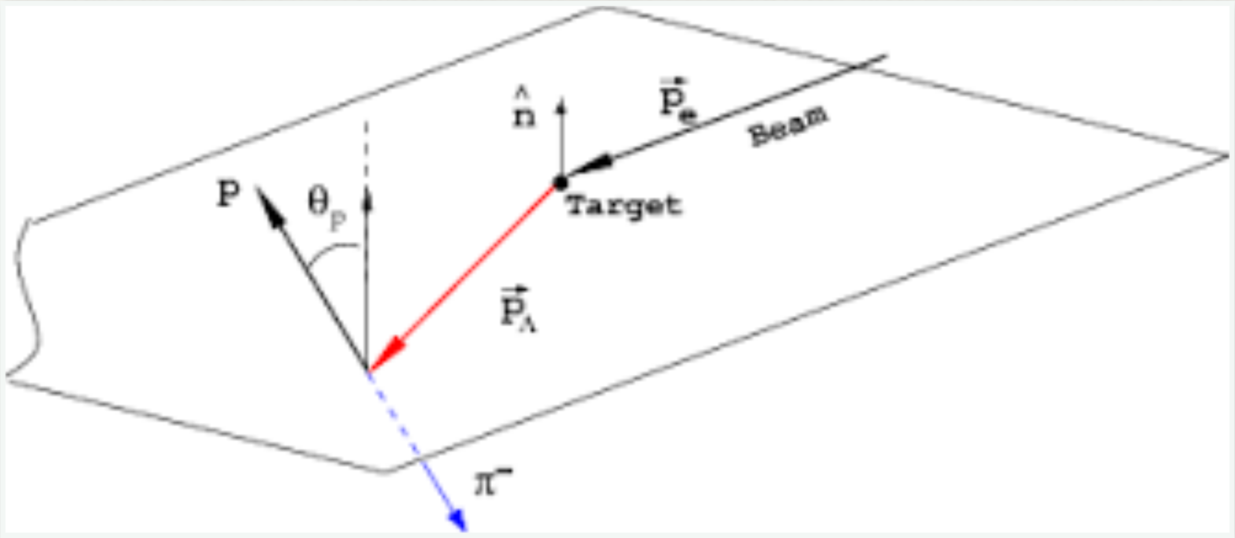
“Probing Nucleons and Nuclei in High Energy Collisions”, Oct. 11, 2018, INT, Seattle, WA

Collinear twist-3 formalism for Λ polarization

Marc Schlegel
Department of Physics
New Mexico State University

Measurement of Λ-spin through decay $\Lambda^0 \longrightarrow p\pi^-$



- Proton preferentially emitted along Λ-spin
- In Λ rest frame: pol. decay distribution

$$\left(\frac{dN}{d\Omega_p}\right)_{\text{pol}} = \left(\frac{dN}{d\Omega_p}\right)_{\text{unpol}} (1 + \alpha\, P_n^\Lambda \cos(\theta_p))$$

P^Λ: Transverse Lambda Polarization

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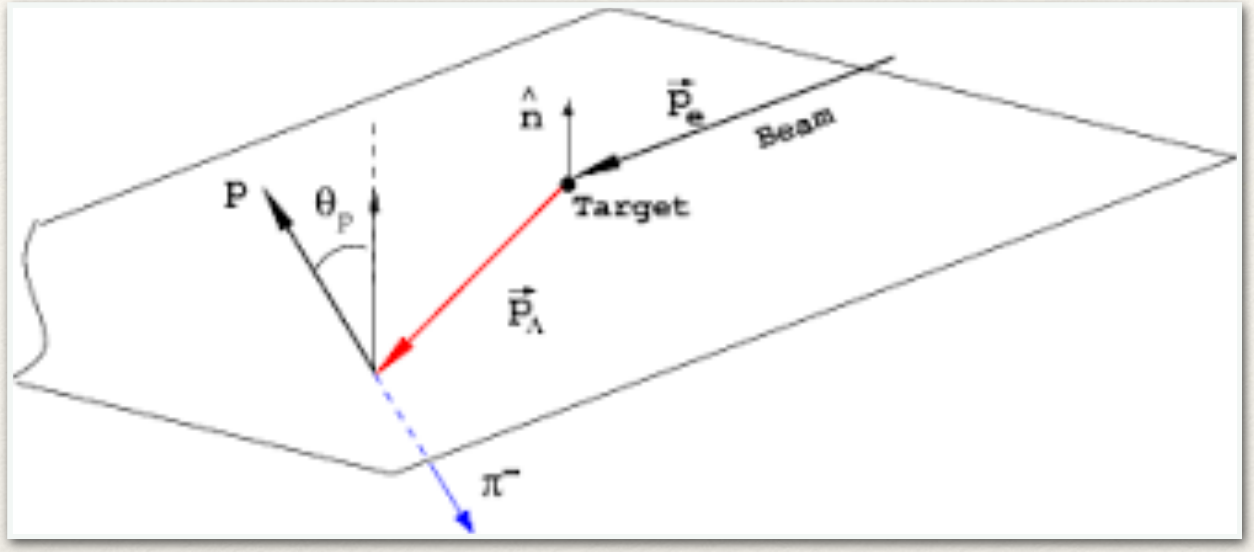

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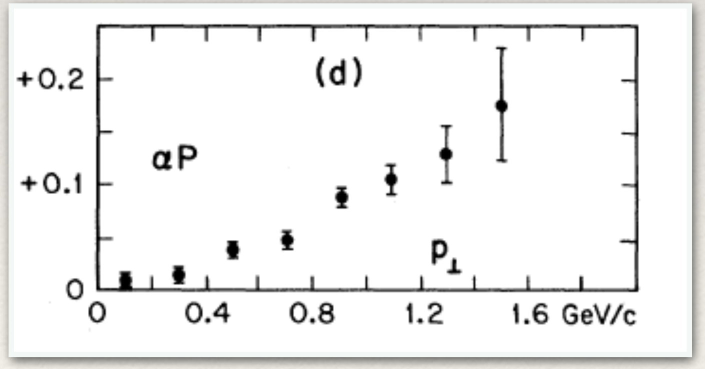
Transverse Λ polarization in pA: long history…

One of the first transverse spin effects at Fermilab (1976): $p+Be \longrightarrow \Lambda^0+X$

and many more follow-up measurements, also at CERN SPS (NA48), HERA-B



Λ polarization was found to be sizeable!

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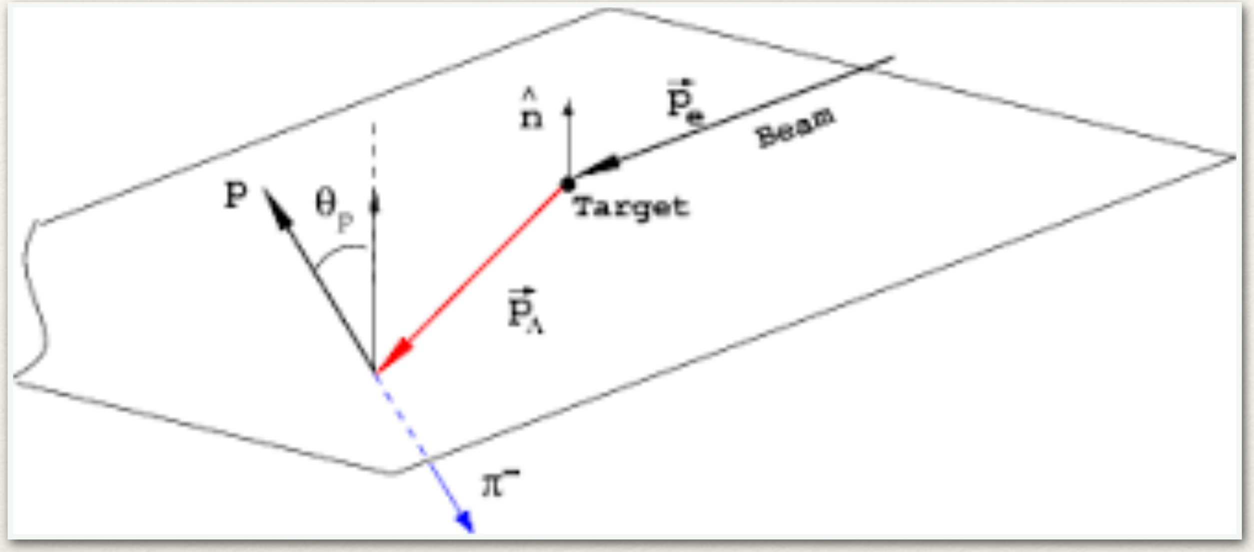

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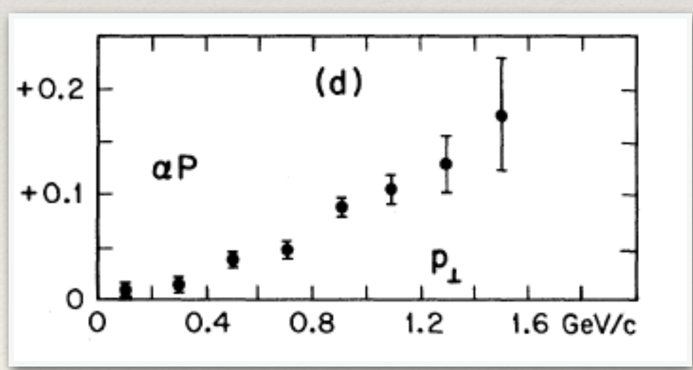

Λ polarization was found to be sizeable!

HERMES (2007, 2014): [0704.3133,1406.3236]

non-zero polarization in quasi-real photoproduction of $\Lambda^\uparrow$ off nuclei!

What about LHC? Is it feasible at a high energy collider?

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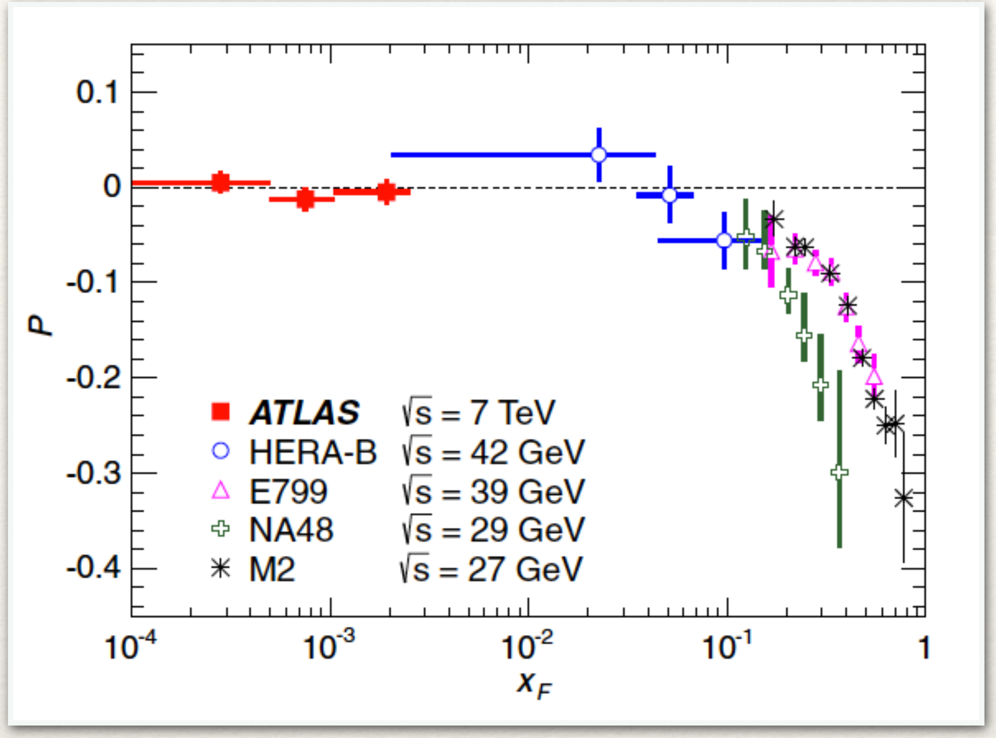

<u>Recent ATLAS measurement at √S = 7 TeV</u>

[ATLAS, PRD 91, 032004 (2015)]

Polarization small at mid-rapidity

Λ polarization at LHC possible

Can Λ polarization be useful for LHC physics?

Tool in particle physics?

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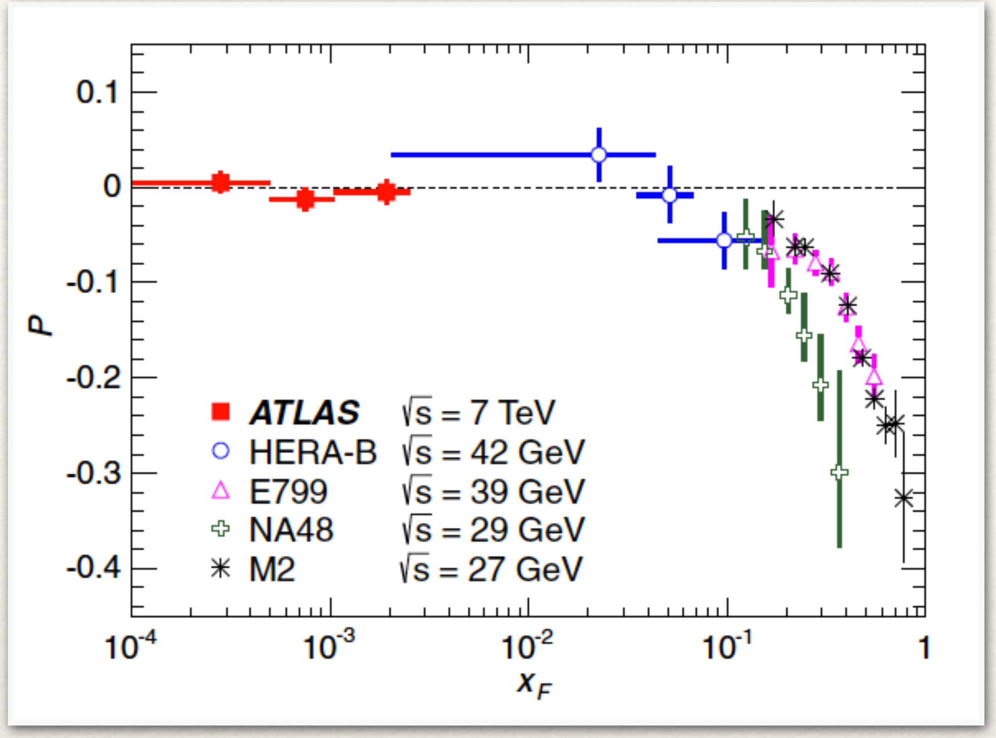

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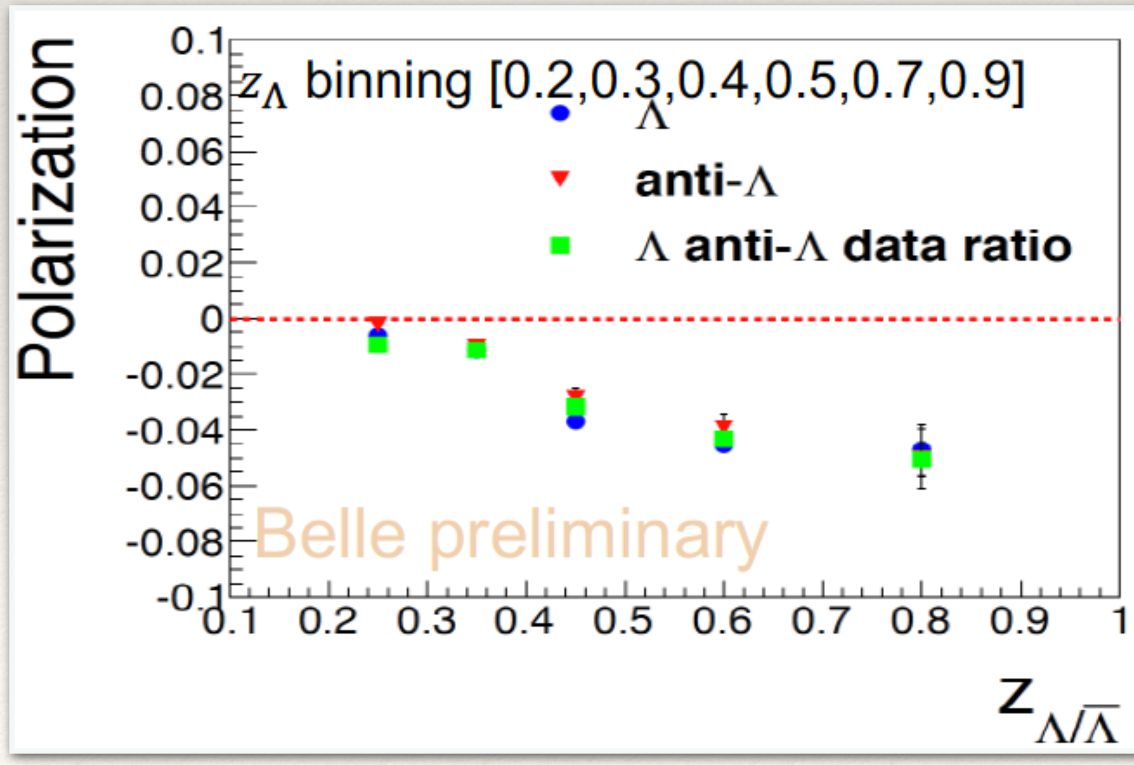
Tool in particle physics?

Simplest and cleanest process (like DIS): $e^+ e^- \longrightarrow \Lambda^\uparrow X$

- OPAL at LEP on Z-pole [Eur.Phys.J C2, 49 (1998)]:
  Longitudinal Polarization,
  no significant Transverse Polarization
- Preliminary Belle data: Transverse Polarization
  [Yinghui Guan, SPIN 2016]
  → talk by A. Vossen

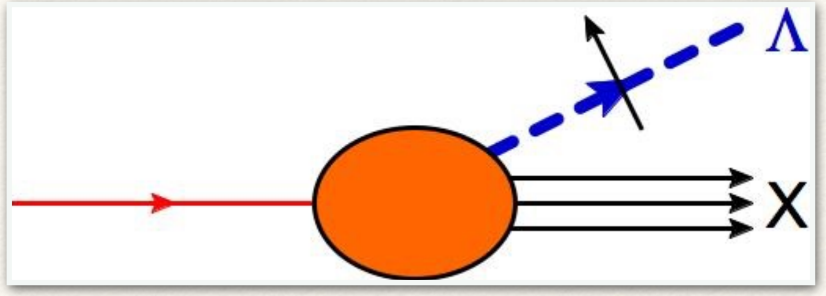
⇒ significant transverse polarization

(measured w.r.t. thrust axis !)



$\Lambda^{\uparrow}$ - production in pQCD

Perturbative QCD at leading twist: Λ fragmentation



parton $\longrightarrow \Lambda$ + X transition:

$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

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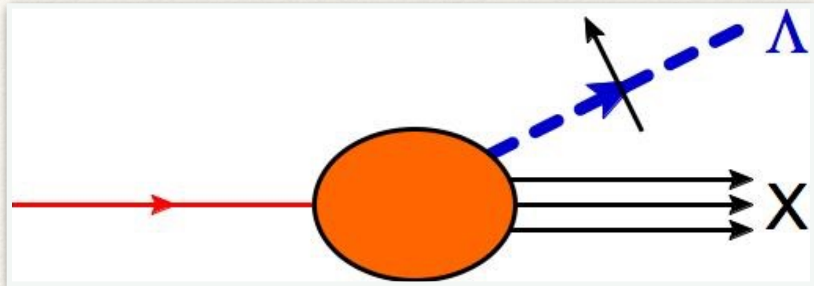

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‘square of the amplitude’

$$\Delta_{ij}(z) = \frac{1}{N_c} \sum_X \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}}\, \langle 0|\, [\infty m, 0]\, q_i(0) |P_\Lambda, S_\Lambda; X\rangle \langle P_\Lambda, S_\Lambda; X|\, \bar{q}_j(\lambda m) [\lambda m, \infty m]\, |0\rangle$$

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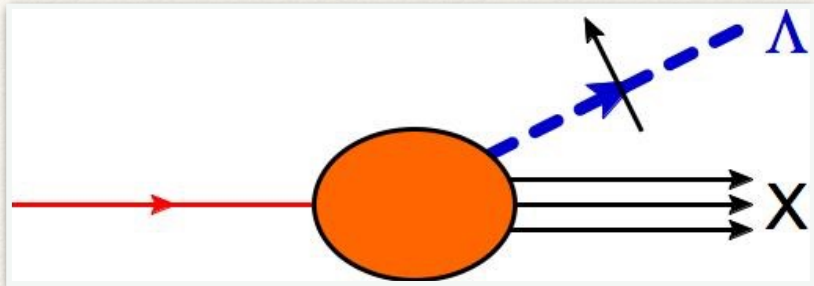

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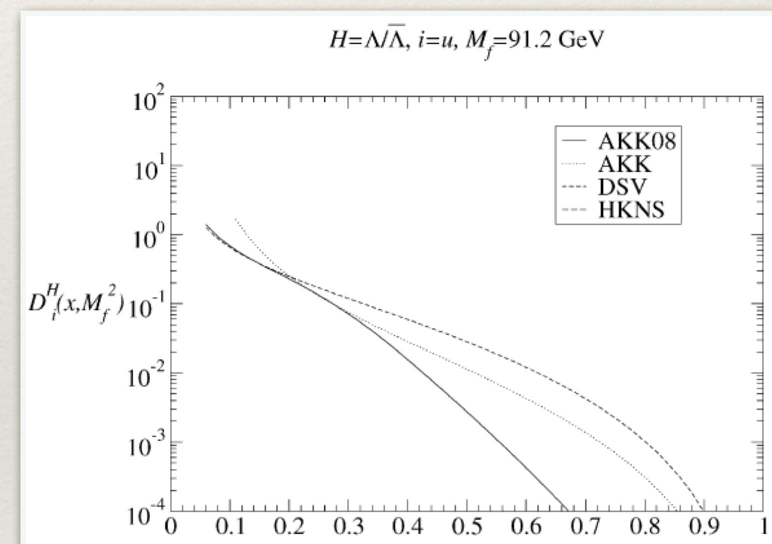
Λ fragmentation functions
at leading twist

$$D_1^{\Lambda/q}(z)$$

FF of unpolarized q $\longrightarrow$ Λ:

fairly known [fits by AKK08, DSV, …]



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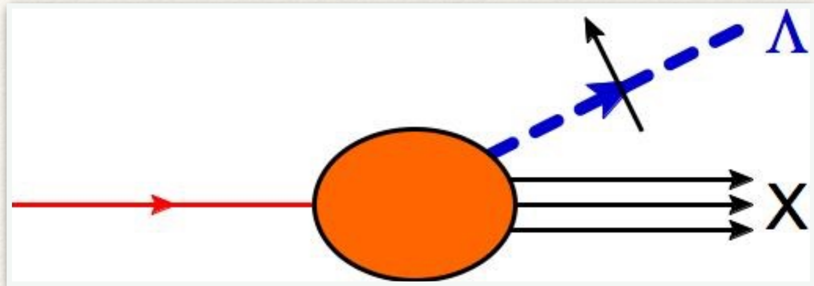

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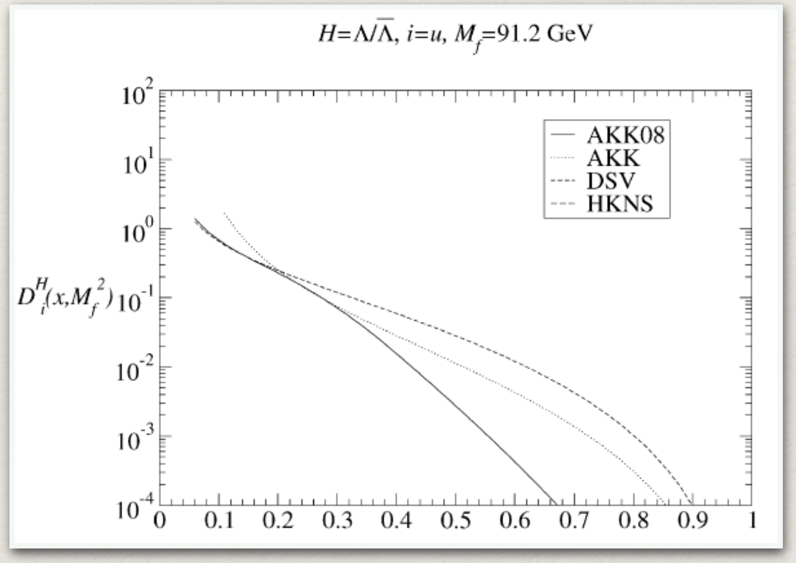

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FF of longitudinally pol. q $\longrightarrow$ Λ:

poorly known [attempts by DSV to fit LEP data]

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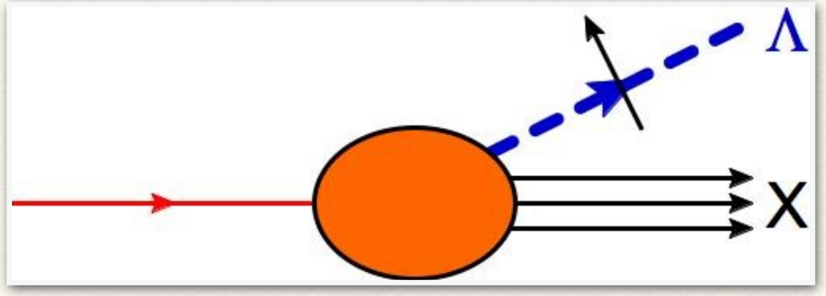

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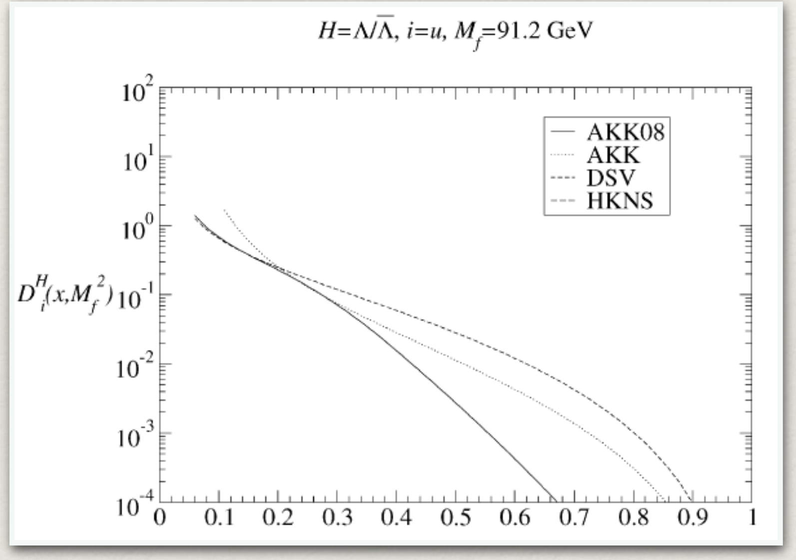

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FF of unpolarized q $\longrightarrow$ Λ:

fairly known [fits by AKK08, DSV, …]

$G_1^{\Lambda/q}(z)$

FF of longitudinally pol. q $\longrightarrow$ Λ:

poorly known [attempts by DSV to fit LEP data]

$H_1^{\Lambda/q}(z)$

FF of transversely pol. q $\longrightarrow$ Λ:

unknown, chiral-odd, hard to extract from single-inclusive processes

Candidate to explain large transverse Λ polarization?

Collinear Twist-3 formalism

'intrinsic' twist-3 FF with transverse spin: $G_T^{\Lambda/q}(z)$ $D_T^{\Lambda/q}(z)$

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'kinematic' twist-3 FF with transverse spin:

$$\Delta_\partial^\alpha(z) = \int d^2 p_T \, p_T^\alpha \, \Delta(z, z p_T) \longrightarrow G_{1T}^{\perp(1),\Lambda/q}(z) \quad D_{1T}^{\perp(1),\Lambda/q}(z)$$

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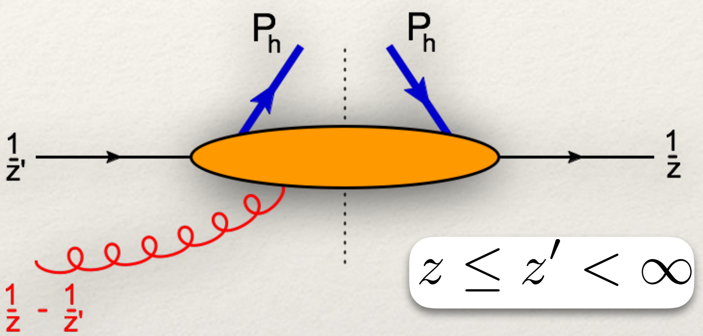
'dynamical' twist-3 FF with transverse spin:

$$\Delta_F^\alpha(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$
$$\Longrightarrow \hat{D}_{FT}^{\Lambda/q}(z,z')\,,\, \hat{G}_{FT}^{\Lambda/q}(z,z')$$



$z \le z' < \infty$

complex functions: $FF(z,z) = 0$ $\quad FF(z,0) = 0$ $\quad \frac{\partial}{\partial z'} FF(z,z')\Big|_{z'=z} = 0$

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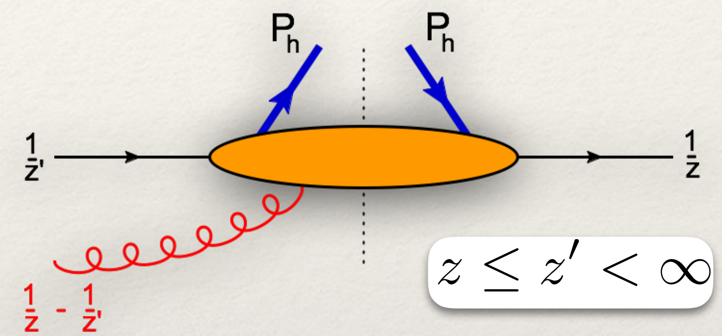

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Relations: Equation of Motion & Lorentz-Invariance

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 93, 054024 (2016)]

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right) G_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

Two equations, three functions → eliminate 'intrinsic & kinematical twist-3'

Single-inclusive Hard Processes suitable for $\Lambda^{\uparrow}$ - production

$\Lambda^{\uparrow}$ in pp - collisions ($p\, p \to \Lambda^{\uparrow} X$):

complete LO formulae not yet available, complicated!

[Koike, Metz, Pitonyak, Yabe, Yoshida, PRD 2017]

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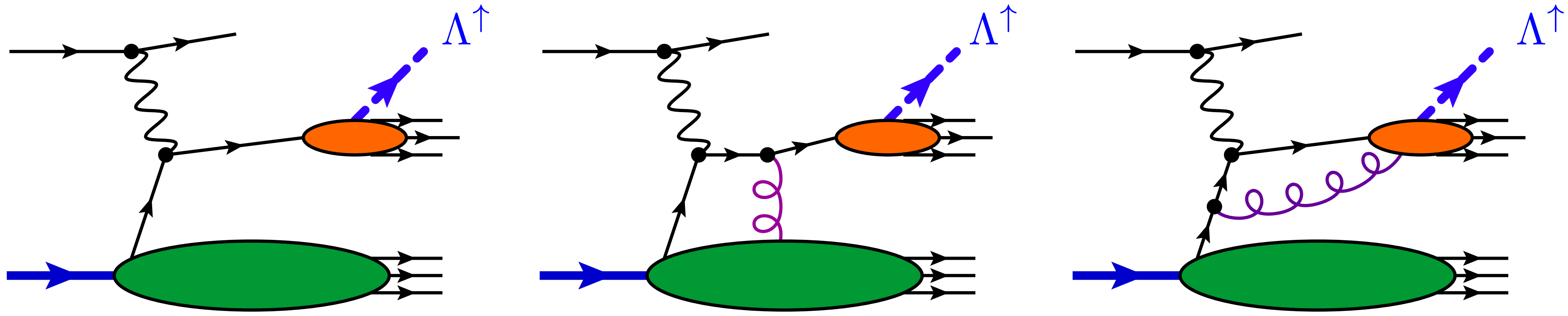
Single-inclusive Λ - production (e p → $\Lambda^\uparrow$ X)



LO-formula (including EoM & LIR)

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016]

$$\sigma(S_\Lambda) \propto \int \hat{\sigma}_1 \otimes f_1^q \otimes \Im[\hat{D}_{FT}^q] + \int \hat{\sigma}_5 \otimes f_1^q \otimes \Im[\hat{G}_{FT}^q] + \int \hat{\sigma} \otimes \frac{d\hat{H}_{FU}^q(x,x)}{dx} \otimes H_1^q(z)$$

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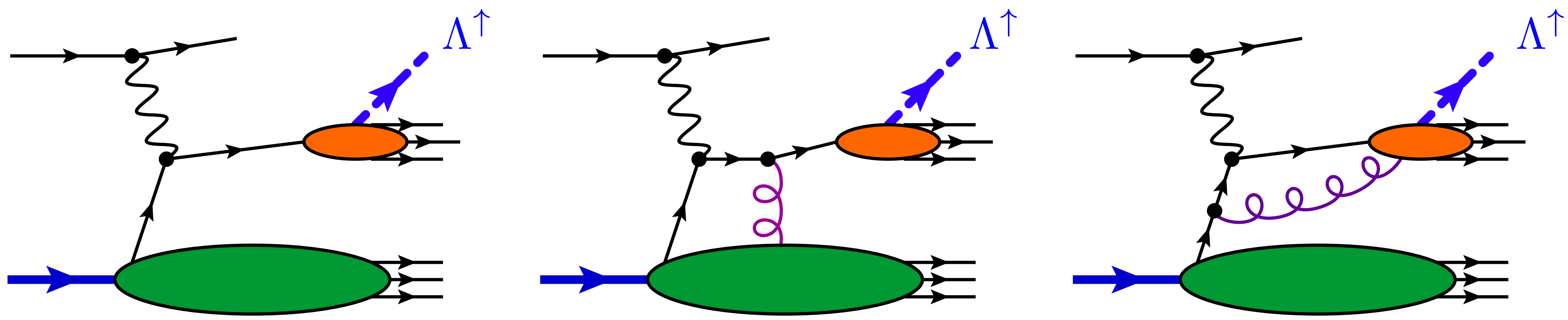

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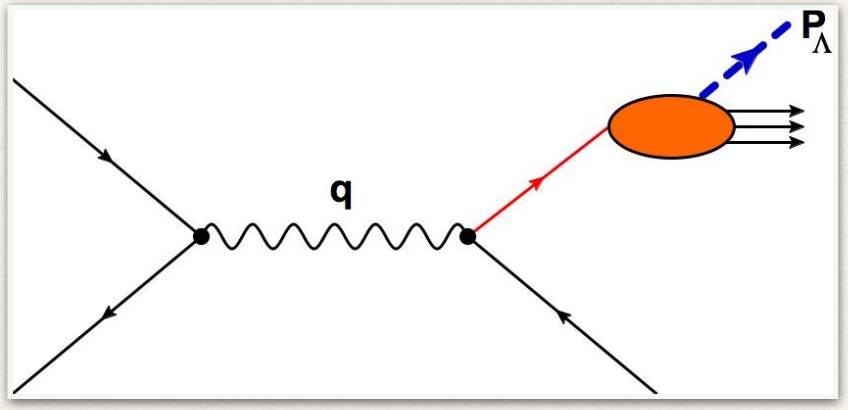
chiral-odd: may be small ?!

Physics opportunity at EIC with P_T > a few GeV!
might help to solve a 40-year old puzzle…

Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

"Parton Model like" at LO



$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)$$

$$z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

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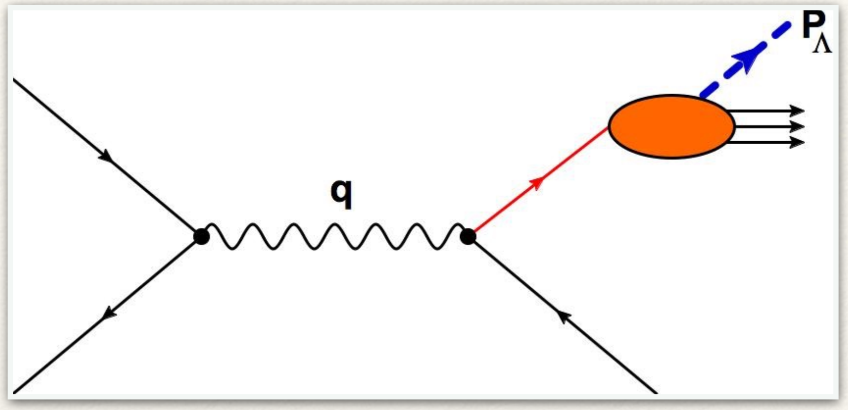

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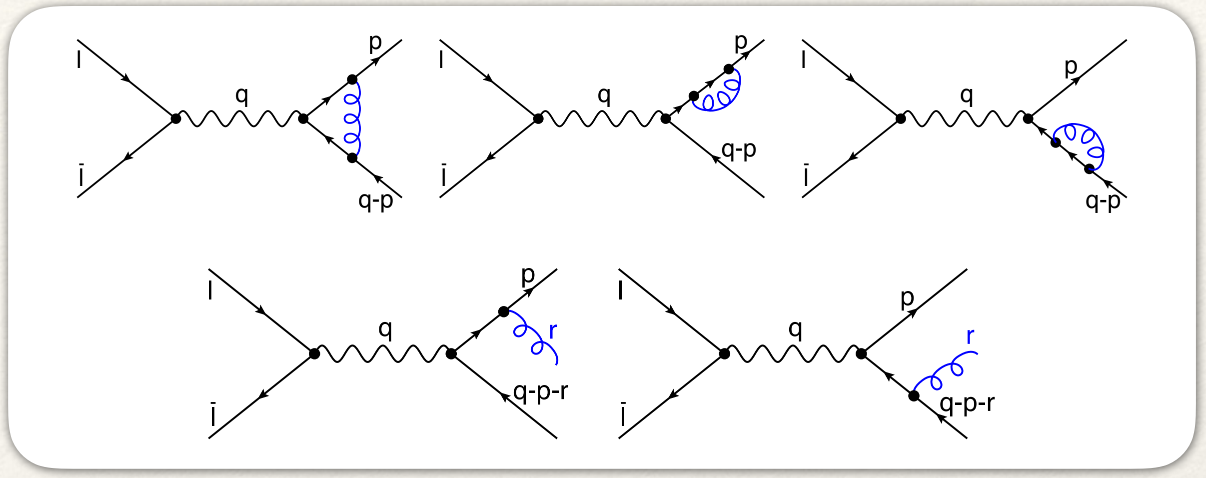
NLO



$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[\hat{\sigma}^{\overline{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w, \mu) + \hat{\sigma}^{\overline{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w, \mu) \Big]$$

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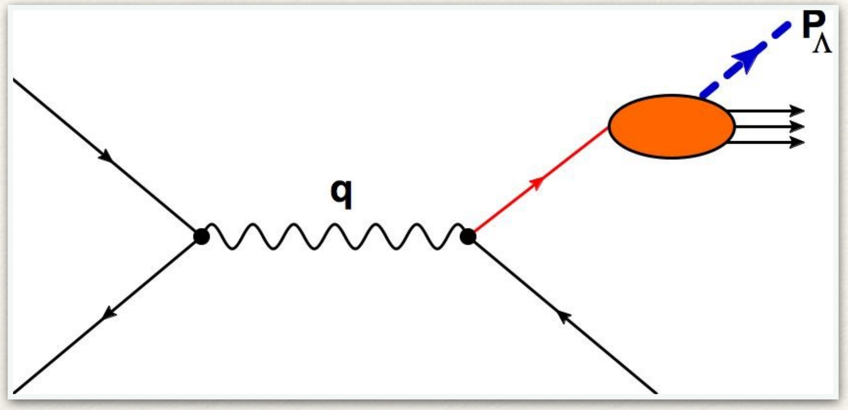

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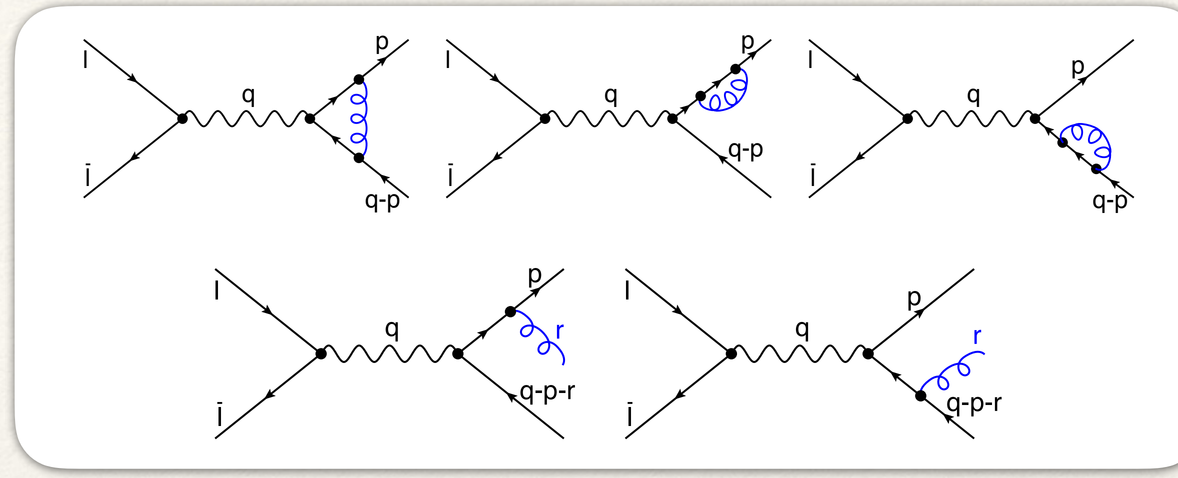

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Typical NLO features:

- infrared safe (cancellation of $1/\varepsilon^2$ - poles in dim. reg.)

$$\hat{\sigma}_{\rm virt} + \hat{\sigma}_{\rm real} = \mathcal{O}(1/\varepsilon)$$

$$\hat{\sigma}^{q/g} \propto -\frac{1}{\varepsilon} P_{q/g\,q}(w) + \mathcal{O}(\varepsilon^0)$$

Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

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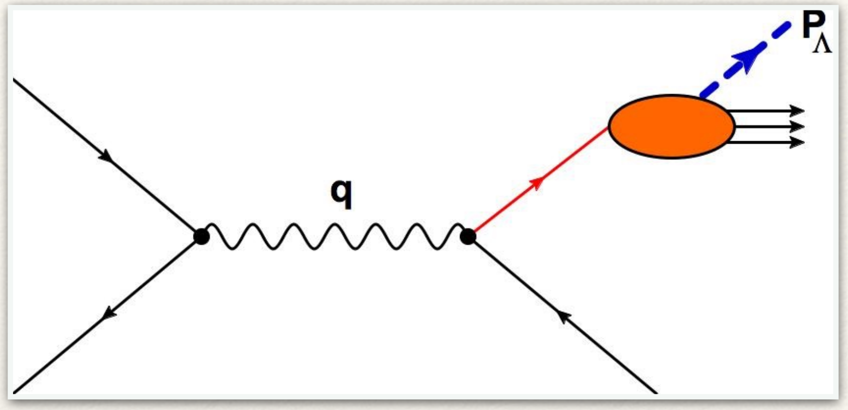

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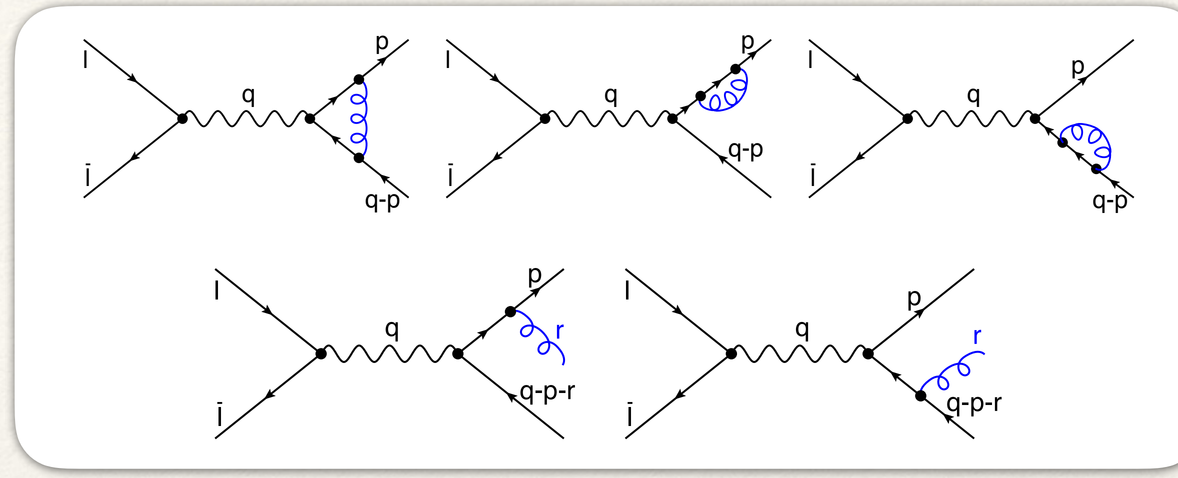

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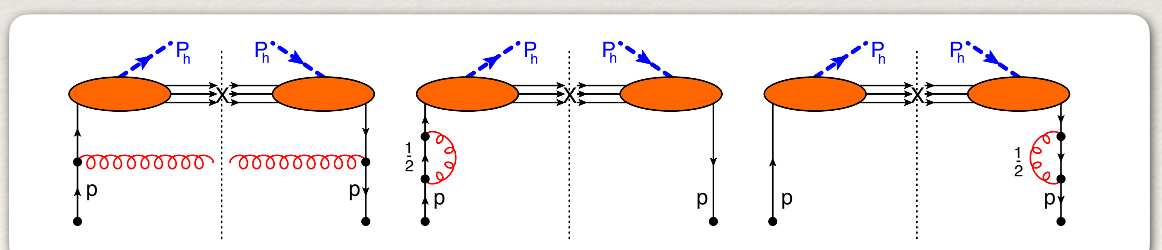
- MSbar renormalization of fragmentation functions → DGLAP evolution

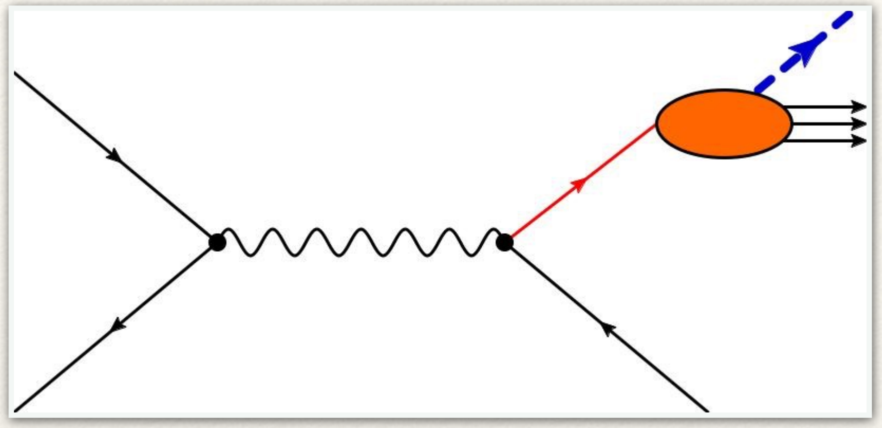


$$D_{1,\text{bare}}^{\Lambda/q}(z) = D_{1,\text{ren}}^{\Lambda/q}(z) + \frac{\alpha_s}{2\pi} \frac{S_\varepsilon^{\overline{\text{MS}}}}{\varepsilon} \sum_{i=q,g} \int_z^1 \frac{dw}{w} P_{iq}(w) D_{1,\text{ren}}^{\Lambda/i}\left(\frac{z}{w}\right) + \mathcal{O}(\alpha_s^2)$$

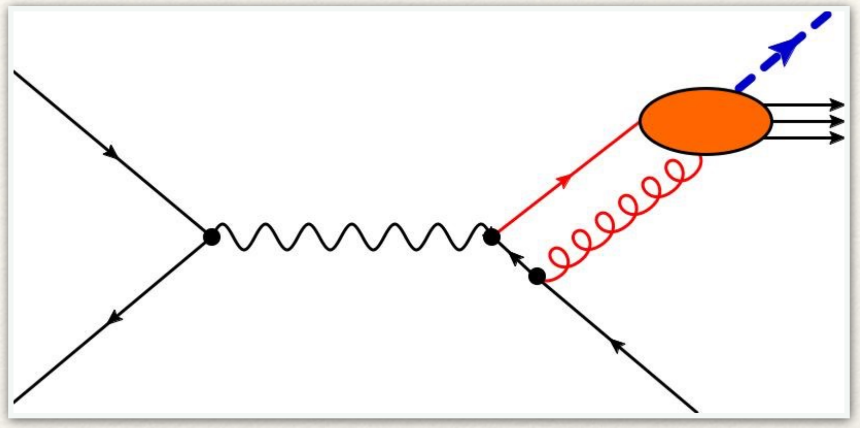
O(1/ε) cancels, necessary condition for one-loop factorization!

Transverse Λ polarization at LO

'intrinsic' & 'kinematical' twist-3 FF:



'dynamical' twist-3 FF:



$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\, \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

→ Equation of Motion:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2\, \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

or:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[-2\, D_{1T}^{\perp(1)\Lambda/q}(z_h) + 2 \int_0^1 d\beta\, \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

Single-Transverse Λ spin asymmetry

- Unique effect driven by a single fragmentation function D_T → absent in DIS (1γ)
- EoM needed at LO to preserve e.m. current conservation of hadronic tensor ($q_\mu W^{\mu\nu} = 0$) (EoM not optional!)

Transverse Λ polarization at NLO

[Gamberg, Kang, Pitonyak, M.S., Yoshida, released soon]

- Study the NLO dynamics for twist-3 fragmentation in the simplest process
- Different compared to twist-3 distributions (no pole contributions)

Transverse Λ polarization at NLO

[Gamberg, Kang, Pitonyak, M.S., Yoshida, released soon]

- Study the NLO dynamics for twist-3 fragmentation in the simplest process
- Different compared to twist-3 distributions (no pole contributions)

Virtual & Real diagrams (qg/q - channel here, gg/g, qqb/g not shown)

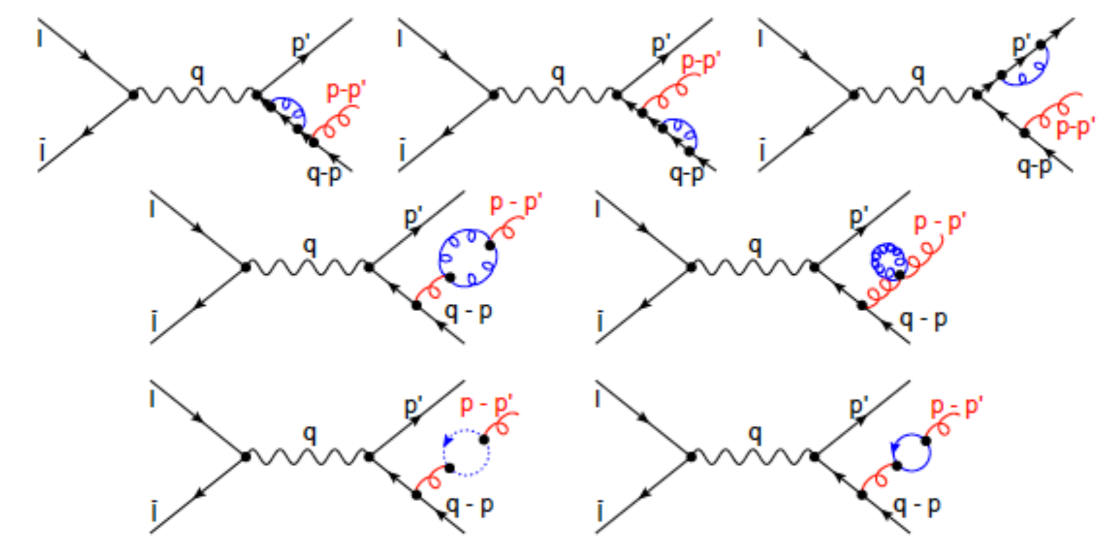


Figure 6: Self-energy corrections

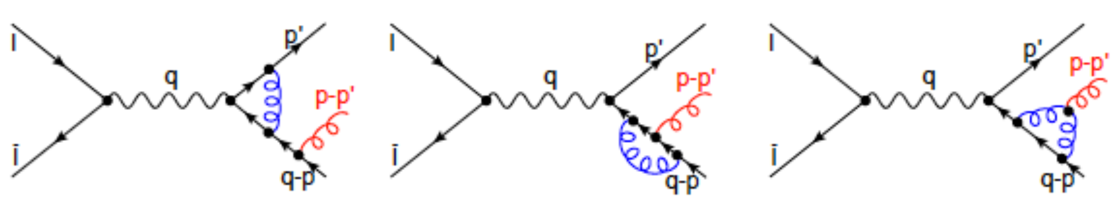


Figure 7: Vertex corrections

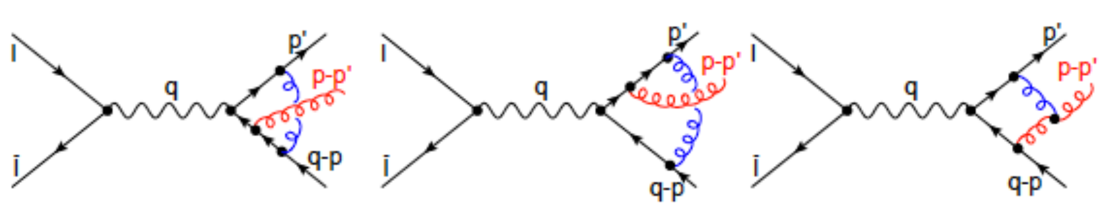


Figure 8: Box corrections

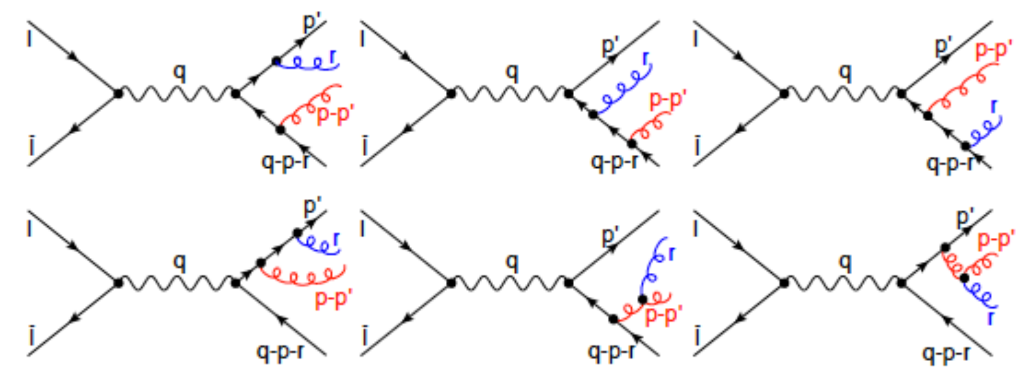


Figure 9: Real corrections

- E.o.M. - relations are crucial: Eliminate 'intrinsic' twist-3 contributions: only then color gauge invariance at NLO! ✔
- Imaginary parts: In the dynamical fragmentation process & loop diagrams
- Infrared $1/\varepsilon^2$ - poles cancel ✔
- $1/\varepsilon$ - poles of imaginary parts of loops cancel through E.o.M. ✔
- $1/\varepsilon$ - collinear poles of real parts of loops through MSbar - renormalization (?)

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

LO

Complete structure of the NLO result w/o intrinsic twist-3

$$
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times
$$

$$
\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}
$$

$$
+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})
$$

LO

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^\varepsilon \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\bar{u},\dots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+\frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

LO

NLO

2-quark correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times
$$

$$
\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}
$$

$$
+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})
$$

$$
+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}_{H_1^{(1)}}^{g;\mathrm{EoM}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})
$$

LO

NLO

2-quark correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^\varepsilon \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times
$$

$$
\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}
$$

$$
+\frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})
$$

$$
+\hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{d\sigma_U^{\mathrm{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

$$+ \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})$$

$$+ \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\overline{u},\dots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}
\end{aligned}
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) =\ & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})
\end{aligned}
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{d\sigma_U^{\mathrm{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^\varepsilon \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\dots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})
\end{aligned}
$$

[redacted]

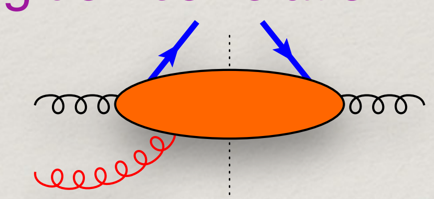
LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+\frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

$$+\hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})$$

$$+\hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}$$

$$+\hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})$$

$$+\hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})$$

$$+\hat{\sigma}^{q\overline{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\overline{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right)$$

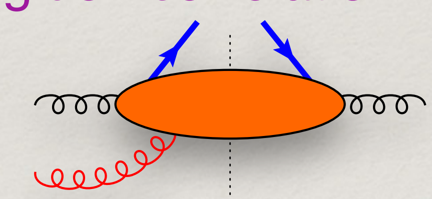
LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right)
\end{aligned}
$$

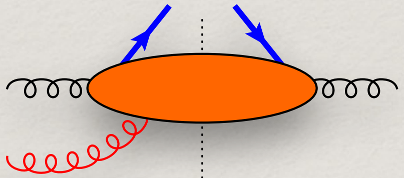
LO

NLO
2-quark correlation w/ EoM

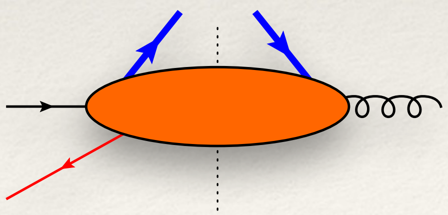
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon}\, \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2}\, \frac{2M_h \epsilon^{P_h m l S_h}}{s}\, (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta\, \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta)\, \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta)\, \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right) \\
& + \hat{\sigma}^q_{\Re}(w,\beta)\, \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
\end{aligned}
$$

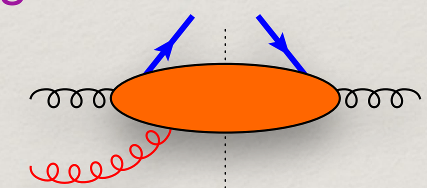
LO

NLO
2-quark correlation w/ EoM

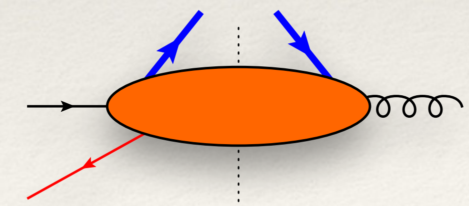
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) =\ & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
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& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
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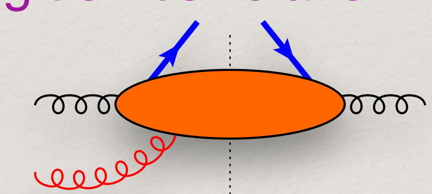
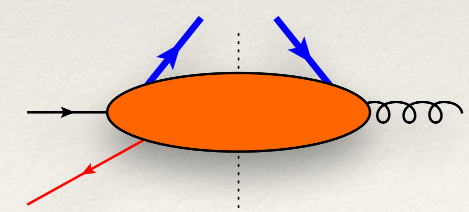


NLO
qq-gluon correlation w/ EoM



NLO
imaginary parts of loops

Complete structure of the NLO result w/o intrinsic twist-3

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& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \Bigg(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \Bigg) \\
& + \hat{\sigma}^{q}_{\Re}(w,\beta)\, \frac{\Re[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
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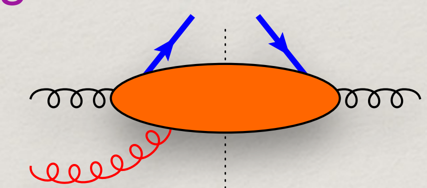

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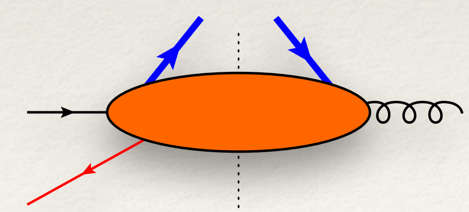

NLO
imaginary parts of loops

<u>All</u> partonic factors calculated in Feynman gauge & Light-cone gauge, both calculations *agree*!

If we assume that twist-3 factorization holds…

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read off evolution equations from collinear divergences for quark twist-3 FF $D_T(z)$

$$\frac{\partial}{\partial \ln \mu^2}\Big(D_T^f(z;\mu)\Big) = \frac{z}{2}\int_z^1 \frac{dw}{w^2}\int_0^1 d\beta \Bigg[P^{[1]}_{1,f\to f}(w)\, D_{1T}^{\perp(1),f}(\tfrac{z}{w};\mu) + P^{[1]}_{1,f\to g}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z}{w};\mu)$$
$$+ P^{[1]}_{2,f\to fg}(w,\beta)\,\frac{\Im[\hat{D}^{fg}_{FT} - \hat{G}^{fg}_{FT}](\tfrac{z}{w},\beta;\mu)}{1-\beta} + P^{[1]}_{3,f\to fg}(w,\beta)\,\frac{2\,\Im[\hat{D}^{fg}_{FT}](\tfrac{z}{w},\beta;\mu)}{(1-\beta)^2}$$
$$+ \sum_{f'=q',\bar{q}'} P^{[1]}_{4,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{D}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)] + \sum_{f'=q',\bar{q}'} P^{[1]}_{5,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{G}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)]$$
$$+ P^{[1]}_{6,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^s(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{7,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^a(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{8,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_1(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2}\Bigg]$$

ordinary DGLAP splitting functions

$$P^{[1]}_{1,f\to f}(w) = -2\frac{C_F\alpha_s}{2\pi}\left(\frac{1+w^2}{(1-w)_+} + \frac{3}{2}\delta(1-w)\right)$$

$$P^{[1]}_{1,f\to g}(w) = 4\frac{C_F\alpha_s}{2\pi}\left(\frac{1+(1-w)^2}{w}\right)$$

Others: more complicated

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read off evolution equations from collinear divergences for quark twist-3 FF $D_T(z)$

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Others: more complicated

<u>Final proof of one-loop factorization:</u>

Need to derive evolution equation directly from correlator!

Previous work on *unpolarized* chiral-odd twist-3 fragmentation:

[Belitsky, Kuraev, NPB 1996; Ma, Zhang, PLB 2017]

"The Gribov-Lipatov reciprocity fulfilled for two-particle cut-vertices only!"

Summary & Outlook

- Λ Polarization: Long history, measured in pp-collisions, recently at ATLAS → feasible at a high-energy collider
- Recent measurement at Belle in e^+e^-: clean processes to determine polarized Λ fragmentation functions
- Theory for e^+e^-: Transverse Λ single-spin asymmetry through (LO) D_T, consequence of missing T-reversal → unique feature
- Outlook/Implication: NLO completed,
  → calculate 'splitting functions' for polarized Λ fragmentation function
  → Double Spin Asymmetry A_{LT} equally important…

  Then: extension to other processes, e.g. e^+e^- to be studied (Λ+π - final state)