“Probing Nucleons and Nuclei in High Energy Collisions”, Oct. 2, 2018, INT, Seattle, WA

Definition of generalized TMDs

Marc Schlegel
Department of Physics
New Mexico State University

talk based on

M. G. Echevarria, A. Idilbi, K. Kanazawa, C. Lorcé, A. Metz, B. Pasquini, M. S.,
Phys. Lett. B 759 (2016), 336-341, [arXiv:1602.06953]

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Generalized Transverse Momentum Dependent Parton Distributions ?

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Naive matrix element

$$W^{[\Gamma]}_{\lambda\lambda'}(x, k_T, \Delta) = \int \frac{d\eta\, d^2 z_T}{2(2\pi)^3}\, \mathrm{e}^{i\eta x + i k_T \cdot z_T} \langle p', \lambda' | \bar{q}(-\tfrac{\eta\, n + z_T}{2}) \Gamma\, \mathcal{W}\, q(\tfrac{\eta\, n + z_T}{2}) | p, \lambda \rangle$$

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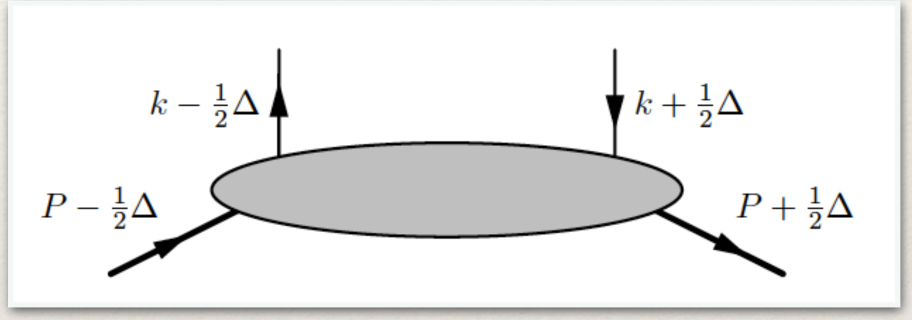
off-diagonal kinematics like GPDs



$$P = \frac{1}{2}(p + p') \qquad \Delta = p' - p$$

$$\Delta^+ = -2\xi P^+ \qquad t = \Delta^2 = -\frac{(1+\xi^2)\,\vec{\Delta}_T^2 + 4\xi^2 M^2}{1-\xi^2}$$

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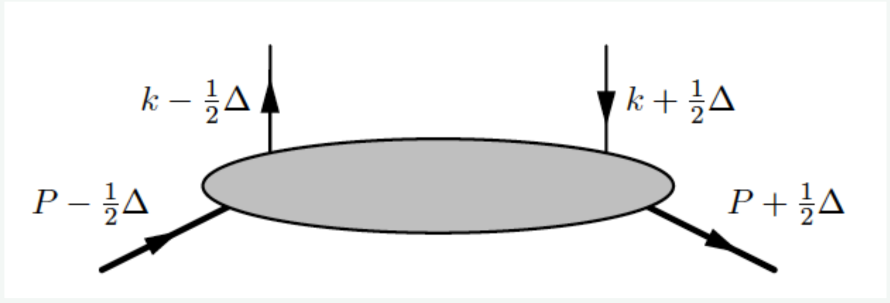

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Parametrization:

[Spin-0: Meißner, Metz, M.S., Goeke, JHEP 0808 (2008), 038 ; Spin-1/2: Meißner, Metz, M. S., JHEP 0908 (2009), 056; Gluons: Lorcé, Pasquini, JHEP 1309 (2013), 138]

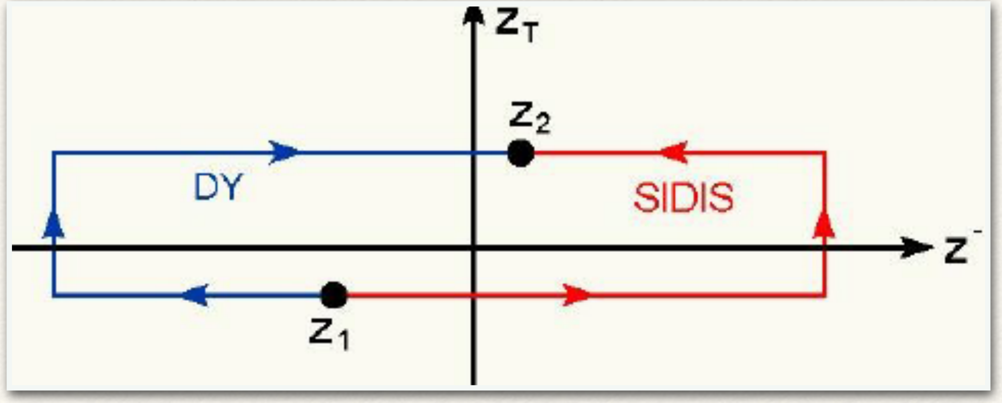
$$W^{[\gamma^+]} = \frac{1}{2M}\bar{u}(p')\left[F_{1,1} + \frac{k_T^\alpha}{P^+}\, i\sigma^{\alpha+}\, F_{1,2} + \frac{\Delta_T^\alpha}{P^+}\, i\sigma^{\alpha+}\, F_{1,3} + \frac{k_T^\alpha\, \Delta_T^\beta}{M^2}\, i\sigma^{\alpha\beta}\, F_{1,4}\right] u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M}\bar{u}(p')\left[-\frac{i\epsilon^{\alpha\beta} k_T^\alpha\, \Delta_T^\beta}{M^2} G_{1,1} + \frac{k_T^\alpha}{P^+}\, i\sigma^{\alpha+}\gamma_5\, G_{1,2} + \frac{\Delta_T^\alpha}{P^+}\, i\sigma^{\alpha+}\gamma_5\, G_{1,3} + i\sigma^{+-}\gamma_5\, G_{1,4}\right] u(p)$$

GTMDs functions of x, k_T, ξ, Δ_T

Wilson line

$$\mathcal{W}[a;b] = \mathcal{P}\mathrm{e}^{-ig\int_a^b ds\cdot A(s)}$$



GTMDs with staple-like Wilson line ⇒ complex functions

$$\Im[\mathrm{GTMD}]\Big|_{\mathrm{SIDIS}} = -\Im[\mathrm{GTMD}]\Big|_{DY}$$

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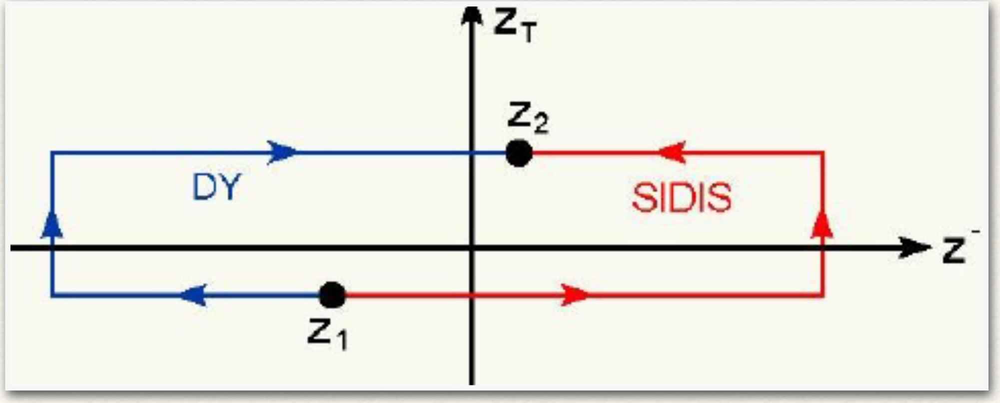

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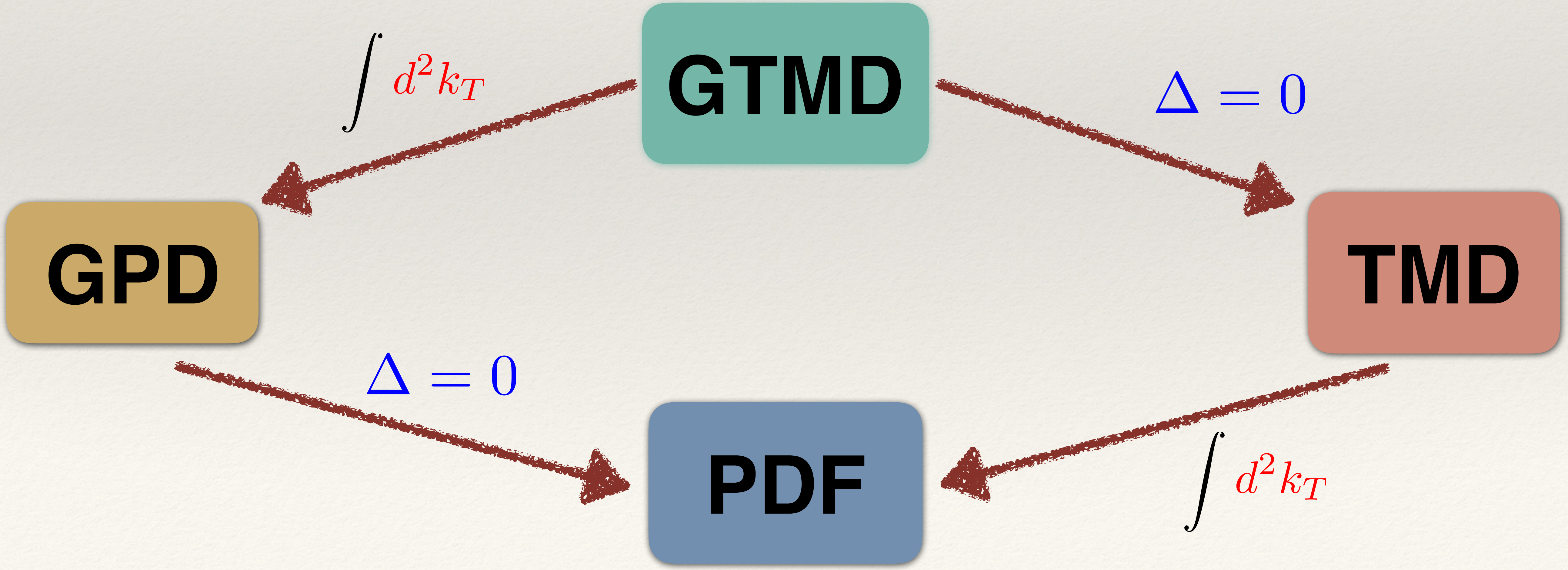
GTMDs with staple-like Wilson line: <u>'Mother functions' of GPDs and TMDs</u>



Relation to Wigner distributions

[Ji, PRL 91, 062001 (2003); Belitsky, Ji, Yuan, PRD 69, 074014 (2004); Lorcé, Pasquini, PRD 84, 014015 (2011)]

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Wigner function in Quantum Mechanics:

$$P(x,p) = \frac{1}{2\pi\hbar}\int dy\,\psi^*(x-\tfrac{y}{2})\,\psi(x+\tfrac{y}{2})\,\mathrm{e}^{-ipy/\hbar}$$

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Wigner Distributions in QCD:

$$\rho^{[\Gamma]}(x_T,p_T,x,S) = \int\frac{d^2\Delta_T}{(2\pi)^2}\,\langle p',S|\left(\int\frac{dy^-\,d^2y_T}{2(2\pi)^3}\,\bar{q}(x_T-\tfrac{y}{2})\,\Gamma\,\mathcal{W}\,q(x_T+\tfrac{y}{2})\,\mathrm{e}^{ip\cdot y}\right)_{y^+=0}|p,S\rangle\Big|_{\xi=0}$$

⇒ Fourier transform of GTMDs

$$\rho^{[\Gamma]}(B_T,k_T,x,S) = \int\frac{d^2\Delta_T}{(2\pi)^2}\,\mathrm{e}^{-i\Delta_T\cdot B_T}\,W^{[\Gamma]}(x,\xi=0,k_T,\Delta_T)$$

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Formulation of Quark OAM and Spin-Orbit Correlations:

$$\langle\hat{L}_z^q\rangle=\int dx\,d^2k_T\,d^2B_T\,(\vec{B_T}\times\vec{k_T})_z\,\rho^{[\gamma^+]}(B_T,k_T,x,S)=-\int dx\,d^2k_T\,\frac{k_T^2}{M^2}\,F_{1,4}^q(x,\xi=0,k_T,\Delta_T=0)$$

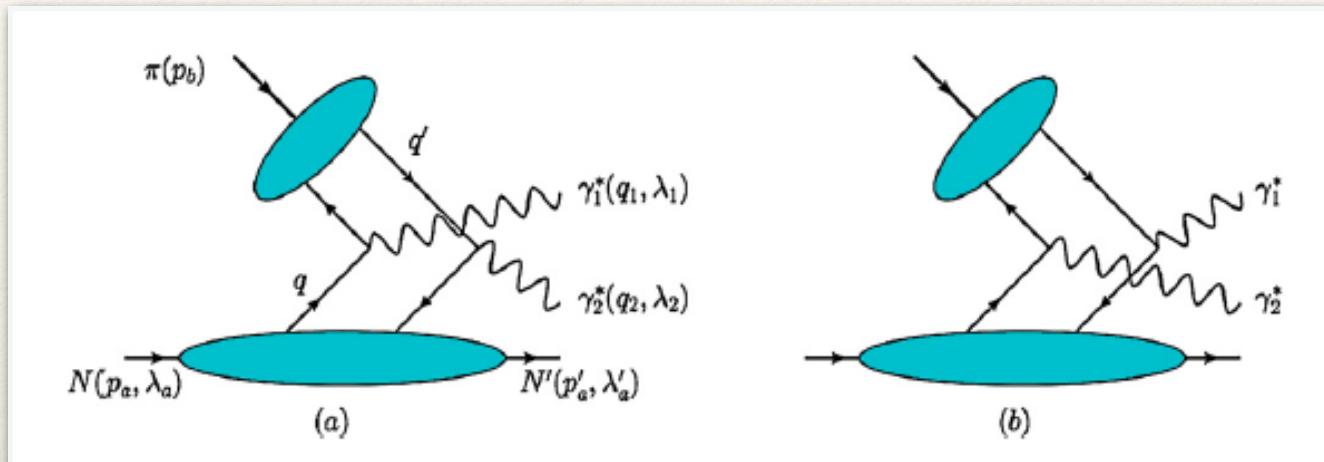
$$C_z^q=\int dx\,d^2k_T\,d^2B_T\,(\vec{B_T}\times\vec{k_T})_z\,\rho^{[\gamma^+\gamma_5]}(B_T,k_T,x,S)=\int dx\,d^2k_T\,\frac{k_T^2}{M^2}\,G_{1,1}^q(x,\xi=0,k_T,\Delta_T=0)$$

(Tree-level) Processes sensitive to GTMDs

Exclusive Double-DY: $\pi N \rightarrow (l^+l^-)\,(l^+l^-)\,N$

[Bhattacharya, Metz, Zhou, PLB 771, 396]

"double parton scattering"



Gauge Link: staple-like past-pointing

$$\tau \sim \int d^2k_{NT} \int d^2k_{\pi T}\, \delta^{(2)}\left(\tfrac{1}{2}\Delta q_T - k_{NT} - k_{\pi T}\right)$$
$$w(k_{NT}, k_{\pi T})\,\mathrm{GTMD}(\ldots, k_{NT})\,\mathrm{LCWF}(\ldots, k_{\pi T})$$

Quark GTMD in ERBL region $-\xi < x < \xi$

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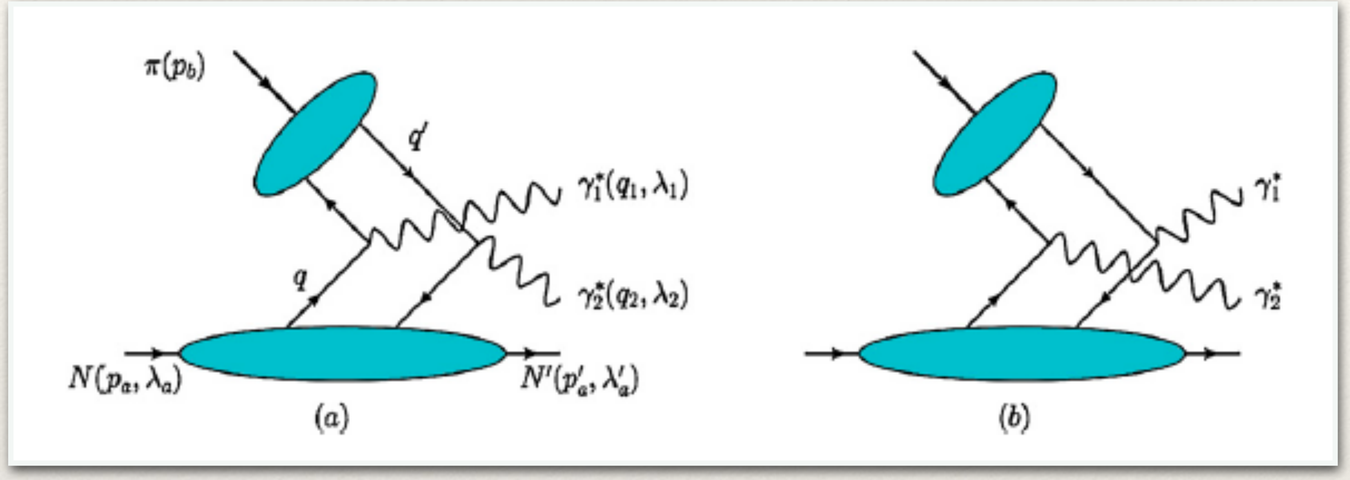

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Exclusive Quarkonium pair production: NN → (Q) (Q) NN

[Bhattacharya, Metz, Kumar, Tsai, Zhou, 1802.10550]

→ Gluon GTMD in ERBL region $-\xi < x < \xi$

Wigner functions in the small - x framework:

[Hatta, Yuan, …]

→ talk by F. Yuan

Large k_T - behaviour of GTMDs

[Kanazawa, Lorcé, Metz, Pasquini, M.S., PRD 90, 014028 (2014)]

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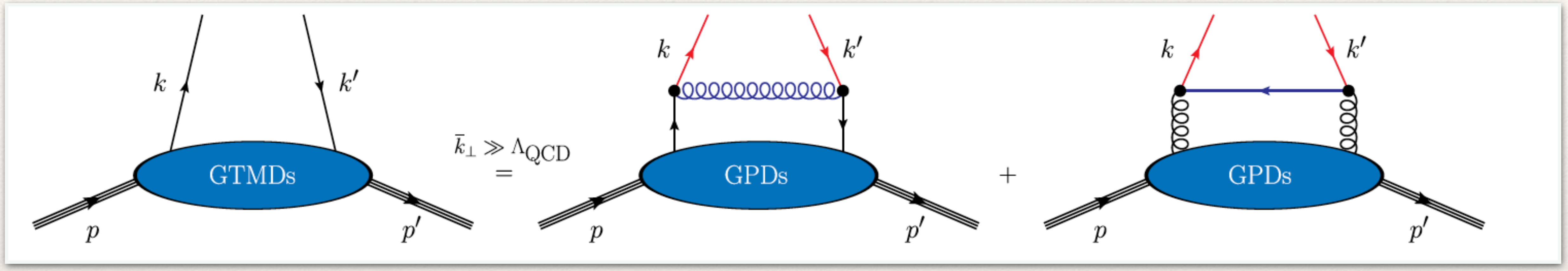
perturbative QCD

$\Longrightarrow$ behaviour of GTMDs are large transverse momenta $k_T \gg \Lambda_{QCD}$



naive GTMD definition in gauge v · A(x) = 0

[cf. TMDs: Bacchetta, Boer, Diehl, Mulders, JHEP 0808, 023 (2008)]

$$n^\mu = [1^+, 0^-, 0_T] \to v^\mu = [1, -\zeta/(2(P^+)^2)]$$

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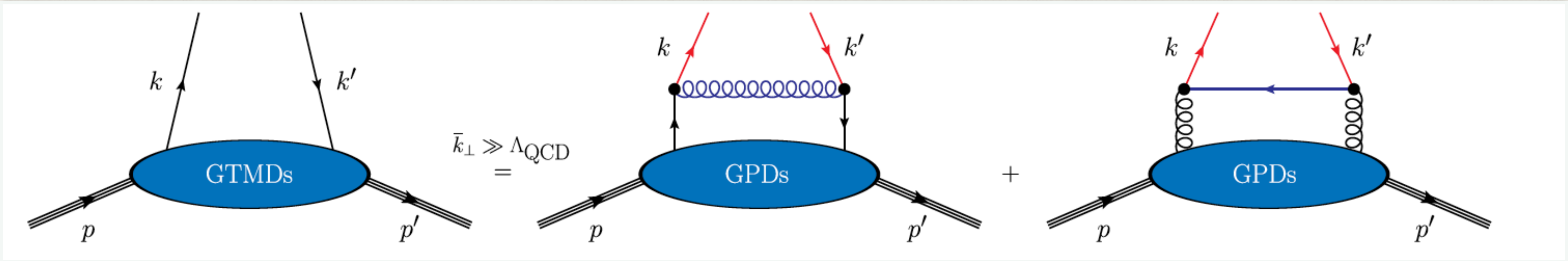

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$$F^q_{1,4}(x, \xi = 0, k_T \gg \Lambda_{QCD}, \Delta_T) = \frac{\alpha_s}{2\pi^2} \int_x^1 \frac{y}{y} \frac{M^2 \left(C_F\, \tilde{H}^q(x/y, 0, \Delta_T) - T_R(1-y)^2\, \tilde{H}^g(x/y, 0, \Delta_T)\right)}{\prod_\pm \left[(k_T \pm (1-y)\Delta_T/2)^2 + y(1-y)\Delta_T^2/4\right]}$$

seems to work for $F_{1,4}$ (at this order α_s), but…

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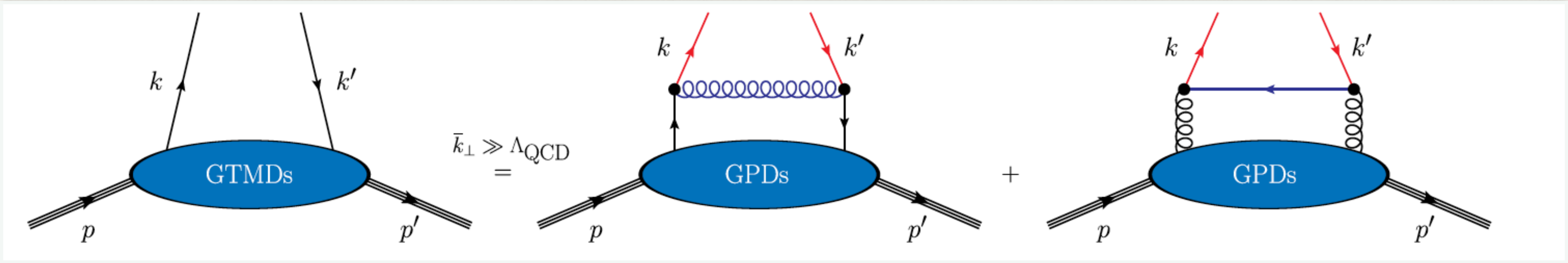

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$$F^q_{1,1} = \frac{\alpha_s}{2\pi^2} \left[C_F \left(\log \left| \frac{x^2 \zeta}{k_T^2} \right| - \frac{3}{2} \right) \frac{1}{k_T^2}\, H^q(x, 0, \Delta) + \int_x^1 \frac{dy}{y}\, C(y, k_T, \Delta_T)\, \mathrm{GPDs}(x/y, 0, \Delta_T) \right]$$

Logarithmic divergence for Wilson lines along the light-cone!

Modified Definition of GTMDs

[Echevarria, Idilbi, Kanazawa, Lorcé, Metz, Pasquini, M.S., Phys. Lett. B 759 (2016), 336-341]

Modify definition of GTMDs in the same way as TMDs (same operator!)

1) Renormalizable Matrix Element
2) Wilson Coefficients without divergences

Inclusion of Soft Function $\Longrightarrow$

$$S(z_T) = \frac{\mathrm{Tr}_c}{N_c} \langle 0 | \mathcal{W}_n^\dagger(-z_T/2)\, \mathcal{W}_{\bar{n}}(-z_T/2)\, \mathcal{W}_{\bar{n}}^\dagger(z_T/2)\, \mathcal{W}_n(z_T/2)\,] 0 \rangle$$

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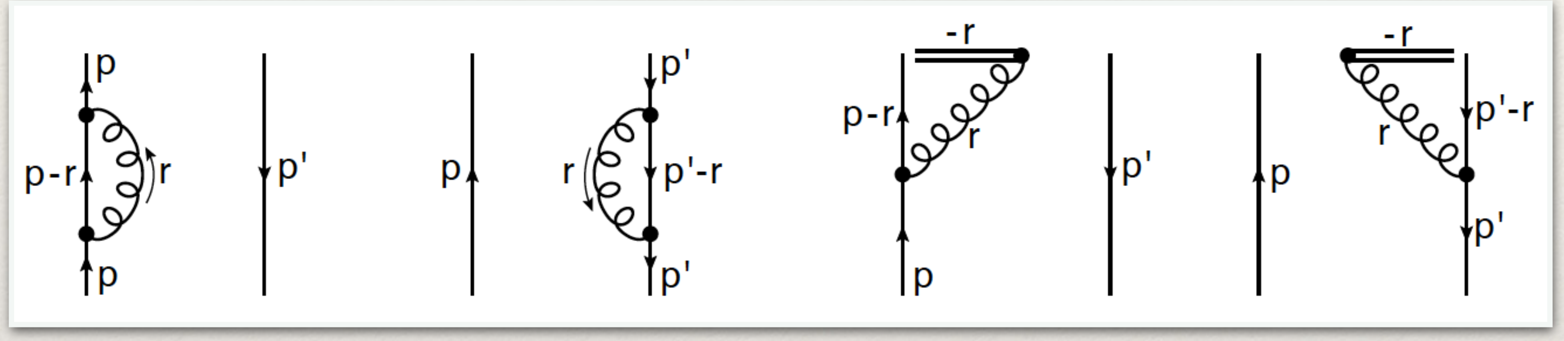
Renormalization of GTMDs with massless quark targets:



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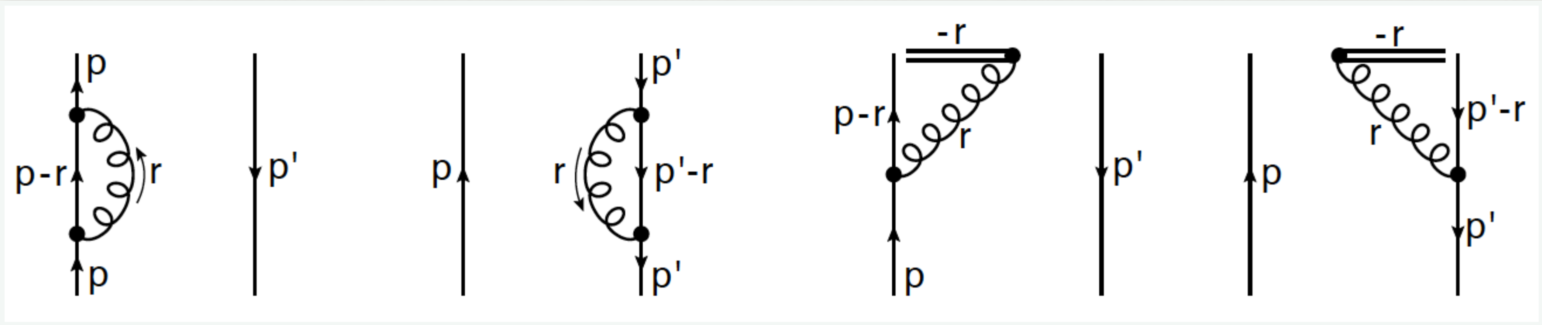

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overlapping UV and soft divergence

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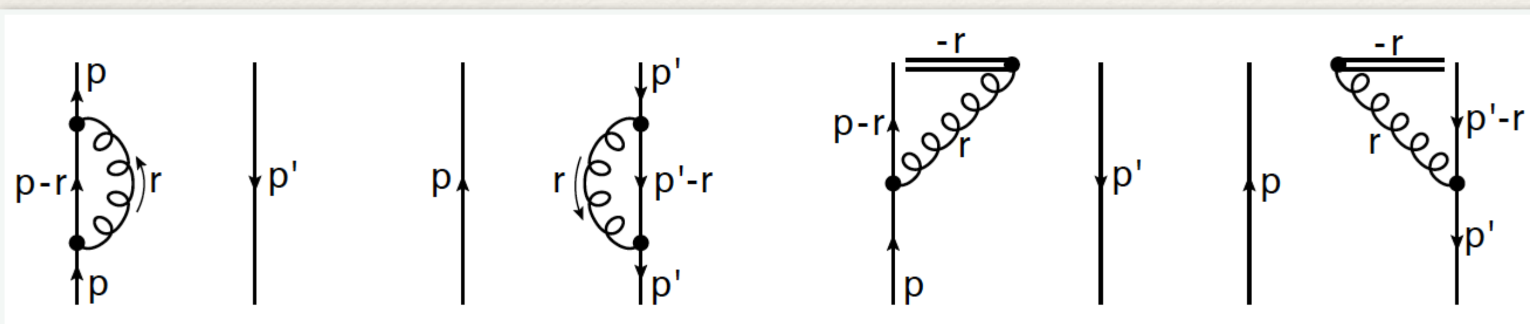

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overlapping UV and soft divergence

Soft function:

$$W^{[\Gamma],\mathrm{SF}} = \frac{\bar{u}(p')\,\Gamma\, u(p)}{2P^+}\, \delta(1-x)\, \delta^{(d-2)}(k_T) \left(1 + \frac{C_F\, \alpha_s}{2\pi} S_\varepsilon \left[\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon} \ln\left(\frac{\delta}{\Lambda\, \mu}\right) + \mathcal{O}(\varepsilon^0)\right] + \mathcal{O}(\alpha_s^2)\right)$$

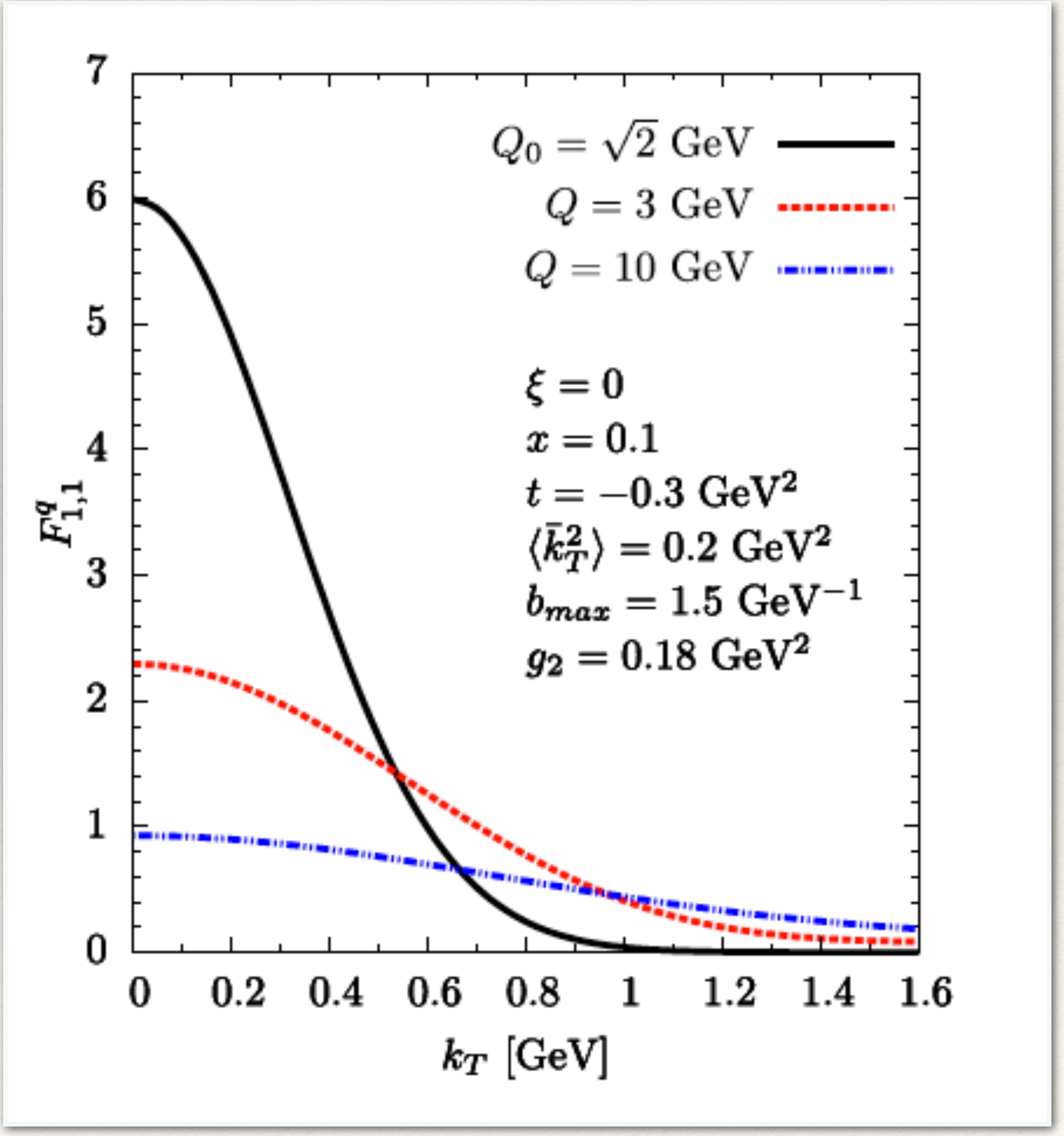
Renormalization Constant:

$$Z^{\overline{\mathrm{MS}}} = 1 - \frac{C_F\, \alpha_s(\mu)}{2\pi} \left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{3}{2} - 2\ln(\Lambda/\mu)\right) + \mathcal{O}(\alpha_s^2)\right)$$

Evolution of GTMDs

Operator of GTMDs = Operator of TMDs ⇒ identical evolution

$$F^q_{1,1}(...;\mu,\Lambda) = e^{-S_{\rm pert}} e^{-S_{\rm non-pert}\ln(\Lambda/\Lambda_0)} F^q_{1,1}(...;\mu_0,\Lambda_0)$$



Input function:

Gaussian model at low Q_0

$$F^q_{1,1}(x,\xi=0,k_T,\Delta_T;\mu=Q_0,\Lambda=Q_0)$$
$$= H^q(x,\xi=0,t=-\Delta_T^2;\mu=Q_0)\,\frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle}$$

GPD model:

$$H^q(x,\xi=0,t;\mu=Q_0) = f_1^q(x;Q_0)\,e^{\lambda t}$$

Evolution:

k_T - distribution flattens out at larger Q just as TMDs do…

Wilson Coefficients

Relation to collinear GPD: Operator Product Expansion in coordinate space $z_T = b_T$

$$F_{1,1}^{q,\mathrm{ren}}(x, \xi = 0, b_T, \Delta_T) = \int_x^1 dy\, C^{q/q}(y, b_T, \Delta_T, \mu, \Lambda)\, H^{q,\mathrm{ren}}(x/y, 0, \Delta_T)$$

Wilson Coefficient C should not depend on regulator δ!

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Calculate Wilson Coefficient for quark target:

$$C^{q/q} = \delta(1-y) + \frac{C_F\,\alpha_s}{2\pi} C^{[1]}(y) + \ldots \implies C^{[1]} = F_{1,1}^{[1]} - H^{[1]}$$

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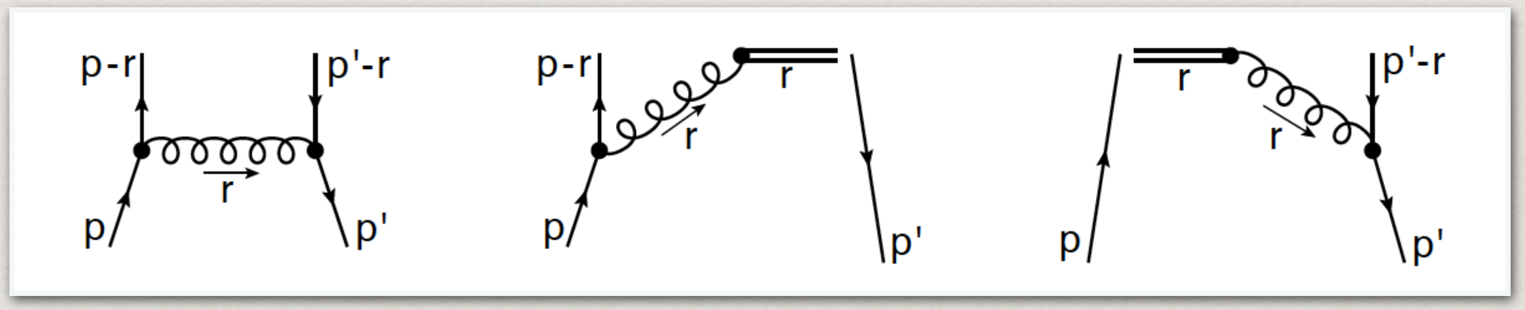
Need GTMD $F_{1,1}$ in coordinate space and GPD H $\Longrightarrow$ real diagrams



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Transverse Momentum Transfer $\Delta_T \Longrightarrow$ complicates Fourier - Transform

Wilson Coefficients

Relation to collinear GPD: Operator Product Expansion in coordinate space $z_T = b_T$

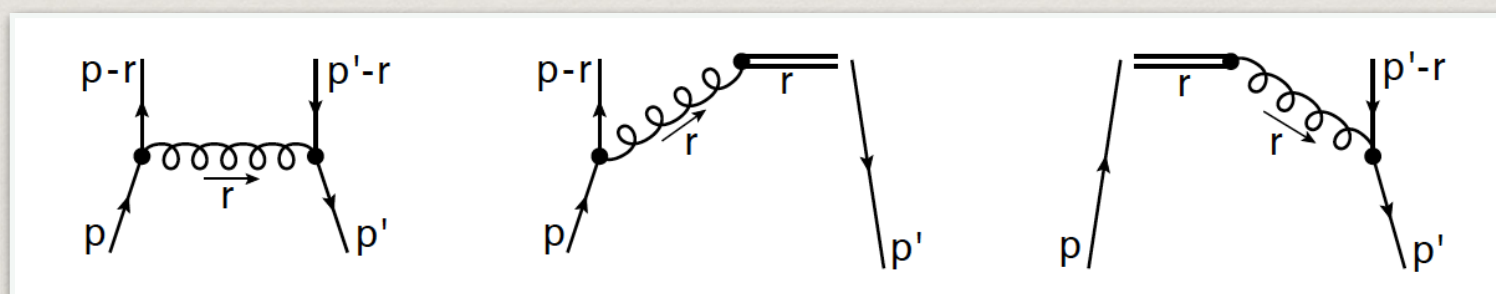
$$F_{1,1}^{q,\mathrm{ren}}(x,\xi=0,b_T,\Delta_T) = \int_x^1 dy\, C^{q/q}(y,b_T,\Delta_T,\mu,\Lambda)\, H^{q,\mathrm{ren}}(x/y,0,\Delta_T)$$

Wilson Coefficient C should not depend on regulator δ!

Calculate Wilson Coefficient for quark target:

$$C^{q/q} = \delta(1-y) + \frac{C_F\,\alpha_s}{2\pi} C^{[1]}(y) + \ldots \implies C^{[1]} = F_{1,1}^{[1]} - H^{[1]}$$

Need GTMD $F_{1,1}$ in coordinate space and GPD H $\Longrightarrow$ real diagrams



$$F_{1,1}^{q,\mathrm{real}}(x,\xi=0,k_T,\Delta_T) = \frac{C_F\,\alpha_s}{2\pi^2}\Bigg[\frac{\frac{1-x}{(1-x)^2+\delta/\Lambda^2}\left(k_T^2(1+x^2)+2x(\delta/\Lambda^2)(k_T\cdot\Delta_T)-(1-x)^2\Delta_T^2/2\right)}{[(k_T-(1-x)\Delta_T/2)^2-(1-x)i\delta]\,[(k_T+(1-x)\Delta_T/2)^2+(1-x)i\delta]} \quad -\delta(1-x)\frac{2\ln(k_T\,\Lambda/\delta)}{k_T^2-\delta^2/\Lambda^2}\Bigg]$$

Transverse Momentum Transfer $\Delta_T \Longrightarrow$ complicates Fourier - Transform

1) Perform k_T - integration $\Longrightarrow$ GPD H $\Longrightarrow$ analytic expression 😀

2) Perform Fourier - Transform $\Longrightarrow$ GTMD $F_{1,1} \Longrightarrow$ numerics 🙄

$$C^{[1]}(x, b_T, \Delta_T, \mu, \Lambda) = \text{Plus Distributions indep. of } \delta + \chi(x, b_T, \Delta_T, \mu)$$

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$$+ \int_0^1 d\alpha \, \frac{\mathrm{e}^{i(1-2\alpha)(1-x) b_T \cdot \Delta_T/2}}{S(\alpha, x, \Delta_T^2, \delta)} \left(a + b \frac{1}{b_T} \frac{\partial}{\partial b_T} + c \frac{\partial^2}{\partial b_T^2} \right) (b_T \, K_1(b_T \, S(\alpha, x, \Delta_T, \delta))) \Big]$$

$$S(\alpha, x, \Delta_T, \delta) = \sqrt{\alpha(1-\alpha)(1-x)^2 \Delta_T^2 - (1-2\alpha)(1-x) i\delta}$$

modified Bessel function

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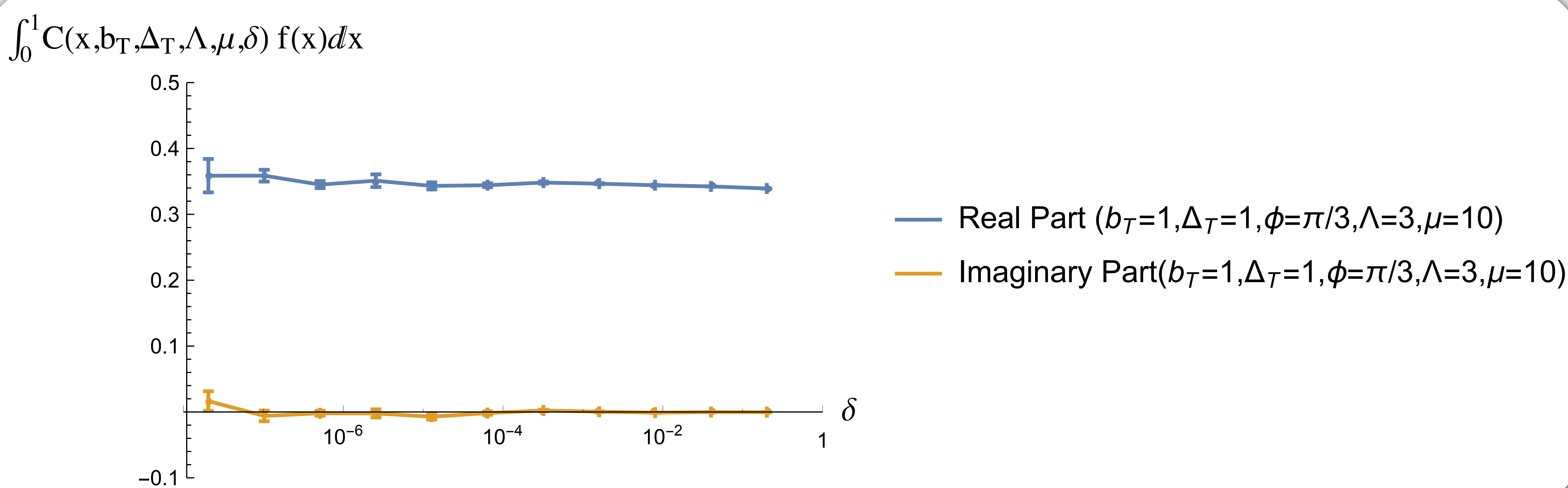
modified Bessel function

What can be shown analytically:

1) Wilson Coefficients are real (ξ=0)
2) $\Delta_T = 0$: Recover the TMD result

What can be shown numerically:

1) Limit δ ⟶ 0 seems to exist



Summary & Outlook

- GTMDs with staple-like Wilson line:
  Unifying functions of GPDs and TMDs
- GTMDs allow for a quantitative and intuitive formulation of OAM and Spin - Orbit Correlations in the nucleon
- perturbative QCD: renormalization & evolution identical to TMDs, Soft Function crucial
- Wilson Coefficients for small b_T OPE (= large k_T): more complicated due to Δ_T (& ξ) - dependence
- Outlook: Implement Wilson coefficients in GTMD evolution, matching procedure of pert. and nonpert. input (b^*-prescription)
- Study numerically the ξ-dependence: ERBL region
- Try other regulators inspired from SCET