of di-jets have been similar

-> no large phase space for extra gluon emissions

Cross sections build from products of $V_x(Y)$



Separating di-jets by $\Delta Y \sim 1/\alpha_S$ opens up phase space for BFKL like emissions

-> additional source of angular de-correlation

Note that unlike in BFKL, the additional gluons can interact strongly with the target

Cross sections depend on $V_x(Y)$, $V_x(Y_0)$ and non-linear evolution between Y_0 and Y0

Di-jet production at unequal rapidities

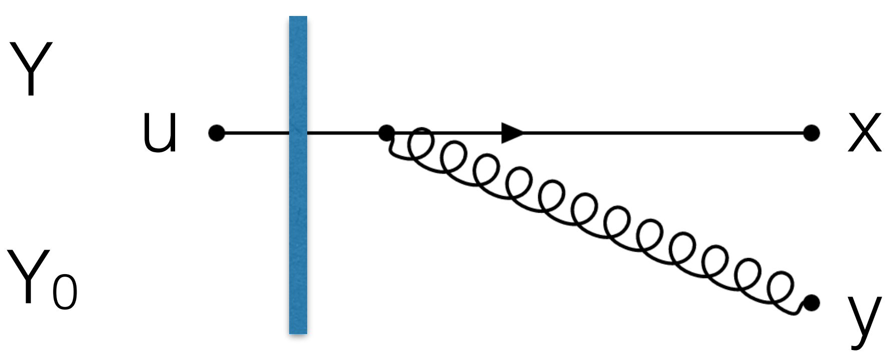
Basic formalism developed by Iancu,Trianta* (2014)

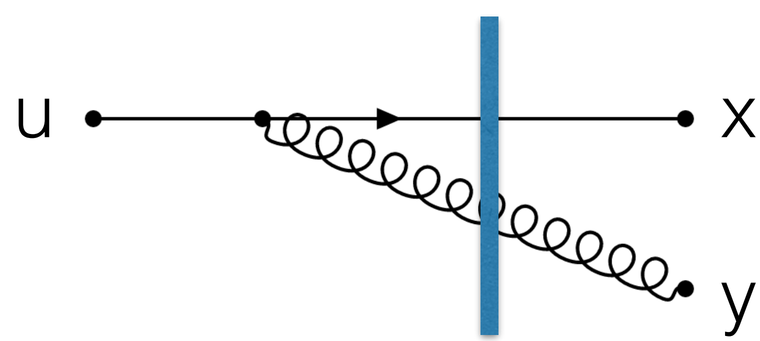
Calculate cross-section formally by the action of Lie derivative

$$L^a_{\mathbf{u}}(Y_0)\; V_{\mathbf{x}}(Y) = \int_{\mathbf{v}} G^{Y_0\to Y}_{L,\,ab}(\mathbf{u},\mathbf{v})\; L^b_{\mathbf{v}}(Y) V_{\mathbf{x}}(Y) = G^{Y_0\to Y}_{L,\,ab}(\mathbf{u},\mathbf{x})\; (+igt^b)\; V_{\mathbf{x}}(Y)$$

as

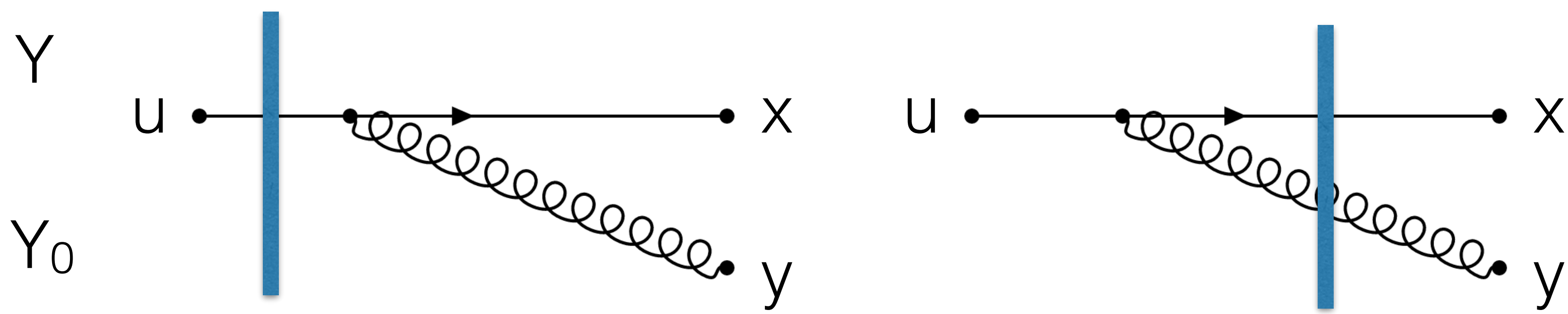
$$\frac{d\sigma(q+A\to q+g+A)}{dy_p dy_k d^2p d^2k} \propto \int_{\mathbf{x}\bar{\mathbf{x}}\mathbf{y}\bar{\mathbf{y}}} e^{-i\mathbf{p}(\mathbf{x}-\bar{\mathbf{x}})} e^{-i\mathbf{k}(\mathbf{y}-\bar{\mathbf{y}})} \int_{\mathbf{u}\bar{\mathbf{u}}} K^i_{\mathbf{y}\mathbf{u}} K^i_{\bar{\mathbf{y}}\bar{\mathbf{u}}}$$
$$\left\langle \left[L^a_{\mathbf{u}}(Y_0) - V^{ab}_{\mathbf{y}}(Y_0) R^b_{\mathbf{u}}(Y_0)\right]\left[\bar{L}^a_{\bar{\mathbf{u}}}(Y_0) - \bar{V}^{ac}_{\bar{\mathbf{y}}}(Y_0)\bar{R}^c_{\bar{\mathbf{u}}}(Y_0)\right] S^a_{12}(\mathbf{x},\bar{\mathbf{x}})(Y)\right\rangle_{V=\bar{V}}$$





Di-jet production at unequal rapidities

$$\frac{d\sigma(q+A\to q+g+A)}{dy_p dy_k d^2p d^2k} \propto \int_{\mathbf{x}\bar{\mathbf{x}}\mathbf{y}\bar{\mathbf{y}}} e^{-i\mathbf{p}(\mathbf{x}-\bar{\mathbf{x}})} e^{-i\mathbf{k}(\mathbf{y}-\bar{\mathbf{y}})} \int_{\mathbf{u}\bar{\mathbf{u}}} K^i_{\mathbf{y}\mathbf{u}} K^i_{\bar{\mathbf{y}}\bar{\mathbf{u}}}$$
$$\left\langle \left[L^a_{\mathbf{u}}(Y_0) - V^{ab}_{\mathbf{y}}(Y_0) R^b_{\mathbf{u}}(Y_0)\right]\left[\bar{L}^a_{\bar{\mathbf{u}}}(Y_0) - \bar{V}^{ac}_{\bar{\mathbf{y}}}(Y_0)\bar{R}^c_{\bar{\mathbf{u}}}(Y_0)\right] S^a_{12}(\mathbf{x},\bar{\mathbf{x}})(Y)\right\rangle_{V=\bar{V}}$$



Since at $Y=Y_0$ gluon can only be radiated from quark

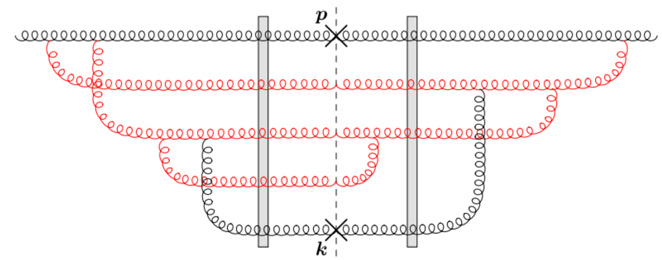
$$G^{Y_0\to Y}_{L,\,ab}(\mathbf{u},\mathbf{v})\Big|_{Y=Y_0} = \delta^{ab}\,\delta^{(2)}(\mathbf{u}-\mathbf{x})\,,$$

cross section for $Y=Y_0$ reduces to z<<1 limit of Marquet (2007)

Di-jet production at unequal rapidities

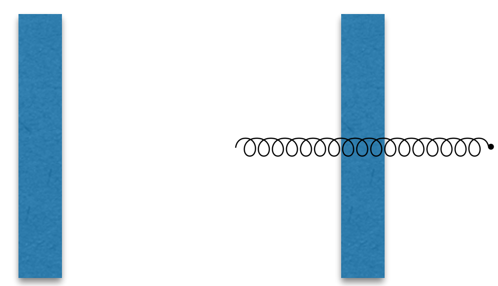
Now if $Y\text{-}Y_0 \sim 1/\alpha_S$ need to account for extra gluon emissions in addition to measured di-jet pair

Projectile evolution:



-> Projectile becomes complicated, real gluon measured at Y_0 can be emitted from various intermediate states

Target evolution:



-> Need to account for presence of one additional real gluon at Y_0

Will be reflected by non-trivial action of Lie derivative encoded in Green's function $G^{Y_0 \to Y}_{L,\, ab}(\mathbf{u}, \mathbf{v})$

Di-jet production at unequal rapidities

Evolution equation for inclusive target fields V_x given by stochastic JIMWLK

$$V_{\mathbf{x}}(Y+dY) = e^{+\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} t^a \sqrt{dY}} V_{\mathbf{x}}(Y) e^{-\frac{ig}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} V^\dagger_{\mathbf{z}}(Y) t^a V_{\mathbf{z}}(Y)} ,$$

Extra gluon satisfies “linearized JIMWLK equation”

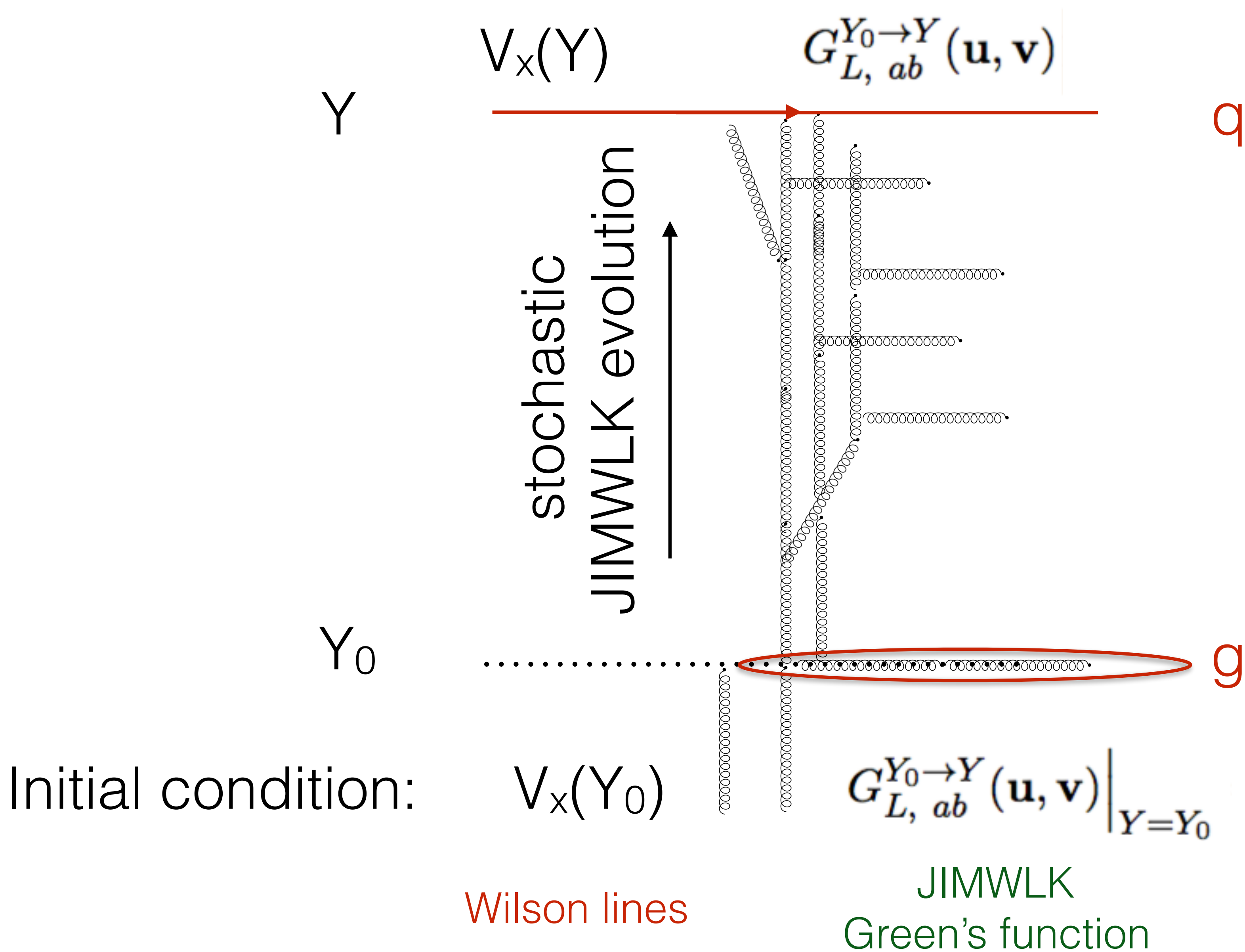
$$G^{Y_0\to Y+dY}_{R,\,b}(\mathbf{u},\mathbf{x}) = e^{+\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} V^\dagger_{\mathbf{z}}(Y) t^a V_{\mathbf{z}}(Y)}\, G^{Y_0\to Y}_{R,\,b}(\mathbf{u},\mathbf{x})\, e^{-\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} V^\dagger_{\mathbf{z}}(Y) t^a V_{\mathbf{z}}(Y)} ,$$

$$-\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}} e^{+\frac{1}{2}\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} V^\dagger_{\mathbf{z}}(Y) t^a V_{\mathbf{z}}(Y)} \int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} \left[V^\dagger_{\mathbf{z}}(Y) t^a V_{\mathbf{z}}(Y), G^{Y_0\to Y}_{R,\,b}(\mathbf{u},\mathbf{z})\right] e^{-\frac{1}{2}\frac{ig\sqrt{dY}}{\sqrt{4\pi^3}}\int_{\mathbf{z}} K^i_{\mathbf{xz}}\xi^{i,a}_{\mathbf{z}} V^\dagger_{\mathbf{z}}(Y) t^a V \ldots}$$

with identical noise terms (additional real/virtual gluons) and Wilson lines V_x on a configuration by configuration basis

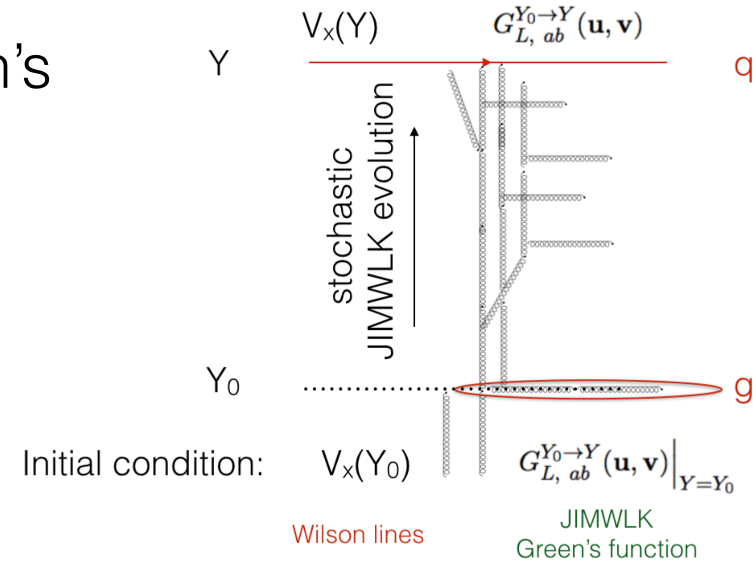
NB: If re-scattering of gluons produced between Y and Y_0 is ignored G^2 becomes BFKL Greens function

Di-jet production at unequal rapidities



Di-jet production at unequal rapidities

Numerically, keeping track of the full Green's function G(u,v) is extremely challenging



However, if we know the momentum k of the gluon measured at Y_0 can directly keep track of mixed space Green's function

$$\tilde{G}_{R,\ ia}^{Y_0\to Y}(\mathbf{k},\mathbf{x}) = \int_{\mathbf{y}} e^{-i\mathbf{k}\mathbf{y}} \int_{\mathbf{u}} K_{\mathbf{yu}}^{i}\left(V_{\mathbf{u}}^{ab} - V_{\mathbf{y}}^{ab}\right)_{Y_0} G_{R,\ b}^{Y_0\to Y}(\mathbf{u},\mathbf{x}) \ ,$$

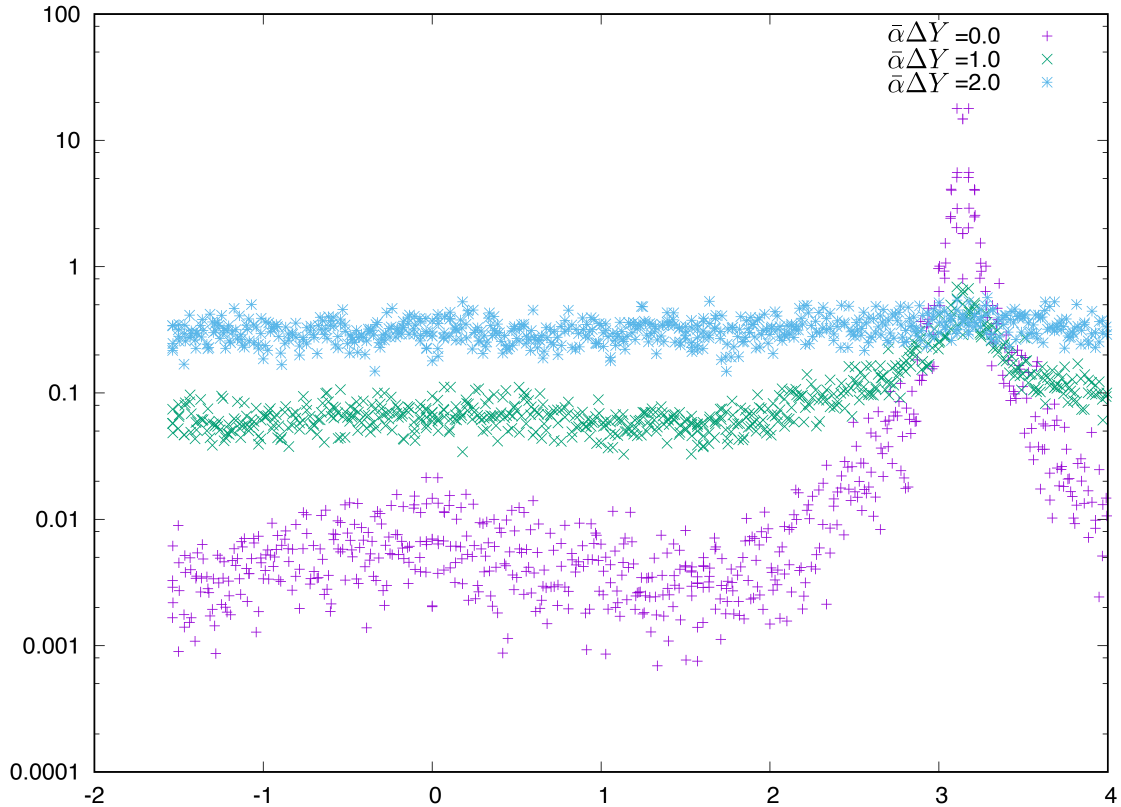
Since evolution equation for Green's function is linear, mixed space version satisfies same evolution equation

$$\frac{d\sigma(q+A\to q+g+A)}{dy_p dy_k d^2p d^2k} \propto \int_{\mathbf{x}\bar{\mathbf{x}}} e^{-i\mathbf{p}(\mathbf{x}-\bar{\mathbf{x}})}\ \mathrm{tr}_r\left[V_{\mathbf{x}}(Y)\tilde{G}_{R,\ ia}^{Y_0\to Y}(\mathbf{k},\mathbf{x})\tilde{G}_{R,\ ia}^{Y_0\to Y}(\mathbf{k},\bar{\mathbf{x}})V_{\mathbf{x}}^{\dagger}(Y)\right]$$

Cross-section calculation also simplifies dramatically, but initial condition for G(k,x) is still expensive

Di-jet production at unequal rapidities

Color algebra complicated; numerics still expensive
-> first proof of principle calculation for SU(2)



Cross-checks of numerics (maybe against BFKL?) still pending

Conclusions & Outlook

Di-jet production in p+A provides important process to search for non-linear small x dynamics

- experimental measurements at RHIC & LHC
- lots of progress in phenomenology by many people

New results for small-x TMDs at finite N_c

First direct evaluation of CGC cross-section

- clear signs of non-linear effects for p_T around Q_s

First steps towards di-jet productions at unequal rapidities

- new possibilities to look at non-linear evolution effects

Stay tuned for more results!